

## Paper 020-2011

## Cash Flow Modeling for Fixed-Income Securities: A New Approach Using the SAS® Risk Management for Banking Solution

Sunny Zhang, Jeff Hasmann, and Tom Kimner, SAS Institute Inc., Cary, NC

### ABSTRACT

Cash flow modeling is critical to valuation and analysis of fixed-income securities including ABS, MBS, and CDO. Traditionally, due to the complexity of cash flow structure of various loans and receivables, the cash flow modeling and validation process is often a black box and is difficult for the investors to understand how their portfolio performance and securities values are associated with, and impacted by, the performance of underlying assets such as credit card receivables or mortgage. As a result, the investors cannot obtain the insights about the future performance trend of the assets to effectively design and execute their investment and risk mitigation strategies. This is also one of the lessons learned from the financial crisis. The paper describes a new and transparent approach that utilizes SAS® Risk Management for Banking to address the above issues. This approach can help both banks and the investors to build an effective early warning system so as to better monitor the underlying asset performance and valuation process for maximum investment returns and optimal risk management process.

### INTRODUCTION

Cash flow analysis is critical to many areas of the financial services industry. Critical applications include funding strategy, evaluation of fixed income portfolios, and analysis of structured finance products.

Traditionally, cash flow analysis has been performed at an aggregate level, using broad assumptions, due to modeling complexity, deficiency in data, and methodology. However, what is missing is the rich cash flow information at the account level, which determines overall liquidity and cash flow adequacy. As a result, senior management often has difficulties understanding how the portfolio's cash flows and liquidity are impacted by the risk characteristics of the underlying assets (loan receivables, mortgage, etc.), and market and credit events. In addition, the aggregate-level approach might not be able to capture the tail risk caused by external market and credit events.

Given the significant liquidity and funding crises experienced during the 2007–2009 financial crisis, more and more financial institutions are realizing that it is not sufficient to model cash flows at the aggregate level, that more sophisticated analysis is required to model at the account or collateral level if an analyst wants to obtain a more effective indication, and projection, of the cash flows and performance of the portfolio. Analyzing cash flows at a more granular level presents some challenges to many financial institutions. First, it requires more sophisticated tools and access to data/information about the underlying accounts and positions in the portfolio. Second, it requires robust modeling techniques to effectively and accurately forecast account-level cash flows under uncertainty. Third, it requires a systematic and proven methodology so the underlying relationship between a portfolio's cash flow stream and securities valuation can be captured in a transparent manner.

SAS Risk Management for Banking provides financial institutions with new capabilities to deal with the challenges associated with cash flow analysis and modeling. It contains the methodology and techniques that allow banks and the investors to better monitor the underlying cash flow characteristics at a granular level, thus enhancing the risk-management process. Specifically, this solution can be used to improve cash flow analysis from the following perspectives:

- Greater transparency of asset securitization and valuation process. The user can easily monitor and evaluate the cash flow characteristics of the underlying assets, and readily connect them to security prices.

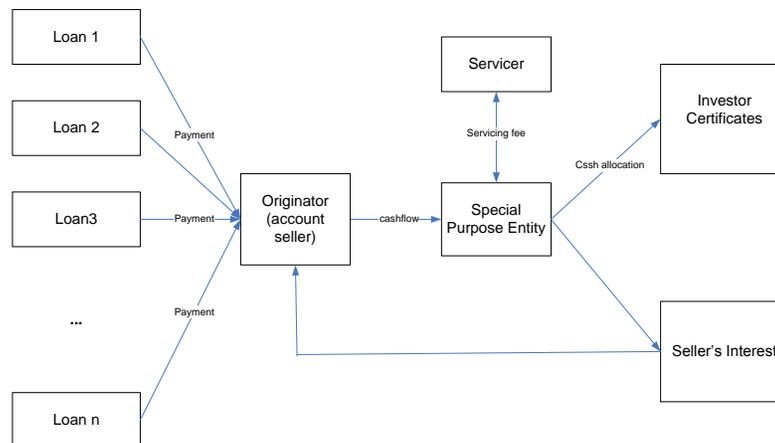
- More accurate pricing of loan and fixed income portfolios based on discounted cash flow methods. SAS incorporates economic and credit risk factors to quantify the uncertainty around loan and fixed-income portfolio cash flows. The solution can effectively incorporate cash allocation rules, credit enhancements, and market dynamics in valuation process with various future scenarios.
- Better securities valuation. Different security pricing results and methods are available for easy comparisons so the investor can understand how different model assumptions affect security pricing, using tools such as stress testing. The investors can also easily estimate the implied default probabilities and effective discount rate from the actual or calculated security prices.
- Improved insight into funding requirements for the bank's treasury group. SAS can assist bank treasurers in monitoring cash flow characteristics of the bank's portfolio on an intra-day basis, thus facilitating improved, and less costly, funding decisions.

In this paper, we provide a high-level introduction of how SAS Risk Management for Banking can help achieve the above goals. We analyze a typical Asset Backed Security (ABS) as a case study. In this case, the ABS represents a portfolio of underlying credit card accounts that are grouped into different tranches with different credit characteristics. Although we are using an ABS case study, the methodologies shown here can be applied to analyze the stochastic cash flows from any portfolio of bonds or loans.

We first describe the typical cash flow analysis process and how it is applied in ABS securitization. Then we present the SAS cash flow modeling process and capabilities. For illustration purposes, we also provide some practical examples and sample SAS code and output.

