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The Comparative Analysis of Predictive Models for Credit Limit Utilization Rate with SAS/STAT®

Denys Osipenko, the University of Edinburgh;

Professor Jonathan Crook, the University of Edinburgh

ABSTRACT

Credit card usage modelling is a relatively innovative task of client predictive analytics compared to risk modelling such as credit scoring. The credit limit utilization rate is a problem with limited outcome values and highly dependent on customer behavior. Proportion prediction techniques are widely used for Loss Given Default estimation in credit risk modelling (Belotti and Crook, 2009; Arsova et al, 2011; Van Berkel and Siddiqi, 2012; Yao et al, 2014). This paper investigates some regression models for utilization rate with outcome limits applied and provides a comparative analysis of the predictive accuracy of the methods. Regression models are performed in SAS/STAT® using PROC REG, PROC LOGISTIC, PROC NLMIXED, PROC GLIMMIX, and SAS® macros for model evaluation. The conclusion recommends credit limit utilization rate prediction techniques obtained from the empirical analysis.

INTRODUCTION

A credit card as a banking product has a dual function both as a convenient loan and a payment tool. This fact makes the task of the profitability prediction for this product more complex than for standard loans. Moreover, a credit card has a fluctuating balance, and its accurate forecast is an urgent problem for credit risk management, liquidity risk, business strategies, customer segmentation and other aspects of bank management. The use of traditional techniques gives acceptable empirical results, but a majority of the industrial models are simplified and make a lot of assumptions. Whilst risk modelling techniques like credit scoring give reliable predictive results and are accepted by industry, the credit business parameters modelling like credit card usage and profitability are less accurate in practice. In particular, this is caused by more complex customer behaviour types – 'wishing to use and how to use' - in comparison with customer risk model – 'must to pay'. Credit products, especially credit cards are sensitive both to macroeconomic trends, cycles and fluctuating factors – systematic components, and to individual behavioural patterns of the customer like desire to spend and personal financial literacy.

The initial research into customer behaviour arose from works dedicated to the economic organization of households (for instance, Awh et al, 1974; Bryant W. K., 1990) and has become the basis for further investigations in the credit products usage and risk modelling literature. The first set of the investigations in the area of credit cards usage paid attention to the consumer credit demand (White, 1976; Dunkelberg and Stafford, 1971; Duca, Whitesell, 1995) and to the probability of credit card use (White, 1975; Dunkelberg, Smiley, 1975; Heck, 1987). The splitting by customer usage can also be applied to improve the predictive accuracy of the scoring model (Banasik J.et al, 2001).

There is a lack of fundamental works dedicated to the lines of credit and credit cards utilization rate prediction in academic literature. Kim and DeVaney (2001) applied the Ordinary Least Squares (OLS) method for the outstanding balance prediction with the Heckman procedure. It has been found that the sets of characteristics related to the probability of having an outstanding balance and to the amount of outstanding balance are different. Agarwal et al (2006) made a point that there is dependence between credit risk and credit line utilization. So limit changes in our investigation show correlation with risk level and previous customer behaviour.

The proportion prediction techniques are widely used for Loss Given Default estimation in credit risk modelling (Belotti and Crook, 2009; Arsova et al, 2011; Van Berkel and Siddiqi, 2012; Yao et al, 2014).

This paper is dedicated to the cross-sectional analysis at the account level with use of a number of approaches for proportion prediction. We perform a comparative analysis of a set of methods for the utilization rate (usage proportion) prediction: i) five direct estimation techniques such as ordinary linear regression, beta regression, beta transformation plus general linear models (GLM), fractional regression (quasi-likelihood estimation), and weighted logistic regression for binary transformed data, ii) two-stage models with the same direct estimation methods and the logistic regression at the first stage for the probability of full use estimation.

DATA SAMPLE

The data set for the current research contains information about credit card portfolio dynamics on account level and cardholders applications. The data sample is uploaded from the data warehouse one of the European commercial banks. The account level customer data sample consist of three parts: i)application form data such as customer socio-demographic, financial, registration parameters , ii) credit product characteristics such as credit limit and interest rate time-dependent, and iii) behavioural characteristics on the monthly basis such as the outstanding balance, days past due, arrears amount, number and types of transactions, purchase and payment turnovers. The macroeconomic data is collected from open source and contain the main macroindicators such as GDP, CPI, unemployment rate, and foreign to local currency exchange rate. The data sample is available for 3 year period. The total number of accounts available for the analysis for whole lending period is 153 400, but not all accounts have been included into the data sample.

Month numeration is calculated in backward order. For example, Month 1 – current month, observation and calculation point in time, Month 2 – previous month (or -1 month).

Month name	Jan	Feb	Mar	Apr	May	Jun
Month Num	6	5	4	3	2	1

June is current month, month of characteristics calculation and prediction. Thus, AvgBalance (1-6) is average balance for Jan-Jun, AvgBalance (1-3) is average balance for Apr-Jun. The characteristics are presented in the Table 1. List of characteristics. The dictionary is not full.

Characteristic	Description
Behavioural characteristics (transactional) – Time Random	
b_AvgBeop13_to_AvgBeop46	Average Balance EOP in the last 3 month to Average Balance in months 4-6
b_maxdpd16	Maximum days past due in the last 6 months
b_Tr_Sum_deb_to_Crd_16	Sum of Debit transactions amounts to Credit transactions amounts for months 1-6
b_Tr_Sum_deb_to_Crd_13	Sum of Debit transactions amounts to Credit transactions amounts for months 1-3
b_Tr_Avg_deb_to_Crd_16	Average Debit transactions amounts to Average Credit transactions amounts for months 1-6
b_Tr_Avg_deb_to_Crd_13	Average Debit transactions amounts to Average Credit transactions amounts for months 1-3
b_TR_AvgNum_deb_16	Average monthly number of debit transactions for months 1-6
b_TR_AvgNum_Crd_16	Average monthly number of credit transactions for months 1-6
b_TR_MaxNum_deb_16	Maximum monthly number of debit transactions for months 1-6
b_TR_MaxNum_Crd_16	Maximum monthly number of credit transactions for months 1-6
b_TR_max_deb_to_Limit16	Amount of maximum debit transaction to limit for months 1-6
b_TR_sum_crd_to_OB13	Sum of credit transaction to average outstanding balance for month 1-3
b_TRsum_deb16_to_TRcrd16	Sum of debit ti sum of credit transactions for month 1-6
b_NoAction_NumM_16	Number of month with no actions for months 1-6
b_NoAction_NumM_13	Number of month with no actions for months 1-3
b_pos_flag_0	POS transaction indicator for current month
b_pos_flag_13	POS transaction indicator for the previous 3 month

Characteristic	Description
b_atm_flag_0	ATM transaction indicator for current month
b_atm_flag_13	ATM transaction indicator for the previous 3 month
b_pos_flag_used46vs13	POS transaction in month 4-6 but no transaction in month 1-3
b_pos_flag_use13vs46	POS transaction in month 1-3 but no transaction in month 4-6
b_atm_flag_used46vs13	ATM transaction in month 4-6 but no transaction in month 1-3
b_atm_flag_use13vs46	ATM transaction in month 1-3 but no transaction in month 4-6
No_dpd	Flag if account was in delinquency
Application characteristics – Time fixed	
Age	As of the date of application
Gender	Assumption that status constant in time
Education	Assumption that status constant in time
Marital status	Assumption that status constant in time
Position	The position occupied by an applicant
Income	As of the date of application
Macroeconomic characteristics – Time random	
Unemployment Rate ln lag3	Log of unemployment rate with 3 month lag
GDPCum_ln yoy	Log of cumulative GDP year to year to the same month
UAH-EURRate_ln yoy	Log of exchange rate of local currency to Euro in compare with the same period of the previous year
CPIYear_ln yoy	Log of the ratio of the current Consumer Price Index to the previous year the same period CPI

Table 1. List of characteristics

MODEL BUILDING

The usage of credit limit may be changed during a lifetime period. The utilization rate (Ut) is defined as the outstanding balance (OB) divided by credit limit (L) $Ut = OB/L$.

