Paper 199-26

The Spectrum of Broadway: A SAS® PROC SPECTRA Inquiry

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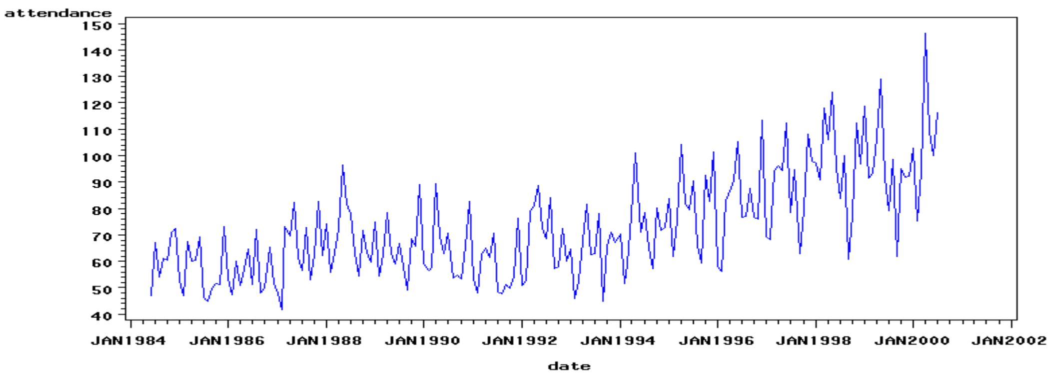
Abstract

This paper describes how to use the sophisticated SAS procedure PROC SPECTRA to detect periodicity in a time series. The series selected for analysis is paid attendance to Broadway theatre performances. This paper illustrates how to interpret the periodogram, spectral density, Fisher's Kappa statistic and other summary statistics output from PROC SPECTRA.

Introduction

The SAS System offers a procedure, PROC SPECTRA, which may be used to detect periodicities in data. Above is a SAS gplot of the amount of monthly paid attendance to Broadway theatre performances in New York City, from June, 1984 through August, 2000. There are several SAS procedures such as PROC ARIMA and PROC X11/X12 which allow the researcher to determine whether or not a time series exhibits any periodicity or seasonality. This paper uses the PROC SPECTRA procedure for this purpose.



Seasonality Detection and PROC SPECTRA

The development of spectral analysis may be traced to the work of Jean Baptiste Joseph Fourier (1768-1830), a French physicist and mathematician. In 1822 Fourier published *Theorie Analytique de la Chaleur*, in which he showed that any periodic function could be modeled by a trigonometric function of sine and cosine components. Spectral analysis allows the researcher to detect the existence of periodicity in a time series. A straightforward way to determine whether or not a series has a periodic component is to plot the periodogram or spectral density of the series onto either the period or the frequency. A significant ordinate value at the period or frequency indicates the numerical value of that periodic element.

Spectral Analysis

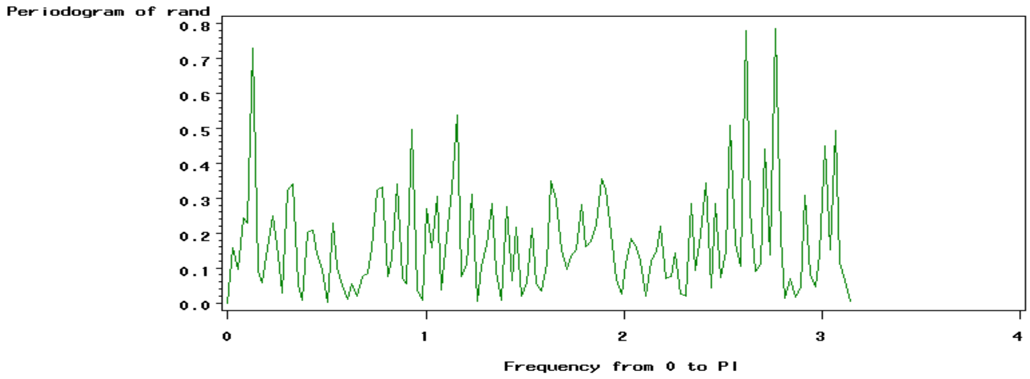
In addition to other output, the SPECTRA procedure outputs estimates of the spectral and cross-spectral densities of multiple time series. These estimates are produced using a finite Fourier transform, which decomposes the series into a sum of sine and cosine waves of varying amplitudes and wavelengths. The Fourier transform for the series X_t may be written as:

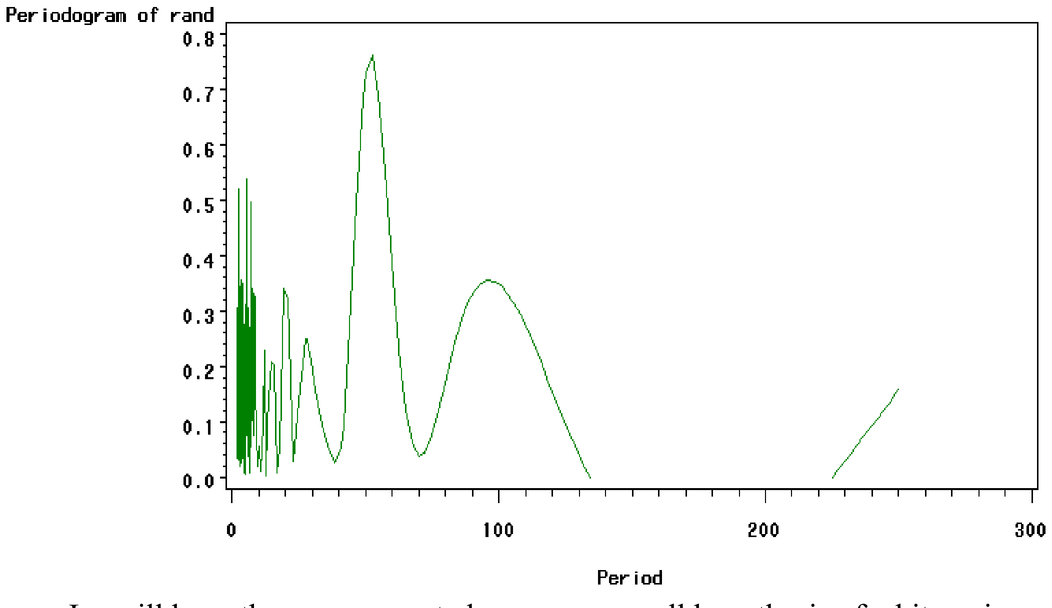
$$X_t = a_0/2 + \sum_{k=1}^{m} [a_k \cos(\omega_k t) + b_k \sin(\omega_k t)]$$

where:
- a_o mean term
- a_k cosine coefficients
- b_k sine coefficients
- ω_k Fourier frequencies ($= 2\pi k/n$)
- m number of frequencies in Fourier decomposition

In a general sense, PROC SPECTRA regresses the time series under analysis onto the sine and cosine variates for frequencies varying from 0 to π by small increments. The plotted periodogram or spectral density function is then the sum of squares of the regression model associated with each frequency. Periodicity is then determined by a high value for the ordinate of the periodogram or spectral density.

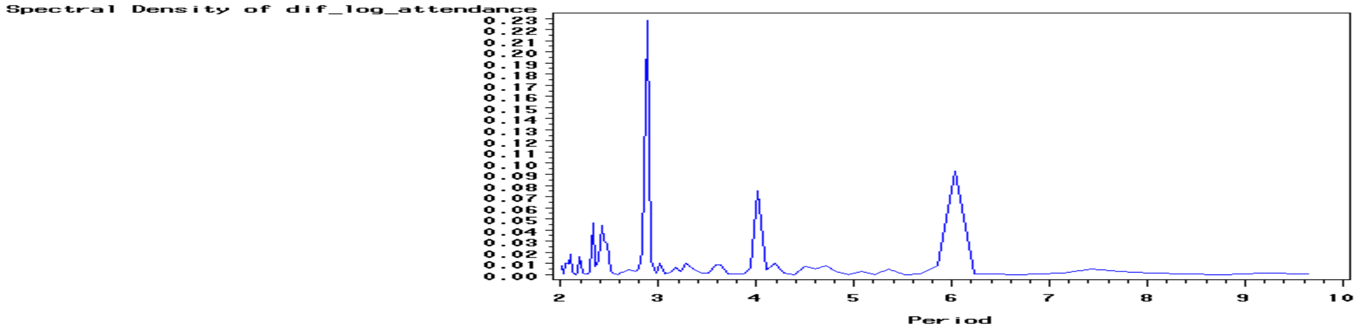
Spectrum of a Random Time Series
In order to illustrate how PROC SPECTRA may be used to detect hidden periodicity, a series of 500 random observations was selected from a uniform probability distribution with parameters a = 0 and b = 1. PROC SPECTRA was then used to estimate the periodogram of the series. Below are plots of the periodogram versus frequency and the periodogram versus the period. If the series were white noise, the values of the amplitude

