APPLYING A MARKOV PROCESS TO ORGANIZATIONAL DYNAMICS

Mark Edmondson, International Business Machines Corporation

INTRODUCTION

A Markov process model has been applied to forecast the distribution of skills for an employee population. Its objective is to forecast skill imbalances to help management formulate retraining and recruiting strategies.

The model accesses an organization's employee data files that contain current and historical information concerning employee position and skill. From this data, "transition matrices" are constructed that contain the following information:

- Conditional transition probabilities representing flows between skills during one time period
- The skill distribution of the population
- The attrition rate for each skill.

In a dynamic organization, future transition rates can be expected to differ from historical rates. Therefore, in order to anticipate changes in transition rates, the theory of the internal labor market is used to modify the historical transition rates. By using these modified transition rates to represent the dynamics of a population, the model iteratively projects skill distributions one year at a time. Projected skill distributions are compared to target levels to determine skill imbalances. Projected skills that are over or under the targeted levels are then balanced by appropriate retraining or recruiting strategies.

The application uses the Statistical Analysis System (SAS) developed by the SAS Institute to extract data, categorize and sort the observations, and perform the operations used in the routine.

PROJECT OBJECTIVE

The project objective is to enhance the current planning process to include balancing planned workload with available skills. The techniques to plan workload demand are assumed to be established. Consequently, the techniques discussed are:

1. Depicting the skill distribution of an organization by using historic trends of skill transitions and attrition
2. Using the concept of the internal labor market to estimate changes in future trends of skill transitions
3. Using this information to forecast the skill distribution and skill imbalances of an organization
4. Comparing the forecast skill distribution to planned workload demand
5. Balancing forecast skills with respect to planned workload demand by developing retraining and recruiting strategies.

The theory that forms the basis for these techniques is well established and documented in the literature of human resource planning (see Bibliography, in particular, Price, Martel, and Lewis for an overview of previous work). This discussion emphasizes the comprehensive use of these techniques to develop a human resource planning process for a dynamic, full-employment environment.

METHODOLOGY

The technique used to forecast skill distributions is to model organization dynamics as a Markov process. As Figure 1 illustrates, three factors influence the current skill distribution over time:

- Attrition, which reduces the number of observations for each skill
- Internal skill transitions, which represent employees whose primary skill has changed
- Recruitment, which is largely controlled by management to alter the skill distribution in a desired manner.

This model works iteratively over time so that the skill distribution at time $t$, is directly implied by the skill distribution at time $t$ plus attrition, transition, and recruitment rates during the time interval $[t,t+1)$.

Central to modeling organizational dynamics as a Markov process is the construction of a transition matrix (also known as a "Markov" or "P" matrix) that shows skill transitions during one time period. Figure 2 shows an example of an organizational transition matrix. The numbers labeling the rows represent skills at time $t$; the numbers labeling the columns represent skills at time $t+1$. The element $p(i,j,t)$ is the conditional probability of having skill $j$ at time $t+1$, given the possession of skill $i$ at time $t$.

The transition matrix projects the skill distribution of an organization by applying a Markov process as follows:

If $n(i,t)$ represents the number of employees with skill $i$ at time $t$, then:

$$n(j,t+1) = \sum_{i} n(i,t) p(i,j,t)$$

and

$$1 \leq i \leq n; 1 \leq j \leq n.$$ 

In vector notation,

$$n(t+1) = n(t) \times P(t).$$
where \( \hat{r}(t) \) is a \( 1 \times n \) row vector, and \( P(t) \) is a \( n \times n \) transition matrix whose elements are \( p(i,j,t) \).

To include skills that are recruited outside the organization, a second term, \( \hat{r}(t) \), is added:

\[
\hat{r}(t+1) = \hat{r}(t) x P(t) + \hat{r}(t).
\]  
(3)

Attrition is handled as a skill in the \( P(t) \) matrix, and is an absorbing state. For example, in Figure 2, if skill \( n \) is labelled as attrition, then \( p(n,n,t) \) would represent the probability of skill \( n \) leaving the population during \([t,t+1)\).