## CASH FLOW ANALYSIS: THE CASE OF ABS SECURITIZATION

To understand the importance of cash flow modeling and how it can be used in liquidity and portfolio management areas, we first examine a basic loan securitization process. Securitization is a process of transforming illiquid assets such as mortgages, auto loans, or credit card receivables into securities through financial engineering. Loan securitization usually involves multiple market participants.<sup>1</sup> The basic setup for asset securitization (Figure 1) is similar to securitizing mortgages and other loan obligations:



<sup>1</sup> For a basic setup of loan securitization, please also refer to FDIC 2007.

**Figure 1: ABS Securitization Process**

At a high level, the process generally consists of the following key points:

- Loan issuers (banks) have accumulated a significant volume of cash flow accounts (such as credit card receivables, mortgages, etc.) and sold them to a securitization vehicle (typically a trust).
- The trust packages the receivables and sells certificates to investors. The investor certificates are generally issued with senior/subordinated structure.
- The cash flows generated from the underlying loan receivables support interest payments/principal payments to the investors. The cash flows are allocated to the certificates based on seniority.
- The proceeds from the sale of the investor certificates are used to pay the financial institution for the purchase of the underlying loan receivables.

In the diagram in Figure 1, the cash flows involved in every step throughout the entire process<sup>2</sup> impact all participants. The expected cash flows and actual cash flows will reflect the funding cost and profitability. It can be affected by various market factors such as interest rates, and credit events such as defaults or delinquency. The main goal of cash flow analysis and modeling is to identify the key factors such as interest rates and credit events and determine how they affect cash flows in order to predict future cash flows. This is critical for financial institutions, as they need to accurately evaluate asset-backed securities to generate maximum investment returns and create optimal risk-management processes. The cash flow analysis process we show in the next section describes how our approach can help achieve this goal.

## BASIC CASH FLOW ANALYSIS AND MODELING PROCESS

The overall valuation process for SAS involves the following steps:

1. *Account-level data processing, analysis, and modeling*: The data is taken from various sources and prepared for analyses. The data includes account-level loan payment history and patterns. Forecasting is performed on monthly cash flows. Modeling includes default probability based account behavior data, borrower's attributes, and applicable economic data.
2. *Creating securitization pools*: This step uses the account-level cash flows that were estimated in the previous step using simulation segmentation techniques. In this step, a new SAS procedure can be used to create effective pools that contain homogenous loan attributes.
3. *Allocating cash flows*: This step allocates cash flows from one or more securitization pools to tranches/claimants based on the rules specified in the allocation methods. In this step, a new SAS procedure can also be used to effectively allocate cash flows based on pre-determined allocation methods.
4. *Simulating market states*: This step is important for pricing securities using simulation techniques and comparing different methods against promised cash flows on the certificates. A stress-testing technique is used to help determine security prices using pre-determined scenarios to see how security prices as well as effective credit enhancements are affected.
5. *Security/Portfolio valuation*: This step involves (1) calculating implied probability of default (PD) and effective discount rates based on the base price and observed/calculated market prices and establishing a curve to represent the relationship between the implied default rate and the observed market prices—the

---

<sup>2</sup> The definition of cash flow can vary. For retail loans such as credit card receivables, the cash flow typically consists of financial charges and principal payments.

implied PD can be calculated at tranche level, and decomposed and reconciled with pool or account PDs—and (2) reporting security pricing and market state simulation results, which include security prices and methods, implied probability of default curves, and how that changes with bond prices and credit-related performance indicators (delinquency, early payment, attrition, etc.).

These steps can be represented with the following diagram:

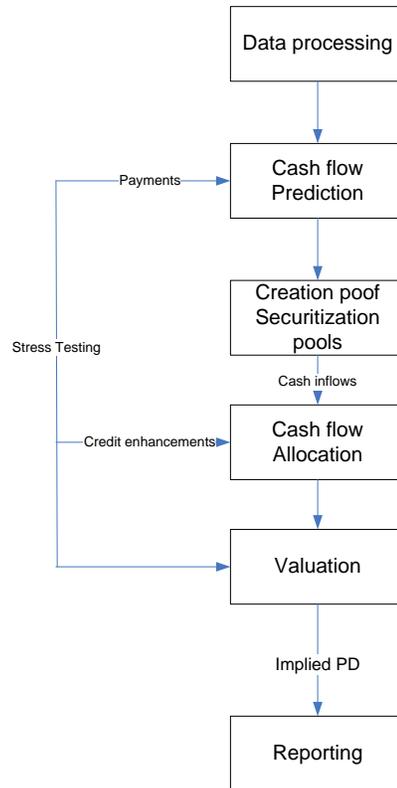


Figure 2: General ABS Valuation Process

## CASH FLOW ANALYSIS FOR LOAN SECURITIZATION—A PRACTICAL EXAMPLE

### ACCOUNT-LEVEL DATA PROCESSING AND ANALYSIS

In this step, these types of data are processed and integrated to create analytic-ready tables for cash flow forecasting and PD prediction based on historical payment and performance. Typically, the input data for loan securitization can include the following data sources:

1. Borrower's payment history: Usually a minimum of two years of monthly payment history is required. Key variables include outstanding balance, principal payment & finance charges, credit limit, payment date, fees, and so on.
2. Account data: This data includes the borrower's account information, credit history, and payment history, such as:
  - Bureau score (e.g., FICO) and behavior score
  - Payment patterns: 30, 60, 90 days past due (DPD), as well as early payment
  - Amounts owed: mortgage and installment trade-lines
  - Length of credit history: new or established
  - New credit: credit inquiries, loan purposes
  - Types of credit in use
3. Reference data: Reference curves include U.S. Treasury yield curve and London Interbank Offered Rate (LIBOR) curve.
4. Macro-economic data (if applicable): consumer price index (CPI), unemployment rate, market indices, and so on.

