Paper 256-28

Using the SAS® System to Construct and Operate Control Charts with Randomized Control Limits

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ABSTRACT

Attribute control charts (e.g., charts for counts and proportions) have discrete jumps in the quantities plotted on the charts, which means that the chart designer has only a discrete set of unique control limits to choose from when designing a chart. These charts have a small set of in-control Average Run Lengths (ARLs) from which to determine the appropriate control scheme. In a chart with randomized control limits the operational control limit in an epoch is determined by a random choice from among a set of control limits. The resulting control chart will then have a predetermined in-control ARL. These charts can be thought of as ones with optimal ARL properties. SAS® programs for designing and operating these charts will be provided.

INTRODUCTION

A process is a collection of activities or occurrences that produce observable (measurable) output. Statistical Process Control (SPC) is the art and science of using observation of the process to determine, in a particular epoch, whether the process is in an in-control state or an out-of-control state. An in-control process is one that is operating in a stable and predictable manner. In this context, "stable and predictable" has a stochastic interpretation. An extreme definition of an in-control process is one which during any epoch i , observations from the process

x_{ij} for $j = 1, \ldots, J$

are distributed *F(x)*. That is, samples from in-control epochs are identically distributed. In practice it is usually sufficient for this to be only approximately so.

When observation of the process produces count or categorical data, the process is said to be an attribute process (the data are attribute data). One method for controlling attribute processes is the control chart (see Montgomery (2000) for a more complete discussion of control charts). A p-chart can be used to control the rate at which non-conforming units arise in a process. Let *p* be the proportion of units that are nonconforming. Note that *p* must either be known or be estimated. The control chart limits are

$$UCL = p + k\sqrt{\frac{p(1-p)}{N}}$$

$$CL = p$$

$$LCL = p - k\sqrt{\frac{p(1-p)}{N}} \qquad (1)$$

At each epoch calculate the sample proportion of nonconforming units

$$\hat{p} = y / N$$

where y is the number of nonconforming units in a sample of *N* units. An out-of-control signal occurs whenever the sample proportion plots outside the control chart limits given in (1). This chart is appropriate whenever *y* can be modeled as Binomial (*N*,*p*).

Similarly, a *c*-chart can be employed to control the rate at which nonconformities (often the less politically correct designation "defects" is used) occur, where c is the average number of nonconformities per sampling unit. The control limits are

$$UCL = c + k\sqrt{c}$$

$$CL = c$$

$$LCL = c - k\sqrt{c} \qquad (2)$$

At each sampling epoch the number of nonconformities, x, is plotted and an out-of-control signal occurs whenever x plots outside the control limits. The c-chart is appropriate whenever the number of nonconformities per sampling unit is Poisson with mean c. The parameter c must be known or must be estimated.

The performance (capabilities) of a particular control chart are usually evaluated by means of the chart's Average Run length (ARL) properties. The ARL is the average number of samples that must be selected until an out-of-control signal occurs.   Note that the ARL is a function of the true state of the process. The in-control ARL is the average run length when the process is in-control.

ATTRIBUTE CONTROL CHART GRANULARITY

When designing an attribute control chart, the designer has only a finite (usually small) number of distinct control limits from which to choose. By distinct control limits we mean control limits for which the ARL properties are different.

To illustrate the idea consider a binomial process which is to be controlled with a p-chart. Assume that at each epoch samples of size 50 are to be taken from a process for which 2.5% of the units are nonconforming (when the process is under control). For any k (refer to formula (1)) in the interval (2.0,3.81) the in-control ARL is 122.96 (see Table 1, where arl1_0 is the in-control ARL). Observe that in practice only two k's are likely to be useful in this example: k in the interval (2.0,3.81) and k in the interval (3.82,4.71); that is, for example, k = 3.0 and k=4.0 (the k with the "next higher" ARL).

Table 1: ARLs for k's with Distinct ARLs

n	p	k	arl1_0
50	0.025	2.00000	27.62
50	0.025	2.90582	122.96
50	0.025	3.81164	661.91
50	0.025	4.71746	4213.88
50	0.025	5.62329	31187.62

Note that for k=3 the control limits are

$$UCL_1 = .025 + 3\sqrt{\frac{.025(1-.025)}{50}} = .025 + .066 = .091$$

$$CL = .025$$

There is no lower control limit since the formula in (1) produces a negative LCL.. For k=4 the Upper Control Limit is UCL_2=.113. For many quality engineers the ARL0 of 122.96 might be too small (too many false alarms). The price associated with choosing the k with the next higher ARL is loss of power. That is, the chart with UCL of .113 will take longer on average to detect (signal) a process shift than the alternative based on k=3. Charts with randomized control limits offer a solution to this problem.