Under the assumption that the credit limit is fixed the utilization rate dynamics is completely dependent on the outstanding balance. In current investigation we are concerned with to the utilization rate that is the percentage of the balance relative to a customer credit limit. In our opinion the utilization rate approach is able to give more adequate customer behaviour estimation than direct outstanding balance prediction in sense of consumption habits and customer demand on money, and also is corresponding with bank credit policy.

The credit limit depends on credit policy rules and is defined particularly according to the customer risk profile. The same behavioural customer segments have various outstanding balances correlated particularly with the credit limit. Thus customer segment does not have a typical outstanding balance, but a typical utilization rate as proportion of the credit limit.

The prediction of utilization rate instead of the outstanding balance amount is used to avoid the possible disproportion in behaviour modelling caused by i) different start terms such as bank's credit policy and product parameters changes which may affect on the initial credit limit for the same category of customers, ii) hypothesis that credit card customer behaviour is affected mainly by the available balance as part of the credit limit and then by the amount of the available balance.

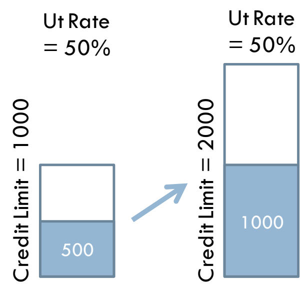


Figure 1. Utilization rate and limit changes

Because of the inconsistencies in the behavioural characteristics calculation (lack of history) and the differences in the utilization rate dynamic at the early and late credit history stages we allocated a separate model for the low MOB period. In our case two periods have been chosen: MOB from 1 to 5 and MOB more than 6. The utilization rate has rapid growth during first 5 month on book showing in Fig.3, but after a peak at 6-8 month on book it stabilized and had slight monotonic decrease.

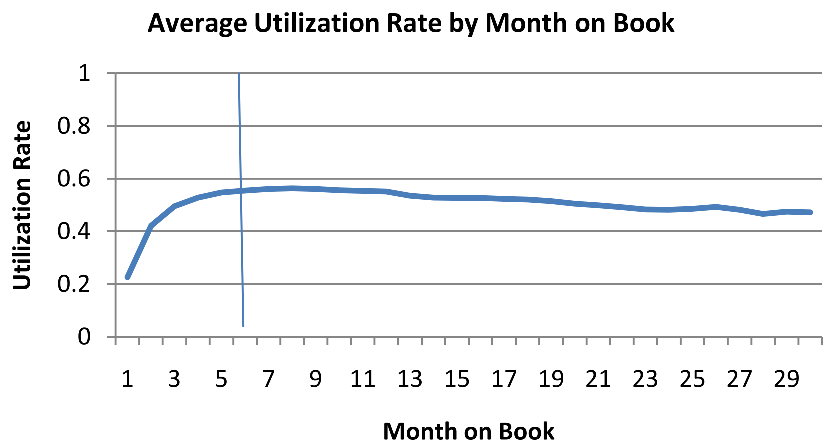


Figure 2. Average utilization rate depending on Month on Book

As it can been seen from the Figure 3. The utilization rate distribution slightly less than 20% of the observations have the utilization rate from 0% to 5% and approximately 20% of observations have the utilization rate from 95% to 100%. Other cases are distributed almost uniformly from 5% to 80%, and the slight increase of the observations can be observed after 85% rate value. However, in 5% cases, the utilization rate exceeds one hundred percent indicates that the use of loan funds is higher than the set credit limit. This can be explained by i) credit overlimit of the outstanding balance; ii) technical accrual of the interest rates, fees and commissions on the principal account. For the further analysis purposes this cases are replaced by 100% utilization to avoid misinterpretation of the utilization rate and according to the business logic. However, the set of cases can be allocated as a separate category to investigate the reasons and drivers of the credit overlimit.

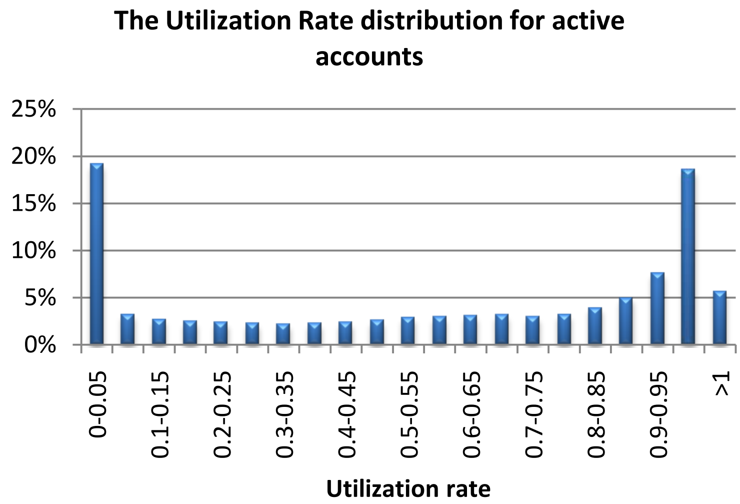


Figure 3. The utilization rate distribution

Figure 4. The Utilization Rate, balance and limit distribution by Position type (example) shows the examples of dependence between utilization rate and client characteristics. Top managers have the highest limits, but the lowest utilization rate in compare with other positions. This means that the outstanding balance is not as different as the credit limit.

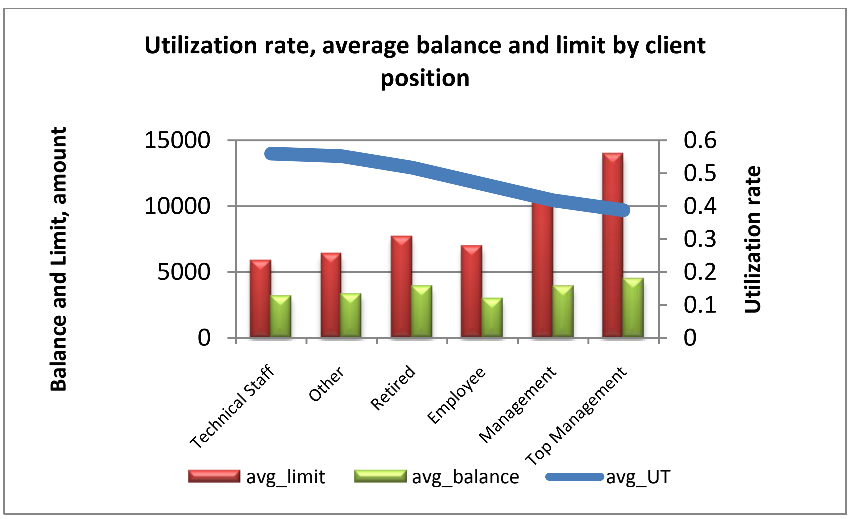


Figure 4. The Utilization Rate, balance and limit distribution by Position type (example)

METHODOLOGY AND MODELS

We apply five proportions prediction methods for the utilization rate both for one-stage and two-stage models. They are the following:

i) linear regression (OLS)

ii) fractional regression (quasi-likelihood)

iii) beta-regression (non-linear)

iv) beta-transformation + OLS/GLM;

v) weighted logistic regression with data binary transformation.

This section describes methods overview and used SAS code. The comparative analysis of results is presented in the section 'Modelling results'.

