SOME PRELIMINARY TESTS IN A REPEATED MEASURES DESIGN

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ABSTRACT

This paper deals with the analysis of variance or covariance in a repeated measures design with at most two grouping factors and one repeated measures factor. Using the MATRIX procedure, a macro is written to test the hypothesis of equal cell VC (variance-covariance) matrices and the hypothesis of equal cell VC matrices with a common matrix having Type-H structure. The macro also computes \( \epsilon \), the multiplicative adjustment factor for the degrees of freedom for the F-tests in the analysis of a mixed model.

1. INTRODUCTION

We first consider the analysis of covariance model in a repeated measures design with two grouping factors (e.g., blocking and treatment factors) and one repeated measures factor. Let \( Y_{ijk} = (Y_{1ijk}, \ldots, Y_{pijk}) \) be a vector of \( p \) responses obtained at successive time points \( t_1, \ldots, t_p \) and \( X_{ijk} \) be a vector of \( q \) covariates for the \( k \)th subject receiving the \( j \)th treatment in the \( i \)th center \( (k = 1, \ldots, n_{ij}; i = 1, \ldots, r; j = 1, \ldots, c) \). Here the centers stand for the blocks. We assume that the conditional distribution of \( X \) given \( \boldsymbol{\gamma} \) is multivariate normal. More specifically, we assume that

\[
Z_{ijk} | X_{ijk} \sim \mathcal{N}_p \left( \mu_{ij}, \Sigma_{ij} \right),
\]

i.e., \( Z_{ijk} \) is a random sample from the \( p \)-variate normal population with conditional mean vector \( \mu_{ij} = (\mu_{ij1}, \ldots, \mu_{ijq}) \) and conditional VC (variance-covariance) matrix \( \Sigma_{ij} : p \times p \).

Let us restrict our attention to the within-subject tests, i.e., the test of hypothesis of no time differences, the test of hypothesis of no treatment x time interaction, etc. The multivariate analysis of variance of (1) requires that the hypothesis

\[
H_0^a : \Sigma_{11} = \Sigma_{12} = \cdots = \Sigma_{rc},
\]

holds. This type of analysis is given in Cole and Grizzle (1966). The validity of F-tests in the usual univariate analysis of variance (through a mixed model approach) requires a more stringent condition than (2). Following Mendoza (1980), this condition can be stated as

\[
H_0^b : C' \Sigma_{11} C = C' \Sigma_{12} C = \cdots = C' \Sigma_{rc} C = \Lambda_{(p-1)}^b
\]

where \( C \) is a \( p \times (p-1) \) matrix of \((p-1)\) normalized orthogonal column contrasts, \( \Lambda > 0 \) and \( \Lambda_{(p-1)}^b \) is the \((p-1) \times (p-1)\) identity matrix. This is equivalent to saying that all \( \Sigma_{ij} \) have a type-H structure and they are all equal (Huynh and Feldt, 1970).

When \( H_0^b \) does not hold, but \( H_0^a \) holds, one can perform the approximate univariate analysis of variance with \( \epsilon \)-adjusted d.f. for the within subject F-tests (for further details, see Geisser and Greenhouse, 1958).

Using the MATRIX procedure of SAS, a macro is written to test \( H_0^a \) and \( H_0^b \) and compute \( \epsilon \). The name of the macro is \( HMATRIX \). \( HMATRIX \) also treats the repeated measures designs with one grouping factor \( (r = 1 \text{ or } c = 1) \) or no grouping factor \( (r = c = 1) \) as special cases. The computer program is listed in the appendix. Section 2 describes the test procedures and Section 3 deals with the input, output and limitations of the program. A numerical example is given in Section 4.

2. TEST PROCEDURES

For the sake of convenience, we introduce the \( i \)th cell VC matrix and \( S_q \) be an unbiased estimate of \( \Sigma_q \) after fitting the \( i \)th center and the \( j \)th treatment by the equation \( \xi = (i - 1) c + j \) for \( i = 1, \ldots, r \) and \( j = 1, \ldots, c \). Obviously \( n_q = n_{ij} \). Now we describe the test procedures for testing \( H_0^a \) and \( H_0^b \) for the model described in (1).

