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A SAS® Macro for Generating Random Numbers of Skew Normal and Skew *t* Distributions

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ABSTRACT

This paper aims to show a SAS® macro for generating random numbers of skew normal and skew *t* distributions as well as the quantiles of these distributions. The results are similar to those generated by 'sn' package of R software.

INTRODUCTION)

The pdf of skew normal is given by (Azzalini, 1985):

$$f(x) = 2\emptyset(x)\Phi(\lambda x) \quad (1)$$

where $\emptyset(x)$ denotes the standard normal probability density function given by

$$\emptyset(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad (2)$$

and $\Phi(\,)$ is the normal cumulative distribution function given by

$$\Phi(x) \int_{-\infty}^{x} \emptyset(t)dt \quad (3)$$

This distribution was first introduced by O'Hagan and Leonard (1976).

To add location and scale parameters to (1), just let $x = \frac{x-\mu}{\sigma}$ and (1) becomes

$$f(x) = \frac{2}{\sigma} \emptyset\left(\frac{x-\mu}{\sigma}\right) \Phi\left(\lambda \frac{x-\mu}{\sigma}\right) \quad (4)$$

Let $\delta = \frac{\lambda}{\sqrt{1+\lambda^2}}$, then the mean and variance of (4) is given by:

$$\mu_{SN} = \mu + \sigma\delta\sqrt{\frac{2}{\pi}} \quad (5)$$

$$\sigma_{SN}^2 = \sigma^2\left(1 - \frac{2\delta^2}{\pi}\right) \quad (6)$$

Azzalini (2015) shows a simple way to generate random number of a skew normal distribution, as follows:

1. Sample u_0, u_1 having marginal distribution $N(0,1)$ and correlation δ. A simple way to achieve this is to generate u_0, v as independent $N(0,1)$ variates and define $u_1 = \delta u_0 + \sqrt{1-\delta^2}v$.
2. Then

$$z = \begin{cases} u_1, \ if\ u_0 > 0 \\ -u_1, otherwise \end{cases} \quad (7)$$

is a random number sampled from the SN distribution with shape parameter $\lambda = \delta/\sqrt{1-\delta^2}$.

3.To change the location and scale from (0,1) to (a,b) with $b > 0$, say, set $y = a + bz$.

The pdf of skew *t* is given by (Azzalini, 1985):

$$f(x) = 2t_v(x)T_{v+1}\left(\lambda x\sqrt{\frac{1+v}{v+x^2}}\right) \quad (8)$$

where $t_v(x)$is the standard *t* density probability function with v degrees of freedom and $T_{v+1}()$ is the cumulative distribution function of a *t* distribution with $v + 1$ degrees of freedom.

To obtain a ST variate, generate $V \sim \chi_\vartheta^2$ and put $w = \frac{z}{\sqrt{V/\vartheta}}$; then w has ST distribution with ϑ degrees of freedom and shape parameter equal to the one of z described in (7).

RESULTS

The SAS® macros for generating random numbers of SN and ST are called %SN and %ST, respectively, and the parameters of the macros are:

```
%SN(n=,seed1=123,seed2=321,shape=0,mean=0,var=1,out=SN);
%ST(n=,seed1=123,seed2=321,shape=0,df=200,mean=0,var=1, out=ST);
```

n = amount of random numbers to be generated;
seed1 = seed to be used in the first random normal distribution;
seed2 = seed to be used in the second random normal distribution;
shape = asymmetry parameter λ;
df = degree of freedom for *t* distribution;
mean = desired average of the distribution;
var = desired variance of the distribution;
out = output for the random numbers.

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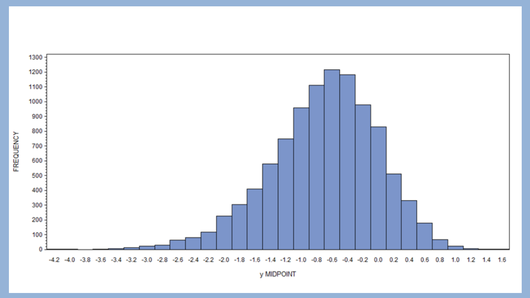
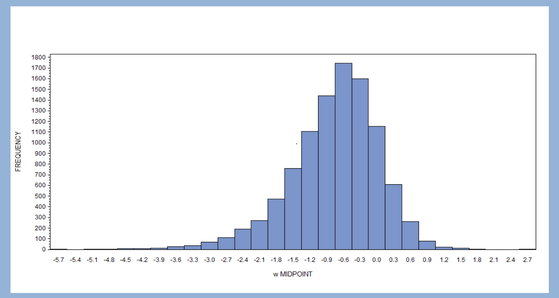
RESULTS

Shape	%qSN Macro		‘sn’ Package	
	0.975	0.025	0.975	0.025
0	1.959964	-1.959964	1.959964	-1.959964
2	2.241402	-0.503389	2.241402	-0.503389
-2	0.503389	-2.241402	0.503389	-2.241402

Table 1. Results of %qSN macro and ‘sn’ Package for quantiles 0.975 and 0.025



Distribution of Skew Normal with SHAPE $= -2$



Distribution of Skew t with SHAPE $= -2$ and DF = **10**

df	Shape	%qST Macro		‘sn’ Package	
		0.975	0.025	0.975	0.025
200	0	1.9718963	-1.971896	1.971896	-1.971896
	2	2.2584026	-0.505154	2.258403	-0.505153
10	0	2.2281389	-2.228139	2.228139	-2.228139
	-2	0.5412621	-2.633543	0.541262	-2.633543

Table 2. Results of %qSN macro and ‘sn’ Package for quantiles 0.975 and 0.025

df	Shape	%qST Macro		%ST Macro	
		0.975	0.025	0.975	0.025
200	0	1.9718963	-1.971896	1.961276	-1.8744130
	-2	0.5051542	-2.258402	0.530769	-2.5815742
10	0	2.2281389	-2.228139	1.338404	-2.256904
	-2	0.5412621	-2.633543	0.446271	-2.467777

Table 4. Quantiles 0.975 and 0.025 Generated by %qST and %ST macros

CONCLUSIONS

This paper showed SAS® macros for generating random numbers and quantiles of skew normal and skew *t* distributions. The results were close that those generated by ‘sn’ package of R software.

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Thank you!!

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