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# An Introduction to Multiple Time Series Analysis and the VARMAX Procedure

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## ABSTRACT

To understand the past, update the present, and forecast the future of a time series, you must often use information from other time series. This is why simultaneously modeling multiple time series plays a critical role in many fields. This paper shows how easy it is to use the VARMAX procedure to estimate and interpret several popular and powerful multivariate time series models, including the vector autoregressive (VAR) model, the vector error correction model (VECM), and the multivariate GARCH model. Simple examples illustrate Granger causality tests for identifying predictive causality, impulse response analysis for finding the effect of shocks, cointegration and its importance in forecasting, model selection for dealing with the trade-off between bias and variance, and volatility forecasting for risk management and portfolio optimization.

## INTRODUCTION

When you analyze a time series, the type of analysis you use usually depends on the nature of the time series: **When a time series' current value depen**ds on its past values, you might use autoregression (AR). When current values in a vector of multiple time series depend on their past values and even on **each other's current** values, you might use vector autoregression (VAR), which systemically captures the dynamics in multiple time series. When long-run equilibrium exists between multiple nonstationary time series, you might use a vector error correction model (VECM) to exploit both the long-run and short-run relationships. When you need to forecast volatility, which is very important in risk management and portfolio optimization, you might use an autoregressive conditional heteroscedasticity (ARCH) or a generalized autoregressive conditional heteroscedasticity (GARCH) model. These models are often used in modeling and forecasting volatility for both univariate and multivariate cases.

The next three sections show examples of some of the models you can use. The first is a simple VAR example that illustrates Granger causality tests for identifying predictive causality, impulse response analysis for finding the effect of shocks, and model selection for dealing with the trade-off between bias and variance. The second example shows how to incorporate the concept of purchasing power parity (PPP) in a VECM and then how well it leads to some good forecasts. The third example applies a multivariate dynamic conditional correlation (DCC) GARCH model on Japanese and US stock market returns and then uses the forecasted volatility in portfolio optimization to achieve a trading strategy that has a better long-run return and return per risk unit than three other benchmark trading strategies.

## VECTOR AUTOREGRESSIVE MODEL

Since Sims[1] (1980) introduced the VAR model into macroeconomic econometrics, it has been widely used in forecasting and policy analysis. This section uses simulated data on

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[1] Christopher Sims and Thomas Sargent were awarded a Nobel Prize in 2011 "for their empirical research on cause and effect in the macroeconomy" (Nobel Media AB 2020b).

inflation, unemployment rate, and interest rate to discuss some basic concepts of using a VAR model, namely Granger causality tests, impulse response functions, and model selection. For a real-data example and further discussion of the advantages and disadvantages of VAR models, see Stock and Watson (2001).

The following statements read the simulated data and plot three time series, which are shown in Figure 1:

```
data simul1;
    input date mmddyy10. inflation interestRate unemploymentRate;
    format date yyq.;
datalines;
01/01/1950 10.0088 11.9544 5.28514
04/01/1950 11.3452 12.7174 5.45672
   ... more lines ...
07/01/1999 9.1981 11.6830 5.02263
10/01/1999 10.6646 13.0948 4.57442
;
proc sgplot data=simul1;
    series x=date y=inflation / legendlabel="Inflation";
    series x=date y=unemploymentRate / legendlabel="Unemployment Rate";
    series x=date y=interestRate / legendlabel="Interest Rate";
    yaxis label="Time Series"; xaxis label="Date";
run;
```
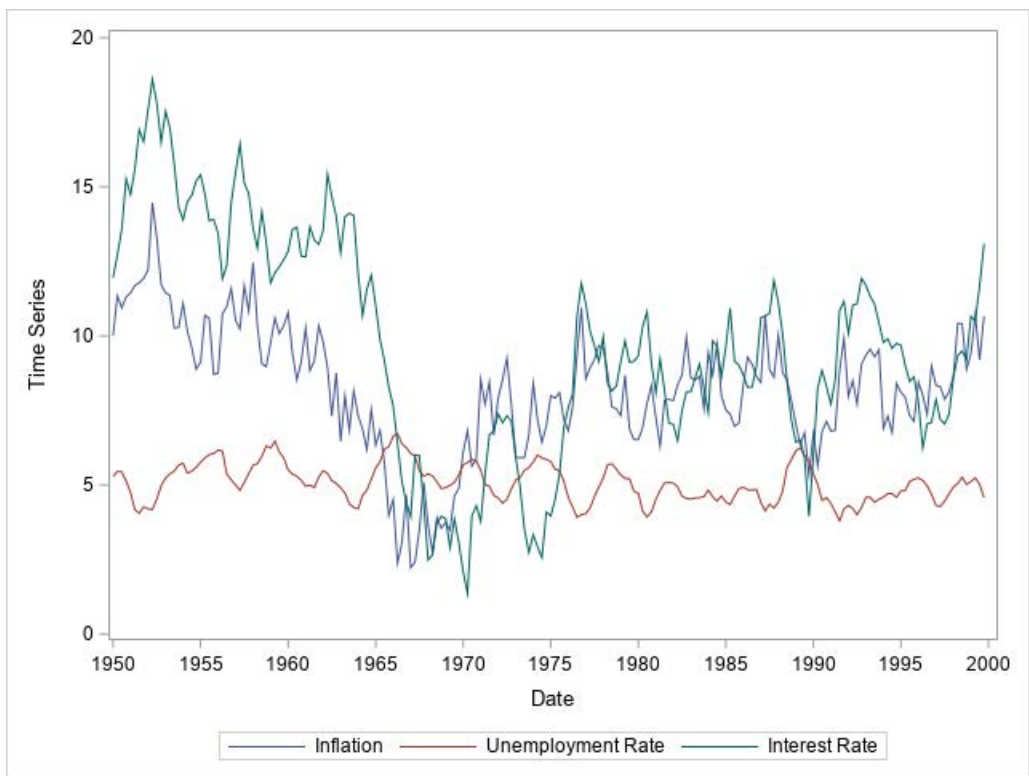


Figure 1. Plot of Three Time Series

It might be difficult to tell how these three time series affect each other from Figure 1. You can use the VARMAX procedure to estimate a VAR model in order to find out whether a relationship exists among these three time series.

