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Paper 5079-2020

Estimating Jump Diffusion and VaR Models of REITs Returns Using High-Frequency Data in SAS®

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Abstract

This paper examines the roles of jumps in the time series of Real Estate Investment Trust (REIT) returns. Using realized variance, bipower variation, and high-frequency REITs data, the paper shows how SAS® programming can be used to estimate the intensity and magnitude of jumps in the returns and the volatility of the returns of a portfolio of REITs securities. The paper also shows how value at risk (VaR) can be estimated for a portfolio of REITs securities when jumps or discontinuity exist in the data. The forecasting accuracy of jump augmented VaR specifications is also compared with other VaR specifications, including those generated from some built-in procedures in SAS, such as the SEVERITY and SURVEYSELECT procedures.

Introduction

There is a consensus amongst financial economists that financial data often exhibit non-normal features such as jumps, fat tails and skewness. In this paper, the impact of jumps on the performance of a portfolio of Real Estate Investment Trust (REIT) securities is examined. Jumps have significant implications for asset pricing (Merton, 1976) and for portfolio management. If jumps are widespread in the dynamics of asset prices, then their risk premia must not only account for the continuous sample path variances, but also the arrival of discrete shocks which can cause large movements in asset prices and their volatilities. Jump risk cannot be diversified away; therefore, investors may need to earn large risk premia to hold such risk. In the dynamics of asset returns, jumps allow for the impact of the release of large information to quickly dissipate in the return generating process, but its effect on volatility is more persistent due its impact on the diffusion process. Real Estate Investment Trust (REITs) are entities which own or invest primarily in real estate related assets. Shares of publicly traded REITs trade on stock exchanges, just like the shares of other publicly traded companies. However, REITs differ from the generally known publicly listed companies in the sense that they represent ownership interest in a trust, which manages investments in real estate related assets. [1] Assets held REITs may include direct investment in commercial real estate, such as: malls, office buildings, warehouses, and vacation resorts, or investment in commercial real estate mortgages and mortgage backed securities.

In line with the goals of the paper, we address two objectives. The first objective is to explore whether non-parametric models can detect jumps in high-frequency profit and loss (P/L) data of a portfolio of REITs securities.[2] In the paper, we show how the statistical framework of the jump detection technique developed in Barndorff-Nielsen and Shephard (2004, 2006), (which is based on realized volatility and bipower variation measure) can be used to identify jumps in the volatility and returns of an Equity REITs index. The paper also highlights stylized facts about jumps in REITs returns and show why financial models that do not account jumps may be unsuitable for managing the risk of portfolios of REITs securities. For most risk managers, value-at-Risk (also known as VaR) is an important tool for financial risk assessment and governance. VaR of a long position is the minimum loss the position can incur in a given time period, with α level of confidence.[3] Since its introduction by the RiskMetrics Group in 1994, the use of VaR measures of market risk has increased exponentially because they have several appealing features.[4] A key assumption when forecasting VaRs for portfolios is that the distribution of probable loss in the value of the portfolio will arise from "normal" market risk, as opposed to all possible market risks. Consequently, most VaR specifications do not typically account for the acute tail risks, which are caused by the arrival of "unusual" news into the financial market. However, the consensus in academic and practitioner literatures is that the scales of the higher-order moments seen in most financial data are much higher than expected for a normal distribution.[5] Therefore, risk managers who rely on these of types of VaR measures are ill-equipped to deal with significant and potentially damaging events that are rare in a data generating process described by a normal distribution, but appear to occur frequently in the real world. Motivated by these findings, the paper then proceeds to show that jump augmented volatility specifications results in improved accuracy of VaR forecasts for portfolios of REITs securities.

[1] The U.S. tax code requires REITs to hold most (75%) of their assets in commercial property, to generate most (75%) of their revenue from property rents, and to return most (90%) of that income to investors in dividends.

[2] Here on, P/L is defined as the return or the percentage change in the value of the portfolio over a fixed period of time.

[3] Alternatively, VaR can also be described in terms of the maximum probable loss in this position, over a given period of time, with 1-α level of confidence.

[4] VaRs are conceptually simple. They can be implemented using both parametric and non-parametric models. VaRs also have direct and practical applications for multiple business and regulatory purposes.

[5] See for example Maheu and McCurdy (2004), Giot and Laurent (2003), Andersen et al. (2007), Bali et al (2008), and Odusami (2009).

I. Estimating Realized Volatility and Jumps

To extract realized jumps from the time series of REITs returns, this paper employs the realized variance measures highlighted in the works of Barndorff-Nielsen and Shephard (2004, 2006). The model starts by assuming that the return generating process is described by two distinct news processes: normal news events following a standard Brownian process; and unusual news events caused by the arrival of information which has significantly large impact, into the REITs markets. By assuming that the logarithm of the intraday prices of REITs portfolio evolves in a continuous time, the standard jump-diffusion process for REITs portfolio returns is defined as:

$$dp_t = \mu_t + \sigma_t dW_t + J_t dq_t \qquad (1)$$

Where dq_t is a Poisson jump process with a jump intensity λ_j and log jump size J_t, which are drawn from a normal distribution $J_t \sim NID(\mu_j, \sigma_j)$. The specification above can be extended to accommodate conditional jump intensity λ_t, conditional jump mean μ_t, and conditional jump variance σ_t once the actual jump incidence are extracted from the data. Note that time t is measured in daily units and that the intra-day geometric returns of the REITs portfolio is defined as:

$$r_{t,j} = p_{t,j.\Delta} - p_{t,(j-1).\Delta} \qquad (2)$$

Where $r_{t,j}$ refers to the j^{th} intra-day return on day t, and $\Delta = \left(1/M\right)$ is the sampling frequency within each day. As such, the daily return at the culmination of each trading equals $r_t = \sum_{j=1}^{M} r_{t,j.\Delta}$.

As noted in Barndorff-Nielsen and Shephard (2004), the quadratic variation of the return process in equation (1) can be estimated by the realized volatility (RV) and the bi-power variation (BV), which converges (as $\Delta \to 0$ or $M \to \infty$) to different quantities of the underlying stochastic process,

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2 \to \int_{t-1}^{t} \sigma_s^2 ds + \sum_{t-1<s<t} J_s^2 \qquad (3)$$

$$BV_t = \frac{\pi}{2} \frac{M}{M-1} \sum_{j=1}^{M} \left|r_{t,j}\right| \left|r_{t,j-1}\right| \to \int_{t-1}^{t} \sigma_s^2 ds. \qquad (4)$$

Barndoff-Nielsen and Shephard (2004) also posit that in the absence of a jump incidence, the realized variance will produce consistent estimate of the integrated variance $\int_{t-1}^{t} \sigma_s^2 ds$. Therefore, jumps can be inferred from the data, by examining the difference between the estimate of the realized volatility and the bi-power variation. When there is no jump, the difference between the two quantities is essentially zero. However, when jump occurs, the difference between the measures is delineated as follows:

$$\frac{RV_t - BV_t}{RV_t} \xrightarrow{M \to \infty} RVJ_t \tag{5}$$

Where RVJ_t is the realized jump ratio, which converges to the standard normal distribution with the Z_t statistics given below as:

$$Z_t = \frac{RVJ_t}{\sqrt{\left(\frac{\pi^2}{4} + \pi - 5\right)\frac{1}{M} Max\left(1, \frac{TP_t}{BV_t^2}\right)}} \xrightarrow{d} NID(0,1) \tag{6}$$

