Paper 156-2013

Smarter Grid Operations with SAS/OR®

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ABSTRACT

Between the time electricity leaves the utility generators and reaches your home or business, 7% of the energy has been dissipated as heat. For the average utility, this represents a total loss of more than $75 million each year.

Some of the electric current producing these losses do not result in the actual production of power, and can be minimized with the proper switching of devices that are located at strategic points in the distribution system. These devices can also conserve energy by reducing voltages in the distribution system and still provide a continuous supply of electricity to the customer.

This paper discusses the use of SAS/OR to control device switching to optimize the operations of the electrical distribution system.

INTRODUCTION

Accessible, abundant, and affordable electric power is one of the cornerstones of all economies. Economic prosperity, national security, and public health and safety cannot be achieved without it.

In 1940, 10% of energy consumption in America was used to produce electricity. In 1970, that fraction was 25%. Today it is over 40%, showing electricity's growing importance as a source of energy supply.

Our current electric grid was built in the 1890s and improved upon as technology advanced through each decade. It was built on a concept of centralized generation connected to transmission and distribution networks that deliver power to homes and industries, and bill them once a month. This limited, one-way interaction makes it difficult for this patched together grid to respond to the energy needs of the 21st century.

The smart grid introduces a communication layer that allows information from customers, sensors, and control devices within the grid to be exchanged between the utility, grid devices, and the customers. This creates a real-time awareness of the status of the grid and allows for significant improvements in grid efficiency and reliability.

By understanding the state of the grid at any given time, the operator can take advantage of today's advanced analytical algorithms and computing capabilities to optimize the settings of control devices to allow the grid to operate more efficiently, thus:

- reducing waste and loss,
- deferring the need for capital investments in new power plants and grid infrastructure,
- reducing the use of fossil fuels and associated emissions,
- reducing peak demand,
- reducing the cost of electricity, and
- facilitating the integration of new technology, such as wind, solar, storage and micro generation.

THE ELECTRIC ENERGY DELIVERY SYSTEM

Electric power is generated at power plants and then moved by alternating current (AC) to substations by transmission lines, which are large, high-voltage power lines. These transmission lines are terminated at a substation where the voltage is dropped to a medium-voltage through a large power transformer. From here, the electricity is fed through lines to a distribution transformer that steps the voltage down to a level for consumption by a customer. The step up and the step down of the voltage have a specific effect on the delivery system. As voltage is increased, the same amount of required power can be served with a lower current, and vice versa. This simple relationship is defined by:

$$Power = Volts \times Amps\ (Current) \tag{1}$$

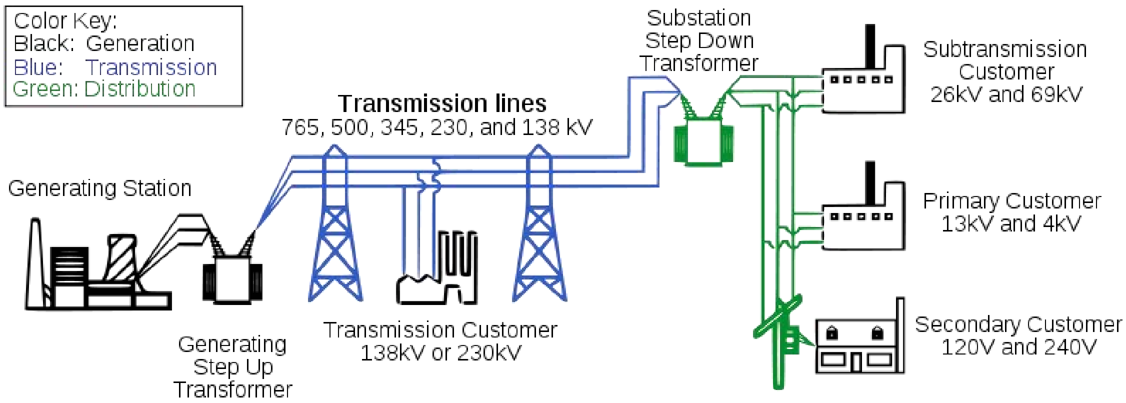

Figure 1. Electric Energy Delivery System [1]

As electricity travels from the generating station to the customers, losses occur. The conductors that are moving the electricity have a resistance. Modern overhead distribution conductors are typically made of stranded aluminum wire, sometimes with a steel reinforcing core. The resistance (R_L) of the conductor is about 0.3 ohms per mile, and decreases with cross-sectional area.

