Paper 250

How to Display Correlated ROC Curves with the SAS® System

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ABSTRACT

Diagnosis is an essential part of clinical practice. Much medical research is carried out to improve methods of diagnosis. Medical tests are developed for diagnosis purpose. Thus statistical methods of evaluating and comparing the performance of such tests are of great importance. A test is associated with an observed variable which is able to discriminate between a diseased and non-diseased population. If this variable lies on an ordinal or metric scale an assessment of the overall performance of the test can be made through the use of the **R**eceiver **O**perating **C**haracteristic (ROC) curve. Correlated ROC curves arise when two or more different tests are performed on the same individuals.

The poster presents an approach to display correlated ROC curves with the SAS Software. SAS code using Base SAS, SAS/STAT and SAS/GRAPH is reported.

INTRODUCTION

The purpose of a medical test is to classify individuals in two groups (eg. diseased or nondiseased). A medical test is associated with an observed variable. Such variables cannot only be used for determining the presence or absence of a medical condition -**diagnosis**- but also for predicting an event of interest -**prognosis**-.
The evaluation and comparison of medical tests requires knowledge about the true medical condition of an individual. It has to be determined by means independent of the test (Gold Standard) that an individual undergoes the condition or event. For example autopsy, biopsy or surgical inspection can be taken as gold standards. If the variable which is associated with the test is dichotomous that means T^+ : test positive, T^-: test negative , then sensitivity and specificity reflect the quality of a test. It is assumed that T^+ is associated with the desease (condition) and T^- with non-disease (non-condition).
Sensitivity is the probability that the test result is positive under the condition that the subject comes from the diseased population.
Specificity is the probability that the test result is negative under the condition that the subject comes from the non-diseased population.

In practice sensitivity and specificity are estimated as the true positive rate and true negative rate - empirical sensitivity and empirical specificity -.
Let D be the event that a subject undergoes the condition and $\overline{D}$ the event that a subject does not have the condition then:

Sensitivity : sens = $P(T^+/ D)$
Specificity: spez = $P(T^-/ \overline{D})$

When a test is based on an observed variable that lies on a continuous or graded scale any outcome value z can serve as a cutpoint to dichotomize the problem. For values greater or equal than the cutpoint the outcome of the test is positiv and for smaller values than the cutpoint the test result is taken as being negativ or vice versa. It depends on whether higher or lower values are associated with the condition of interest.
Sens(z) is the empirical sensitivity of a test that is derived by dichotomizing the metric or ordinal variable into positiv or negativ results on the basis of the cutpoint z. spez(z) is the corresponding empirical specificity.
The receiver operating characteristic (ROC) curve is theoretically given as a plot of sensitivity (Y-Axis) vs. (1-specificity) (X-Axis) by varying the variable on which the test is based over all possible values.The empirical ROC curve is a graphical display of sens(z) vs. 1-spez(z) as the sample values z vary over all measured values.
An interesting property of the ROC curve is that it is invariant under orderpreserving transformations of the test variable; that means if the test variable is transformed by a monotone function the resulting ROC curve will be the same.

CONSTRUCTION OF THE ROC CURVE WITH THE SAS SYSTEM

There are several possibilities to compute sens(z) and spez(z) for each of the sample values z using the SAS System. Since Version 6.10 an option in PROC LOGISTIC is available that allows computation of sens(z) and spez(z) and output them into a SAS data set.

PROC LOGISTIC DATA= *input- SAS-data-set* ;

```
   MODEL status = test / OUTROC =  output-SAS-data-set ;
RUN;
```

The *input- SAS-data-set* consists of at least the two variables *status* and *test*. One record per observational unit (eg. patient) is used:
status : dichotomous event (condition) variable eg. 1 deseased, 2 non deseased
test: variable which is associated with the medical test

Amongst other variables the *output-SAS-data-set* contains the variables:
SENSIT sens(z)
1MSPEC 1-spez(z)

PROC LOGISTIC uses the estmated probability p of the event from the logistic model to compute sensitivity and specificity. For $p = \dfrac{\exp(a + bz)}{1 + \exp(a + bz)}$ is a monotone transformation of the values z of the test variable the ROC curve cunstructed from the probabilities p will be the same as generated from the sample values z. An advantage of this transformation is that higher probabilities reflect the condition of interest. Therefore one need not take care of whether higher or lower values of the test variable are associated with the condition of interest.