LINEAR MODEL BASED APPROACH

The linear regression is tested for proportion modelling (Belotti and Crook, 2012; Arsova et al, 2011)

$$y_i = \beta_0 + \boldsymbol{\beta}^T \mathbf{x}_i + \varepsilon_i$$
$$\varepsilon_i \sim N(0, \sigma^2)$$

We assume the utilization rate depends on behavioural, application, macroeconomic characteristics, and also on the previous periods utilization rate with time lag.

$$UT_{it+T} = \phi_n UT_{i(t-l)} + \sum_{k=1}^{K} \beta_b \cdot B_{bit} + \sum_{l=1}^{L} \alpha_a \cdot A_{ai} + \sum_{m}^{M} \gamma_1 M_{mt}$$

φ, α, β, γ are regression coefficients (slopes)

B_{bit} is behavioural factor b for case i in period t, for example, average balance to maximum balance, maximum debit turnover to average outstanding balance, maximum number days in delinquency – time varying;

A_{ai} is application factor a for case i, for example, client's demographic, financial and product characteristics such as age, education, position, income, property, interest rate – time constant;

M_{mt} is macroeconomic factor m in period t such as GDP, FX, Unemployment rate changes – time varying;

$UT_{i\,t+T}$ is the utilization rate for case i in period t;

l is time lag between current point in time and characteristics calculation slice;

T is the period of prediction in months.

However, time parameter t for this task is used to identify the point in time for lag an each behavioural characteristics and in case the cross sectional analysis the behavioural characteristics and macroeconomic variables are not time varying.

One of the weaknesses of the linear regression application for the utilization rate modelling is the unlimited range of the function outcome. It can be fixed with the conditional function as the following

$$f(x) = \begin{cases} 0, & f(x) < 0 \\ f(x), & f(x) \in [0;1] \\ 1, & f(x) > 1 \end{cases}$$

Linear regression approach can be a reason of high concentration of the rate values on bounds 0 and 1. Moreover, such shape of the distribution function is not continuous and has broken points of inflection what is often non-corresponding with real economic processes. However, this approach is easy to use and it can be applied for quick preliminary analysis of correlations and general trends.

Hereinafter SAS code describe step-by-step all stages of the modelling with the same data source, target and predictors.

```
/* Predictors variable setup */
%let RegAMB_nl =
mob
Limit
UT0
avg_balance
b_AvgOB16_to_MaxOB16_ln
...
CPI_lnqoq_6
SalaryYear_lnyoy_6
;

/* Ordinary Least Squares */

proc reg data=DevSample outest = Est_OLS;
model Target_UT_plus6 = &RegAMB_nl / tol vif collin corrb;
output p=UT_plus6_OLS out=DevSample_pred;
run;
```

BETA-REGRESSION APPROACH

One of the ways to set bounds for target variable is to apply a transformation of the empirical distribution to the theoretical with appropriate limits. A beta distribution can be applied to match the distribution shape with borders, in our case it is 0 and 1. The approach proposed by Ferrari and Cribari - Neto (2004) for LGD modelling is applied for the utilization rate prediction.

The Beta-distribution can be bounded between two values and parameterized by two positive parameters, which define the shape of the distribution. Generally the empirical probability density function of the utilization rate distribution is U-shape. The parameters $\alpha$and β should be set up to match the density function shape to minimize the residuals between the empirical and theoretical distribution.

The beta distribution probability density function is

$$f_X(x) = \frac{1}{\mathrm{B}(\alpha,\beta)}\, x^{\alpha-1}(1-x)^{\beta-1}$$

where α, $\beta > 0$.

The parameters α and β are estimated to match the theoretical distribution close to empirical one. Beta distribution function can be represented via Gamma function as

$$Beta(y,\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}$$

Because the outcome is in the range between 0 and 1 the logistic transformation is used to find the dependences between predictors x(a) and regressor

$$\mu(a) = L(\mathbf{x}(a)'\beta) = \frac{e^{\mathbf{x}(a)'\beta}}{1+e^{\mathbf{x}(a)'\beta}}.$$

Log-likelihood function is the following:

$$l_t(\mu_t,\phi_t) = \log\Gamma(\phi) - \log\Gamma(\mu_t\phi) - \log\Gamma((1-\mu_t)\phi) + (\mu_t\phi - 1)\log y_t + \{(1-\varpi_t)\phi - 1\}\log(1-y_t)$$

```
/* Beta regression */
/* Predictors equation for PROC NLMIXED */
%let eta_nl_beh =
b0+
mob_e*mob+
Limit_e*Limit+
UT0_e*UT0+
...
CPI_lnqoq_e*CPI_lnqoq+
SalaryYear_lnyoy_e*SalaryYear_lnyoy;

%let prm_nl_beh =
b0 = 0
mob_e=0
Limit_e=0
UT0_e=0
...;

proc nlmixed data = DevSample;
parms &prm_nl_beh;
 mu = exp(&eta_nl_beh)/(1 + exp(&eta_nl_beh));
 phi = exp(d0);
 w = mu*phi;
 t = phi - mu*phi;
 ll = lgamma(w+t) - lgamma(w) - lgamma(t) + ((w-1)*log(Target_UT_plus6_p)) +
((t-1)*log(1 - Target_UT_plus6_p));
 model Target_UT_plus6_p ~ general(ll);
 predict mu out=betareg_result_nl_beh_mu (keep=Target_UT_plus6_p pred);
 predict phi out=betareg_result_nl_beh_phi (keep=Target_UT_plus6_p pred);
run;
```

The inverse beta-transformation with cumulative distribution function is applied to find real rate value according to the estimated one. After this transformation the logistic regression is applied to estimate dependence between beta-function and predictors, and then inverse transformation from cumulative probability function is applied to get the utilization rate.

BETA-TRANSFORMATION PLUS OLS

The algorithm uses the beta distribution to transform the original target. First stage is to find the beta-distribution coefficients (alpha and beta) to fit the development sample distribution using the non-linear regression procedures. Secondly, replace real target variable by the ideal beta-distributed. Thirdly, find appropriate normal distributed value. Then, run OLS or Generalized Linear Mixed Model to find regression coefficients. After this stage it is need to perform the inverse transformation for normal distribution and then inverse for the beta regression. To obtain a prediction it is necessary to transform linear regression results with normal and then beta regression with the constant alpha and beta coefficient found at the first stage.

```
/* Beta Transformation PLUS OLS */

proc sql;
      create table test_MM_Est as
      select *,
            samp_mean*(samp_mean*(1-samp_mean)/samp_var-1)as alpha_moments,
            (1-samp_mean)*(samp_mean*(1-samp_mean)/samp_var-1) as
beta_moments
```

```
      from
      (select     mean(Target_UT_plus6_p) as samp_mean,
                  var(Target_UT_plus6_p) as samp_var
      from DevSample);
run;

proc nlmixed data=&r.Tr_plus6m_nl_beh;
      /*using MM estimates as starting points*/
      parms /bydata data=test_MM_Est(drop=samp_mean samp_var
                        rename=(alpha_moments=alpha_mle
beta_moments=beta_mle));
      ll=log(gamma(alpha_mle+beta_mle))-log(gamma(alpha_mle))-
log(gamma(beta_mle))
            +(alpha_mle-1)*log(Target_UT_plus6_p)+(beta_mle-1)*log(1-
Target_UT_plus6_p);
      model Target_UT_plus6_p~general(ll);
run;

/* Put alpha and beta for Beta distribution from PROC NLMIXED results */
%let alpha = 0.2909;
%let beta = 0.3049;

/* Transform to Beta distribution */

data DevSample;
set DevSample;
Target_UT_plus6_p_beta = cdf("BETA", Target_UT_plus6_p, &alpha, &beta);
run;

/* Find quantile for Normal distribution for Beta distribution value */

data DevSample;
set DevSample;
Target_UT_plus6_p_beta_prob = quantile("NORMAL", Target_UT_plus6_p_beta);
run;

/* Run GLIMMIX or REG procedures */

proc glimmix data=DevSample;
      model Target_UT_plus6_p_beta_prob = &RegAMB_nl;
      output out = DevSample pred=Target_UT_plus6_p_beta_prob_pr;
run;

/*or*/

proc reg data=DevSample;
      model Target_UT_plus6_p_beta_prob = &RegAMB_nl;
      output out = DevSample pred=Target_UT_plus6_p_beta_ols;
run;

/* Inverse transformation from Normal to Beta distribution for results*/

data DevSample;
set DevSample;
Target_UT_plus6_p_beta_prob_res =
quantile("BETA",cdf("NORMAL",Target_UT_plus6_p_beta_prob_pr),&alpha,&beta);
run;
```

Beta transformation approaches widely used for LGD modelling, but empirical researches show not high predictive power results (Arsova et al., 2011; Loterman et al., 2012; Bellotti and Crook, 2009).

