MULTIVARIATE MULTIPLE COMPARISONS USING SAS

A. Lawrence Gould, Merck Sharp & Dohme Research Laboratories

Introduction

Much has been written about univariate multiple comparisons [10]. Much less has been written about multivariate multiple comparisons. Similarly, many statistical computing packages, including SAS, provide a way to perform univariate multiple comparisons, but few (if any) provide a convenient procedure for multivariate multiple comparisons. There is a need for some convenient way to perform multivariate multiple comparisons because in many areas of experimentation, including clinical trials, relevant distinctions between (e.g.) treatments may depend upon a set of measurements considered together. The purposes of this paper are to describe how SAS may be used to perform the calculations for multivariate multiple comparisons among the levels of the main effects in a factorial-type design, to provide a subroutine using the MATRIX procedure for performing these calculations, and to illustrate them with data from a clinical trial of an analgesic.

Confidence region tests for multivariate problems were introduced by Aitchison [1, 2]. Gabriel described a systematic theory of simultaneous test procedures [1] and showed how they could be used to develop multivariate multiple comparison procedures for one-way designs and the general multivariate linear hypothesis [5]. Mudholkar et al. [11] extended Gabriel's procedure to cover hypotheses not falling within the class of general hypotheses defined by Gabriel. However, Gabriel's procedure does include the sorts of hypotheses considered here.

Gabriel's Procedure

The basic model assumed is

$$\mathbf{Y} = \mathbf{X}\mathbf{B} + \mathbf{E}$$  \hspace{1cm} (1)

where \(\mathbf{Y}\) denotes an \(n \times p\) matrix whose rows constitute a sample of \(n\) realizations of a \(p\)-element variate, \(\mathbf{X}\) denotes a known \(n \times m\) design matrix, \(\mathbf{B}\) denotes an \(m \times p\) matrix of unknown parameters, and \(\mathbf{E}\) denotes an \(n \times p\) matrix of random errors satisfying

$$\mathbf{E} = \mathbf{G}$$

$$\mathbf{S} = \mathbf{E}^\prime \mathbf{E} = \begin{cases} \mathbf{I} & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

where \(\mathbf{E}\) denotes the \(i\)-th row of \(\mathbf{E}\). For testing purposes, the rows of \(\mathbf{E}\) are assumed to be independent realizations from a \(p\)-variate normal distribution with mean vector \(\mathbf{0}\) and unknown covariance matrix \(\mathbf{I}\).

Linear null hypotheses are taken to have the form

$$\mathbf{H}_i : \mathbf{g}_i' = \mathbf{C}_i' \mathbf{B}_i = \mathbf{0}$$  \hspace{1cm} (2)

where \(\mathbf{C}_i\) denotes an \(m \times v_i\) matrix of rank \(v_i\) and \(\mathbf{B}_i\) denotes a \(p \times p_i\) matrix of rank \(p_i\). The least squares estimator of \(\mathbf{g}_i\) is

$$\mathbf{g}_i = \mathbf{C}_i' \mathbf{B}_i$$

where \((\mathbf{\cdot})^\prime\) denotes a generalized inverse. The sum of squares and cross products (SSCP) matrix corresponding to the hypothesis \(\mathbf{H}_i\) is

$$\mathbf{S}_{\mathbf{H}_i} = \mathbf{g}_i' \mathbf{C}_i' \mathbf{C}_i \mathbf{C}_i' \mathbf{g}_i$$

Also \(\mathbf{B}_i\) is a nonsingular \(p_i \times p_i\) matrix, and the corresponding SSCP matrix for error for that hypothesis is

$$\mathbf{S}_{\mathbf{E}} = \mathbf{g}_i' \mathbf{C}_i' \left(\mathbf{C}_i' \mathbf{C}_i \mathbf{C}_i' - \mathbf{B}_i' \mathbf{B}_i\right) \mathbf{B}_i'$$

also \(p_i \times p_i\) and nonsingular. In practice, the model (1) usually will include all the relevant parameters; the hypothesis (2) amounts to a statement about the equality of some specified subset of these parameters.