## MODEL SELECTION OF AR ORDER

How to select the AR order in a VAR model is the first important question for model selection. You can use the MINIC= option in the VARMAX procedure to help you answer this question, as in the following statements, which also use the DATA=(WHERE=) option to base the VAR model on the data before 1998. (Using the data before 1998 to estimate the model leaves the last eight quarters of data as out-of-sample data.)

```
proc varmax data=simul1(where=(date<'01JAN1998'd));
    id date interval=quarter;
    model inflation unemploymentRate interestRate /
        minic=(type=aicc p=5 q=0);
run;
```

Notice that the P= option (which specifies the AR order) is not directly specified in the MODEL statement. Instead, the MINIC= option in the MODEL statement includes the P=5 suboption, which requests that the information criterion specified in the TYPE= suboption be calculated for orders 0 through 5. As shown in Figure 2, the AR 4 row corresponds to the smallest AICC, which indicates that 4 should be the selected AR order. In the analysis in the next section, P=4 is explicitly specified as an option in the MODEL statement.

| Minimum Information Criterion Based on AICC | |
| --- | --- |
| Lag | MA 0 |
| AR 0 | 2.4620192 |
| AR 1 | -3.078406 |
| AR 2 | -3.484704 |
| AR 3 | -3.63753 |
| AR 4 | -3.683988 |
| AR 5 | -3.624124 |

**Figure 2. Minimum Information Criterion Based on AICC**

## GRANGER CAUSALITY TESTS AND IMPULSE RESPONSE FUNCTIONS

The following statements perform the Granger causality tests and plot the impulse response functions:

```
proc varmax data=simul1(where=(date<'01JAN1998'd)) plots=all;
    id date interval=quarter;
    model inflation unemploymentRate interestRate / p=4 lagmax=24;
    causal group1=(inflation) group2=(unemploymentRate);
    causal group1=(inflation) group2=(interestRate);
    causal group1=(unemploymentRate) group2=(inflation);
    causal group1=(unemploymentRate) group2=(interestRate);
    causal group1=(interestRate) group2=(inflation);
    causal group1=(interestRate) group2=(unemploymentRate);
run;
```

In the preceding statements, each of six CAUSAL statements checks the Granger causality between two variables (that is, whether the variable listed in the GROUP2= option contributes to the forecast variable listed in the GROUP1= option). The testing result is shown in Figure 3. The *p*-values shown in the "Pr > ChiSq" column clearly indicate that unemployment rate Granger-causes both inflation and interest rate at the 5% significance level, but neither inflation nor interest rate Granger-causes unemployment rate at the 5% significance level. From this view, unemployment rate can even be treated as an exogenous variable and then moved to the right-hand side of the equation, which is beyond the scope of this paper.

| Granger-Causality Wald Test | | | |
|---|---|---|---|
| Test | DF | Chi-Square | Pr > ChiSq |
| 1 | 4 | 36.87 | <.0001 |
| 2 | 4 | 14.25 | 0.0065 |
| 3 | 4 | 1.01 | 0.9083 |
| 4 | 4 | 2.55 | 0.6356 |
| 5 | 4 | 16.88 | 0.0020 |
| 6 | 4 | 32.74 | <.0001 |

| Test 1: Group 1 Variables: | inflation |
|---|---|
| Group 2 Variables: | unemploymentRate |

| Test 2: Group 1 Variables: | inflation |
|---|---|
| Group 2 Variables: | interestRate |

| Test 3: Group 1 Variables: | unemploymentRate |
|---|---|
| Group 2 Variables: | inflation |

| Test 4: Group 1 Variables: | unemploymentRate |
|---|---|
| Group 2 Variables: | interestRate |

| Test 5: Group 1 Variables: | interestRate |
|---|---|
| Group 2 Variables: | inflation |

| Test 6: Group 1 Variables: | interestRate |
|---|---|
| Group 2 Variables: | unemploymentRate |

**Figure 3. Granger Causality Tests**

The PLOTS=ALL and LAGMAX=24 options in the preceding statements request all plots of three types of impulse response functions (IRFs)—namely standard, accumulated, and orthogonalized IRFs—in 24 lags. Figure 4 shows the impact of an inflation shock (impulse) on three variables over time: **inflation's response decreases** over time, unemployment rate has almost no response, and **interest rate's response** increases in the first three years and then decreases at a very slow rate. Be cautious about these IRFs, which are based on simulated data rather than on real data.
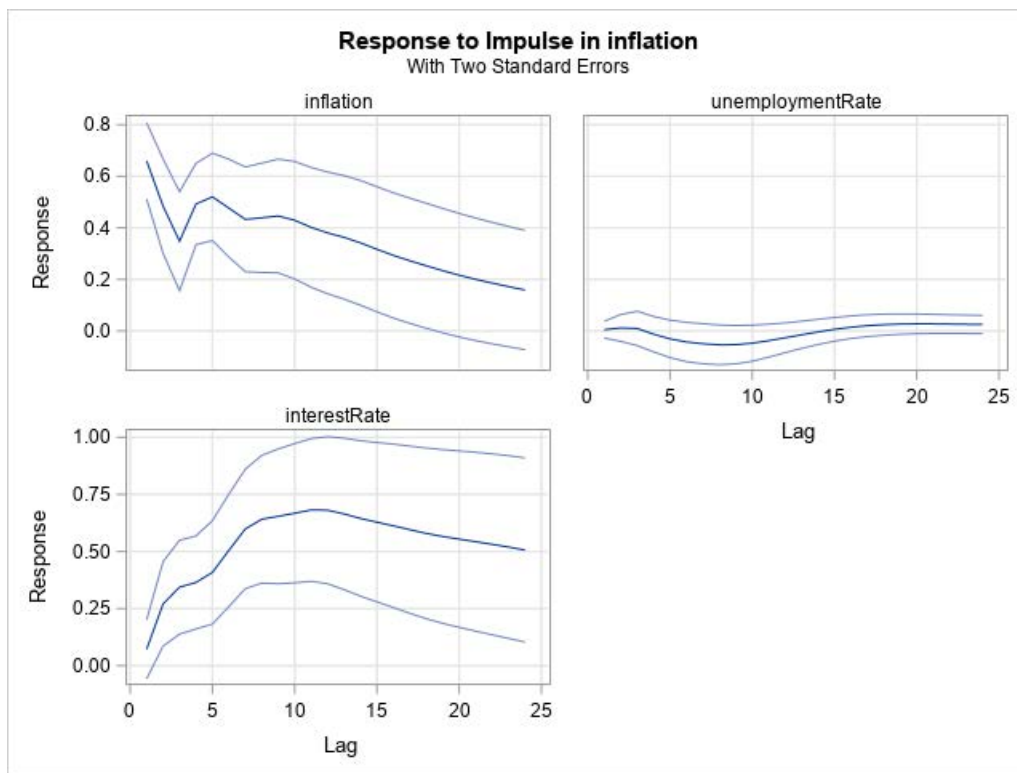
**Figure 4. Impulse Response Functions**