Since skill \( n \) is absorbing, \( p(n,n,t) = 1 \) and \( p(i,n,t) = 0 \) for \( i = 1 \) to \( n-1 \); consequently, if there is an exit from the organization, the observation can only re-enter the organization via the recruitment term.

One assumption of a Markov process is that transition probabilities are independent of time; hence, the time subscript is often dropped from the transition matrix to reflect this assumption:

\[
\hat{r}(t+1) = \hat{r}(t) x P(t).
\]  
(4)

Eq. (4) is the relationship used in a Markov process to project skill distributions.

**OBTAINING THE DATA**

The data needed for this analysis is obtained from employee data files, which need to contain both current and historical information concerning position and skill. For our example, four files are used representing data during four years, 1979 through 1982. To extract the information required by the model, a Statistical Analysis System (SAS) program was written to create one new data file from the four original files. Two major routines are performed by this program: in the first, observations from the four original data files are categorized by skill; in the second, the four files are merged into one data file that contains five variables: EMPLOYEE NUMBER, POS79, POS80, POS81, and POS82 (see Figure 3). POS79 to POS82 have data values that represent the skill category (position) for each observation during the year and designated. This SAS data set is then accessed by a SAS routine to construct three numerical-transition matrices, one for each 12-month interval between year-end 1979 and year-end 1982. A numerical-transition matrix is an array whose elements \( n(i,j,t) \) are the number of transitions between skills \( i \) and \( j \) during the time interval \([t,t+1)\).

For example, Figure 4 shows the numerical-transition matrix that is derived from the sample data of Figure 3.

The probability-transition matrix, with elements \( p(i,j,t) \), is calculated from the numerical-transition matrix by dividing each element by the respective row sum:

\[
p(i,j,t) = n(i,j,t) / \sum_j n(i,j,t)
\]  
(5)

\( 1 \leq i \leq n; 1 \leq j \leq n. \)

The skill distribution for the organization at time \( t \) is also obtained from the numerical-transition matrix:

\[
\hat{n}(i,t) = \hat{n}(i,j,t).
\]  
(6)

**FORECASTING SKILL DISTRIBUTIONS**

Eqs. (5) and (6) are used to obtain the transition matrices and skill distributions observed for year-end 1979 through year-end 1982. To forecast skill distributions, eq. (4) is used:

\[
\hat{n}(t+1) = \hat{n}(t) x P(t).
\]  
(4)

For \( \hat{n}(t) \), the skill distribution for year-end 1982 is used. Recruitment, represented by \( \hat{r}(t) \), is currently assumed to be zero. Since the Markov model assumes that the transition probabilities are independent of time, each historical-transition matrix should be statistically equivalent. However, for many applications, the historical-transition matrices may be significantly different. One possible solution in this situation is to use a weighted average of the historical-transition matrices, but in many applications the transition probabilities change significantly over time. In such cases, a weighted average is meaningless, requiring the use of the most recent transition matrix to project skill distributions. For a discussion on techniques for developing a transition matrix from historical data, refer to Mahoney and Milkovich [9].

Once the skill distributions, \( \hat{n}(t) \), have been projected for each year through the plan horizon, they are compared to planned skill demand to determine the skills that are imbalanced. If \( d(t) \) is a \( 1 \times n \) vector whose elements, \( d(i,t) \), represent the demand for skill \( i \) at time \( t \), then

\[
\delta(t) = \hat{n}(t) - d(t)
\]  
(7)

gives the skill imbalances at time \( t \). The result is a set of vectors, \( \delta(t) \), that identify projected skill overages and shortages for each skill at time \( t \). Note that the sum of the elements in \( \delta(t) \) is the planned organization population at time \( t \) (skill demand), while the sum of the elements in \( \hat{n}(t) \) is the organization population projected by the model (skill supply).

