SAS® GLOBAL FORUM 2017

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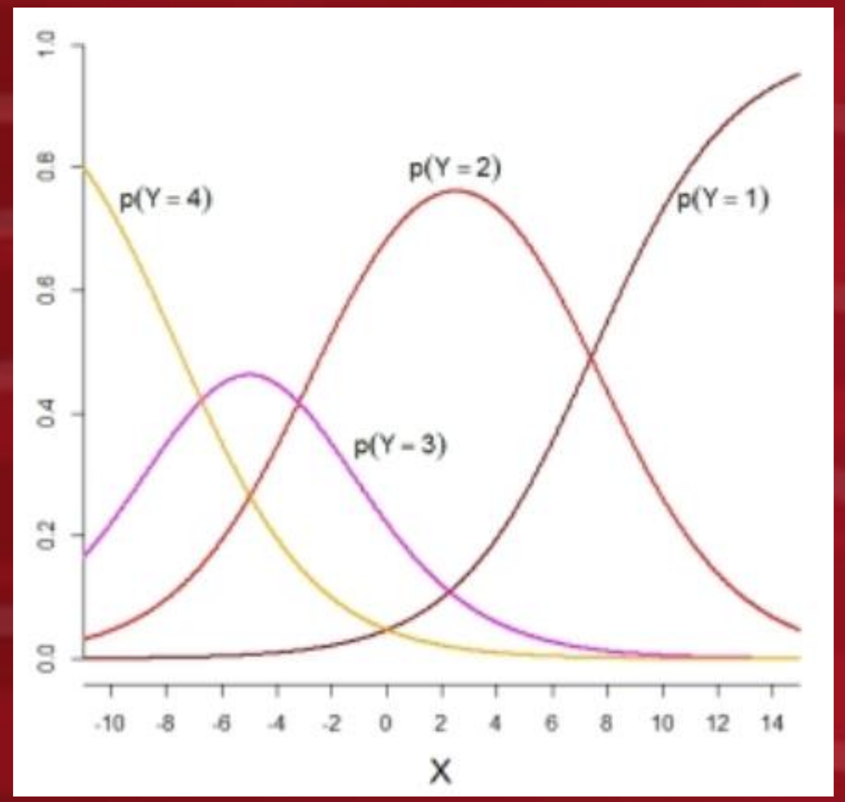
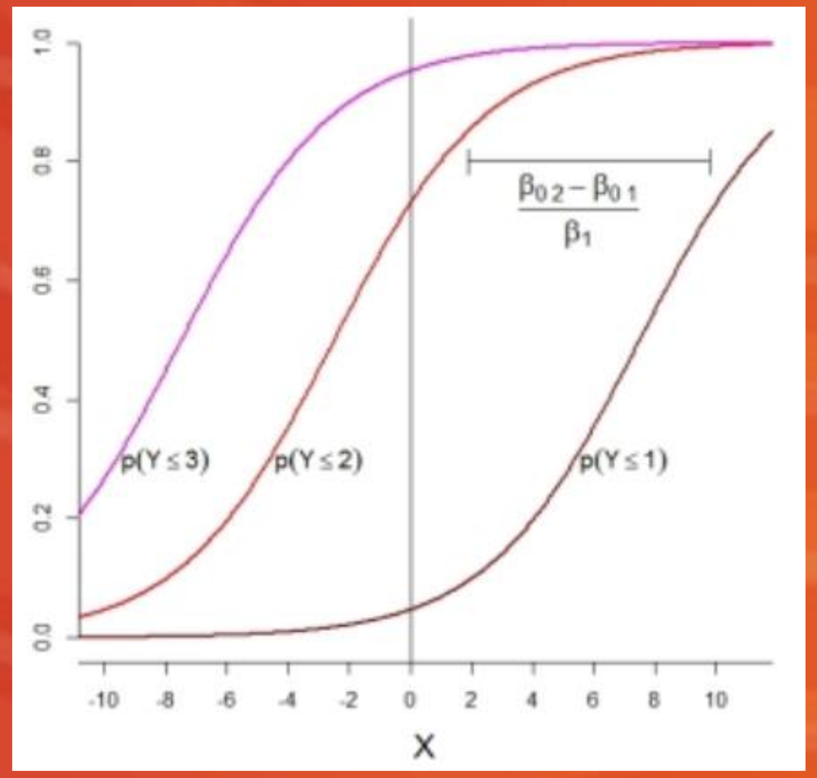
Using New SAS® 9.4 Features for Cumulative Logit Models with Partial Proportional Odds