2.1 Test for \( H_0^a \)

Here we modify the likelihood ratio test (Box, 1969) for the analysis of covariance situation. Let \( \Sigma_q \) be the \( i \)th cell VC matrix and \( S_q \) be an unbiased estimate of \( \Sigma_q \) after fitting the \( i \)th cell model, i.e., after fitting the regression of \( \xi \) on \( X_{1i}, \ldots, X_{qi} \) for the \( i \)th cell. Note that \( S_q \) is based on \( w_q = n_{ij}^{-1} q \) d.f. Let \( S = \Sigma w \Sigma S_q / \Sigma \), where \( w = \Sigma w_q = \frac{n_{ij} q}{\xi} \).

Then \( G_1 = \frac{n_{ij} q}{\xi} \) with \( n = 2 n_{ij} \). Then

\[
H_0^b : C' \Sigma_{11} C = C' \Sigma_{12} C = \cdots = C' \Sigma_{rc} C = \Lambda_{(p-1)}^b
\]
\[(1-m)[\log(S) - \sum w_x \log[S_x^2]] \text{ is approximately distributed as chi-square with} \]
\[v = p(p+1)(g-1)/2 \text{ d.f., where} \]
\[m = (2p^2+3p-1)\{\sum (1/\omega_x^2) - 1/\omega^2\}/\omega^2 \]
\[6(p+1)(g-1)]. \]

A more precise approximation can be obtained using F-distribution (Box, 1950). If \(m\) and \(\omega^2\) are as defined before,
\[(1-m-\omega^2/v_0) \{\log(S) - \sum w_x \log[S_x^2]\}/v_0 \]
\[\text{approx} \]
\[F(v, v_0), \]
where \(v_0 = (v+2)/[(p-1)(p+2)\{\sum (1/\omega_x^2) - 1/\omega^2\}/\omega^2 \]
\[6(g-1)-m^2]. \]

If \(H_0\) holds, we can compute the multiplicative adjustment factor \(\epsilon\) as follows: Let \(\lambda_1, \ldots, \lambda_{p+1}\) be positive eigenvalues of \(G(I_g - \lambda/J_g)\), where \(G\) is a pooled VC matrix and \(J\) is a \(p \times p\) matrix with all elements being equal to unity. Then \(\epsilon = \sum (\lambda_i^2)/(\sum (\lambda_i^2))\) is the multiplicative adjustment factor for the degrees of freedom.

Note that if \(S\) is singular, the \(g\)th cell is omitted from the test of homogeneity of cell VC matrices. However, it is not omitted from the computation of \(G\) from which \(\epsilon\) is calculated. When all \(S\) matrices are nonsingular, \(S = G\).

2.2 Test for \(H_{0b}\)

Mendoza (1980) has given the test for \(H_{0b}\) when \(q\), the number of covariates, is zero. We extend his results to the analysis of covariance situation, i.e., when \(q > 0\). Let \(A_g = \mathbf{C}^\top \mathbf{G} \mathbf{C}\) be the SS and SP matrix of the transformed vector, where \(\mathbf{C}\) is as defined in (3). Then
\[G_g = m^2(2w_k(p-1)\log w_k^2 \]
\[+ w(p-1)\log[tr(A)/(p-1)]) \]
\[- w(p-1)\log - \sum \omega \log[A_x]] \]
\[G_g \text{ has approximate chi-square distribution with } gp(p-1)/2 - 1 \text{ d.f., where } A = \sum \omega \text{ and} \]
\[1 - m^2 = (w(p-1)^2(2p^2-p-2)(1/\omega_k^2)-4)/(6w(p-1)[gp(p-1)-2]) \]

Note that as in the test of homogeneity of cell VC matrices, the cells with singular VC matrices are omitted from the computation of the \(G_g\)-statistic.