### MONTHLY CASH FLOW AND CREDIT LOSS PROJECTIONS

Cash flows from loan receivables mainly consist of two sources, and they are finance charges and principal payment. Finance charges include interest charged to borrower, and various types of fees such as membership charge, servicing fees, over-limit fees, late payment fee, and so on. Principal payments can be those required (included in monthly minimum payment) or excess payment (early payment). Fluctuations in cash flows can be caused by many factors such as delinquency, early payment, or prepayment of principal.

In this step, both option-theoretic models and econometric models are available to predict the cash flow. In an econometric model, the borrower's monthly payment history is associated with other related information (economic data and account data) to forecast the borrower's future monthly outstanding balance and payment (usually 6–12 months). In an option-theoretic model, the borrower's options for early payment as well as the probability of default are also used to predict monthly cash flows.

The forecasted cash flows in general will be adjusted with the borrower's credit quality and estimated credit losses. Credit quality including charge-offs and delinquency can have a significant impact on the value of cash flows. In this step, the borrower's historical performance data, credit information (bureau score, behavioral score, etc.), and repayment capacity (income, debt, etc.) are used to predict the borrower's PD and associated charge-off rate. The standard approach is to use predictive modeling (more specifically, the logistic regression model) to predict PD. A two-stage model can be used to predict both PD and credit losses.

Often times, delinquency curves and loss curves need to be derived from historical payment data. These two curves are important for producing projected delinquency/loss curves. They can be created using a vintage—either a periods-out or a dates approach. Factors to consider include growth adjustment, timing/cumulative timing, cumulative loss rate, and so on. Those factors will be plugged into the payment patterns for cash flow projection.

Once PD is predicted at the account level, the following key indicators can be calculated accordingly:

- Yield:  $(\text{financial charge} + \text{fees}) / (\text{outstanding principal balance})$
- Principal payment rate:  $(\text{principal payment received each month}) / \text{outstanding principal balance}$

- **Charge-offs:** Charge-offs is the amount of uncollected loan balances removed from a bank's books and charged against its loss reserve. The charge-off rate is the amount of charge-offs divided by the average outstanding loan balances owed to the issuer. Since in most cases, there are now sufficient charge-offs data to make meaningful forecasting, PD is usually estimated instead to calculate charge-off rate.
- **Excess spread:** yield – base rate (investor coupon) – charge-off rate, where base rate = (coupon paid to investors)/outstanding balance + %servicing fee
- **Pool stability:** (beginning outstanding balance – principal payment – credit losses)/ beginning outstanding balance. The underlying assets can completely turn over every few months. The customers who are paying off their account balances can be replenished by the customers who are building balances through purchases and balance transfers.

In addition, the account-level PD can be used to estimate pool-level (joint) PD using default correlation. Figure 3 shows an example of account-level cash-flow forecasting results, which will be used for creating securitization pools. In this case, statistical/econometric models are applied to the borrowers' payment and credit history to predict the monthly cash flow for the next 6–12 months.

Account Number	mon_yr	Type of Observation	Number of Periods into the Forecast	Total Balance	monthly_payment_rate	monthly_yield	total_payment	annual_int_rate	Principal payment	monthly_util_rate
1	01MAY2006	FORECAST	1	\$2,721.23	2.1%	1.6%	\$56.18	19%	\$10.53	19%
1	02MAY2006	FORECAST	2	\$2,713.43	2.1%	1.6%	\$56.07	20%	\$10.67	18%
1	03MAY2006	FORECAST	3	\$2,705.43	2.1%	1.6%	\$55.96	19%	\$10.80	18%
1	04MAY2006	FORECAST	4	\$2,697.28	2.1%	1.6%	\$55.85	20%	\$10.94	18%
1	05MAY2006	FORECAST	5	\$2,688.99	2.1%	1.6%	\$55.74	20%	\$11.08	17%
1	06MAY2006	FORECAST	6	\$2,680.61	2.1%	1.6%	\$55.63	19%	\$11.22	17%
1	07MAY2006	FORECAST	7	\$2,672.14	2.1%	1.6%	\$55.52	19%	\$11.36	17%
1	08MAY2006	FORECAST	8	\$2,663.61	2.1%	1.6%	\$55.42	19%	\$11.50	16%
1	09MAY2006	FORECAST	9	\$2,655.03	2.1%	1.6%	\$55.31	19%	\$11.64	16%
1	10MAY2006	FORECAST	10	\$2,646.40	2.1%	1.6%	\$55.20	19%	\$11.78	16%
1	11MAY2006	FORECAST	11	\$2,637.74	2.1%	1.5%	\$55.09	18%	\$11.92	16%
1	12MAY2006	FORECAST	12	\$2,629.06	2.1%	1.6%	\$54.98	19%	\$12.06	15%
70	01MAY2006	FORECAST	1	\$-98.28	22%	11%	\$167.70	1.8%	\$209.89	2.0%
70	02MAY2006	FORECAST	2	\$-209.76	23%	11%	\$166.84	1.8%	\$216.13	4.4%
70	03MAY2006	FORECAST	3	\$-330.57	34%	12%	\$165.98	1.8%	\$222.36	6.9%
70	04MAY2006	FORECAST	4	\$-457.95	35%	12%	\$165.11	1.8%	\$228.60	9.5%
70	05MAY2006	FORECAST	5	\$-589.97	31%	12%	\$164.25	1.8%	\$234.83	12%
70	06MAY2006	FORECAST	6	\$-725.27	32%	13%	\$163.39	1.8%	\$241.07	15%
70	07MAY2006	FORECAST	7	\$-862.88	27%	13%	\$162.53	1.8%	\$247.31	18%
70	08MAY2006	FORECAST	8	\$-1,002.12	27%	13%	\$161.67	1.8%	\$253.54	21%
70	09MAY2006	FORECAST	9	\$-1,142.51	27%	14%	\$160.81	1.8%	\$259.78	24%
70	10MAY2006	FORECAST	10	\$-1,283.71	28%	14%	\$159.95	1.8%	\$266.01	27%
70	11MAY2006	FORECAST	11	\$-1,425.48	31%	14%	\$159.09	1.8%	\$272.25	30%
70	12MAY2006	FORECAST	12	\$-1,567.66	32%	15%	\$158.23	1.8%	\$278.48	33%
71	01FEB2006	FORECAST	1	\$11,536.34	1.9%	2.0%	\$223.81	23%	\$-299.04	98%
71	02FEB2006	FORECAST	2	\$11,669.08	2.1%	2.0%	\$242.44	23%	\$-347.41	97%
71	03FEB2006	FORECAST	3	\$11,774.73	2.0%	2.0%	\$237.83	23%	\$-395.78	97%
71	04FEB2006	FORECAST	4	\$11,995.43	2.0%	2.0%	\$244.28	23%	\$-444.15	97%
71	05FEB2006	FORECAST	5	\$12,219.18	2.0%	2.0%	\$245.46	23%	\$-492.52	98%
71	06FEB2006	FORECAST	6	\$12,440.87	2.0%	2.0%	\$249.16	24%	\$-540.89	98%
71	07FEB2006	FORECAST	7	\$12,672.79	1.9%	2.1%	\$251.65	24%	\$-589.26	98%
71	08FEB2006	FORECAST	8	\$12,862.07	1.9%	2.1%	\$254.72	24%	\$-637.63	99%
71	09FEB2006	FORECAST	9	\$13,027.09	1.9%	2.1%	\$257.52	24%	\$-686.00	99%
71	10FEB2006	FORECAST	10	\$13,179.88	1.9%	2.1%	\$260.44	24%	\$-734.37	99%
71	11FEB2006	FORECAST	11	\$13,321.80	1.9%	2.1%	\$263.30	24%	\$-782.74	100%