Paul J. Hilliard, Educational Testing Service (ETS)

USERS PROGRAM

Using New SAS 9.4 Features for Cumulative Logit Models with Partial Proportional Odds

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INTRODUCTION

Multicategory Logit Models extend the techniques of logistic regression to response variables with three or more categories.

For response variables in which a natural ordering of the categories exists, a ***Cumulative Logit Model*** is employed. The simplest form assumes that the effect of an explanatory variable (predictor) is identical for all modeled logits (known as the assumption of proportional odds). However, as the sample size and number of predictors increase, it is unlikely that proportional odds can be assumed across all predictors.

Partial Proportional Odds Models are an emerging method to effectively model this relationship, which fits a model with unique parameter estimates at each level of the modeled relationship ***only*** for the predictors in which proportionality cannot be assumed. It is often a much simpler model than the "general" model that generates separate parameter estimates across all response levels for each predictor.

In **SAS® 9.4, PROC LOGISTIC** now offers model-building functionality in which all equal and unequal slope parameters are available for effect selection, whereas previously the statistician was required to assess predictor non-proportionality *a priori* through likelihood tests or graphical means.

No other commercially available software (SPSS, R, Stata etc.) offer this unique solution

This presentation uses a public-use data set to examine the new functionality in PROC LOGISTIC using stepwise variable selection methods.

THE NEED FOR A PARTIAL PROPORTIONAL ODDS MODEL

- Modeled relationships are often complex and based upon a variety of both categorical and continuous predictors. The Score Test of the Proportional Odds Assumption frequently ***rejects*** the null hypothesis that proportionality can be assumed.
- The test is anti-conservative and ***nearly always*** results in rejection, particularly when the number of explanatory variables is large, the sample size is large, or if there are continuous predictors in the model.

DATA

- A publicly available dataset ("Adolescent Placement Study") was used for model building.
- Contains data for 508 adolescents across 11 predictors related to the placement location following psychiatric evaluation. A total of 4 placement locations exist (dependent variable) which are ordered by restrictiveness.
- Contains a mix of dichotomous, polytomous, and quantitative predictors.

MODEL BUILDING

- **PROC LOGISTIC** with **LINK=CLOGIT** was used to build Cumulative Logit Models.
- Stepwise Selection Method utilized (though forward and backward selection methods are also permitted).
- The method begins with an intercept-only model, then evaluates each available predictor to determine which predictor most improves model fit. After each step, the method re-examines the significance of all predictors in the model and removes any predictor which, after controlling for other predictors in the model, is no longer significant at the alpha level that is user-specified, iteratively until no more predictors meet the significance level for entry.
- **α = 0.10** designated as level for entry or retention in the models

MODELS BUILT INCLUDE:

- Proportional Odds Model (labeled **PropOdds**), which is the default method unless user-specified
- Partial Proportional Odds Model (**PPODDS≠**)
- Partial Proportional Odds Model with all equal slope predictors forced into the model (**PPODDSwith**=)

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RESULTS

PROPORTIONAL ODDS MODEL (PropOdds)