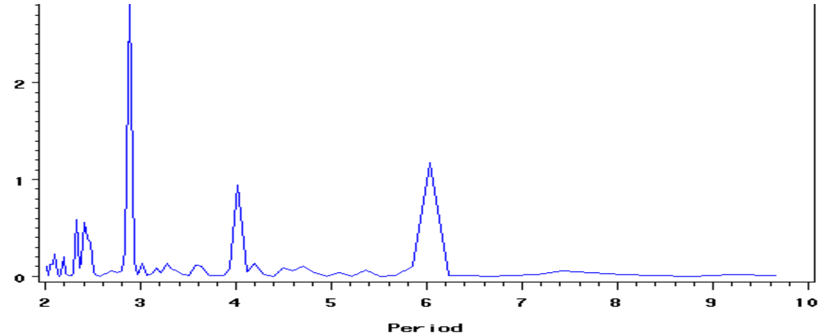




periodogram, J_k , will have the same expected values. If there is autocorrelation in the series, each J_k will have a different expected value. Then, in order to test if the series is random, PROC SPECTRA provides two tests: the Fisher Kappa test and the Bartlett Kolmogorov -Smirnov test of the null hypothesis of white noise. As shown below, this series does in fact appear to be random, as judged by these tests.

SPECTRA Procedure Test for White Noise for Variable: Random: M-1 124 Max(P(*)) 0.786098 Sum(P(*)) 21.67942 Fisher's Kappa: (M-1)*Max(P(*))/Sum(P(*)) Kappa 4.496254 versus Table Value of 7.832 Bartlett's Kolmogorov-Smirnov Statistic: Maximum absolute difference of the standardized partial sums of the periodogram and the CDF of a uniform(0,1) random variable. Test statistic 0.072885 versus 0.12213 from table. Result: The null hypothesis of white noise can't be rejected. Since it is widely acknowledged that the periodogram is only a fair approximation to the true spectrum that it is trying to estimate, the researcher will often use a weighted average of the periodogram ordinates in order to smooth out the periodogram. The justification for this is that adjacent frequencies are highly correlated and the resulting spectral density will be easier to interpret. Numerous alternatives are used for developing the weighting method - selected according to how smooth the spectral density is desired to be. These weights are often referred to as the spectral window.





Empirical Results

The results of applying PROC SPECTRA to the paid attendance to Broadway performances time series are illustrated below. The first-difference of the natural logarithm was calculated in order to achieve stationarity. A gplot of the dif_log_attendance is shown. Next, both the periodogram and the spectral density of dif_log_attendance are plotted against the period.

The SPECTRA Procedure

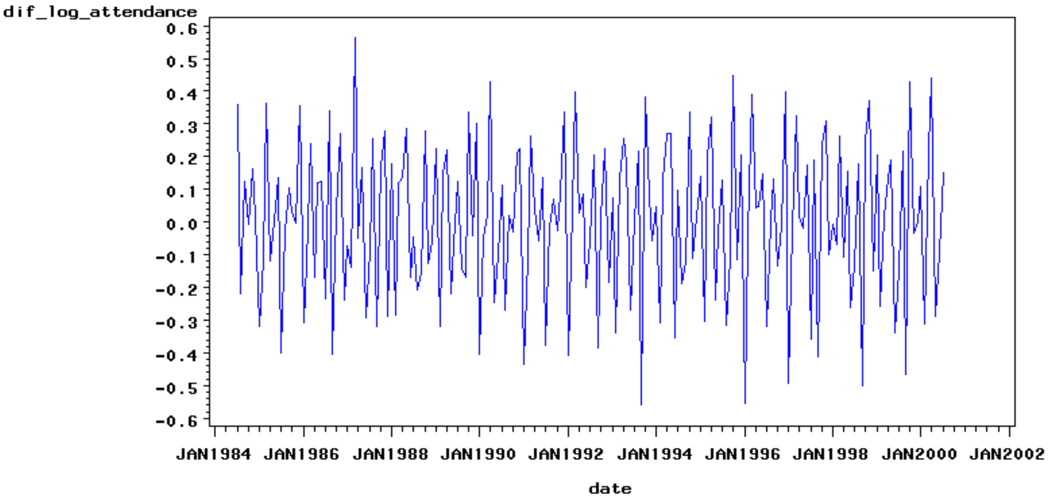
The SAS PROC SPECTRA procedure output is divided into major parts, as follows: Test for White Noise for Variable dif_log_attendance M = 96 Max(P(*)) 2.872207 Sum(P(*)) 10.76142 Fisher's Kappa: M*MAX(P(*))/SUM(P(*)) Kappa 25.62225 Bartlett's Kolmogorov-Smirnov Statistic: Maximum absolute difference of the standardized partial sums of the periodogram and the CDF of a uniform(0,1) random variable. Test Statistic 0.29967

