Using SAS Software to Design and Evaluate Lot Acceptance Sampling Plans

Dennis W. King, PhD, STATKING Consulting Inc., Fairfield, OH

ABSTRACT

Although the trend in modern quality control is a move towards on-line statistical process control through the use of control charts and control algorithms, lot acceptance sampling is still heavily used in industry particularly by governmental suppliers. Lot acceptance sampling consists of sampling a lot of finished product and determining whether the lot is of acceptable quality based on some quality characteristic. One of the most used sampling plans for lot acceptance sampling is Military Standard 105. In this presentation, SAS code sequences and examples will be described that show how to use the SAS DATA step along with functions available in base SAS to automate the design of sampling plans shown in this military standard.

Introduction

Although acceptance sampling is thought by some quality practitioners to be an outdated method for quality improvement, the technique is still widely used in industry today. Lot acceptance sampling is a first step towards quality improvement through quantitative statistical methods. Using SAS software, performance characteristics of sampling plans can be computed and used to make decisions on the appropriate sampling plan to be used on a particular process.

In this paper, one of the most widely used set of sampling plans, US Department of Defense Military Standard 105 (1989), will be discussed. It will be seen that with the help of SAS software a much broader range of sampling plans than is shown in the standard can be designed and evaluated.

The basis for acceptance sampling is quite simple. A group of homogeneous product called a lot is produced by a supplier. In order to determine if the quality of the manufactured goods is acceptable to the consumer, a sample of product is taken from the lot and a quality characteristic is measured on each item in the sample. If the quality of the sample is at an acceptable level then the lot is accepted by the consumer. If the quality of the sample is unacceptable then the lot is rejected and returned to the supplier.

There are four key performance measures for every sampling plan. These are the probability of acceptance of the plan, denoted \( P_a \), the operating characteristic (OC) curve, the average total inspection (ATI) and the average outgoing quality (AOQ). In this paper, these characteristics will be discussed in terms of attribute single sampling plans. Single sampling plans are those for which lot quality is judged from a single sample of size \( n \) from a lot of size \( N \). This is the most basic of sampling plans. An excellent summary of the computing formulas for these performance characteristics for other sampling schemes is given in Schilling (1982).

For an attribute, single sampling plan, a lot is accepted when \( c \) or less defects are found in the sample. The probability of acceptance, \( P_a \), for this type of sampling plan is the probability of accepting a lot given that the stream of lots coming through the inspection process has a proportion of items defective \( p \). For a lot size \( N \) containing \( k=NP \) defectives, \( P_a \) is equal to the probability of finding \( c \) or less defectives in a sample of size \( n \). The correct probability model for this situation is the hypergeometric model,

\[
P_a = \sum_{x=\min(0, n-(N-k))}^{c} \binom{k}{x} \binom{N-k}{n-x} / \binom{N}{n}
\]

which can be computed using the PROBYFR function in base SAS software. Since \( k=NP \), \( P_a \) is a function of \( p \). A plot of \( P_a \) over a range of proportion defectives is called an operating characteristic (OC) curve shown for the sampling plan. The OC curve gives the probability of passing a lot over a range of lot percent defectives.

From Figure 1, it can be seen that for a single sampling plan with \( N=10,000, n=100, c=2 \), the probability of acceptance at \( ACL=0.01 \) is \( .92 \) while the probability of acceptance at LTPD is \( .12 \). These two values, \( P_a \) at ACL and \( P_a \) at LTPD, are the basis for constructing most if not all sampling plans used in industry.

Another performance characteristic for sampling plans is the average total inspection (ATI). The ATI is important in terms of overall sampling costs and rework costs. For a specified fraction defective \( p \), \( n \) of the items in a lot are always inspected. The lot will be rejected \( (1 - LTPD) \) of the time and submitted to 100% rectifying inspection. Therefore, for a single sampling plan,

\[
ATI = n + (1 - LTPD) (N - n)
\]
Since $P_d$ is a function of $p$, $ATI$ can be calculated for a range of proportions defective and plotted as a smooth curve on a graph.

The final measure of performance for sampling plans is the average outgoing quality (AOQ). This is the level of quality seen by the consumer. For a single sampling plan,

$$AOQ = \frac{P_d p(N-n)}{N}.$$  

Since $P_d$ is a function of $p$, this quantity can be calculated for a range of proportion defective. The AOQ will be maximized at a certain proportion defective, say $p_m$. The maximum AOQ of a sampling plan is called the average outgoing quality limit (AOQL). This value gives a worst case scenario of the proportion of defective product seen by the consumer.