RANDOMIZED CONTROL LIMITS

A control chart with randomized control limits (henceforth denoted an RCL chart) will be illustrated with a p-chart. Consider k-sigma limits $k_1 < k_2$ and let the control limits associated with k_1 be denoted by (LCL$_1$,UCL$_1$) with corresponding signal probability ρ_1. The signal probability is he probability of an out-of-control signal from a single sample. Note that for the cases described in this paper, the signal probability is the reciprocal of the ARL. Hence, for the example above,

$$\rho_1 = 1/122.96 = .00813$$

For k_2 let the control limits be denoted by (LCL$_2$,UCL$_2$) and the signal probability be denoted by ρ_2. The following control scheme will be called a control chart with randomized control limits.

- At sample *t* calculate the sample proportion nonconforming.
- If the sample proportion nonconforming is greater than UCL2 or less than LCL2, an out-of-control signal occurs.
- If the sample proportion nonconforming is less than UCL1 and greater than LCL1, then no out-of-control signal occurs.
- If the sample proportion nonconforming is in the interval (UCL1,UCL2) or in the interval (LCL2,LCL1), which is denoted the randomization zone, then with probability *β* decide the process is out-of-control.

The chart is depicted in Figure 1. The color coding indicates the intensity of the signal.
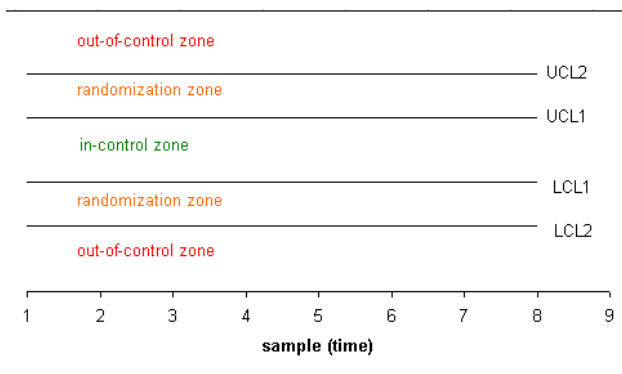

Figure 1:Schematic RCL Chart

At any epoch t, the signal probability for the RCL chart is

$$\rho = \beta \rho_1 + (1 - \beta) \rho_2 \qquad (3)$$

In order to achieve a target in-control ARL of ARL_0^* a user need only determine *β* using the formula below.

$$\rho = 1 / ARL_0^*$$

$$\beta = \frac{\rho - \rho_2}{\rho_2 - \rho_1} \qquad (4)$$

Note that the ARL for the RCL chart will always lie between the ARLs for the two standard charts used to determine *β*.

