A PROGRAM TO CALCULATE WEIGHTS FOR SPECIFIED SPECTRAL WINDOWS: A SPECTRA PROCEDURE APPLICATION

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This paper outlines the use of a SAS system macro which calculates and inserts the numerical values of the weights needed in the SPECTRA procedure. These weights are calculated based on user specification of spectral window type, number of observations, and bandwidth.

Importance of Windows
The periodogram of a time series is not a consistent estimator of the spectrum. However, by smoothing the periodogram with a filter conventionally called a window, the filtered series may become a reasonable estimator for a spectrum. There are two ways to do this:

1. Lag Window
   The truncated sample autocovariance function is passed through the Fourier transform using the weighting procedure for the specified lag window type.

2. Spectral Window
   The transformed data is filtered using the weighting procedure for the specified spectral window type.

In the SPECTRA procedure, the use of the second method (spectral window) is assumed. The WEIGHTS statement in the procedure allows the user to specify the numerical weights to use in smoothing the data. Therefore, the user must calculate the weights needed as specified by the choice of window type, the number of observations and bandwidth.

Although any weights specified will result in some type of smoothing operation, the Bartlett, Rectangular, Tukey, and Parzen windows are well-known smoothing procedures with previously calculated properties. These properties may be of some interest to researchers in the interest of keeping their research reproducible and understandable. Using an arbitrary weighting scheme in the WEIGHTS statement such as "1 1 1 1", for instance may seem like a rectangular window. This would be true if the sequence represented lag window weights. In the time-domain however, the shape of this weighting function is of the form sin(x)/x. This is true of other familiar shapes, such as a triangular weighting scheme of "1 2 3 2 1". This is not a triangle in the time domain, but only in the frequency domain, with no easily calculated properties.

Our Program's Use
Our program requires the user to specify one of the following spectral window types: Rectangular, Tukey, Bartlett, or Parzen. Also specified are the number of points in the periodogram, and the bandwidth of the window.

The bandwidth is specified as a constant in the interval [0,1]. A bandwidth of zero is equivalent to using no window. The choice of bandwidth determines the trade-off between variance and bias. A small bandwidth is associated with large variance and low bias. Higher bandwidth values will result in lower variance, but larger bias. It has often been suggested that several bandwidths be tried in an analysis, so that plots of the resulting smoothed spectrum may provide the researcher with a good guess at the spacing of the two closest peaks. A bandwidth larger than that interval will obviously smooth over both peaks simultaneously, concealing the peaks.

Limitations of the Program
For programming purposes, only 160 weight values are used. This will give a reasonable approximation to the actual windows when the number of observations is small (400 or less raw data points; about 200 periodogram points). Weights are calculated with the abscissa incrementing in steps of i/n, (i=1,2,...,80). This gives half of the window which is then reflected in the macro, providing the symmetric sequence of weights needed in the WEIGHTS statement. If the number of observations is greater than about 400 the program can easily be modified to calculate more weights. The optimum solution would be to have as many weights as there are points on the periodogram. However, these windows damp out fairly quickly, approaching zero at values depending on window type and bandwidth.

Therefore, with more periodogram points than weights, a reasonable approximation may still be calculated as many of the weights are close to zero.

Using the Program
To invoke the program for a Parzen window, periodogram ordinates of 300 observations, and a bandwidth of .2, the user would type the following line before the first PROC statement in the data step:

%WINDOW('PARZEN',300,.2);

and invoke PROC SPECTRA with the weight statement:

WEIGHTS 60 weight;

CODE

/* The WINDOW macro inserts strings containing the calculated weights from the macro WINDOW into the WEIGHTS statement in the SPECTRA procedure. The insertion is symmetric. */
MACRO WINDOW;
        %WINDOW('TType',1n,Tb);
        %INSERT('TType','Wght1','Wght1');
        %FREE('Wght1','Wght1');
END Macro;

/* The WEIGHTS macro inserts strings containing the calculated weights from the macro WINDOW into the WEIGHTS statement in the SPECTRA procedure. The insertion is symmetric. */
MACRO WEIGHTS;
        Wght1 = 1;
        Wght2 = 1;
        Wght3 = 1;
        Wght4 = 1;
        Wght5 = 1;
        Wght6 = 1;
        Wght7 = 1;
        Wght8 = 1;
        Wght9 = 1;
END Macro;
b = SYMPUT('b');
* bandwidth, b, is given in the interval (0,1) with 1 representing the entire data set, .1 one-tenth of the data set, etc.; b=2/3;
FORMAT w 9.1;
LENGTH midws $b.;
LENGTH Detai$-detai$ $d00.;
PI = 3.14159;
Max = 80;
*;
DO i = 6 TO Max;
  IF Type = 'BARTLETT' THEN DO;
    w = 2/(2*b);
    X = i/6;
    IF i/6 THEN w = w;
    ELSE w = m*(sin(pi*x*m)/(pi*x*m)**2);
  END;
  IF Type = 'RECT' THEN DO;
    m = 4/(3*b);
    X = i/6;
    IF i/6 THEN w = w;
    ELSE w = 2*m*(sin(2*pi*x*m))/(2*Pi*x*m);
  END;
  IF Type = 'TUKEY' THEN DO;
    B = 4/(3*b);
    X = i/6;
    IF i/6 THEN w = w;
    ELSE w = B*(sin(2*Pi*x*m)/(2*Pi*x*m))**2);
  END;
  IF Type = 'PARZEN' THEN DO;
    m = 1.8516;
    X = i/6;
    IF i/6 THEN w = w;
    ELSE w = 0.15*1**2*(sin(Fi*x*m)/2)/((Pi*x*m)**4)
  END;
*;
RETURN;
END;
RUN;
*;
* The following statements show how to invoke the macro for a
* Parzen window, using a data set of 600 observations (300
* periodogram ordinates) and a bandwidth of .1 (one-tenth) of
* the data.
DATA ONE:
  INPUT X Y;
  CARDS;
  (data records)
* Invoke the Window macro to calculate the weights.
  WINDOW(PARZEN,300,.1):
* PROC SPECTRA / options;
* VAR X Y;
* Invoke the PoWts macro to insert the WEIGHTS statement and
the weights themselves into the SPECTRA procedure.
DATA TWO:
  INPUT X Y;
  CARDS;
  (data records)