**THE EFFECT OF INTERNAL LABOR-MARKET FORCES ON TRANSITION RATES**

The skill distributions forecast from eq. (4) assume that transition rates are independent of time. However, in a dynamic organization that experiences changes in personnel levels, changes in transition rates over time can be significant. The means used to estimate future changes in transition rates is to consider the organization as an internal labor market (ILM). Refer to Mahoney and Milkovich [9] for a discussion on this subject.

As an example, consider a population with two skills, \( i \) and \( j \), that contain \( n(i) \) and \( n(j) \) employees, respectively. Also consider the function \( U(n(i), n(j)) \) that is equal to the utility of having skills \( i \) and \( j \) staffed at these
levels. This two-skill system is said to be in equilibrium if \( U(n(i), n(j)) \) is at a local or global maximum. At equilibrium, there are no labor-market forces inducing inter-skill transitions. Inter-skill transitions are induced by ILM forces when the system is at disequilibrium (i.e., \( U(n(i), n(j)) \) is not at a local maximum). Disequilibrium may be a result of:

1. A change in staffing levels due to reasons other than ILM forces (e.g., attrition). This is a movement along the current utility function.

2. A change in the business environment, such as output demand, business strategy, cost of capital, or the external labor market. This is a shift in the utility function.

In both cases, ILM forces will result, inducing changes in skill staffing levels in a manner that will move towards equilibrium.

The means to model the effect of ILM forces in a Markov process is via the transition rates in the \( P \) matrix. For example, if a significant attrition of computer operators unexpectedly occurs, the system will be placed in disequilibrium, resulting in ILM forces that will encourage the placement of additional computer operators. These ILM forces are simulated as an increase in the transition rates to the computer operator skill category.

**FORECASTING SKILL IMBALANCES**

It is known that future ILM forces will change the transition rates and affect skill imbalances. However, predicting the magnitude of such a change with accuracy is difficult. Because of the uncertainty involved, instead of using point estimates for the transition rates, upper and lower bounds are established. The model is run with each of these bounds resulting in upper and lower bounds for the supply of each skill.

For the initial run, the transition rates are obtained from historical data. The result is \( \delta(t) \) from eq. (7) that shows projected skill overages and shortages. Both the transition rates and \( \delta(t) \) from the initial run are analyzed to estimate the effect of ILM forces on future transition rates.

While performing this analysis, four possible situations for each pair of skills, \( i \) and \( j \), must be considered:

### Situation 1: \( s(i,t) > 0 \) and \( s(j,t) > 0 \)

Both the skill of origin \((i)\) and the skill of destination \((j)\) are in a state of overage. Any change in the corresponding transition rate \( p(i,j,t) \) will shift the surplus from one skill to another. Without knowing the specific utility function, no clear ILM forces result from such a condition. Therefore, no change is made to the transition rate.

### Situation 2: \( s(i,t) > 0 \) and \( s(j,t) < 0 \)

The skill of origin \((i)\) is in a state of overage and the skill of destination \((j)\) is in a state of shortage. Although it is expected that ILM forces will increase the transition rate, such an increase may require an emphasis on retraining from skill \( i \) to \( j \). Retraining, along with recruiting, is addressed separately as a solution to residual skill imbalances. Therefore, no change is made to the transition rate.

### Situation 3: \( s(i,t) < 0 \) and \( s(j,t) > 0 \)

The skill of origin \((i)\) is in a state of shortage and the skill of destination \((j)\) is in a state of overage. ILM forces will decrease the future transitions from skill \( i \) to \( j \). To establish a lower bound of such a decrease, \( p(i,j) \) is set to zero, thereby disallowing any transitions from skill \( i \) to \( j \).

### Situation 4: \( s(i,t) < 0 \) and \( s(j,t) < 0 \)

Both the skill of origin and the skill of destination are in a state of shortage. Unless the shortage in one skill is much greater than in the other, ILM forces cannot be estimated without knowing the utility function. Therefore, no change is made to the transition rate.

