Paper 263-26

New Methods for Modeling Reliability Using Degradation Data

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ABSTRACT

Engineers and researchers use life tests in a wide variety of industrial applications to determine how well critical components and materials will perform under different operating conditions. By estimating the failure time distribution, decisions can be made that influence the design, improvement, and storage of products so they meet customer requirements. Manufacturer warranty decisions may also be influenced by knowledge of product reliability performance.

Traditional life tests are often not the most efficient way to obtain reliability information because few, if any, actual failures are observed. On the other hand, it is often possible to obtain pseudo-failure data using degradation measurements. Consequently, there is growing interest in studying the degradation of product performance or material properties over time.

In this paper we describe two methods for the analysis of such data. The first uses the SAS/QC® RELIABILITY procedure with pseudo-failure times to predict a given percentile of the failure time distribution, and the second uses the SAS/STAT® NLMIXED procedure to fit concave degradation models, and some SAS® macros to estimate the failure distribution. These methods are illustrated with a degradation study on the strength of an electronic material, which was subjected to four different temperatures and measured over time. We compare the two approaches, and show the advantages of using concave degradation models in this setting.

1. INTRODUCTION

An engineering team was put together to determine the shelf life of a dielectric used in electronic components. The study and understanding the shelf life of this product, as well as how it degrades with time and temperature, will allow us to define internal practices and make customer recommendations based on reliability data. It was the goal of the team to answer the following questions:

1. What is the proper temperature for storing the material?, and
2. How long will the material last at a given temperature?

An experiment was designed to answer these questions using degradation data, and is described in Section 2. In Section 3 we show how degradation data can be analyzed using traditional reliability methods, while in Section 4 we show how to fit degradation models using non-linear mixed models, we then use these models to obtain an estimate of the failure time distribution. In section 5 we compare the two approaches.

2. EXPERIMENTAL DESIGN

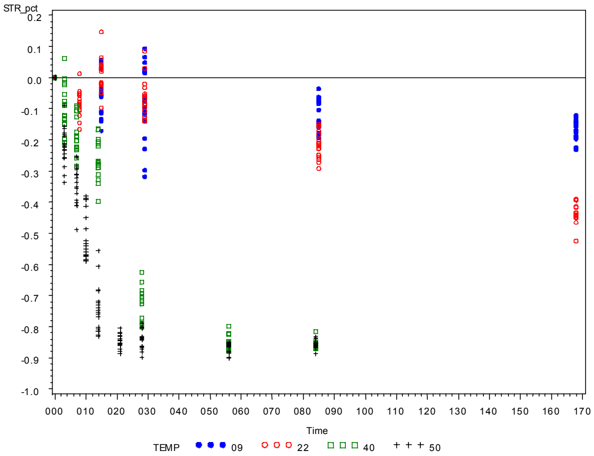
For the accelerated degradation study (ADT) four different temperatures were selected. Two temperatures that are common storage conditions: refrigeration (9°C) and room temperature (22°C), and two temperatures (40°C & 50°C) chosen to accelerate the degradation process. Sampling in time was less frequent at low temperatures, more frequent at high temperatures. The study was designed to last a year but enough information to answer the above questions was obtained before that.

Three lots of material were used in the study, and six units per lot were randomly selected for a given temperature and time interval. All the samples were sealed in metalized Mylar bags to avoid issues with humidity.

This experiment was designed to track several key responses of the dielectric as a function of time and temperature, but in this paper will concentrate on one of the strength properties of this product. An internal test was developed to measure the strength of the material.

Strength Test: To measure the strength the material is held across a jig using double-stick tape. A ball probe is lowered until contact is made with the sample. Once in contact, the probe is accelerated through the sample at 10 mm/sec. The force required by the probe to rupture the material is then recorded. The properties of the material change quite rapidly with temperature. To adjust for material variability the initial strength of each sample was measured, and used as a baseline. The response is the amount of strength loss, in percent, from the initial condition.