TP_t is the Tri-Power Quarticity, robust to jumps, and is estimated as specified below:

$$TP_t = \frac{M}{M-2} \cdot \frac{M}{4\left[\Gamma(7/6)/\Gamma(1/2)\right]^3} * \sum_{j=3}^{M} \left|r_{t,j}\right|^{4/3} \left|r_{t,j-1}\right|^{4/3} \left|r_{t,j-2}\right|^{4/3} \tag{7}$$

Significant jumps are delineated from the data by the realization of Z_t in excess of some critical value Φ_α , such that volatility jumps RJ_t and jumps in returns J_t are estimated by the following specifications:

$$RJ_t = I\left(Z_t \geq \Phi_\alpha^{-1}\right) \cdot \left[RV_t - BV_t\right] \tag{8}$$

$$J_t = sign(r_t) \cdot \sqrt{RJ_t} \tag{9}$$

Φ_α is the cumulative distribution function of a standard normal distribution, α is the significance level and $I_{\left(Z_t \geq \Phi_\alpha^{-1}\right)}$ is an indicator function for the identification of jumps on a trading day. On a given trading day, the function equals 1, when a jump is detected, and 0 otherwise. In deciding the appropriate threshold for significant jump, let us follow existing literature and select 0.99.[6]

[6] See Andersen et al (2010)

Given the above, the integrated variance of the REITs return process can also be derived as:

$$IV_t = I\left(Z_t \geq \Phi_{\alpha}^{-1}\right) \cdot BV_{i,t} + I\left(Z_t < \Phi_{\alpha}^{-1}\right) \cdot RV_t \tag{10}$$

II. Data

The data used in this paper are fifteen-minute log difference in the values of the Dow Jones Equity All REIT index, which was obtained from Bloomberg Professional Services. The data spans the sample period, beginning on June 26th, 2012 and ending on June 30th, 2018. The fifteen-minute returns obtained for the REITs series cover the typical daily trading period from 9:30AM to 4:00PM (EST) and yields a total of M= 27 daily intraday returns. In total, the dataset is comprised of 44,793 intraday observations, which when aggregated sums up to 1,659 daily observations. Higher sampling frequency may yield finer measures, but it often comes at the expense of increased microstructure noise in the data. The Dow Jones Equity All REIT index tracks the aggregate performance of REITs which primarily own and operate income-producing real estate. The index is comprised of 176 REITs securities.

Table 1:

Daily REITs Returns, Volatilities, and Jumps
The MEANS Procedure

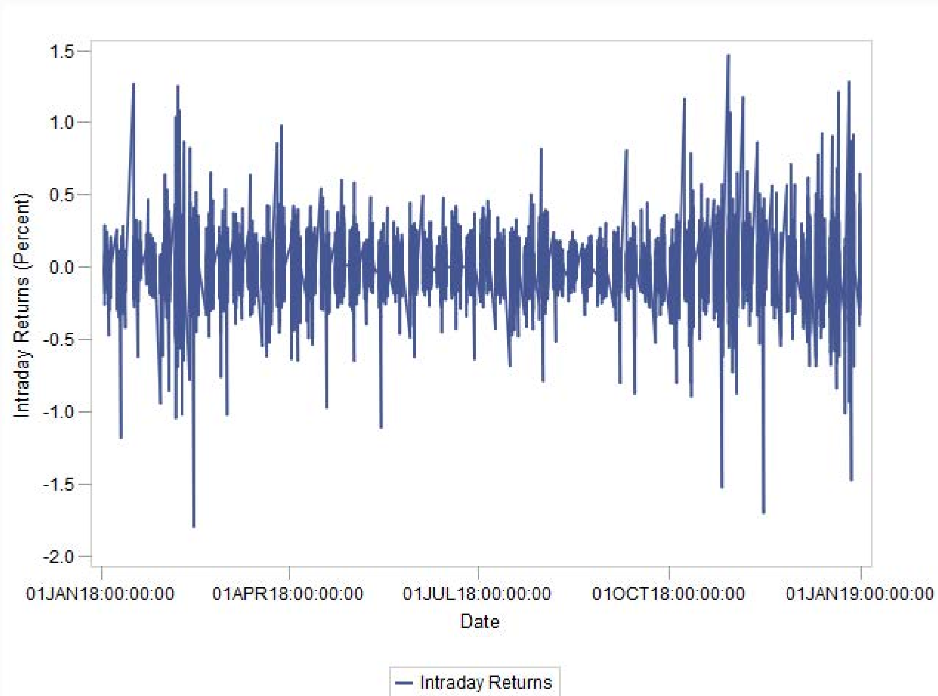
Variable	Label	N	Mean	Std Dev	Minimum	Maximum
r_t	Daily Returns	1,659	0.02	0.89	-4.82	3.32
RV_t	Daily Realized Volatility	1,659	0.71	0.93	0.00	14.60
RJ_t	Realized Volatility Jumps	324	0.64	0.78	0.03	7.17
J_t	Realized Jumps	324	0.08	0.80	-2.68	1.91

As shown in Table 1, the average daily returns for the Equity REIT index is 0.02%, while the average realized jump for the same is 0.08%. The coefficient of variation (which is the ratio of standard deviation to the mean) for the jump series imply that there is a great degree of dispersion in the magnitudes of jumps that have been identified in the data. Negative jumps led to reductions in portfolio values by as much as -2.68%, while positive jumps led to increases by as much as 1.91%. Going by the critical value selected for the realization of jumps, it is evident from the results that jumps occurs on average every five days. Figure 1 highlights the daily returns r_t of the REITs portfolio, which are by definition, the daily open-to-close changes in the prices of the Equity REITs index.

In Figure 2, daily returns and daily realized volatility of returns are also graphed to show the effect of the diffusion process on returns. While in Figure 3, the contributions of jumps to realized volatility is graphed alongside the daily returns on the Equity REITs index. On the graphs, two issues are manifest: first, jumps appear to be clustered around specific periods in time, which implies that jumps in one period may have some predictive value for jumps in other periods.[7] Second, jumps are clearly associated with large moves in REITs returns, however the distribution of jumps are not necessarily symmetric. For the case of Equity REITs, there are more positives (189) than negative (135) jumps in the sample period.

Figure 1

Intraday Equity REITs Return.



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[7]Jump arrival rate (intensities) can be model using Duration models and Autoregressive jumps intensity models.