As current flows through the lines, the resistance causes energy to be lost in the form of heat. As the current increases, so does the heat. Line losses are proportional to the square of the current. This is why transmission lines are at such a high voltage.

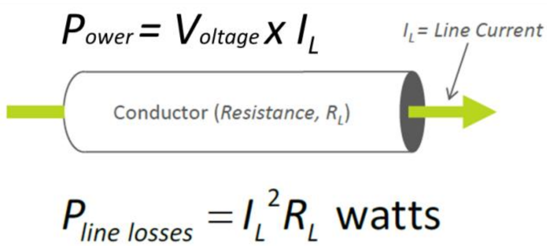

Figure 2. Current Flow and Power Loss in a Conductor [5]

It is estimated that up to 10% of the electricity that is generated is lost through transmission and distribution of the electricity to the customer, and over 40% of this is attributed to distribution losses.

CHARACTERISTICS OF ELECTRICITY DISTRIBUTION

The distribution system begins as the circuit leaves the substation and ends when the circuit terminates into the customers' meters. The substation transformer steps the high voltage down to a medium voltage that is appropriate for shorter distances in the distribution system. Voltage is typically stepped down again prior to termination at the customer site via a distribution transformer.

An individual distribution circuit can serve many customers and it may contain a mix of customer types. Customer types include industrial, commercial, and residential. The circuit is typically three-phase, higher voltage (4kv to 69kv) for large industrial customers and single-phase, lower voltage (240v/120v) for residential customers. The ultimate goal of the system is to serve the load at these customer sites. Loads can vary in type, and these different types can have a significant impact on the performance of the distribution system. For example, an incandescent light bulb is a purely resistive load and the resulting current is in phase with the voltage. However, a motor or a compressor is an inductive load and it can cause the current wave form to lag the voltage wave form. In order to supply the load of the system, higher current is then drawn, which increase the line losses. In addition to the line losses, the lagging current also results in power that is not used to serve the load, known as reactive power or VARs (volt-amperes reactive). Utilities inject reactance into the system using capacitors to compensate for lagging effects of different load types. Prior to the smart grid communications layer, these compensating devices (capacitors, regulators) were designed into the network at a specific setting and very rarely were adjusted. By receiving real-time feedback from the network, system operators can run optimization routines to adjust these devices to keep the voltage and current synchronized and to reduce losses. The industry calls this volt-VAR optimization.

THE CASE FOR VOLT-VAR OPTIMIZATION

REAL, REACTIVE POWER AND LOSSES

Inductive loads, such as motors, solenoids, and relays have coils that produce a magnetic field. The current that is used to produce the magnetic field is called magnetizing current. These devices also have a load current that is used in producing output. This output can only be produced in the presence of the magnetic field.

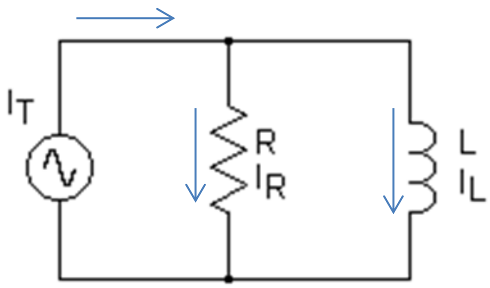

Figure 3. Load and Magnetizing Currents

$\mathbf{I_L}$= Magnetizing Current

$\mathbf{I_R}$ = Load Current

$\mathbf{I_T}$ = Total Current

Magnetizing current has these properties:

- It establishes the magnetic field so the motor will spin.
- The amount of magnetizing current that a motor draws depends on how the motor was made and does not vary with the load.
- It uses no energy. Magnetizing current consumes no power.
- When voltage (a sine wave) is applied to the motor, the magnetizing current (also a sine wave) doesn't rise until 90 degrees after the voltage rises. In other words, magnetizing current lags voltage by 90 degrees.