3. INPUT AND OUTPUT

3.1 Cards Setup

DATA ABC;

INPUT statement;
CARDS;

3.2 Output

1. Cell VC matrices are printed.
2. Pooled VC matrix is printed under the same \(G\). If any cell VC matrix is singular, the pooled VC matrix is computed after excluding the corresponding data and renamed as \(S\).
3. It prints a chi-square value CHI-SQR for testing the equality cell VC matrices with degrees of freedom D.F. and associated probability level P-VALUE.
4. It prints an F-value for testing the equality of cell VC matrices with degrees of freedom D.F.1 and D.F.2 and associated probability level P-VALUE.
5. It prints the pooled VC matrix $G$ from all cells and prints EPSILON which is the multiplicative adjustment factor for the degrees of freedom proposed by Greisser and Greenhouse (1958).

6. It gives a chi-square value CHI-SQR for testing $H_{0b}$ with degrees of freedom D.F. and the associated probability level P-VALUE (Mendoza, 1980).

3.3 Cautions and Limitations

1. The data set FIRST must not have any missing value.

2. The program requires that the data set FIRST be fetched. Hence this data set must be created specifically for this program.

3. KEEP statement must be specified in the order stated in Section 3.1. Make sure that the variables in the KEEP statement physically appear in this order.

4. The number of grouping factors must not exceed two and the number of repeated measures factor must be one.

5. Number of levels of the repeated measures factor must not exceed 15. Note that this is the same as the number of repeated responses obtained during the study drug period.

6. The number of observation vectors in each cell should be larger than $p$ in order to have all VC matrices non-singular. Although there is no limit on the number of covariates (common to all visits), a large number of weak covariates relative to the sample size should be avoided or else it would result in loss of efficiency. It should be noted that the test procedures programmed here are sensitive to non-normality of $p$-dimensional random variable and here we assume the multivariate normality as stated in (1).

4. A NUMERICAL EXAMPLE

The hemoglobin (gm%) data obtained at Visit 1 (baseline) and then at Visits 2, 3 and 4 during the treatment period of a clinical trial are used to illustrate the computer output. This was a multicenter trial in which patients were randomly assigned to one of the three treatments in each of the two centers. The presentation of the means and sample sizes for various center x treatment cells follows:

<table>
<thead>
<tr>
<th>Cell</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>5 13.7 13.9 13.4 13.4</td>
</tr>
<tr>
<td>(1,2)</td>
<td>7 13.7 13.9 13.7 13.6</td>
</tr>
<tr>
<td>(1,3)</td>
<td>9 13.0 13.0 12.9 13.1</td>
</tr>
<tr>
<td>(2,1)</td>
<td>10 12.9 12.3 12.3 12.4</td>
</tr>
<tr>
<td>(2,2)</td>
<td>10 13.0 12.9 12.7 13.0</td>
</tr>
<tr>
<td>(2,3)</td>
<td>10 12.4 12.1 12.4 12.0</td>
</tr>
</tbody>
</table>

*(i,j)*th cell means the $i$th center and the $j$th treatment.

The computer printout for cell VC matrices, pooled VC matrix, computation of $\epsilon$, and the results of the tests for $H_{0a}$ and $H_{0b}$ follows.