To display the ROC curve PROC GPLOT can be used.

```
PROC GPLOT DATA= output-SAS-data-set;
   PLOT  _SENSIT_*_1MSPEC_ ;
RUN;
```

DISPLAYING CORRELATED ROC CURVES

Correlated ROC curves arise when two or more different tests are performed on the same individuals (observational units).
To compare the discriminating abilities of two or more test variables measured on the same individuals graphically, the corresponding ROC curves have to be plotted.
The following SAS macro gives an example how to display several correlated ROC curves within the same graph.
The *input- SAS-data-set* consists of at least the variables *status, test1,test2,...,testn* and one record per observational unit (eg. patient):
status : dichotomous event (condition) variable eg. 1 deseased, 2 non deseased
test1,..,testn: variables associated with the medical test

```
options ls=79  nonumber;
title "ROC curve";

goptions cback=white ctext=black ftext=swiss
hsize=20cm vsize=20cm  target=winprtc;

%let indata = input- SAS-data-set ;
%let vars = test1 test2        testn;

%let pop=status;

%let cvars = "&vars";

/* in order to restructure the input data set
from multivariate to univariate*/
data trans;
set &indata;

array w  &vars;
do over w;
wert=w;
variable=scan(&cvars,_i_);
index=_i_;
output;
end;

run;

proc sort data=trans;
by index;
run;

/** log-reg **/

proc logistic data=trans /*descending*/;
model &pop=wert / outroc=o;
output out=p p=_PROB_;

/* to compute the ROC curve for each
 test variable separately*/

by index variable;
run;

data p;
format _PROB_ 8.6;
set p;
_PROB_=round(_PROB_,.00001);
run;

data o;
set o;
_PROB_=round(_PROB_,.00001);
run;

proc sort data=o;
by index _PROB_;
```

```
run;
proc sort data=p;
by index _PROB_;
run;

/* in order to have the test variables mapped to
sensitivity and 1-specificity */
data p1;
keep _PROB_ wert  _sensit_ _1mspec_ index variable;
merge p o;
if wert^=.;
by index _PROB_;
run;

proc print data=p1 ;
run;

symbol1 c=black w=3 v=none i=join l=1 ;
symbol2 c=blue w=3 v=none i=join l=2  ;
symbol3 c=green w=3 v=none i=join l=3;

/*symboln c=red w=3 v=none i=join l=4;*/

proc gplot data=o;
plot  _sensit_*_1mspec_=variable /
ctext=black caxis=black vaxis=0 to 1 by .1;
run;

quit;
title;
```

To illustrate how the above SAS program works, the following data set (reported in DeLong et al. 1988) is used:

ROC data set

ALB	TP	TOTSCORE	POPIND	POPIND1	PATIENT
3.0	5.8	0	0	2	1
3.2	6.3	5	1	1	2
3.9	6.8	7	1	1	3
2.8	4.8	4	0	2	4
3.2	5.8	7	1	1	5
0.9	4.0	5	0	2	6
2.5	5.7	2	0	2	7
1.6	5.6	5	1	1	8
3.8	5.7	5	1	1	9
3.7	6.7	4	1	1	10
.	.	4	1	1	11
3.2	5.4	6	1	1	12
3.8	6.6	4	1	1	13
4.1	6.6	5	1	1	14
3.6	5.7	5	1	1	15
4.3	7.0	6	1	1	16
3.6	6.7	6	0	2	17
2.3	4.4	4	1	1	18
4.2	7.6	6	0	2	19
4.0	6.6	4	0	2	20
3.5	5.8	4	1	1	21
3.8	6.8	3	1	1	22
3.0	4.7	2	0	2	23
4.5	7.4	5	1	1	24
3.7	7.4	5	1	1	25
3.1	6.6	4	1	1	26
4.1	8.2	4	1	1	27
4.3	7.0	5	1	1	28
4.3	6.5	6	1	1	29
3.2	5.1	5	1	1	30
2.6	4.7	4	1	1	31
3.3	6.8	4	0	2	32
1.7	4.0	3	0	2	33
.	.	4	1	1	34
3.7	6.1	5	1	1	35
3.3	6.3	3	1	1	36
4.2	7.7	4	1	1	37
3.5	6.2	5	1	1	38
2.9	5.7	1	0	2	39
2.1	4.8	3	1	1	40
.	.	2	1	1	41
2.8	6.2	2	0	2	42
.	.	3	1	1	43
.	.	3	1	1	44
4.0	7.0	3	1	1	45
3.3	5.7	4	1	1	46
3.7	6.9	5	1	1	47
2.0	.	3	1	1	48
3.6	6.6	5	1	1	49

POPIND event variable 1 ... benefit from surgery 0 ... no benefit from surgery
POPIND1 event variable 1 ... benefit from surgery 2 ... no benefit from surgery can be used without the descending OPTION in PROC LOGISTIC

ALB albumin, TP total protein, TOTSCORE score variable ... medical test variables
PATIENT ... patient identification number

Part of the program with actual parameters:

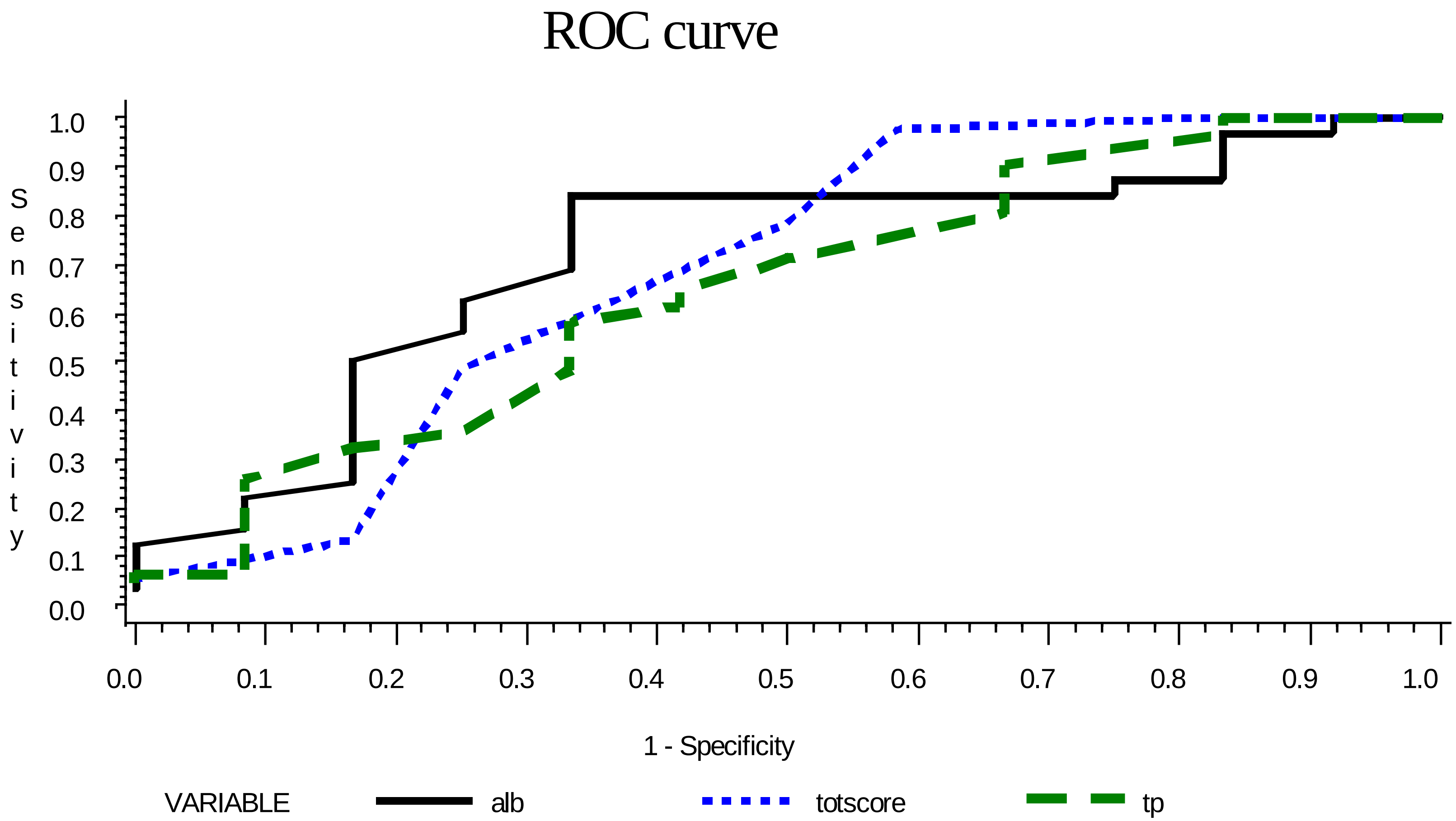
```
options ls=79  nonumber;
title "ROC curve";

goptions cback=white ctext=black ftext=swiss
hsize=20cm vsize=20cm  target=winprtc;

%let indata = ROC ;
%let vars = ALB TP TOTSCORE;

%let pop=POPIND1;
```