Tests of (2) usually are based on some function of the roots of \(\mathbf{S}_{\mathbf{H}_i}^{-1} \mathbf{H}_i\), say \(\phi(\mathbf{S}_{\mathbf{H}_i}^{-1} \mathbf{H}_i)\). Gabriel prefers, for optimality reasons, Roy's maximum root statistic,

$$\phi(\mathbf{S}_{\mathbf{H}_i}^{-1} \mathbf{H}_i) = \begin{cases} \mathbf{H}_i \left(\mathbf{S}_{\mathbf{H}_i}^{-1} \mathbf{H}_i\right) \mathbf{H}_i & \text{maximum eigenvalue of } \mathbf{S}_{\mathbf{H}_i}^{-1} \mathbf{H}_i \end{cases}$$  \hspace{1cm} (3)

Whatever function of the roots is used, the computed value is compared to a specified critical value, say \(\phi_{0.05}\), and the corresponding hypothesis (2) is rejected if the computed value exceeds this critical value. The value of \(\phi_{0.05}\) depends upon the hypotheses considered in the simultaneous test procedure in the following way.

Let \(\mathbf{W}_i = (\mathbf{1} \mid i \in I)\)

denote a family of hypotheses indexed by \(I\) such that if \(\mathbf{W}_0\) is in \(\mathbf{W}_i\), then all other hypotheses \(\mathbf{W}_i\) in \(\mathbf{W}_0\) are implied by \(\mathbf{W}_0\) in the sense that if \(\mathbf{C}_0\) and \(\mathbf{B}_0\) are the matrices in (2) corresponding to \(\mathbf{W}_0\), then the columns of \(\mathbf{C}_0\) and \(\mathbf{B}_0\) for any other
hypothesis $w_i$ in $\Omega$ are linear combinations of the columns of $E_0$ and $Y_0$, respectively. Every hypothesis $w_i$ in $\Omega$ must be implied by $w_0$ in this sense, but $\Omega$ does not have to contain all the hypotheses implied by $w_0$. Then, $\psi_{oa}$ is defined by

$$\Pr(\psi^{-1} H_0 > \psi_{oa} \mid \psi_0) = \alpha.$$  

When $\psi$ is the maximum root statistic $\psi_{MR}$ (8), the critical values $\psi_{oa}$ may be obtained directly from Roy's charts [9]. If $p = 2, 3$, or 4, then the tables provided by Roy and Rees [5] and by Foster [3, 4] may be used. These tables give critical values for

$$\gamma = \frac{\psi}{1+\gamma},$$

so that if $\gamma_{oa}$ is read from these tables, then

$$\psi_{oa} = \frac{1}{\gamma_{oa}} + \psi_0.$$  

(4)

These tables $\gamma_i$ correspond to degrees of freedom associated with $E_0$ and $\gamma_2$ corresponds to the degrees of freedom associated with $Y_0$.

Multivariate simultaneous test procedures based on monotone increasing functions of the roots of $E_0 - H$ have a number of desirable properties; in particular they are coherent and (if $\psi = \psi_{MR}$) consonant:

Coherence: If $w_i$ and $w_j$ are hypotheses in $\Omega$ and $w_i$ implies $w_j$, then rejection of $w_j$ implies rejection of $w_i$ (or, equivalently, acceptance of $w_i$ implies acceptance of $w_j$);

Consonance: If $w_i$ and $w_j$ are hypotheses in $\Omega$ and $w_i$ implies $w_j$, then rejection of $w_i$ implies rejection of $w_j$.

Only the test based on $\psi_{MR}$ is consonant for a family $\Omega$ including $w_0$ and all possible single contrasts among the rows of $E$ and all possible linear combinations of the variables. (The "all" is important; if some of the contrasts implied by $w_0$ are not in $\Omega$, then apparent consonance can occur.)

Because of these properties, decisions from the multiple comparison process regarding the members of $\Omega$ can be summarized in either of two ways:

1. Minimal rejected hypotheses, consisting

of hypotheses $w_i$ such that if $w_i$ implies $w_j$, then $w_i$ is rejected, but $w_j$ is not rejected.

2. Maximal accepted hypotheses, consisting of hypotheses $w_j$ such that if $w_i$ implies $w_j$ and $w_j$ is not rejected, then $w_i$ is rejected.