RCL CHART DESIGN USING SAS

The SAS macro program below produces output useful for designing an RCL *p*-chart with a predetermined in-control ARL.

```
/********************
SAS Macro for p-chart Profile
*
*
*********************/
%macro arlprofp(
    n_in=,      /* sample size */
    p_in=,      /* binomial proportion */
    targetarl0_in= /*target in-control ARL */
    );          /*        */


data getbeta;
    targetarl0=&targetarl0_in;
    rho=1/targetarl0;
    n=&n_in;
    p=&p_in;
    se=sqrt((p*(1-p))/n);
    kjump = 1/(n*se);
    do k=2.0 to 6.0 by kjump;
            k1=k;
            uclp1=(p+k1*sqrt((p*(1-p))/n));
            lclp1=(p-k1*sqrt((p*(1-p))/n));
            uclpy1=(p+k1*sqrt((p*(1-
p))/n))*n;
            lclpy1=(p-k1*sqrt((p*(1-
p))/n))*n;
            if lclpy1 <=0 then powp1_0=1-
probbnml(p,n,floor(uclpy1));
                    else powp1_0 = 1-
probbnml(p,n, floor(uclpy1))

    +probbnml(p,n,floor(lclpy1));
            arl1_0=1/powp1_0;

            k2=k1+kjump;
            uclpy2=(p+k2*sqrt((p*(1-
p))/n))*n;
            lclpy2=(p-k2*sqrt((p*(1-
p))/n))*n;
            if lclpy2 <=0 then powp2_0=1-
probbnml(p,n,floor(uclpy2));
                    else powp2_0 = 1-
probbnml(p,n, floor(uclpy2))

    +probbnml(p,n,floor(lclpy2));
            arl2_0=1/powp2_0;

            if arl1_0 < targetarl0 and arl2_0
> targetarl0 then
                    beta = (rho-
powp2_0)/(powp1_0-powp2_0);
                    else beta=0;
            output;
    end;

    proc print label;
    title1 'ARL0 by k-sigma limits';
    title2 "n = &n_in; p = &p_in";
    title3 "Negative LCL Implies No Lower
Control Limit  ";
    var k arl1_0 uclpy1 lclpy1 uclp1 lclp1;
    format k 4.2 arl1_0 10. uclpy1 6.2 lclpy1
6.2 uclp1 6.4 lclp1 6.4;
    label k = "k"
                    arl1_0 = "In-control ARL"
                    uclpy1 = "UCL(y)"
                    lclpy1 = "LCL(y)"
                    uclp1 = "UCL(p)"
                    lclp1 = "LCL(p)";
```

```
      run;
data usebeta1;
    set getbeta;
    if beta = 0 then delete;
    deltastop = 5*se;
    deltajump = .5*se;
    do delta = 0 to deltastop by deltajump;
    pdelta=p+delta;
    if lclpy1 <=0 then powp1_0=1-
probbnml(pdelta,n,floor(uclpy1));
                   else powp1_0 = 1-
probbnml(pdelta,n, floor(uclpy1))

    +probbnml(pdelta,n,floor(lclpy1));
    arl1_0=1/powp1_0;

    if lclpy2 <=0 then powp2_0=1-
probbnml(pdelta,n,floor(uclpy2));
                   else powp2_0 = 1-
probbnml(pdelta,n, floor(uclpy2))

    +probbnml(pdelta,n,floor(lclpy2));
    arl2_0=1/powp2_0;
    RCLARL = 1/(beta*powp1_0+(1-
beta)*powp2_0);
    ratio_k2_rcl = arl2_0/rclarl;
    output;
           end;

data usebeta2;
    set getbeta;
    if beta = 0 then delete;
    deltastop = -5*se;
    deltajump = -.5*se;
    do delta = 0 to deltastop by deltajump;
    pdelta=max(0.001,p+delta);
    if lclpy1 <=0 then powp1_0=1-
probbnml(pdelta,n,floor(uclpy1));
                   else powp1_0 = 1-
probbnml(pdelta,n, floor(uclpy1))

    +probbnml(pdelta,n,floor(lclpy1));
    powp1_0=max(powp1_0, .000001);
    arl1_0=1/powp1_0;

    if lclpy2 <=0 then powp2_0=1-
probbnml(pdelta,n,floor(uclpy2));
                   else powp2_0 = 1-
probbnml(pdelta,n, floor(uclpy2))

    +probbnml(pdelta,n,floor(lclpy2));
    powp2_0=max(powp2_0, .000001);
    arl2_0=1/powp2_0;
    RCLARL = 1/(beta*powp1_0+(1-
beta)*powp2_0);
    ratio_k2_RCL = arl2_0/rclarl;
    output;
           end;
data plotdat1;
    set usebeta1;


    proc gplot;

                 plot arl1_0*delta=1
                      arl2_0*delta = 2
                      RCLARL*delta=3
/overlay

                 haxis=axis2
```

```
             /* annotate=bars */
             legend;
      symbol1 c=BLUE,i=join, l=14, v=none;
      symbol2 c=BLUE, i=join, l=1, v=none;
      symbol3 c=BLACK, i=join, l=2 v=none;
         title1 "ARL Profile for RCL
Chart:Upward Shift";
       title2 "n = &n_in; p = &p_in";
         title3 "RCL Chart has In-control
ARL at Target ARL = &targetarl0_in";
         title4 "arl1_0 = ARL(k1); arl2_0
= ARL(k2)";


    proc gplot ;
         plot ratio_k2_RCL*delta;
         title1 "ARL Profile for RCL
Chart: Upward Shift";
         title2 "n = &n_in; p = &p_in:
Ratio=ARL(k2)/ARL(RCL)";
         title3 "RCL p-Chart ARL Ratio
with Target In-Control ARL(RCL) =
&targetarl0_in";
         run;


data plotdat2;
    set usebeta2;
    proc gplot;
                 plot arl1_0*delta=1
                      arl2_0*delta = 2
                      RCLARL*delta=3
/overlay
             haxis=axis2
             /* annotate=bars */
             legend;
      symbol1 c=BLUE,i=join, l=14, v=none;
      symbol2 c=BLUE, i=join, l=1, v=none;
      symbol3 c=BLACK, i=join, l=2 v=none;
         title1 "ARL Profile for RCL
Chart:Downward Shift";
       title2 "n = &n_in; p = &p_in";
         title3 "RCL Chart has In-control
ARL at Target ARL = &targetarl0_in";
         title4 "arl1_0 = ARL(k1); arl2_0
= ARL(k2)";

    proc gplot ;
         plot ratio_k2_RCL*delta;
         title1 "ARL Profile for RCL
Chart: Downward Shift";
         title2 "n = &n_in; p = &p_in:
Ratio=ARL(k2)/ARL(RCL)";
         title3 "RCL p-Chart ARL Ratio
with Target In-Control ARL(RCL) =
&targetarl0_in";
         run;
data alldata;
    set usebeta1 usebeta2;
    proc print label;
    title1 "ARL Profile for RCL Chart";
    title2 "n = &n_in; p = &p_in";
    title3 "Ratio = ARL(k2)/(ARL(RCL))";
    title4 "k2 is smallest k such that ARL  >
Target ARL of &targetarl0_in ";

var k1 k2 beta lclpy1 lclpy2 uclpy1
uclpy2 delta pdelta arl1_0 RCLARL arl2_0
         ratio_k2_rcl targetarl0;
    format k1 k2 lclpy1 lclpy2 delta 5.4
pdelta 5.4 arl1_0 7.2 RCLARL 7.2 arl2_0 7.2
         ratio_k2_rcl 5.2 beta targetarl0
7.2;
```

```
    label pdelta = "p + delta"
arl1_0 = "ARL(k1)"
RCLARL = "ARL(RCL)"
arl2_0 = "ARL(k2)"
ratio_k2_RCL = "Ratio:ARL(k2)/(RCL ARL)"
lclpy1 = "LCL(k1)"
uclpy1 = "UCL(k2)"
lclpy2 = "LCL(k2)"
uclpy2 = "UCL(k2)";        ;

    run;
%mend arlprofp;
%arlprofp(n_in=300,p_in=0.025,targetarl0_in=3
70);
```