## RESTRICTED VAR MODEL

Because the Granger causality tests show that past values of inflation and interest rate are not likely to help forecast unemployment rate, a better model might be achieved by restricting those parameters to be 0 in order to exclude them from the model. The restricted VAR(4) model is estimated by using the RESTRICT statement, as follows:

```
proc varmax data=simul1(where=(date<'01JAN1998'd));
   id date interval=quarter;
   model inflation unemploymentRate interestRate / p=4;
   output out=o11 lead=8;
   restrict ar(1:4,unemploymentRate,inflation)=0,
            ar(1:4,unemploymentRate,interestRate)=0;
run;
```

As shown in Figure 5, although the restricted VAR(4) model has a smaller log likelihood than the former VAR(4) model (in the **section** "Granger Causality Tests and Impulse Response Functions"), the restricted VAR(4) model also has a smaller AICC than the former VAR(4) model, which implies that the restricted VAR(4) is likely to be a better model for out-of-sample forecasting.

| Log-likelihood | 106.192 |
|---|---|

| Information Criteria | |
|---|---|
| AICC | -93.2291 |
| HQC | -63.3763 |
| AIC | -122.384 |
| SBC | 23.25582 |
| FPEC | 0.024376 |

| Log-likelihood | 102.226 |
|---|---|

| Information Criteria | |
|---|---|
| AICC | -111.705 |
| HQC | -81.9344 |
| AIC | -130.452 |
| SBC | -10.7036 |
| FPEC | 0.025427 |

**Figure 5. Comparison between VAR (Left) and Restricted VAR (Right) Models**

The following statements plot the eight-quarter out-of-sample forecasts and their confidence intervals together with the actual values. The plot is shown in Figure 6.

```
data o12;
    set o11(where=(date>='01JAN1998'd));
    set simul1(where=(date>='01JAN1998'd));
    id = 1; name = "Inflation          ";
    actual = inflation; forecast = for1;
    lci = lci1; uci = uci1; output;
    id = 2; name = "Unemployment Rate";
    actual = unemploymentRate; forecast = for2;
    lci = lci2; uci = uci2; output;
    id = 3; name = "Interest Rate     ";
    actual = interestRate; forecast = for3;
    lci = lci3; uci = uci3; output;
    keep id name date actual forecast lci uci;
run;
proc sort data=o12; by id; run;
proc sgpanel data=o12;
    panelby name / layout=rowlattice uniscale=column novarname sort=data;
    band x=date upper=uci lower=lci / legendlabel="Confidence Interval";
    series x=date y=forecast / legendlabel="Forecast";
    scatter x=date y=actual / legendlabel="Actual Value";
    rowaxis display=(nolabel);
    colaxis label="Date" type=time interval=quarter;
run;
```
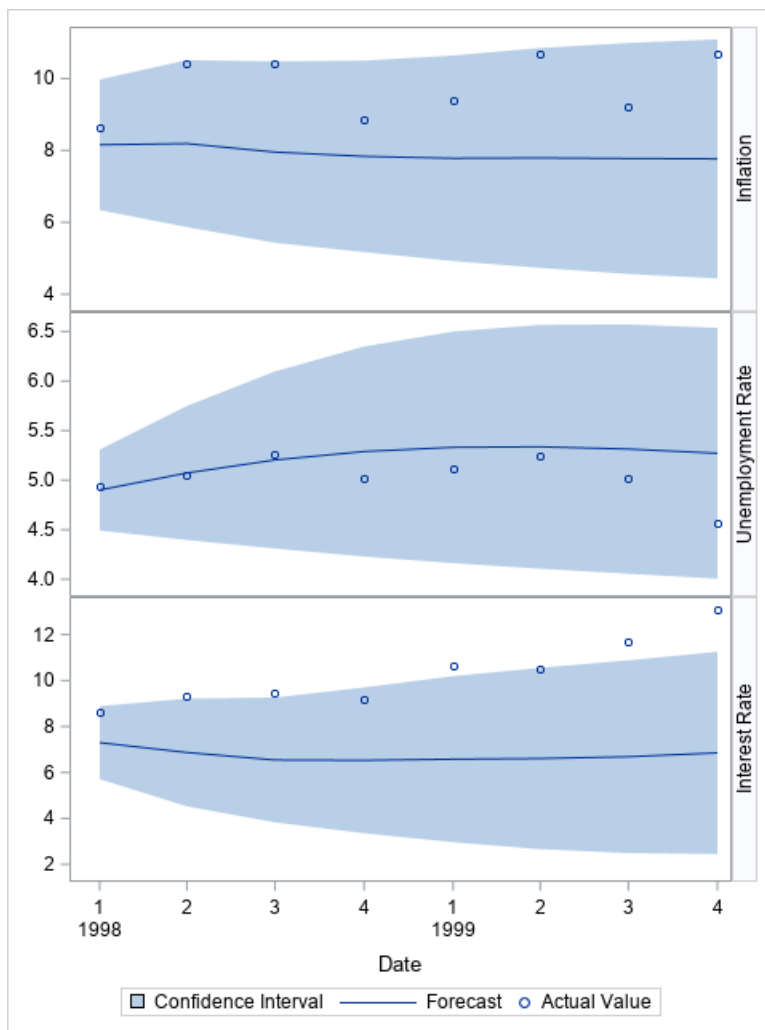
**Figure 6. Out-of-Sample Forecasts of Three Time Series and Their Actual Values**

# VECTOR ERROR CORRECTION MODEL

When some nontrivial linear combination of two or more nonstationary time series leads to a stationary time series, these nonstationary time series are called cointegrated. The cointegration, sometimes also called common trends, is very important in modeling nonstationary time series. Clive Granger was awarded a Nobel Prize in 2003 **"for methods of analyzing economic time series with common trends (cointegration)"** (Nobel Media AB 2020a). The main tool for modeling cointegration is the vector error correction model (VECM), which exploits both the long-run equilibrium and short-run relationships in the nonstationary time series. One popular example is the relationship between exchange rates and price indices (which are both nonstationary time series). According to the concept of purchasing power parity (PPP), the ratio between exchange rate and price index should be a constant at long-run equilibrium. In this section, simulated data mimic the exchange rate between the British pound and the US dollar and the price index of the UK as a relative price versus the US.