**CELL VARIANCE-COVARIANCE MATRICES**

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>.66218</td>
<td>.133398</td>
<td>.312887</td>
</tr>
<tr>
<td>Y2</td>
<td>.312887</td>
<td>.332343</td>
<td>.593649</td>
</tr>
<tr>
<td>Y3</td>
<td>.312887</td>
<td>.332343</td>
<td>.593649</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>.380842</td>
<td>.195263</td>
<td>.171011</td>
</tr>
<tr>
<td>Y2</td>
<td>.195263</td>
<td>.183395</td>
<td>.146916</td>
</tr>
<tr>
<td>Y3</td>
<td>.171011</td>
<td>.183395</td>
<td>.146916</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>.237571</td>
<td>.01688</td>
<td>.184099</td>
</tr>
<tr>
<td>Y2</td>
<td>.01688</td>
<td>.284541</td>
<td>.206891</td>
</tr>
<tr>
<td>Y3</td>
<td>.184099</td>
<td>.206891</td>
<td>.562854</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>.831666</td>
<td>-.00933</td>
<td>.182852</td>
</tr>
<tr>
<td>Y2</td>
<td>-.00933</td>
<td>.369161</td>
<td>.319731</td>
</tr>
<tr>
<td>Y3</td>
<td>.182852</td>
<td>.319731</td>
<td>.412264</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>.157941</td>
<td>.133398</td>
<td>-.00291</td>
</tr>
<tr>
<td>Y2</td>
<td>.133398</td>
<td>.448552</td>
<td>.095017</td>
</tr>
<tr>
<td>Y3</td>
<td>-.00291</td>
<td>.095017</td>
<td>.160974</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>.450425</td>
<td>.424447</td>
<td>.457083</td>
</tr>
<tr>
<td>Y2</td>
<td>.424447</td>
<td>.415382</td>
<td>.612598</td>
</tr>
<tr>
<td>Y3</td>
<td>.457083</td>
<td>.612598</td>
<td>.817459</td>
</tr>
</tbody>
</table>

**FOOTNOTE:** Y1, Y2, Y3... represent the levels of the repeated measures factor
POOLED VC MATRIX, G, BASED ON ALL DATA

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>0.437795</td>
<td>0.179019</td>
<td>0.209709</td>
</tr>
<tr>
<td>Y2</td>
<td>0.179019</td>
<td>0.471928</td>
<td>0.29229</td>
</tr>
<tr>
<td>Y3</td>
<td>0.209709</td>
<td>0.29229</td>
<td>0.447861</td>
</tr>
</tbody>
</table>

CHI-SQUARE VALUE FOR TESTING HOMOGENEITY OF VC MATRICES

TEST1 | CHI-SQR | D.F. | P-VALUE |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>57.8942</td>
<td>50</td>
<td>.152436</td>
</tr>
</tbody>
</table>

F-TEST (BOX'S APPROXIMATION) FOR TESTING HOMOGENEITY OF VC MATRICES

TEST2 | F VALUE | D.F.1 | D.F.2 | P-VALUE |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.22781</td>
<td>30</td>
<td>1379.01</td>
<td>.035949</td>
</tr>
</tbody>
</table>

MULTIPlicative ADJUSTMENT FACTOR TO D.F.

POOLED VC MATRIX, G, COMPUTED FROM ALL CELLS

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>Y1</th>
<th>Y2</th>
<th>Y3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>0.437795</td>
<td>0.179019</td>
<td>0.209709</td>
</tr>
<tr>
<td>Y2</td>
<td>0.179019</td>
<td>0.471928</td>
<td>0.29229</td>
</tr>
<tr>
<td>Y3</td>
<td>0.209709</td>
<td>0.29229</td>
<td>0.447861</td>
</tr>
</tbody>
</table>

