At the 1977 SUGI Conference, Goodnight suggested that the general form of estimable functions provided by GLM may be translated into main effects designs involving any number of qualitative factors and levels. An analysis of his method leads to the conclusion:

A necessary and sufficient condition for a design construction rule is that it generates a prototype design matrix $X^*$ which satisfies the identity $X^*(X'X)X'X=X$, where columns of $X^*$ corresponding to swept rows of $X'X$ are arbitrary. Selection of a particularly simple form of $X^*$ results in the following general rule for construction of design matrices to test specific effects and interactions among qualitative factors, using the general form of estimable functions and based on minimum sample size:

1. Generate the first row of $X$ by setting $L_1=1$ and all other $L_i$'s to zero.
2. Generate all other rows of $X$ by setting $L_1$ and each succeeding free coefficient to one with the provision: If the free coefficient corresponds to an interaction, then free coefficients for the corresponding levels of all effects and interactions contained in this interaction must be set to one (all others set to zero).

The resulting class of designs provides a useful supplement to those traditionally obtained by fractional replication. However, the power of any particular test may vary within the class of equivalent designs, depending on the particular choice of $X^*$.

INTRODUCTION

At the 1977 SUGI Conference, Goodnight (1977) suggested that the general form of estimable functions may be translated into a main effects design involving any number of qualitative factors at any number of levels. The method was illustrated by generating the following $2 \times 2 \times 3$ effects plan in factors A, B, and C. For this design, the general form of estimable functions will always be

$$
U \quad L1 \\
A1 \quad L2 \\
A2 \quad L1+L2 \\
B1 \quad L4 \\
B2 \quad L1+L4 \\
C1 \quad L6 \\
C2 \quad L7 \\
C3 \quad L1+L6+L7
$$

A general rule of construction for any main effects design matrix ($X$) was stated as follows:

(I-M) Generate the first row of $X$ by setting $L_1=1$ and all other $L_i$'s to zero.

(II-M) Generate all other rows of $X$ by setting $L_1$ and each successive free coefficient to one.

When applied to this example, the following design matrix (G-I) resulted, where its subsequent translation into actual treatment levels is also shown.

<table>
<thead>
<tr>
<th>Design matrix G-I</th>
<th>Treatment Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>U A1 A2 B1 B2 C1 C2 C3</td>
<td>A B C</td>
</tr>
<tr>
<td>L1=1 1 0 0 1 0 0 1</td>
<td>2 2 3</td>
</tr>
<tr>
<td>L1=L2=1 1 1 0 0 1 0 0 1</td>
<td>1 2 3</td>
</tr>
<tr>
<td>L1=L4=1 1 0 1 0 0 0 1</td>
<td>2 1 3</td>
</tr>
<tr>
<td>L1=L6=1 1 0 1 0 1 0 0</td>
<td>2 2 1</td>
</tr>
<tr>
<td>L1=L7=1 1 0 1 0 1 0 1</td>
<td>2 2 2</td>
</tr>
</tbody>
</table>

By inspection, it was noted that the intermediate step of constructing $X$ always may be eliminated according to the following derived general rule:

(I-M) For the first observation, set all factors at their highest level.

(II-M) For all succeeding observations, vary each factor, one at a time, through all of its remaining levels.

The most striking feature of what was accomplished is that the minimum number of observations, i.e. $n=5$, has been obtained to test all main effects, corresponding exactly to the degrees of freedom assigned for MODEL. To one versed in the traditional theory of fractional factorial designs, e.g. Kempthorne (1952), this seems surprising in that neither a $1/2$ replication ($n=6$) nor a $1/3$ replication ($n=4$) of a $2 \times 2 \times 3$ factorial design provides the required main effects design without prohibitive confounding. What has been accomplished is the required design at the sacrifice of orthogonality and hence computational convenience, i.e. use of GLM as opposed to ANOVA. This leads to the following questions which we find to be of interest:

(1) What is the theoretical basis of this method of generation?
(2) Are there minimum sample size, partial factorial designs for qualitative factors which admit to tests of certain specified interactions including those ordinarily confounded in traditional fractional factorial designs?
(3) How may these designs be generated?
(4) Is the sacrifice of orthogonality justified?

