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INTRODUCTION

Today the ever advancing technology in the field of quantitative software such as SAS has made the application of theoretical concept in the real business world an easy task. Not long ago marginal analysis in economics textbooks was regarded as an abstract thought just to be learned in the course and to be forgotten after the exam. With the aid of computer and appropriate SAS software, marginal analysis can now be used in business enterprises to determine the optimal production units, price, and level of employment in order to achieve maximization of profit.

This paper intends to show how the use of SAS procedures can assist students of managerial economics to apply conceptual optimization formulas in business decision problems. Although the data used here are hypothetical, the managerial case in this paper can be a real one.

The aim, therefore, is two-fold: (1) application of the marginal concept to efficiently achieve management optimal production goals, and (2) employment of such SAS procedures as PROC UNIVARIATE, PROC MEANS, PROC CORR, and PROC REG to facilitate the process of finding the best solution to the management objectives.

The models used in this managerial case will establish a direct relation between the amount and the price of input and the firm's optimal level of output. This is unique in that it integrates production and employment by linking the two marginal concepts together: the marginal revenue product and the marginal cost of input.

OPTIMAL INTEGRATION OF OUTPUT AND EMPLOYMENT: A HYPOTHETICAL CASE

This is the case of a Gas Heating Systems Company, Inc. GHS is planning to market an attractive and efficient gas heater designed to blend with household furniture. The combustion chamber is made in such a way that only ten percent of the heat escapes with the outgoing fume. For this reason a small amount of gas is consumed to heat up a large space. GHS purchases the component parts from independent sub-contractors and puts them together in its assembly plants. Due to a successful marketing promotion encouraging responses are forthcoming and strong indications point at growing demand and business expansion.

Being a substitute product for the existing gas heaters already in the market, GHS is price sensitive, therefore, the cost of labor plays an important determinat in the company's profit.

GHS's economists and marketing personnel are using a relatively simple economic concept: as production expands additional labor will be employed so long as their contribution as measured by their net marginal product exceeds their wages (the cost of each additional MRP of labor);

\[ \text{MRP of Labor} > \text{Marginal Cost of Labor} \]

DATA AND SAS PROGRAMMING

This case study uses a set of hypothetical demand and cost data with an intention to derive by way of SAS procedures the average and marginal revenue and labor wage rate necessary for GHS to arrive at optimal output and employment. These revenue and cost data constitute the information for SAS/STAT and SAS/ETS econometric analysis.

```
data a;
  input obs p q tc
  tr - p*q;
  are - tc/q;
  q2 - q*q;
  q3 - q**3;
  trl - lag1(tr);
  ql - lag1(q);
  mr - (tr 4r1) / (q - ql);
  profit - tr - tc;
  atc = lag1(tc);
  mc - (tc - atc) / (q - ql);
  cards;
  0 75 0 250000
  1 70 12500 687500
  2 65 25000 1125000
  3 60 37500 1562500
  4 55 50000 2000000
  5 50 62500 2437500
  6 45 75000 2875000
  7 40 87500 3312500
  8 35 100000 3750000
  9 30 112500 4187500
  10 25 125000 4625000
  11 20 137500 5062500
  12 15 150000 5500000
  13 10 162500 5937500
  14 5 175000 6375000
  15 0 187500 6812500
;```

