EXPLORING THE MANDELBROT SET

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ABSTRACT

In the mid 1970's, IBM Fellow Benoit B. Mandelbrot founded a new branch of geometry. Influenced by the unusual shapes occurring in nature, he named this work fractal geometry; fractal meaning "irregular" or "fragmented." His later work in this area led to the development of a truly fascinating collection of points known as the Mandelbrot set.

This set can be constructed by starting with all points in the complex plane within a circle, about the origin, of radius two. Points from this initial collection are successively excluded should they fall outside the circle following a recursive operation. The points remaining after an infinite number of operations represents the Mandelbrot set. The methodology by which this set can be depicted via SAS/GRAPH is presented. This paper also explores the behavior of the points at specific locations at the set boundaries.

INTRODUCTION

Points in the complex plane can be represented via their real and imaginary parts. For example, the number $a + bi$ has a real component, $a$, and an imaginary component, $b$. The $i$ denotes the part of the complex number that is imaginary. The term imaginary is derived from the fact that the $i$ stands for the square root of $-1$.

Complex numbers possess the attribute of size. Size can be thought of as the distance the number is from the origin, $O + Oi$. Consider the complex number $3 - 4i$. The size, or magnitude, of this number is the square root of $(3)^2 + (-4)^2$ or 5.

The starting locus of points lie within a circle of radius two, or in other words, are of magnitude less than two. Let $C$ be a fixed point within this circle and let $Z$ initially be the point $0 + Oi$. Now, replace $Z$ with the expression $Z^2 + C$. That is,

$Z \leftarrow Z^2 + C$.

Op the first iteration, since $Z$ is zero, $Z^2 + C$ is still $C$. This result is then substituted into the expression for $Z$; the new sum is then $C^2 + C$. Repeating this operation again for $Z$, the sum on the next iteration becomes $(C^2 + C)^2 + C$.

The Mandelbrot set is the collection of all complex numbers $C$ for which the size of $Z^2 + C$ is finite even after an infinite number of iterations. Complex-number theory dictates that the above algorithm will drive $Z$ to infinity if and only if $Z$ reaches a size of two or greater. Essentially, once the operation perturbs the number $C$ enough such that the size of its result $Z$ falls outside the original circle, it is excluded from the set.

GENERATING THE MANDELBROT SET

A VS FORTRAN program (Figure 1) was developed which searched for the numbers $C$ that would eventually lie in the Mandelbrot set. First, a 400 by 250 grid was placed over the region of interest: $-3.05 - 1.25i$ to $1.05 + 1.25i$. Using more divisions and a larger axis for the real component was necessary to take into account the shape and size of an individual cell for a rotated SAS/GRAPH plot.

The iterative replacement process previously described was then carried out for each of the grid points. Given a number $C$, this procedure can be outlined as follows:

1. Set the complex number $Z$ to $0 + 0i$ and the variable ITER to 0.
2. Replace $Z$ with $Z^2 + C$, increment ITER by 1, and determine the size of the new number $Z$.
3. If the size of $Z$ is less than two, go to step 2. However, should the size be two or greater, store the value of ITER for the number $C$.

Should the size of $Z$ be less than two, even after 150 iterations (an approximation to an indefinitely large number), the number $C$ is assumed to lie in the Mandelbrot set.

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The output generated from this procedure is 100,000 integer triples. Each triple contains the row (real) and column (imaginary) indicators of the point's position in the grid; the third number is the variable ITER for that number C.

**PLOTTING THE MANDELBROT SET**

The output from the FORTRAN program is sent to and stored in a file. A SAS/GRAPH program was developed (Figure 2) to read the file, assign colors to the grid points based upon the value of ITER, and then produce the plot.

An ANNOTATE data set was created by processing the file of ITER values. For each grid point, the row and column coordinates were translated into screen percentages in the X and Y directions. Having XSYS and YSYS values set to '3' allow for using percentages to pinpoint grid locations in the plotting region.

Rectangles are then drawn for each point using ANNOTATE functions MOVE and BAR. These functions are used in pairs to first specify the lower left-hand and then the upper right-hand corner for each rectangle. The length and height of each rectangle (plot resolution) depends on the number of grid points in each direction. The color of the rectangle is then solely based on the coordinate's ITER value at the lower left-hand corner.

PROC GSLIDE then used this ANNOTATE data set to carry out the actual plotting. A plot of the Mandelbrot set produced by the program can be seen in Figure 3. Vertical and horizontal axes were generated separately for this figure and overlaid.

**EXPLORING THE MANDELBROT SET**

One of the special features of the Mandelbrot set is how the points that lie beyond its surface "behave", i.e., their relative tendency to instability. It is through the plotting of this behavior that the true beauty of the Mandelbrot set can be appreciated.

Figures 4 and 5 contain two magnified areas at the Mandelbrot set boundary. For the specific locations of these two, refer to Figure 3. The magnifications for these two plots, over that of the original, are 200x and 5000x, respectively, for Figures 4 and 5.

There are infinitely many more regions to plot. It is left, and encouraged, for the interested readers to use their imaginations (and color schemes) and further explore the Mandelbrot set. The referenced article in Scientific American is an excellent source for additional information and background material. It served as the inspiration for this effort.

