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Split and Conquer! Using SAS/QC® to Design Quality into Complex Manufacturing Processes

José G. Ramírez
W.L. Gore & Associates, Inc.
501 Vieve's Way
Elkton, MD 21922-1320

Randy Tobias
SAS Institute, Inc.
SAS Campus Drive
Cary, NC 27513

ABSTRACT

A complex manufacturing process has many stages, with different factors active at different stages. How do you discover which factors are important at a particular stage, and what factor, or factors, from an early stage interacts with one, or more, at a later stage (e.g. the source of raw material versus final packaging)? Split-plot experiments come to the rescue! They are a key tool for improving final product quality by studying how all stages affect final product quality. Until recently the design of fractional factorial experiments with more than one stage was not trivial. In this paper we show how the ADX Interface for design of experiments in SAS/QC® makes it very easy to design and analyze the most common experiments of this type. For more complex situations, recent advances in PROC FACTEX enable you to custom design experiments for processes with several stages. Two examples are presented showcasing these new features in ADX and PROC FACTEX.

1. INTRODUCTION

In a perfect world, the one described in Physics fairy tales where there is no friction, and the Sci-Fi one in textbooks where every experiment can be completely randomized, there is no need to worry about split-plot situations. But let's face it; the world is not perfect and as such we run into constraints in the number of experiments that can be run, and how the experiment needs to be conducted. Why are split-plot situations important? Because they are pervasive in science and technology. In fact, they occur more often than we know or are willing to admit—some may say all the time.

2. WHY SPLIT-PLOT DESIGNS?

Three decades ago, Cuthbert Daniel wrote "Nested designs (some factors held constant, others varied in each "nest") are common in industrial research. The *larger the system* under study, the more likely it is that such plans will prove *the more convenient* or *even the only possible* ones." In fact, too many industrial experiments are run in a split-plot mode, but then analyzed as if they were completely randomized designs. (Remember the perfect world?) It is very important to consider the design structure when performing the analysis so you can reach the right conclusions. Ignoring the split-plot structure corrupts the estimates of underlying noise, and thus confuses the statistical judgment as to which effects are significant relative to noise. This point is illustrated with the data in Table 1, for an experiment designed to compare the performance, based on a given quality characteristic, of an old and a new film type, and three manufacturing pressures (P1, P2, and P3). The experiment consisted of 6 different treatment combinations (2 films × 3 pressures). The 6 different combinations were repeated 8 times, as shown in Table 1, for a total of 48 runs.

Time Block (t)	Film Type	Pressure P1	P2	P3
1	Old	15.39	15.49	15.39
	New	15.40	15.62	15.14
2	Old	15.97	15.79	14.99
	New	15.25	15.37	15.55
3	Old	15.88	15.91	15.48
	New	15.92	15.26	15.43
4	Old	15.36	15.51	15.47
	New	15.30	15.53	15.66
5	Old	15.86	15.19	14.93
	New	15.42	15.03	15.26
6	Old	15.53	15.61	15.49
	New	15.32	15.55	15.50
7	Old	15.91	16.06	15.53
	New	15.75	15.54	15.68
8	Old	16.03	15.55	15.49
	New	15.75	15.31	15.62

Table 1. 2×3 Film Type x Pressure Experimental Design

Table 2 below shows the ANalysis Of Variance (ANOVA) table, obtained using the ADX Interface for design of experiments, for the data in Table 1 assuming that the experiment was run as a completely randomized design (CRD); i.e., assuming that the 48 runs were completely randomized.

ADX: Fit details for Y

Select effects to be included in pre

Fixed Effects | Variance Comp

Effect	Num DF	Den DF	F Value	Pr > F
FILM	1	42	2.53	0.1194
PRESSURE	2	42	3.17	0.0520
FILM*PRESSURE	2	42	3.09	0.0559

Table 2. ANOVA for Completely Randomized Design Analysis

According to this analysis ***none*** of the effects—neither the main effects nor the interaction—are statistically significant at the 5% level. In other words, no terms in the model would usually be judged to have an effect on the response. The truth is that the experiment could not be completely randomized because Film Type was applied to entire rolls of material, while the 3 Pressures were applied to portions of each roll. In other words Film Type and Pressure were applied to different sizes of experimental units. The implication is that one needs to first randomize the assignment of Film Type (old, new) to a given roll, *and then* randomize the assignment of the 3 Pressures to the thirds of the given roll as shown below

Film Type		**Pressure**		
	OLD	P3	P1	P2
	NEW	P1	P3	P2

Figure 1. Two Step Randomization for Film Type x Pressure Experiment.

In this way the experiment was repeated 8 times (the 8 time blocks in Table 1). An analysis of this data taking into account the split-plot structure, Table 3, reveals that Pressure and its interaction with Film Type are statistically significant at the 5% level.

ADX: Fit details for Y

Select effects to be included in pre

Fixed Effects | Variance Comp

Effect	Num DF	Den DF	F Value	Pr > F
FILM	1	7	3.36	0.1093
PRESSURE	2	28	4.23	0.0249
FILM*PRESSURE	2	28	4.11	0.0271

Table 3. ANOVA for Split-Plot Design Analysis

Therefore, ignoring the split-plot structure induced by the different sizes of experimental units could lead to wrong conclusions. Note that it is also possible to produce the opposite behavior by ignoring the split-plot structure, inappropriately declaring effects to be statistically significant when they are not. Experimenters should be aware of the dangers of not understanding the split-plot nature of most industrial experiments.

3. WHEN DO SPLIT-PLOT SITUATIONS ARISE?

The question then is, when do we encounter split-plot experiments? Split-plot designs occurred primarily in two situations:

1. There is a *restriction in randomization* for an experimental factor.
   For example, running all treatment combinations associated with "low" temperatures followed by all treatment combinations associated with "high" temperatures.
2. There are *different sizes of experimental units*.
   For example, in the example above Film Type, was applied to a 300-meter roll (experimental unit 1); while Pressure was applied to 50-meter pieces within the roll (experimental unit 2). The 300-meter roll was "split" into three 50-meter pieces for the experimental factor Pressure.

It is very important to realize that these designs have two error terms, reflecting the variation between whole-plots, and the variation within whole-plots (between subplots). Estimates of effects at the split-plot level typically have more precision than estimates of effects at the whole-plot level. Notice that 35 of the 42 denominator degrees-of-freedom (Den DF) for error in Table 2 are assigned to whole-plots (7) and sub-plots (28) in Table 3. The remaining (7) are taken up with estimating the Time Block effect (not shown). In short, not all effects are created equal for split-plot designs. Hence, for situations where some effects are of particular importance, the use of split plot designs is advantageous.