UTILIZATION RATE MODELLING WITH FRACTIONAL LOGIT TRANSFORMATION (QUASI-LIKELIHOOD)

The utilization rate is bounded between 0 and 1 and required appropriate methods to keep the predicted value in this range. One of the techniques is fractional logit regression proposed by Papke & Wooldridge (1996). The Bernoulli log-likelihood function is given by

$$l_i(\mathbf{b}) \equiv y_i \log[G(\mathbf{x}_i\mathbf{b})] + (1 - y_i)\log[1 - G(\mathbf{x}_i\mathbf{b})]$$

The quasi-likelihood estimator of β is obtained from the maximization of

$$\max_{\mathbf{b}} \sum_{i=1}^{N} l_i(\mathbf{b})$$

Crook and Bellotti (2009) apply the Fractional logit transformation for the Loss Given Default parameter modelling:

$$T_{RR} = \log(RR) - \log(1 - RR)$$

where RR – is recovery rate.

The LGD parameter has the same features as the Utilizaiton rate such as U-shape and bimodal distribution. Thus the similar techniques can be applied for the modelling and parameters estimations.

The utilization rate UT is transformed to UT_{TR} for regression estimation

$$UT_{TR} = \ln(UT) - \ln(1 - UT).$$

The inverse transformation of the predicted value is the following:

$$UT = \frac{\exp(UT_{TR})}{1 + \exp(UT_{TR})}$$

The quasi-likelihood methods used to estimate the parameters in the model.

The SAS procedure GLIMMIX is used for the regression coefficients estimation. Procedure GLIMMIX performs estimation and statistical inference for generalized linear mixed models (GLMMs). A generalized linear mixed model is a statistical model that extends the class of generalized linear models (GLMs) by incorporating normally distributed random effects.

```
/*Fractional regression with Quasi-likelihood methods */
/* Preparation for fractional - quasi-likelihood regression */

if Target_UT_plus6 = 0 then Target_UT_plus6_p = 0.0001;
else if Target_UT_plus6 = 1 then Target_UT_plus6_p = 0.9999;
else Target_UT_plus6_p = Target_UT_plus6;
Target_UT_plus6_log=log(Target_UT_plus6_p)-log(1-Target_UT_plus6_p);

proc glimmix data=DevSample;
_variance_ = _mu_**2 * (1-_mu_)**2;
      model Target_UT_plus6_p = &RegAMB_nl / link=logit;
      output out=DevSample pred(ilink)=ut0_quasi_pred;
run;

Alternatively can be used PROC NLMIXED for fractional response from Yao(2014)
as following:

Program 2. Fractional response regression
proc nlmixed data=MyData tech=newrap maxiter=3000 maxfunc=3000 qtol=0.0001;
 parms b0-b14=0.0001;
 cov_mu=b0+b1*Var1+b2*Var2+…+b14*Var14;
 mu=logistic(cov_mu);
 loglikefun=RR*log(mu)+(1-RR)*log(1-mu);
 model RR~general(loglikefun);
 predict mu out=frac_resp_output (keep=instrument_id RR pred);
run;
```

WEIGHTED LOGISTIC REGRESSION WITH BINARY TRANSFORMATION APPROACH

The relatively innovative approach is the use of the weighted logistic regression with binary transformation of the data sample. The logit function is bounded between 0 and 1 and traditionally applied for the probability prediction. To apply logistic regression which used binary distribution the target proportion variable need to be transformed from continuous to binary form. The utilization rate can be considered as the probability to use the credit limit by 100%. For example, the utilization rate 75% can be presented as 75% probability to use full credit limit and 25% probability not to use the credit limit. This approach is used by Siddiqi (2012) for the Loss Given Default prediction. Each observation is presented as two observations (or two rows) with the same set of predictors according to the good/bad or 0/1 definition used in logistic regression. The outcome with target 1 corresponds to the rate r, which determines the weight equal to this rate r. The outcome with target 0 corresponds to the rate 1-r, which determines the weight equal to 1-r. The logistic regression probability of event is the utilization rate estimation.

The data sample is transformed according to the following scheme:

Utilization Rate	Binary recovery – target	Weight
1	1	1
0	0	1
R, 0<r<1	1	r
	0	1-r

The set of methods is used for the rates modelling as LGD account level prediction. Stoyanov (2009) investigated, in particular, the following approaches to LGD account level modelling as binary transformation of the LGD using uniform random numbers, binary transformation of the LGD using manual cut-off. Arsova et al (2011) applied both direct approaches to the LGD modelling such as OLS regression, beta regression and fractional regression and indirect approaches such as logistic regression with binary transformation of LGD by random number, logistic regression with binary transformation of LGD by weights, and also multi-stage models like Ordinal Logistic Regression with nested Linear Regression.

This study uses weighted logistic regression with binary transformed sample for rate estimations. In common logistic regression matches the log of the probability odds by a linear combination of the characteristic variables as

$$\operatorname{logit}(p_i) = \ln\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \boldsymbol{\beta} \cdot \mathbf{x}_i^{\,T},$$

where

pi is the probability of particular outcome, β0 and **β** are regression coefficients, **x** are predictors.

$P_i = E(Y_i \mid \mathbf{x}_i, \beta) = \Pr(Y_i = 1 \mid \mathbf{x}_i, \beta)$ is the probability of event for *i*th observation.

In general this approach can be interpreted as the following: the utilization equal to 10% is the same as from accounts 10 accounts have utilization equal 100% and 90 accounts have utilization equal 0%. Thus the weighted logistic regression with proportions transformed to weights can be applied in SAS/STAT.

```
/*Weighted Logistic Regression with data sample binary transformation */
/* --- Create binarized sample --- */
/* Development Sample Binary Tranformation*/

data DevSample_wb;
set DevSample;
good=0;
bad=1;
ut_weight=Target_UT_plus6; run;

data DevSample_wb_ne;
set DevSample;
good=1;
bad=0;
ut_weight=1-Target_UT_plus6; run;

data DevSample_wb_reg;
set DevSample_wb DevSample_wb_ne; run;

/* --- Run Model --- */

proc logistic data=DevSample_wb_reg outest = r_Ut_wb_est;
model good = &RegAMB_nl /
selection = stepwise
slentry = 0.05
slstay = 0.05;
output out=r_ut_wb_out predicted=pr_ut;
weight ut_weight; run;

/* Extract real sample for analysis - drop doubles after binary
transformation step */

data wlog_nl_dev (keep = Target_UT_plus6_p pr_ut);
set r_ut_wb_out;
if good=0; run;
```

The predicted outcome from the logistic regression is used as the utilization rate estimation. For validation purposes we need to come back to original data set by dropping the doubled rows (account). For example, if we define the full utilization as 'bad = 1' and 'good=0' we need to keep this cases in sample and drop 'good=1' cases where the outcome estimation is equal to the 1-utilization rate.

MODELLING RESULTS

ONE-STAGE MODELLING APPROACHES SUMMARY

We applied five methods for direct proportion prediction for three types of models: Less than 5 MOB, MOB 6+ with No changes in Limit, and MOB 6+ with Limit Changes.

