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Ordered weighted averaging in SAS: A MCDM application

Ali Emrouznejad, Aston Business School, Birmingham, UK

ABSTRACT

This paper explores the use of the optimization procedures in SAS/OR software with application to the ordered weighted averaging (OWA) operators of decision-making. OWA was originally introduced by Yager (1988) has gained much interest among researchers, hence many applications in the areas of decision making, expert systems, data mining, approximate reasoning, fuzzy system and control have been proposed.

To facilitate the use of OWA operator by SAS users, a code was implemented. An example is given to illustrate the features of the proposed macro.

INTRODUCTION

The ordered weighted averaging (OWA) operator which was initially introduced by Yager (1988) has attracted much interest among researcher. Since then several applications of the OWA operators are reported in different areas, such as decision making, expert systems, neural networks, group decision making and fuzzy systems and control. Many more applications of OWA are recently reported in multiple criteria decision making and preference ranking.

OWA provides a general class of parameterized aggregation operators that include the min, max, average, and several other operators that recently introduced to the literature. The orness measure reflects the 'andlike' or 'orlike' aggregation result of an OWA operator, which is very important both in theory and applications. The extension of Yager's orness concept to other aggregation operators has recently been focused by some researchers. O'Hagan(1988) suggested the problem of constraint nonlinear programming with a maximum entropy procedure, the solution is called a MEOWA (Maximum Entropy OWA) operator. Liu (2006) developed an OWA operator generating method with the equidifferent OWA operator, and discussed its properties; Wang and Parkan (2005) proposed a linear programming model with minimax disparity approach to get the OWA operator under desired orness level. Amin and Emrouznejad (2006) and Amin (2007) extended the minimax disparity approach using different optimization model.

On the other hand, SAS is recognized as a one of the lead packages for statistical analysis and as a powerful tool for database management systems in many organizations. Many large industry including banks, pharmaceuticals, manufacturers, telecommunications companies, ... are suing SAS, all with the same basic needs to make better strategic decisions. There are wide rages of applications in SAS that the users recognized them as powerful tools in organizational management. For example the SAS/OR System has numerous optimization procedures which handle the standard problems such as linear and non-linear programming with all types of constraints, including integer, mix-integer and combinatorial optimization (SAS/OR User's Guide: Mathematical Programming; 2007). These capabilities are exposed to the SAS users in a variety of places such as in procedures LP and NLP (Kearney; 1999) PROC OPTMODEL (Hughes and Kearney; 2007). In addition to the standard procedures available in SAS System applications such as neural network, simulation and control project management are introduced. Recently SAS/DEA and SAS/MALM macros were introduced by Emrouznejad (2000, 2002, and 2005) for measuring efficiency and productivity of Decision Making Units.

This paper aims to introduce a new application in SAS System and with the aim for evaluation of a set of alternatives using OWA operators.

OWA OPERATORS

An OWA operator of dimension n is a mapping:

$$F : R^n \to R$$

that is associated with weighting vector

$$W = [w_1, w_2, \ldots, w_n]^T$$

such that

$$\sum_{i=1}^{n} w_i = 1; w_i \in [0, 1]$$

and where

$$F(a_1,\ldots,\ a_n)=\sum_{j=1}^{n} w_j b_j$$

In this formula b_j is the j^{th} largest element of the collection of the aggregated objects

$$(a_1,\ldots,\ a_n).$$

The function value

$$F(a_1,\ldots,\ a_n)$$

determines the aggregated value of arguments

$$(a_1,\ldots,\ a_n).$$

A fundamental aspect of the OWA operator is the re-ordering step, in particular an argument a_i is not associated with a particular weight w_i but rather a weight w_i is associated with a particular ordered position i of the arguments. A known property of the OWA operators is that they include the Max, Min and arithmetic mean operators for the appropriate selection of the vector W:

$$W=[1,0,\ldots,0]^T,\quad F(a_1,\ldots,\ a_n)=\underset{i}{Max}(a_i).$$

$$W=[0,0,\ldots,0]^T,\quad F(a_1,\ldots,\ a_n)=\underset{i}{Min}(a_i).$$

$$W=\left[\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\right]^T,\quad F(a_1,\ldots,\ a_n)=\frac{1}{n}\sum_{i=1}^{n} a_i.$$