Load current has these properties:

- It develops when something tries to prevent the motor from spinning (like a compressor attached to the motor). The resistance to the spinning is the load. The load current increases with increased load.
- This is what is doing the work (i.e., driving the compressor).
- When voltage is applied to the motor, the load current rises and falls perfectly in step with the voltage. It doesn't lag at all. This is referred to as being in phase; a 0 degree difference. Load current is in phase with voltage.

Total motor current has these properties:

- Total motor current is the sum of magnetizing current plus load current ($\vec{A} = \vec{B} + \vec{C}$). We know that magnetizing current lags the voltage by 90 degrees and is constant, regardless of the load, while load current is in phase with the voltage and grows as the load increases.
- The sum of those two, which is the motor current delivered by the cable, should be offset from voltage somewhere between 0 and 90 degrees, depending on the amount of load current being drawn.
- Adding a capacitor to the system injects current that is 90 degree leading, supplies the magnetizing current, and reduces the phase angle between voltage and total current.

The resulting power relationship between voltage, magnetizing current, and load current is typically represented by the following terms:

- kW is working power (also called actual power or active power or real power). It is the power that actually powers the equipment and performs useful work.
- kVAR is reactive power. It is the power that magnetic equipment (transformer, motor, and relay) needs to produce the magnetizing flux. It does no work.
- kVA is apparent power. It is the vectorial summation of kVAR and kW and its magnitude is $\sqrt{kW^2 + kVAR^2}$
- PF is the power factor. It is the ratio of kW to kVA and can also be expressed as cos(ϴ), where ϴ is the angle between the real and apparent power vectors.

This relationship can also be described by the power triangle.

The following example shows how adding a capacitor to the system can reduce the amount of power that is required to serve the same load. Consider a motor load of 200kW. Because of the inductive nature of the load, say it requires a reactive power component of 150 kVARs. This results in a power factor of 80%. If the utility uses a capacitor to inject a reactance of 100 kVARs, the power factor would be improved to 97%.

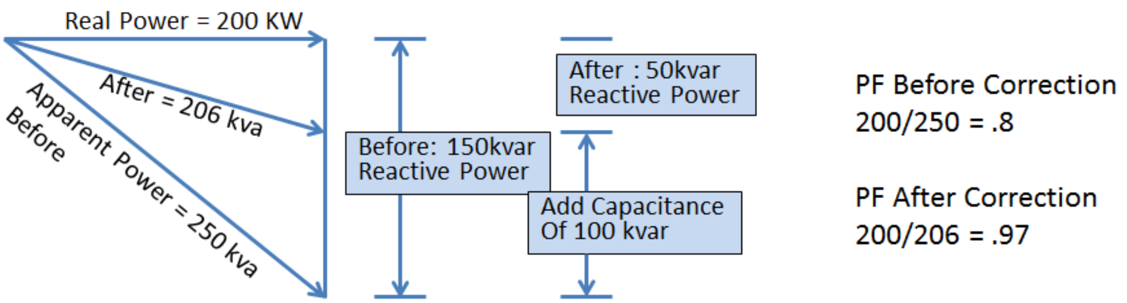

Figure 4. Power Factor Correction

As shown in the example, the greater the value of reactive power (kVAR), the lower the power factor (PF) and the higher the total or apparent power required to serve the load.

A system designer endeavors to select equipment and design a system that reduces the drop in PF. This is typically done by selecting devices such as capacitors to supply the required kVARs. This is a static view, but the system is ever changing. Volt-VAR optimization takes this changing nature into account and tells the operator at any given time what the optimal setting of capacitance should be.

POWER AND VOLTAGE RELATIONSHIPS AND PEAK LOADING CHARACTERISTICS

During times of peak energy usage, utilities have traditionally had several expensive options for meeting this peak load demand. These include starting up high cost peak load generation, purchasing expensive power from the interconnected grid, or shedding load through load control actions (e.g., shutting off air conditioning units). The smart grid provides the utility another option, voltage reduction.

Previously, we showed the relationship between power, voltage, and current as follows:

$$Power = Volts \times Amps\ (Current)$$

From this simple equation, it is recognized that if voltage is decreased, so is power. This is true for constant impedance and constant current loads (not constant power). In the US, regulation requires that voltage falls between 114 volts to 126 volts. Prior to the smart grid, the utility did not have visibility into the network, and they would design it to operate on the high end to ensure that they met the 114-volt limit.