The factors chosen for the model validation are R square, mean absolute error (MAE), root-mean-square deviation (RMSE) and mean absolute percentage error (MAPE). The key factor are R squre and MAE. The highest value of the R2 for all 3 types of the models : MOB 6 and more with no limit change – 0.5522, MOB 6 and more with limit changes – 0.5066 and MOB 1-5 – 0.4535) is given by weighted logistic regression approach. However, the fractional regression approach has shown close coefficient of determination results. Other methods such as OLS, Beta regression (non-linear mixed procedure) and beta-transformation + OLS have shown weaker results. The least coefficient of determination value has Beta-regression plus OLS approach, but on contrary this approach has the least value of MAE coefficient (0.1779, 0.1831 and 0.2051 for MOB 6+ no limit change, MOB 6+ limit change and MOB 1-5 respectively). For comparison the weighted logistic regression has MAE 0.1922, 01941 and 0.2169 for the same types of models. In fact OLS gives results which are not significantly worse than the fractional regression, but the outcome value does not have limitation in range from 0 to 1, and OLS method can give results out of the defined range of permissible values.

One-Stage Model		Method	Development Sample				Validation Out-of-sample			
			R2	*MAE*	*RMSE*	*MAPE*	*R2*	*MAE*	*RMSE*	*MAPE*
Month on Book 6 or more	Limit NO Change	OLS	0.5498	0.1930	0.2537	316.3440	0.5498	0.1930	0.2537	3.1134
		Fractional (Quasi-Likelihood)	***0.5502***	0.1922	0.2544	315.9280	***0.5509***	0.1919	0.2534	313.1440
		Beta regression (nlmixed)	0.5341	0.2076	0.2589	321.1190	0.5344	0.2071	0.2580	318.4330
		Beta transformation + OLS	0.4698	0.1779	0.2761	174.0870	0.4707	0.1781	0.2751	172.0360
		Weighted Logistic Regression	**0.5522**	0.1921	0.2538	311.1860	**0.5533**	0.1917	0.2527	308.6320
	Limit Changed	OLS	0.5010	0.1967	0.2552	3.2235	0.5064	0.1955	0.2527	2.9666
		Fractional (Quasi-Likelihood)	***0.5040***	0.1950	0.2544	252.6080	***0.5099***	0.1937	0.2518	256.1710
		Beta regression (nlmixed)	0.4877	0.2080	0.2586	247.3150	0.4911	0.2071	0.2566	250.5250
		Beta transformation + OLS	0.4246	0.1831	0.2740	168.9060	0.4350	0.1810	0.2704	172.4160
		Weighted Logistic Regression	**0.5066**	0.1941	0.2538	244.2070	**0.5136**	0.1926	0.2509	247.1390
Month on Book 1-5		OLS	0.4481	0.2200	0.2820	3.2976	0.4474	0.2180	0.2802	3.1635
		Fractional (Quasi-Likelihood)	***0.4513***	0.2171	0.2812	427.6390	***0.4494***	0.2154	0.2796	421.1560
		Beta regression (nlmixed)	0.4075	0.2431	0.2922	378.2450	0.4085	0.2400	0.2898	373.4210
		Beta transformation + OLS	0.3324	0.2051	0.3102	226.6340	0.3287	0.2048	0.3088	230.1630
		Weighted Logistic Regression	**0.4535**	0.2169	0.2806	3.2565	**0.4547**	0.2146	0.2783	3.1102

Table 2. Summary validation of the regression methods for three utilization rate models

The comparative analysis results given from development and out-of-sample validation have shown that the best proportion prediction model for the utilization rate direct estimation are fractional regression (quasi-likelihood) and weighted logistic regression with binary transformation of the data sample.

Arsova et al (2011), Loterman et al (2012) have shown the same best methods for Loss Given Default prediction, thus our utilization rate modelling results with regressions confirm obtained before for another proportion predictions.

We provide the detailed results for Limit No Change model. For Limit Change and MOB 1-5 models the distributions and proportions keep similar trends and mainly differ in scales only.

Statistic	OLS	Fractional	Beta regression	Beta+OLS	Weighted Logistic Regrssion
Mean	0.53520	0.53893	0.50482	0.53220	0.53516
Std Deviation	0.28105	0.28297	0.25019	0.37851	0.28243
Skewness	-0.36165	-0.34751	-0.26532	-0.23848	-0.35561
Uncorrected SS	76937	78008	66833	89795	77090
Coeff Variation	52.5123	52.5070	49.5595	71.1215	52.7744
Sum Observations	112680	113465	106284	112048	112671
Variance	0.07899	0.08007	0.06259	0.14327	0.07976
Kurtosis	-1.19212	-1.37168	-1.35845	-1.59755	-1.35032
Corrected SS	16630	16859	13178	30163	16793
Std Error Mean	0.00061	0.00062	0.00055	0.00082	0.00062

Table 3. Statistic parameters for predicted distributions for Limit NO Change Model

As it can be seen the mean values are around the 0.53 value for all approaches exclude beta regression which has in average underestimated results. On the other hand Beta transformation with OLS shows the highest standard deviation and variation.

Quantile	OLS	Fractional	Beta regression	Beta+OLS	Weighted Logistic Regrssion
100% Max	1.1187	0.9821	0.9495	1.0000	0.9829
99%	0.9440	0.9251	0.8713	0.9962	0.9188
95%	0.8960	0.8845	0.8287	0.9789	0.8804
90%	0.8647	0.8626	0.8053	0.9663	0.8581
75% Q3	0.7843	0.7987	0.7346	0.9079	0.7940
50% Median	0.5985	0.6153	0.5543	0.6285	0.6103
25% Q1	0.2753	0.2551	0.2608	0.0997	0.2568
10%	0.1177	0.1152	0.1333	0.0057	0.1103
5%	0.0770	0.0904	0.1143	0.0025	0.0850
1%	-0.0045	0.0562	0.0872	0.0007	0.0446
0% Min	-0.3780	0.0055	0.0232	0.0000	0.0018

Table 4. Outcome Distributions for five prediction methods for Limit NO Change Model

The predicted with OLS utilization rate distribution has U-shape with some values below zero and higher than 1 (see Figure 5. Linear regression predicted outcome distribution). To avoid the values out of interval without use of conditional function like if value less than 0 let it be 0 we apply another approaches with outcome values in range.

Four other approaches have outcome values strictly between 0 and 1. Figure 6. Beta regression, Beta-transformation, Fractional regression, and Weighted logistic regression distributions. The U-shape distribution corresponds with original utilization rate distribution in the data sample. However, at the right side (high proportion values) pick is not in the value 1 as expected, but in the area of 0.85-0.9 what has made the prediction biased. Beta regression (NLMIX) distribution is similar to fractional one but the left pick of low utilization is higher than the area of high utilization. This means that the prediction can be underestimated. The U-shape of the beta-transformation approximated with OLS has the most fitted shape with the original distribution. However, the validation results are weakest among all models (Table 2. Summary validation of the regression methods for three utilization rate models). An approach with weighted logistic regression also has right peak not at the high bound, but in 0.85-0.9 area. The predicted outcome can be underestimated for high utilization values, but the statistical validation results have shown the best values. Weighted logistic and Fractional response regressions give similar distributions and higher predictive power in compare with other approaches.

