Beating Student’s t and the Bootstrap -- Using SAS® Software to Generate Conditional Confidence Intervals for Means

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Abstract

We illustrate a method for generating conditional confidence intervals for means of symmetric distributions. A simulation study demonstrates that the conditional confidence interval (CCI) has parameter coverage rates at the nominal level of the confidence interval where the Bootstrap procedure has coverage rates below the nominal level. The CCI produces average interval lengths which are equal to or shorter than those produced by using Student’s t procedure. The CCI, which can be computed by inverting a permutation test, is easily computed. Although the CCI is computationally intensive, it only requires a few seconds of CPU time to execute. SAS/IML® code (implemented under a CMS operating system) is available from the authors for the one and two sample cases and for a linear regression setting.

Method

The derivation of the CCI can be found in Vos and Chenier (1995). The CCI procedure is motivated by conditioning arguments, but it is equivalent to a resampling plan. In contrast to other resampling procedures such as the bootstrap or jackknife, the CCI subsamples may differ in the total number of observations used, and the actual number of observations used in any one subsample will follow a binomial distribution. We chose to generate 4000 subsamples since Efron and Tibshirani (1993) suggest this many subsamples may be required for the bootstrap procedure to generate reasonable estimates.

Each of the 4000 subsamples is chosen by giving each of the observations in the original sample a 50% chance of being included. The subsample means generate a sampling distribution from

Introduction

With the increases in the computing power available to researchers has come a corresponding increase in the number of methods available to do certain analyses. Methods which rely on intensive computations or resampling plans have become much more popular in the past few years. In this paper, we illustrate how one may use such an approach to obtain a confidence interval for a population mean of a symmetric distribution which is superior in many cases to analogous approaches using the standard t-interval or the bootstrap procedure.

The conditional confidence interval (CCI) is obtained by inverting a permutation test. Although it is computationally intensive, the SAS code needs only a few seconds to generate results via this procedure, making the CCI suitable for interactive use. The code for the one sample case, the two sample case and for a linear regression setting all are available from the authors.
which confidence limits for the mean can be obtained by taking the appropriate percentile values of the subsampling distribution.

**Simulation Results**

To compare the performance of the CCI to intervals based on Student's t or the bootstrap, we conducted a simulation study. Data were generated using a variety of symmetric and skewed distributions based on original sample sizes of either 10 or 20 observations. For purposes on brevity, the results for the smaller sample size are summarized below.

For the one sample case, we initially compared how these procedures perform on symmetric distributions. Next, we compared their behavior for skewed distributions. Finally, we looked at distributions having heavy tails and a class of "contaminated normal" distributions where the data are normally distributed, but a certain percentage of the observations have a different variance from the majority of the population.

For normally distributed data, the coverage rate and average interval length of the CCI is equivalent to that of the standard t-interval. As the amount in the tails of the distribution increased, the CCI maintained its coverage at the nominal level. The standard t-interval tended to overcover in relation to the nominal level, and the average interval lengths of the t-intervals were longer than those of the CCI. With the larger sample size of 20, this finding was still present, although it was less dramatic.

In contrast, the intervals of the bootstrap procedure consistently underestimated the true nominal level. This finding was true even though the average lengths of the bootstrap intervals were longer than those of the CCI for the distributions having the heaviest tails.

A comparison of the coverage rates of all three methods is found in Figure 1. Due to the consistent undercoverage of the bootstrap relative to the nominal rate, only the relative lengths of the CCI and the t-interval are presented in Figure 2.

Next, we compared the behavior of these methods for a variety of skewed distributions. These results are presented in Figures 3 and 4. As the skewness of the population increased, the coverage rates of all three methods fell off dramatically. For those distributions which were slightly to moderately skewed, the CCI had equal coverage rates to the standard t-interval with shorter average interval lengths when the sample size was small. With the larger sample size, there was little difference in the performance of the CCI and the standard t-interval. As before, the coverage of the bootstrap intervals was too low for both sample sizes. One reason for the undercoverage of the bootstrap intervals was that the average interval length tended to be too small.

For heavy-tailed distributions, another possible option is to use the trimmed mean as a basis for producing a confidence interval. We compared the performance of intervals using a trimmed mean approach versus the CCI where the same proportion of trimming took place prior to the computation of the interval. Figures 5 and 6 show a comparison of these two approaches versus the unadjusted CCI. The coverage rates of the CCI, the trimmed CCI and the interval based on the trimmed mean were all equivalent. The trimmed CCI produced shorter intervals than the usual CCI in all cases except the normal distribution. In all cases, the length of the trimmed CCI interval was equal to or less than the interval length derived from the trimmed mean procedure. For the cases where the distribution had heavy tails (e.g. the Cauchy and the Slash), the trimmed CCI length was approximately 10%
shorter than the trimmed mean interval.

Summary

The conditional confidence interval (CCI) is an attractive approach to implement when one wishes to produce confidence intervals for a population mean. It has less restrictive assumptions than the usual t-procedure, yet it performs up to the standards of the t-interval for the case where the population is normally distributed. The performance of the CCI also exceeds that of the t-interval when the population was slightly to moderately skewed.

The CCI also consistently outperformed the bootstrap. The coverage rates of the CCI were at the nominal levels, while the bootstrap tended to have coverage rates below the nominal levels. Also, the average interval length of the CCI tended to be shorter than those of the bootstrap when the coverage of the two methods were approximately equal.

For symmetric distributions having heavy tails, the CCI outperformed both the t-interval and the bootstrap procedures. However, a version of the CCI used in conjunction with trimming produced even shorter intervals than the usual CCI while maintaining coverage rates. The trimmed CCI also met or exceeded the performance of the trimmed mean procedure in terms of coverage rates and interval lengths. For distributions having the heaviest tail densities, the trimmed CCI was clearly superior to all other methods considered.

References


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Figure 1. Coverage rates of 90% CI for symmetric distributions

Figure 2. Relative lengths of 90% CI for symmetric distributions
Figure 3. Coverage rates of 90% CI for skewed distributions

Figure 4. Relative lengths of 90% CI for skewed distributions
Figure 5. Coverage rates of 90% CI for heavy-tailed distributions

*Note: The values for the unadjusted CCI are off the scale for the t(1) and slash distributions.

Figure 6. Relative lengths of 90% CI for heavy-tailed distributions