Application

A clinical trial was performed comparing the effects of three dosages of a new analgesic agent with aspirin and placebo on post-epiophotomy pain. Each patient was given her assigned medication once upon demand following surgery, and was asked to evaluate the degree of pain (and of relief) initially and at hourly intervals following medication, for eight hours. Initial examination of the data [8] suggested that two derived statistics could summarize usefully the observed sequences of pain scores:

$$SPID = \sum_{t=1}^{T} (PID)_t,$$

$$DPID = (PID)_t + (PID)_{t-1} - (PID)_{t-2} - (PID)_t,$$

where $(PID)_t$ denotes the difference between the pain score at hour $t$ and the initial (pretreatment) score.

One hundred fifty-nine patients entered the trial, of whom 154 provided data through hour 6. The computations for the multivariate multiple comparison method will be illustrated by application to the SPID and DPID scores at hour 6, i.e.,

$$X_i = (SPID_6, DPID_6), i = 1, \ldots, 154.$$  

A two-way factorial model was used for the analysis, the factors being treatment and pretreatment score (i.e., whether the initial pain was moderate or severe). The model also included the treatment x pretreatment score interaction term.

The processing of the data proceeded through four phases:

1. Input of the data and preliminary processing to compute the SPID and DPID statistics.

2. Set-up of the datasets called by the multiple comparison procedure, namely NVALS, the matrix of values of the independent variables defining the main effects; YVALS, the matrix of dependent variable values; VINC, containing the maximum level of interaction to incorporate into the model (1) (as the first observation), and the subsets of the vector of response variables (elements of $Y_v$) for which the calculations are to be done (the routine always does the calculations for the complete
vectors $Y_i$, so they don't have to be specified; and NAME, containing as successive observations the names of the main effects in eight characters or less.  

3. Execution of the MANOVA option of GLM on the data to provide a check on the subroutine calculations and also to provide a test for the significance of the interaction (if the interaction is significant, the comparisons among the main effect levels may be meaningless).

4. Execution of the multivariate multiple comparison routine.

If a given main effect has $m$ levels, then the multiple comparison routine tests for equality of all the levels, then each pair of levels, then each triple of levels, etc., for all the elements of $Y_i$ and for the various subsets of the elements of $Y_i$ specified in VINC (which is referred to in the subroutine as the matrix VARINC).

Phase 1 was entirely dependent upon the dataset being analyzed. Its details are immaterial, and it is sufficient to say that it produced a dataset called ORIG whose variables for each observation included $T$ (the treatment group corresponding to the observation), PRE (the corresponding pretreatment score), and $SPD_6$ and $DPD_6$ (the observed values of the dependent variable).

For Phase 2, the following steps generated XVALS:

```
DATA XVALS; SET ORIG;
KEEP T PRE;
IF SP6 = 1 AND DP6 = 1 THEN OUTPUT;
RETURN;
```

the following steps generated YVALS:

```
DATA YVALS; SET ORIG;
KEEP SP6 DP6;
IF SP6 = 1 AND DP6 = 1 THEN OUTPUT;
RETURN;
```

the following steps generated VINC:

```
DATA VINC;
INPUT V1 V2; CARDS
2
1
2 ;
```

Note that in this case at most two-factor interaction terms were to be included in the model, and the multiple comparison calculations were to be carried out on $SPD_6$ and $DPD_6$ separately as well as on these two variables together. The following steps generated NAME:

```
DATA NAME;
INPUT NAME $; CARDS;
```

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta_{0.05}$</th>
<th>$\psi_{0.05}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>0.085</td>
<td>0.093</td>
</tr>
<tr>
<td>.10</td>
<td>0.073</td>
<td>0.079</td>
</tr>
<tr>
<td>.15</td>
<td>0.066</td>
<td>0.071</td>
</tr>
</tbody>
</table>