The hierarchical structure of split plot designs ─namely, whole-plots split into subplots─ has a wide variety of applications that can work to your advantage. We discuss two specific advantages below.

3.1 Robust Design

In robust product experimentation the study of interactions between the environmental (noise) and design (control) factors is of particular interest. A deeper understanding of such interactions will ultimately aid the design of products that are less sensitive to changes in the environmental conditions. In such studies, interactions between environmental factors only, or similarly interactions between design factors only, are usually not of absolute importance. Due to their error structure, split plot designs provide better estimates (more precision) of interactions between environmental and design factors than do completely randomized designs. See Box and Jones (1992) for further detail.

3.2 Sequential Processes

In sequential processes some experimental factors occur at a given process step, while another set of factors occurs at a subsequent step. One way of designing experiments for processes where the output from one process is the input from another process is to design experiments for each of the steps independently from each other. However if the dynamics of the process and the interdependence of the steps are being studied, a more holistic approach is called for. Once again, split-plot designs offer excellent opportunities by providing precise estimates of the interactions between the factors from the first step and the factors from the second step. Of course, situations occur with more than two process steps and this will lead to more complex split-plot type situations.

4. DO YOU NEED TO WORRY ABOUT SPLIT-PLOT SITUATIONS?

You bet you do! Since most of the experiments run in real-life are subjected to one or more constraints, split-plot situations are the norm rather than the exception. As discussed above, taking into account the split-plot nature of most experiments can work to your advantage, and save you from making the wrong conclusions.

Apart from having to take into account restrictions in randomization, or different sizes of experimental units, experiments also face constraints on the number of resources available. Fractional factorial designs are a very efficient way to run experiments subject to constraints on the number of observations. The idea of constructing fractional factorials to take into account the split-plot confounding is not new. Kempthorne (1952) called these types of designs fractional factorial split-plot, and Addelman (1964) provided a table of factorial plans with split-plot confounding for different number of Whole-Plot and Split-Plot factors. However, in recent years there has been a wave of research on fractional factorial split-plots (FFSP). Huang et al (1998), and Bingham and Sitter (1999, 2001) have applied the concept of *minimum aberration* (Fries and Hunter (1980)) to split-plot designs, giving comprehensive tables for small to moderately sized minimum aberration split-plot designs, while Kulahci et al (2006) have discussed alternative criteria to minimum aberration for designing FFSP.

5. SAS/QC TO THE RESCUE

Useful catalogs of FFSP have appeared in the pages of various journals. For example, Huang, Chen, and Voelkel (1998) give a catalog of minimum aberration fractional split-plot designs. Starting with SAS® Version 9.1 the ADX interface for the design and analysis of experiments provides users with point-and-click access to two-stage FFSP designs, including the entire catalog of Huang et al. ADX also makes it easy to perform a complete and appropriate mixed-model analysis of FFSP experiments.

In addition to ADX's point-and-click access to standard FFSP designs, in SAS® Version 9.2 PROC FACTEX enables you to construct custom FFSP designs for situations with *multi-step* processes, or with more complex restrictions in randomization, and multiple sizes of experimental units. The intuitive syntax allows multiple specifications of whole-plots and subplots, making it possible to cover a wide array of situations. The new features consist of the BLOCK UNIT=() option for describing the split-plot structure of the experiment, and the UNITEFFECT statement for specifying where effects of interest should be estimable within this scheme.

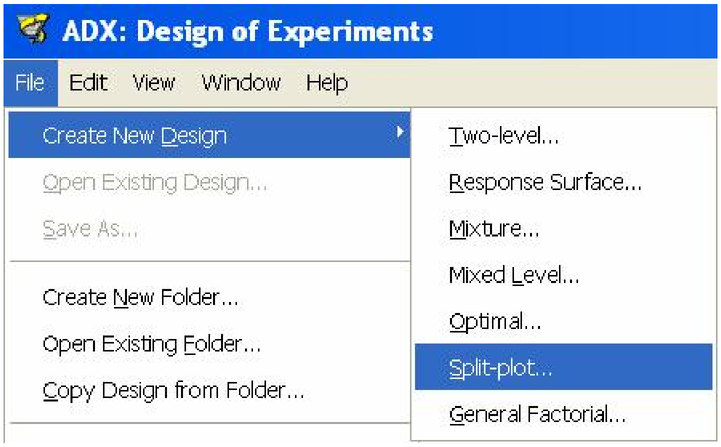
In the next section we use ADX to design a two-stage split-plot experiment, and after that we demonstrate the new features of FACTEX on a more complex design.

6. A 2-STEP PROCESS EXAMPLE

To show the power and flexibility of the ADX interface, consider a situation where a design is required for an experiment on a process that involves two sequential steps. Four process factors (L1, L2, L3, L4) are first applied to the experimental units ***before*** three other factors (O1, O2, O3) are applied to complete the manufacturing process. A full factorial design required $2^7 = 128$ runs; but engineering allocated no more than 50 experimental units for this experiment because each experimental run costs about $500. Thus, a fractional design is required. The design will also necessarily involve split-plots, since there is a restriction in the randomization: you can randomize L1-L4, and then randomize O1-O3 *within* L1-L4, but you cannot completely randomize all the seven factors L1-L4 & O1-O3 simultaneously. What follows shows how you can use the ADX interface to design a 32-run FFSP for this situation. This is only 25% of the full factorial, or a saving of 75% of experimental resources—about $48,000!

6.1 Designing FFSP Using ADX

The first step in designing an experiment using ADX is to define the experimental factors and responses. This can be done by selecting Create New Design > Split-Plot from the File Menu



First we specify the factors in the whole-plot; i.e., the factors in the first step of the process L1-L4, as well as the experimental settings as shown in Table 4.

