

Paper 105-28

Randomized Rounding

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ABSTRACT

How do you apply a small growth, e.g. 3% on small integer values? Standard rounding distorts the results: any value up to a certain point (in our case 16) is rounded down, any value above that point is rounded up, and so on. Randomized rounding provides a method to simulate a growth for integer values that prevents this distortion.

INTRODUCTION

In the airline industry, business class passengers are very important. Their numbers per flight are normally rather low, a few tens at most, but the revenue they bring in is quite substantial. In a recent study I had to estimate the effect of a growth of 3% of the business class traffic. In this study I replayed the reservations process of a series of actual flights, but with a passenger demand which was 3% higher than the actual demand. The straightforward approach is to take all the flights, add 3% to the number of passengers, round the figure and use that as input in the analysis and that is where the trouble starts...

STANDARD ROUNDING

As mentioned in the introduction, the straight-forward approach to the problem is to take the original demand, apply the growth and round it:

```
NewDemand = ROUND(OriginalDemand * 1.03);
```

Table 1 includes some statistics of the original flights and the unrounded and rounded result after applying 3% growth.

	Mean	STD	Median	Sum
Original number of passengers	14.23	3.29	14	3201
3% growth, before rounding	14.65	3.39	14.42	3297.03
3% growth after normal rounding	14.46	3.63	14	3253

Table 1: Summary statistics regarding the original number of passengers per flight, the effects after applying a 3% growth and what happens to them after normal rounding.

What you see in table 1 is that the rounded numbers, after applying the growth, do not represent the original. In comparison with the un-rounded numbers, the mean went down, the STD went up and the total number of passengers only represents a growth of 1.6% rather than the planned 3%.

The simple explanation is that for all flights with 16 or less passengers the new number is rounded down and for all flights with 16 or more passengers the number is rounded up. So the mix of flights determines the result: in the extreme situation that all flights would have less than 17 passengers, all growth is lost!

RANDOMIZED ROUNDING

To avoid this problem you can apply "randomized rounding": rounding up or down occurs with a probability equal to the fraction of the new number, e.g. 14.3 has a probability of .3 to be rounded up and .7 to be rounded down.

In the SAS System you can achieve that with the following statement:

```
Random_rounded = FLOOR(unrounded) +
  (UNIFORM(0) < unrounded - FLOOR(unrounded));
```

The statement makes use of the fact that TRUE and FALSE have

the value of respectively 1 and 0 in SAS. UNIFORM(0) draws a uniformly distributed random number between 0 and 1. If the drawn number is smaller than the fraction of the unrounded number the comparison is TRUE and the result is rounded up, otherwise it is rounded down.

Table 2 contains the same flights as table 1, but now with the randomized-rounded figures.

	Mean	STD	Median	Sum
Original number of passengers	14.23	3.29	14	3201
3% growth, before rounding	14.65	3.39	14.42	3297.03
3% growth after randomized rounding	14.61	3.48	14	3287

Table 2: The same figures as in table 1, but the last row now shows the results of randomized rounding. It matches the unrounded figures much better than the standard rounded figures.

To make the randomized rounding easier to use, you can of course put it in a macro:

```
%macro RanRound(roundvar);
  FLOOR(&roundvar) +
  (UNIFORM(0) lt &roundvar -
  FLOOR(&roundvar));
%mend;
```

which can be used as in:

```
Ranrounded = %RanRound(unrounded);
```

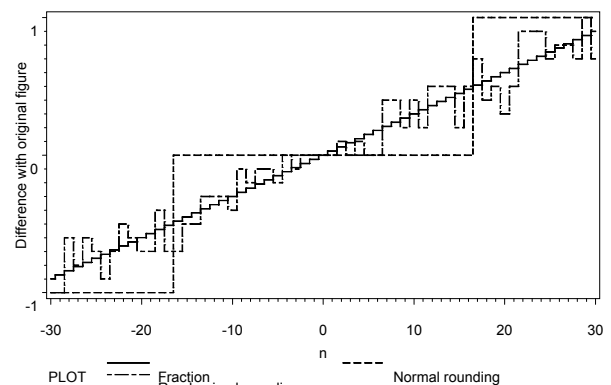
SIMULATION

To illustrate the effect in another way, we did an experiment where the original value stepped from -30 to 30 and applied a growth of 3% to it.

Next we looked at the average difference from the original value of the unrounded, the rounded and the randomized-rounded values resulting from 10 repetitions of the experiment.

The result is in plot 1. You see the fractions grow with little steps from -0.9 to +0.9 and the normal rounded result in three steps from -1 to 0 to +1.

The average value of the randomized rounded result follows the original unrounded steps, but with fluctuations that represent the random effects.



Plot 1: The difference between the original value (n) and the unrounded new value and the various rounded results. This is based on the mean values of 10 experiments with the original values stepping from -30 to 30 and applying a growth of 3%.

CONCLUSION

The randomized rounding method is a simple and effective method to apply small growth factors on small integer values. By nature it is a simulation method, with the random effects that come with that.

The randomness of the method has as an implication that running the same model again, will lead to a slightly different result. A solution for that is to determine the seed for the UNIFORM function yourself, rather than let SAS determine it, as is the case when the argument in the function is zero.

REFERENCES

More information about the FLOOR and UNIFORM functions:
SAS Language Reference: Dictionary, Version 8, Volume 1:
page 367 (FLOOR) and page 574 (UNIFORM)
More information about the SAS macro language:
SAS Macro Language: Reference, Version 8

An introduction to both can be found in the BBU publication:
Working with the SAS System.

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CONTACT INFORMATION

Your comments and questions are valued and encouraged.

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