Consequently, out of four possible situations, only situation 2 results in a transition rate adjustment. After these adjustments are made, the model is rerun resulting in a set of skill distributions, \( n(t) \), which differ from the skill distributions \( n(t) \) from the initial run. These two sets of skill distributions are used to establish upper and lower bounds for the supply of each skill as follows:

**Upper bound for skill \( i \) at time \( t \)**

\[
UB(i,t) = \max [n(i,t), n'(i,t)]
\]

**Lower bound for skill \( i \) at time \( t \)**

\[
LB(i,t) = \min [n(i,t), n'(i,t)]
\]

Once the upper and lower bounds for supply have been projected for each year through the plan horizon, they are compared to planned skill demand to determine skill imbalances. If \( d(t) \) is a \( 1 \times n \) vector whose elements \( d(i,t) \) represent the demand for skill \( i \) at time \( t \), then it is said that skill \( i \) is balanced if:

\[
LB(i,t) \leq c(i,t) \leq UB(i,t).
\]

Similarly, skill imbalances are given by:

**Skill Shortages:**

\[
UB(i,t) - d(i,t) \quad \text{when} \quad d(i,t) > UB(i,t);
\]

**Skill Overages:**

\[
LB(i,t) - d(i,t) \quad \text{when} \quad d(i,t) < LB(i,t).
\]
A set of \( n \times n \) vectors \( \mathbf{z}(t) \) can be constructed to show skill imbalances as follows:

\[
\begin{align*}
UB(i,t) - d(i,t) & \quad \text{if} \quad d(i,t) > UB(i,t); \\
LB(i,t) - d(i,t) & \quad \text{if} \quad d(i,t) < LB(i,t); \\
0 & \quad \text{otherwise}. 
\end{align*}
\]  

Note that \( \mathbf{z}(t) \) represents the skill imbalances expected when ILM forces are considered, while \( \mathbf{s}(t) \) from eq. (7) reflects I.M forces.

**Using the results as a staffing tool**

The elements of \( \mathbf{z}(t) \), which give projected shortages and overages by skill, can be used to devise retraining and recruiting strategies. As a general guideline, skill overages are considered as a source of personnel for shortages by matching available skills with needed skills as closely as possible. Usually, skill matching is done so as to minimize retraining; however, restaffing strategies may require retraining to successfully facilitate skill transitions. Although costly, retraining may result in long-term benefits, such as reduced recruiting requirements and improved employee relations.

After matching and retraining have been pursued, residual imbalances may still exist. Residual shortages can be expected due to normal attrition; since the model projects residual shortages by skill and year, this information can be used to plan specific recruiting requirements to fill these shortages. Residual overages generally occur in skills that cannot easily be retrained to fill needed positions. Although residual overages may be a significant short-term problem, it is expected that they will be minimized in the future because of the recruiting and retraining strategies being implemented.

**Conclusion**

A Markov process can be used to estimate the effect of internal labor-market forces on an organization's skill distribution. This analysis develops a strategy for retraining and recruiting that reduces future skill imbalances. Since a skill imbalance is projected only if the demand for a skill is not within the upper and lower bounds of the supply for that skill, some future imbalances may not be predicted. This method is therefore conservative in determining skill imbalances, with only the larger imbalances detected; short-term or small imbalances may not be projected with this method. For these reasons, this analysis should be used in conjunction with conventional personnel management techniques to achieve an effective human resource planning strategy.
Figure 2. Organizational Transition Matrix

Time $t+1$

\[
\begin{array}{cccc}
\text{SKILL} & 1 & j & n \\
1 & p(1,1,t) & \cdots & p(1,n,t) \\
\vdots & & \ddots & \vdots \\
i & p(i,1,t) & \cdots & p(i,j,t) & \cdots \\
\vdots & & & \ddots & \vdots \\
n & & & & p(n,n,t)
\end{array}
\]

Figure 3. Sample Data Set

<table>
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<th>EMPLOYEE NUMBER</th>
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<th>POS80</th>
<th>POS81</th>
<th>POS82</th>
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NOTE: '*' denotes a missing value.
Figure 4. Numerical Transition Matrix for 1980-1981 (Sample Data from Figure 3)

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BIBLIOGRAPHY