Figure 2
Daily Returns and Realized Volatility

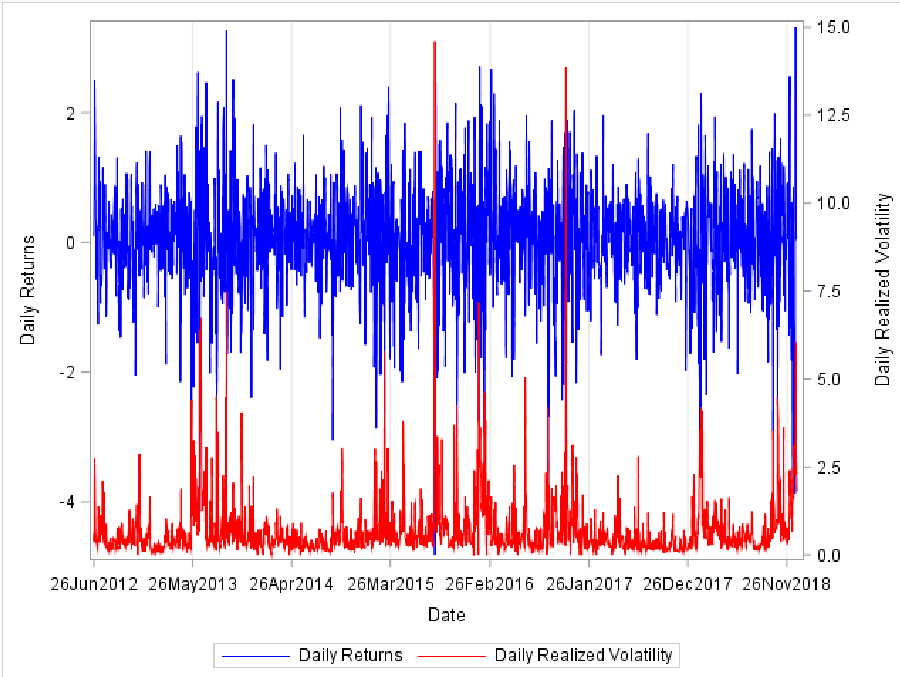
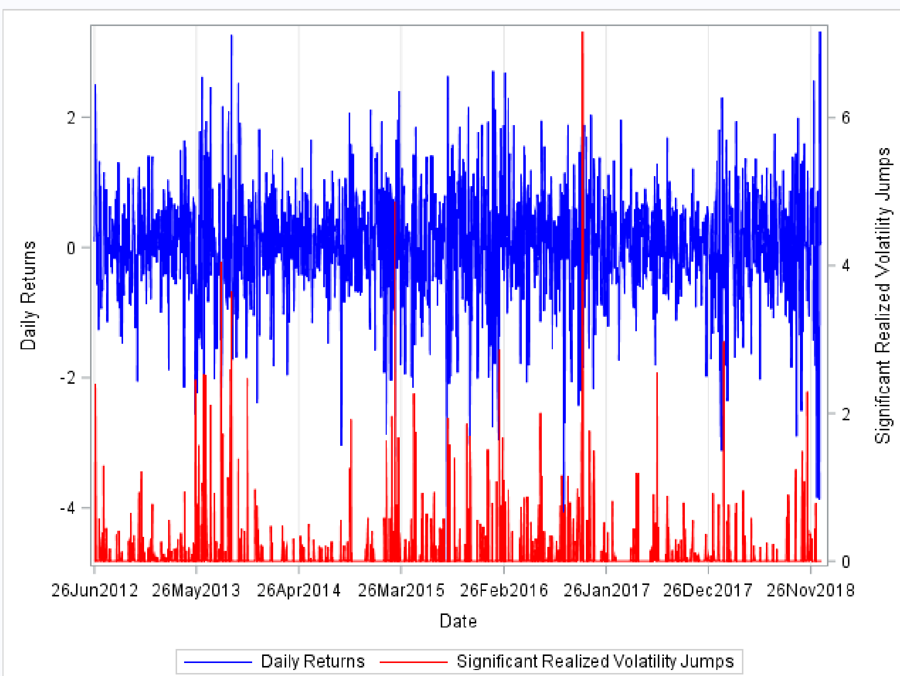


Figure 3
Daily Returns and Volatility Jumps



III. Accounting for Realized Volatility and Jumps in VaR

This section presents the framework for estimating the realized volatility and jump augmented (HAR-RVJ) VaR specification. Given a coverage probability α between 0 and 1, the one-day conditional VaR threshold for the values of the Equity REIT index is specified as follows:

$$\mathrm{VaR}_{t,\alpha} = \inf\left\{r \mid P\left(r_t \le r \mid \Phi_{t-1}\right) \le \mathrm{VaR}_{t,\alpha}\right\} \tag{11}$$

Furthermore, let $\Pi_{t,\alpha}$ denote the conditional threshold for a given probability density function at α.[8] Since VaR specifications are generally constructed around the first and second moments of the distribution function, it is worthwhile to consider the normal distribution as a suitable probability density function for estimating our HAR-RVJ VaR measures. Given a coverage probability α and an information set Φ_{t-1}, we can now establish the forecast of our VaR specification as follows:

$$VaR_{t+1} = \mu_{t+1} + \Pi_{t,\alpha}\sigma_{t+1} \tag{12}$$

where

$$\alpha = \int_{-\infty}^{\Pi_{t,\alpha}} f\left(r_t \mid \Phi_{t-1}\right) dr_t \tag{13}$$

In the HAR-RVJ VaR specification, $f\left(r_t \mid \Phi_{t-1}\right)$ is the conditional normal density function and is specified as a function of μ_{+}, and σ_{+}, which are the one-step ahead forecast of the conditional mean and conditional variance of returns. For the case of the normal density function, the conditional threshold variable $\Pi_{t,\alpha}$ can be obtained by estimating the numerical value of the threshold which equalizes the area under the density function $f\left(r_t \mid \Phi_{t-1}\right)$ to the coverage probability α. There are various methods for producing the variance forecasts for the VaR specification. An efficient approach is to augment the Heterogenous Autoregressive Realized (HAR) Volatility model of Corsi (2009) with the realized jump variable to account for the effect of jumps on the sample path of realized volatility. In the HAR framework, realized volatility is specified as a least square function of daily, weekly, and monthly realized volatility. The augmented HAR model, which now includes the additional jump covariate is specified as follows:

[8] α is the confidence level of the VaR forecast.

$$RV_{t+1} = \beta_0 + \beta_D RV_t + \beta_w RV_{t,t-5} + \beta_M RV_{t,t-22} + \beta_j RJ_t + \varepsilon_{t+1} \quad (14)$$

Where RV_t denotes daily realized volatility, $RV_{t,t-5} = \frac{1}{5}\sum_{i=1}^{5} RV_{t-i}$ is the weekly realized volatility, and $RV_{t,t-22} = \frac{1}{22}\sum_{i=1}^{22} RV_{t-i}$ is the monthly realized volatility. The logarithm of realized volatility has been shown to be approximately normally distributed, therefore forecasts of realized volatility were also derived from the logarithmic HAR-RVJ model using the specification below.

$$\begin{aligned} \log\left(RV_{t+1}\right) = \beta_0 + \beta_D \log\left(RV_t\right) + \beta_w \log\left(RV_{t,t-5}\right) + \beta_M \log\left(RV_{t,t-22}\right) \\ + \beta_J \log\left(1 + RJ_t\right) + \varepsilon_{t+1} \end{aligned} \quad (15)$$

With $\varepsilon_{t+1} \sim i.i.d.N\left(0, \sigma_\varepsilon^2\right)$. To generate the forecast of RV, we undo the transformation in Equation (15) by employing the expression below:

$$RV_{t+1} = \exp\left(\ln\left(RV_{t+1}\right) + \sigma_\varepsilon^2 / 2\right) \quad (16)$$

Table 2 reports the statistics of the 99% VaR thresholds for long positions in the Equity REITs index. These thresholds were obtained from three VaR specifications: The Linear HAR-RVJ; the Log-Linear HAR-RVJ; and the Log-Normal Severity specification. Also included are the results of VaR forecast obtained from the historical distributions of the portfolio returns using the PROC Univariate procedure, and from bootstrapping 10,000 return observations from the daily series using the PROC SURVEYSELECT procedure.