Figure 3: Account-Level Cash Flow Forecasting

## CREATE SECURITIZATION POOLS FROM THE PORTFOLIO OF ESTIMATED CASH FLOWS

In this step, the estimated account-level cash flows will be used to create securitization pools. This is mainly driven by how these pools are structured (start date, number of issues/series, receivable values and coupons, as well as how

cash inflows (forecasted) are distributed to those pools. Payments, principal balances, and interest rates from individual accounts are aggregated with similar attributes.<sup>3</sup>

This is implemented within the SAS Risk Management for Banking environment using some special procedures including the RDPOOL procedure.

PROC RDPOOL, which is included in the solution, can be used to create securitization pools from a large loan portfolio with predicted cash flows. The pools can be created according to any specified variables such as loan maturity, counterparty, credit quality, and so on. By using this special procedure, the users can significantly reduce the number of exposures in a portfolio while retaining the key information associated with loan performance. The built-in copula techniques (such as Gaussian copula or Student's t copula) also consider default correlation at account, pool, or tranche levels to enhance simulation for more accurate and realistic results.

Figure 4 shows the securitization pools created by PROC RDPOOL. This table shows that each individual loan account is assigned to a particular pool that represents a meaningful and unique level of risk. Within each pool, all accounts have maximum level of homogeneity in terms of payment patterns, credit quality and risk, as well as other similar account-level attributes. For details about how to use PROC RDPOOL, please refer to *SAS Risk Dimensions 5.2: Procedures Guide*, pages 55–61.

Account #	Pool #	Date	total balance	%total balance	principal payment	Yield_Amt	Yield	cash flow (pred.)
1	001	01Dec2010	\$110,429,017.00	10%	\$11,042,901.70	\$759,199.49	7%	\$644,169.27
2	001	01Jan2010	\$108,269,312.00	10%	\$10,826,931.20	\$744,351.52	7%	\$631,570.99
3	001	01Feb2010	\$82,091,369.00	10%	\$8,209,136.90	\$564,378.16	7%	\$478,866.32
4	001	01Mar2010	\$82,851,876.00	10%	\$8,285,187.60	\$569,606.65	7%	\$483,302.61
80	004	01Nov2010	\$61,367,946.00	16%	\$9,818,871.36	\$319,624.72	5%	\$204,559.82
81	004	01Dec2010	\$68,286,979.00	16%	\$10,925,916.64	\$355,661.35	5%	\$227,623.26
82	004	01Jan2010	\$86,748,551.00	16%	\$13,879,768.16	\$451,815.37	5%	\$289,161.84
83	004	01Feb2010	\$98,425,344.00	16%	\$15,748,055.04	\$512,632.00	5%	\$328,084.48
84	004	01Mar2004	\$222,602,361.00	16%	\$35,616,377.76	\$1,159,387.30	5%	\$742,007.87
85	004	01Apr2010	\$237,495,421.00	16%	\$37,999,267.36	\$1,236,955.32	5%	\$791,651.40
99	005	01Jan2010	\$67,194,490.00	16%	\$10,751,118.40	\$349,971.30	5%	\$223,981.63
100	005	01Feb2010	\$81,650,526.00	38%	\$13,064,084.16	\$425,263.16	5%	\$272,168.42
101	005	01Mar2010	\$89,677,269.00	38%	\$14,348,363.04	\$467,069.11	9%	\$298,924.23
102	005	01Apr2010	\$212,349,119.00	38%	\$33,975,859.04	\$1,105,984.99	9%	\$707,830.40
103	008	01Dec2010	\$445,366,039.00	38%	\$71,258,566.24	\$2,319,614.79	9%	\$1,484,553.46
104	008	01Jan2010	\$342,363,649.00	38%	\$54,778,183.84	\$1,783,144.01	9%	\$1,141,212.16
105	008	01Feb2010	\$414,833,440.00	38%	\$66,373,350.40	\$2,160,590.83	9%	\$1,382,778.13
106	008	01Mar2010	\$377,215,416.00	38%	\$60,354,466.56	\$1,964,663.63	9%	\$1,257,384.72

