ABSTRACT

This paper presents a method for obtaining hypergeometric attribute sampling plans. This method, an extension to a single attribute sampling plan developed by Alexander (1985), is useful in developing specialized plans for isolated lots. Examples are provided.

INTRODUCTION

Attribute acceptance sampling can be used to accept or reject lots based upon the number of nonconforming items found in a random sample of the lot. Traditional attribute sampling plans, like MIL-STD-105D, are based upon the binomial probability distribution or the Poisson approximation to the binomial. Such sampling plans assume that lots are formed from processes in continuous production at some average fraction of nonconforming items. If the formation of the lot is noncontinuous or "isolated" in nature (which can happen if production is irregular or performed on an infrequent basis), then sampling plans based on the binomial or Poisson distribution become inaccurate because the sampled universe is finite rather than infinite. For isolated lots, the probability of lot acceptance is the proportion of lots having the same quality which would be accepted from an infinite series of lots, not series of lots drawn at random from an infinite process.

In other words, the emphasis is on individual lot quality rather than on process quality (See Duncan, 1974, pp. 157-160). The appropriate probability distribution to use with isolated lots is the hypergeometric distribution, however hypergeometric sampling plans are difficult to find because of the computational effort in producing hypergeometric probabilities of acceptance.

Though hypergeometric sampling plan development has received little attention in the past, the Department of Defense and the International Organization of Standardization's (ISO) subcommittees on statistical methods have shown relatively recent interest in the hypergeometric sampling plan (See Duncan et. al., 1980 and Godfrey and Mundel, 1984).

Derivation of hypergeometric sampling plans usually involves the availability of tables like those found in Lieberman and Owen (1961), Odoh and Owen (1983), or Military Specification MIL-M-28510F for Microcircuits (1983). Using these tables can be unwieldy when the interpolation of one or more parameters are involved. If such tables are not available, one has to use a programmable calculator or computer. This paper describes a hypergeometric extension to Alexander's (1985) SAS* routine, SSPLAN. SSPLAN is an adaptation of Gunther's (1969) iterative search procedure which determines sample sizes for two-point single attribute sampling plans.

DESCRIPTION AND OPERATION

Exhibits 1 and 2 are the SAS data statements and flowchart respectively used to determine sample sizes and acceptance numbers for the hypergeometric distribution. The user:

- Logs onto TSO,
- Allocates files,
- Invokes SAS and
- Inputs the lot size, Acceptable Quality Level (AQL), probability of acceptance for the AQL, Limiting Quality Level (LQL), and the probability of acceptance for the LQL. (See the Appendix for the definitions of these terms).

In Alexander's SSPLAN routine, TYPE took on values of "0" for the Poisson distribution or "1" for binomial. TYPE could represent the lot size for values greater than "1" with the hypergeometric distribution.

The bottom part of Exhibit 1 gives the statements necessary to compute:

- Acceptance probabilities for operating characteristics,
- Average outgoing quality,
- Average total inspection.
Three examples are presented. The results of each example are in agreement with the results of the sampling plans derived by the authors. The inputs are lot size (variable type in SSPLAN), AQL, LQL, and $\alpha$. They are entered from the terminal in free format on one line. Each example has an input listing and output listing.

Example 1: The first example is from Guenther (1969, p. 108) for lot size ($TYP=50$) of 100. The number of nonconforming items in the lot ($k=5$ and $k=20$) were converted into the AQL and LQL, respectively, by dividing by the lot size. The input is given in Input Listing 1. The output is given in Output Listing 1.

**INPUT LISTING 1**

THIS PROGRAM DETERMINES MINIMUM SAMPLE SIZE, ACCEPTANCE NOS., OPERATING CHARACTERISTICS (OC), AND AVERAGE OUTGOING QUALITY (AQO) CURVES FOR SINGLE ATTRIBUTE SAMPLING PLANS.

INPUT THE PROBABILITY DISTRIBUTION TYPE (0 FOR POISSON, 1 FOR BINOMIAL, LOT SIZE > 1 FOR HYPERGEOMETRIC) FOLLOWED BY THE ACCEPTABLE QUALITY LEVEL (AQL), PROBABILITY OF ACCEPTANCE GIVEN THE AQL (I.E. PRODUCER RISK), LIMITING QUALITY LEVEL (LQL), AND ACCEPTANCE PROBABILITY GIVEN THE LQL (I.E. CONSUMER RISK).

NOTES: SEPARATE EACH ENTRY BY A SPACE NOT COMMAS.