- For ordinal logistic regression, the default method of model building uses a Proportional Odds Model. Generally, a user would assume proportionality of the predictors unless statistical or graphical evidence suggests otherwise.

```
proc logistic data= MB outest=SASGlobal_est_4PROPODDS;
      model PLACE = AGE RACE GENDER NEURO EMOT DANGER ELOPE LOS BEHAV CUSTD VIOL /
            LINK=CLOGIT SELECTION=STEPWISE SLE=0.10 SLS=0.10;
      score data= XV out=SasGlob.Pred_XV_4PROPODDS;
      ods output association=SasGlob.SASGlobal_assoc_4PROPODDS;
run;
```

Analysis of Maximum Likelihood Estimates

Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	0	1	3.8320	0.9915	14.9364	0.0001
Intercept	1	1	5.3762	1.0107	28.2949	<.0001
Intercept	2	1	7.3393	1.0444	49.3814	<.0001
AGE		1	-0.1142	0.0604	3.5777	0.0586
RACE		1	-0.6136	0.2025	9.1809	0.0024
NEURO		1	-0.3507	0.1028	11.6421	0.0006
DANGER		1	0.2543	0.1215	4.3798	0.0364
LOS		1	-0.0532	0.00686	60.0603	<.0001
BEHAV		1	-0.3195	0.0627	25.9971	<.0001
CUSTD		1	-2.3526	0.2342	100.9056	<.0001

- ***Seven*** of the eleven predictors are significant to the modeled relationship.
- Since proportional odds are assumed, the same parameter estimate is used at all response levels for a given predictor
- With the exception of DANGER, all parameter estimates are negative. This indicates that, controlling for all other predictors in the model, as the value of a given predictor increases, the probability that the observation is more likely to be classified in a lower category level decreases (a less restrictive treatment setting).

RESULTS

PROPORTIONAL ODDS MODEL (PropOdds)

- PROC LOGISTIC automatically generates a Score Test to assess the proportionality assumption. For the modeled relationship, the following results were obtained:

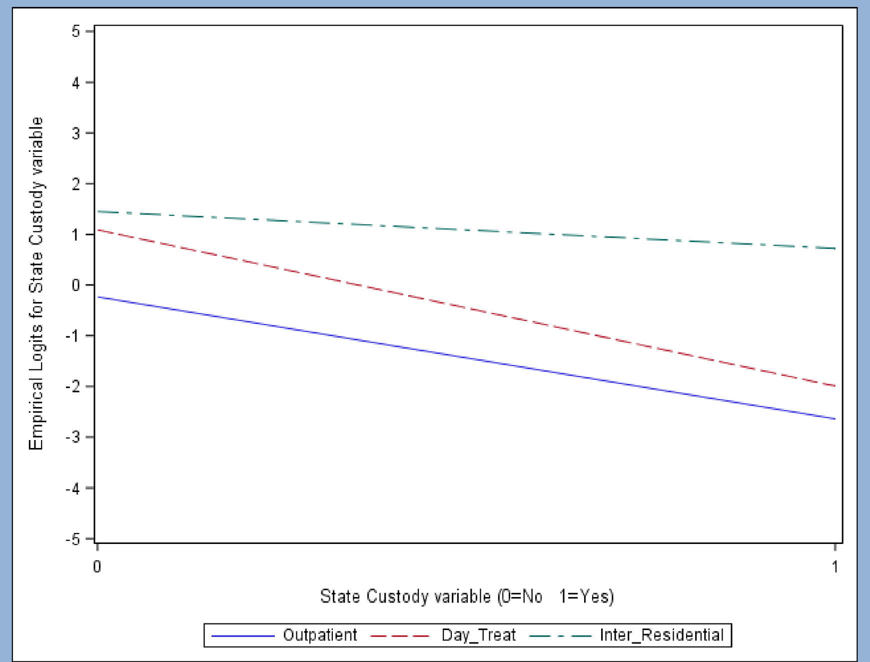
Score Test for the Proportional Odds Assumption