The sampling plan characteristics just discussed can be calculated using the PROBHYP function as illustrated in the following code. For the single sampling plan discussed above with $N=10,000$, $n=100$ and $c=2$, values of the probability of acceptance, average total inspection and the average outgoing quality are generated using the DATA step and PROB PRINT and shown in Output 1 in the appendix. Using the SASGRAPH procedure GPLOT, plots of the ATI and AOQ are shown in Outputs 2 and 3 in the appendix.

```sas
data single;
  n=100; c=2; lotn=10000;
  do p=.007 to .067 by .005;
    if p<.01 then pa=1;
    else pa=probhyp(lotn,k,n,c);
    ati = n + (1-pa)*(lotn-n);
    aqi = (pa*p*(lotn-n))/lotn;
    output;
  end;
run;

proc print data=single(obs=15);
  title 'Performance Characteristics for Single Sampling Plan';
  title2 'N=10000, n=100, c=2';
run;

proc gplot data=single;
  plot ati*p / vaxis=axis1 vzero;
  axis1 label=(a=90 'Units Inspected');
  symbol i=join v=none;
  title 'Average Total Inspection for Single Sampling Plan';
  title2 'N=10000, n=100, c=2';
run;

proc gplot data=single;
  plot aqi*p / vaxis=axis1 vzero;
  axis1 label=(a=90 'Average Outgoing Quality');
  symbol i=join v=none;
  title 'Average Outgoing Quality';
  title2 'N=10000, n=100, c=2';
run;
```

For this plan, the probability of acceptance is about 92% at $p=0.01$ and about 5.5% at $p=0.06$. The average outgoing quality is at worst 1.34% defective. This AOQL value is reached at a defective rate of approximately 2.5%.

**Military Standard 105 Plans**

Military Standard 105 has long been used to design sampling plans for quality characteristics measured as conforming/nonconforming (attribute data). The plans shown in this standard are based on the probability of passing a lot at AQL. Although the plans involve various switching rules for tightened and normal inspection, the tables and graphs shown in this standard are based on AQL at normal inspection levels.

The OC curves of any of the plans in Military Standard 105 can be verified using SAS software. For example, with $N=2,000$, the use of sample size code letter $K$ is indicated from Table I of the Standard. From Table II-A of the Standard $n=125$ and $c=3$ for AQL=.01. To verify the values of $P_a$ for this plan, use the following SAS code:

```sas
data oc105;
  n=125; c=3; lotn=2000;
  do p=.00658, .01, .0109, .014, .0203, .0294, .0409, .062, .0804;
    k=ceil(lotn*p);
    pa=probhyp(lotn,k,n,c);
    output;
  end;
run;

proc print;
  title 'OC Curve Values for MIL-STD-105 Plan';
  title2 'N=2000, n=125, c=3';
run;
```

As can be seen from Output 4 in the appendix, the probability of acceptance at AQL=.01 is 0.96775. The probability of accepting a lot containing 6.2% defectives with this plan is 0.04055. These values correspond to Table X-K-1 in the Standard. Any plan in Military Standard 105 can be evaluated in this way.

Due to the relationship of the lot size to the sample sizes in Military Standard 105, attribute sampling plans other than those given in the 105 tables can be constructed for any specified lot size and AQL. For instance, to find a plan which guarantees that the probability of acceptance at AQL = 0.8% defective is 90%, use the following SAS code:

```sas
data newoc;
  lotn=2000; aql=.008; c=-1;
  if lotn < 1200 then logn=2.05+.37*log(lotn);
  else logn=-.63+.75*log(lotn);
  n=int(exp(logn));
  do while (pa<.90);
    n=n-1;
    pa=probhyp(lotn,k,n,c);
  end;
  keep lotn aql n pa;
run;
```

```sas
proc print;
  title 'Sample Size and Acceptance Number';
  title2 'to Achieve Specified PA at AQL';
run;
```

1218
From Output 5 in the appendix, the sample size and acceptance number necessary to guarantee that \( P_a \geq 0.90 \) at AQL = 0.08 are \( n = 129 \) and \( c = 2 \).

**Summary**

In this paper, it has been shown that the values of the OC curves, the ATI and the AOQ performance statistics for attribute sampling plans can be generated using the SAS data step. The design of specific attribute sampling plans such as those shown in Military Standard 105 can be automated using SAS software.