<table>
<thead>
<tr>
<th>INPUT</th>
<th>AQL</th>
<th>LQL</th>
<th>PRODUCER RISK</th>
<th>CONSUMER RISK</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0047 0.1000</td>
<td>0.0097</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
<tr>
<td>0.0152 0.1000</td>
<td>0.0095</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
<tr>
<td>0.0250 0.1000</td>
<td>0.0096</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
<tr>
<td>0.0352 0.1000</td>
<td>0.0095</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
</tbody>
</table>

**OUTPUT LISTING 1**

MINIMUM SAMPLE SIZE= 29

<table>
<thead>
<tr>
<th>OBS</th>
<th>P</th>
<th>NACE</th>
<th>AQL</th>
<th>LQL</th>
<th>PRODUCER RISK</th>
<th>CONSUMER RISK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0047</td>
<td>0.1000</td>
<td>0.0097</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0152</td>
<td>0.1000</td>
<td>0.0095</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0250</td>
<td>0.1000</td>
<td>0.0096</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0352</td>
<td>0.1000</td>
<td>0.0095</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
</tbody>
</table>

Example 2: The second example is from Odah and Owen (1983, pp. 8-9) for a lot size ($TYP=400$) of 0.005, $\alpha=0.99$, $LQL=0.035$, and $\alpha=0.20$. Input in this example is given in Input Listing 2. The sampling plan generated and performance characteristics are given in Output Listing 2.

**INPUT LISTING 2**

THIS PROGRAM DETERMINES MINIMUM SAMPLE SIZE, ACCEPTANCE NOS., OPERATING CHARACTERISTICS (OC), AND AVERAGE OUTGOING QUALITY (AQO) CURVES FOR SINGLE ATTRIBUTE SAMPLING PLANS.

INPUT THE PROBABILITY DISTRIBUTION TYPE (0 FOR POISSON, 1 FOR BINOMIAL, LOT SIZE > 1 FOR HYPERGEOMETRIC), FOLLOWED BY THE ACCEPTABLE QUALITY LEVEL (AQL), PROBABILITY OF ACCEPTANCE GIVEN THE AQL (I.E. PRODUCER RISK), LIMITING QUALITY LEVEL (LQL), AND ACCEPTANCE PROBABILITY GIVEN THE LQL (I.E. CONSUMER RISK).

NOTES: SEPARATE EACH ENTRY BY A SPACE NOT COMMAS.

<table>
<thead>
<tr>
<th>INPUT</th>
<th>AQL</th>
<th>LQL</th>
<th>PRODUCER RISK</th>
<th>CONSUMER RISK</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0047 0.1000</td>
<td>0.0097</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
<tr>
<td>0.0152 0.1000</td>
<td>0.0095</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
</tbody>
</table>

**OUTPUT LISTING 2**

MINIMUM SAMPLE SIZE= 33

<table>
<thead>
<tr>
<th>OBS</th>
<th>P</th>
<th>NACE</th>
<th>AQL</th>
<th>LQL</th>
<th>PRODUCER RISK</th>
<th>CONSUMER RISK</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0047</td>
<td>0.1000</td>
<td>0.0097</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0152</td>
<td>0.1000</td>
<td>0.0095</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0250</td>
<td>0.1000</td>
<td>0.0096</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0352</td>
<td>0.1000</td>
<td>0.0095</td>
<td>0.20</td>
<td>0.0073 233.4</td>
<td></td>
</tr>
</tbody>
</table>
SINGLE ATTRIBUTE SAMPLING PLANS. THIS PROGRAM DETERMINES MINIMUM SAMPLE SIZE, AND AVERAGE OUTGOING QUALITY (AOQ) CURVES FOR ACCEPTANCE NOS. OPERATING CHARACTERISTICS (OC). FOR EXAMPLE, SEPARATE EACH ENTRY QUALITY LEVEL (LO). THE LOT IS CONSIDERED ACCEPTABLE BY THE ACCEPTABLE QUALITY LEVEL (AQL), PROBABILITY OF ACCEPTANCE GIVEN THE AQL, I.E., PRODUCER RISK (α), LIMITING QUALITY LEVEL (LQL), AND ACCEPTANCE PROBABILITY GIVEN THE LQL, I.E., CONSUMER RISK (β). NOTE: INPUT EACH ENTRY BY A SPACE NOT COMMA.

Example 3: The third example, from Duncan (1974, pp.169-170), uses a lot size(TYPE) of 200, AQL=0.01, L=α=0.95, LQL=0.08 and β=0.10. Input Listing 3 and Output Listing 3 give the input and output, respectively.

