SAS/STAT® 9.3 User's Guide

The DISTANCE Procedure

(Chapter)

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Chapter 33

The DISTANCE Procedure

Contents

Overview: DISTANCE Procedure

The DISTANCE procedure computes various measures of distance, dissimilarity, or similarity between the observations (rows) of an input SAS data set, which can contain numeric or character variables, or both, depending on which proximity measure is used.

The proximity measures are stored as a lower triangular matrix or a square matrix (depending on the SHAPE= option) in an output data set that can then be used as input to the CLUSTER, MDS, and MODECLUS procedures.

The number of rows and columns in the output data set equals the number of observations in the input data set. If the input data set contains BY groups, an output matrix is computed for each BY group with the size determined by the maximum number of observations in any BY group.

The output data set is of type TYPE=DISTANCE or TYPE=SIMILAR, depending on the value of the METHOD= option. See the METHOD= option for more information about the association between the method and the output data set type.

Data set types do not persist when you copy or modify a data set. You must specify the TYPE= data set option for the new data set, as in the following example:

```
data dist2(type=distance);
   set dist;
run;
```

See the OUT= option for more information about data set type persistence.

PROC DISTANCE also provides various nonparametric and parametric methods for standardizing variables. Different variables can be standardized with different methods.

Distance matrices are used frequently in data mining, genomics, marketing, financial analysis, management science, education, chemistry, psychology, biology, and various other fields.

Levels of Measurement

Measurement of some attribute of a set of objects is the process of assigning numbers or other symbols to the objects in such a way that properties of the numbers or symbols reflect properties of the attribute being measured. There are different *levels* of measurement that involve different properties (relations and operations) of the numbers or symbols. Associated with each level of measurement is a set of transformations of the measurements that preserve the relevant properties; these transformations are called *permissible* transformations. A particular way of assigning numbers or symbols to measure something is called a *scale* of measurement.

The most commonly discussed levels of measurement are as follows:

Nominal Two objects are assigned the same symbol if they have the same value of the attribute. Permissible transformations are any one-to-one or many-to-one transformation, although a many-to-one transformation loses information.

Ordinal Objects are assigned numbers such that the order of the numbers reflects an order relation defined on the attribute. Two objects x and y with attribute values $a(x)$ and $a(y)$ are assigned numbers $m(x)$ and $m(y)$ such that if $m(x) > m(y)$, then $a(x) > a(y)$. Permissible transformations are any monotone increasing transformation, although a transformation that is not strictly increasing loses information.

Interval Objects are assigned numbers such that differences between the numbers reflect differences of the attribute. If $m(x) - m(y) > m(u) - m(v)$, then $a(x) - a(y) > a(u) - a(v)$. Permissible transformations are any affine transformation $t(m) = c * m + d$, where c

and d are constants; another way of saying this is that the origin and unit of measurement are arbitrary.

Log-interval Objects are assigned numbers such that ratios between the numbers reflect ratios of the attribute. If $m(x)/m(y) > m(u)/m(v)$, then $a(x)/a(y) > a(u)/a(v)$. Permissible transformations are any power transformation $t(m) = c * m^d$, where c and d are constants.

Ratio Objects are assigned numbers such that differences and ratios between the numbers reflect differences and ratios of the attribute. Permissible transformations are any linear (similarity) transformation $t(m) = c * m$, where c is a constant; another way of saying this is that the unit of measurement is arbitrary.

Absolute Objects are assigned numbers such that all properties of the numbers reflect analogous properties of the attribute. The only permissible transformation is the identity transformation.

Proximity measures provided in the DISTANCE procedure accept four levels of measurement: nominal, ordinal, interval, and ratio. Ordinal variables are transformed to interval variables before processing. This is done by replacing the data with their rank scores, and by assuming that the classes of an ordinal variable are spaced equally along the interval scale. See the RANKSCORE= option in the section "PROC DISTANCE Statement" on page 2064 for choices on assigning scores to ordinal variables. There are also different approaches for how to transform an ordinal variable to an interval variable. See Anderberg (1973) for alternatives.

Symmetric versus Asymmetric Nominal Variables

A binary variable contains two possible outcomes: 1 (positive/present) or 0 (negative/absent). If there is no preference for which outcome should be coded as 0 and which as 1, the binary variable is called *symmetric*. For example, the binary variable "is evergreen?" for a plant has the possible states "loses leaves in winter" and "does not lose leaves in winter." Both are equally valuable and carry the same weight when a proximity measure is computed. Commonly used measures that accept symmetric binary variables include the Simple Matching, Hamann, Roger and Tanimoto, Sokal and Sneath 1, and Sokal and Sneath 3 coefficients.