THEORETICAL BASIS

Mathematically, any design matrix $X$ must satisfy the key identity

$$X(X'X)X'X=X \quad (1)$$

where any $g$-inverse $(X'X)^{-1}$ of $X'X$ will suffice. GLM produces a $g$-inverse as a result of the SWEEP operation in solving the normal equations. Upon request, GLM will print the
non-zero rows of \((X'X)^{-1}X'X\) as a symbolic vector, representing the general form of any estimable function, e.g. as given above for the class of all possible 2 \(\times\) 2 \(\times\) 3 main effects designs. Since the integer part of each \(L\) coefficient indicates at what stage a new, linearly independent row of \(X'X\) has been found by SWEEP, the non-zero rows of \((X'X)^{-1}X'X\) may be constructed by successively setting each \(L\) coefficient to one. All intervening rows implicitly are set to zero by SWEEP. This is illustrated below for the 2 \(\times\) 2 \(\times\) 3 factorial example where we obtain \((X'X)^{-1}X'X =\):

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
<th>B1</th>
<th>B2</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>(L1=1)</td>
</tr>
<tr>
<td>0 1</td>
<td>-1</td>
<td>0 0 0 0 0 0 (L2=1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 0 0 1</td>
<td>-1 0 0 0 (L4=1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 0 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 0 0 0 0 1</td>
<td>-1 (L7=1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 0 0 0 0 0 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From general theory of the linear model \(y = X\beta + \epsilon\), \(X\) is always estimable, so that any admissible \(X\) is expressible as a linear combination of the non-zero rows of \((X'X)^{-1}X'X\). Let us express the linear combinations suggested by rule (I-M) and (II-M) in the form of a matrix product \(X^*(X'X)^{-1}X'X\), where

\[
\begin{array}{cccccccc}
1 0 & * & 0 & 0 & 0 & * & (L1=1) \\
0 & 1 & * & 0 & 0 & 0 & * & (L1=L2=1) \\
1 0 & * & 1 & 0 & 0 & * & (L1=L4=1) \\
1 0 & * & 0 & * & 1 & 0 & * & (L1=L6=1) \\
1 0 & * & * & 0 & 1 & * & (L1=L7=1) \\
\end{array}
\]

and * indicates that the corresponding element may be chosen arbitrarily since it corresponds to a zero row of \((X'X)^{-1}X'X\). Completing the multiplication yields the design matrix for \(C=1\). Comparing \(X^*\) and \(X\), we see that the non-arbitrary columns are identical.

**Conclusion:** A necessary and sufficient condition for a design construction rule is that it generates a prototype design matrix \(X^*\) which satisfies the identity \(X^*(X'X)^{-1}X'X = X\), analogous to equation 1.

For the 2 \(\times\) 2 \(\times\) 3 factorial example, an alternative rule for a main effects design could be easily based on the prototype matrix

\[
\begin{array}{cccccccc}
1 1 & * & 0 & 0 & * & 1 0 & * \\
1 0 & * & 0 & * & 1 0 & * \\
1 1 & * & 0 & * & 0 1 & * \\
1 1 & * & 0 & 0 & 0 0 & * \\
\end{array}
\]

Goodnight's rule, of course, results in identifiable symmetry in the sample plan, and suggests the immediate generalization expressed by steps (1-M) and (2-M).

**DESIGNS FOR TESTING INTERACTION**

The basic conclusion of the previous section clearly applies to all experimental designs, regardless of whether or not they admit to tests for interaction. The general approach is as follows: 1) Generate the non-zero rows of \((X'X)^{-1}X'X\) from the general form of estimable functions for the specified configuration of main effects and interactions to be tested. 2) Construct the simplest possible prototype design matrix \(X^*\) which reflects the presence of the specified configuration of main effects and interactions, being aware that GLM will SWEEP the last level of each main effect and any interactions which involve these levels.

In order to illustrate the method of construction, suppose we consider an actual application. A graduate student, M. Durbin (1980), wished to perform a behavioral study of the remarkable fright reaction of hognose snakes, e.g. swelling, rolling over, bleeding at the mouth, etc. When confronting each snake with a stimulus animal (SA), she wished to vary the following four factors:

- Motion (M): SA stationary (1) or moving (2),
- Vision (V): Snake's vision blocked (1) or open (2),
- Olfactory (O): Snake's smell blocked (1) or open (2),
- Contact (C): Contact made (1) or not made (2) with SA.

Taken in all combinations, this would require \(2^4 = 16\) trials, but since these were to be repeated for each of three stimulus animals (predator, prey and neutral) \(n=48\) trials per snake seemed prohibitive (they learn!). In actual fact, she required only tests for the main effects (M, V, O, C) and the interactions of these with SA, i.e. potentially \(n=21\) observations provided such a design existed. Consider the non-zero rows of \((X'X)^{-1}X'X\) as reconstructed from the general form of estimable functions

\[
\begin{array}{cccccc}
U & L1 & M1 & V1 & O1 & C1 \\
M1 & L2 & M2 & V2 & O2 & C2 \\
L1 & L2 & M2 & V2 & O2 & C2 \\
V1 & L4 & M2 & V2 & O2 & C2 \\
O1 & L6 & M2 & V2 & O2 & C2 \\
C1 & L8 & M2 & V2 & O2 & C2 \\
\end{array}
\]
Non-zero rows of \((X'X)^{-1}X'X\):

\[
\begin{array}{ccccccc}
U & M & V & O & C & MV & VO \\
1 & 2 & 1 & 2 & 1 & 2 & 11 & 12 & 21 & 22 & 11 & 12 & 21 & 22 \\
1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & (L_1=1) \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 & 0 & (L_2=1) \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 1 & 0 & -1 & (L_4=1) \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (L_6=1) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & (L_{10}=1) \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & (L_{14}=1) \\
\end{array}
\]