INPUT LISTING 3

<table>
<thead>
<tr>
<th>TYPE</th>
<th>LOT SIZE</th>
<th>AQL</th>
<th>LO</th>
<th>LQL</th>
<th>α</th>
<th>β</th>
<th>MINIMUM SAMPLE SIZE</th>
<th>AOQ</th>
<th>AOQ</th>
<th>AOQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>0.01</td>
<td>0.08</td>
<td>0.029</td>
<td>0.032</td>
<td>0.174</td>
<td>200</td>
<td>0.01</td>
<td>0.08</td>
<td>0.029</td>
</tr>
</tbody>
</table>

OUTPUT LISTING 3

<table>
<thead>
<tr>
<th>OPERATING CHARACTERISTICS AND AVERAGE OUTGOING QUALITY FOR THE SINGLE ATTRIBUTE SAMPLING PLAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONE P</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>5</td>
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<tr>
<td>7</td>
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<tr>
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<td>31</td>
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<tr>
<td>33</td>
</tr>
<tr>
<td>35</td>
</tr>
<tr>
<td>37</td>
</tr>
</tbody>
</table>

CONCLUSION

Although the current trend in quality control is to de-emphasize sampling inspection as a means of assuring product quality, Godfrey and Mandel (1984) provided guidelines when acceptance sampling is appropriate. Some situations in which the above sampling plan could apply are:

- Destructive testing,
- When dealing with infrequent or first-time suppliers,
- When little prior knowledge is known about the quality of products or individual lots;
- When the customer wishes to confirm whether the lot quality is as the supplier claims;
- When the customer suspects the wrong shipment of good quality parts were received;
- When product damage occurs due to mishandling or transport from the supplier (See Vardeman, 1986).

Sampling plans like the one described above may be useful in making the transition from inspection to statistical process control by providing a quick and efficient method for developing specific sampling plans without being bogged down with table look-ups or computations usually associated with traditional acceptance sampling plans.

In order to generate customized sampling plans, the above plan may be integrated with other quality control software, such as SAS/QC (See Rodriguez, 1986).

ACKNOWLEDGMENTS

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(301) 765-7586

REFERENCES


* SAS is the registered trademark of SAS Institute Inc., SAS/QC is a trademark of SAS Institute Inc., Cary, NC USA

APPENDIX

Definition of Terms

Lot Size: Collection of items where each item was manufactured or formed under conditions as homogeneous as possible.

Acceptable Quality Level (AQL): The desired level of quality which should be accepted most of the time. Expressed as the minimum fraction of nonconforming items in the lot.

Probability of Acceptance at the AQL \((1-\alpha)\): Level of protection that the sampling plan provides against rejecting lots up to the desired level of quality. The Producer's risk \(\alpha\) is the risk of rejecting lots at the desired AQL.

Limiting Quality Level (LQL): The undesired level of quality which should be rejected most of the time. Expressed as the maximum fraction of nonconforming items in the lot.

Probability of Acceptance at the LQL \(\beta\): Risk of accepting lots at the undesired level of quality. Sometimes referred to as the Consumer's risk where poor quality lots are at or beyond the LQL.

Measures of Sampling Plan Performance

Operating Characteristic (OC) curve: Plot of the probability of lot acceptance for different values of the true fraction of nonconforming items in the lot. Provides the degree of discrimination between the risks of accepting poor quality lots and rejecting good quality lots. Complements the statistical power curve used in testing hypotheses.

Average Outgoing Quality (AOQ) curve: Plot of the average fraction of nonconforming items after rejected lots are screened 100 percent where nonconforming items are replaced with conforming items.

Average Total Inspection (ATI): The total number of units inspected plus the average number of units screened.

EXHIBIT 1: SAS DATA Statements to add to Alexander's SSPLAN routine to obtain Hypergeometric Sampling Plans.

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EXHIBIT 1: SAS DATA Statements to add to Alexander's SSPLAN routine to obtain Hypergeometric Sampling Plans.

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THIS MATERIAL MAY BE REPRODUCED BY OR FOR THE U.S. GOVERNMENT PURSUANT TO THE COPYRIGHT LICENSE UNDER DOD FAR SUPPL. CLAUSE 52.227-7013 AUGUST 1984.
EXHIBIT 2. Sampling Plan Determination Algorithm for Isolated Lots following the Hypergeometric Distribution.

Input:
- Lot Size
- \( l = a \) (Producer's Risk)
- AQL
- LQL
- \( \beta \) (Consumer's Risk)

\[
\begin{align*}
n & = 1 \\
c & = 0 \\
\end{align*}
\]

\[
P_n = \sum_{j=0}^{j=n} \left[ \left( \frac{N-LQL}{N} \right) \left( \frac{N-N+LQL}{N-2} \right) \right] / (n) \\
\]

If \( P_n \leq a \)

\[
P_c = \sum_{j=0}^{j=c} \left[ \left( \frac{N-LQL}{N} \right) \left( \frac{N-N+LQL}{N-2} \right) \right] / (c) \\
\]

If \( P_c \geq 1 - \beta \)

Minimum \( n \) and \( c \) have been found

Generate Operating Characteristics (OC) and Average Outgoing Quality (AOQ) Curves