If the outcomes of a binary variable are not equally important, the binary variable is called *asymmetric*. An example of such a variable is the presence or absence of a relatively rare attribute, such as "is color-blind" for a human being. While you say that two people who are color-blind have something in common, you cannot say that people who are not color-blind have something in common. The most important outcome is usually coded as 1 (present) and the other is coded as 0 (absent). The agreement of two 1's (a present-present match or a positive match) is more significant than the agreement of two 0's (an absent-absent match or a negative match). Usually, the negative match is treated as irrelevant. Commonly used measures that accept asymmetric binary variables include Jaccard, Dice, Russell and Rao, Binary Lance and Williams nonmetric, and Kulcynski coefficients.

When nominal variables are employed, the comparison of one data unit with another can only be in terms of whether the data units score the same or different on the variables. If a variable is defined as an asymmetric nominal variable and two data units score the same but fall into the absent category, the absent-absent match is excluded from the computation of the proximity measure.

Standardization

Since variables with large variances tend to have more effect on the proximity measure than those with small variances, it is recommended that you standardize the variables before the computation of the proximity measure. The DISTANCE procedure provides a convenient way to standardize each variable with its own method before the proximity measures are computed. You can also perform the standardization by using the STDIZE procedure, with the limitation that all variables must be standardized with the same method.

Mandatory Standardization

Variable standardization is not required if any of the following conditions is true:

- if there is only one level of measurement
- if only asymmetric nominal and nominal levels are specified
- if the NOSTD option is specified in the PROC DISTANCE statement

Otherwise, standardization is mandatory.

When standardization is mandatory and no standardization method is specified, a default method of standardization will be used. This default method is determined by the measurement level. In general, the default method is STD for interval variables and is MAXABS for ratio variables except when METHOD=GOWER or METHOD=DGOWER is specified. See the STD= option in the section "VAR Statement" on page 2071 for the default methods for GOWER and DGOWER as well as methods available for standardizing variables.

When standardization is mandatory, PROC DISTANCE ignores the REPONLY option, if it is specified.

Getting Started: DISTANCE Procedure

Creating a Distance Matrix as Input for a Subsequent Cluster Analysis

The following example demonstrates how you can use the DISTANCE procedure to obtain a distance matrix that will be used as input to a subsequent clustering procedure.

The following data, originated by A. Weber and cited in Hand et al. (1994, p. 297), measure the amount of protein consumed for nine food groups in 25 European countries. The nine food groups are red meat (RedMeat), white meat (WhiteMeat), eggs (Eggs), milk (Milk), fish (Fish), cereal (Cereal), starch (Starch), nuts (Nuts), and fruits and vegetables (FruitVeg). Suppose you want to determine whether national figures in protein consumption can be used to determine certain types or categories of countries; specifically, you want to perform a cluster analysis to determine whether these 25 countries can be formed into groups suggested by the data.

The following DATA step creates the SAS data set Protein:

```
data Protein;
   input Country $14. RedMeat WhiteMeat Eggs Milk
                      Fish Cereal Starch Nuts FruitVeg;
   datalines;
Albania        10.1  1.4  0.5   8.9  0.2  42.3  0.6  5.5  1.7
Austria         8.9 14.0  4.3  19.9  2.1  28.0  3.6  1.3  4.3
Belgium        13.5  9.3  4.1  17.5  4.5  26.6  5.7  2.1  4.0
Bulgaria        7.8  6.0  1.6   8.3  1.2  56.7  1.1  3.7  4.2
Czechoslovakia  9.7 11.4  2.8  12.5  2.0  34.3  5.0  1.1  4.0
Denmark        10.6 10.8  3.7  25.0  9.9  21.9  4.8  0.7  2.4
E Germany       8.4 11.6  3.7  11.1  5.4  24.6  6.5  0.8  3.6
Finland         9.5  4.9  2.7  33.7  5.8  26.3  5.1  1.0  1.4
France         18.0  9.9  3.3  19.5  5.7  28.1  4.8  2.4  6.5
Greece         10.2  3.0  2.8  17.6  5.9  41.7  2.2  7.8  6.5
Hungary         5.3 12.4  2.9   9.7  0.3  40.1  4.0  5.4  4.2
Ireland        13.9 10.0  4.7  25.8  2.2  24.0  6.2  1.6  2.9
Italy           9.0  5.1  2.9  13.7  3.4  36.8  2.1  4.3  6.7
Netherlands     9.5 13.6  3.6  23.4  2.5  22.4  4.2  1.8  3.7
Norway          9.4  4.7  2.7  23.3  9.7  23.0  4.6  1.6  2.7
Poland          6.9 10.2  2.7  19.3  3.0  36.1  5.9  2.0  6.6
Portugal        6.2  3.7  1.1   4.9 14.2  27.0  5.9  4.7  7.9
Romania         6.2  6.3  1.5  11.1  1.0  49.6  3.1  5.3  2.8
Spain           7.1  3.4  3.1   8.6  7.0  29.2  5.7  5.9  7.2
Sweden          9.9  7.8  3.5   4.7  7.5  19.5  3.7  1.4  2.0
Switzerland    13.1 10.1  3.1  23.8  2.3  25.6  2.8  2.4  4.9
UK             17.4  5.7  4.7  20.6  4.3  24.3  4.7  3.4  3.3
USSR            9.3  4.6  2.1  16.6  3.0  43.6  6.4  3.4  2.9
W Germany      11.4 12.5  4.1  18.8  3.4  18.6  5.2  1.5  3.8
Yugoslavia      4.4  5.0  1.2   9.5  0.6  55.9  3.0  5.7  3.2
;
```