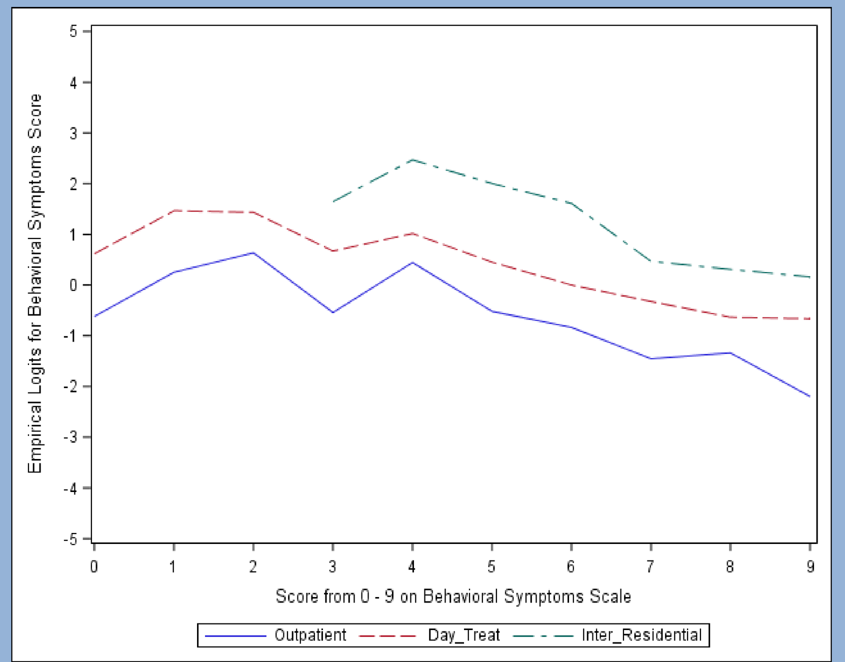
Chi-Square	DF	Pr > ChiSq
136.0528	14	<.0001

- The test statistic is ***highly*** significant. This suggests that the proportionality of the predictors in the modeled relationship cannot be assumed.
- The assumption can be assessed graphically for a given predictor by plotting the predictor versus the "Empirical Logits", which are based upon the observed counts of the dependent variable
- For response level *i* when there are *J* levels from 0 to J, the Empirical Logit can be calculated by:

$$\text{EmpLogit } y_i = \log\left(\frac{y_0 + y_1 + \ldots + y_i}{y_{i+1} + y_{i+2} + \ldots + y_J}\right)$$

- Using the study data, PROC SGPLOT displays the empirical logits for the State Custody variable (CUSTD) and Behavioral Symptoms Scale (BEHAV)





- For CUSTD, while the empirical logits for Placement levels 0 and 2 (Outpatient and Inter-Residential) are fairly parallel to one another, the empirical logit for Placement level 1 is not parallel to the others. A lack of "parallel-ness" exists for BEHAV as well.
- Taken in sum, the plot indicates a lack of proportionality.

$$\text{EmpLogit } y_i = \log\left(\frac{y_0 + y_1 + ... + y_i}{y_{i+1} + y_{i+2} + ... + y_J}\right\}$$

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RESULTS

PARTIAL PROPORTIONAL ODDS MODEL (PPODDS≠)

- To build a Partial Proportional Odds Model, the following code was used.
- The "EQUALSLOPES" option along with the "UNEQUALSLOPES" option need to be included in the MODEL statement.

```
proc logistic data=SasGlob.SASGlobal_MB outest=SASGlobal_est_4PPODDS;
      model PLACE = AGE RACE GENDER NEURO EMOT DANGER ELOPE LOS BEHAV CUSTD VIOL /
            LINK=CLOGIT SELECTION=STEPWISE SLE=0.10 SLS=0.10
      equalslopes unequalslopes details maxiter=10000;
      score data=SasGlob.SASGlobal_XV out=SasGlob.Pred_XV_4PPODDS;
      ods output association=SasGlob.SASGlobal_assoc_4PPODDS;
run;
quit;
```

- All predictors in both their equal and non-equal (general) slope forms across the response levels are available for selection. These predictors are then ***simultaneously evaluated*** in a stepwise manner to build the model.