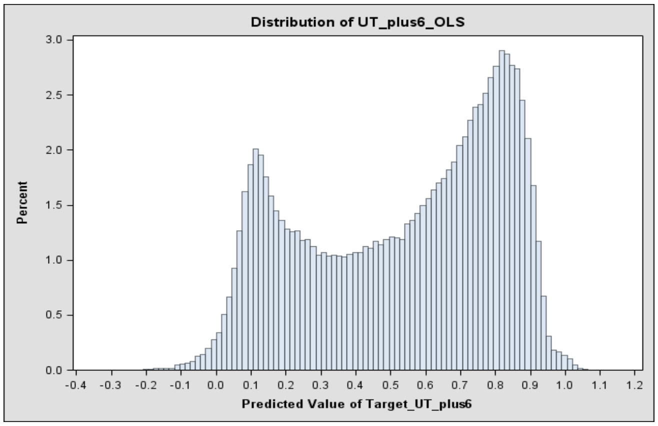
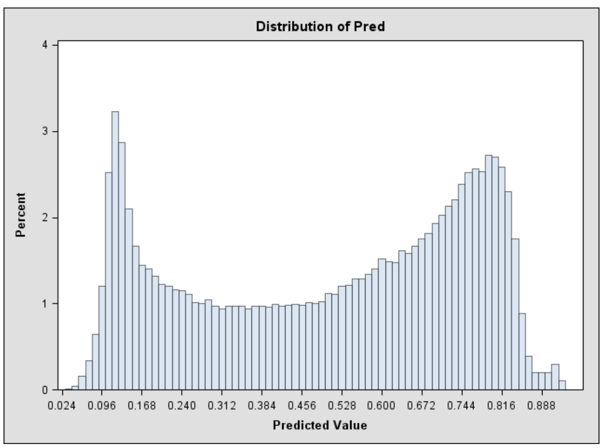
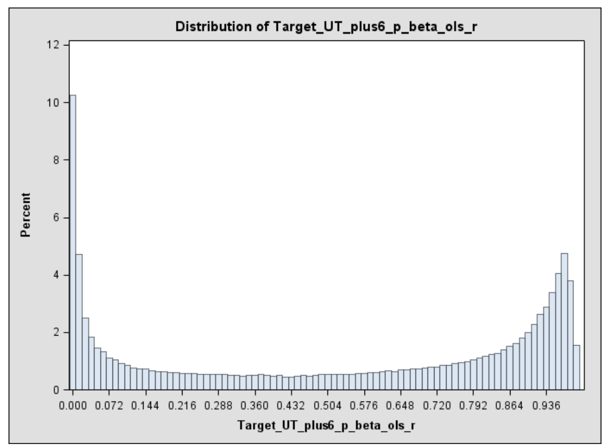


Figure 5. Linear regression predicted outcome distribution

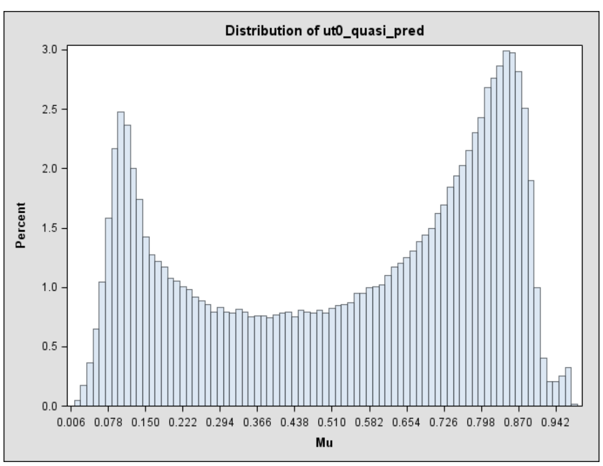
Beta regression



Beta transformation + OLS



Fractional regression (quasi-likelihood)



Weighted Logistic Regression

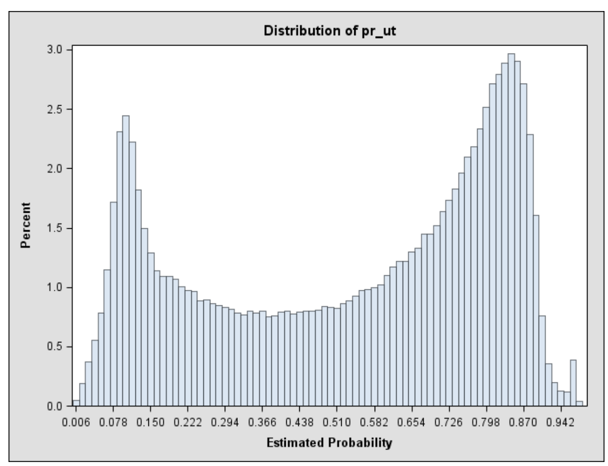


Figure 6. Beta regression, Beta-transformation, Fractional regression, and Weighted logistic regression distributions