SAS - OUTPUT:


ROC curve
Sensitivity
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1 - Specificity
VARIABLE
alb
totscore
tp

Printed output is only given for variable ALB

```
                          ROC curve

---------------- INDEX=1 VARIABLE=alb ----------------
--

                    The LOGISTIC Procedure

    Data Set: WORK.TRANS
    Response Variable: POPIND1
    Response Levels: 2
    Number of Observations: 44
    Link Function: Logit

                    Response Profile

            Ordered
              Value  POPIND1     Count

                  1        1        32
                  2        2        12

WARNING: 5 observation(s) were deleted due to missing
values for the response
         or explanatory variables.

      Model Fitting Information and Testing Global Null
Hypothesis BETA=0

                             Intercept
                 Intercept      and
  Criterion        Only      Covariates    Chi-Square for Covariates

  AIC             53.564       51.021           .
  SC              55.348       54.590           .
  -2 LOG L        51.564       47.021         4.542 with 1 DF (p=0.0331)
  Score             .            .            4.700 with 1 DF (p=0.0302)

                   Analysis of Maximum Likelihood Estimates

              Parameter Standard    Wald       Pr >     Standardized     Odds
 Variable DF   Estimate   Error  Chi-Square Chi-Square    Estimate      Ratio

INTERCPT 1    -1.9418   1.4606     1.7675     0.1837          .          .
WERT     1     0.9099   0.4508     4.0742     0.0435      0.403372     2.484

          Association of Predicted Probabilities and Observed
                              Responses

          Concordant = 71.1%          Somers' D = 0.438
          Discordant = 27.3%          Gamma     = 0.444
          Tied       =  1.6%          Tau-a     = 0.178
          (384 pairs)                 c         = 0.719
```