By knowing that the voltage is closer to the load, the utility can operate the system closer to the limit. During peak times they can reduce the voltage and reduce the demand on the system. By monitoring the system and by running optimization routines, the utility can essentially defer the need to bring on an expensive peaking unit or defer the cost of building a new generating station.

OPTIMIZING DISTRIBUTION OPERATIONS

THE MATHEMATICAL MODEL

The Power System Equations

The basic element of an electrical system can be represented by an impedance z, which consists of a real component r and an imaginary component x, thus:

$$z = r + jx \tag{2}$$

The complex voltage v across this element is equal to the product of z and the complex current i flowing through it, as follows:

$$v = zi \tag{3}$$

or

$$i = yv \tag{4}$$

where

$$y = z^{-1} = g + jb \tag{5}$$

The real power P and reactive power Q across a component of the power system are related to the voltage v across and the current i flowing through the component as follows:

$$P + jQ = vi^* \tag{6}$$

where $i*$ is the conjugate of i, or

$$P + jQ = vy^*v^* \tag{7}$$

The power system is made up of several of these elements, which are connected together. These connection points are called nodes or buses. The currents that are injected into the system through each bus are related to the bus voltages by a system of equations that is analogous to equation (3). As such, the following matrix equation results for a system having three buses:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \tag{8}$$

The matrix of Y_{ij} values is known as the wye matrix or the bus admittance matrix.

The Three-Phase Bus Admittance Matrix

Power systems generally consist of three phases (a, b, and c) with the phase voltages being shifted 120˚ from each other. When all the elements of the system are equal, the ensuing currents are similarly balanced. In such a case, equation (8) is sufficient to represent the three-bus system. Usually, however, especially in distribution systems, the three-phase system is not balanced. The numbers of rows and columns of the bus admittance matrix will then be three times larger. Each element in the bus admittance matrix in equation (8) is now replaced by a sub-matrix that represents the three phases separately:

$$Y_{Bus} = \begin{bmatrix} Y_{11}^{3\phi} & Y_{12}^{3\phi} & Y_{13}^{3\phi} \\ Y_{21}^{3\phi} & Y_{22}^{3\phi} & Y_{23}^{3\phi} \\ Y_{31}^{3\phi} & Y_{32}^{3\phi} & Y_{33}^{3\phi} \end{bmatrix} \tag{9}$$

where

$$Y_{ik}^{3\phi} = \begin{bmatrix} Y_{ik}^{aa} & Y_{ik}^{ab} & Y_{ik}^{ac} \\ Y_{ik}^{ba} & Y_{ik}^{bb} & Y_{ik}^{bc} \\ Y_{ik}^{ca} & Y_{ik}^{cb} & Y_{ik}^{cc} \end{bmatrix} \tag{10}$$

and

$$Y_{ik}^{pm} = G_{ik}^{pm} + jB_{ik}^{pm} \tag{11}$$

p and m are phase indices, and i and k are bus indices.

Typically, a distribution system will include the distribution lines, transformers, voltage regulators, and shunt capacitor devices. Each element of the three-phase system is normally modeled as a six-by-six bus admittance matrix representing the terminal buses and the three phases of each bus. The total system admittance matrix is formed by adding all the matching elements of the individual bus-admittance matrices.

A system of equations analogous to equation (6) can then be written for the overall distribution system using the three-phase total bus-admittance matrix of the system.

Using the total bus-admittance matrix of the distribution system, we can now create a system of equations for power that is analogous to equation (7). Writing separate equations for the real and imaginary components yields the following sets of equations:

$$P_{Gi}^{p} - P_{Li}^{p} = V_i^p \sum_{k=1}^{n} \sum_{m=1}^{3} V_k^m \left[G_{ik}^{pm} cos\theta_{ik}^{pm} + B_{ik}^{pm} sin\theta_{ik}^{pm} \right] \quad (12)$$

$$Q_{Gi}^{p} - Q_{Li}^{p} = V_i^p \sum_{k=1}^{n} \sum_{m=1}^{3} V_k^m \left[G_{ik}^{pm} sin\theta_{ik}^{pm} - B_{ik}^{pm} cos\theta_{ik}^{pm} \right] \quad (13)$$

where

P_{Gi}^{p}	is the per unit real power injection at bus i, phase p
P_{Li}^{p}	is the per unit real power load at bus i, phase p
Q_{Gi}^{p}	is the per unit reactive power injection at bus i, phase p
Q_{Li}^{p}	is the per unit reactive power load at bus i, phase p
V_i^p	is the voltage at bus i, phase p
Θ_{ik}^{pm}	is the angle between voltages V_i^p and V_k^m
G_{ik}^{pm}, B_{ik}^{pm}	are the real and imaginary components (also known as the conductance and the susceptance) of the elements of the bus admittance matrix