Recently, Amin and Emrouznejad (2006) (AE-OWA; hereafter) introduced an alternative linear programming to determine the OWA weights. In this model each OWA vector is associated with an orness function; usually refer to as degree of OWA; i.e. α = orness(w) (for mathematical presentation of AE-OWA model see Amin and Emrouznejad; 2006).

OWA IN SAS

One important issue in the theory of OWA aggregation is the determination of the associated weights. As we discussed a number of approaches have been suggested in the literature for obtaining the weights. One of the aims of this paper is to provide user friendly and powerful software that can provide various OWA operators; hence the user can easily choose the one that is more suitable for their purpose.

The proposed code in this paper computes ordered weighted averaging using several types of OWA operators including min, arithmetic mean, max and minimax AE-OWA. The users' family with SAS can easily add other type of OWA operators if needed. This SAS code first calculates the weights of criteria, and then calculates the rank of the alternatives based on the weighted criteria. The main procedure used for OWA problem is PROC OPTMODEL in SAS/OR.

```
*An example of OWA problem in SAS;
%let _altdata='c:\sasowa\OWAdata.txt';
%let _alpha=0.3;
%let _ncriteria=6;
%let _nalternative=7;

* To import text tab delimited data file to SAS data file;
proc import
  datafile=&_altdata
  out=alts
  dbms=tab
  replace;
  getnames=yes;
run;

proc optmodel;
set CRITERIA = 1..&_ncriteria;
```

```
set CRITERIA1 = 1..&_ncriteria-1;
set ALTERNATIVE = 1..&_nalternative;
number c{ALTERNATIVE, CRITERIA};
number rankAE{ALTERNATIVE} ;
number rankMIN{ALTERNATIVE} ;
number rankMAX{ALTERNATIVE} ;
number rankMEAN{ALTERNATIVE} ;
number corder{ALTERNATIVE, CRITERIA};
number wMAX{ALTERNATIVE};
number wMIN{ALTERNATIVE};
number wMEAN{ALTERNATIVE};
number ctemp;
number altName{ALTERNATIVE};
number alpha=&_alpha;
number n=&_ncriteria;
var w{CRITERIA} ;
var delta;

read data alts
into [ALTERNATIVE]
{j in CRITERIA} <c[ALTERNATIVE,j]=col("C"||j)> ;

min obj =delta;

con orness: (1/(n-1))* sum{i in CRITERIA} (n-i)*w[i] = alpha;

con wji{i in 1..n, j in i+1..n }:
w[j]-w[i]+delta >= 0;

con wij{i in 1..n, j in i+1..n }:
w[i]-w[j]+delta >= 0;

con swumw:
sum{i in 1..n} w[i]= 1;

solve ;

* MAX-OWA;
for  {i in CRITERIA}
if i=1 then wMAX[i]=1; else wMAX[i]=0;

* MIN-OWA;
for  {i in CRITERIA}
if i=n then wMIN[i]=1; else wMIN[i]=0;

* MEAN-OWA;
for  {i in CRITERIA}
wMEAN[i]=1/n;

*Order the critaria;
for {k in ALTERNATIVE}
    do;
    for  {i in CRITERIA}
           for {j in CRITERIA}
           do;
           if c[k,j]<c[k,i] then
                  ctemp=c[k,i]; c[k,i]=c[k,j];c[k,j]=ctemp;
           end;
    end;
```

```
*Rank by AE-OWA model;
for {k in ALTERNATIVE}
    do;
    rank[k]=0;
    for  {i in CRITERIA}
           rank[k]=rank[k]+c[k,i]*w[i];
    end;
print rank;

*Rank by MIN-OWA model;
for {k in ALTERNATIVE}
    do;
    rankMIN[k]=0;
    for  {i in CRITERIA}
           rankMIN[k]=rankMIN[k]+c[k,i]*wMIN[i];
    end;
print rankMIN;

*Rank by MAX-OWA model;
for {k in ALTERNATIVE}
    do;
    rankMAX[k]=0;
    for  {i in CRITERIA}
           rankMAX[k]=rankMAX[k]+c[k,i]*wMAX[i];
    end;
print rankMAX;

*Rank by MEAN-OWA model;
for {k in ALTERNATIVE}
    do;
    rankMEAN[k]=0;
    for  {i in CRITERIA}
           rankMEAN[k]=rankMEAN[k]+c[k,i]*wMEAN[i];
    end;
print rankMEAN;

quit;
```