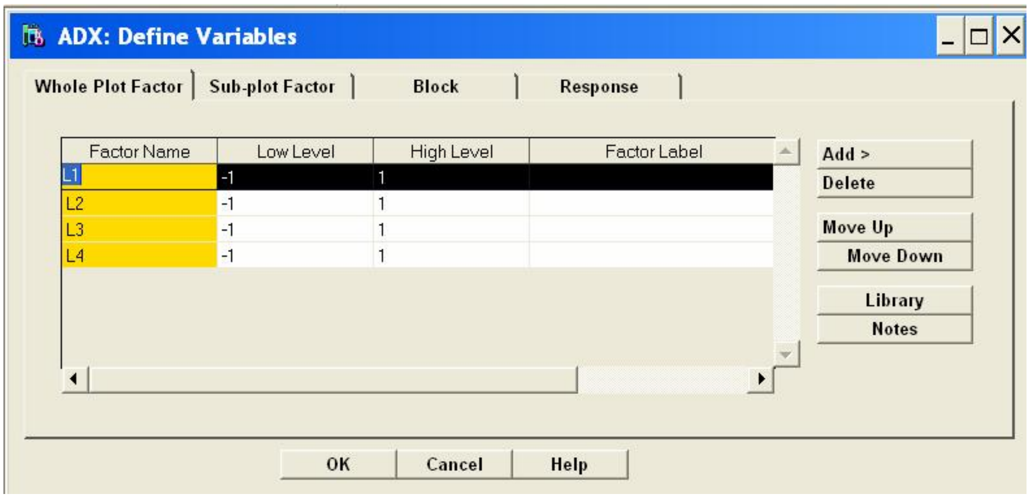


Table 4. Whole Plot Factors Specification

Next we specify the 3 factors, O1-O3, in the sub-plot or second step in the process settings as shown in Table 5.

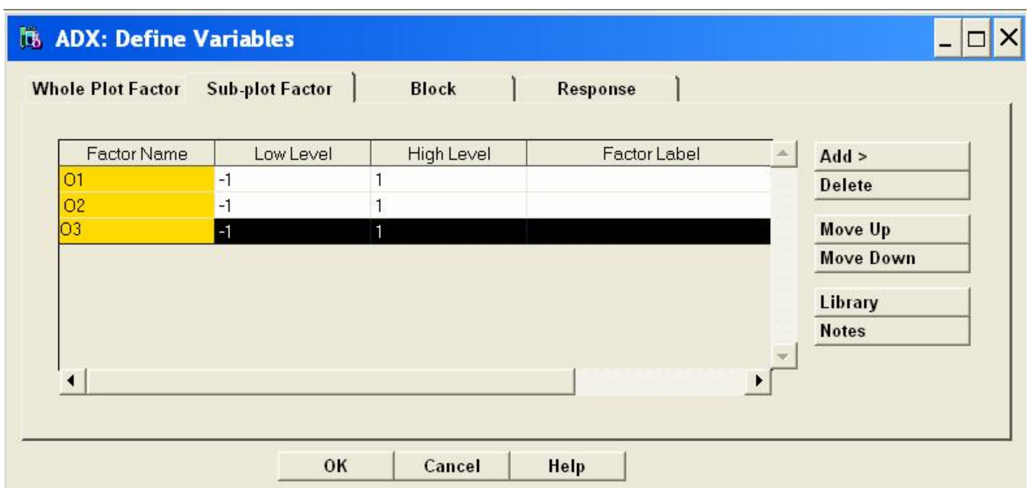
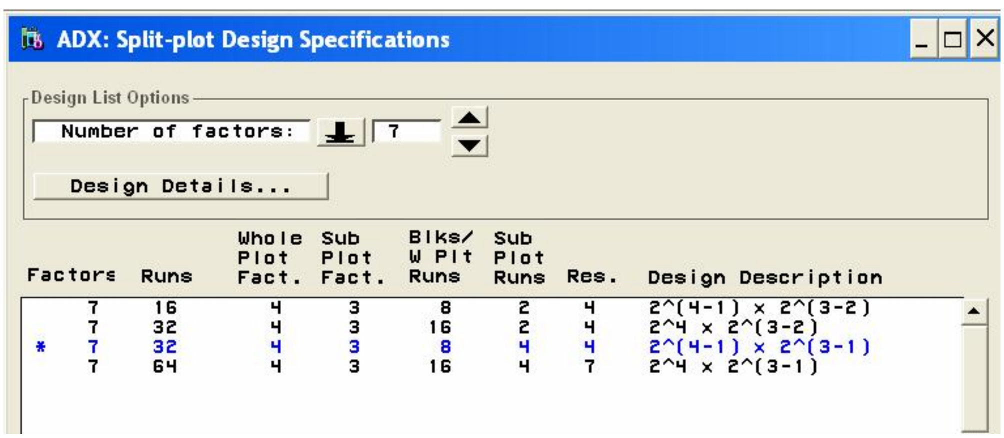


Table 5. Sub-Plot Factors Specification

Responses can be defined in the same way. Once the whole-plot and subplot factors have been defined we can select an appropriate FFSP by clicking on the Select Design button

The "Split-plot Design Specifications" window, Table 6, shows all the FFSP available for 4 whole-factors and 3 subplot factors. This window shows the total number of runs, as well as the whole-plot number of runs, and the subplot number of runs. We select a 32-run design with 8 whole-plot runs and 4 subplot runs for each whole-plot run. In other words, each whole-plot run is "split" into four subplot runs for a total of 32 runs. The "Design Description" shows that we can think of this design as a combination of two fractional factorials $2^{4-1} \times 2^{3-1}$. However, more information is needed to see if this is a good choice.



	Factors	Runs	Whole Plot Fact.	Sub Plot Fact.	Blks/ W Plt Runs	Sub Plot Runs	Res.	Design Description
	7	16	4	3	8	2	4	2^(4-1) x 2^(3-2)
	7	32	4	3	16	2	4	2^4 x 2^(3-2)
*	7	32	4	3	8	4	4	2^(4-1) x 2^(3-1)
	7	64	4	3	16	4	7	2^4 x 2^(3-1)

Table 6. $2^{4-1} \times 2^{3-1}$ Fractional Factorial Design Specifications

The "Design Information" tab in the "Design Details" window, Table 7, shows that this design comes from Huang et al. (1998), and also shows the number of whole-plot and subplot factors and runs.

ADX: Design Details

Design Information | Design Listing | Design Properties | Alias Structure

Design Attribute	Value
Design Name	Untitled
Date Created	04AUG2006
DESIGN PROPERTIES	
Design Type	Split-plot design
Design Description	Huang: 2^(4-1) x 2^(3-1)
Factors	7
Responses	1
Runs	32
Blocks	8
Whole Plot Factors	4
Sub-plot Factors	3
Whole Plot Runs	8
Sub-plot Runs	4

Table 7. $2^{4\text{-}1} \times 2^{3\text{-}1}$ Fractional Factorial Design Information

Kulahci et al. (2006) have introduced a performance criterion that is helpful when selecting FFSP. In these situations we normally are not able to estimate all the two-factor interactions (2fi) free and clear of main effects or of each other. The question is, how many 2fi can we estimate, and how many of the estimable 2fi are clear from confounding with other 2fi? For each FFSP we can compute the number of estimable 2fi and the number of clear 2fi. The "Design Properties" tab, Table 8, shows that we can estimate 18 2fi, 15 of which are clear from confounding. Not bad for a 32-run design that preserves the sequential nature of the manufacturing process!