Program Output

Table 2 :In Control ARLs for Available k

```
                  ARL0 by k-sigma limits                    08:54
                     n = 300; p = 0.025
          Negative LCL Implies No Lower Control Limit

          In-control
      k       ARL      UCL(y)    LCL(y)    UCL(p)    LCL(p)
   2.00        17       12.91      2.09    0.0430    0.0070
   2.37        41       13.91      1.09    0.0464    0.0036
   2.74       101       14.91      0.09    0.0497    0.0003
   3.11       244       15.91     -0.91    0.0530   -.0030
   3.48       592       16.91     -1.91    0.0564   -.0064
   3.85      1519       17.91     -2.91    0.0597   -.0097
   4.22      4108       18.91     -3.91    0.0630   -.0130
   4.59     11702       19.91     -4.91    0.0664   -.0164
   4.96     35029       20.91     -5.91    0.0697   -.0197
   5.33    109989       21.91     -6.91    0.0730   -.0230
   5.70    361635       22.91     -7.91    0.0764   -.0264
```

Suppose the process has known proportion nonconforming 0.025, that samples of size 300 are to be selected at each epoch, and target in-control ARL is 370.. Then the macro statement is
`%arlprofp(n_in=300,p_in=0.025,targetarl0_in=370);`
From Table 2 one can see that a chart with k = 2.0 has p-chart control limits (0.007,0.043) and the in-control ARL of 17. Note that the control limits in terms of the number of nonconforming units (y) is also given. In order to get an in-control ARL of reasonable size a p-chart with n=300 will be one sided (indicated by the negative LCL). Likely standard p-chart choices for k are 3.11 (ARL0 = 244) and k = 3.48 (ARL0 = 592). If a target in-control ARL of at least 370 is desired then the latter choice is necessary and the corresponding in-control ARL is 592.