In general, the values of the historical VaR, the bootstrapped VaR, are close. Since both specifications are unconditional to the flow of information into the financial markets, neither will be efficient with respect to accounting for market risk, particularly as market dynamics evolve over time. The same limitation is evident in the Log-Normal Severity VaR, which has an unconditional VaR of 2.041%. The VaRs of the linear and Log-Linear HAR-RVJ specifications range from -0.632% to -5.514% and -0.865% to -6.159%, respectively. The flexibility exhibited by both specifications allows them to respond to changes in risk levels as new information is disseminated into the financial market.

Table 2					
99% VaR Statistics					
VaR Specifications	N	Mean	Std Dev	Minimum	Maximum
Historical	1,659	-2.607	N/A	-2.607	-2.607
Bootstrap Distribution	10,000	-2.564	N/A	-2.564	-2.564
Linear HAR-RVJ	1,659	-1.847	0.666	-5.508	-0.749
Log-Linear HAR-RVJ	1,659	-2.202	0.754	-6.305	-0.896
Log-Normal Severity	1,659	-2.041	N/A	-2.041	-2.041

Figure 4A
Conditional VaR and Daily REITs Returns
(Linear Realized Volatility-Jump Specification)

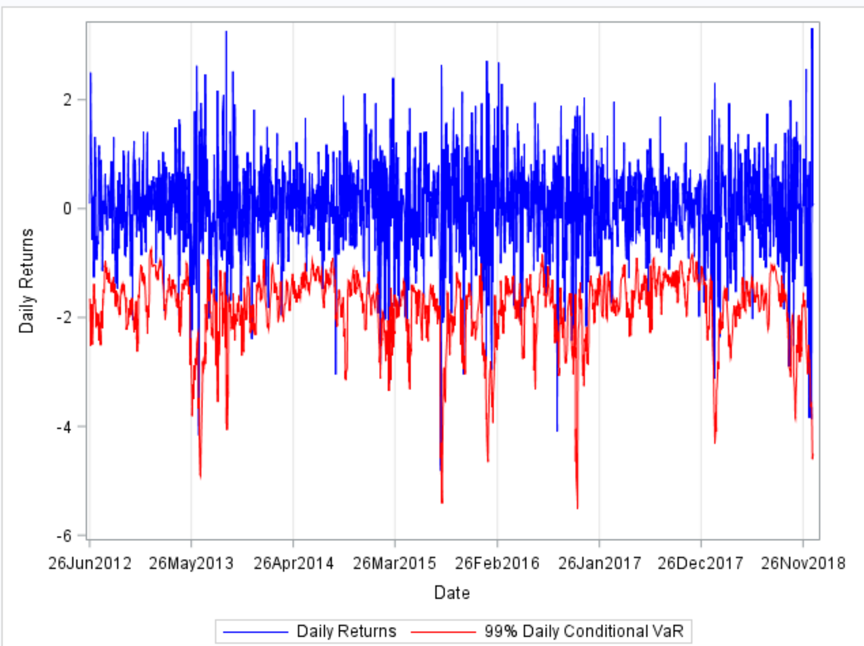


Figure 4B
Conditional VaR and Daily REITs Returns
(Log-Linear Realized Volatility-Jump Specification)

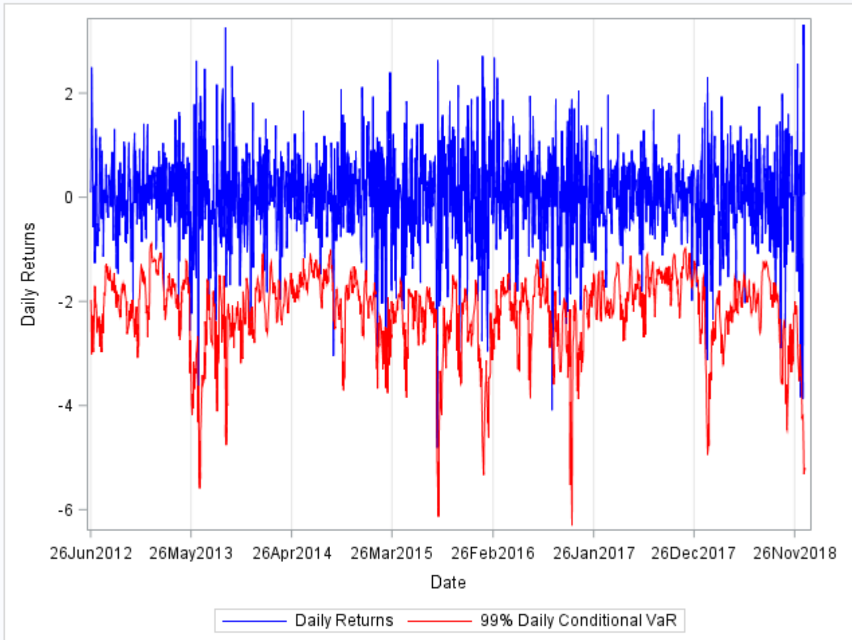
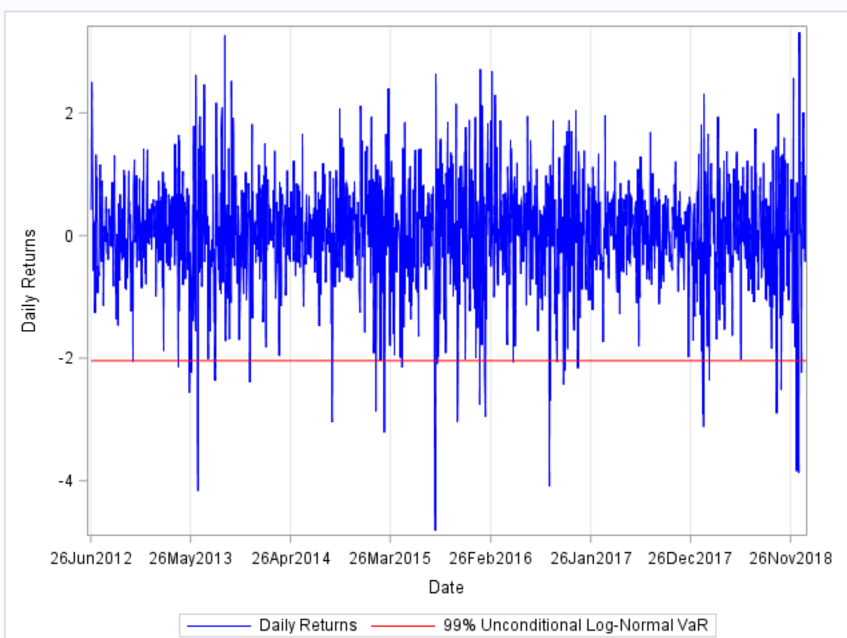