Figure 1 shows the percent reduction in strength as a function of time for the four different temperatures. The graph shows the typical concave degradation behavior at high temperatures; i.e., 40ºC and 50ºC.



Fig

Figure 1. Percent reduction in strength as a function of time for four temperatures.

Since this is a destructive test a “sample” was defined as a collection of siblings from the same locations within a lot. This within sample variability is considered to be negligible.

3. FAILURE-TIME ANALYSIS OF ADT DATA

Traditional failure-time analysis of accelerated degradation data can be performed using pseudo-failure times. Remember that in ADT studies we monitor the degradation or performance of a material or product over time. In other words, we do not record actual failure times but the value of a response over time. By selecting a critical degradation point for the response of interest pseudo-failure times can be computed by observing when a given sample reaches this critical point. It is important to note that these pseudo-failure times are dependent on the chosen cutoff point; i.e., changing the cutoff point changes the pseudo-failure time for a given sample. The decision on what the critical degradation point is should be based on engineering considerations. One also needs to consider the direction of the degradation process over time; i.e., does the response increase or decrease with time?

The analysis using pseudo-failure times has basically two steps:

1. For each sample one calculates the pseudo-failure time at which the sample crosses the given cutoff point.
2. These pseudo-failure times are then used to estimate the failure distribution F(t).

A SAS macro was written to interpolate the pseudo-failure time at which a given sample was less than, or greater than, a critical degradation level. This macro takes a SAS data set with the degradation information and creates another SAS data set with pseudo-failure times and censoring information. The macro uses linear interpolation between the two times at which the crossing may have occurred.

Figure 2 shows the resulting pseudo-failure times for a cutoff point of 35% reduction in initial strength. Note that at high temperatures, 40ºC and 50ºC, the pseudo-failure times are less than 30 time units, while at 9ºC all the pseudo-failure times are censored; i.e., none of the samples had responses less than the cutoff point.

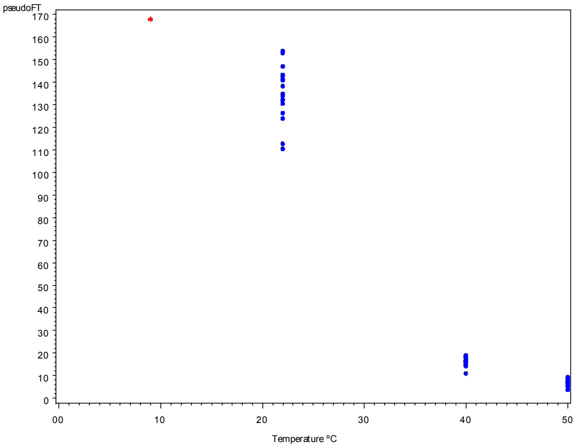


Figure 2. Pseudo-Failure times for a 35% reduction in initial strength.

Once the pseudo-failure times are calculated PROC RELIABILITY can be used to estimate the failure time distribution F(t). The first step in the analysis is to investigate which distribution fits the data best.

Figure 3 shows Weibull probability plots for each temperature. Note that for each temperature the pseudo-failure times seem to follow a straight line, indicating that the Weibull distribution does a good job in fitting the data for all temperatures. Based on this we can now estimate the “life” of the dielectric for a given percentile.

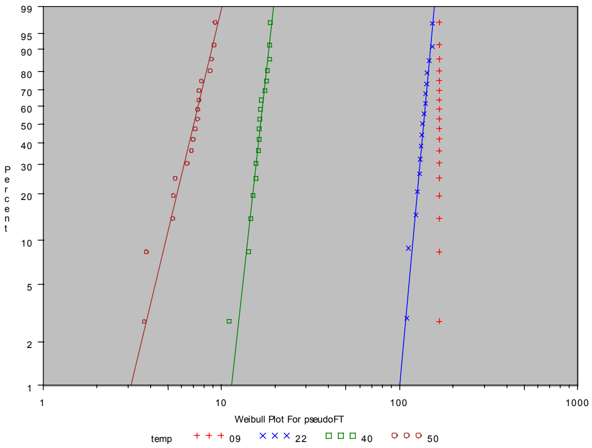


Figure 3. Weibull probability plots for pseudo-failures.