The data set Protein contains the character variable Country and the nine numeric variables representing the food groups. The $14. in the INPUT statement specifies that the variable Country has a length of 14.

The following statements create the distance matrix and display part of it:

```
title 'Protein Consumption in Europe';
proc distance data=Protein out=Dist method=Euclid;
   var interval(RedMeat--FruitVeg / std=Std);
   id Country;
run;

proc print data=Dist(obs=10);
   title2 'First 10 Observations in Output Data Set from PROC DISTANCE';
run;
title2;
```

An output SAS data set called Dist, which contains the distance matrix, is created through the OUT= option. The METHOD=EUCLID option requests that Euclidean distances (which is the default) should be computed and produces an output data set of TYPE=DISTANCE.[1]

The VAR statement lists the variables (RedMeat—FruitVeg) along with their measurement level to be used in the analysis. An interval level of measurement is assigned to those variables. Since variables with large variances tend to have more effect on the proximity measure than those with small variances, each variable is standardized by the STD method to have a mean of 0 and a standard deviation of 1. This is done by adding "/ STD=STD" at the end of the variables list.

The ID statement specifies that the variable Country should be copied to the OUT= data set and used to generate names for the distance variables. The distance variables in the output data set are named by the values in the ID variable, and the maximum length for the names of these variables is 14.

There are 25 observations in the input data set; therefore, the output data set Dist contains a 25-by-25 lower triangular matrix.

The PROC PRINT statement displays the first 10 observations in the output data set Dist as shown in Figure 33.1.

[1] Data set types do not persist when you copy or modify a data set. You must specify the TYPE= data set option for the new data set. See the METHOD= and OUT= options for more information about data set types.