Voltage regulators and some substation transformers have tap changers that can vary their transformation ratios so that the voltage downstream can be adjusted. In some cases, shunt capacitors can be switched on and off in order to reduce the reactive current flowing in the lines, and to adjust voltages as well. When these devices are switched, the corresponding elements of the bus admittance matrix likewise change. These elements can thus be used in optimizing the system defined by the power and voltage relationships in equations (12) and (13).

THE OPTIMIZATION PROBLEM FORMULATION

Distribution operators and planners control the switching of capacitors, voltage regulators, and transformer load tap changers in order to either minimize losses or to minimize the power drawn from the source while maintaining acceptable voltage levels. The setting of these control variables (described formally below) affects the key measures like the power at various points in the distribution system via the bus-admittance matrix. This results in the following problem formulation.

1. Control variables

- Along the distribution feeder, switched capacitors are normally simply ON or OFF. The susceptance B of the capacitor is added to the diagonal of the bus admittance matrix to reflect this:

$$B_{ii}^{pm} = \begin{cases} B, capacitor\ is\ ON \\ 0, capacitor\ is\ OFF \end{cases} \quad (14)$$

- Load tap changers and regulators usually have a range of ± 10%:

$$0.9 \leq \ \tau_i \ \leq 1.1 \quad (15)$$

where τ_i is the tap ratio.

A transformer has the following transformer bus-admittance matrix [6]:

$$Y_{Bus\_T} = \begin{bmatrix} Y_{\rho\rho} & Y_{\rho\varsigma} \\ Y_{\varsigma\rho} & Y_{\varsigma\varsigma} \end{bmatrix} \quad (16)$$

where $Y_{\rho\rho}$ and $Y_{\varsigma\varsigma}$ are the sub-matrices representing the primary and secondary sides of the three-phase transformer, and $Y_{\rho\varsigma}$ and $Y_{\varsigma\rho}$ represent the mutual admittances between them.

Tap changers modify the transformer sub-matrices as follows:

- Divide the self-admittance $Y_{\rho\rho}$ of the primary by τ_1^2
- Divide the self-admittance $Y_{\varsigma\varsigma}$ of the secondary by τ_2^2
- Divide the mutual admittance matrices $Y_{\rho\varsigma}$ and $Y_{\varsigma\rho}$ by $\tau_1\tau_2$.

where τ_1 and τ_2 are the primary and secondary tap ratios, respectively.

2. Objective functions

- Loss Minimization - since all powers are assumed to be injections, the sum of all bus powers is the total power loss P_L :

$$Min \quad P_L = \sum_{i=1}^{n}\sum_{p=1}^{3} P_i^p \tag{17}$$

where $n = number\ of\ buses$
$p = \ phase\ index$

- Conservation Voltage Reduction (CVR) – the defined objective is to minimize the total power supplied:

$$Min \ \ P_{CVR} = \sum_{j=1}^{n_G}\sum_{p=1}^{3} P_j^p \tag{18}$$

where $n_G = number\ of\ source\ buses$

3. Constraints

- The real power injection at each bus is given by the following equation:

$$P_i^p = v_i^p \sum_{k=1}^{n}\sum_{m=1}^{3} v_k^m \left[G_{ik}^{pm}\cos\theta_{ik}^{pm} + B_{ik}^{pm}\sin\theta_{ik}^{pm}\right] \tag{19}$$

where $n = number\ of\ buses$
$v_k^m = voltage\ at\ bus\ k, phase\ m$
$\theta_{ik}^{pm} = the\ angle\ between\ the\ voltages\ at\ bus\ i, phase\ p\ and\ bus\ k, phase\ m$
$p, m = \ phase\ indices$