ADX: Design Details

Design Information | Design Listing | Design Properties | Alias Structure

Design Attribute	Value
Design description	Huang: 2^(4-1) x 2^(3-1)
Defining relation	L1*L2*L3*L4 = L1*L2*O1*O2*O3
Resolution	4
ABERRATION	
Number of 4-letter words in defining relation	1
Number of 5-letter words in defining relation	2
Number of 6-letter words in defining relation	0
Number of 7-letter words in defining relation	0
MODEL ESTIMATION (2FI model)	
Number of estimable terms	18
Number of clear terms	15

Table 8. $2^{4\text{-}1} \times 2^{3\text{-}1}$ Fractional Factorial Design Properties

The “Alias Structure” tab, Table 9, shows that all 7 main effects are clear from confounding and that ALL 2fi between Whole-plot factors (L1-L4) and Subplot factors (O1-O3), as well as ALL the 2fi between the Subplot factors (O1-O3) are free from confounding. The six 2fi between the Whole-plot factors are confounded with each other in three strings of size two; i.e., L1*L2 is confounded with L3*L4, L1*L3 is confounded with L2*L4, and L1*L4 is confounded with L2*L3.

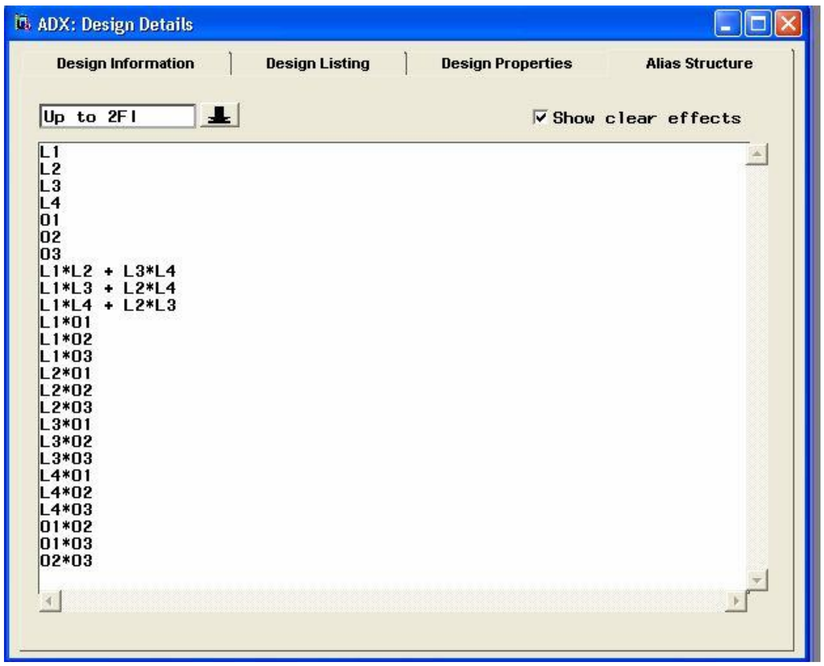
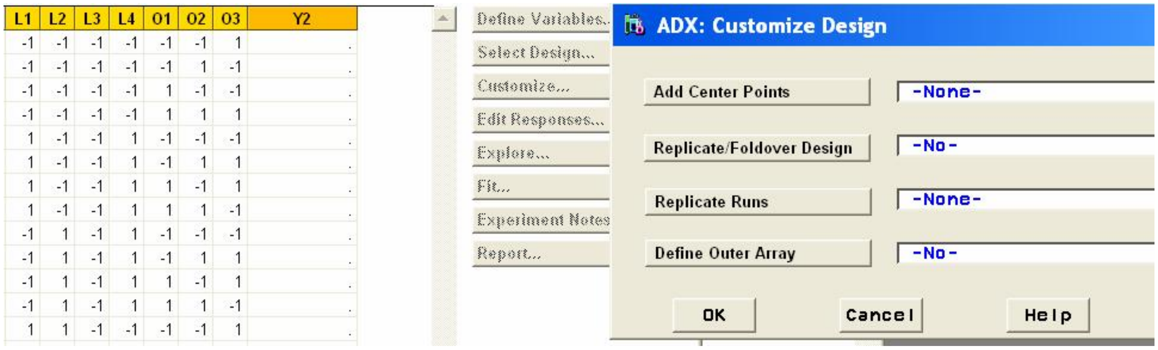


Table 9. $2^{4-1} \times 2^{3-1}$ Fractional Factorial Design Alias Structure

One of the main goals in this experiment was to study the interactions between the first step factors (L1, L2, L3, and L4) with the step two factors (O1, O2 and O3). Note that, from the alias table, all 15 of these interactions are clear from confounding with other two-factor interactions.

As we mentioned in Section 2 split-plot designs require two error terms: one to test for the whole-plot factors and their interactions, and another one to test for the sub-plot factors, their interactions, and their interactions with the whole-plot factors. In order to accomplish this we need to replicate the whole-plots. This can be easily done by using the “Customize” feature of the ADX interface for design of experiments. Clicking on the

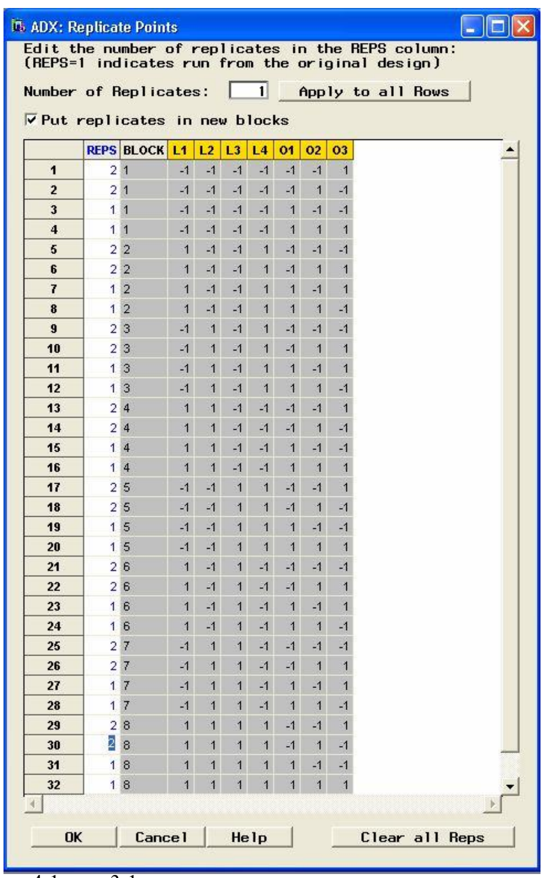
"Customize" button, Table 10, gives you options for adding center points, replicating the entire design, including fold-over, replicating particular runs, or defining an outer array.