Figure 4C
Unconditional Log-Normal VaR and Daily REITs Returns
(Log-Normal Severity Distribution VaR Specification)



IV. Backtesting VaR specifications

Under the Basel II Accord, covered financial institutions are required to hold capital reserves against market risks. The amount of reserve is determined by approved internal measurements of the maximum loss over 10 trading days at the 99% confidence level. Acceptable internal models are chosen based on the ability of the model to adequately predict the market risk of covered institutions through backtesting of the model's output. There are no specific recommendations as to the exact methodology to be used for backtesting the quality of VaR specifications. However, capital requirement is determined from a multiplier derived from the number of instances where actual losses are greater than the values forecasted by the VaR specification.[9]

In this section, an assessment of the suitability of the HAR-RVJ VaR specification for the above requirements is done. The tests employed here evaluate the precision of the specifications, in terms of how well can their VaRs predict the sensitivity of portfolio returns to market risks. Let us compare the accuracy of the VaR forecasts obtained from the HAR-RVJ specification against the VaRs derived from the alternative models, (namely: the VaR derived from historical simulation and bootstrap distribution, and VaR estimates produced by the application of the severity loss distribution model).

For the first VaR specification tests, let us calculate the proportion P^N of VaR exceptions I_t, obtained for each VaR specification. P^N is calculated as follows:

$$P^N = \frac{N}{T} \times 100\%, \quad N = \sum_{t=1}^{T} I_t \tag{17}$$

and

$$I_t = \begin{cases} 1, & \text{if } r_t < VaR_t \\ 0, & \text{otherwise.} \end{cases} \tag{18}$$

N is the total number of VaR exceptions (i.e., days in which actual losses exceed the magnitude predicted by the VaR measure), and *T* is the total number of observations. The observed accuracy of a reliable VaR measure should essentially be the same as the confidence level used to generate the VaR estimates (i.e. $N = \alpha T$ or $\alpha = P^N$). To validate this proposition, we will implement the unconditional coverage test of Kupiec (1995). The null hypothesis under the unconditional coverage test is that the actual number of exceptions *N* and the expected

[9] Updated requirements for capital reserve and market risk management are specified in the Revisions to the Basel II market Risk Framework in 2009.

number of exceptions αT are statistically the same. The likelihood ratio for this test has an asymptotic Chi-square distribution with one degree of freedom and is given as:

$$LR^{Unc} = 2\left[N\ln\left(\frac{N}{\alpha T}\right) - (T-N)\ln\left(\frac{T-N}{T-\alpha T}\right)\right] \sim \chi^2(1) \quad (19)$$

Furthermore, let's conduct a joint test of the null hypothesis of coverage and independence, where a joint test of the null is that actual and expected numbers of exceptions are the same, and that VaR exception in one period does not contain any predictive information about future VaR exceptions. The existence of a systematic pattern of VaR exceptions implies that exceptions cluster into specific periods. This will diminish the usefulness of such models for risk management because one of the underlying assumptions of VaR calculations is that risk events are *i.i.d* in occurrence. We will evaluate the null described above by applying the conditional coverage test of Chrisoffersen (1998). The likelihood ratio of this test is as follows:

$$LR^{Cond} = 2\left[n_{00}\ln\left(\frac{\Lambda_{00}}{1-\Lambda}\right) + n_{01}\ln\left(\frac{1-\Lambda_{00}}{\Lambda}\right) + n_{10}\ln\left(\frac{\Lambda_{10}}{1-\Lambda}\right) + n_{11}\ln\left(\frac{1-\Lambda_{10}}{\Lambda}\right)\right] \sim \chi^2(2) \quad (20)$$

where n_{ij} is the number of observations with value *i* followed by value *j*[10],

$\Lambda_{00} = n_{00}/(n_{00}+n_{01})$, $\Lambda_{10} = n_{10}/(n_{10}+n_{11})$, and $\Lambda = (n_{01}+n_{11})/T$. The conditional coverage test has an asymptotic Chi-Square distribution with two degrees of freedom. One significant drawback of the conditional coverage test is its reliance on the frequency with which consecutive VaR exceptions occur. Since exceptions are inherently rare, the ability of the test to yield informative results is limited.[11]

V. Comparing the Results of VaR Specifications

Table 3 presents the number of exceptions (exceedances) for the 99% VaRs for each specification. Also shown are the likelihood ratios for the unconditional coverage and the conditional coverage tests for the 99% VaRs. For the 99% VaR threshold, the P^N values for the Historical, Bootstrap, and Log-Linear HAR-RVJ specification are for the most part equivalent to the coverage probability of 1%. The P^N values for the Linear HAR-RVJ is slightly

[10] For example, n_{11} is the number times a VaR exception is followed another VaR exception.

[11] The SAS codes for both tests are also provided in the appendix.

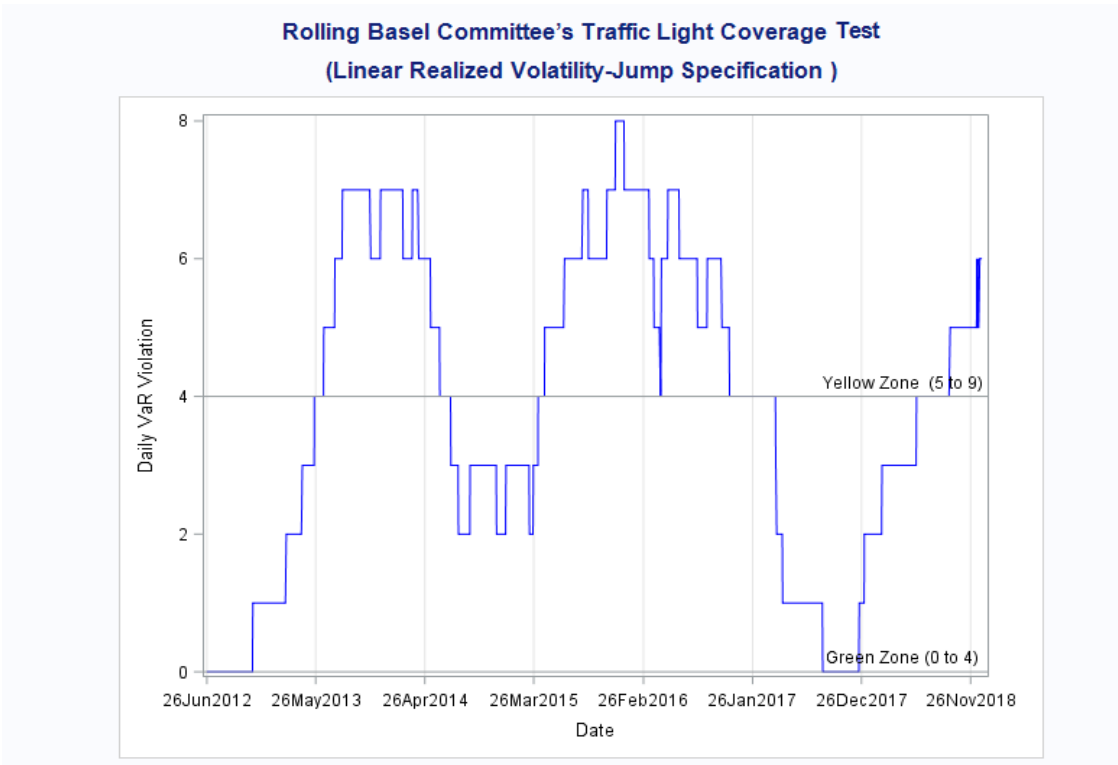
higher, with a value of 1.7%, while the P^N values for Log-Normal Severity specifications ranked lowest in terms of deviation from the coverage probability (P^N of 2.0%).