- The reactive power injection at each bus is given by the following equation:

$$Q_i^p = v_i^p \sum_{k=1}^{n}\sum_{m=1}^{3} v_k^m \left[G_{ik}^{pm}\sin\theta_{ik}^{pm} - B_{ik}^{pm}\cos\theta_{ik}^{pm}\right] \tag{20}$$

- There is a limit to the capacity of the power sources:

$$P_{j\,min} \leq \ P_j \ \leq P_{j\,max} \tag{21}$$

- There is a limit to the capacity of the reactive power sources:

$$Q_{j\,min} \leq \ Q_j \ \leq Q_{j\,max} \tag{22}$$

- The bus voltages cannot exceed the allowable operating range:

$$v_{i\,min} \leq \ v_i \ \leq v_{i\,max} \tag{23}$$

- Although not very critical in the distribution system, the angles between bus voltages are not allowed to exceed a certain range:

$$\theta_{ij\,min} \leq \ \theta_{ij} \ \leq \theta_{ij\,max} \tag{24}$$

IMPLEMENTATION

TECHNOLOGY AND TOOLS

Based on the optimization formulation introduced in the previous section, it is clear that this problem is a nonlinear programming (NLP) problem since both choices for objective function as well as the constraints contain some nonlinear terms such as sine and square. In addition, the variables related to the capacitors are binary variables, representing the capacitor being ON or OFF. PROC OPTMODEL and the NLP solver in SAS/OR are used to model and solve this distribution optimization problem, and we use heuristic techniques to handle the integer decision variables in order to solve a continuous nonlinear optimization problem. The OPTMODEL modeling language provides a modeling environment for building, solving, and maintaining optimization models. This makes the process of translating the symbolic formulation of an optimization model into the OPTMODEL modeling language virtually transparent since the modeling language mimics the symbolic algebra of the formulation as closely as possible [7]. It also integrates seamlessly with solvers—including the NLP solver—which can be invoked repeatedly, if needed, to solve the problem after the model is formulated. We will introduce a few key sections of the code in this paper.

MODELING THE DECISION VARIABLES

To formulate a problem in PROC OPTMODEL, the first step is to define parameter sets, which in this particular problem is a collection of indices that are based on the power distribution network topology. The sample PROC OPTMODEL code of defining sets is shown as follows. For example, the set named BUS_BY_PHASE represents a two-dimensional set, where the first element of the set denotes the bus, and the second element represents the power distribution phase.

```
set <num, num> BUS_BY_PHASE;                       /*Set of bus index plus phase id*/
set <num> BUS;                                     /*Set of bus*/
set <num, num, num, num, num> BRANCH_BY_PRI_SEC;   /*Set of branch index, primary bus
                                                     index, secondary bus index,
                                                     primary phase id, secondary
                                                     phase id*/
set <num, num, num> BRANCH;                        /*Set of branch index, primary bus,
                                                     secondary bus*/
set <num, num, num, num> BRANCH_PHASE;             /*Set of branch and phase*/
set PHASE = {1,2,3};                               /*Set of three distribution phases*/

/*Set of physical branches and virtual branches that represents distribution network*/
set NETWORK_PHASE = (setof{<i,t> in BUS_BY_PHASE}<i,i>)
                     union
                    (setof{<l,k,m,p,q> in BRANCH_BY_PRI_SEC}<k,m>)
                     union
                    (setof{<l,k,m,p,q> in BRANCH_BY_PRI_SEC}<m,k>);
```

Next, we need to define decision variables and dependent variables that are related to the optimal settings of available switchable devices in the network. The variables that are defined in this optimization problem include transformer/regulator taps and capacitor positions. As a result of tap-setting changes, the values of bus voltages and bus angles might be different. The values of bus admittance matrices also depend on the device settings. The decision variables and dependent variables in the optimization formulation are as follows.

- Transformer and regulator taps can take on a range of ±10% in 16 increments each (0.00625 per step). For this implementation it is assumed that this can be represented by a continuous function. At the end of the optimization, the tap is rounded to the nearest step increment. This variable is named tap_low in our PROC OPTMODEL formulation.
- The switching of capacitors is a binary decision, and the variable is named bus_b_shunt in our PROC OPTMODEL formulation. Although it is declared as a set of binary variables, we use heuristic techniques to relax the variables and solve a sequence of continuous approximations to the nonlinear optimization problem.
- The bus voltage variable is named bus_voltage and the bus angle variable is named bus_angle in our PROC OPTMODEL formulation. The bus voltage and bus angle are not controlled directly, but are changed by controlling the transformer and regulator taps and by switching the capacitors.
- The bus admittance matrix is not a decision variable, but a dependent variable with a real part named G and imaginary part named B in our PROC OPTMODEL formulation. G and B are declared using the IMPVAR statement in PROC OPTMODEL since they are quantities of interest that are completely determined by other variables.