A simple prototype design Matrix \(X^*\) appears below:

\[
\begin{array}{ccccccccccccccc}
U & M & V & O & C & MV & VO \\
1 & 2 & 1 & 2 & 1 & 2 & 11 & 12 & 21 & 22 & 11 & 12 & 21 & 22 \\
1 & * & 0 & * & 0 & * & 0 & * & * & * & * & * & * & * & * & *(U) \\
1 & * & 0 & * & 0 & * & 0 & * & * & * & * & * & * & * & * & *(M) \\
1 & * & 1 & * & 0 & * & 0 & * & * & * & * & * & * & * & * & *(V) \\
1 & * & * & 0 & 1 & * & 0 & * & * & * & * & * & * & * & * & *(O) \\
1 & * & * & 0 & * & 1 & * & 0 & * & * & * & * & * & * & * & *(C) \\
1 & * & * & * & * & 1 & * & 0 & * & * & * & * & * & * & * & *(MxV) \\
1 & * & * & 0 & 1 & 0 & * & 0 & * & * & * & * & * & * & * & *(VxO) \\
\end{array}
\]

Using identity (2), we obtain

\[
\begin{array}{ccccccccccccccc}
U & M & V & O & C & MV & VO \\
1 & 2 & 1 & 2 & 1 & 2 & 11 & 12 & 21 & 22 & 11 & 12 & 21 & 22 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & (L_1=1) \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & (L_2=L_1=1) \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & (L_4=L_6=1) \\
1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & (L_6=L_{10}=1) \\
1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (L_4=L_6=L_{10}=1) \\
\end{array}
\]

Sample Plan for Design D-1

(Actually used by Durham)

<table>
<thead>
<tr>
<th>Observation</th>
<th>M</th>
<th>V</th>
<th>O</th>
<th>C</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>(U)</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>(M)</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>(V)</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>(O)</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>(C)</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>(MxV)</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>(GxO)</td>
</tr>
</tbody>
</table>

By deduction from the above pattern, a general rule of construction for any interaction design matrix using the general form of estimable functions may be stated as follows:

(I-H) Generate the first row of \(X\) by setting \(L_1=1\) and all other \(L\)'s to zero.

(II-H) Generate all other rows of \(X\) by setting \(L\) and each succeeding free coefficient to one with the provision: If the free coefficient corresponds to an interaction, then free coefficients for the corresponding levels of all effects and interactions contained in this interaction must be set to one.

By inspection of the sampling plan which emerged from Design D-1, clearly the following simplified, general rule may be stated:

(1-H) For the first observation, set all factors at their highest level.

(2-H) For all main effects to be tested, generate additional observations by varying each factor, one at a time, through all its remaining levels.

(3-H) For each \(k\)-factor interaction to be tested, generate additional observations by varying the \(k\) included factors through all possible combinations of levels, excluding all combinations which include the highest level of any factor.

As in the case of designs for main effects, these rules are based on the simplest possible form of prototype design matrix \(X^*\). Other rules could be formulated in terms of alternative \(X^*\), but the resulting sampling plans would lack the natural symmetry of the current version.
In retrospect, one might have used a $\frac{1}{2}$ replication of a 24 factoral as a basis for the hognose snake experiment. By confounding with respect to the MVOC interaction, possible treatment combinations could have been

<table>
<thead>
<tr>
<th>Observation</th>
<th>M</th>
<th>V</th>
<th>O</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

for a total of $n=388=24$ compared to 21 trials per snake. The complete pattern of confounding is $M=VOC$, $V=MOC$, $O=MVC$, $C=MVO$, $MV=OC$, $MO=VC$, and $MC=VQ$ so that the design would have been acceptable in the sense that both $M \times V$ and $V \times O$ are testable (as well as either $M \times O$ or $V \times C$). But suppose Ms. Durham had also required that $O \times C$ be testable. Use of the traditional $\frac{1}{2}$ replication is nullified because $M \times V$ and $O \times C$ are completely confounded, yet Design D-1 may be modified by adding an eighth observation

<table>
<thead>
<tr>
<th>Observation</th>
<th>M</th>
<th>V</th>
<th>O</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

to accomplish the task.

