Examination of the utilization of geographic space is of importance to such disciplines as ecology, social science, and policy and planning. Characterizing the intensity of space use from point data is central to this process. A harmonic mean estimator is one technique for estimating space-use, and is less biased than other estimators which use arithmetic means. A series of SAS programs have been developed which read geographic locations as Cartesian coordinates, identifies the coordinates for the harmonic mean, and projects a three-dimensional representation of the intensity of use (from the first areal moments).

Application of the programs are demonstrated using locational data from research projects on bighorn sheep and broad-winged hawks. Utility programs were also developed for managing data prior to the analysis, and exporting harmonic mean results to other systems for further spatial and statistical analyses.

The objective of this paper is to present a series of programs for analyzing spatial use data using the harmonic mean estimator. These programs estimate the harmonic mean of spatial activity and provide estimates of area included within the boundaries derived from the estimate of dispersion of the loci. Graphic visualizations are provided to depict the distribution of the data loci, the overlay grid used to calculate the inverse first areal moments, isopleths which comprise the boundaries of the loci under consideration, and 3-dimensional representation of the overlay grid and inverse first areal moments. In addition, several utility programs were developed for managing spatially referenced point data and exporting the results of the harmonic mean analysis in a format appropriate for further analysis with other software.

DATA ENTRY AND MANIPULATION

The data can be introduced to the harmonic program from a variety of sources. The harmonic program requires the X and Y coordinates (Cartesian coordinates) in a permanent SAS data set (HARMONIC.LOCI). Full-screen data entry templates were created with FSEDIT for directly entering X and Y coordinates, and

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Data analysis showed that the group of observations that are to be analyzed. All observations will be included in some cases, whereas other situations require subsets of observations for intergroup comparisons. The information that is recorded with the activity loci can be utilized for subsetting the observations.

**DATA ANALYSIS**

Estimation of the intensity of space use is accomplished with several steps, all of which originate from a matrix of the location data. The first step is to calculate an overlay grid that includes all of the location data points. This is performed by taking the difference of the maximum and minimum abscissa values and the difference of the maximum and minimum ordinate values. The shorter of these two differences is then divided by 10 to form the grid interval. The corners of the grid are then calculated using combinations of the maxima and minima of the ordinate and abscissa values. The grid is then laid over the location loci. Inverse first areal moments are calculated for each grid intersection by estimating the harmonic mean of the distances from the intersection to all location loci.

The harmonic mean center is the location of the maximum value of the inverse first areal moment. Other peaks may exist on the resulting surface of the grid intersections and inverse first areal moments. The location of these peaks is also saved. Isolines, the lines which comprise boundaries of the location loci by including a specified percentage of the loci points with the harmonic mean center as the mean, are estimated as the confidence ellipses of the principle and minor axes (Sokal and Rohlf 1981).

Isopleths, lines formed as the average distance from the isoline to the location loci, are estimated to determine the area patterns of activity. The area within this isopleth boundary is then estimated using Simpson's rule.

Further analyses can be conducted to elucidate how other factors may relate to the distribution of activity in geographic space. Other types of data are commonly collected with the loci or are available for the entire geographic area. The association of these factors with distribution of activity can be analyzed by statistical analysis or spatial coincidence. The simplest approach would be to statistically characterize the original loci by data collected at the same locations. If geographically continuous data is available for the study area, the data can be extracted for the coordinates of the overlay grid and utilized in a statistical model with the first areal moments. Finally, the overlay grid coordinates and first areal moments can be ported to a geographic information system for further spatial analyses.

**DATA VISUALIZATION**

Several graphical outputs were developed to aid in the interpretation of the activity data and harmonic analysis. A scattergram of the activity loci is produced, and geographically referenced to the intersections of the overlay grid. The surface of the first areal moments and inverse first areal moments can be viewed with 3-dimensional plots (G3D) and contour plots (GCONTOUR). Both types of plots aid in identifying and interpreting activity centers. Isopleth plots provide a standardized approach for identifying an area of activity. Criteria commonly employed include 95% and 75% isopleths, which respectively include 95% and 75% of the observations. The area enclosed by the isopleth can be totaled for intergroup comparisons or analyzed for geographic makeup.

**EXAMPLE APPLICATIONS**

The programs that were developed in this manuscript were tested with two widely disparate sets of animal location data. Free-ranging bighorn sheep were monitored with radio-telemetry (Layne 1987), and geographic location determined from visual observations. Universal Transmercator (UTM) coordinates were recorded from USGS Topographic maps. Figure 1 shows the visual results of the analysis conducted on locational information for a herd of ewes. Note the two centers of activity (or peaks) on the 3-dimensional plot of the inverse first areal moments.

The spatial activity of broad-winged hawks was also monitored with radio-telemetry equipment (Steblein 1989). However, the geographic location of the
Hawks was largely determined by triangulation. Harmonic mean analysis was conducted on location data from a male hawk during the breeding season. The contour and 3-dimensional plots of the inverse areal moments (Figure 2) indicate a dominant peak and very minor sub-peak.

DISCUSSION

The emphasis in this manuscript has been with application of the harmonic mean estimator to the spatial distribution of vertebrate activity. The original description of the technique was by Neft (1966) and seems appropriate for characterizing areal distributions in many other disciplines.

LITERATURE CITED


APPENDIX

The formulae for calculating the harmonic mean estimator (Sokal and Rohlf 1981) and inverse first areal moment (Dixon and Chapman 1980) are listed below:

**Harmonic Mean Estimator**

\[ \frac{1}{H} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{X_i} \]

**Inverse First Areal Moment**

\[ \frac{1}{\sqrt{M^{-1}}} = \frac{1}{p} \sum_{x=1}^{p} \frac{1}{r_{jx}} \]

where \( H \) is the harmonic mean estimator, \( n \) is the number of observations, and \( X \) is the variate; and \( P \) is the number of loci, \( r \) is the distance between the grid intersections and loci, \( j \) is the grid coordinate, and \( x \) is the loci number.

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Figure 1. Grid intersections plotted as X-Y and first area moment plotted as Z axis for bighorn sheep (Ovis canadensis).

Figure 2a. Contour plot of inverse first area moment for broad-winged hawk (Buteo platypterus).
Figure 2c. Grid intersections plotted as X-Y and first areal moment plotted as Z axis for Broad-winged Hawk (Buteo platypterus).