Figure 4: Securitization Pools

<sup>3</sup> Segmentation methods are typically used for this purpose. For example, Abrahams and Zhang (2011) introduced a new segmentation approach for creating securitization pools with homogenous risk for all the loans within each pool.

## CASH FLOW ALLOCATION

Typically, the cash flow allocations for each master trust are specified in the pooling and servicing agreement and determine whether the bank is conforming to the requirements as servicer and when applying its valuation models. In this step, cash from one or more securitization pools will be allocated to the tranches based on seniorities (waterfall) and pre-specified rules (maturity, payment frequency, coupon rate, and principal). In the SAS ABS valuation process, cash flow allocation is implemented through a set of allocation methods, which can readily incorporate the following factors:

- **Seller's interest:** The seller's interest is designed to absorb minor receivable fluctuations. Any factors that decrease the pool of receivables beyond the maximum amount that can be absorbed by the seller's interest require the issuing bank to provide additional accounts to the trust. Seller's interest can be specified as a percentage of total value of securitization pools.
- **Excess spread:** This equals yield – base rate (investor coupon and serving fee) – charge-off rate. It is typically one of the first defenses against loss. Even if some of the underlying loan payments are late or default, the coupon payment can still be made. In some extreme cases, excess spread is applied to outstanding classes as principal.
- **Credit tranches:** According to the tranche seniority, any credit losses will be absorbed by the most junior class of bondholders until the principal value of their investment reaches zero. If this occurs, the next class of bonds absorbs credit, and so on, until finally the senior bonds begin to experience losses.
- **Over-collateralization:** This means that the face value of the underlying credit receivables is larger than the security it backs. As a result, even if some of the payments from the borrowers are late or default, principal and interest payments on ABS can still be made.
- **Reserve account:** This is created to reimburse the issuing trust for losses up to the amount allocated for the reserve. The reserve account will often be non-declining throughout the life of the security, meaning that the account will increase proportionally up to some specified level as the outstanding debt is paid off.

Cash flow allocation for financial charges is a multi-step process:

- On a pro-rata basis between the seller's interest and the investor's interests
- Applied to the investor certificate holders based on pro-rata
- Applied to each series based on priority (waterfall): In most cases, the pooling and servicing agreement will allow or require cash flow sharing among the various series in a master trust. In this situation, any excess finance charge cash flows in one series are available to cover any finance charge cash flow shortfalls in another series on a subordinated basis.

Principal payment rate affects the pool size of underlying loan receivables. The average life of a loan receivable is generally much shorter than the life of a particular series. The principal payment rate varies month to month, but on average, for the past few years, approximately 15 to 18% of borrower principal balance is paid down each month in a "normal" economy. Accordingly, an average borrower's receivable balance turns over about every five to seven months. Investor certificates issued in each series are typically interest only with maturities ranging from three to ten years. Therefore, the pooling and servicing agreement must allow for account additions to ensure that the underlying receivables do not shrink to a level below the amount required for the investors' certificates. Loan receivables represent open-end, revolving credit, and monthly outstanding balances on the designated accounts can vary from month to month due to seasonal spending patterns, changing principal payment rates, dilution, and attrition.

Principal payments can be either expected principal collections, or excess principal collections (borrower's early payment). If a security issue is in a revolving phase, early payment will be used to purchase new receivables from the loan issuing bank, or allocated pro-rata to each series. If it is in the accumulation/amortization phase, principal payment can be used to pay down other series, or accumulated for bullet payments.

Figure 5 provides a simplified allocation example in which three series are issued within the master trust and each series has three investor certificate classes (class A, B, and C).<sup>4</sup> This example assumes that the pooling and servicing agreement requires finance charge and principal cash flow sharing between each series.