In version 8 of SAS, PROC RELIABILITY is used to predict the life of the dielectric at a given temperature for a given percentile, assuming a Weibull distribution and an Arrhenius relation between time and temperature. For this application we wanted to know the predicted time at which 1% and 95% of the population would fail, when the material was stored at 9ºC. A 95% confidence interval for the predicted times was required to assess our predictions.

The results indicate that for 9ºC storage temperature, 1% of the dielectric material will survive for 313 time units, with a 95% confidence interval [262; 373]. The predicted failure time for 95% of the population is 742 time units, with a 95% confidence interval [664; 830]. The output also shows the parameter estimates for the Arrhenius-Weibull model.

```
                              Model Information

                Input Data Set                          WORK.SUGI26_FT
                Analysis Variable                              pseudoFT
                Relation                   Arrhenius (Activation Energy)
                Censor Variable                                  censor
                Distribution                                    Weibull


                             Algorithm converged.


                                Summary of Fit

                      Observations Used                    71
                      Uncensored Values                    53
                      Right Censored Values                18
                      Missing Observations                  2
                      Maximum Loglikelihood          16.84626


                          Weibull Parameter Estimates
                               Asymptotic Normal
                                      Standard      95% Confidence Limits
       Parameter          Estimate       Error         Lower         Upper

       Intercept          -28.6692      0.5013      -29.6516      -27.6867
       temp                 0.8537      0.0134        0.8275        0.8799
       EV Scale             0.1519      0.0162        0.1233        0.1871
       Weibull Shape        6.5848      0.7007        5.3452        8.1118


                            Observation Statistics
    pseudoFT      censor        temp       PCNTL      STDERR       LOWER       UPPER
       .             .            9   312.51637     28.3986   261.53116   373.44109  1%
       .             .            9   742.39798    42.07849   664.34152   829.62564 95%
```

A very useful tool for assessing the fit of the model, and also to predict failure times, is a relation plot (see Nelson (1990), Chapter 3). This plot, Figure 4, shows again the good fit obtained by using the Weibull distribution. In the right-hand-side plot the vertical line shows were the 1% and 95% lines intersect the 9ºC temperature. By projecting onto the vertical axis one can read-off the 313 and 742 failure time predictions for these two percentiles.

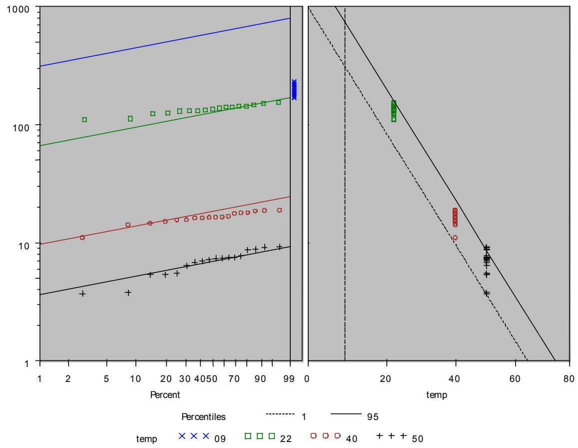


Figure 4. Relation plot for dielectric strength data.