Obs	FREQ	PERIOD	P_01	S_01
1	0.00000	.	0.00000	0.000164
2	0.03256	193.000	0.00207	0.000164
3	0.06511	96.500	0.00121	0.000096
4	0.09767	64.333	0.00016	0.000013
5	0.13022	48.250	0.00268	0.000213
6	0.16278	38.600	0.00593	0.000472
7	0.19533	32.167	0.01039	0.000827
8	0.22789	27.571	0.00344	0.000274
9	0.26044	24.125	0.01070	0.000851
10	0.29300	21.444	0.00685	0.000545
11	0.32555	19.300	0.00405	0.000322
12	0.35811	17.545	0.01072	0.000853
13	0.39066	16.083	0.01825	0.001453
14	0.42322	14.846	0.00492	0.000392
15	0.45578	13.786	0.01133	0.000902
16	0.48833	12.867	0.00153	0.000121
17	0.52089	12.063	0.08627	0.006866
18	0.55344	11.353	0.01696	0.001350
19	0.58600	10.722	0.03565	0.002837
20	0.61855	10.158	0.01538	0.001224
21	0.65111	9.650	0.00781	0.000622
22	0.68366	9.190	0.01872	0.001490
23	0.71622	8.773	0.00151	0.000120
24	0.74877	8.391	0.00509	0.000405
25	0.78133	8.042	0.01479	0.001177
26	0.81388	7.720	0.03404	0.002709
27	0.84644	7.423	0.05924	0.004714
28	0.87899	7.148	0.01408	0.001120
29	0.91155	6.893	0.00673	0.000536
30	0.94411	6.655	0.00069	0.000055
31	0.97666	6.433	0.00407	0.000324
32	1.00922	6.226	0.00318	0.000253
33	1.04177	6.031	1.17221	0.093281
34	1.07433	5.848	0.10021	0.007975
35	1.10688	5.676	0.00940	0.000748
36	1.13944	5.514	0.00051	0.000041
37	1.17199	5.361	0.06753	0.005374
38	1.20455	5.216	0.00265	0.000211
39	1.23710	5.079	0.03782	0.003009
40	1.26966	4.949	0.00115	0.000091
41	1.30221	4.825	0.03232	0.002572
42	1.33477	4.707	0.10356	0.008241
43	1.36733	4.595	0.05990	0.004766
44	1.39988	4.488	0.08819	0.007018
45	1.43244	4.386	0.00202	0.000161
46	1.46499	4.289	0.01527	0.001215
47	1.49755	4.196	0.12927	0.010287
48	1.53010	4.106	0.05073	0.004037
49	1.56266	4.021	0.94289	0.075033
50	1.59521	3.939	0.08054	0.006409
51	1.62777	3.86000	0.00712	0.00057
52	1.66032	3.78431	0.00780	0.00062
53	1.69288	3.71154	0.01092	0.00087
54	1.72543	3.64151	0.10858	0.00864
55	1.75799	3.57407	0.11046	0.00879
56	1.79055	3.50909	0.01199	0.00095
57	1.82310	3.44643	0.01529	0.00122
58	1.85566	3.38596	0.04263	0.00339
59	1.88821	3.32759	0.07754	0.00617
60	1.92077	3.27119	0.12979	0.01033
61	1.95332	3.21667	0.03408	0.00271
62	1.98588	3.16393	0.08058	0.00641

63	2.01843	3.11290	0.02462	0.00196
64	2.05099	3.06349	0.01000	0.00080
65	2.08354	3.01563	0.13118	0.01044
66	2.11610	2.96923	0.02101	0.00167
67	2.14865	2.92424	0.15164	0.01207
68	2.18121	2.88060	2.87221	0.22856
69	2.21376	2.83824	0.26360	0.02098
70	2.24632	2.79710	0.06031	0.00480
71	2.27888	2.75714	0.03249	0.00259
72	2.31143	2.71831	0.04622	0.00368
73	2.34399	2.68056	0.05583	0.00444
74	2.37654	2.64384	0.02516	0.00200

X11 Procedure

For comparison purposes the final seasonal factor results of the PROC X11 procedure applied to Broadway attendance are presented. Seasonal factors above 100 occurred at March, May, June, November and December. A low index of 78.895 occurred for the month of September.