Figure 33.1 First 10 Observations in the Output Data Set from PROC DISTANCE

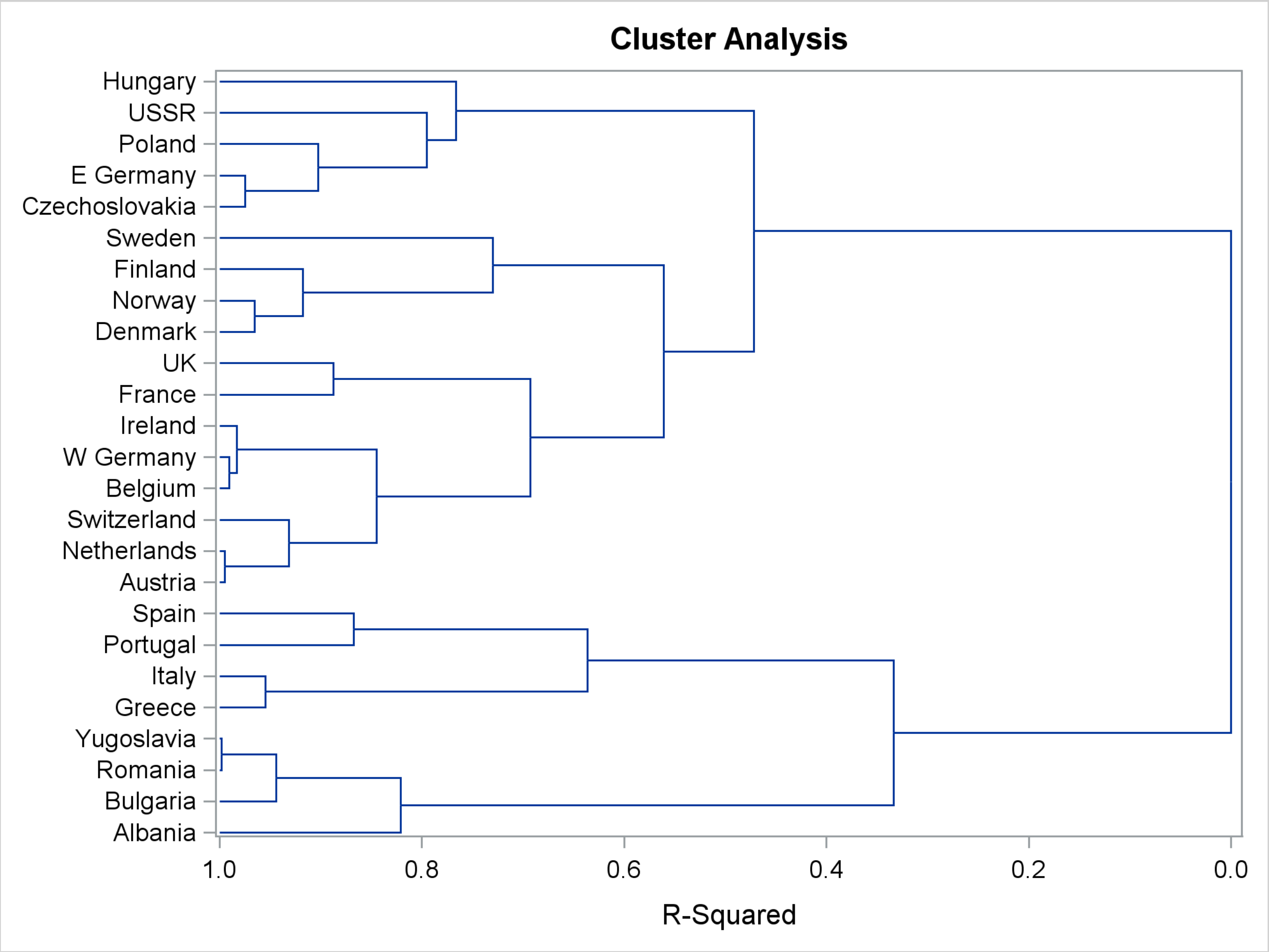
```
                        Protein Consumption in Europe
         First 10 Observations in Output Data Set from PROC DISTANCE

                                                               C
                                                               z
                                                               e
                                                               c
                                                               h
                                                               o
                                                     B         s
         C             A         A         B         u         l         D
         o             l         u         e         l         o         e
         u             b         s         l         g         v         n
         n             a         t         g         a         a         m
 O       t             n         r         i         r         k         a
 b       r             i         i         u         i         i         r
 s       y             a         a         m         a         a         k

 1 Albania          0.00000    .         .         .         .         .
 2 Austria          6.12388   0.00000    .         .         .         .
 3 Belgium          5.94109   2.44987   0.00000    .         .         .
 4 Bulgaria         2.76446   4.88331   5.22711   0.00000    .         .
 5 Czechoslovakia   5.13959   2.11498   2.21330   3.94761   0.00000    .
 6 Denmark          6.61002   3.01392   2.52541   6.00803   3.34049   0.00000
 7 E Germany        6.39178   2.56341   2.10211   5.40824   1.87962   2.72112
 8 Finland          5.81458   4.04271   3.45779   5.74882   3.91378   2.61570
 9 France           6.29601   3.58891   2.19329   5.54675   3.36011   3.65772
10 Greece           4.24495   5.16330   4.69515   3.74849   4.86684   5.59084




                                          N                    S
                                          e                    w           Y
      E                                   t                    i        W  u
      _                                   h        P           t        _  g
      G        F                 H  I     e        o  R        z        G  o
      e        i        F     G  u  r     r  N  P  r  o     S  e        e  s
      r        n        r     r  n  e  I  l  o  o  t  m  S  w  r        r  l
      m        l        a     e  g  l  t  a  r  l  u  a  p  e  l     U  m  a
 O    a        a        n     e  a  a  a  n  w  a  g  n  a  d  a     S  a  v
 b    n        n        c     c  r  n  l  d  a  n  a  i  i  e  n  U  S  n  i
 s    y        d        e     e  y  d  y  s  y  d  l  a  n  n  d  K  R  y  a

 1  .        .        .       .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 2  .        .        .       .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 3  .        .        .       .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 4  .        .        .       .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 5  .        .        .       .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 6  .        .        .       .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 7 0.00000   .        .       .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 8 3.99426  0.00000   .       .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
 9 3.78184  4.56796  0.00000  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
10 5.61496  5.47453  4.54456  0  .  .  .  .  .  .  .  .  .  .  .  .  .  .  .
```

The following statements produce the dendrogram in Figure 33.2:

```
ods graphics on;

proc cluster data=Dist method=Ward plots=dendrogram(height=rsq);
   id Country;
run;
```

The CLUSTER procedure performs a Ward's minimum-variance cluster analysis based on the distance matrix created by PROC DISTANCE. PROC CLUSTER, along with ODS Graphics, produces the dendrogram shown in Figure 33.2. The option PLOTS=DENDROGRAM(HEIGHT=RSQ) specifies the squared multiple correlation as the height variable in the dendrogram.