The following statements read the simulated data of the exchange rate and price index and then generate their natural logarithm values for further modeling:

```
data simul2;
   input xr p;
   date=intnx('year', '01jan1950'd, _n_-1 ); format date year.;
   lp = log(p); lx = log(xr);
   datalines;
   0.38972 62.941
   0.38961 63.445
   ... more lines ...
   0.59267 134.171
   0.54500 144.752
;
```

The following statements estimate the VECM model on data before 2000 (which leaves eight years of data from 2000 to 2007 as out-of-sample data):

```
proc varmax data=simul2(where=(date<'01JAN2000'd));
   id date interval=year;
   model lp lx / p=2;
   cointeg rank=1 ectrend normalize=lx;
   output out=o21 lead=8;
run;
```

The estimation result of long-run parameters, as shown in Figure 7, indicates that the difference between the logarithm of the exchange rate and the logarithm of the price index are close to some constant, which confirms the economic theory.

| Alpha and Beta Parameter Estimates | | | | | | |
|---|---|---|---|---|---|---|
| Equation | Parameter | Estimate | Standard Error | t Value | Pr > \|t\| | Variable |
| D_lp | ALPHA1_1 | 0.20543 | 0.04067 | 5.05 | <.0001 | Beta[,1]*_DEP_(t-1) |
| | BETA1_1 | -1.27176 | | | | lp(t-1) |
| D_lx | ALPHA2_1 | -0.18997 | 0.03964 | -4.79 | <.0001 | Beta[,1]*_DEP_(t-1) |
| | BETA2_1 | 1.00000 | | | | lx(t-1) |
| _ECTREND_ | ECCONST1 | 6.38085 | | | | 1 |

**Figure 7. Long-Run Parameter Estimates**

The following statements plot the out-of-sample forecasts together with their actual values. The plots are shown in Figure 8 and Figure 9.

```
data o22;
    set o21(where=(date>='01JAN2000'd));
    set simul2(where=(date>='01JAN2000'd));
run;
proc sgplot data=o22;
    band x=date lower=lci1 upper=uci1 / legendlabel="Confidence Interval";
    series x=date y=for1 / legendlabel="Forecast";
    scatter x=date y=lp / legendlabel="Actual Value";
    yaxis label="log(Price Index)"; xaxis label="Date";
run;
proc sgplot data=o22;
    band x=date lower=lci2 upper=uci2 / legendlabel="Confidence Interval";
    series x=date y=for2 / legendlabel="Forecast";
    scatter x=date y=lx / legendlabel="Actual Value";
    yaxis label="log(Exchange Rate)"; xaxis label="Date";
run;
```

As shown in Figure 8 and Figure 9, the forecasts are very good. For further discussion about VECMs and their model selection, see Chen and Kechagias (2017).
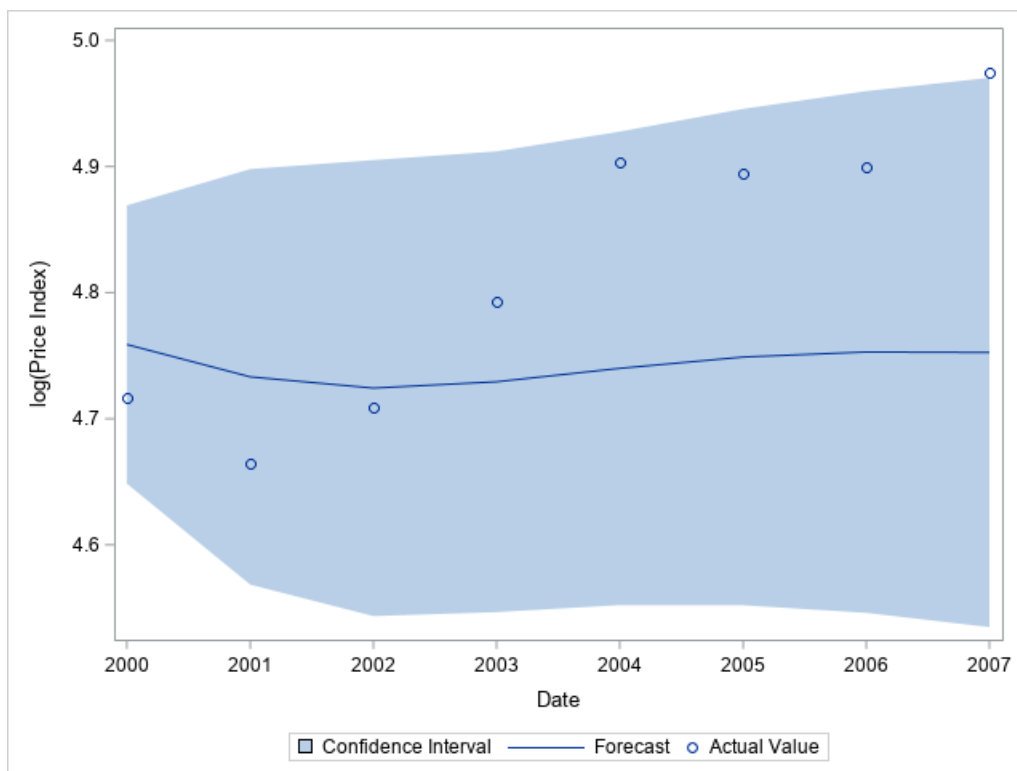