The parameter estimates from the Arrhenius-Weibull model can be used to estimate the temperature needed so that a given percentile of the population will survive a given number of time units. For example, let us suppose that we want to predict what the required temperature would be so that 1% of the population of our dielectric material will survive at least 280 time units. Again, we are assuming that a failure is defined as the point in time at which a sample was below the cutoff point of 35% reduction in initial strength. These type of calculations are useful when, for example, one needs to figure out appropriate storage condition for a given product. The required temperature, so that a given percentile of the population will survive a given number of time units, can be easily computed using the values of intercept (-28.6692), temperature (0.8537), and Weibull shape (6.5848) obtained from the PROC RELIABILITY fit.

```
/**Temperature for Arrhenius-Weibull Fit for a
given percentile**/
%let b0      = -28.6692 ;
%let b1      =   0.8537 ;
%let wshape =   6.5848 ;
%let D_time = 280       ;
%let pct     =   1      ;
data _null_ ;
D_TEMP=round(11605*&b1/(log(&D_Time/(-log(1-
&pct/100))**(1/&wshape))
             - &b0) - 273.15 , 1.0) ;
put "Design Temperature for &pct% Life at
&D_Time Days= " D_TEMP "C";
run ;
```

giving a predicted storage temperature of 10°C.

```
Design Temperature for 1% Life at 280 Days= 10 C
NOTE: DATA statement used:
      real time           0.05 seconds
      cpu time            0.00 seconds
```

4. ANALYSIS OF ADT DATA USING CONCAVE DEGRADATION MODELS

Degradation data can provide more information than the traditional censored-failure data. Figure 1 shows that the percent reduction in strength decreases with time until it reaches an asymptote or limit degradation, for 40ºC and 50ºC. This type of degradation over time can be approximately described by a concave model of the type:

$$Degradation(time) = -e^{\beta_2} * \left(1 - e^{-e^{\beta_1} * AF(Temp) * time}\right) \qquad (1)$$

Here e^{β_2} describes the asymptote; e^{β_1} is the rate reaction at a baseline temperature (9ºC in our case); and AF(*Temp*) is the Arrhenius acceleration factor for a given temperature, *Temp,* with respect to the baseline temperature (9ºC in our case), and activation energy β_3:

$$AF(Temp) = e^{\beta_3\left[\frac{11605}{9+273.15} - \frac{11605}{Temp+273.15}\right]} \qquad (2)$$

Traditionally PROC NLIN can be used to fit the non-linear concave degradation model of equation (1). However, in this type of studies the samples are not identical; i.e., there is sample-to-sample variability that needs to be taken into account to get a better estimate of the standard deviations of parameter estimates. These non-linear degradation models are very similar in nature to non-linear growth curves in which different subjects are observed over time, and the subject-to-subject variation needs to be taken into account.

In version 8e of SAS we have available a new procedure, PROC NLMIXED, to fit non-linear mixed models in which fixed and random effect can enter non-linearly into a given model. For the percent reduction in strength, the parameters of Equation (1) were estimated using PROC NLMIXED. The PROC NLMIXED output gives parameter estimates for the parameters β_1, β_2, and β_3; the variances and covariance for β_1 and β_2; as well as an estimate for the overall variance. These are summarized in the table below.

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
mb1	-7.0378	0.07999	70	-87.98	<.0001	0.05	-7.1974	-6.8783	0.020196
sb1	0.02192	0.01817	70	1.21	0.2317	0.05	-0.01432	0.05816	15.23754
mb2	-0.09659	0.0102	70	-9.47	<.0001	0.05	-0.1169	-0.07625	-0.0875
sb2	0.000014	0.000074	70	0.18	0.8546	0.05	-0.00013	0.000161	3364.48
c21	0.000545	5.97E-06	70	91.41	<.0001	0.05	0.000533	0.000557	-332.319
b3	0.9039	0.01927	70	46.9	<.0001	0.05	0.8655	0.9424	-0.0965
s2e	0.005148	0.000381	70	13.5	<.0001	0.05	0.004387	0.005908	-10.9751

From these values we get an estimate of the asymptote, e^{β_2}, equal to -exp(-0.0965) = –90% reduction in strength, and an activation energy, β_3, of 0.9039.