```
proc print;
run;
proc plot;
plot p*q = 'd';
plot mr*q = 'm';
plot tc*q = 'c';
plot are*q = 'a';
```

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plot tr\*q - 'r';
plot profit\*q - 'p';
run;

proc reg;
model p - q;
model m\(r\) = q;
model tc - q;
model tr - q;
model profit - q q2;
run;

proc plot;
plot profit\*q - 'p' tr\*q - 'r' tc\*q - 'c' / overlay;
run;

LABOR SUPPLY AND PRODUCTIVITY RATIO

A highly competitive local labor market suggests a perfect elastic labor supply curve. GHS's department of human resources estimates that the supply of labor is a function of the wage rate: \(L = K\cdot P\), and projects that the value of \(K\) would approximate \(10,000\). Thus the supply of labor would be: \(L = 10,000\ P\). Furthermore, labor productivity is expected to be one labor hour to two assembled gas heating units. In other words, \(L = 2Q\) or \(Q = 0.5L\).

THEORETICAL MODEL

In order to describe this firm in a statistical manner, it is necessary to specify the following relationships:

Demand function: \(Quantity = f(\ Price)\ ceteris paribus\)
\(P = a - bQ\)

Cost function: \(Cost = f(\ Quantity)\)
\(Cost = a + bQ + cQ^2\)

Average Cost function: \(ATC = \frac{f(\ Q)}{Q}\)
\(ATC = aQ + b + cQ\)

Total Revenue function: \(TR = PQ\)
\(TR = aQ + bQ^2\)

Profit function: \(\pi = a - bQ + cQ^2 - dQ^3\)

ANALYTICAL SOLUTION OF THE GHS DECISION PROBLEM

From the SAS estimates of the econometric functions used here, GHS faces the following demand and cost conditions:

\(P = 75 - 0.0004\ Q\)

and since each unit of labor is expected to produce two units of output then:

\(TC = 250,000 + 15\ Q + 2\ P_i\ Q\)

GHS's profit function can now be obtained:

\(\pi = TR - TC\)

\(\pi = 75\ Q - 0.0004\ Q^2 - 250,000 - 15\ Q - 2\ P_i\ Q\)

The optimal level of output can be determined by setting the marginal profit equal to zero and solving for \(Q\).

\(\frac{d\pi}{dQ} = 0.0008\ Q + 60 - 2\ P_i = 0\)

or \(P_i = 30 - 0.0004\ Q\)

since \(MRP_i = P_i\) then \(MRP_i = P_i = 30 - 0.0004Q\)

on the other hand, \(Q = 0.5L\)

\(P_i = MRP_i = 30 - 0.0004\ (0.5L) = 30 - 0.0002L\) or \(L_i = 150,000 - 5,000\ P_i\)

The employment level and the equilibrium wage rate can be determined by setting the demand for and supply of labor equal to each other.

\(150,000 - 5000P_i = 10,000\ P_i\) thus \(P_i = 10\)

and \(L_i = 10,000\ (10) = 100,000\) work hours.

The optimal level of output would be:

\(Q = 0.5\ (10,000) = 50,000\) gas heaters, with the profit maximizing price of:

\(P = 75 - 0.0004\ (50,000) = 55\)

The maximum profit can now be determined by:

\(\pi = TR - TC - 0.0004\ (50,000^2) + 60\ (50,000) - 2\ (10)(50,000) - 250,000 = 75,000\)

GRAPHICAL OVERVIEW OF FUNCTIONS

Plot of \(P_i\). Symbol used is 'd'.

Plot of MRP_i. Symbol used is 'x'.
Plot of TC*Q. Symbol used is 'c'.

Plot of ATC*Q. Symbol used is 'a'.

Plot of TR*Q. Symbol used is 'r'.

Plot of PROFIT*Q. Symbol used is 'p'.

ECONOMETRIC ESTIMATION USING SAS PROCEDURES

Model: MOLE1
Dependent Variable: P

Analysis of Variance

Parameter Estimates

Model: MOLE2
Dependent Variable: QA

Analysis of Variance

Parameter Estimates

Model: MOLE3
Dependent Variable: TC

Analysis of Variance

Parameter Estimates

Model: MOLE4
Dependent Variable: ARC

Analysis of Variance

Parameter Estimates

Model: MOLE5
Dependent Variable: TR

Analysis of Variance

Parameter Estimates

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CONCLUSION

The SAS procedure estimated the econometric functions of revenue and cost. Even though the data was hypothetical, the actual real world data could just as well have been used in estimating relevant econometric functions. A researcher would be able to arrive at an optimal solution for an actual firm. The difference is that the real data might not have produced as clear cut a solution as was presented above.

REFERENCES


SUMMARY OF SOLUTIONS

Optimal Levels of Output and Employment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Level</td>
<td>50,000 GH</td>
</tr>
<tr>
<td>Price</td>
<td>55.00</td>
</tr>
<tr>
<td>Wage Rate</td>
<td>10</td>
</tr>
<tr>
<td>Employment Level</td>
<td>10,000 work hours</td>
</tr>
<tr>
<td>Profit</td>
<td>75,000</td>
</tr>
<tr>
<td>Rate of Productivity</td>
<td>2 labor hours per one unit of heater</td>
</tr>
</tbody>
</table>