L1	L2	L3	L4	O1	O2	O3	Y2
-1	-1	-1	-1	-1	-1	1	.
-1	-1	-1	-1	-1	1	-1	.
-1	-1	-1	-1	1	-1	-1	.
-1	-1	-1	-1	1	1	1	.
1	-1	-1	1	-1	-1	-1	.
1	-1	-1	1	-1	1	1	.
1	-1	-1	1	1	-1	1	.
1	-1	-1	1	1	1	-1	.
-1	1	-1	1	-1	-1	-1	.
-1	1	-1	1	-1	1	1	.
-1	1	-1	1	1	-1	1	.
-1	1	-1	1	1	1	-1	.
1	1	-1	-1	-1	-1	1	.

Table 10. $2^{4-1} \times 2^{3-1}$ Fractional Factorial Design Customize

Since we have a constraint of no more than 50 experimental runs replicating the whole 32-run design will put us over the limit; i.e., 64 runs. In this case we want to replicate only the first two runs in each block for a total of 32 + 16 = 48 runs (just below our limit), as shown in Table 11 below.

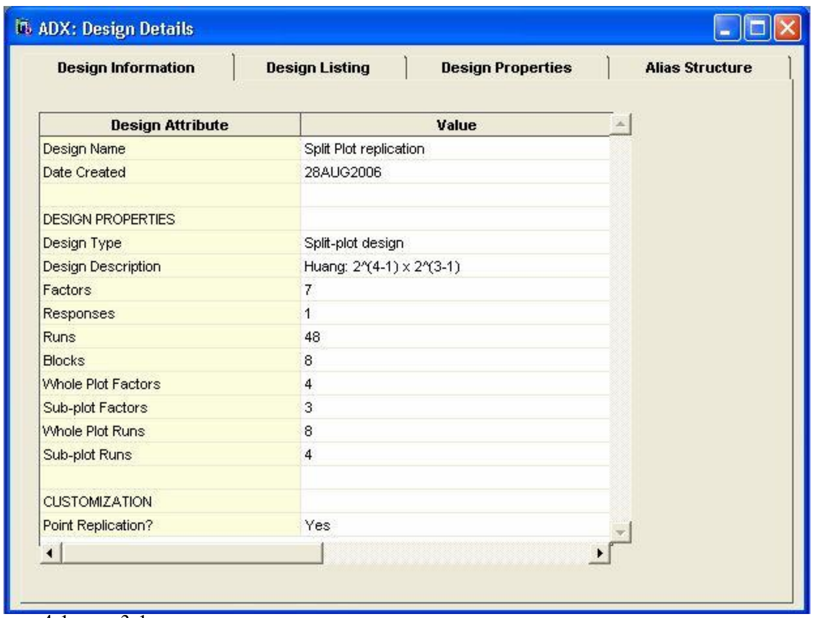


	REPS	BLOCK	L1	L2	L3	L4	O1	O2	O3
1	2	1	-1	-1	-1	-1	-1	-1	1
2	2	1	-1	-1	-1	-1	-1	1	-1
3	1	1	-1	-1	-1	-1	1	-1	-1
4	1	1	-1	-1	-1	-1	1	1	1
5	2	2	1	-1	-1	1	-1	-1	-1
6	2	2	1	-1	-1	1	-1	1	1
7	1	2	1	-1	-1	1	1	-1	1
8	1	2	1	-1	-1	1	1	1	-1
9	2	3	-1	1	-1	1	-1	-1	-1
10	2	3	-1	1	-1	1	-1	1	1
11	1	3	-1	1	-1	1	1	-1	1
12	1	3	-1	1	-1	1	1	1	-1
13	2	4	1	1	-1	-1	-1	-1	1
14	2	4	1	1	-1	-1	-1	1	-1
15	1	4	1	1	-1	-1	1	-1	-1
16	1	4	1	1	-1	-1	1	1	1
17	2	5	-1	-1	1	1	-1	-1	1
18	2	5	-1	-1	1	1	-1	1	-1
19	1	5	-1	-1	1	1	1	-1	-1
20	1	5	-1	-1	1	1	1	1	1
21	2	6	1	-1	1	-1	-1	-1	-1
22	2	6	1	-1	1	-1	-1	1	1
23	1	6	1	-1	1	-1	1	-1	1
24	1	6	1	-1	1	-1	1	1	-1
25	2	7	-1	1	1	-1	-1	-1	-1
26	2	7	-1	1	1	-1	-1	1	1
27	1	7	-1	1	1	-1	1	-1	1
28	1	7	-1	1	1	-1	1	1	-1
29	2	8	1	1	1	1	-1	-1	1
30	2	8	1	1	1	1	-1	1	-1
31	1	8	1	1	1	1	1	-1	-1
32	1	8	1	1	1	1	1	1	1

Table11. $2^{4-1} \times 2^{3-1}$ Fractional Factorial Design Replication

Note how the column labeled "REPS" contains the number of replications, 1 or 2, of each of the 32 runs in the $2^{4-1} \times 2^{3-1}$ FFSP. This allows for greater flexibility when constructing FFSP. The final step is to ask the ADX interface to put the replications in new blocks. The final customization is shown below indicating that 16 new runs will be added to the 32-run FFSP.

The "Design Information" tab, Table 12, in the "Design Details" window shows the customized design indicating 48 runs and point replication.



Design Attribute	Value
Design Name	Split Plot replication
Date Created	28AUG2006
DESIGN PROPERTIES	
Design Type	Split-plot design
Design Description	Huang: 2^(4-1) x 2^(3-1)
Factors	7
Responses	1
Runs	48
Blocks	8
Whole Plot Factors	4
Sub-plot Factors	3
Whole Plot Runs	8
Sub-plot Runs	4
CUSTOMIZATION	
Point Replication?	Yes

Table 12. $2^{4-1} \times 2^{3-1}$ Fractional Factorial Design Properties (after customization)

6.2 Analyzing FFSP using ADX

Once the data comes in it can be easily analyzed within ADX. See for example Ramírez and Ramírez (2001). Here we show the analysis for one response of interest, Y2. The Fit Details for Y2 table, Table 13, shows the statistically significant effects at the 10% level of significance. Note that all the factors in the first step of the process, L1-L4, are statistically significant, as well as factors O2 & O3 of the second processing step. In addition, three whole-plot-by-subplot interactions (L2×O2, L3×O2, L3×O3), and one subplot-by-subplot interaction, O2×O3 are significant. This is of particular importance since one of the goals of the experiment was to be able to identify interactions between the two processing steps, to better understand the manufacturing process. Note that two of the Whole-plot factors, L2 & L3, interact with one Subplot factor O2. What this means is that the settings of some of the first step factors cannot be independently set from some of the second step parameters.