An example of data file is:

'c:\sasowa\OWAdata.txt'

Alternative	c2	c3	c1	c1	c2	c6
1	1	3	2	3	1	2
2	3	1	1	3	1	3
3	2	2	1	3	3	1
4	1	3	3	1	1	3
5	3	3	1	3	3	2
6	1	3	1	3	1	1
7	1	2	3	1	2	2

CONCLUSION

This paper developed a new application in SAS/OR using PROC OPTMODEL for ranking alternatives using OWA operators. The code implemented in this paper has no limitation on the number of alternatives or dimension in the OWA vector. The software is also very flexible so experienced SAS users can easily add other OWA operators to the code.

REFERENCES

A. Emrouznejad (2002) A SAS Application for Measuring Efficiency and Productivity of Decision Making Units, in the proceeding of the 27th SAS International Conference, pp 259-27, April 2002, USA.

A. Emrouznejad (2000) An extension to SAS/OR for decision system support. SAS Users Group International, 25th Annual Conference, Indiana Convention Centre, Indianapolis, IN, USA; April 9-1.

A. Emrouznejad (2005), Measurement efficiency and productivity in SAS/OR, Computers & Operations Research, 32(7)1665-1683.

Ed Hughes and Trevor Kearney (2007), Optimization with SAS/OR®: What It Is, What's New, and How It Adds Value, Proceedings of SAS Global Forum 33, SAS Institute Inc., Cary, NC

G.R. Amin, A. Emrouznejad (2006), An extended minimax disparity to determine the OWA operator weights, Computers and Industrial Engineering 50 (3) 312–316.

G.R. Amin (2007), Notes on properties of the OWA weights determination model, Computers and Industrial Engineering, 52(4): 533-538.

M. O'Hagan (1988), Aggregating template or rule antecedents in real-time expert systems with fuzzy set, in: P. Grove (Ed.), Proc. 22nd Annu. IEEE Asilomar Conf. on Signals, Systems, Computers, CA, pp. 681–689.

R.R. Yager (1988), On ordered weighted averaging aggregation operators in multicriteria decision making, IEEE Transactions on Systems, Man and Cybernetics 18 (1) 183–190.

SAS Institute Inc (2007). "SAS/OR User's Guide: Mathematical Programming

Trevor Kearney (1999), "Advances in Mathematical Programming and Optimization in the SAS System", SUGI24 Proceedings, Cary, NC: SAS Institute Inc.

X. Liu (2006), On the properties of equidifferent OWA operator, International Journal of Approximate Reasoning 43 (1) 90–107.

Y. Wang, C. Parkan (2005), A minimax disparity approach for obtaining OWA operator weights, Information Sciences 175 (1) 20–29.

CONTACT INFORMATION (HEADER 1)

Your comments and questions are valued and encouraged. Contact the author at:

Ali Emrouznejad,
Operations and Information Management,
Aston Business School,
Aston University,
Birmingham B4 7ET,
United Kingdom
Email: a.emrouznejad@aston.ac.uk
Web: www.DEAzone.com