The estimated values along with Equation (1) can be used to obtain predicted curves for %reduction in strength, for each of the four temperatures. Figure 5 shows the fitted curves using the predicted values from the concave degradation mixed model fitted with PROC NLMIXED. Note how the 50ºC curve has an asymptotic behavior of about –90% reduction in strength.

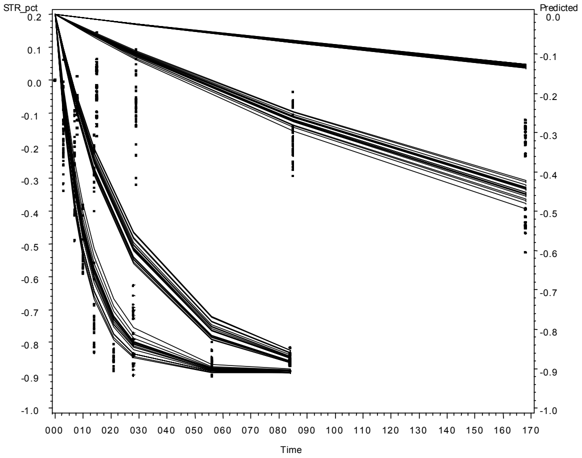


Figure 5. Concave degradation curves from PROC NLMIXED

Computing the Failure Time Distribution F(t).

One can compute the failure time distribution F(t) of the pseudo-lifetimes by inverting Equation (1) and using algorithm 13.1, page 328. of Meeker and Escobar (1998). Bootstrap confidence limits for F(t) using the estimated parameters from model (1) can also be computed by following the procedure described in Algorithm 13.3, page 332, of Meeker and Escobar (1998), as outlined in the following steps.

1. Simulate a large number of bootstrap samples of degradation data using the same experimental design as in the original experiment.
2. Fit the model (1) to each sample using PROC NLMIXED and compute F(t) for each sample using the procedure described previously.
3. For each t, sort the estimated values of F(t) in increasing order.
4. The upper and lower bootstrap confidence limits are the *u*th and *l*th ordered values of F(t), where the indices *u* and *l* are computed from formulas given by Meeker and Escobar (1998).

A SAS macro was written to compute F(t) and bootstrap confidence limits, using DATA step programs, IML, and PROC NLMIXED.

Figure 6 displays the resulting estimated failure time distribution F(t) along with 95% bootstrap confidence bands for a cutoff point of 35% strength reduction and a temperature of 9ºC. The 1% percentile of the estimated failure distribution is 386 time units, with a 95% confidence interval [199, 479], and the 95% percentile of the estimated failure distribution is 714 time units, with a 95% confidence interval [608, 796].

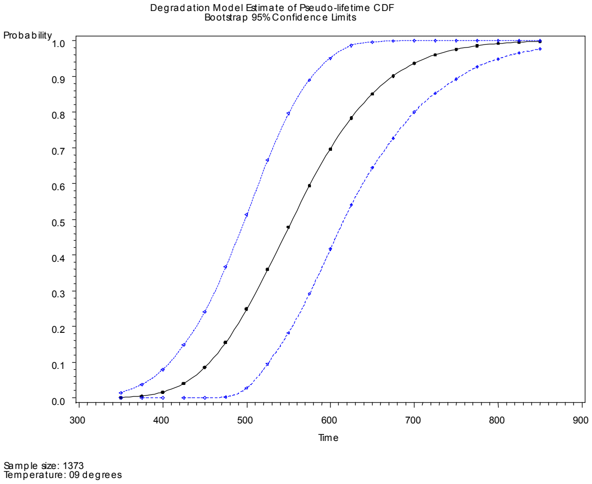


Figure 6. Degradation model estimate of pseudo-lifetime distribution with two-sided 95%
Bootstrap confidence bands.