ROC curve

OBS	_PROB_	WERT	VARIABLE	INDEX	_SENSIT_	_1MSPEC_
1	0.245480	0.9	alb	1	1.00000	1.00000
2	0.380860	1.6	alb	1	1.00000	0.91667
3	0.402540	1.7	alb	1	0.96875	0.91667
4	0.469560	2.0	alb	1	0.96875	0.83333
5	0.492270	2.1	alb	1	0.93750	0.83333
6	0.537700	2.3	alb	1	0.90625	0.83333
7	0.582500	2.5	alb	1	0.87500	0.83333
8	0.604450	2.6	alb	1	0.87500	0.75000
9	0.647040	2.8	alb	1	0.84375	0.75000
10	0.647040	2.8	alb	1	0.84375	0.75000
11	0.667530	2.9	alb	1	0.84375	0.58333
12	0.687410	3.0	alb	1	0.84375	0.50000
13	0.687410	3.0	alb	1	0.84375	0.50000
14	0.706620	3.1	alb	1	0.84375	0.33333
15	0.725120	3.2	alb	1	0.81250	0.33333
16	0.725120	3.2	alb	1	0.81250	0.33333
17	0.725120	3.2	alb	1	0.81250	0.33333
18	0.725120	3.2	alb	1	0.81250	0.33333
19	0.742880	3.3	alb	1	0.68750	0.33333
20	0.742880	3.3	alb	1	0.68750	0.33333
21	0.742880	3.3	alb	1	0.68750	0.33333
22	0.776090	3.5	alb	1	0.62500	0.25000
23	0.776090	3.5	alb	1	0.62500	0.25000
24	0.791500	3.6	alb	1	0.56250	0.25000
25	0.791500	3.6	alb	1	0.56250	0.25000
26	0.791500	3.6	alb	1	0.56250	0.25000
27	0.806120	3.7	alb	1	0.50000	0.16667
28	0.806120	3.7	alb	1	0.50000	0.16667
29	0.806120	3.7	alb	1	0.50000	0.16667
30	0.806120	3.7	alb	1	0.50000	0.16667
31	0.819950	3.8	alb	1	0.37500	0.16667
32	0.819950	3.8	alb	1	0.37500	0.16667
33	0.819950	3.8	alb	1	0.37500	0.16667
34	0.832990	3.9	alb	1	0.28125	0.16667
35	0.845270	4.0	alb	1	0.25000	0.16667
36	0.845270	4.0	alb	1	0.25000	0.16667
37	0.856800	4.1	alb	1	0.21875	0.08333
38	0.856800	4.1	alb	1	0.21875	0.08333
39	0.867610	4.2	alb	1	0.15625	0.08333
40	0.867610	4.2	alb	1	0.15625	0.08333
41	0.877710	4.3	alb	1	0.12500	0.00000
42	0.877710	4.3	alb	1	0.12500	0.00000
43	0.877710	4.3	alb	1	0.12500	0.00000
44	0.895940	4.5	alb	1	0.03125	0.00000

PROC LOGISTIC does not compare ROC curves but issues the area under the curve (`c = .719`, ALB from the above example) in the following portion of the output:

```
     Association of Predicted Probabilities and Observed
                         Responses

     Concordant = 71.1%          Somers' D = 0.438
     Discordant = 27.3%          Gamma     = 0.444
     Tied       =  1.6%          Tau-a     = 0.178
     (384 pairs)                 c         = 0.719
```

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REFERENCES

Elizabeth R. DeLong, David M. DeLong, Daniel L. Clarke-Pearson (1988) *Comparing the Areas Under Two or More Correlated Receiver Operating Characteristic Curves: A Nonparametric Approach,* BIOMETRICS 44, 837-845, September 1988

SAS ® Language: Reference, Version 6, First Edition
SAS ® Procedures Guide, Version 6, Third Edition
SAS/GRAPH ® Software: Reference, Version 6, First Edition, Volumes 1 and 2
SAS/IML® Software: Usage and Reference, Version 6, First Edition
SAS/STAT ® Software: Reference, Version 6, First Edition, Volumes 1 and 2

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