ADX: Fit details for Y2

Select effects to be included ir

Fixed Effects | Variance Compor

Effect	Num DF	Den DF	F Value	Pr > F
L1	1	8	46.82	0.0001
L2	1	8	237.69	<.0001
L3	1	8	28.48	0.0007
L4	1	8	23.00	0.0014
L1*L2	1	8	0.11	0.7479
L1*L3	1	8	0.36	0.5675
L1*L4	1	8	2.17	0.1788
O1	1	14	0.60	0.4504
O2	1	14	3.24	0.0934
O3	1	14	4.22	0.0591
L1*O1	1	14	1.08	0.3154
L1*O2	1	14	0.11	0.7424
L1*O3	1	14	0.56	0.4677
L2*O1	1	14	0.84	0.3743
L2*O2	1	14	13.22	0.0027
L2*O3	1	14	0.27	0.6113
L3*O1	1	14	0.04	0.8353
L3*O2	1	14	4.94	0.0431
L3*O3	1	14	4.07	0.0631
L4*O1	1	14	1.32	0.2706
L4*O2	1	14	1.66	0.2185
L4*O3	1	14	1.95	0.1840
O1*O2	1	14	0.01	0.9309
O1*O3	1	14	1.75	0.2069
O2*O3	1	14	3.18	0.0964

Table 13. $2^{4-1} \times 2^{3-1}$ Fractional Factorial Design ANOVA Results

Figure 2 below shows a boxplot of the main effect for factor L1 showing an increase in the response when going from the low to the high level.

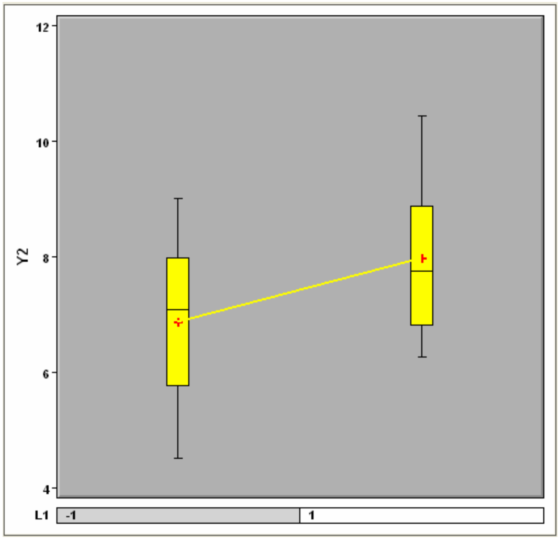


Figure 2. Main effect Boxplot for First Step Factor L1.

Figure 3 shows the interaction plot for L2×O2. Note that for the Low level of L2 the response increases with O2, but for the High level of L2 the response does not change much when O2 changes.

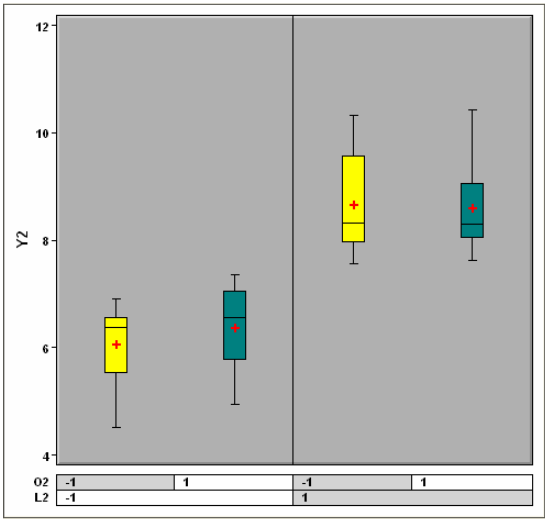


Figure 3. Boxplot Showing the Interaction of L2 and O2.

Figure 4 shows the interaction plot for L3×O2. For the Low level of L2 the response does not change with O3, but for the High level of L2 the response increases with O3.

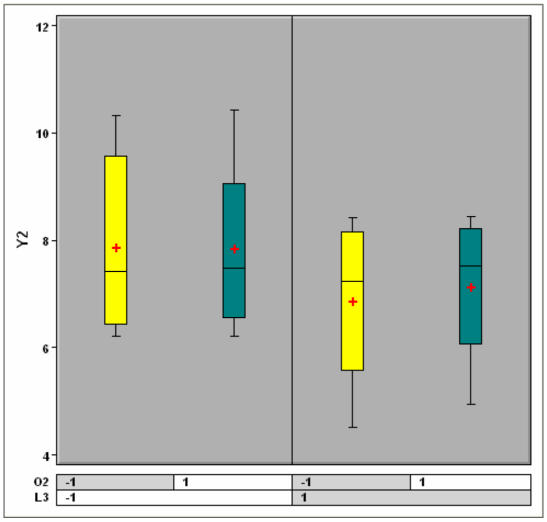


Figure 4. Boxplot Showing the Interaction of L3 and O2.

7. USING FACTEX FOR MORE COMPLEX SPLIT-PLOT DESIGNS

Process complexity usually gives rise to situations with more than two steps, with several factors at each process step. Recently, one of us was faced with designing an experiment for a 4-Step process that needed to be optimized. The process consists of 1 factor in Step 1, 4 factors in Step 2, 2 factors in Step 3, and 3 factors in Step 4. A full factorial design will require $2^1 \times 2^4 \times 2^2 \times 2^3 = 1024$ runs—an impractical, if not impossible, number in most situations. What we needed was a fractional factorial design that preserved the split-split-split-plot structure of the 4-Step process. Note that a 4-Step process gives rise to three levels of "split" which accounts for the different restrictions in randomization, the different number of experimental units involved, or both. We use the notation FFS^{3}P to the note a fractional factorial split-plot with 3 levels of splitting.