Figure 33.2 Dendrogram of R Squared



After inspecting the dendrogram in Figure 33.2, you will see that when the countries are grouped into six clusters, the proportion of variance accounted for by these clusters is slightly less than 70% (69.3%). The 25 countries are clustered as follows:

- Balkan countries: Albania, Bulgaria, Romania, and Yugoslavia
- Mediterranean countries: Greece and Italy
- Iberian countries: Portugal and Spain
- Western European countries: Austria, Netherlands, Switzerland, Belgium, former West Germany, Ireland, France, and U.K.
- Scandinavian countries: Denmark, Norway, Finland, and Sweden
- Eastern European countries: former Czechoslovakia, former East Germany, Poland, former U.S.S.R., and Hungary

Syntax: DISTANCE Procedure

The following statements are available in the DISTANCE procedure:

```
PROC DISTANCE < options > ;
    BY variables ;
    COPY variables ;
    FREQ variable ;
    ID variable ;
    VAR level(variables < / opt-list >) ;
    WEIGHT variable ;
```

Both the PROC DISTANCE statement and the VAR statement are required.

PROC DISTANCE Statement

PROC DISTANCE < *options* > **;**

The options available with the PROC DISTANCE statement are summarized in Table 33.1 and discussed in the following section.

Table 33.1 Summary of PROC DISTANCE Statement Options

Option	Description
Standardize variables	
ADD=	Specifies the constant to add to each value after standardizing and multiplying by the value specified in the MULT= option
FUZZ=	Specifies the relative fuzz factor for writing the output
INITIAL=	Specifies the method for computing initial estimates for the A-estimates
MULT=	Specifies the constant to multiply each value by after standardizing
NORM	Normalizes the scale estimator to be consistent for the standard deviation of a normal distribution
NOSTD	Suppresses standardization
SNORM	Normalizes the scale estimator to have an expectation of approximately 1 for a standard normal distribution
STDONLY	Standardizes variables only (suppresses computation of the distance matrix)
VARDEF=	Specifies the variances divisor
Generate distance matrix	
ABSENT=	Specifies the value to be used as an absence value for all the asymmetric nominal variables
METHOD=	Specifies the method for computing proximity measures
PREFIX=	Specifies a prefix for naming the distance variables in the OUT= data set
RANKSCORE=	Specifies the method of assigning scores to ordinal variables
SHAPE=	Specifies the shape of the proximity matrix to be stored in the OUT= data set
UNDEF=	Specifies the numeric constant used to replace undefined distances
Replace missing values	
NOMISS	Omits observations with missing values from computation of the location and scale measures, if standardization applies; outputs missing values to the distance matrix for observations with missing values

Table 33.1 continued

Option	Description
REPLACE	Replaces missing data with zero in the standardized data
REPONLY	Replaces missing data with the location measure (does not standardize the data)
Specify data set details	
DATA=	Specifies the input data set
OUT=	Specifies the output data set
OUTSDZ=	Specifies the output data set for standardized scores

These options and their abbreviations are described (in alphabetical order) in the remainder of this section.

ABSENT=*number* | *qs*

specifies the value to be used as an absence value in an irrelevant absent-absent match for *all* of the asymmetric nominal variables. If you want to specify a different absence value for a particular variable, use the ABSENT= option in the VAR statement. See the ABSENT= option in the section "VAR Statement" on page 2071 for details.

An absence value for a variable can be either a numeric value or a quoted string consisting of combinations of characters. For instance, ., -999, and "NA" are legal values for the ABSENT= option.

The default absence value for a character variable is "NONE" (notice that a blank value is considered a missing value), and the default absence value for a numeric variable is 0.

ADD=*c*

specifies a constant, c, to add to each value after standardizing and multiplying by the value you specify in the MULT= option. The default value is 0.

DATA=*SAS-data-set*

specifies the input data set containing observations from which the proximity is computed. If you omit the DATA= option, the most recently created SAS data set is used.

FUZZ=*c*

specifies the relative fuzz factor for computing the standardized scores. The default value is 1E–14. For the OUTSDZ= data set, the score is computed as follows:

$$\text{if } |\text{standardized scores}| < m \times c, \text{ then standardized scores} = 0$$

where m is the numeric constant specified in the MULT= option, or 1 if MULT= option is not specified.

INITIAL=*method*

specifies the method of computing initial estimates for the A-estimates (ABW, AWAVE, and AHUBER). The following methods are not allowed for the INITIAL= option: ABW, AHUBER, AWAVE, and IN.

The default value is INITIAL=MAD.

METHOD=*method*

specifies the method of computing proximity measures.

For use in PROC CLUSTER, distance or dissimilarity measures such as METHOD=EUCLID or METHOD=DGOWER should be chosen.