5. PSEUDO-FAILURES OR DEGRADATION MODELS?

Figure 5 shows that many of the samples had not reached the 35% cutoff reduction in initial strength at the end of the experiment, especially at the lower temperatures. Therefore these units were treated as censored data in the traditional failure-time analysis of pseudo-lifetimes described in Section 3. In other words, the distribution of the degradation paths with degradation less than the 35% cutoff, was ignored in the analysis, thus information was lost. The degradation analysis described in Section 4 uses all of the information in the degradation data, since it directly models the degradation process. The degradation analysis approach should provide more accurate results, especially when extrapolation beyond the cutoff point is required.

On the other hand, the degradation analysis approach is more computationally intensive. Fitting the model of Equation (1) by maximum likelihood requires the solution of a difficult optimization problem. PROC NLMIXED automates the estimation procedure to a great extent, but there still can be problems in finding good initial parameter estimates so the fitting algorithm will converge. Bootstrap confidence limits require fitting the model to a large number of simulated data sets, which can be time-consuming.

The following table shows the 1% and 95% estimates, along with 95% confidence intervals for the two approaches at a storing temperature of 9ºC.

Method	Percentile	Lower 95% CI	Estimate	Upper 95% CI
Pseudo-Failures	1%	262	313	373
Degradation Model	1%	199	386	479
Pseudo-Failures	95%	664	742	830
Degradation Model	95%	608	714	795

Note that the intervals for the degradation analysis are wider than those obtained using the pseudo failure times because they contain the sample-to-sample variation. This is especially apparent for the 1 percentile where a lot of censoring is present.

As a general recommendation we suggest using:

1) The pseudo failure time analysis when a quick and approximate analysis is desired, and there is little censoring involved, and
2) The degradation analysis approach when a more detailed analysis of the degradation data is required, at the expense of a more computationally burdensome approach.

See section 13.8 of Meeker and Escobar (1998) for a useful discussion on the limitations of the pseudo-failure time analysis as compared to the degradation analysis.

REFERENCES

Meeker, W.Q., and Escobar, L.A. (1998), *Statistical Methods for Reliability Data*, New York: John Wiley & Sons.

Nelson, W. (1990) *Accelerated Testing: Statistical Models, Test Plans, and Data Analysis*, New York: John Wiley & Sons.

SAS Institute Inc. (2000), *SAS/QC User's Guide, Version 8*, Cary, NC: SAS Institute Inc

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CONTACT INFORMATION

We welcome comments and questions.

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APPENDIX

```
/**            Figure 3               **/
/** Weibull Probability Plots **/
symbol1 c=red v=plus;
symbol2 c=blue v=x;
symbol3 c=green v=square;
symbol4 c=brown v=circle;
proc reliability data=SUGI26_ft ;
distribution weibull;
pplot pseudoFT=temp  / overlay
                       noconf
                       cframe = ligr;
run;
symbol ;
title ;



/**            Figure 4               **/
/** Arrhenius-Weibull Fit    **/
proc reliability data=SUGI26_ft ;
distribution weibull ;
model pseudoFT*censor(1) = temp /
                         relation = arrhenius2
                         obstats( q= .01 .95
control=cntrl) ;
rplot pseudoFT*censor(1) = temp /
      noconf
      pplot
      fit = model
      lrclper
      relation = arrhenius2
      plotfit 01 95
      lupper = 1000
      slower = 0
      sref = 9
      plotdata
      cframe = ligr ;
run ;
title ;

/** PROC NLMIXED Code    **/
proc nlmixed data=SUGI26_str tech=QUANEW
update=BFGS inhess ebopt;
   parms / data=brt_init ;
   temp0 = 09;
   AF =  exp( b3 * ( 11605/(temp0 + 273.15) -
11605/(temp + 273.15) ));
   Dt = -exp( b2 )  * ( 1 - exp( - exp( b1 ) *
AF * days ) );
   predict Dt out = Brittle_Prd;
   model   brit_pct ~ normal( Dt , s2e );
   random  b1 b2 ~ normal( [ mb1, mb2 ], [ sb1,
c21, sb2 ] ) subject =
           sample out=brt_ran;
   id af ;
run;
```