MULTIPlicative ADJUSTMENT FACTOR TO THE DEGREES OF FREEDOM

EPSILON VALUE

.927715

REFERENCE


REFERENCES


APPENDIX

64 DATA FIRST; SET A;
65 KEEP CENTER DRUG XL Yi-3 NFACtor NCOV; NFACtor=2; NCOV=1;
66 PROC SORT; BY CENTER DRUG;
67 ?PROC PRINT;
68 PROC MEANS MAXDEC=3; BY CENTER DRUG;
69 VAR X1 Y1 Y2 Y3;
70 TITLE CENTER BY DRUG MEAS;
71 MACRO HMATRIX
72 PROC MATRIX FW=7;
73 ** THIS PROGRAM: **
74 ** (1) TESTS HOMOGENEITY OF CELL VC MATRICES IN ANOVA OR ANCOVA. **
75 ** (2) CALCULATES EPSILON-MULTIPLICATIVE FACTOR TO D.F., AND **
76 ** (3) TESTS THE JOINT HYPOTHESIS USING MENDOZA'S (1980) TEST **
77 ** LIMITED TO TWO GROUPING FACTORS, ONE TRIAL FACTOR, AND **
78 ** 15 VISITS--NUMBER OF LEVELS OF TRIAL FACTOR. **
79 ** NFACtor = NUMBER OF GROUPING FACTORS **
80 ** NCOV = P2 = NUMBER OF COVARIATES **
81 ** DATA FIRST MUST BE SORTED BY GROUPING FACTORS **
82 ** NLEVEL = NUMBER OF LEVELS OF GROUPING VARIABLE(S) **
83 ** NLEVEL = NUMBER OF LEVELS FORMED BY GROUPING VARIABLE(S). **
84 85 NC=NCOL(C); NFCT=NC-1;P2=C(1,NCF-1); NR=NROW(C);
86 P1=NC-2-NFCT-P2; Z=C(NFCT+1;NC-2);
87 N = [P] = NUMBER OF LEVELS OF THE TRIAL FACTOR;
88 IF NFCT=1 THEN DR=DESIGN(C(*,1));
89 IF NFCT=2 THEN DR=DESIGN(C(*,1))\DESIGN(C(*,2));
90 NLEVEL=NCOL(K);
91 NLEVEL = NUMBER OF LEVELS FORMED BY GROUPING VARIABLE(S).
ORTH =

1  -1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -2  0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -3  0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -4  0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -5  0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -6  0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -7  0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -8  0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -9  0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -10 0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -11 0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -12 0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -13 0  0  0  0  0  0  0  0  0  0  0  0  0  0
1  -14 0  0  0  0  0  0  0  0  0  0  0  0  0  0

P3 = P1-1;
ORTH = ORTH(1;P3;P1); ORTH = INV(SORT(ORTH+ORTH'))*ORTH;
CUMN = 0; T3=S-CUM(1;P1); P2=0; P4=0; M=0; MA=0; FA3=0; MNEW =0;
NRR=E; CCYX=E; RRR=E; NR1=E; NOTE CELL VARIANCE-COVARIANCE MATRICES;
IF NFCT>1 THEN NOTE SKIP=2 (CELL 1 IS (1,1)TH CELL--FIRST BLOCK;
IF NFCT>1 THEN NOTE AND FIRST TREATMENT,CELL 2 IS (1,2)TH CELL--;
IF NFCT>1 THEN NOTE FIRST BLOCK AND SECOND TREATMENT,ETC.;
IF NFCT>1 THEN NOTE NOTE: CELL 1 IS FOR FIRST LEVEL OF RESPONSE;
IF NFCT>1 THEN NOTE VARIABLE: CELL 2 IS FOR SECOND LEVEL OF RESPONSE;
IF NFCT>1 THEN NOTE VARIABLE: AND SO FORTH;
LVL=LEVEL;
DO K = 1 TO LVL;
NR1 = NNR(K+1); CUMN = CUM +NR1; V = ZCUMN-NR1+1:CUMN;)
RES = (1NR1-J(NR1)+1)/NR1*V; SUM=RES*RES;
SII=SUM(P21+P21+P21+P21); P2=0 THEN SII=SUM(1-P2); P2=1 THEN P2=SUM;
IF P2>0 THEN CYX=SUM;
IF P2>0 THEN CYX=SII-SII*INV(SUM>1-P2,1-P2))#S12;
N=NR1-1-P2;
VAR = CYX/#N; x VAR = VC MATRIX;
DEF=DEF(VAR); IF DEF=0.001 THEN GO TO READ;
M=NG#LOG(DEF#CCYX) + M; FA1=11/#NG+FA1;
THEN = N#LOG(log(DEF#CCYX)) + MHEW;
FA2 = IF(/(NG)#K + FA2; TSS=CYX+TSS; CYX IS THE SS • SP RESIDUAL MATRIX;
VARIABLEVAR; VARIABLE=CELL VC MATRIX;
RES=RES-(I(P2)-J(P2))#K;)
RESDF = NR-LEVEL-I; S=TSS/RESDF; S=POOLED VC MATRIX BASED ON ALL DATA;
M=RESDF#LOG(DEF#CCYX) + MA; FA3 = N#P3#LOGCHG + FA3; GO TO RUN;
READ: NOTE SKIP=2 THE FOLLOWING VC MATRIX IS SINGULAR. THEREFORE;
NOTE THE CORRESPONDING CELL IS EXCLUDED;
NOTE THE FOLLOWING 3 VALUES ARE REQUIRED IN MENDOZA'S TEST;
AG = ORTH#CCYX*ORTH';
MA = NG#LOG(DEF#CCYX) + MA; FA3 = N#P3#LOGCHG + FA3; GO TO RUN;
GO TO RUN;
RESDF = NR-LEVEL-I; M=RESDF#LOG(DEF#CCYX) + MA; FA3 = N#P3#LOGCHG + FA3; GO TO RUN;
**TEST FOR HOMOGENEITY OF CELL VC MATRICES**

