I. INTRODUCTION

Additional time series capability has been added to the procedures AUTOREG, MATRIX and ARIMA for the 79.6 release. The additions to ARIMA will be available in test form in a procedure called TARIMA for the 79.6 release and will be added to the ARIMA procedure for the 82 release.

The AUTOREG procedure will iterate the correction for autocorrelation and the calculation of the residual autocorrelation until convergence is attained. The iteration is requested by specifying the ITER option either on the MODEL statement or on the PROC statement. The iterations will be printed if the ITPRINT option is specified.

Three additional MATRIX functions have been added. They are:

- ARMACOV: autocovariance of an ARMA model
- ARMALIK: likelihood function, sum of squares and residuals for an ARMA model
- MRATIO: inverts matrix generating functions.

Subset ARMA models, intervention and transfer function models can be fit using the TARIMA procedure.

II. PROC AUTOREG

Currently the AUTOREG procedure does a two stage fitting sequence. Ordinary least squares is used to compute estimates of the residuals which are in turn used to estimate an autocovariance function. Based on this autocovariance function and the specified lag structure the regression model is refit using generalized least squares. In SAS 79.6 this process may be iterated until the estimates of the regression model parameters and the autoregressive parameters are consistent. Because the estimates of the autoregressive parameters are based on the Yule-Walker equations the resulting estimates of the parameters are not unconditional least squares estimates. However, for long series they will be nearly identical to unconditional least squares estimates.

The equations which are iterated can also be derived by considering the minimization of a quadratic form. This quadratic form is obtained by using a finite portion of the inverse of the infinite variance matrix corresponding to the autoregressive model for the residuals. The unconditional sum of squares corresponds to using the inverse of a finite portion of the variance matrix.

An example of the iteration feature in AUTOREG is given below for the General Motors series from Grunfeld's data as given by Maddala (p. 214). There are twenty observations on three variables I, gross investment, F, lagged value of the firm, and C, lagged value of stock and equipment. I is taken as the dependent variable and F and C are the independent variables. A first order autoregressive process is assumed to model the residuals. The estimates after one iteration, the final iterated estimates, the unconditional least squares estimates and the maximum likelihood estimates are given below with their estimated standard errors given in parentheses. INT labels the intercept parameter, F and C the corresponding regression coefficients and RHO the lag one autoregressive parameter.

PARAMETERS

<table>
<thead>
<tr>
<th></th>
<th>INT</th>
<th>F</th>
<th>C</th>
<th>RHO</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONE</td>
<td>-51.90</td>
<td>.0936</td>
<td>.404</td>
<td>-.426</td>
</tr>
<tr>
<td>ITERATION</td>
<td>(95.3)</td>
<td>(.022)</td>
<td>(.045)</td>
<td>(.202)</td>
</tr>
<tr>
<td>LAST</td>
<td>-22.81</td>
<td>.0862</td>
<td>.420</td>
<td>-.646</td>
</tr>
<tr>
<td>ITERATION</td>
<td>(96.5)</td>
<td>(.019)</td>
<td>(.055)</td>
<td>(.171)</td>
</tr>
<tr>
<td>UNCOND'L</td>
<td>-16.40</td>
<td>.0849</td>
<td>.425</td>
<td>-.712</td>
</tr>
<tr>
<td>SUM OF SQ</td>
<td>(102.7)</td>
<td>(.019)</td>
<td>(.063)</td>
<td>(.191)</td>
</tr>
<tr>
<td>MLE</td>
<td>-20.30</td>
<td>.0856</td>
<td>.422</td>
<td>-.672</td>
</tr>
<tr>
<td></td>
<td>(97.56)</td>
<td>(.019)</td>
<td>(.059)</td>
<td>(.198)</td>
</tr>
</tbody>
</table>

III. MATRIX Functions

Three new functions have been added to the MATRIX procedure for use with the analysis of time series.

The first computes the covariance function of an ARMA process from the ARMA parameters. The syntax is:

CALL ARMACOV(AUTO,CROSS,MOVE,PHI,THETA,N);
The first row of PHI and THETA is ignored and assumed to be one. The parameterization is such that the ARMA(1,1) model

\[ y_t = \phi y_{t-1} + \theta \epsilon_{t} \]

has PHI=-0.8 and THETA=0.9.