Characteristic	MOB 6+ - Limit NO Change				MOB 6+ - Limit Changed				MOB 1-5			
	Parameter Estimate	Standard error	t Value	Pr > |t|	Parameter Estimate	Standard error	t Value	Pr > |t|	Parameter Estimate	Standard error	t Value	Pr > |t|
Intercept	0.19868	0.00823	24.14	<.0001	0.14837	0.01252	11.85	<.0001	0.2773	0.0156	17.78	<.0001
Account info												
mob	-0.00328	0.0001604	-20.44	<.0001	-0.00188	0.0002648	-7.09	<.0001				
limit	1.59E-07	1.33E-07	1.19	0.2345	-2.7E-06	2.31E-07	-11.67	<.0001	-2.4E-06	2.93E-07	-8.2	<.0001
UT	0.53061	0.00312	170.19	<.0001	0.51333	0.00492	104.28	<.0001	0.43759	0.00581	75.3	<.0001
avg_balance	2.07E-06	2.37E-07	8.73	<.0001	3.84E-06	3.57E-07	10.74	<.0001	0.0000032	5.61E-07	5.7	<.0001
Behavioural - dynamic												
b_AvgOB16_to_MaxOB16_ln	0.04088	0.00134	30.59	<.0001	0.04039	0.00233	17.34	<.0001				
b_TRmax_deb16_To_Limit_ln	0.00699	0.00049122	14.22	<.0001	0.01649	0.0008275	19.93	<.0001	-0.00552	0.00045667	-12.09	<.0001
b_TRavg_deb16_to_avgOB16_ln	-0.01841	0.00068774	-26.76	<.0001	-0.03006	0.00125	-24.08	<.0001	-0.01085	0.00105	-10.31	<.0001
b_TRsum_deb16_to_TRsum_crd16_ln	0.01087	0.00061894	17.55	<.0001	0.01241	0.00119	10.46	<.0001	0.02698	0.00074873	36.04	<.0001
b_UT1_to_AvgUT16ln	-0.00282	0.00040175	-7.01	<.0001	-0.00195	0.0006926	-2.82	0.0048	0.00912	0.00068481	13.32	<.0001
b_UT1to2ln	0.00178	0.00030936	5.76	<.0001	0.0009643	0.0005288	1.82	0.0682	-0.00734	0.00021089	-34.79	<.0001
b_UT1to6ln	-0.0048	0.00025297	-18.99	<.0001	-0.00647	0.0004217	-15.34	<.0001				
b_NumDeb13to46ln	0.00545	0.00033788	16.14	<.0001	0.00937	0.0006162	15.21	<.0001				
b_inactive13	0.0824	0.00464	17.77	<.0001	0.16916	0.00812	20.84	<.0001				
b_avgNumDeb16	0.0001071	0.00004173	2.57	0.0103	-0.00149	0.0003031	-4.91	<.0001	0.00679	0.00034267	19.83	<.0001
b_OB_avg_to_eop1ln	-0.00132	0.00033009	-4	<.0001	-0.000629	0.00052	-1.21	0.2262	-0.00927	0.00064119	-14.45	<.0001
b_DelBucket16	0.02571	0.00235	10.96	<.0001	0.03251	0.00441	7.36	<.0001	0.02082	0.00821	2.54	0.0112
b_pos_flag_0	0.01847	0.00174	10.63	<.0001	0.01697	0.00244	6.94	<.0001	0.021	0.00263	7.99	<.0001
b_pos_flag_13	0.03935	0.00213	18.51	<.0001	0.04299	0.003	14.32	<.0001				
b_atm_flag_0	0.053	0.00152	34.91	<.0001	0.05427	0.00213	25.44	<.0001	0.09135	0.00279	32.7	<.0001
b_atm_flag_13	0.05824	0.00246	23.7	<.0001	0.03996	0.00355	11.25	<.0001				
b_pos_flag_used46vs13	0.02783	0.0019	14.64	<.0001	0.02612	0.0027	9.68	<.0001				
b_pos_flag_use13vs46	-0.02589	0.00203	-12.78	<.0001	-0.03041	0.00286	-10.61	<.0001				
b_atm_flag_used46vs13	0.01634	0.0022	7.43	<.0001	0.00351	0.00329	1.07	0.2861				
b_atm_flag_use13vs46	-0.02562	0.00214	-11.96	<.0001	-0.01679	0.00306	-5.48	<.0001				
b_pos_use_only_flag_13	0.01185	0.00275	4.31	<.0001	0.00375	0.00397	0.95	0.3441	-0.03498	0.0047	-7.45	<.0001
no_dpd	-0.00524	0.00413	-1.27	0.2039	0.00506	0.00694	0.73	0.4657				
max_dpd_60	0.02693	0.00698	3.86	0.0001	0.03216	0.01178	2.73	0.0063				
Application- static												
AgeGRP1	0.01494	0.00189	7.9	<.0001	0.01191	0.0025	4.77	<.0001	0.02992	0.00353	8.47	<.0001
AgeGRP3	-0.00166	0.00172	-0.97	0.3323	0.00783	0.00234	3.34	0.0008	-0.01675	0.0032	-5.23	<.0001
customer_income_ln	-0.03324	0.00165	-20.1	<.0001	-0.02355	0.00258	-9.11	<.0001	-0.04541	0.00327	-13.87	<.0001
Edu_High	-0.02094	0.00175	-11.94	<.0001	-0.0196	0.00249	-7.86	<.0001	-0.04343	0.00326	-13.33	<.0001
Edu_Special	-0.00198	0.00169	-1.17	0.2415	-0.00226	0.00243	-0.93	0.3518	-0.01095	0.00314	-3.48	0.0005
Edu_TwoDegree	-0.01803	0.00389	-4.63	<.0001	-0.01047	0.00539	-1.94	0.052	-0.0494	0.00731	-6.76	<.0001
Marital_Civ	0.00494	0.00268	1.84	0.0653	0.00403	0.0038	1.06	0.2898	0.03416	0.00501	6.82	<.0001
Marital_Div	0.00377	0.0019	1.98	0.0478	0.00829	0.00279	2.97	0.003	0.02359	0.00352	6.71	<.0001
Marital_Sin	0.00625	0.00201	3.11	0.0019	0.00631	0.00282	2.24	0.0253	0.019	0.00376	5.06	<.0001
Marital_Wid	0.02133	0.00349	6.12	<.0001	0.03034	0.00613	4.95	<.0001	0.03118	0.00642	4.85	<.0001
position_Man	0.01174	0.00183	6.43	<.0001	0.01479	0.00272	5.45	<.0001	0.01727	0.00342	5.06	<.0001
position_Oth	0.00891	0.00181	4.91	<.0001	0.00718	0.00257	2.79	0.0052	0.00821	0.00337	2.43	0.015
position_Tech	0.00673	0.0017	3.96	<.0001	0.01045	0.00245	4.26	<.0001	0.01417	0.00315	4.49	<.0001
position_Top	0.00989	0.00336	2.94	0.0033	0.01065	0.00568	1.87	0.0611	0.00769	0.00629	1.22	0.2211
sec_Agricult	0.00428	0.00327	1.31	0.1905	0.01956	0.00485	4.03	<.0001	-0.0196	0.00608	-3.22	0.0013
sec_Constr	-0.00428	0.00437	-0.98	0.3279	-0.0252	0.00651	-3.87	0.0001	-0.01599	0.00802	-2	0.046
sec_Energy	-0.00213	0.00281	-0.76	0.4483	-0.00362	0.00401	-0.9	0.3664	-0.00917	0.00522	-1.76	0.0792
sec_Fin	-0.02624	0.00211	-12.45	<.0001	-0.03388	0.00316	-10.72	<.0001	-0.04477	0.00388	-11.55	<.0001
sec_Industry	0.00344	0.00552	0.62	0.5328	0.00516	0.008	0.64	0.5195	-0.000521	0.01043	-0.05	0.9602
sec_Manufact	0.00805	0.0043	1.87	0.0613	0.00217	0.00622	0.35	0.7266	-0.02594	0.00785	-3.3	0.001
sec_Mining	0.00362	0.00299	1.21	0.2259	0.00344	0.00436	0.79	0.4303	0.01126	0.00563	2	0.0456
sec_Service	-0.00867	0.00158	-5.49	<.0001	-0.01918	0.00232	-8.27	<.0001	-0.01405	0.00296	-4.75	<.0001
sec_Trade	-0.00604	0.00212	-2.84	0.0045	-0.00638	0.003	-2.13	0.0332	-0.00529	0.00395	-1.34	0.18
sec_Trans	-0.00676	0.00414	-1.63	0.1026	-0.01358	0.00608	-2.23	0.0256	-0.01666	0.00761	-2.19	0.0286
car_Own	-0.01099	0.0015	-7.34	<.0001	-0.00857	0.00219	-3.91	<.0001	-0.01934	0.00279	-6.93	<.0001
car_coOwn	0.00212	0.00228	0.93	0.3517	0.00259	0.00332	0.78	0.4353	-0.00269	0.00425	-0.63	0.5277
real_Own	0.0005113	0.00145	0.35	0.7248	0.00244	0.00205	1.19	0.2341	-0.00849	0.00269	-3.15	0.0016
real_coOwn	-0.00257	0.00154	-1.68	0.0939	-0.00185	0.00214	-0.87	0.3866	0.00291	0.00286	1.02	0.3097
reg_ctr_Y	-0.00803	0.00223	-3.6	0.0003	-0.01008	0.00316	-3.19	0.0014	-0.00798	0.00256	-3.12	0.0018
reg_ctr_N	-0.00653	0.00224	-2.92	0.0035	-0.00913	0.00318	-2.87	0.0041	0.00625	0.00419	1.49	0.1358
child_1	0.01064	0.0019	5.61	<.0001	0.01041	0.00266	3.91	<.0001	0.0208	0.00356	5.84	<.0001
child_2	0.0065	0.00108	6.02	<.0001	0.00441	0.00157	2.8	0.0051	0.01498	0.00203	7.39	<.0001
child_3	0.03175	0.00362	8.78	<.0001	0.02034	0.00564	3.61	0.0003	0.03036	0.00672	4.52	<.0001
Macroeconomic - dynamic												
Unempl_lnyoy_6	0.26643	0.02401	11.1	<.0001	0.52425	0.03853	13.61	<.0001	0.57361	0.14279	4.02	<.0001
UAH_EURRate_lnmom_6	-0.07515	0.02953	-2.54	0.0109	-0.09468	0.04392	-2.16	0.0311	-0.61495	0.0459	-13.4	<.0001
UAH_EURRate_lnyoy_6	0.16824	0.0185	9.09	<.0001	0.19912	0.02704	7.37	<.0001	0.22877	0.04324	5.29	<.0001
CPI_lnqoq_6	0.62416	0.04887	12.77	<.0001	1.46551	0.07027	20.86	<.0001	-0.60753	0.07393	-8.22	<.0001
SalaryYear_lnyoy_6	-0.26247	0.04234	-6.2	<.0001	0.13688	0.06107	2.24	0.025	0.01177	0.08418	0.14	0.8888
Credit Limit Changes												
l_ch1_ln (limit change month ago)					0.05093	0.00673	7.57	<.0001				
l_ch6_ln (limit change 6 month ago)					0.09616	0.00382	25.17	<.0001				

Table 5. Parameters estimation comparative analysis for Ordinary Least Squares example

Models for APP, BEH NL and BEH CL used different sets of characteristics is shown in Table 5. Parameters estimation comparative analysis for Ordinary Least Squares example. The application parameters are used for all segments, but long term behavioural predictors are not used for APP and limit changes are used for BEH CL only. We provide OLS method estimations here for the comparative analysis of the model segments. As it can be seen from the table 4 the same characteristics have different estimations and t-values for APP, BEH NL and BEH CL. For example, current utilization rate is less significant for limit changed account than for the account without limit changes (170 and 104 t values respectively). Some characteristics can have opposite trend for behavioural and application model such as education level and the indicator of the POS use only for the last 3 month. Thus the same drivers impact on the utilization rate and customer behaviour in the different way.