In this, as in most multiple-comparison testing situations, there is some argument as to the appropriate value of $\alpha$ to choose. The $\alpha$ chosen will reflect the experiment-wise error rate, i.e., the probability of falsely rejecting any of the null hypotheses of which, in this case, there are at least 75 (25 combinations of treatment effects x three choices of sets of variables). However, the comparison-wise error rate, i.e., the probability of falsely rejecting a particular null hypothesis will be much less. This problem has been considered extensively in the literature (see [10] for references), with no clear-cut solution. We adopt here the compromise of setting the experiment-wise error rate fairly high (say $\alpha = 0.10$ or 0.15), accepting the increased risk of falsely rejecting at least one of the null hypotheses implied by the multiple comparisons in order to have some reasonable chance of correctly rejecting some, and rely upon interpretation of the observed pattern of rejections to reach a proper conclusion. As things turn out, however, the same conclusion results whether one chooses $\alpha = 0.05$ or $\alpha = 0.10$: Treatment groups 3 (high dosage of new analgesic) and 4 (aspirin) differed significantly from treatment group 5 (placebo), but not from each other or from the
other two dosages (low and mid) of the new analgesic. Moreover, the differences were due essentially to differences with respect to the SPID₆ score, and not to the DPID₆ score. However, jointly considering the SPID₆ and DPID₆ scores did increase the sensitivity of the test somewhat in that the test statistic values always were higher when the two variables were considered together than when SPID₆ was considered alone.

Concluding Remarks

The subroutine listed in Table 1 should suffice for many of the experimental designs used in practice. In a randomized block situation, for example, given reason to accept the absence of a block x treatment interaction, multiple comparisons among the treatment levels would be accomplished by specifying the interaction level to be 1 (i.e. fitting a model containing only main effects). However, there are designs for which the subroutine would be inappropriate as it stands. For such designs, variations of the routine could be written.

Also, to encompass a wider class of hypotheses than those of the form (2), the subroutine would have to be modified. Mudholkar et al. [11] consider hypotheses of the form

\[ \mathbf{X}' \mathbf{y}(\theta) = 0, \]

where \( \mathbf{X} \) is a \( p \times m \) matrix and \( \mathbf{y}(\theta) \) is the vector of elements of \( \theta \), i.e.

\[ \mathbf{y}(\theta) = \left( \theta_1 \theta_2 \cdots \theta_p \theta_1 \theta_2 \cdots \theta_p \cdots \right), \]

and show that for this class of hypotheses, there is no uniformly "best" test statistic corresponding to the maximum root statistic for hypotheses of the form (2). Hypotheses of the form (5) could be accommodated within the logical flow of the routine, but at the cost of more complicated input requirements.

References

LISTING OF MVMULC, THE SUBROUTINE THAT PERFORMS THE CALCULATIONS FOR MULTIVARIATE MULTIPLE COMPARISONS (CONTINUED ON THE NEXT PAGE)

MACRO MVMULC OPTIONS NOCENTER; PROC MATRIX;

THIS SUBROUTINE REQUIRE THAT FOUR DATASETS HAVE BEEN DEFINED PRIOR TO BEING INVOKED. THESE DATASETS ARE XVALS YVALS VINC FNAME. THEY ARE ASSUMED TO CONTAIN THE FOLLOWING INFORMATION:

EACH COLUMN OF XVALS CORRESPONDS TO A MAIN EFFECT AND CONTAINS THE VALUES FOR THAT MAIN EFFECT ASSOCIATED WITH THE OBSERVATIONS. THE ROWS OF YVALS CONTAIN THE VECTORS OF RESPONSES (DEPENDENT VARIABLES) CORRESPONDING TO THE OBSERVATIONS (SAMPLE MEMBERS).

VINC SPECIFIES THE SUBSETS OF THE RESPONSES TO BE INCLUDED IN THE COMPUTATIONS OF THE MULTIPLE COMPARISONS. THE SUBSET CONSISTING OF ALL THE RESPONSES DOES NOT HAVE TO BE INCLUDED IN VINC BECAUSE THE ROUTINE WILL PERFORM THE CALCULATIONS FOR THIS SUBSET AUTOMATICALLY.

THE FIRST ROW OF VINC MUST CONTAIN ONLY THE LEVEL OF INTERACTION TO BE INCLUDED IN THE MODEL (THAT IS, THE HIGHEST LEVEL), PLUS MISSING VALUE SYMBOLS TO FILL OUT THE ROW. THE REMAINING ROWS OF VINC CONTAIN THE INFORMATION DESCRIBED ABOVE.

FRAME IS A CHARACTER DATASET CONTAINING ONE VARIABLE PER ROW. THE FIRST ROW CONTAINS THE NAME OF THE FIRST MAIN EFFECT, THE SECOND ROW OF FRAME CONSISTS OF NO MORE THAN 8 CHARACTERS.