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**REFERENCE**


**FOR ADDITIONAL INFORMATION . . .**

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Figure 1. FORTRAN Program Listing

THIS VS FORTRAN PROGRAM EXPLORES THE EFFECTS OF ITERATED
OPERATION ON NUMBERS IN THE COMPLEX PLANE. THIS WILL LEAD
TO THE MANDELBROT SET. THIS PROGRAM IS BASED ON AN
ARTICLE APPEARING IN THE AUGUST 1985 ISSUE OF "SCIENTIFIC
AMERICAN", THE PLOT GENERATED HERE IS THE ENTIRE
MANDELBROT SET. THE BOUNDARIES ARE (-3.05, -1.25i) TO
(1.05, 1.25i).

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IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 Z, C
INTEGER*2 MATRIX(400,250)

SET THE LOCATION IN THE COMPLEX PLANE TO EXPLORE. SCALE
THE REAL PART OF THE INCREMENT TO ACCOUNT FOR ASPECT
RATIO ON PLOT.

II NC = 400
JINC = 250
C = (-3.050000,-1.2500000)
RINC = (21.00*2.500000)/(12.00*FLOAT(IINC))
CINC = 2.500000/FLOAT(JINC)

ITERATE ON EACH POINT IN THE SELECTED REGION. ASSUME
ANY POINT STABLE AFTER 150 ITERATIONS WILL REMAIN STABLE
AND IS IN THE MANDELBROT SET.

DO 30 I = I, II NC
   DO 20 J = J, JINC
      Z = (0.00, 0.00)
      ITER = 0
      10 Z = Z*Z + C
      ITER = ITER + 1
      ZSIZE = CBABS(Z)
      IF (ITER .LE. 150 .AND. ZSIZE .LT. 2.00) GO TO 10
      MATRIX(I,J) = ITER
      C = C + CMPLX(0.00, CINC)
   20 CONTINUE
   CREAL = REAL(C) + RINC
   C = CMPLX(CREAL,-1.2500000)
30 CONTINUE

OUTPUT RESULTS TO EXTERNAL FILE.

KINC = JINC/5
DO 40 I = I, IINC
   DO 40 K = K, KINC
      JS = 5*K - 4
      JE = 5*K
      WRITE (6,6000) (I, J, MATRIX(I,J), J=JS,JE)
40 CONTINUE

FORMAT STATEMENT.

6000 FORMAT (S(' ',13, IX, 13, IX, 14))
STOP
END
/* Figure 2. SAS/GRAPH Program Listing */

GOPTIONS DEVICE=LAS6500 CHARTYPE=2 GACCESS=GSASFILE GPROTOCOL='TRANTAB=GTABTCAII;
OPTIONS NOTEXT82;
DATA BROT;
  LENGTH COLOR $ 8;

* THIS PROGRAM READS FROM THE FORTRAN-GENERATED IN-FILE AND 
* SUBSEQUENTLY ASSIGNS COLORS BASED ON THE NUMBER OF ITERATIONS 
* TO INSTABILITY. PROC GSLIDE IS THEN USED TO PLOT. THIS 
* PROGRAM SPECIFICALLY PLOTS THE ENTIRE MANDELBROT SET. THE 
* BOUNDARIES OF THE PLOT ARE:
* (-3.05, -1.251) TO (1.05, 1.251).

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INFILE IN400SET;
INPUT I J ITER @8;

* SET PLOTTING SYSTEM AND DETERMINE LOWER LEFT-HAND CORNER OF 
* EACH AREA. MANDELBROT SET GENERATED FROM A 400 BY 250 
* ARRAY. THEREFORE, THE REAL SIDE OF A PLOTTED AREA WILL BE 
* OF SIZE 0.25, WHILE THE IMAGINARY SIDE WILL BE OF SIZE 0.40.
* XSYS = '3';
* YSYS = '3';
* STYLE = 'SOLID';
* FUNCTION = 'MOVE';
* X = (I - 1)*0.25;
* Y = (J - 1)*0.40;
* OUTPUT;

* COLOR REGIONS BASED ON ITERATIONS. BLACK AREA WILL BE THE 
* SET OF POINTS THAT LIE IN THE MANDELBROT SET. THEY ARE ASSUMED 
* TO BE STABLE AFTER 150 ITERATIONS.
* USE BAR FUNCTION TO SET UPPER RIGHT-HAND CORNER OF AREA.
* THEN FILL IN REGION.
* IF ITER > 150 THEN DO;
*   COLOR = 'BLACK'; FUNCTION = 'BAR';
*   X = X + 0.25; Y = Y + 0.40; OUTPUT;
* END;
ELSE IF ITER > 9 AND ITER <= 30 THEN DO;
  COLOR = 'YELLOW'; FUNCTION = 'BAR';
  X = X + 0.25; Y = Y + 0.40; OUTPUT;
END;
ELSE IF ITER = 9 OR ITER = 7 OR ITER = 5 OR ITER = 3 OR ITER = 1 THEN DO;
  COLOR = 'CYAN'; FUNCTION = 'BAR';
  X = X + 0.25; Y = Y + 0.40; OUTPUT;
END;
ELSE IF ITER = 8 OR ITER = 6 OR ITER = 4 OR ITER = 2 THEN DO;
  COLOR = 'BLUE'; FUNCTION = 'BAR';
  X = X + 0.25; Y = Y + 0.40; OUTPUT;
END;
* PRODUCE PLOT VIA ANNOTATE AND PROC GSLIDE.
* PROC GSLIDE ANNO=BROT;
* TITLE1 ' ';

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Figure 3. Plot of Mandelbrot Set
Figure 4. Magnification Plot of Area #1

Figure 5. Magnification Plot of Area #2