This situation is beyond the capacity of the ADX Interface, which only accommodates one level of splitting. But the new split-plot features of FACTEX in Version 9.2 give intuitive syntax for finding such a design. We will discuss these features in general first, and then use them in the context of this FFS^{3}P example.

The syntax for defining the split-plot structure has the following form

BLOCK UNITS = (*unit-specifications* **)**

where each *unit-specification* has the form

unit-factor = *nlev*

The value of *nlev* is the number of levels for the unit factor. Unit factors are not involved in the model structure of the design. Instead, you use a `BLOCK UNITS=()` specification in conjunction with one or more `UNITEFFECT` statements to constrain how the factor levels can change across the runs of the experiment.

Unit effects are specified by the `UNITEFFECTS` statement, which has the following form

UNITEFFECT *unit-effect* / < **WHOLE = (** *whole-unit-effects* **)** >
< **SUB = (** *sub-unit-effects* **)** > **;**

Unit effects constrain how the factor levels can change across the runs of the experiment. A *unit-effect* is an interaction between *unit-factors* defined in the BLOCKS UNITS=() specification.

unit-factor * … * *unit-factor*

It defines a partition of the runs on which to define whole-unit and sub-unit effects of the factors named in the FACTORS statement. Whole-unit effects are those that must be constant within groups of runs defined by the *unit-effect*; sub-unit effects are those that should be estimable from differences within groups of runs defined by the unit-effect. The nature of the whole-unit effects is typically a necessary feature of how the experiment must be designed, and for this reason Ramírez, Kulahci, and Tobias (2007) call these “design constraints”. In contrast, the SUB=() option says what plot-mean contrasts will be used to compute the sub-effects and random error terms that will be used to test them, and thus these are called “model constraints”. For both whole-effects and sub-effects, the *effects* listed must be enclosed within parentheses, as in the MODEL statement.

Thus for example, the following two statements give the split-plot structure and modeling constraints for a 16-run design consisting of 8 whole plots, defined by the experimental unit Wire, and two subplots (16 = 8 × 2), with three whole-plot factors and three subplot factors

```
size design=16;            /* Total no. runs        */
blocks units=(Wire=8);     /* Number of whole plots */
uniteffect
   Wire / whole=(W1 W2 W3) /* Whole plot factors    */
          sub  =(S1 S2 S3); /* Subplot    factors    */
```

Further examples below demonstrate how to use this syntax to construct designs with more complicated split-plot structures.

7.1 Designing FFSP Using PROC FACTEX

Returning to the FFS^{3}P situation discussed before, the PROC FACTEX code needed to generate this design is:

```
proc factex ranorder time=10;
   factors Z        /* Stage 1 */
           A B C D  /* Stage 2 */
           Q R      /* Stage 3 */
           U V W;   /* Stage 4 */
   model r=4 / minabs;
   size design=64;
   blocks units=(Stage1=2 Stage2=8 Stage3=2);
   uniteffect Stage1
      / whole=(Z      )  /* Stage 1 */
        sub  =(A B C D   /* Stage 2 */
               Q R       /* Stage 3 */
               U V W  ); /* Stage 4 */
   uniteffect Stage1*Stage2
      / whole=(A B C D)  /* Stage 2 */
        sub  =(Q R       /* Stage 3 */
               U V W  ); /* Stage 4 */
   uniteffect Stage1*Stage2*Stage3
      / whole=(Q R    )  /* Stage 3 */
        sub  =(U V W  ); /* Stage 4 */
   examine aliasing confounding;
   output out=Design;
run;
```

This generates a 64-run design—*only* 1/16 of the full factorial. Even in this relatively tiny fraction, this design allows you to estimate all 10 main effects clear from confounding with 2fi, as well as 39 2fi , 33 of which are free from confounding with other 2fi. In other words, we can estimate, free from confounding, 33 2fi out of the possible 45 2fi, or 73.33% of the 2fi. Not bad for only 64 runs! The other 6 estimable 2fi are confounded with other 2fi as shown below.

In this design three 2fi between the Step 1 factor and 3 Step 2 factors are confounded with other 2fi involving Step 2 factors. The other three confounded 2fi involve Step 2, Step 3 and Step 4 factors as shown in Table 14.

2fi	Alias
Z*B	Z*B + C*D
Z*C	Z*C + B*D
Z*D	Z*D + B*C
A*Q	A*Q + U*W
A*U	A*U + Q*W
A*W	A*W + Q*U

Table 14. Alias Structure for 4-Step Example

In order to be able to estimate the experimental error or 'noise' the experiment needs to be replicated. The experiment was carried out in two cycles of 64 runs each. This can be easily generated with a Data step

```
data Design; set Design;
   do Cycle=1 to 2; output; end;
run;
```

The Cycle-to-Cycle variation, or replication of the whole-plots, provides the estimate of error to test factor Z in Step 1. The error estimate for the Step 2 factors A, B, C, and D is provided by the replications of subplots, within whole-plots, for the 2 cycles.

Replications of Sub-subplots, within subplots, within whole-plots, for the 2 cycles provide the estimate of error against which to test the Step 3 factors Q and R.

7.2 Analyzing FFSP using PROC MIXED

PROC MIXED is a procedure that allows the user to fit a variety of linear mixed models. A mixed model generalizes the traditional linear model by modeling not only the means of the data but also the variances and covariances. Split-plot designs fall into the category of mixed models because the restrictions in randomization, or the different sizes of experimental units, induce different levels of variability and correlations between experimental units within a level. The analysis of the 4-Step example can be carried out using PROC MIXED by specifying the appropriate signal and noise structures.

```
proc mixed data=SGF2007 ;
class Cycle Z A B C D Q R U V W;
model RY = Z|A|B|C|D|Q|R|U|V|W@1
           /* interactions with whole-plot */
           Z*A    Z*B   Z*C   Z*D Z*Q     Z*R   Z*U   Z*V   Z*W
           /* split-unit interactions  */
           A*B    A*C   A*D   A*Q   A*R   A*U   A*V
           B*Q    B*R   B*U   B*V   B*W
           C*Q    C*R   C*U   C*V   C*W
           D*Q    D*R   D*U   D*V   D*W
           Q*R    Q*U   Q*V
           R*U    R*V   R*W
           U*V    V*W
/* 14 3fi are contained below*/
            Z*A*B*C*D*Q*R*U*V*W
;
random  Cycle(Z) A*B*C*D*Cycle(Z) A*B*C*D*Q*R*Cycle(Z) ;
run ;
```