Table 3				
VaR Exceptions				
VaR Specifications	N	Proportion	Unconditional Coverage Test	Conditional Coverage Test
Historical VaR	17	1.01%	0.44	81.12
Bootstrapped VaR	16	0.95%	3.51	73.54
Linear HAR-RVJ	29	1.74%	57.17	-180
Log-Linear HAR-RVJ	15	0.90%	6.17	N/A
Log-Normal Severity	34	2.00%	81.68	-126
Critical value of unconditional coverage test at 1% significance is 6.634. Critical Value for the conditional coverage test at 1% significant is 9.21				

In terms of the coverage tests, the VaRs derived from the Log-Linear HAR-RVJ display remarkably superior performance relative to the other specifications. For 99% VaR threshold, we fail to reject the null of the unconditional coverage test at the 1% level of significance for the Log-Linear VaRs. This implies that the expected and actual numbers of exceptions are statistically the same for the Log-Linear HAR-RVJ specification. In comparison, the null of the unconditional coverage test for the Linear HAR-RVJ and the Log-Normal Severity VaR specification are rejected, given that the test statistics obtained for both specifications are significantly higher than the critical value. The main reason the proportion of exceptions for the historical and bootstrap VaRs are equal to the coverage probability is that they both suffer from lookback bias.[12] Therefore, it is likely that they may perform very poorly in out-of-sample scenarios. The Log-Linear HAR-RVJ VaR outshines other specifications, based on the results of the conditional coverage tests. The null for the conditional coverage tests is rejected for

[12] Both VaRs were drawn from the 1% tail of the return distribution for the entire sample period. Coverage test results for both the historical and bootstrap VaRs are presented for comparison purposes.

the historical, bootstrap, Linear HAR-RVJ and Log-Normal Severity VaR.[13] Overall, all the specifications (with the exception of the Log-Linear HAR-RVJ), appear to be deficient in terms of their ability to adequately forecast the VaRs of the Equity REIT index.



Under the Basel Committee's supervisory framework for backtesting VaR specification, three classes of VaR exceptions exists for the 99% threshold in a 250-trading window. Between 0 and four exceptions $\left(0 \le P^N \le 1.6\%\right)$ is classified as the green zone, for which the number of exceptions generated by the model is considered to be adequate for regulatory purposes; between five and nine exceptions $\left(1.6\% \le P^N \le 3.6\%\right)$ is classified as the yellow zone, in which the supervisor is required to investigate the reasons for the exceptions. VaR specifications with ten or more exceptions $\left(P^N > 3.6\%\right)$ in a 250-day window are deemed to be in the red zone, which is indicative of a modeling problem. Figures 5A-C graphs the 250-day rolling-window Traffic light tests for Log-Linear and Linear HAR-RVJ VaR, and Log-Normal

[13] The result for the Log-Linear HAR-RVJ is not reported because consecutive exceptions $\left(n_{11}\right)$ was not observed in the data for the specification.

Severity VaR specifications. The rolling cumulative exceptions P^N values obtained from the Log-Linear HAR-RVJ (Figure 5A) models for REITs are for the most part in the green zone, making them suitable for regulatory purpose. The rolling cumulative exceptions P^N for the Linear HAR-RVJ (Figure 5B) are for the most part in the lower spectrum of yellow zone, while the rolling cumulative exceptions for Log-Normal Severity (Figure 5C) periodically reaches the upper edges of the yellow zone, which makes it highly suspect for VaR forecasting.

Figure 5A
Rolling Basel Committee's Traffic Light Coverage Test
(Log-Linear Realized Volatility-Jump Specification)

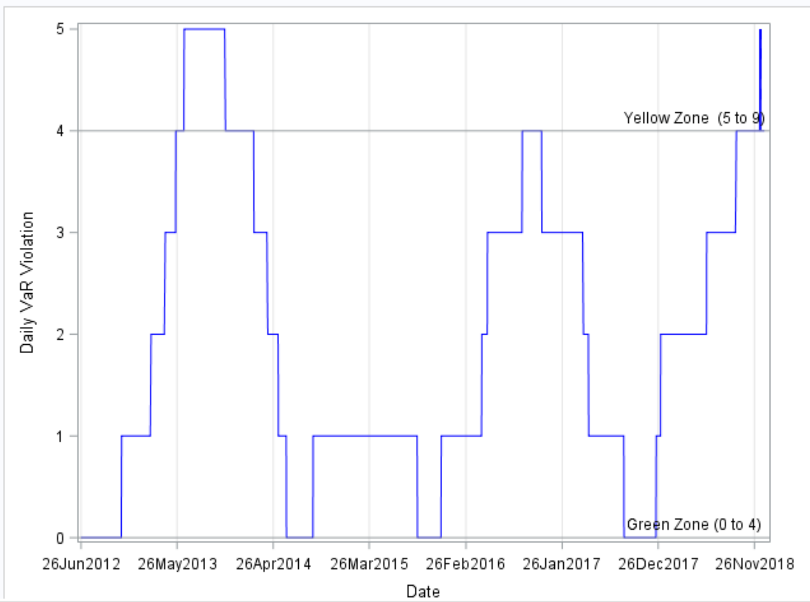


Figure 5B
Rolling Basel Committee's Traffic Light Coverage Test
(Linear Realized Volatility-Jump Specification)