**Figure 8. Out-of-Sample Forecasts of Log Price Index and Their Actual Values**
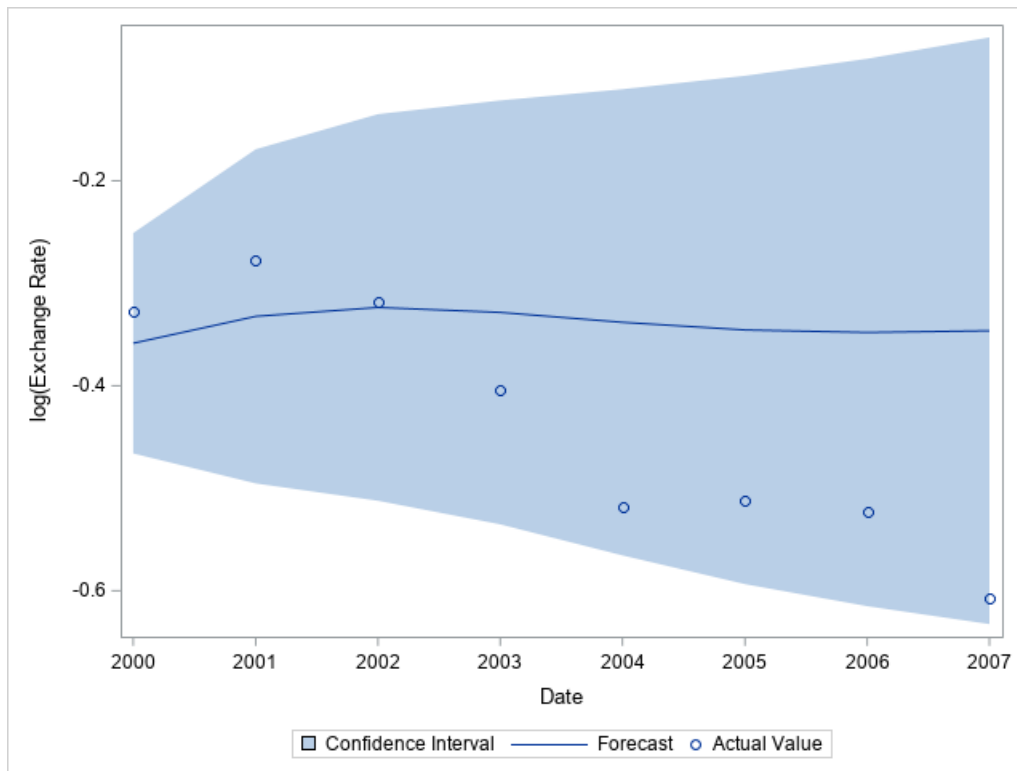
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**Figure 9. Out-of-Sample Forecasts of Log Exchange Rate and Their Actual Values**


## MULTIVARIATE GARCH MODEL

Time-varying volatility and volatility clustering, which are common in financial time series, play an important role in risk management, portfolio optimization, and other fields. Robert Engle was awarded a Nobel Prize in 2003 **"for methods of analyzing economic time series with time-varying volatility (ARCH)"** (Nobel Media AB 2020a). The ARCH and GARCH models have been widely used in volatility modeling and forecasting. This section models simulated data for the Japanese and US stock market weekly returns by using a DCC-GARCH model, and then uses the forecasted volatility in the portfolio optimization to generate a trading strategy that is far better than three benchmark trading strategies.

The following statements read simulated data and plot the Japanese and US stock market weekly returns in Figure 10:

```
data simul3;
    input date mmddyy10. jpWeeklyReturn usWeeklyReturn;
    format date mmddyy10.;
datalines;
01/13/1965 1.8863 1.4196
01/21/1965 -0.8785 0.7890
   ... more lines ...
12/17/2019 2.7634 1.8973
12/24/2019 -0.9835 0.9620
;
proc sgplot data=simul3;
    series x=date y=jpWeeklyReturn;
    yaxis label="Japanese Stock Market Weekly Return"; xaxis label="Date";
run;
```

```
proc sgplot data=simul3;
    series x=date y=usWeeklyReturn;
    yaxis label="US Stock Market Weekly Return";
    xaxis label="Date";
run;
```
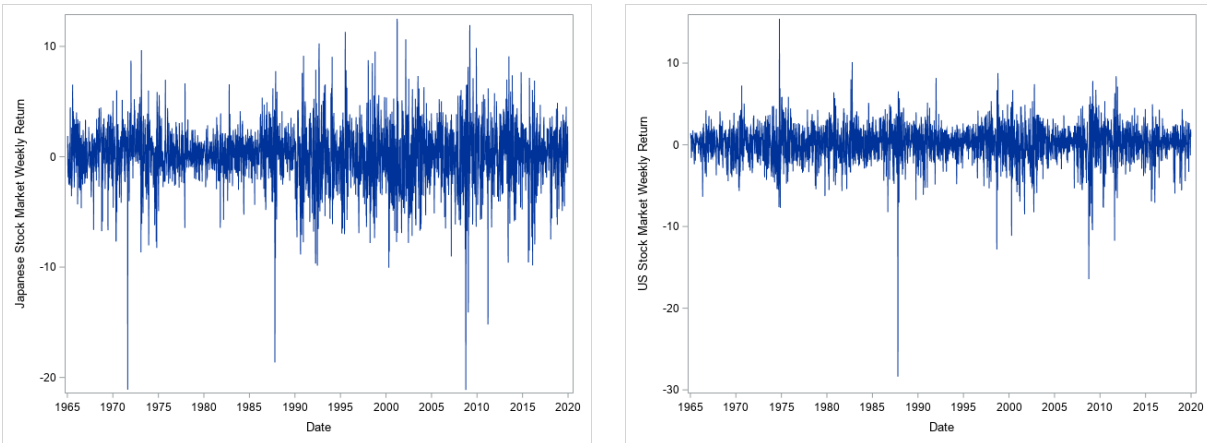


**Figure 10. Japanese and US Stock Market Weekly Returns**

In the VARMAX procedure, you can use the GARCH statement to specify a multivariate GARCH model. The FORM=DCC option specifies the dynamic conditional correlation (DCC) GARCH model. The SUBFORM=GJR option together with P=1 and Q=1 options specify the GJR GARCH(1,1) model for each time series in order to address the asymmetric news impact: negative and positive shocks of the same magnitude have different effects on future volatility. The OUTHT= option specifies the output table for the forecasted volatility. The TECH=TR option in the NLOPTIONS statement is used to specify the trust region optimization algorithm, which leads to the correct estimation in this example.

```
ods output GARCHParameterEstimates = ogarchparm31 CovInnovation=ocov31;
proc varmax data=simul3;
    model jpWeeklyReturn usWeeklyReturn;
    garch form=dcc p=1 q=1 subform=gjr outht=oh31;
    nloptions tech=tr;
run;
```

The GARCH model parameter estimation result (as shown in Figure 11) indicates that all parameters are significant at the 5% significance level, which means that the conditional correlation is likely to be time-varying (DCC vs. CCC) and that the news impact is asymmetric (GJR GARCH vs. standard GARCH).