The advantage of this design principle becomes more apparent as the number of levels per factor is increased. Suppose in a $3 \times 3 \times 3 \times 3$ experimental situation involving factors $A$, $B$, $C$, and $D$, we wished to test all main effects, and the $A \times B$, $C \times D$, and $A \times B \times C$ interactions, based on a minimum number of trials. As Kempthorne (1952, page 412-413) points out, a $\frac{1}{3}$ replication of a 3$^4$ experiment requiring 27 observations has little practical value since all 2-factor interactions are confounded, e.g., $A \times B$ with $C \times D$, etc. In absence of our proposed alternative, we might recommend that the full $n=3^8=81$ observations be made. Yet, by use of our simplified design rule, we obtain the following sample plan which admits to tests of the required effects and interactions (assuming no $A \times C$ or $B \times C$ interactions), based on a minimum possible sample size of $n=25$ observations:

<table>
<thead>
<tr>
<th>Observation</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Possibly $n=25$ could be reduced still further by sacrificing observations and hence degrees of freedom associated with the $A \times B \times C$ interaction. We haven't fully examined the implications of using only a fraction of the traditional degrees of freedom. What has been gained is that the experimenter can proceed with his work. What has been lost is orthogonality, i.e. all subsequent analysis must be accomplished in GLM rather than ANOVA. Yet, the estimable functions being tested by Type III or IV SS are identical to those we ordinarily test based on a full complement of observations.

As a final point, it is relevant to ask if it makes any difference which member of a design class is chosen, e.g. the class of $3 \times 3 \times 3 \times 3$ designs to test $A$, $B$, $C$, $D$, $A \times B$, $C \times D$, and $A \times B \times C$. Though this is a many-sided question, suppose we use the power of a particular test as a criterion. For the $i$'th effect or interaction, $(i=1, \ldots, m)$, GLM will always formulate a null hypothesis, $H_0(i)$: $G_i(L)$ is of full row rank corresponding to the degrees of freedom $(b)$ assigned to $H_i$, and $L=(X'X)^{-1}X'$ is invariant over the class. Since $G$ is fixed by GLM (type III or IV), $K_i$ also must be invariant.

Under usual assumptions, however, the noncentrality parameter takes the form

$$
\lambda_i = (2^{i-1})^2 G_i' \{G_i(X'X)^{-1}G_i\}^{-1}G_i' G_i
$$

and clearly this and the resultant power will vary as a function of $(X'X)$ and hence $X$, even within a class.
\[ G(X'X)^{-1} G' \] and we may define
\[ D = |G(X'X)^{-1} G'|, \]
proportional to the generalized variance. Note that a search of designs to minimize \( D \) is equivalent to the familiar problem of minimizing \( |X'X| \) in the search for optimal response surface designs, e.g. Box and Draper (1971). Clearly, \( D \) represents the kernel, in some sense, of a composite noncentrality parameter analogous to equation (3).

In order to illustrate quantitatively the range of variability to be expected among designs within the same model class, suppose we consider \( n=8 \) point designs for factors at two levels which may be generated by the following Hadamard matrix:

\[
\begin{array}{cccccccc}
U & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\
\end{array}
\]

By using obvious aliasing relationships, Box and Draper (1971) concluded that rows of the above array provide D-optimal design points for the following models. Values of \( D \) for analogous Goodnight-type designs are shown for comparison.

<table>
<thead>
<tr>
<th>#Factors/Model</th>
<th>D-criterion</th>
<th>Goodnight</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>K=4 ( X_1=A, X_2=B, X_3=C, X_4=D, X_5=AD = RC, X_6=AR = CD, X_7=AC = RD )</td>
<td>8, 0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K=5 ( X_1=A, X_2=B, X_3=C, X_4=D, X_5=E, X_6=AB = CD, X_7=AC = BD )</td>
<td>8, 0.125</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K=6 ( X_1=A, X_2=B, X_3=C, X_4=D, X_5=E, X_6=F, X_7=AB = CD )</td>
<td>8, 0.03125</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K=7 ( X_1=A, X_2=B, X_3=C, X_4=D, X_5=E, X_6=F, X_7=G )</td>
<td>8, 0.0078125</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Clearly, the Goodnight-type designs are less than D-optimal. The prevalence of a single level in each column of a Goodnight-type design matrix is contradictory to the equal balance required in order to diagonalize \( G(X'X)^{-1} \) corresponding to minimum \( D \). On the other hand, it was this very property which had appeal in the hognose snake experiment, namely to minimize as much as possible those experimental trials which imposed special stress, e.g., obstructing vision and smell. In spite of complex computer algorithms now available which iterate to D-optimal designs, e.g., Cook and Nachtsheim (1980), perhaps this sort of practicality still has bearing on the ultimate experimental design.

REFERENCES