The sample code of variable declarations in PROC OPTMODEL is as follows. The INIT expression in PROC OPTMODEL specifies initial values for variables if they exist. The values could be read from positions of switchable devices or meters before the optimization process.

```
variable tap_low {<l,k,m,p> in BRANCH_BY_PHASE} init tap_low0[l,k,m,p];
variable bus_b_shunt {<i,t> in BUS_BY_PHASE}  init bus_b_shunt_init[i,t] binary;
variable bus_voltage {<i,t> in BUS_BY_PHASE}  init bus_voltage0[i,t];
variable bus_angle {<i,t> in BUS_BY_PHASE}  init bus_angle0[i,t];
impvar G{<k, m> in NETWORK_PHASE, p in PHASE, q in PHASE};
impvar B{<k, m> in NETWORK_PHASE, p in PHASE, q in PHASE};
```

OBJECTIVES AND CONSTRAINTS

By equations (17-18), we can formulate the objectives of Loss Minimization and Conservation Voltage Reduction (CVR) with the keyword minimize <Objective Name> in PROC OPTMODEL. The example code of loss minimization is shown below.

```
minimize LOSS = sum {<l,k,m,q> in BRANCH_BY_PHASE}(
                    (bus_voltage[k,q]*cos(bus_angle[k,q])
                      -bus_voltage[m,q]*cos(bus_angle[m,q]))
                    * sum {<(l),(k),(m),(q),p> in BRANCH_BY_PRI_SEC}(
                           branch_g[l,k,m,q,p]
                           *(bus_voltage[k,p]*cos(bus_angle[k,p])
                              -bus_voltage[m,p]*cos(bus_angle[m,p]))
                          -branch_b[l,k,m,q,p]
                           *(bus_voltage[k,p]*sin(bus_angle[k,p])
                              -bus_voltage[m,p]*sin(bus_angle[m,p]))
                           )
                    + (bus_voltage[k,q]*sin(bus_angle[k,q])
                        -bus_voltage[m,q]*sin(bus_angle[m,q]))
                      * sum{<(l),(k),(m),(q),p> in BRANCH_BY_PRI_SEC}(
                            branch_b[l,k,m,q,p]
                            *(bus_voltage[k,p]*cos(bus_angle[k,p])
                               -bus_voltage[m,p]*cos(bus_angle[m,p]))
                           +branch_g[l,k,m,q,p]
                            *(bus_voltage[k,p]*sin(bus_angle[k,p])
                               -bus_voltage[m,p]*sin(bus_angle[m,p]))
                            )
                    );
```

Similarly, by equations (19-24), we can formulate constraints in PROC OPTMODEL. For example, for each bus, working power injection and consumption need to be balanced for each distribution phase. The PROC OPTMODEL code below covers all the working power balance constraints for each bus and phase by indexing the set BUS_BY_PHASE. For more details about how to construct objectives and constraints in PROC OPTMODEL, please refer to the *SAS/OR User's Guide* [7].

```
con p_load_con {<k,q> in BUS_BY_PHASE}:
    p_gen[k,q] - bus_p_load[k,q]
       = bus_voltage[k,q]
            * sum{<(k),m> in NETWORK_PHASE, p in PHASE}(
                  bus_voltage[m,p]
                     * ( G[k,m,q,p] * cos(bus_angle[k,q]-bus_angle[m,p])
                       + B[k,m,q,p] * sin(bus_angle[k,q]-bus_angle[m,p]))
                  );
```

After the model is formulated in PROC OPTMODEL, the SOLVE statement is used to invoke the NLP solver to solve the problem. To accommodate problems of a larger scale or problems with difficult non-convexity features, a heuristic approach that combines the relaxing and fixing of variables with local search techniques might be necessary, and the NLP solver may be called multiple times. For small, simple problems, a single SOLVE statement may be sufficient.