| GARCH Model Parameter Estimates | | | | |
|---|---|---|---|---|
| Parameter | Estimate | Standard Error | t Value | Pr > \|t\| |
| DCCA | 0.01621 | 0.00605 | 2.68 | 0.0074 |
| DCCB | 0.98095 | 0.00813 | 120.70 | 0.0001 |
| DCCS1_2 | 0.37104 | 0.10914 | 3.40 | 0.0007 |
| GCHC1_1 | 0.35878 | 0.08425 | 4.26 | 0.0001 |
| GCHC2_2 | 0.24050 | 0.04635 | 5.19 | 0.0001 |
| ACH1_1_1 | 0.07073 | 0.01519 | 4.66 | 0.0001 |
| ACH1_2_2 | 0.02835 | 0.01388 | 2.04 | 0.0412 |
| TACH1_1_1 | 0.08215 | 0.02463 | 3.34 | 0.0009 |
| TACH1_2_2 | 0.16742 | 0.02663 | 6.29 | 0.0001 |
| GCH1_1_1 | 0.84642 | 0.02160 | 39.18 | 0.0001 |
| GCH1_2_2 | 0.84627 | 0.01758 | 48.14 | 0.0001 |

**Figure 11. GARCH Model Parameter Estimates**

The following statements plot the conditional correlation against time in Figure 12 and plot the news impact curves for the Japanese and US stock markets in Figure 13:

```
data oh32; set oh31; set simul3; corr = h1_2 / sqrt(h1_1*h2_2); run;
proc sgplot data=oh32;
   series x=date y=corr;
   yaxis max=1 label="Correlation"; xaxis label="Date";
run;
proc iml;
   use ogarchparm31; read all var {estimate} into garchParm; close;
   use ocov31; read all into cov; close;
   omega1 = garchParm[4]; alpha1 = garchParm[6]; gamma1 = garchParm[8];
   beta1 = garchParm[10]; sigma1 = cov[1,1];
   omega2 = garchParm[5]; alpha2 = garchParm[7]; gamma2 = garchParm[9];
   beta2 = garchParm[11]; sigma2 = cov[2,2];
   eps = (-100:100)`/10; h1=J(nrow(eps),1,0); h2=J(nrow(eps),1,0);
   do i = 1 to nrow(eps);
      if(eps[i]<0) then h1[i] = omega1 + beta1*sigma1
                                  + (alpha1+gamma1)*eps[i]*eps[i];
      else h1[i] = omega1 + beta1*sigma1 + (alpha1)*eps[i]*eps[i];
      if(eps[i]<0) then h2[i] = omega2 + beta2*sigma2
                                  + (alpha2+gamma2)*eps[i]*eps[i];
      else h2[i] = omega2 + beta2*sigma2 + (alpha2)*eps[i]*eps[i];
   end;
   epsh = eps || h1 || h2;
   cn = {'eps' 'jpH' 'usH'};
   create o33 from epsh[colname=cn]; append from epsh; close;
quit;
```

```
proc sgplot data=o33;
    styleattrs datacontrastcolors=(black blue);
    scatter x=eps y=jpH / legendlabel="JP";
    scatter x=eps y=usH / legendlabel="US";
    refline 0 / axis=x lineattrs=(color=gray thickness=2);
    refline 0 / axis=y lineattrs=(color=gray thickness=2);
    yaxis label="Current Volatility" min=0;
    xaxis label="Previous Return Shock";
run;
```
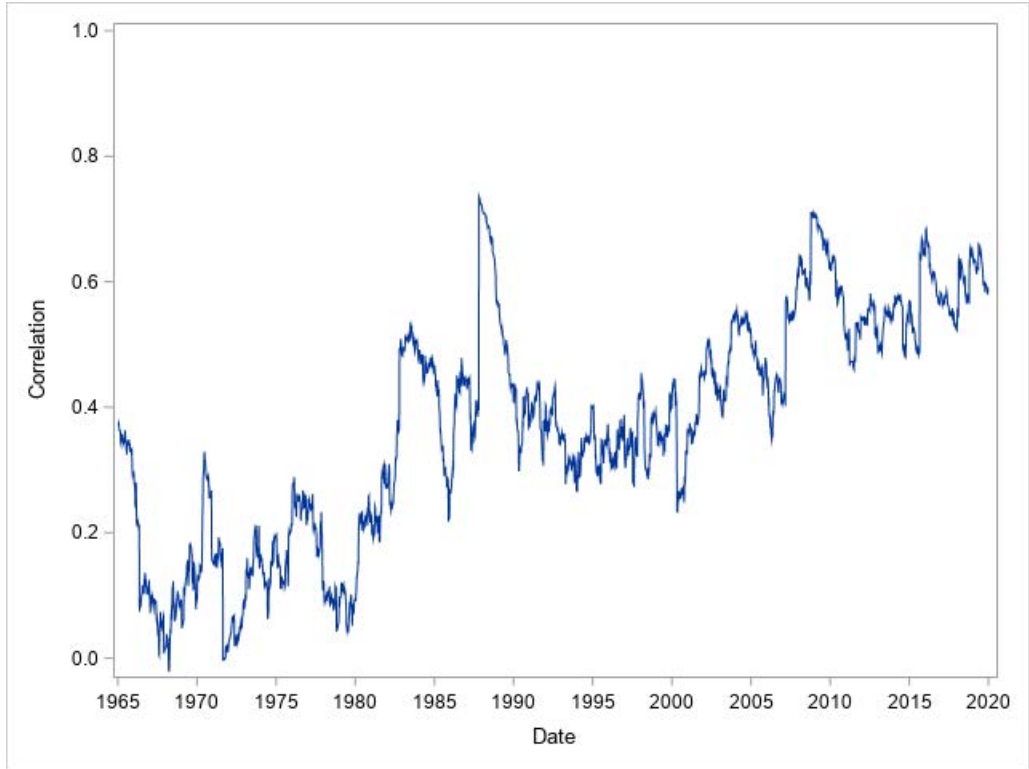