Additional SAS output is displayed in Table 3. The output has been split to fit on the printed page.
1. The basic effectiveness of the sampling plan
2. the benefit associated with an RCL chart
3. the parameters and control limits for the RCL chart
Values of k for two standard charts are presented (k1 = 3.109 and k2=3.479). The p-chart upper control limits are in Table 2 (Table 3 has the control limits for y). and are 0.0530 and 0.0564, respectively.   The table displays ARLs for the in-control process (delta = 0 and p =0.025) and ARLs for an upward shift of delta. For example, for a process shift of 0.009 (the process is out-of-control and currently 3.4% of units being produced are nonconforming) the k1 p-chart has ARL 18.85 and the k2 p-chart has ARL 34.08. An RCL p-chart with

Table 3: ARL Profile

```
                ARL Profile for RCL Chart
                   n = 300; p = 0.025
                Ratio = ARL(k2)/(ARL(RCL))
     k2 is smallest k such that ARL  > Target ARL of 370


    k1     k2    beta  LCL(k1)  LCL(k2)  UCL(k1)  UCL(k2)  delta
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0000
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0045
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0090
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0135
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0180
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0225
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0270
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0315
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0361
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0406
  3.109  3.479  .4221   -.908    -1.91    15.91    16.91    .0451

                                                   Ratio:
              p +                                  ARL(k2)/(
   delta     delta    ARL(k1)    ARL(RCL)   ARL(k2)  RCL ARL)

   .0000     .0250    244.39     370.00    592.36    1.60
   .0045     .0295     56.44      80.43    116.65    1.45
   .0090     .0340     18.85      25.42     34.08    1.34
   .0135     .0385      8.27      10.60     13.34    1.26
   .0180     .0430      4.46       5.46      6.53    1.20
   .0225     .0475      2.82       3.31      3.80    1.15
   .0270     .0520      2.02       2.29      2.53    1.11
   .0315     .0565      1.59       1.74      1.88    1.08
   .0361     .0611      1.34       1.44      1.52    1.06
   .0406     .0656      1.20       1.26      1.31    1.04
   .0451     .0701      1.11       1.15      1.18    1.03
```

beta = 0.4221 has ARL of 25.42 for a process shift of 0.009 (in-control ARL is 370). A key point is the loss of power associated with using the k2 limits. The ratio ARL(k2)/ARL(RCL) measures the loss of power associated with using the standard p-chart with longer than required in-control ARL.. For a shift of 0.009 the ratio is 1.34, indicating that if this shift occurs it will take on average 34% longer to detect the shift using the k2 charts compared to using the RCL chart.
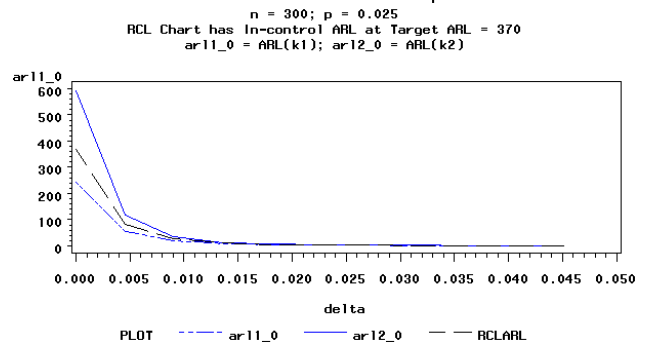

Figure 2: ARL Profile Graph

Using Figure 2 one can visually assess the ARL differences between the three alternatives. The RCL p-chart can be more easily assessed using a ratio graph such as the one depicted in Figure 3, below.
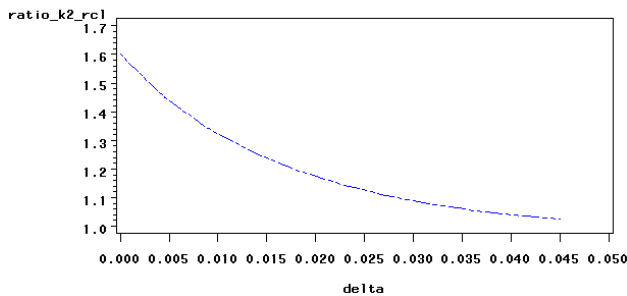
Figure 3: Ratio Graph

Note that for a shift of 0.015 that the ratio is about 1.25 (that is, it takes about 25% more samples to detect this shift using the k2 chart).