The following six tables outline the proximity measures available for the METHOD= option. These tables are classified by levels of measurement accepted by each method. Each table contains four or five columns: the Method column shows the proximity measures, one or two Range columns show the upper and lower bounds, and the TYPE= column shows the type of proximity. The TYPE= column contains SIMILAR if a method generates similarity measures or DISTANCE if a method generates distance or dissimilarity measures. The output data set is of the type shown. For more information about the output data set, see the OUT= option.

For formulas and descriptions of these methods, see the section "Details: DISTANCE Procedure" on page 2078.

Table 33.2 shows the range and output matrix type of the GOWER and DGOWER methods. These two methods accept all measurement levels including ratio, interval, ordinal, nominal, and asymmetric nominal. METHOD=GOWER or METHOD=DGOWER always implies standardization. Assuming all the numeric (ordinal, interval, and ratio) variables are standardized by their corresponding default methods, the possible range values for both methods are from 0 and 1, inclusive. For more information about the default methods of standardization for METHOD=GOWER or METHOD=DGOWER, see the STD= option in the section "VAR Statement" on page 2071.

Table 33.2 Methods That Accept All Measurement Levels

Method	**Description**	**Range**	**TYPE=**
GOWER	Gower and Legendre (1986) similarity	0 to 1	SIMILAR
DGOWER	1 minus GOWER	0 to 1	DISTANCE

Table 33.3 shows methods that accept ratio, interval, and ordinal variables.

Table 33.3 Methods That Accept Ratio, Interval, and Ordinal Variables

Method	Description	Range	TYPE=
EUCLID	Euclidean distance	≥ 0	DISTANCE
SQEUCLID	Squared Euclidean distance	≥ 0	DISTANCE
SIZE	Size distance	≥ 0	DISTANCE
SHAPE	Shape distance	≥ 0	DISTANCE
COV	Covariance	≥ 0	SIMILAR
CORR	Correlation	–1 to 1	SIMILAR
DCORR	Correlation transformed to Euclidean distance	0 to 2	DISTANCE
SQCORR	Squared correlation	0 to 1	SIMILAR
DSQCORR	One minus squared correlation	0 to 1	DISTANCE
L(p)	Minkowski (L_p) distance, where p is a positive numeric value	≥ 0	DISTANCE
CITYBLOCK	L_1, city-block, or Manhattan distance	≥ 0	DISTANCE
CHEBYCHEV	L_∞	≥ 0	DISTANCE
POWER(p, r)	Generalized Euclidean distance where p is a positive numeric value and r is a nonnegative numeric value. The distance between two observations is the rth root of sum of the absolute differences to the pth power between the values for the observations.	≥ 0	DISTANCE

Table 33.4 shows methods that accept ratio variables. Notice that all possible range values are nonnegative, because ratio variables are assumed to be positive.

Table 33.4 Methods That Accept Ratio Variables

Method	Description	Range	TYPE=
SIMRATIO	Similarity ratio (if variables are binary, this is the Jaccard coefficient)	0 to 1	SIMILAR
DISRATIO	One minus similarity ratio	0 to 1	DISTANCE
NONMETRIC	Lance and Williams nonmetric coefficient	0 to 1	DISTANCE
CANBERRA	Canberra metric distance coefficient	0 to 1	DISTANCE
COSINE	Cosine coefficient	0 to 1	SIMILAR
DOT	Dot (inner) product coefficient	≥ 0	SIMILAR
OVERLAP	Overlap similarity	≥ 0	SIMILAR
DOVERLAP	Overlap dissimilarity	≥ 0	DISTANCE
CHISQ	Chi-squared coefficient	≥ 0	DISTANCE
CHI	Squared root of chi-squared coefficient	≥ 0	DISTANCE
PHISQ	Phi-squared coefficient	≥ 0	DISTANCE
PHI	Squared root of phi-squared coefficient	≥ 0	DISTANCE

Table 33.5 shows methods that accept nominal variables.

Table 33.5 Methods That Accept Nominal Variables

Method	Description	Range	TYPE=
HAMMING	Hamming distance	0 to v	DISTANCE
MATCH	Simple matching coefficient	0 to 1	SIMILAR
DMATCH	Simple matching coefficient transformed to Euclidean distance	0 to 1	DISTANCE
DSQMATCH	Simple matching coefficient transformed to squared Euclidean distance	0 to 1	DISTANCE
HAMANN	Hamann coefficient	–1 to 1	SIMILAR
RT	Roger and Tanimoto	0 to 1	SIMILAR
SS1	Sokal and Sneath 1	0 to 1	SIMILAR
SS3	Sokal and Sneath 3	0 to 1	SIMILAR

Note that v denotes the number of variables (dimensionality).

Table 33.6 shows methods that accept asymmetric nominal variables. Use the ABSENT= option to create a value to be considered absent.