TWO-STAGE MODEL SUMMARY

Two-stage model means that at the first stage the probability to get a boarder value as 0 and 1 is calculated, and then the proportion estimation in the interval (0;1) are applied.

At the first stage the probability that an account has zero utilization (Pr (Ut=0) and then that an account has full utilization (Pr (Ut=1)) in the performance period is calculated with binary logistic regression. At the second stage the proportion between 0 and 1 excluding 0 and 1 values is calculated according to the set of the approaches used for one-stage direct estimation (Figure 7. Two-stage regression model schema).

The two-stage model utilization rate is calculated with the following formula:

$$Ut = (1 - \Pr(Ut = 0))(\Pr(Ut = 1) + (1 - \Pr(Ut = 1)) \cdot E(Ut \mid Ut \neq 0, Ut \neq 1))$$

Where Pr(Ut=0) and Pr(Ut=1) are the probability the utilization rate is equal to 0 or 1 respectively.

$E(Ut \mid Ut \neq 0, Ut \neq 1)$ is the utilization rate proportion estimation for the utilization rates not equal zero and not equal to 1.

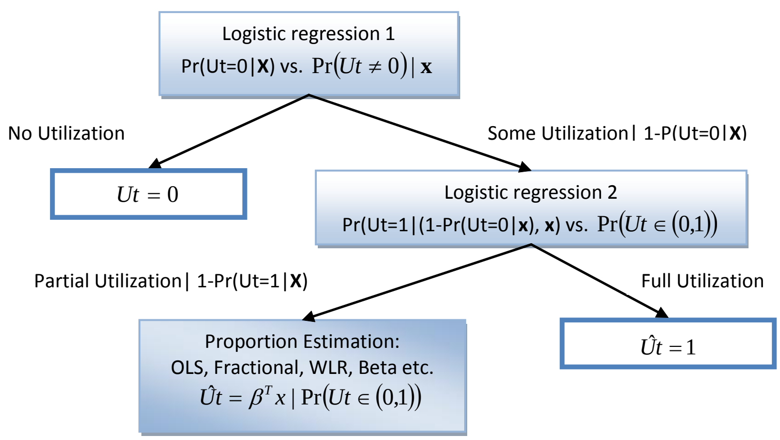


Figure 7. Two-stage regression model schema

Two-stage model consist of two parts: the probability of zero utilization and full utilization with use of logistic regression and the proportion estimation with use of the set of the same methods as for one-stage model.

In general two-stage models have shown better model accuracy and prediction results for development and validation samples, but the difference in forecasts errors are insignificant. For example, for Limit No Change model for OLS method for one-stage and two-stage approaches R^2 = 0.5498 and 0.5534, MAE= 0.1930 and 0.1913 respectively. However, if we compare Stage 2 model with one-stage direct estimation it can be seen that one-stage model gives better results. For example, one-stage and stage 2 of two-stage model R^2 = 0.5498 and 0.4310, MAE= 0.1930 and 0.1948 respectively. But this difference is compensated high prediction performance of the Stage 1 model – logistic regression, which has KS = 0.6262 and 0.5931, GIni = 0.7479 and 0.7243 for probability the utilization rate is equal to zero and the utilization rate is equal to 1 respectively.

Month on Book	Limit Changes	Stage	Method	Development Sample				Validation Out-of-sample			
MOB 6 or more	Limit NO change	*Stage 1*	*Probability*	*KS*	*Gini*	*ROC*		*KS*	*Gini*	*ROC*	
		Pr(UT=0)	Logistic Regression	0.6262	0.7479	0.8739		0.6331	0.7547	0.8774	
		Pr(UT=1)	Logistic Regression	0.5931	0.7243	0.8622		0.6036	0.7355	0.8678	
		Stage 2	*Proportion Estimation*	*R2*	*MAE*	*RMSE*	*MAPE*	*R2*	*MAE*	*RMSE*	*MAPE*
		0<UT<1	OLS	0.4310	0.1948	0.2462	4.9151	0.4235	0.1950	0.2462	4.8260
			Fractional(Quasi-Likelihood)	0.4309	0.1946	0.2463	4.9683	0.4235	0.1950	0.2462	4.8260
			Beta regression (nlmixed)	0.4183	0.2102	0.2506	5.0499	0.4108	0.2104	0.2507	4.9075
			Beta transformation + OLS	0.3680	0.1802	0.2673	2.7377	0.3618	0.1809	0.2673	2.6513
			Weighted Logistic Regression	**0.4325**	0.1945	0.2457	4.8937	0.4253	0.1948	0.2456	4.7564
		Two-stage	*Aggregate*	*R2*	*MAE*	*RMSE*	*MAPE*	*R2*	*MAE*	*RMSE*	*MAPE*
		0<= UT <=1	OLS	0.5534	0.1913	0.2535	3.1366	0.5536	0.1910	0.2526	3.0784
			Fractional(Quasi-Likelihood)	0.5527	0.1915	0.2536	3.1590	0.5529	0.1912	0.2528	3.0979
			Beta regression (nlmixed)	0.5366	0.2068	0.2581	3.2109	0.5364	0.2063	0.2574	3.1502
			Beta transformation + OLS	0.4720	0.1773	0.2754	1.7407	0.4724	0.1774	0.2745	1.7019
			Weighted Logistic Regression	**0.5548**	0.1914	0.2531	3.1116	0.5553	0.1910	0.2521	3.0532

Table 6. Two-stage models comparative analysis

The two-stage models for other two segments: Limit Changed and Month on Book less than 6 have shown approximately the same results for models quality assessment and validation. Difference is in scale only. For example, MOB 1-5 model shows low KS and Gini parameters (~0.30 and ~0.40 respectively) what is normal and expected for the application scoring.

CONCLUSION

The main task of this paper is to find more accurate method for the credit limit utilization rate. We applied a number of methods already used for proportions prediction as Loss Given Default and compared obtained results with published. The general proportion models accuracy evaluations for LGD are confirmed for the utilization rate too (Belotti and Crook, 2009; Arsova et al, 2011; Yao et al, 2014). We applied five methods: i) linear regression (OLS), ii) fractional regression (quasi-likelihood), iii) beta-regression (non-linear), iv) beta-transformation + OLS/GLM, v) weighted logistic regression with data binary transformation in one-stage direct model and two-stage model with logistic regression for the probability of bounded values estimation.

The best validation results have been shown by for both one- and two-stage models are:

i) fractional regression and

ii) weighted logistic regression with data binary transformation.

However, OLS results are not differ dramatically and describe the similar distribution shape. Beta transformation has the most similar distribution shape but has the worst validation results.

Two-stage models show slight better result for all five approaches than one-stage model. The probabilities estimation models for the utilization rate bound values 0 and 1 have high performance results for credit risk behavioural models.

We also segmented our population and use three separate groups of models for customer with less than 6 month on balance, customer with 6 and more month on balance and no changes limit, and customer with 6 and more month on balance and increased limit. These three model segment has different sets of characteristics. For example, additional limit changes parameters or limited number of behavioural characteristics for MOB less than 6 accounts. Models for changed limit are slight stronger than MOB 6+

without limit change rather because of additional parameters, models for MOB less than 6 show weaker predictive power because of short behavioural history and these models rather based on application data.

Business contribution is in use of the utilization rate for the profitability estimation in credit limits strategy and marketing strategies at the account level. Credit limit utilization rate depends on the customer behavioural pattern and revolvers and transactors can have different utilization rates.

The next stages of the investigation for the utilization rate modelling can be dedicated to the use of other methods prediction like discrete choice, CHAID, SVM etc. and based on LGD modelling experience can give even higher performance results than regression.

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CONTACT INFORMATION

Your comments and questions are valued and encouraged. Contact the author at:

Denys Osipenko
The University of Edinburgh Business School
29 Buccleuch Place, Edinburgh, Lothian EH8 9JS
denis.osipenko@gmail.com

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