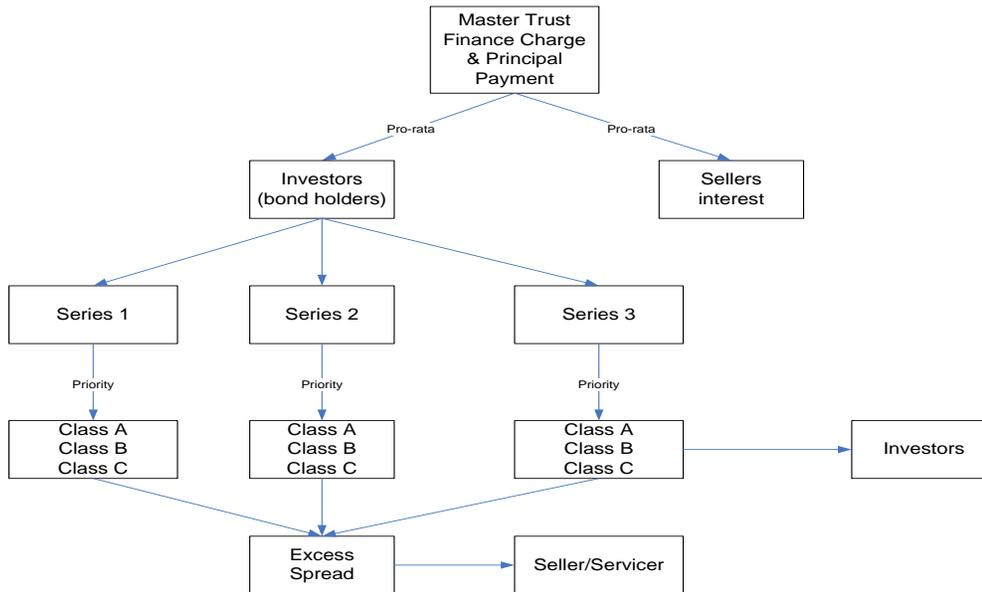


Figure 5: Cash Flow Allocation

In SAS Risk Management for Banking, cash flow allocation is conveniently controlled by the allocation table and associated method programs. For example in Figure 6, allocation methods as well as principal and interest payment orders and date all can be specified and accessed by a method program:

```

proc rdsec env=ss_env.sec cfdata=secpool allocdata=alloctable
  out=securitization;
  variables statenumber = statenumber trancheorder = (principalord interestord)
  other = (startd payment balance ncf secpercent) ;
run;

```

Here allocdata = alloctable specifies the allocation table name to be used in allocation process. The allocation method, which is 'allocType 1' as shown in Figure 6, specifies the method program used to allocate the cash flows. The RDSEC procedure, available within this solution, can apply the specified cash flow allocation table and allocation methods to allocate cash flows from securitization pools to the tranches for valuation and analysis purposes.

<sup>4</sup> See FDIC 2007.

Securitization ID	Tranche Id	Cash flow start Date	balance	payment	unit	Region	Allocation Method	Tranche type	Principal order	Interest order
sec1	tranche1	06Jan2011	\$186,164.92	\$14,074.68	Unit1	NC	allocType1	tranche	1	1
sec1	tranche2	06Jan2011	\$59,758.58	\$3,802.12	Unit1	NC	allocType1	tranche	3	2
sec1	tranche3	06Jan2011	\$68,259.81	\$4,715.15	Unit1	NC	allocType1	tranche	6	3
sec2	tranche1	06Jan2011	\$9,630.23	\$717.30	Unit2	NC	allocType1	tranche	1	1
sec2	tranche3	06Jan2011	\$13,584.60	\$421.17	Unit2	NC	allocType1	tranche	6	3
sec3	tranche1	06Jan2011	\$61,507.16	\$5,087.79	Unit3	NC	allocType1	tranche	1	1
sec3	tranche2	06Jan2011	\$8,401.68	\$505.87	Unit3	NC	allocType1	tranche	3	2
sec3	tranche3	06Jan2011	\$24,924.76	\$1,131.17	Unit3	NC	allocType1	tranche	6	3
sec1	reserve	06Jan2011	\$31,418.33	\$6,777.58	Unit1	NC	allocType1	reserve	4	
sec2	reserve	06Jan2011	\$2,321.48	\$341.54	Unit2	NC	allocType1	reserve	4	
sec3	reserve	06Jan2011	\$9,483.36	\$2,017.45	Unit3	NC	allocType1	reserve	4	
sec1	fee1	06Jan2011			Unit1	NC	allocType1	fee	2	
sec2	fee1	06Jan2011			Unit2	NC	allocType1	fee	2	
sec3	fee1	06Jan2011			Unit3	NC	allocType1	fee	2	
sec1	fee2	06Jan2011			Unit1	NC	allocType1	fee	5	
sec2	fee2	06Jan2011			Unit2	NC	allocType1	fee	5	
sec3	fee2	06Jan2011			Unit3	NC	allocType1	fee	5	

Figure 6: Cash Flow Allocation Table

In addition, the amount allocated to the pools is calculated and discounted with associated risk factors such as PD, charge-off rate, principal payment rate, and so on. Part of the sample pricing method code can look like:

```

_cash flow _matamt[i] =max((payment) * ( 1+usd1m) *(1-avg_prin_pmt_rate)
* (1-charge_off_rate), 0);
_cash flow _matamt[i] =max((payment) * ( 1+usd1m) *(1-avg_prin_pmt_rate)
* (1-charge_off_rate), 0)/2;
_cash flow _cfint[i] = (payment)/2; = (payment)/2;

```

Here `_cash flow _matamt[i]` and `_cash flow _cfint[i]` are built-in arrays and defined as cash flow maturity amount and cash flow interest, respectively, and `i` is number of the cash flows.

## VALUATION PROCESS

This is implemented within the SAS Risk Management for Banking environment using both pricing methods and simulation methods. Simulation techniques (such as historical, Monte Carlo, or scenarios) are used to create various market states to show how values of pool receivables are impacted by changes in related market risk factors. The built-in copula techniques (such as Gaussian copula or Student's t copula) also consider default correlation at account, pool, or tranche levels to enhance simulation for more accurate and realistic results. Typically, With SAS Risk Management for Banking, we can perform a Monte Carlo simulation of prepayments/early payment and defaults to discount the cash flows to obtain the expected values for the MBS.