Table 33.6 Methods That Accept Asymmetric Nominal Variables

Method	Description	Range	TYPE=
DICE	Dice coefficient or Czekanowski/Sorensen similarity coefficient	0 to 1	SIMILAR
RR	Russell and Rao	0 to 1	SIMILAR
BLWNM	Binary Lance and Williams nonmetric, or Bray-Curtis coefficient	0 to 1	DISTANCE
K1	Kulcynski 1	≥ 0	SIMILAR

Table 33.7 shows methods that accept asymmetric nominal and ratio variables. Use the ABSENT= option to create a value to be considered absent. The table contains five columns. The third column contains possible range values if only one level of measurement (either ratio or asymmetric nominal but not both) is specified; the fourth column contains possible range values if both levels are specified.

The JACCARD method is equivalent to the SIMRATIO method if there is no asymmetric nominal variable; if both ratio and asymmetric nominal variables are present, the coefficient is computed as the sum of the coefficient from the ratio variables and the coefficient from the asymmetric nominal variables. See “Proximity Measures” in the section “Details: DISTANCE Procedure” on page 2078 for the formula and descriptions of the JACCARD method.

Table 33.7 Methods That Accept Asymmetric Nominal and Ratio Variables

Method	Description	Range (One Level)	Range (Two Levels)	TYPE=
JACCARD	Jaccard similarity coefficient	0 to 1	0 to 2	SIMILAR
DJACCARD	Jaccard dissimilarity coefficient	0 to 1	0 to 2	DISTANCE

MULT=*c*

specifies a numeric constant, *c*, by which to multiply each value after standardizing. The default value is 1.

NOMISS

omits observations with missing values from computation of the location and scale measures when standardizing; generates undefined (missing) distances for observations with missing values when computing distances. Use the UNDEF= option to specify the undefined values.

If a distance matrix is created to be used as an input to PROC CLUSTER, the NOMISS option should not be used because PROC CLUSTER does not accept distance matrices with missing values.

NORM

normalizes the scale estimator to be consistent for the standard deviation of a normal distribution when you specify the option STD=AGK, STD=IQR, STD=MAD, or STD=SPACING in the VAR statement.

NOSTD

suppresses standardization of the variables. The NOSTD option should not be specified with the STDONLY option or with the REPLACE option.

OUT=*SAS-data-set*

specifies the name of the SAS data set created by PROC DISTANCE. The output data set contains the BY variables, the ID variable, computed distance variables, the COPY variables, the FREQ variable, and the WEIGHT variables.

If you omit the OUT= option, PROC DISTANCE creates an output data set named according to the DATA*n* convention.

The output data set is of type TYPE=DISTANCE or TYPE=SIMILAR. See the METHOD= option for more information about the association between the method and the output data set type. Data set types do not persist when you copy or modify a data set. You must specify the TYPE= data set option for the new data set, as in the following example:

```
data dist2(type=distance);
   set dist;
run;
```

If you do not specify the TYPE=DISTANCE data set option, the new data set is the default TYPE=DATA. If you use the new data set in a procedure that accepts both TYPE=DATA or TYPE=DISTANCE data sets (such as PROC CLUSTER or PROC MODECLUS), the results will be incorrect.

OUTSDZ=*SAS-data-set*

specifies the name of the SAS data set containing the standardized scores. The output data set contains a copy of the DATA= data set, except that the analyzed variables have been standardized. Analyzed variables are those listed in the VAR statement.

PREFIX=*name*

specifies a prefix for naming the distance variables in the OUT= data set. By default, the names are

Dist1, Dist2, ..., Dist*n*. If you specify PREFIX=ABC, the variables are named ABC1, ABC2, ..., ABCn. If the ID statement is also specified, the variables are named by appending the value of the ID variable to the prefix.

RANKSCORE=MIDRANK | INDEX

specifies the method of assigning scores to ordinal variables. The available methods are listed as follows:

MIDRANK	assigns consecutive integers to each category with consideration of the frequency value. This is the default method.
INDEX	assigns consecutive integers to each category regardless of frequencies.

The following example explains how each method assigns the rank scores. Suppose the data contain an ordinal variable ABC with values A, B, C. There are two ways to assign numbers. One is to use midranks, which depend on the frequencies of each category. Another is to assign consecutive integers to each category, regardless of frequencies.

Table 33.8 Example of Assigning Rank Scores

ABC	MIDRANK	INDEX
A	1.5	1
A	1.5	1
B	4	2
B	4	2
B	4	2
C	6	3

REPLACE

replaces missing data with zero in the standardized data (to correspond to the location measure before standardizing). To replace missing data with something else, use the MISSING= option in the VAR statement. The REPLACE option implies standardization.