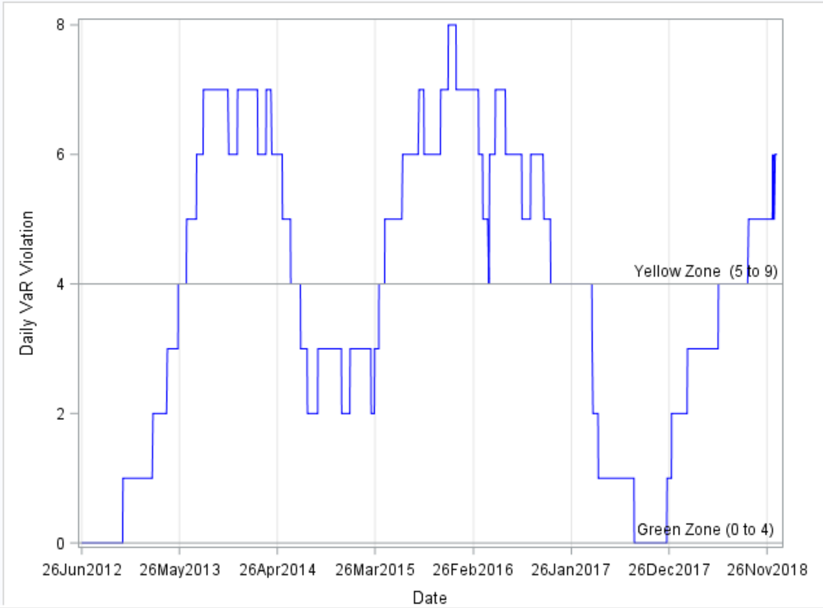
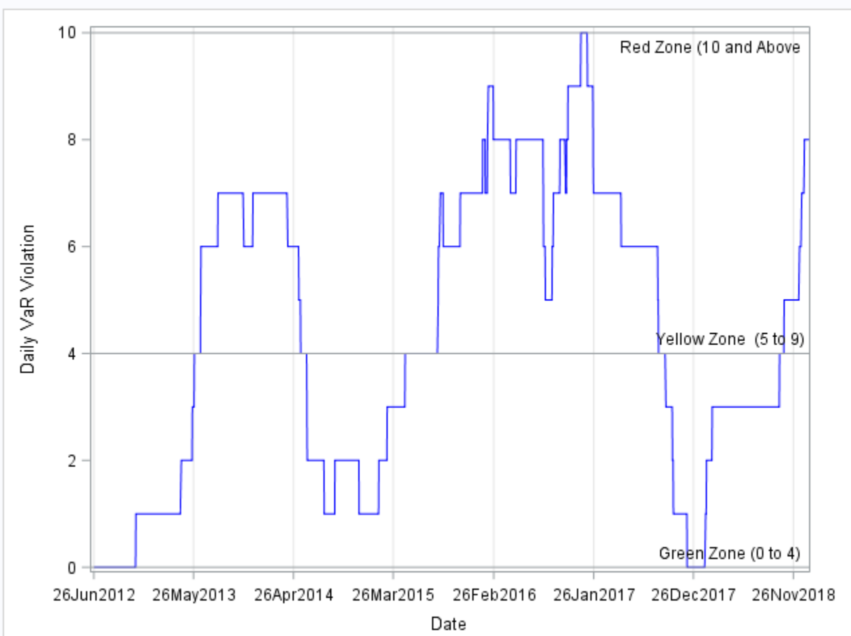


Figure 5c
Rolling Basel Committee's Traffic Light Coverage Test
(Log-Normal Severity Distribution Specification)



VI. Conclusion

The use of conditional distributions for estimating VaRs yield significant improvement over time-invariant specifications because of their innate flexibility. Conditional distributions rely on the forecast of the conditional variance of the underlying stochastic process. Thus, these types of specifications are better able to reflect market risk as they evolve over time. In this paper, the ability of non-parametric models to detect jumps in high-frequency REITs returns is investigated. The paper also evaluates whether jump augmented realized volatility models provide better forecast of Value at Risk for a portfolio of REITs securities. Statistical programs for all models and specifications was done in the SAS.