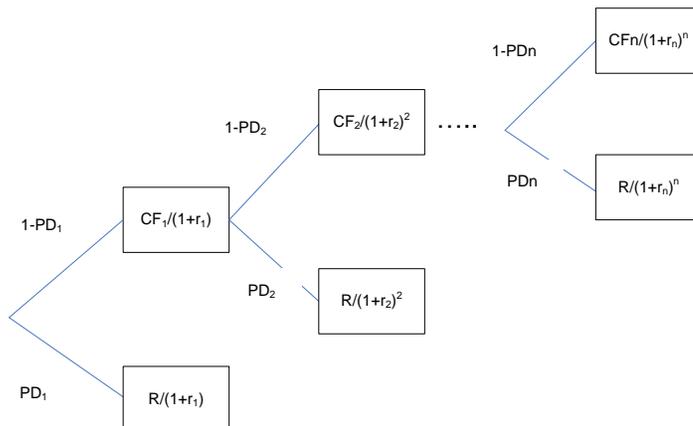
Figure 7 shows a sample output table from the solution pricing environment. It compares both expected cash flow amount and predicted cash flow amount by securitization ID with simulated market states.

Securitization ID	Tranche ID	State Number	Cash Flow Number	Maturity Date	Maturity Amount	Expected Amount
sec1	tranche1	0	1	August 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche1	0	2	November 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche1	0	3	February 01, 2011	\$222,773.38	\$460,000.00
sec1	tranche2	0	1	August 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche2	0	2	November 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche2	0	3	February 01, 2011	\$0.00	\$470,000.00
sec1	tranche3	0	1	August 01, 2010	\$1,000.00	\$1,000.00
sec1	tranche3	0	2	November 01, 2010	\$1,000.00	\$1,000.00
sec1	tranche3	0	3	February 01, 2011	\$0.00	\$321,000.00
sec1	tranche1	1	1	August 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche1	1	2	November 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche1	1	3	February 01, 2011	\$222,773.38	\$460,000.00
sec1	tranche2	1	1	August 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche2	1	2	November 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche2	1	3	February 01, 2011	\$0.00	\$470,000.00
sec1	tranche3	1	1	August 01, 2010	\$1,000.00	\$1,000.00
sec1	tranche3	1	2	November 01, 2010	\$1,000.00	\$1,000.00
sec1	tranche3	1	3	February 01, 2011	\$0.00	\$321,000.00
sec1	tranche1	2	1	August 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche1	2	2	November 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche1	2	3	February 01, 2011	\$222,773.38	\$460,000.00
sec1	tranche2	2	1	August 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche2	2	2	November 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche2	2	3	February 01, 2011	\$0.00	\$470,000.00
sec1	tranche3	2	1	August 01, 2010	\$1,000.00	\$1,000.00
sec1	tranche3	2	2	November 01, 2010	\$1,000.00	\$1,000.00
sec1	tranche3	2	3	February 01, 2011	\$0.00	\$321,000.00
sec1	tranche1	3	1	August 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche1	3	2	November 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche1	3	3	February 01, 2011	\$222,773.38	\$460,000.00
sec1	tranche2	3	1	August 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche2	3	2	November 01, 2010	\$10,000.00	\$10,000.00
sec1	tranche2	3	3	February 01, 2011	\$0.00	\$470,000.00

Figure 7: Simulation Table

Given observed or market prices of the bond, we can solve for the implied probability of default. It is commonly assumed that a bond has  $t$  cash flows remaining. The probability of default is same for each of the coupon periods. Thus, each period's PD will be multiplied by the probability of no default in all the previous periods. As shown in Figure 8, this equation can also be better understood with a binomial tree structure, in which upon maturity the bond could default with probability  $P$  and recovery rate  $R$ , or no default with probability  $1-P$ . The price of the bond is simply the expected value of the two possible outcomes discounted at the risk-free interest rate  $r$ .<sup>5</sup>

<sup>5</sup> For a detailed description, see Duffie and Singleton 2003.



Where

- r= default-free short rate (discount rate)
- L= default loss rate. Recovery rate (R)=1-L
- P= Default probability
- FV= principal value
- CFi = cash flows from coupon payment

**Figure 8: Implied PD Calculation Method—Binomial Tree of Default Probabilities**

For simplicity, let's consider an ABS tranche as an n-year defaultable, zero-coupon bond with a face value of \$100. Assume that the initial risk-free default rate is 6%, and recovery rate is 0.4. Assume that the observed or calculated market price is \$86.07, which is lower than its promised bond value (\$100) and can be considered as the result of higher credit risk (or deteriorating credit quality). Then we can calculate the implied default rate or PD as the expected value of the two possible outcomes discounted at the risk-free interest rate r, or  $86.07 = (100 * (1 - PD) + 100 * 0.4 * PD) / (1 + 0.06)$ . We can solve for  $PD = 0.1461$ , or 14.61% as the implied default rate, and the default-adjusted interest rate  $= (100 - 86.07) / 86.07 = 16.2\%$ . The difference  $(16.2\% - 14.61\% = 1.59\%)$  reflects the default timing risk.<sup>6</sup>

In general, bonds exhibit positive convexity of price to default probability, and vice versa. An increase in the implied PD can be due to the volatility of PD. With an implied default probability for each of the market prices, a curve can be established to represent this relationship. As an indicator of risk, the PD curve exhibits an upward sloping shape with bond maturity, and a downward sloping shape with bond price.<sup>7</sup>

The solution can value ABS tranches (or bonds) using different valuation methods and simulation approaches specified in the solution's risk engine (Risk Dimension). Figure 9 shows a sample output table from the solution that lists two different bonds prices and associated implied PD. This provides the investors more insights and confidence how their securities are priced and the related risks.