An example of a single SOLVE statement with the NLP solver is as follows. Introduced in SAS/OR version 12.1, the new option MS (Multiple Start) enables the solver to tackle the problem from multiple starting points in different threads simultaneously, which possibly results in a better local minimum with shorter run time.

```
SOLVE with nlp obj LOSS / MS tech=as printfreq=1 opttol=1.0e-6 maxiter=100
                          msmaxstarts=50 seed=12345;
```

THE IEEE FOUR-BUS SYSTEM

For purposes of illustration, the optimization algorithm was applied on a test system developed by the IEEE. Since this system was designed to test load flow algorithms, the model was validated by fixing the decision variables at their original settings and comparing the results against the reference. The decision variables were then allowed to vary and the results were compared with the simulation case to verify that the desired improvement was achieved.

The IEEE four-bus test system consists of a three-phase transformer that is set between two line segments; this system has no capacitors. The system is feeding a three-phase load. The transformer is configured in various ways to mimic the different connections (wye and delta) that are used in the field. The load can be balanced or unbalanced.

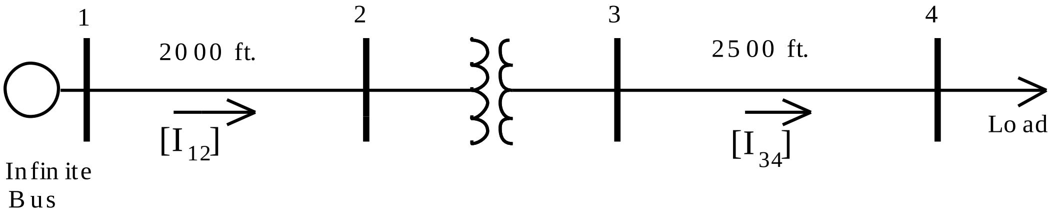

Figure 5. IEEE 4 Node Test Feeder [8]

RESULTS

Table 1 shows the comparison of load flow simulation results for the IEEE four-bus system with the grounded wye - grounded wye transformer configuration. It shows close agreement between SAS simulation and the reference case. Optimal transformer settings are shown in Table 2 as the result of PROC OPTMODEL runs. Results for objective loss minimization are also given in Table 3, indicating an improvement of 23.87% for loss minimization.

Bus	Phase	Voltage from Simulated Load Flow	Voltage from IEEE Load Flow	Difference
1	A	1.00000	1.00000	0.000%
1	B	1.00000	1.00000	0.000%
1	C	1.00000	1.00000	0.000%
2	A	0.99502	0.99506	-0.004%
2	B	0.98763	0.98756	0.007%
2	C	0.98367	0.98367	0.000%
3	A	0.95991	0.95971	0.021%
3	B	0.93875	0.93889	-0.015%
3	C	0.91715	0.91724	-0.010%
4	A	0.90554	0.90558	-0.004%
4	B	0.80352	0.80357	-0.006%
4	C	0.76300	0.76318	-0.024%

Table 1. Load Flow Results for IEEE Four-Bus System with Grounded Wye - Grounded Wye Configuration

Branch_index	Bus_primary	Bus_secondary	Phase	Optimal_tap_position	Initial_tap_position
3	2	3	A	11	0
3	2	3	B	16	0
3	2	3	C	16	0

Table 2. Optimal Transformer Tap Settings for IEEE Four-Bus System with Grounded Wye - Grounded Wye Configuration

Power loss after optimization(kW)	Power loss before optimization(kW)	Improvement
361.50	474.83	23.87%

Table 3. Optimization Results for IEEE Four-Bus System with Grounded Wye - Grounded Wye Configuration

CONCLUSION

In this paper we have shown that it is possible to formulate the distribution optimization problem in SAS/OR to optimize distribution device switching for loss minimization and conservation voltage reduction. This avoids the set and test method that is in common use. This has the benefit of avoiding multiple load flow runs that do not promise reaching the global optimum. It was also shown that, instead of having to run a separate load flow simulation, the same optimization model can be used by simply fixing the decision variables at their original set points.

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ACKNOWLEDGMENTS

The authors acknowledge the help of Michelle Opp and Manoj Chari in reviewing this paper.

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