75	2.40910	2.60811	0.01746	0.00139
76	2.44165	2.57333	0.00009	0.00001
77	2.47421	2.53947	0.00484	0.00039
78	2.50676	2.50649	0.03893	0.00310
79	2.53932	2.47436	0.35217	0.02803
80	2.57187	2.44304	0.38950	0.03100
81	2.60443	2.41250	0.55722	0.04434
82	2.63698	2.38272	0.16583	0.01320
83	2.66954	2.35366	0.08731	0.00695
84	2.70210	2.32530	0.58454	0.04652
85	2.73465	2.29762	0.04490	0.00357
86	2.76721	2.27059	0.00605	0.00048
87	2.79976	2.24419	0.00286	0.00023
88	2.83232	2.21839	0.02305	0.00183
89	2.86487	2.19318	0.20054	0.01596
90	2.89743	2.16854	0.03330	0.00265
91	2.92998	2.14444	0.00124	0.00010
92	2.96254	2.12088	0.02648	0.00211
93	2.99509	2.09783	0.23069	0.01836
94	3.02765	2.07527	0.11904	0.00947
95	3.06020	2.05319	0.12536	0.00998
96	3.09276	2.03158	0.00496	0.00039
97	3.12531	2.01042	0.09939	0.00791

Proc SPECTRA Syntax:

```
PROC SPECTRA options;
      BY variables;
      VAR variables;
      WEIGHTS constants;
```

D10 Final Seasonal Factors

Year	JAN	FEB	MAR	APR	MAY	JUN
1984	.	.	.	.	.	113.244
1985	91.635	78.485	110.683	103.101	117.186	112.454
1986	93.464	78.862	108.785	104.452	116.950	110.824
1987	95.429	79.652	104.981	107.760	116.638	108.096
1988	96.013	80.234	102.534	112.771	115.777	104.303
1989	96.110	81.016	99.336	116.545	115.797	101.741
1990	96.046	81.239	98.098	119.324	116.131	100.406
1991	96.384	81.105	95.832	119.479	118.333	100.695
1992	97.625	80.138	95.686	119.413	120.779	100.681
1993	99.374	78.405	94.999	117.775	123.190	103.186
1994	101.355	77.460	96.156	116.951	122.148	106.520
1995	101.523	77.850	98.673	115.804	120.697	109.399
1996	101.909	80.336	102.385	115.183	118.832	110.158
1997	102.543	82.756	105.450	113.500	118.614	109.698
1998	104.080	85.408	106.845	112.588	119.560	107.986
1999	105.707	87.118	107.095	111.501	121.487	105.345
2000	107.068	88.724	107.136	111.626	122.713	102.465
Avg	99.142	81.174	102.167	113.611	119.052	106.306

D10 Final Seasonal Factors

Year	JUL	AUG	SEP	OCT	NOV	DEC
1984	88.281	90.521	90.292	93.701	111.457	111.544
1985	89.555	91.209	88.798	94.256	110.599	110.821
1986	91.992	92.271	85.920	95.379	108.790	111.150
1987	96.084	92.383	83.295	95.747	106.849	112.406
1988	98.520	92.301	81.740	95.474	104.804	115.913
1989	100.657	90.872	82.066	94.304	103.221	118.259
1990	100.736	89.510	82.390	93.972	102.410	119.924
1991	101.571	88.592	82.634	93.851	102.773	116.964
1992	101.715	88.574	81.547	95.706	103.566	113.055
1993	102.686	88.921	79.553	97.777	103.033	108.540
1994	102.642	90.252	76.278	99.475	102.734	106.515
1995	101.316	92.464	73.274	98.014	103.007	105.328
1996	97.793	95.141	70.814	95.956	103.776	105.594
1997	94.458	98.339	69.225	93.020	104.862	104.765
1998	91.911	100.967	67.644	91.164	106.262	103.596
1999	91.248	102.725	66.856	89.612	107.243	102.026
2000	91.001	.	.	.	.	.
Avg	96.598	92.815	78.895	94.838	105.337	110.400

Conclusion

The SAS procedure PROC SPECTRA provides the researcher with a powerful statistical tool for determining whether or not a time series exhibits any periodicity. The results of the spectral analysis of paid attendance to Broadway theatre in New York City indicate that this series exhibits both a quarterly and a half-yearly cycle. Analysis of the periodogram or the spectral density plotted against the frequency or period provides a quick efficient way to determine periodicity.

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