<sup>6</sup> See the technical note appendix at the end of the paper for general calculation of implied PD.

<sup>7</sup> For a detailed description regarding implied PD, please refer to Fitch Solutions 2007 and Perry 1995.

Securitization ID	Tranche ID	Start Date	Evaluation Date	Value - Simulation 1	% Change for Simulation 1	Implied PD from method 1	Value - Simulation 2	Base Value	Implied PD from method 2
sec1	tranche1	08Oct2010	06Jan2011	\$321,465.99	(\$0.03)	3.448%	\$332,551.02	\$332,551.02	0.000%
sec1	tranche2	08Oct2010	06Jan2011	\$93,718.04	(\$0.03)	3.448%	\$96,949.70	\$96,949.70	0.000%
sec1	tranche3	08Oct2010	06Jan2011	\$109,295.11	(\$0.03)	3.448%	\$113,063.91	\$113,063.91	0.000%
sec2	tranche1	07Nov2010	06Jan2011	\$16,510.39	(\$0.03)	3.448%	\$17,079.71	\$17,079.71	0.000%
sec2	tranche3	07Nov2010	06Jan2011	\$16,181.90	(\$0.03)	3.448%	\$16,739.89	\$16,739.89	0.000%
sec3	tranche1	07Dec2010	06Jan2011	\$111,039.69	(\$0.03)	3.448%	\$114,868.65	\$114,868.65	0.000%
sec3	tranche2	07Dec2010	06Jan2011	\$12,863.42	(\$0.03)	3.448%	\$13,306.99	\$13,306.99	0.000%
sec3	tranche3	07Dec2010	06Jan2011	\$33,549.70	(\$0.03)	3.448%	\$34,706.59	\$34,706.59	0.000%

Figure 9: Price Comparison and Implied PD

## CONCLUSION

In this paper, we have described how SAS Risk Management for Banking solution can be used to analyze and model cash flow. We have used the loan securitization process as an example for illustration. As shown in this paper, SAS Risk Management for Banking has the following benefits when used for cash flow analysis and modeling:

- The solution improves transparency for investors to monitor and evaluate the underlying asset performance, and understand the valuation process and results.
- The solution increases flexibility and scalability for expansion and integration of third-party or proprietary pricing models. Also by creating new allocation methods to implement the appropriate allocation rules, the same process can be used to value other types of ABS.
- The solution contains both built-in method programs for robust industry standard valuation functions and unique simulation methods for efficient, accurate, and consistent valuation.

## REFERENCES

- Abrahams, Clark, and Mingyuan Zhang. Prepublication draft, 2011. "A New Framework for Asset-Backed Securities (ABS)" in *The Risk of Investment Products: From Product Innovation to Risk Compliance*. Hong Kong: World Scientific Publishing.
- Duffie, Darrell, and Kenneth J. Singleton. 2003. *Credit Risk: Pricing, Measurement, and Management*. Princeton, NJ: Princeton University Press.
- FDIC. March 2007. *Credit Card Securitization Manual*. Washington, DC: FDIC.
- Fitch Solutions. 2007. *Fitch Equity Implied Rating and Probability of Default Model*. Quantitative Research Special Report. NY, NY: Fitch Inc. Available at [http://www.fitchratings.com/web\\_content/product/eir\\_method.pdf](http://www.fitchratings.com/web_content/product/eir_method.pdf)
- Furletti, Mark J. December 2002. *An Overview of Credit Card Asset-Backed Securities*. Discussion Paper. Philadelphia, PA: Federal Reserve Bank of Philadelphia. Available at <http://ssrn.com/abstract=927489>
- Perry, Clayton. April 26, 1995. *Implied Default Probabilities: A New Approach to Risky Debt*, Fixed Income Research Bulletin, Credit Suisse First Boston.
- SAS Institute Inc. 2009. *SAS Risk Dimensions 5.2: Procedures Guide*. Cary, NC: SAS Institute Inc.
- SAS Institute Inc. 2009. *SAS Risk Management for Banking 2.1: Administrator's Guide*. Cary, NC: SAS Institute Inc.

## APPENDIX: TECHNICAL NOTE

Bond price can be expressed as the sum of the two discounted expected values, weighted by the probabilities of default or no default:

$$B = \sum_{i=1}^t \frac{CF_i(1-PD)^i + FV * R * PD(1-PD)^{i-1}}{(1+r)^i}$$

Here  $r$  = default-free short rate (discount rate)  
 $L$  = default loss rate. Recovery rate ( $R$ )= $1-L$   
 $PD$  = default probability  
 $FV$  = principal value  
 $CF_i$  = cash flows from coupon payment

Alternatively, the above equation can be rewritten as

$$B = \sum_{i=1}^t \frac{CF_i e^{-i*PD}}{(1+r)^i} + FV \left( \frac{e^{-t*PD}}{(1+r)^t} \right) + FV * R * \sum_{i=1}^t \frac{e^{-(i-1)*PD} - e^{-i*PD}}{(1+r)^i}$$

## CONTACT INFORMATION

Your comments and questions are valued and encouraged. Contact the author:

Mingyuan Zhang  
 SAS Institute  
 100 SAS Campus Drive  
 Cary NC 27513 USA  
 919-531-9248  
 Fax: 919-531-4444  
 E-mail: sunny.zhang@sas.com

SAS and all other SAS Institute Inc. product or service names are registered trademarks or trademarks of SAS Institute Inc. in the USA and other countries. ® indicates USA registration.

Other brand and product names are trademarks of their respective companies.