Chart Operation

A generic control chart with randomized control limits is given in Figure 1. Other options include a chart that has only one upper and one lower control limits line (the line is broken appropriately. If a process engineer develops the RCL chart for use by operations personnel then a chart which identifies which set of control limits are in effect for each epoch in a set of periods (a week, a months, whatever). When software is used so that data is entered from terminals the program can provide the appropriate control limits after performing a random experiment. Proc Shewhart in SAS-QC can be modified to produce RCL charts (see Wludyka (2003)).

RCL C-CHART

When one wants to control the number of nonconformities per sampling unit a c-chart may be used (see equation (2). A SAS Macro that produces an ARL profile for a c-chart follows. Input for the macro is c, the (known) average rate at which nonconformities occur, and the target in-control ARL. The macro is similar to the one for the p-chart

```
/********************
SAS Macro for c-chart Profile
*
*
********************/
%macro arlprofc(
   n_in=,      /* sample size (use 1) */
   c_in=,      /* Poisson average */
   targetarl0_in= /*target in-control ARL */
   );          /*       */


data getbeta;
     targetarl0=&targetarl0_in;
     rho=1/targetarl0;
     n=&n_in;
     c=&c_in;
     se=sqrt(c);
     kjump = 1/se;
     do k=2.0 to 6.0 by kjump;
             k1=k;
             uclc1=c+k*se;
             lclc1=c-k*se;
             if lclc1 <=0 then powc1_0=1-
poisson(c,floor(uclc1));
                    else powc1_0 = 1-
poisson(c,floor(uclc1))
```

```
             +poisson(c,floor(lclc1));
        arl1_0=1/powc1_0;

        k2=k1+kjump;
        uclc2=c+k2*se;
        lclc2=c-k2*se;
        if lclc2 <=0 then powc2_0=1-
poisson(c,floor(uclc2));
                else powc2_0 = 1-
poisson(c,floor(uclc2))
                +poisson(c,floor(lclc2));
        arl2_0=1/powc2_0;

        if arl1_0 < targetarl0 and arl2_0
> targetarl0 then
                beta = (rho-
powc2_0)/(powc1_0-powc2_0);
                else beta=0;
        output;
   end;

   proc print label;
   title1 'c-Chart: ARL0 by k-sigma limits';
   title2 "n = &n_in; c = &c_in";
   title3 "Negative LCL Implies No Lower
Control Limit   ";
   var k arl1_0 lclc1 uclc1   ;
   format k 4.2 arl1_0 10. uclpy1 6.2 lclpy1
6.2 uclp1 6.4 lclp1 6.4;
   label k = "k"
                arl1_0 = "In-control ARL"
                uclc1 = "UCL(c)"
                lclc1 = "LCL(c)"
                ;
   run;
data usebeta1;
   set getbeta;
   if beta = 0 then delete;
   deltastop = 5*se;
   deltajump = .5*se;
   do delta = 0 to deltastop by deltajump;
   cdelta=c+delta;
   if lclc1 <=0 then powc1_0=1-
poisson(cdelta,floor(uclc1));
                else powc1_0 = 1-
poisson(cdelta,floor(uclc1))

   +poisson(cdelta,floor(lclc1));
        arl1_0=1/powc1_0;

   if lclc2 <=0 then powc2_0=1-
poisson(cdelta,floor(uclc2));
                else powc2_0 = 1-
poisson(cdelta,floor(uclc2))

   +poisson(cdelta,floor(lclc2));
        arl2_0=1/powc2_0;
   RCLARL = 1/(beta*powc1_0+(1-
beta)*powc2_0);
   ratio_k2_rcl = arl2_0/rclarl;
   output;
        end;

data plotdat1;
   set usebeta1;



   proc gplot;

             plot arl1_0*delta=1
                   arl2_0*delta = 2
```

```
                            RCLARL*delta=3
    /overlay


             haxis=axis2
             /* annotate=bars */
             legend;
      symbol1 c=BLUE,i=join, l=14, v=none;
      symbol2 c=BLUE, i=join, l=1, v=none;
      symbol3 c=BLACK, i=join, l=2 v=none;
            title1 "ARL Profile for RCL c-
Chart:Upward Shift";
         title2 "n = &n_in; c = &c_in";
            title3 "RCL Chart has In-control
ARL at Target ARL = &targetarl0_in";
            title4 "arl1_0 = ARL(k1); arl2_0
= ARL(k2)";


     proc gplot ;
            plot ratio_k2_RCL*delta;
            title1 "ARL Profile for RCL c-
Chart: Upward Shift";
            title2 "n = &n_in; c = &c_in:
Ratio=ARL(k2)/ARL(RCL)";
            title3 "RCL p-Chart ARL Ratio
with Target In-Control ARL(RCL) =
&targetarl0_in";
            run;


data alldata;
     set usebeta1 ;
     proc print label;
     title1 "ARL Profile for RCL c-Chart";
     title2 "n = &n_in; c = &c_in";
     title3 "Ratio = ARL(k2)/(ARL(RCL))";
     title4 "k2 is smallest k such that ARL  >
Target ARL of &targetarl0_in ";

     var k1 k2 beta lclc1 lclc2 uclc1 uclc2
            delta cdelta arl1_0 RCLARL arl2_0
            ratio_k2_rcl targetarl0;
     format k1 k2 beta lclc1 lclc2 uclc1 uclc2
            delta cdelta arl1_0 RCLARL arl2_0
            ratio_k2_rcl targetarl0;
     label pdelta = "c + delta"
              arl1_0 = "ARL(k1)"
              RCLARL = "ARL(RCL)"
              arl2_0 = "ARL(k2)"
              ratio_k2_RCL =
"Ratio:ARL(k2)/(RCL ARL)"
                   lclc1 = "LCL(k1)"
                   uclc1 = "UCL(k1)"
                   lclc2 = "LCL(k2)"
                   uclc2 = "UCL(k2)";

     run;
%mend arlprofc;
%arlprofc(n_in=1,c_in=2.5,targetarl0_in=370);
```

