Overview

The well-studied and widely used VAR and VARMA series are short-range dependent (SRD) in the sense that their autocorrelation function dies out exponentially fast with increasing lag. This behavior implies that the series’ distant past has a negligible effect on its present values. However, in many financial and macroeconomic applications, stationary yet persistent series arise—for example, a shock in the economy can have a long-lasting effect on inflation or on currency exchange rates. Such persistent series are called long-range dependent (LRD) series or long-memory series. They are characterized by a slowly decaying sample autocorrelation function. Modeling long-memory series requires special treatment, because the typical VARMA approach can lead to highly parameterized or misspecified models.

Details

The FI option is a new addition to the VARMAX procedure that enables you to fit a vector autoregressive fractionally integrated moving average (VARFIMA) model to the data. This model can capture both long- and short-range dependence in a multivariate time series and also allow for exogenous variables, as well as seasonal dummies and time trends. Like other models available in the VARMAX procedure, the VARFIMA model can exploit both the serial dependence of individual series and the interrelationships between different series to provide improved in-sample fit and out-of-sample forecasts.

A main characteristic of a VARFIMA model is the slowly decaying autocorrelation functions (ACFs) of the individual series. In contrast to the ACFs of a VARMA series, which decay exponentially fast with increasing lag, the ACFs of a VARFIMA series decay very slowly. Figure 1, for example, depicts the different rates of decay between the ACFs of an ARFIMA series and an AR series.

Example: US CPI and PCE Inflation Rates

This example illustrates how you can use the VARMAX procedure to analyze a bivariate series of two commonly used inflation measures: the consumer price index (CPI) and the personal consumption expenditure (PCE) price index. The data, depicted in Figure 2, are collected by the Bureau of Labor Statistics and the Bureau of Economic Analysis, respectively, and consist of seasonally adjusted US inflation rates from January 1982 to November 2016. Although the CPI and the PCE both gauge consumer inflation, they are inherently different measures of prices, and being able to model and forecast their co-movements might be useful to analysts and policymakers.
Figure 3 depicts the sample ACFs of the two series, which indeed decay slowly with the increasing lag. Hence the VARFIMA model is a suitable choice for these data.

**Figure 3: Autocorrelation Functions of CPI and PCE Series**

![Autocorrelation Functions](image)

You can use the following statements to fit a VARFIMA(1, D, 0) model with no intercept to post-2000 data and compute 12-month-ahead forecasts:

```sas
proc varmax data=cpipce(firstobs=97) plots=forecasts;  
model cpi pce / noint fi p = 1;  
output out=for back=12 lead=12;  
restrict ar(1,2,1)=ar(1,1,2)=0;  
run;
```

Figures 4 shows the parameter estimates of the model. Both of the LRD parameters are significant, verifying the existence of long-memory behaviour in the inflation rates. Moreover, the estimate of the cross covariance is also significant, implying that there is dependence between the two rates.

**Figure 4: VARFIMA(1, D, 0) Parameter Estimates**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>t Value</th>
<th>Pr &gt;</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>cpi</td>
<td>AR1_1</td>
<td>-0.16530</td>
<td>0.07370</td>
<td>-2.25</td>
<td>0.0607</td>
<td>cpi(l-1)</td>
</tr>
<tr>
<td></td>
<td>AR1_2</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>cpi(l-1)</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>0.46180</td>
<td>0.03067</td>
<td>15.06</td>
<td>0.0001</td>
<td>pce(l-1)</td>
</tr>
<tr>
<td>pce</td>
<td>AR1_2</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>cpi(l-1)</td>
</tr>
<tr>
<td></td>
<td>AR1_2</td>
<td>-0.52316</td>
<td>0.06239</td>
<td>-8.30</td>
<td>0.0001</td>
<td>pce(l-1)</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>0.45603</td>
<td>0.03070</td>
<td>14.86</td>
<td>0.0001</td>
<td>pce(l-1)</td>
</tr>
</tbody>
</table>

Figures 5 shows the CPI rates forecasts; the PCE forecast plot looks similar to the CPI plot and is omitted here for brevity’s sake.

**Figure 5: Multistep-Ahead Forecasts for CPI Rates**

![Multistep-Ahead Forecasts](image)

The BOUND, INITIAL, and TEST statements, as well as the RESTRICT statement in the example, are also supported for the VARFIMA model. These statements give you additional flexibility in modeling and forecasting long-memory series.

**System Requirements**

Estimation as demonstrated in these examples requires:
- Base SAS® 9.4 or later
- SAS/ETS 14.2 or later