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Appendix

```
/*Primary Datafile is Ireits_Data */
/*Macrovariables for Bipower Values*/
%let ret=rrei; /*15 Mins Intraday returns*/
%Let alpha=0.99; /*Critical Value for Identification of Jumps */
%Let p= %sysevalf(4.0/3.0);/*Quotient*/
%Let mu=
%sysevalf((2.0**(&p/2.0))*%sysfunc(gamma(0.5*(&p+1)))/%sysfunc(gamma(0
.5))); /*mean function*/
%put &alpha &p &mu;

PROC SORT Data=ireits_data out=reits_idata;
 by date;
RUN;

DATA reits_ijdata;;
 set reits_idata;
 DDate=datepart(date);            /*Extract Daily Dates from Intraday
 Returns*/
 format DDate date9.;
 r=&ret*100;                      /*Intraday Return in Percent */
 retsq=r*r;
```

```
 bvv=abs(r*lag(r));
 qq=(abs(lag2(r))**&p)*(abs(lag(r))**&p)*(abs(r)**&p);
 label r = 'Intraday Returns';
RUN;
/* Cumulate Intraday Returns; Calculate Volatilities, & Bipower
Variation in Daily Obs */
DATA reits_rjdata;
set reits_ijdata;
by DDate;
  if first.DDate then  do;
  rv=0; /*Realized Volatility */
  bvi=0; /*Sum of Lagged Squared Intraday variations*/
  rt=0;
  q=0;
  mn=0;
  end;
  rv+retsq;
  bvi+abs(r)*lag(abs(r));
  rt+r;
  q+qq;
  mn+1; /* # of Intraday Returns in Each Day */
  if last.DDate then output;
  keep DDate rv bvi rt q mn;
RUN;

DATA reits_rijdata;
set reits_rjdata nobs=l;
 rename DDate=Date;
m=mn-1;
bv=0.5*constant("pi")*(m/(m-1))*bvi;
/*Bipower Variations */
rjr=(rv-bv)/rv;
tp= m*&mu*(-3.0)*(m/(m-2))*q;
/* Realized Tripower Quarticity */
zj=rjr/(sqrt((((0.5*constant("pi"))**2)+constant("pi")-
5.0)*(1.0/m)*max(1,tp/(bv**2))));/
*Test Statistics for Identification of Jumps */
if zj>= quantile("normal",&alpha) then rjp=(rv-bv); else rjp=""; /*
Significant Realized Volatility Jump Count */
if zj>= quantile("normal",&alpha) then js=sign(rt)*sqrt(rv-bv); else
js="";/* Jumps In Returns Measure */
if zj>= quantile("normal",&alpha) then rj=(rv-bv); else rj=0; /*
Significant Realized Volatility Jump */
if zj>= quantile("normal",&alpha) then iv=rv; else iv=bv;
/*Instantaneous Variance */
pj=cdf("normal",zj);/*Jump Probabilities  */
Jplus = IFN(sign(js)=1, 1,'','');
Jminus = IFN(sign(js)=-1,1,'','');
Label rt = 'Daily Returns';
Label rv = 'Daily Realized Volatility';
Label rjp = 'Significant Realized Volatility Jumps'; /*Record values
only days with Jumps Days*/
```

```
Label rj = 'Significant Realized Volatility Jumps';/*set values to
zero on days without Jumps Days*/
Label js = 'Realized Jumps';
Label pj= 'Daily Probability of Jumps';
Label Jplus = 'Number of Positive Jumps';
Label Jminus = 'Number of Negative Jumps';
drop m mn;
RUN;


/*Descriptive Statistics */
Proc means data=reitsout;
var rt rv rjp js jplus jminus;
run;



/*Estimating Value-at-Risk VaR */

/* Forecasting Realized Volatility with Inputs from Corsi (2009) HAR-J
Model */
PROC EXPAND Data = reits_rijdata OUT = reitsout;
 convert rv = rvw / METHOD = none TRANSFORMOUT = (movave 5);
 /*5-Day Rolling Average of Realized Volatility */
 convert rv = rvm / METHOD = none TRANSFORMOUT = (movave 22);
/*22-Day Rolling Average of Realized Volatility */
 convert rv = rvd / METHOD = none TRANSFORMOUT = (lag 1);
/*1-Day lag of Realized Volatility */
 convert rvj = rvjd / METHOD = none TRANSFORMOUT = (lag 1);

convert rj = rjd / METHOD = none TRANSFORMOUT = (lag 1);
RUN;


/*Linear and Log-Linear HAR-RVJ With Significant Volatility Jump
Models */

/*Linear Significant Volatility Jump Model*/
Proc reg data=reitsout outest=charjest;

/*Future Volatility is a function of lagged, 5-Day Average, and One
Month Average Volatility, and Realized Significant Volatility Jumps*/
rvhat:model rv =  rvd rvw rvm rjd ;
     output out= crvpred
           predicted=predicted_rv ;
run;

/*Generate Forecast of Realized Volatility Using Regression Parameters
*/
proc score data=reitsout score=charjest out=LHARVJScore type=parms;
   var rvd rvw rvm rjd;
run;
```

```
/*Generate Daily VaR Forecast (SJVaR99) */
Data SJVaR99;
if(_n_ =1) then  set charjest (rename=(_rmse_= mse))  ;
set LHARVJScore ;
SJVaR99 = quantile('normal', &varset)*rvhat**0.5;
if rt<SJVAR99 then vio=1; else vio=0;
id=1;
keep id SJVaR99 rt rv rvf rjd vio Date;
Label SJVAR99 = '99% Daily Conditional VaR';
Label Vio = 'Daily VaR Violation';
run;



/*Log HAR-RVJ Model- Transform RV into Log(RV) and RJ in Log (1+RJ) */

Data reits_out;
set reitsout;
lrv = log(rv);
lrvd = log(rvd);
lrvw = log(rvw);
lrvm = log(rvm);
lrvj = log(1+rjd);
run;


/*Log-Linear HAR-RVJ With Significant Volatility Jump Model -Default
Model */
Proc Reg data=reits_out outest=lgharjest;
rvfhat:model lrv = lrvd lrvw lrvm lrvj;
      output out=lgrvpred;
      ods output parameterestimates=lgparest anova=lganova;
run;

/*Generate Forecast of Realized Volatility Using Regression Parameters
*/
proc score data=reits_out score=lgharjest out=LGHARVJScore type=parms;
   var lrvd lrvw lrvm lrvj;
run;

/*Extract Parameter Estimates into Macrovariable for log
transformation*/
 proc sql noprint;
select count(*)into :nobs from lgparest;
select Estimate into :Beta_1 - :Beta_%sysfunc(strip(&nobs)) from
lgparest;
 quit;


/*Generate Daily VaR Forecast (LGVaR99) */
Data LGVaR99;
if(_n_ =1) then  set lgharjest(rename=(_rmse_= rmse))  ;
set LGHARVJScore ;
```

```
rvhat = exp(rvfhat-&beta_5+0.5*rmse**2); /*Transform Log(RV) into RV
and adjust for (1) in (1+rvj) */
LGVaR99 = quantile('normal', &varset)*rvhat**0.5;
if  rt<LGVAR99 then vio=1; else vio=0;
id=1;
keep LGVaR99 rt rv rvf rvj vio id Date;
Label LGVAR99 = '99% Daily Conditional VaR';
Label Vio = 'Daily VaR Violation';
run;
proc means data=LGVaR99;
var LGVaR99 Vio;
run;
```

```
/*Proc Severity Approach. Code was adapted from SAS User Guide (Proc
Severity Procedure) */

proc fcmp library=sashelp.svrtdist outlib=work.means.scalemod;
   function BURR_MEAN(x, Theta, Alpha, Gamma);
      if not(Alpha * Gamma > 1) then
         return (.); /* first moment does not exist */
      return (Theta*gamma(1 + 1/Gamma)*gamma(Alpha -
1/Gamma)/gamma(Alpha));
   endsub;
   function EXP_MEAN(x, Theta);
      return (Theta);
   endsub;
   function GAMMA_MEAN(x, Theta, Alpha);
      return (Theta*Alpha);
   endsub;
   function GPD_MEAN(x, Theta, Xi);
      if not(Xi < 1) then
         return (.); /* first moment does not exist */
      return (Theta/(1 - Xi));
   endsub;
   function IGAUSS_MEAN(x, Theta, Alpha);
      return (Theta);
   endsub;
   function LOGN_MEAN(x, Mu, Sigma);
      return (exp(Mu + Sigma*Sigma/2.0));
   endsub;
   function PARETO_MEAN(x, Theta, Alpha);
      if not(Alpha > 1) then
         return (.); /* first moment does not exist */
      return (Theta/(Alpha - 1));
   endsub;
   function WEIBULL_MEAN(x, Theta, Tau);
      return (Theta*gamma(1 + 1/Tau));
   endsub;
   quit;
```

```
%let Dret=Drei;

   Data sreits;
   set reits.dreits;
   lret=exp(&dret*100);
   nret=&dret*100;
   keep date nret lret;
   Label nret = 'Daily Returns';
   Label lret = 'Daily Log-Normal Returns';
   format date date9.;
   run;

   /*----- Fit all distributions and generate scoring functions ------
*/
proc severity data=sreits outest=est print=all plots=none;
   loss lret;
   dist _predefined_ ;
   outscorelib outlib=scorefuncs commonpackage;
run;


/* Set VaR level and forecast VaR*/
%let varLevel=0.01;

data scores;
   set sreits(firstobs=2);
     logn_var  = log(sev_quantile_LOGN(&varLevel));
      if nret<logn_var then vio = 1; else vio = 0;
id=1;
keep Logn_VaR nret Date vio id;
Label logn_var = '99% Unconditional Log-Normal VaR';
Label Vio = 'Daily VaR Violation';
run;

proc means data=scores;
var Logn_VaR Vio;
run;








/* BackTesting VaR Accuracy. Tests are the same for all VaR
Specifications. Code for one example is provided */


/*Exceptions counter */
Data LExcd;
```

```
set SJVaR99  nobs=nob;
by id;
if first.id then do;
n00=0.0;
n01=0.0;
n10=0.0;
n11=0.0;
nvio=0.0;
noob=-1;
end;
noob+1;
if vio=1 then nvio+1;
if lag(vio)=0 and vio=0 then n00+1;
if lag(vio)=0 and vio=1 then n01+1;
if lag(vio)=1 and vio=0 then n10+1;
if lag(vio)=1 and vio=1 then n11+1;
if last.id then output;
keep nvio n00 n01 n10 n11 noob;
run;

/* Unconditional (LRUC) and Conditional Coverage (LRC) Tests. */
Data LBacktest;
set Lexcd;
lam0=n00/(n00+n01);
lam1=n11/(n10+n11);
lam=(n01+n11)/noob;
LRUC = 2 *(nvio*log(nvio/(&varset*noob))-(noob-nvio)*log((noob-
nvio)/(noob-&varset*noob)));
LRC = 2*log(((lam0/(1-lam))**n00)*(((1-lam0)/lam)**n01)*((lam1/(1-
lam))**n10)*((1-lam1)/lam)**n11);
run;

/*Basel Traffic Light Coverage Test*/
proc expand data = SJVaR99 OUT = Lbasel;
 convert vio = bvio / METHOD = none TRANSFORMOUT = (movsum 250);
run;
```

Contact Information

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