Summary of Stepwise Selection

Step	Effect Entered	Effect Removed	DF	Number In	Score Chi-Square	Wald Chi-Square	Pr > ChiSq
1	U_CUSTD		3	1	183.1529		<.0001
2	LOS		1	2	121.9317		<.0001
3	BEHAV		1	3	20.9883		<.0001
4	NEURO		1	4	10.9480		0.0009
5	RACE		1	5	7.7161		0.0055
6	DANGER		1	6	5.4120		0.0200
7	U_EMOT		3	7	8.0716		0.0446
8	AGE		1	8	3.1047		0.0781

- ***Three*** of the predictors entered the model in their ***non-equal*** slope forms. The relationship between these predictors and adolescent placement is non-proportional across placement levels.
- For these predictors, model fit is also improved most when the non-equal slope form is included in the model.

RESULTS

PARTIAL PROPORTIONAL ODDS MODEL (PPODDSwith=)

- Another option is a restriction that requires every model during selection to include the equal slope parameters. The **INCLUDE=EQUALSLOPES** option then tests the unequal slope parameters for a significant effect and retains them in the model if significant at a user-specified alpha level.
- The multiple unequal slope parameters then serve as individual adjustments to the account for the non-proportionality.

```
proc logistic data=SasGlob.SASGlobal_MB outest=SASGlobal_est_4PPODDS_INC;
      model PLACE = AGE RACE GENDER NEURO EMOT DANGER ELOPE LOS BEHAV CUSTD VIOL /
            LINK=CLOGIT SELECTION=STEPWISE SLE=0.10 SLS=0.10
      equalslopes unequalslopes include=equalslopes details maxiter=10000;
      score data=SasGlob.SASGlobal_XV out=SasGlob.Pred_XV_4PPODDS_INC;
      ods output association=SasGlob.SASGlobal_assoc_4PPODDS_INC;
run;
```

- All eleven predictors in their equal slope form plus the unequal slope parameter estimates for three of the predictors are included in the model. These predictors, when controlling for all others already in the model, exhibit non-proportionality across placement levels AND are significant to the modeled relationship.

Summary of Stepwise Selection

Step	Effect Entered	Effect Removed	DF	Number In	Score Chi-Square	Wald Chi-Square	Pr > ChiSq
1	U_CUSTD		2	12	108.2995		<.0001
2	U_EMOT		2	13	6.0004		0.0498
3	U_ELOPE		2	14	4.9389		0.0846

- Contrasting this model to the previous model, unequal slope parameter estimates for ELOPE were added to this model while both models contained the U_CUSTD and U_EMOT parameter estimates.

DISADVANTAGES OF THIS METHOD

1) Forcing all equal slope predictors into the model unnecessarily adds complexity (and parameters). This can be evaluated by comparing the Akaike Information Criterion (AIC) for various models

2) The resulting model has non-significant predictors. Of the eleven predictors in this example, four did not meet the α=0.10 level specified for entry into the model through stepwise selection and would not be included if INCLUDE=EQUALSLOPES was not specified.

3) The inclusion of non-significant parameters into our model tends to ***increase*** standard errors for the parameter estimates, which ***decreases*** the predictive ability of the model.

$$\text{EmpLogit } y_i = \log\left(\frac{y_0 + y_1 + \ldots + y_i}{y_{i+1} + y_{i+2} + \ldots + y_J} \right\}$$

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A COMPARISON OF MODELS

Model Comparison may be conducted several ways:

- ***Log-Likelihood Test*** - when one model contains a subset of predictors from a more complex model, the difference in log-likelihood forms a Chi-Square test statistic that can be tested against a Chi-Square distribution with degrees of freedom equal to the difference in the number of parameters in the full model and reduced model. ***Lower*** values of -2*(log-likelihood) are indicative of more optimal models
- ***Akaike Information Criterion (AIC)*** – a method which uses the log-likelihood values from each model, but adds a "penalty" for including unnecessary predictors to the model that add little to the overall relationship. ***Lower*** AIC values are associated with more optimal models

Model	# Parameters Estimated	AIC	-2 LL (Log-Likelihood)
PropOdds	10 (7 + 3 intercept)	853	833
PPODDS≠	**15 (12 + 3 intercept)**	**780**	750
PPODDSwith=	20 (17 + 3 intercept)	785	**745**

SUMMARY

- The AIC value is lowest for the ***PPODDS≠*** model and substantially lower than for the PropOdds model. The model contains five additional parameter estimates, but their inclusion improved model fit.
- The ***PPODDSwith=*** model included five additional parameter estimates compared to the PPODDS≠ model, yet log-likelihood decreased very slightly and the AIC was higher. This suggests this model contains unnecessary parameters.
- ***Both models*** with the Partial Proportional Odds option represent an improvement over the PropOdds model.
- Since the Score Test for the Proportional Odds Assumption was highly significant, the PropOdds model is not particularly attractive. While SAS provides parameter estimates and fits a model, the estimated probabilities across response categories (response functions) are not proportional to the values of some of the predictors.

EVALUATION SET DATA AND METHODS (A SECONDARY METHOD OF MODEL COMPARISON)

- Of the 508 observations in the Adolescent Placement dataset, 408 observations were used for model-building, leaving 100 aside to serve as an unbiased sample that will be used to evaluate the prediction power of the models.
- **PROC SURVEYSELECT** with **SEED=535113001** and **Simple Random Sampling (SRS)** was used to create a Evaluation Set.

A COMPARISON OF MODELS

PROPORTIONAL ODDS MODEL (PropOdds)

Table of F_PLACE by I_PLACE

Frequency	*I_PLACE(PREDICTED PLACEMENT)*				
F_PLACE(TRUE PLACEMENT)	*0*	*1*	*2*	*3*	*Total*
0	21	6	3	1	31
1	8	3	5	1	17
2	3	3	16	9	31
3	4	2	4	11	21
Total	36	14	28	22	100

FINDINGS:

- The correct placement was predicted 51% of the time
- The placement levels were adjacent to one another (+- 1 level) an additional 35% of the time
- 14% of the placements differed by at least 2 levels (poor prediction)
- Simple Kappa statistic: 0.3302
- Weighted Kappa: 0.5668

- The ***weighted kappa*** value of ***0.5668*** for this model indicates it is of limited predictive value (values of 0.70 or higher are characteristic of highly predictive models while values between 0.60 and 0.69 are considered marginally acceptable and characteristic of models with some predictive value).
- The model correctly placed the adolescent just over half the time and was very poor at predicting a placement level of 1 (Day Treatment), only predicting that 3 of the 17 observations with a true placement level of 1 would be classified as such.

PARTIAL PROPORTIONAL ODDS MODEL (PPODDS≠)

Table of F_PLACE by I_PLACE

Frequency	*I_PLACE(PREDICTED PLACEMENT)*				
F_PLACE(TRUE PLACEMENT)	*0*	*1*	*2*	*3*	*Total*
0	23	5	3	0	31
1	6	7	4	0	17
2	3	2	23	3	31
3	4	3	3	11	21
Total	36	17	33	14	100

FINDINGS:

- The correct placement was predicted 64% of the time
- The placement levels were adjacent to one another (+- 1 level) an additional 23% of the time
- 13% of the placements differed by at least 2 levels (poor prediction)
- Kappa statistic: 0.5054
- Weighted Kappa: 0.6200

- ***Weighted kappa*** value of ***0.6200*** for this model indicates it has some predictive value.
- It correctly predicts placement level in nearly 2 out of 3 observations (13 more adolescents out of 100 total are now correctly placed using the model compared to the PropOdds model).
- Prediction remains somewhat poor at Placement Level 1 . These could be adolescents whose behavioral characteristics are not well-captured by the available predictors in the modeled relationship.