GENERATE THE DESIGN MATRIX

STEP 1: SET UP THE MAIN EFFECT COLUMNS AND INDICES

STEP 2: RUN THROUGH THE LOOPS TO GENERATE THE INTERACTION

COMPUTATIONS OF THE MULTIPLE COMPARISONS.

LEVEL OF INTERACTION MUST BE SPECIFIED ON INPUT.

IF V IS A MATRIX WHOSE ROWS ARE THE SET OF ALL DISTINCT ORDERED COMBINATIONS OF K OF THE INTEGERS FROM 1 TO L, THEN PLETS TRANSFORMS V TO A MATRIX WHOSE ROWS ARE THE SET OF ALL DISTINCT ORDERED COMBINATIONS OF K+1 OF THE INTEGERS FROM I TO L. V MUST BE SUPPLIED EXPLICITLY -- K IS DEFINED IMPLICITLY BY K = NCOL(V). V IS RETURNED UNCHANGED IF K>L.

IF V IS A MATRIX WHOSE ROWS ARE THE SET OF ALL DISTINCT ORDERED COMBINATIONS OF K OF THE INTEGERS FROM 1 TO L, THEN PLETS TRANSFORMS V TO A MATRIX WHOSE ROWS ARE THE SET OF ALL DISTINCT ORDERED COMBINATIONS OF K+1 OF THE INTEGERS FROM I TO L. V MUST BE SUPPLIED EXPLICITLY -- K IS DEFINED IMPLICITLY BY K = NCOL(V). V IS RETURNED UNCHANGED IF K>L.
NOW COMPUTE THE MATRIX OF PARAMETER ESTIMATES AND OVERALL ERROR

COVARIANCE MATRIX FROM MANOVA

\[ \text{EMAT} = V'V; \]

START OF COMPUTATIONS FOR MULTIPLE COMPARISONS AMONG THE LEVELS OF EACH MAIN EFFECT. THE VARIABLES WITH RESPECT TO WHICH THE MULTIPLE COMPARISONS COMPUTATIONS ARE TO BE DONE ARE IN THE MATRIX VARINC.

SET UP THE HYPOTHESIS MATRICES \( C \) EXPLAINING EQUALITY OF FACTOR LEVELS SPECIFIED IN FHYP

SET UP THE HYPOTHESIS MATRIX \( D \) SPECIFYING THE SUBSET OF VARIABLES ON WHICH THE CALCULATIONS ARE TO BE DONE. THESE MUST BE SPECIFIED AS THE ROWS OF THE MATRIX VARINC.

STORE THE COMPUTED ROOT STATISTIC FOR THIS \( C \) MATRIX, THE NEXT \( D \) MATRIX IF THERE IS ONE, OR STORE THE CURRENT HYPOTHESIS MATRIX \( C \).

PRINT THE RESULTS.

PRINT PAGE MULTIVARIATE MULTIPLE COMPARISON RESULTS;

PRINT FACTOR FORMAT=$8. ROWNAME=RR COLNAME=RR;

PRINT LEVELS ROWNAME=CC COLNAME=CC;

PRINT TESTSTATS FORMAT=8.4;

NOTE "(1) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

NOTE "(2) THE NEXT, (L+1)ST COLUMN CONTAINS THE TEST CRITERION VALUES FOR EACH LEVEL";

NOTE "(3) THE REMAINING COLUMNS CONTAIN THE TEST CRITERION VALUES FOR EACH HYPOTHESIS WHEN THE VARIABLES IN THE SETS LISTED BELOW ARE INCLUDED";

NOTE "(4) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

PRINT LEVELS ROWNAME=CC COLNAME=CC;

PRINT TESTSTATS FORMAT=8.4;

NOTE "(1) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

NOTE "(2) THE NEXT, (L+1)ST COLUMN CONTAINS THE TEST CRITERION VALUES FOR EACH LEVEL";

NOTE "(3) THE REMAINING COLUMNS CONTAIN THE TEST CRITERION VALUES FOR EACH HYPOTHESIS WHEN THE VARIABLES IN THE SETS LISTED BELOW ARE INCLUDED";

NOTE "(4) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

PRINT LEVELS ROWNAME=CC COLNAME=CC;

PRINT TESTSTATS FORMAT=8.4;

NOTE "(1) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

NOTE "(2) THE NEXT, (L+1)ST COLUMN CONTAINS THE TEST CRITERION VALUES FOR EACH LEVEL";