If \( LVL = 1 \) then go to adjust:

1. Note page pooled VC matrix, \( G \), based on all data.
2. After excluding:
3. Note singular cell VC matrices.
4. \( R = \{Y1, Y2, Y3\} \)
5. \( C = \{Y1, Y2, Y3\} \)
6. \( R = \{Y1, Y2, Y3\} \)
7. \( C = \{Y1, Y2, Y3\} \)
8. \( R = \{Y1, Y2, Y3\} \)
9. \( C = \{Y1, Y2, Y3\} \)
10. \( R = \{Y1, Y2, Y3\} \)
11. \( C = \{Y1, Y2, Y3\} \)

If \( S = 0 \) then:

1. Note page pooled VC matrix, \( G \), based on all data.
2. After excluding:
3. Note singular cell VC matrices.
4. \( R = \{Y1, Y2, Y3\} \)
5. \( C = \{Y1, Y2, Y3\} \)
6. \( R = \{Y1, Y2, Y3\} \)
7. \( C = \{Y1, Y2, Y3\} \)
8. \( R = \{Y1, Y2, Y3\} \)
9. \( C = \{Y1, Y2, Y3\} \)

**ASSOCIATED P-VALUE:**

1. \( TEST1 = \frac{CHI\_SQ}{DF} \)
2. \( PSI\_VAL = 1 - \text{PROB}(F, DF1, DF2) \)
3. \( TEST2 = \frac{F}{DF1, DF2} \)
4. \( PSI\_VAL = \frac{1 - \text{PROB}(F, DF1, DF2)}{DF1, DF2} \)

**ASSOCIATED R-FRAME:**

1. \( EPSILON = \text{MULTIPLICATIVE ADJUSTMENT FACTOR TO THE DEGREES OF FREEDOM} \)
2. \( \text{EPSILON} = \frac{EPSILON}{DF} \)
3. \( \text{EPSILON} = \frac{EPSILON}{DF} \)
4. \( \text{EPSILON} = \frac{EPSILON}{DF} \)
5. \( \text{EPSILON} = \frac{EPSILON}{DF} \)
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12. \( \text{EPSILON} = \frac{EPSILON}{DF} \)
13. \( \text{EPSILON} = \frac{EPSILON}{DF} \)

**ASSOCIATED MENDOZA'S JOINT TESTS:**

1. \( DF = \text{LEVEL} \times \text{LEVEL} - 1 \)
2. \( \text{EPSILON} = \frac{\text{MULTIPLICATIVE ADJUSTMENT FACTOR TO THE DEGREES OF FREEDOM}}{DF} \)
3. \( \text{EPSILON} = \frac{\text{MULTIPLICATIVE ADJUSTMENT FACTOR TO THE DEGREES OF FREEDOM}}{DF} \)
4. \( \text{EPSILON} = \frac{\text{MULTIPLICATIVE ADJUSTMENT FACTOR TO THE DEGREES OF FREEDOM}}{DF} \)
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