The model statement specifies the “signal” structure of the model; i.e., the main effects and interactions that we can estimate. In this case we have 10 main effects and 39 two factor interactions. The remaining 14 degrees-of-freedom represent high order interactions.

```
model RY = Z|A|B|C|D|Q|R|U|V|W@1
           /* interactions with whole-plot */
           Z*A    Z*B   Z*C   Z*D Z*Q    Z*R   Z*U   Z*V   Z*W
           /* split-unit interactions  */
           A*B    A*C   A*D   A*Q   A*R   A*U   A*V
           B*Q    B*R   B*U   B*V   B*W
           C*Q    C*R   C*U   C*V   C*W
           D*Q    D*R   D*U   D*V   D*W
           Q*R    Q*U   Q*V
           R*U    R*V   R*W
           U*V    V*W
/* 14 df are contained below*/
            Z*A*B*C*D*Q*R*U*V*W
```

The random statement specifies the "noise" structure of the model; i.e., the error terms necessary to test the statistical significance of the effects in the model statement. Here Cycle(Z) represents the 2 df noise used to test Step 1 factor Z; A*B*C*D*Cycle(Z) represents the 14 df noise to test Step 2 factors A, B, C and D, and the 2fi between Step 1 and Step 2 factors; and A*B*C*D*Q*R*Cycle(Z) represents the 16 df noise to test Step 3 factors Q and R, and the 2fi between Step 1and Step 3 factors, and Step and Step 3 factors. The residual, with 32 df, is used to test Step 4 factors U,V and W, and all the other 2fi. Note that the total number of degrees-of-freedom for error, 64, has been "split" into four components corresponding to the split-split-split-plot structure; i.e., 64=2+14+6+32.

```
random  Cycle(Z) A*B*C*D*Cycle(Z) A*B*C*D*Q*R*Cycle(Z) ;
```

Table 15 shows the ANOVA table for 21 out of the 49 terms of the model, and shows the 3 statistically significant (at the 5% level of significance) effects. In this case the Step 2 factor C is significant and it interacts with Step 1 factor Z, and Step 2 factor A. Note that Z*C is confounded with B*D (Table 4) so additional runs may be required to determine which of the two interactions, is significant.

Type 3 Tests of Fixed Effects				
Effect	**Num DF**	**Den DF**	**F Value**	**Pr > F**
Z	1	2	0.01	0.9225
A	1	14	4.22	0.0592
B	1	14	2.29	0.1527
C	1	14	11.53	***0.0044***
D	1	14	0.79	0.3892
Q	1	16	2.18	0.1594
R	1	16	0.01	0.9110
U	1	32	1.93	0.1742
V	1	32	0.05	0.8169
W	1	32	0.01	0.9209
Z*A	1	14	0.01	0.9126
Z*B	1	14	1.63	0.2224
Z*C	1	14	5.57	***0.0333***
Z*D	1	14	0.79	0.3892
Z*Q	1	16	0.07	0.7978
Z*R	1	16	0.21	0.6511
Z*U	1	32	0.17	0.6835
Z*V	1	32	2.39	0.1321
Z*W	1	32	1.47	0.2344
A*B	1	14	0.24	0.6328
A*C	1	14	6.80	***0.0207***

Table 15. Partial ANOVA for 4-Step Example

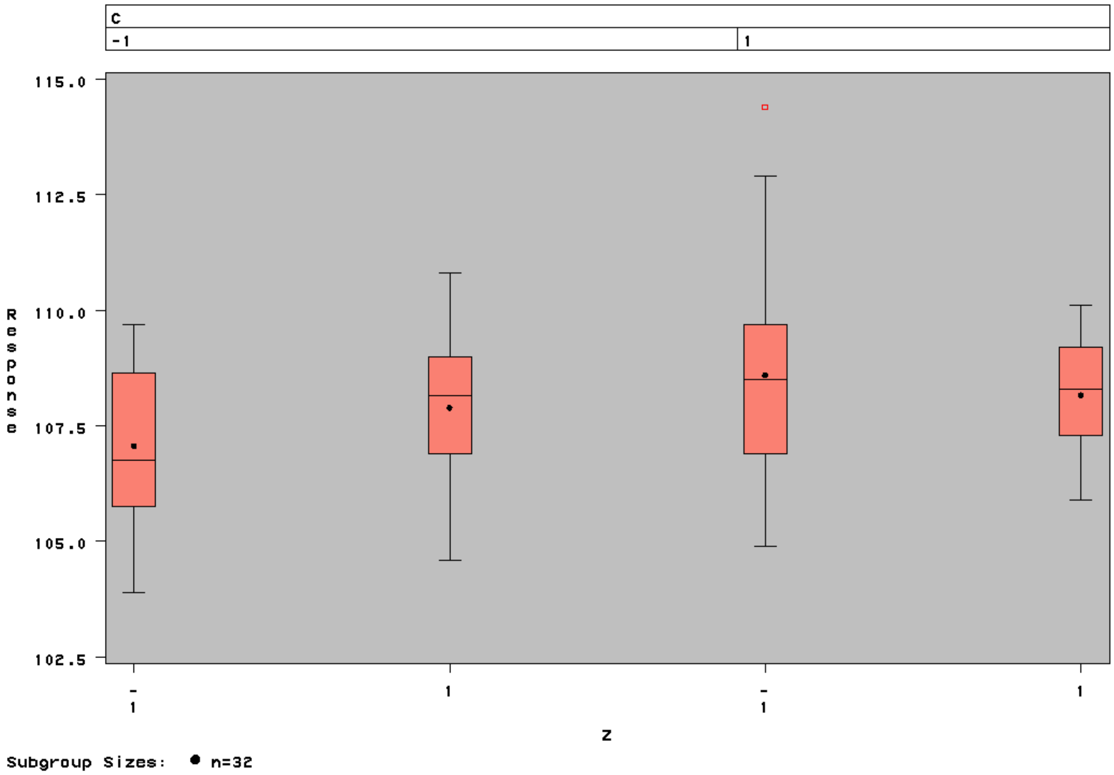


Figure 5. Boxplot Showing the Interaction of Step 1 Factor Z and Step 2 Factor C.

8. CONCLUSIONS

The complexity of many of today's manufacturing processes in addition to the restrictions in the number of experiments that can be performed, and the way they need to be performed, give rise to split-plot type of experimental situations. Recent advances in the design of fractional factorial split-plots, and their incorporation into SAS® 9.2 ADX and PROC FACTEX, makes it now possible to design efficient experiments tailored to meet complex experimental situations. The examples presented in this paper demonstrated the power and versatility of ADX and PROC FACTEX**.**

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