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A COMPARISON OF MODELS

PARTIAL PROPORTIONAL ODDS MODEL (PPODDSwith=)

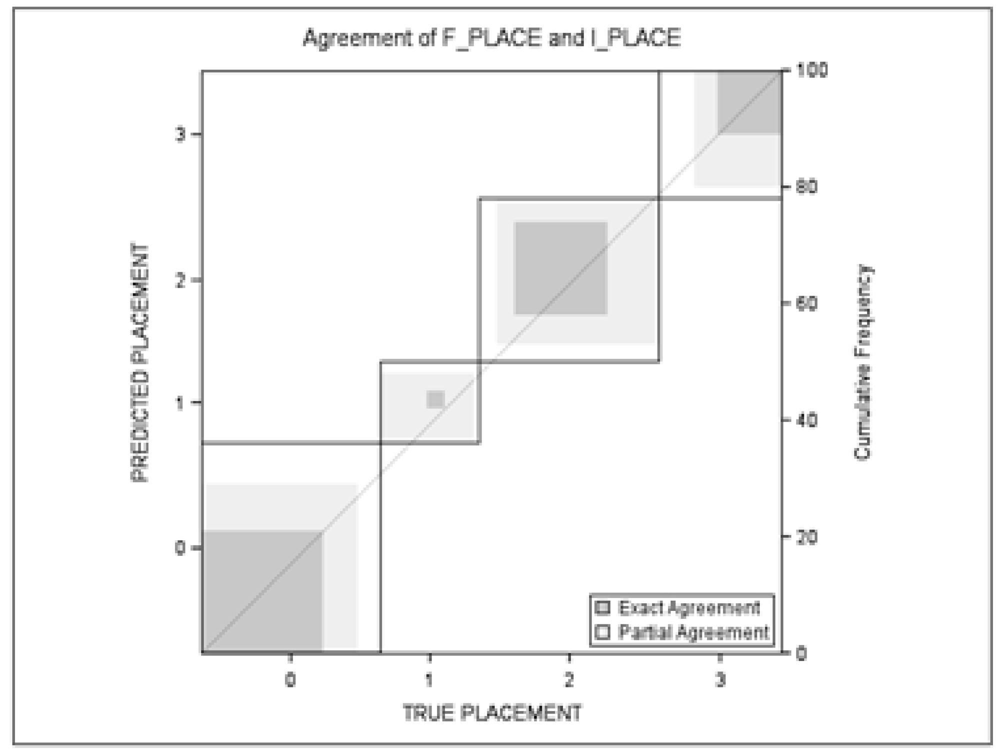
Table of F_PLACE by I_PLACE

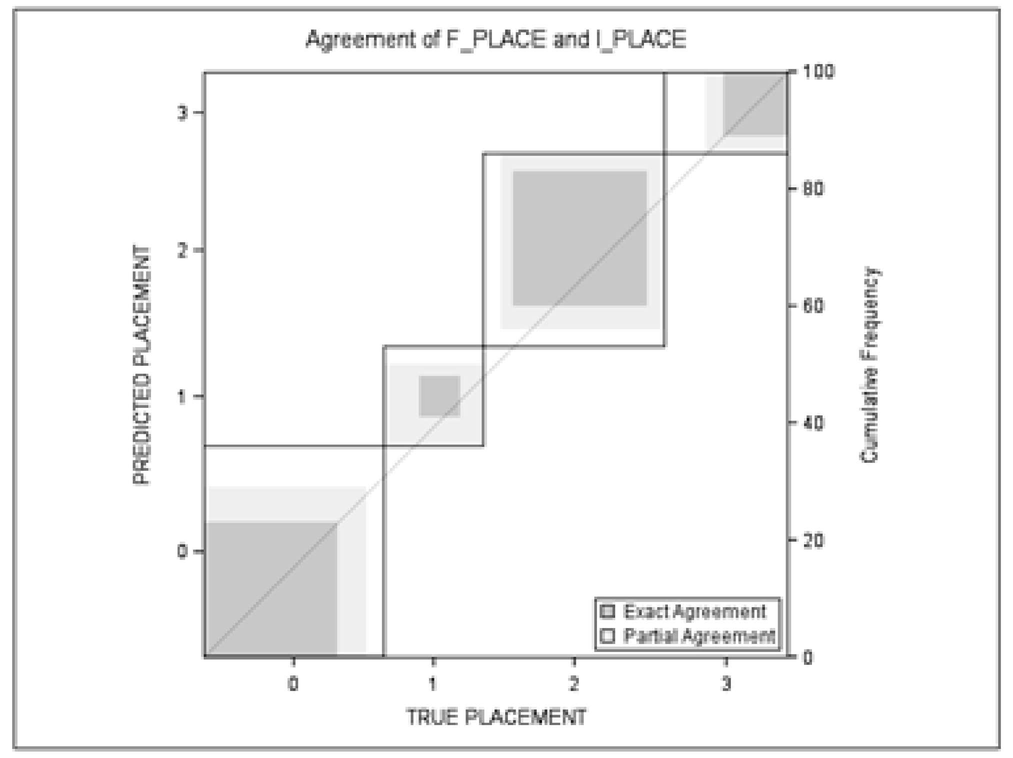
Frequency	I_PLACE(PREDICTED PLACEMENT)				
F_PLACE(TRUE PLACEMENT)	0	1	2	3	Total
0	24	3	3	1	31
1	7	6	3	1	17
2	4	1	21	5	31
3	4	3	4	10	21
Total	39	13	31	17	100

FINDINGS:

- The correct placement was predicted 61% of the time
- The placement levels were adjacent to one another (+- 1 level) an additional 23% of the time
- 16% of the placements differed by at least 2 levels (poor prediction)
- Kappa statistic: 0.4622
- Weighted Kappa: 0.5728

- ***Weighted Kappa*** value of ***0.5728*** is lower than for the PPODDS≠ model.
- The rate of poor predictions (where predicted placement differed by 2+ ordered levels) was actually the highest among the three models.
- The decrease in predictive value of this model could be due to its inclusion of additional non-significant predictors. If several of these predictors are related to one another, the model could suffer from multicollinearity, which has the effect of increasing the standard errors of the estimates.





- The above visuals compare the true and predicted placement levels for the **PropOdds** and the **PPODDS≠** models. The darker grey within each box indicates exact agreement while the lighter shade of grey indicates partial or adjacent agreement.
- At all score levels, the **PPODDS≠** model has a higher rate of exact agreement.

CONCLUSION

- For relationships beyond those that are simple with few predictors and observations, the Proportional Odds assumption is **<u>frequently</u>** violated when employing a cumulative logit link.
- The Partial Proportional Odds Model offers users an effective solution that addresses the assumption and offers an improvement in model fit.
- Improvement can be measured in terms of statistics like the log-likelihood or AIC value, or through classification agreement.
- Classification agreement is of crucial importance if a model is ever used to make or support high-stakes decisions. For example, if these are decisions in which the cost of mis-classification is substantial, a sizable cost savings could be realized by using a partial proportional odds model.
- SAS offers a model building approach ***unique to the industry*** (unavailable in SPSS, R, or Stata). It is flexible and gives the user the ability to specify a complex model with a minimal amount of code or user input.

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CONTACT INFORMATION

Your comments and questions are valued and encouraged. Contact the author at:

Paul J. Hilliard
Educational Testing Service
660 Rosedale Road (MS 20-T)
Princeton, NJ 08541
(609) 759-5748
philliard@ets.org

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