**Figure 12. Dynamic Conditional Correlations between Japanese and US Stock Market Returns**
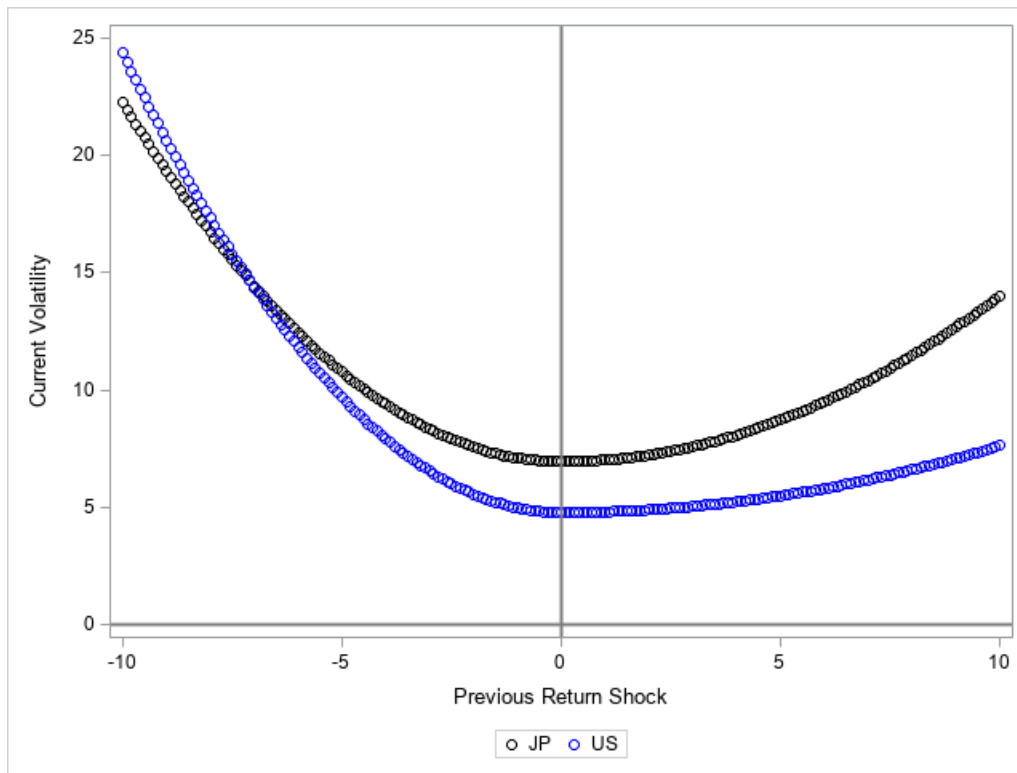
**Figure 13. News Impact Curves for Japanese and US Stock Markets**

The following statements use the forecasted volatility in the portfolio optimization (by maximizing the expected utility function, which is defined by using the Arrow-Pratt risk aversion index to balance risk and return) and then compare this trading strategy (max U) with three benchmark trading strategies: holding Japanese stocks only (JP), holding US stocks only (US), and the well-known equal-weight portfolio (EQ, half-Japanese and half-US):

```
proc iml;
   start maxuWeight(gamma,m,h);
      maxuw = J(nrow(h),2,0);
      do t = 1 to nrow(h);
         * max A * x^2 + B * x + C, where x is between 0 and 1;
         bigA = -gamma/2*(h[t,1]+h[t,3]-2*h[t,2])/10000;
         bigB = m[t,1]-m[t,2]-2*gamma/2*(h[t,2]-h[t,3])/10000;
         bigC = m[t,2] - gamma/2 * h[t,3]/10000;
         if(bigA = 0) then do;
            if(bigB = 0) then maxuw[t,1] =0.5;
            else do;
               if (bigB > 0) then maxuw[t,1] = 1;
               else maxuw[t,1] = 0;
            end;
            maxuw[t,2] = 1 - maxuw[t,1];
         end;
         else do;
            x = -bigB / (2*bigA); * F.O.C.;
            uatx = bigA * x * x + bigB * x + bigC;
            uat1 = bigA + bigB + bigC;
            uat0 = bigC;
```

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```
            if(x>=0 & x<=1) then do;
                if(uatx>uat0 & uatx>uat1) then maxuw[t,1] = x;
                else do;
                    if(uat1>uatx & uat1>uat0) then maxuw[t,1] = 1;
                    else maxuw[t,1] = 0;
                end;
            end;
            else do;
                if(uat1>uat0) then maxuw[t,1] = 1;
                else maxuw[t,1] = 0;
            end;
            maxuw[t,2] = 1 - maxuw[t,1];
        end;
    end;
    return(maxuw);
finish;

start excessReturn(p,w);
    * assume that risk-free return is 0;
    er = J(nrow(w),1,1);
    do t = 1 to nrow(er);
        if(t=1) then er[t] = p[t,]*w[t,]`-1;
        else er[t] = w[t,1]*p[t,1]/p[t-1,1]+w[t,2]*p[t,2]/p[t-1,2]-1;
    end;
    return(er);
finish;

start wealth(er);
    wealth = J(nrow(er),1,0);
    do t=1 to nrow(wealth);
        if(t=1) then wealth[t] = er[t]+1;
        else wealth[t] = wealth[t-1] * (er[t]+1);
    end;
    return(wealth);
finish;

start stat(er,v);
    AV = mean(er);
    SD = std(er);
    IR = AV/SD;
    FW = v[nrow(v)];
    APY = (exp(log(FW)/55)-1)*100;
    return(AV || SD || IR || FW || APY);
finish;

* main program;
use oh32; read all into d; close;
h = d[,1:3]; date = d[,4]; r = d[,5:6];
* restore "stock prices";
p = J(nrow(r),ncol(r),0);
do t = 1 to nrow(r);
    do i = 1 to ncol(r);
        if(t=1) then p[t,i] = exp(r[t,i]/100) * 1;
        else p[t,i] = exp(r[t,i]/100) * p[t-1,i];
    end;
end;
```

```
    jpw = J(nrow(p),ncol(p),0); jpw[,1] = 1;
    jper = excessReturn(p,jpw);
    jpv = wealth(jper);
    jpstat = stat(jper,jpv);

    usw = J(nrow(p),ncol(p),0); usw[,2] = 1;
    user = excessReturn(p,usw);
    usv = wealth(user);
    usstat = stat(user,usv);

    eqw = J(nrow(p),ncol(p),1/ncol(p));
    eqer = excessReturn(p,eqw);
    eqv = wealth(eqer);
    eqstat = stat(eqer,eqv);