NOTE "(3) THE REMAINING COLUMNS CONTAIN THE TEST CRITERION VALUES FOR EACH HYPOTHESIS WHEN THE VARIABLES IN THE SETS LISTED BELOW ARE INCLUDED";

NOTE "(4) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

PRINT LEVELS ROWNAME=CC COLNAME=CC;

PRINT TESTSTATS FORMAT=8.4;

NOTE "(1) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

NOTE "(2) THE NEXT, (L+1)ST COLUMN CONTAINS THE TEST CRITERION VALUES FOR EACH LEVEL";

NOTE "(3) THE REMAINING COLUMNS CONTAIN THE TEST CRITERION VALUES FOR EACH HYPOTHESIS WHEN THE VARIABLES IN THE SETS LISTED BELOW ARE INCLUDED";

NOTE "(4) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

PRINT LEVELS ROWNAME=CC COLNAME=CC;

PRINT TESTSTATS FORMAT=8.4;

NOTE "(1) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

NOTE "(2) THE NEXT, (L+1)ST COLUMN CONTAINS THE TEST CRITERION VALUES FOR EACH LEVEL";

NOTE "(3) THE REMAINING COLUMNS CONTAIN THE TEST CRITERION VALUES FOR EACH HYPOTHESIS WHEN THE VARIABLES IN THE SETS LISTED BELOW ARE INCLUDED";

NOTE "(4) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

PRINT LEVELS ROWNAME=CC COLNAME=CC;

PRINT TESTSTATS FORMAT=8.4;

NOTE "(1) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

NOTE "(2) THE NEXT, (L+1)ST COLUMN CONTAINS THE TEST CRITERION VALUES FOR EACH LEVEL";

NOTE "(3) THE REMAINING COLUMNS CONTAIN THE TEST CRITERION VALUES FOR EACH HYPOTHESIS WHEN THE VARIABLES IN THE SETS LISTED BELOW ARE INCLUDED";

NOTE "(4) COL TO COL(L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS";

PRINT LEVELS ROWNAME=CC COLNAME=CC;

PRINT TESTSTATS FORMAT=8.4;
TABLE 2


MULTIVARIATE MULTIPLE COMPARISON RESULTS

<table>
<thead>
<tr>
<th>LEVELS</th>
<th>LEVELS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROW 1</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 2</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 3</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 4</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 5</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 6</td>
<td>. .</td>
</tr>
<tr>
<td>ROW 7</td>
<td>. .</td>
</tr>
<tr>
<td>ROW 8</td>
<td>. .</td>
</tr>
<tr>
<td>ROW 9</td>
<td>. .</td>
</tr>
<tr>
<td>ROW 10</td>
<td>. .</td>
</tr>
<tr>
<td>ROW 11</td>
<td>. .</td>
</tr>
<tr>
<td>ROW 12</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 13</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 14</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 15</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 16</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 17</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 18</td>
<td>. .</td>
</tr>
<tr>
<td>ROW 19</td>
<td>. .</td>
</tr>
<tr>
<td>ROW 20</td>
<td>. .</td>
</tr>
<tr>
<td>ROW 21</td>
<td>. .</td>
</tr>
<tr>
<td>ROW 22</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 23</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 24</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 25</td>
<td>1.0000</td>
</tr>
<tr>
<td>ROW 26</td>
<td>. .</td>
</tr>
</tbody>
</table>

NOTES:
(1) COL 1 TO COL (L=LEVELS) SPECIFY THE FACTOR LEVELS ASSUMED EQUAL BY THE HYPOTHESIS
(2) THE NEXT, (L+1)-TH COLUMN CONTAINS THE TEST CRITERION VALUES FOR EACH HYPOTHESIS WHEN ALL THE VARIABLES ARE INCLUDED
(3) THE REMAINING COLUMNS CONTAIN THE TEST CRITERION VALUES FOR EACH HYPOTHESIS WHEN THE VARIABLES IN THE SETS LISTED BELOW ARE INCLUDED

VARINC | COL1 | COL2 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ROW1</td>
<td>1</td>
<td>.</td>
</tr>
<tr>
<td>ROW2</td>
<td>2</td>
<td>.</td>
</tr>
</tbody>
</table>