**Appendix**

**Figure 1. Sample OC Curve**

![Sample OC Curve](image)

**Output 1.**

<table>
<thead>
<tr>
<th>OBS</th>
<th>N</th>
<th>C</th>
<th>LOTN</th>
<th>P</th>
<th>K</th>
<th>PA</th>
<th>ATI</th>
<th>AOQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.000</td>
<td>0</td>
<td>1.00000</td>
<td>100.00</td>
<td>0.00000</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.005</td>
<td>50</td>
<td>0.98646</td>
<td>234.05</td>
<td>0.004883</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.010</td>
<td>100</td>
<td>0.92156</td>
<td>876.56</td>
<td>0.009123</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.015</td>
<td>150</td>
<td>0.81046</td>
<td>1976.43</td>
<td>0.012035</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.020</td>
<td>200</td>
<td>0.67672</td>
<td>3305.32</td>
<td>0.017599</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.025</td>
<td>250</td>
<td>0.54157</td>
<td>4638.44</td>
<td>0.023464</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.030</td>
<td>300</td>
<td>0.41867</td>
<td>5955.21</td>
<td>0.029434</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.035</td>
<td>350</td>
<td>0.31453</td>
<td>6886.19</td>
<td>0.036898</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.040</td>
<td>400</td>
<td>0.23069</td>
<td>7716.21</td>
<td>0.045135</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.045</td>
<td>450</td>
<td>0.16577</td>
<td>8358.91</td>
<td>0.053785</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.050</td>
<td>500</td>
<td>0.11703</td>
<td>8841.45</td>
<td>0.065973</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.055</td>
<td>550</td>
<td>0.08134</td>
<td>9194.73</td>
<td>0.071429</td>
</tr>
<tr>
<td>13</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.060</td>
<td>600</td>
<td>0.05576</td>
<td>9447.96</td>
<td>0.078512</td>
</tr>
<tr>
<td>14</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.065</td>
<td>650</td>
<td>0.03775</td>
<td>9626.23</td>
<td>0.086430</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
<td>2</td>
<td>10000</td>
<td>0.070</td>
<td>700</td>
<td>0.02528</td>
<td>9749.77</td>
<td>0.094752</td>
</tr>
</tbody>
</table>

**References**


SAS and SAS/GRAPH is a registered trademark or trademark of the SAS Institute, Inc. in the USA and other countries. © indicates USA registration.
Output 2.

Average Total Inspection for Single Sampling Plan
N=10000, n=100, c=2

Output 3.

Average Outgoing Quality
N=10000, n=100, c=2

Output 4.

OC Curve Values for MIL-STD-105 Plan
N=2,000, n=125, c=3

<table>
<thead>
<tr>
<th>OBS</th>
<th>M</th>
<th>C</th>
<th>LOTN</th>
<th>P</th>
<th>K</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>125</td>
<td>3</td>
<td>2000</td>
<td>0.00658</td>
<td>14</td>
<td>0.99109</td>
</tr>
<tr>
<td>2</td>
<td>125</td>
<td>3</td>
<td>2000</td>
<td>0.01000</td>
<td>20</td>
<td>0.96775</td>
</tr>
<tr>
<td>3</td>
<td>125</td>
<td>3</td>
<td>2000</td>
<td>0.01090</td>
<td>22</td>
<td>0.95577</td>
</tr>
<tr>
<td>4</td>
<td>125</td>
<td>3</td>
<td>2000</td>
<td>0.01400</td>
<td>28</td>
<td>0.90698</td>
</tr>
<tr>
<td>5</td>
<td>125</td>
<td>3</td>
<td>2000</td>
<td>0.02030</td>
<td>41</td>
<td>0.74849</td>
</tr>
<tr>
<td>6</td>
<td>125</td>
<td>3</td>
<td>2000</td>
<td>0.02940</td>
<td>59</td>
<td>0.49010</td>
</tr>
<tr>
<td>7</td>
<td>125</td>
<td>3</td>
<td>2000</td>
<td>0.04090</td>
<td>82</td>
<td>0.23296</td>
</tr>
<tr>
<td>8</td>
<td>125</td>
<td>3</td>
<td>2000</td>
<td>0.06200</td>
<td>124</td>
<td>0.04055</td>
</tr>
<tr>
<td>9</td>
<td>125</td>
<td>3</td>
<td>2000</td>
<td>0.08040</td>
<td>161</td>
<td>0.00659</td>
</tr>
</tbody>
</table>

Output 5.

Sample Size and Acceptance Number to Achieve Specified PA at AQL

<table>
<thead>
<tr>
<th>OBS</th>
<th>LOTN</th>
<th>AQL</th>
<th>C</th>
<th>N</th>
<th>PA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2000</td>
<td>0.008</td>
<td>2</td>
<td>129</td>
<td>0.92078</td>
</tr>
</tbody>
</table>