    * Maximize Expected Utility U(w) = m*w'-gamma*w'*H*w/2;
    * signal m is the momentum factor (Jegadeesh and Titman 1993);
    m = J(nrow(h),2,0);
    do t = 1 to nrow(m);
       if(t<=50) then m[t,] = mean(r[1:50,]/100);
       else m[t,] = mean(r[t-50:t-5,]/100);
    end;

    gamma = 0.1; * Arrow-Pratt risk aversion index;
    maxuw = maxuWeight(gamma,m,h);
    maxuer = excessReturn(p,maxuw);
    maxuv = wealth(maxuer);
    maxustat = stat(maxuer,maxuv);

    values = date || jpv || usv || eqv || maxuv;
    cn = {'date' 'jp' 'us' 'eq' 'maxu'};
    create values from values[colname=cn];
    append from values;
    close;

    stats = (1:5)` || jpstat` || usstat` || eqstat` || maxustat`;
    cn = {'type' 'jp' 'us' 'eq' 'maxu'};
    create stats from stats[colname=cn];
    append from stats;
    close;
quit;
```

The following statements print the measure of the four trading strategies in Figure 14:

```
data stats;
   set stats;
   if(type=1) then stat="Average Return          ";
   if(type=2) then stat="Standard Deviation      ";
   if(type=3) then stat="Sharpe Ratio            ";
   if(type=4) then stat="Final Wealth/Inital Wealth";
   if(type=5) then stat="APY(%)                  ";
run;
proc print data=stats noobs label;
   var stat jp us eq maxu;
   label stat="Measure" jp="JP" us="US" eq="EQ" maxu="max U";
run;
```

| Measure | JP | US | EQ | max U |
|---|---|---|---|---|
| Average Return | 0.0015 | 0.0017 | 0.0016 | 0.0019 |
| Standard Deviation | 0.0277 | 0.0230 | 0.0213 | 0.0245 |
| Sharpe Ratio | 0.0546 | 0.0722 | 0.0746 | 0.0765 |
| Final Wealth/Inital Wealth | 18.9476 | 38.0873 | 34.7839 | 60.6574 |
| APY(%) | 5.4941 | 6.8419 | 6.6658 | 7.7497 |

**Figure 14. Measure of Four Trading Strategies**

As shown in Figure 14, the max U strategy has both the best average return and the best Sharpe ratio (an indicator of risk-adjusted return). Over 55 years, the final wealth that results from using the max U strategy is about 60% more than the final wealth that results from using the second-best strategy (US).

The following statements plot the wealth curve (or portfolio value) of each trading strategy against time in Figure 15:

```
data values; set values; format date mmddyy10.; run;
proc sgplot data=values;
   styleattrs datacontrastcolors=(black blue red green);
   series x=date y=jp / legendlabel="JP";
   series x=date y=us / legendlabel="US";
   series x=date y=eq / legendlabel="EQ";
   series x=date y=maxu / legendlabel="max U";
   yaxis label="Wealth"; xaxis label="Date";
run;
```
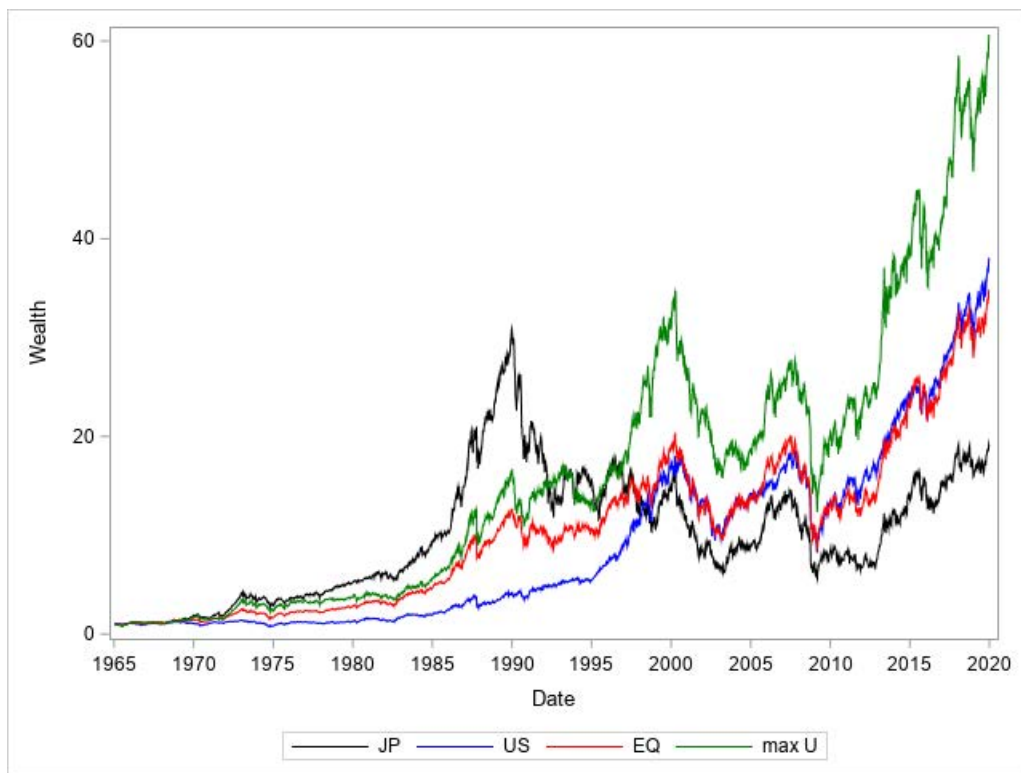
**Figure 15. Wealth Curves of the Four Trading Strategies**

## CONCLUSION

In this paper, three simple examples illustrate how easy it is to use the VARMAX procedure to specify and estimate VAR, VECM, and multivariate GARCH models, and then how to infer results and use them. For other multiple time series tools such as state space models (SSMs) and hidden Markov models (HMMs), see the chapters **"The SSM Procedure"** in *SAS/ETS® 15.1: **User's Guide*** and "The HMM Procedure" in *SAS® Econometrics 8.5: Econometrics Procedures*.

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## ACKNOWLEDGMENTS

## RECOMMENDED READING

- *SAS/ETS 15.1: User's Guide*

- *SAS Econometrics 8.5: Econometrics Procedures*

- *SAS Econometrics 8.5: Programming Guide*

## CONTACT INFORMATION

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