For example, if on average there are 2.5 nonconformities per sampling unit and one wants an in-control ARL of 300, one would use:
```
%arlprofc(n_in=1,c_in=2.5,targetarl0_in=300)
Output:
```

Table 4: C-Chart Profile

The effectiveness of using an RCL c-chart in this example can be assessed using Figure

Table 5: RCL Profile for c-Chart
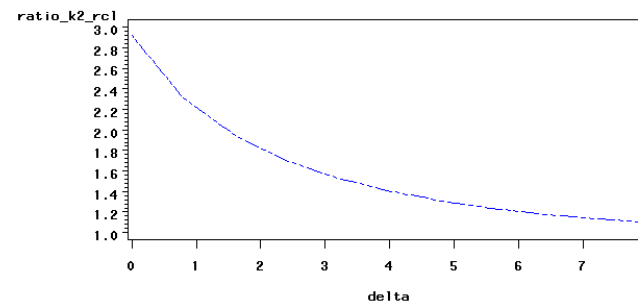
STAGE ONE PROCESS CONTROL
During Stage One of process control the goal is to determine whether the process is under initial control and using measurements collected from the in-control process estimate process parameters required to monitor the process. If initially the process is not in-control, steps must be taken to bring the process under control.

A direct approach is to collect m = 20 or so samples from the process and use this data to determine a subset of in-control observations. Typically one estimates the key parameters from an initial data set, fits control limits based on this data, and then looks retrospectively for points that plot outside the control limits. These are removed and iteratively one establishes a set ob in-control observations.

When RCL control limits are an option the author recommends using a chart in the format of Figure 1.
1. Search for assignable causes for any points that plot in the out-of-control zone. Delete those points for which assignable causes can be found and calculate parameter estimates based on the remaining points.
2. If, based on the control limits from step (1), any points plot in the randomization zone, search for assignable causes for these points; delete those points for which assignable causes can be found. <u>Note: for an in-control process the probability that a point plots in the randomization zone is small</u>.
3. Use those points remaining after step 3 to determine preliminary control limits.

CONCLUSION

Two macros are described that produce output useful in designing RCL control charts for Binomial and Poisson processes.

REFERENCES

Montgomery, D. C. (2000). *Introduction to Statistical Quality Control*, 4th ed., John Wiley and Sons, New York.

Wludyka, Peter. 2003. "SAS Programs for Operating RCL p-Charts and c-Charts." The source code can be downloaded from the University of North Florida Center for Research and Consulting in Statistics web page (www.unf.edu/coas/math-stat/CRCS) as technical report #021503.

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