The second function computes the concentrated log likelihood, the unconditional sum of squares, the standardized residuals and the standard deviations of the residuals for a time series given an ARMA model. The parameterization is the same as for the ARMACOV function. The method used is described in the article by Ansley. The syntax is:

```plaintext
CALL ARMALIK(LNL,RESID,STD,X,PHI,THETA);
```

**INPUT ARGUMENTS**

- `X`: n by 1 or 1 by n matrix of the values of the time series (assumed mean zero)
- `PHI`: p+1 by 1 matrix with the autoregressive parameters
- `THETA`: q+1 by 1 matrix with the moving average parameters

**RETURNED ARGUMENTS**

- `LNL`: 3 by 1 matrix where
  - LNL(1,1) is the log likelihood concentrated with respect to the variance of the innovation process
  - LNL(2,1) is the unconditional sum of squares divided by n
  - LNL(3,1) is the log of the determinant of the normalized variance matrix
- `RESID`: 1 by n matrix of the standardized residuals. If the model is correctly specified these residuals should be constant variance and uncorrelated.
- `STD`: 1 by n matrix of the standard deviations of the actual residuals. The actual residuals can be obtained as STD*RESID

The third function generalizes the RATIO function to matrix arguments. Let B represent the backshift operator. Then if AR(B) is an autoregressive operator and MA(B) is a moving average operator, MRATIO will compute PSI(B) where

\[ PSI(B) = AR(B)^{-1}MA(B) \]

The syntax is:

```plaintext
PSI=MRATIO(AR,MA,T,R);
```

**INPUT ARGUMENTS**

- `AR`: n by n's matrix of autoregressive operator. The first n by n submatrix is the constant term, the second n by n submatrix is the lag one term and so on.
- `MA`: n by m's matrix of moving average operator. The first n by m submatrix is the constant term, the second n by m submatrix is the lag one term and so on.
- `T`: 1 by 1 matrix containing the value for t.
- `R`: 1 by 1 matrix containing the value for r, the number of terms to calculate.

**RETURNED ARGUMENT**

- `PSI`: n by r*m matrix of impulse response values.

IV. PROC ARIMA

The 82 release of the ARIMA program will be able to fit subset ARMA models, intervention and transfer function models. A test release of these features is available in the 79.6 release in a procedure called TARIMA. The syntax for the TARIMA procedure is the same as for the ARIMA procedure with the following additions.

First for the IDENTIFY statement a new parameter CROSSCOR has been added. Specifying CROSSCOR= (list of variable names) will request crosscorrelation plots of these variables with the variable named in the VAR= parameter. It is also necessary that any variable to be used as an input variable in a subsequent ESTIMATE statement must be named in the immediately previous IDENTIFY statement. If a simple ARIMA model has been previously fit to a variable in the CROSSCOR list, then both that variable and the variable named by the VAR= parameter are prewhitened by the ARIMA model before the crosscorrelations are computed.

Along with the crosscorrelation plots, a plot of an estimate of the inverse autocorrelation function has been added. See Cleveland and Chatfield for interpretation of the inverse autocorrelation plot.
Second for the ESTIMATE statement the P and Q parameters may be specified as before or they may be specified as lag lists. The specification as lag lists has the syntax:

\[ P=(\text{lag}_1, \text{lag}_2, \ldots, \text{lag}_K) \]

where the commas are required and more than one parenthesized list may follow.

Also a new parameter INPUT may be used to specify a transfer function or intervention model. This parameter is specified as:

\[ \text{INPUT}=(\text{list of input specifications}) \]

where an input specification is of the form:

\[ S(\text{lag}_1, \ldots, \text{lag}_K) / (\text{lag}_1, \ldots, \text{lag}_I) \text{ name} \]

where S is an integer representing an overall backshift, the parenthesized list before the / represents the lags present in the numerator, the parenthesized list after the / represents the lags present in the denominator of the transfer function and name is the name of the variable to which this transfer function is to be applied.

An example specification is:

\[ \text{INPUT}=(4(1,3)12)/(1) \text{ X} \]

which represents the transfered series:

\[ sB^4 (1-aB-bB^3) (1-cB^{12}) / (1-dB) X_t \]

where B is the backshift operator and a, b, c, d and s are free parameters.

In the 79.6 test release residuals are computed starting at the beginning of the transferred series. In the 82 release this will be changed so that residuals will not be computed until P values are available where P is the order of the numerator of the transfer function. The start up values used to compute the transferred series consist of 0 for the previous values needed for the denominator and either the mean of the series or the first value of the series for the previous values needed for the numerator. If a simple ARIMA model has been previously fit to the series to be transferred then the mean of the series is used for the start up, otherwise the first value is used.

REFERENCES