You cannot specify the following options together:

- both the REPLACE and the REPONLY options
- both the REPLACE and the NOSTD options

REPONLY

replaces missing data with the location measure specified by the MISSING= option or the STD= option (if the MISSING= option is not specified), but does *not* standardize the data. If the MISSING= option is not specified and METHOD=GOWER is specified, missing values are replaced by the location measure from the RANGE method (the minimum value), no matter what the value of the STD= option is.

You cannot specify both the REPLACE and the REPONLY options.

SHAPE=TRIANGLE | TRI | SQUARE | SQU | SQR

specifies the shape of the proximity matrix to be stored in the OUT= data set. SHAPE=TRIANGLE requests the matrix to be stored as a lower triangular matrix; SHAPE=SQUARE requests that the matrix be stored as a squared matrix. Use SHAPE=SQUARE if the output data set is to be used as input to the MODECLUS procedures. The default is TRIANGLE.

SNORM

normalizes the scale estimator to have an expectation of approximately 1 for a standard normal distribution when the STD=SPACING option is specified.

STDONLY

standardizes variables only and computes no distance matrix. You must use the OUTSDZ= option to save the standardized scores. You cannot specify both the STDONLY option and the NOSTD option.

UNDEF=*n*

specifies the numeric constant used to replace undefined distances, such as when an observation has all missing values, or if a divisor is zero.

VARDEF=DF | N | WDF | WEIGHT | WGT

specifies the divisor to be used in the calculation of distance, dissimilarity, or similarity measures, and for standardizing variables whenever a variance or covariance is computed. By default, VARDEF=DF. The values and associated divisors are as follows:

Value	**Divisor**	**Formula**
DF	degrees of freedom	$n-1$
N	number of observations	n
WDF	sum of weights minus 1	$(\sum_i w_i) - 1$
WEIGHT | WGT	sum of weights	$\sum_i w_i$

VAR Statement

VAR *level (variables < / opt-list>)*
< level (variables < / opt-list>) . . . level (variables < / opt-list>) > **;**

where the syntax for the *opt-list* is as follows:

ABSENT=*value*
MISSING=*miss-method | value*
ORDER=*order-option*
STD=*std-method*
WEIGHTS=*weight-list*

The VAR statement lists variables from which distances are to be computed. The VAR statement is required. The variables can be numeric or character depending on their measurement levels. A variable cannot appear more than once in either the same list or a different list.

level is required. It declares the levels of measurement for those variables specified within the parentheses. Available values for *level* are as follows:

ANOMINAL	variables are asymmetric nominal and can be either numeric or character.
NOMINAL	variables are symmetric nominal and can be either numeric or character.

ORDINAL variables are ordinal and can be either numeric or character. Values of ordinal variables are replaced by their corresponding rank scores. If standardization is required, the standardized rank scores are output to the data set specified in the OUTSDZ= option. See the RANKSCORE= option in the PROC DISTANCE statement for methods available for assigning rank scores to ordinal variables. After being replaced by scores, ordinal variables are considered interval.

INTERVAL variables are interval and numeric.

RATIO variables are ratio and numeric. Ratio variables should always contain positive measurements.

Each variable list can be followed by an option list. Use "/ " after the list of variables to start the option list. An option list contains options that are applied to the variables. The following options are available in the option list:

ABSENT= specifies the value to be used as an absence value in an irrelevant absent-absent match for asymmetric nominal variables.

MISSING= specifies the method (or numeric value) with which to replace missing data.

ORDER= selects the order for assigning scores to ordinal variables.

STD= selects the standardization method.

WEIGHTS= assigns weights to the variables in the list.

If an option is missing from the current attribute list, PROC DISTANCE provides default values for all the variables in the current list.

For example, in the VAR statement

```
var ratio(x1-x4/std= mad weights= .5 .5 .1 .5 missing= -99)
    interval(x5/std= range)
    ordinal(x6/order= desc);
```

the first option list defines x1–x4 as ratio variables to be standardized by the MAD method. Also, any missing values in x1–x4 should be replaced by –99. x1 is given a weight of 0.5, x2 is given a weight of 0.5, x3 is given a weight of 0.1, and x4 is given a weight of 0.5.

The second option list defines x5 as an interval variable to be standardized by the RANGE method. If the REPLACE option is specified in the PROC DISTANCE statement, missing values in x5 are replaced by the location estimate from the RANGE method. By default, x5 is given a weight of 1.

The last option list defines x6 as an ordinal variable. The scores are assigned from highest to lowest by its unformatted values. Although the STD= option is not specified, x6 is standardized by the default method (STD) because there is more than one level of measurements (ratio, interval, and ordinal) in the VAR statement. Again, if the REPLACE option is specified, missing values in x6 are replaced by the location estimate from the STD method. Finally, by default, x6 is given a weight of 1.

More details for the options are explained as follows.

STD=*std-method*

specifies the standardization method. Valid values for *std-method* are MEAN, MEDIAN, SUM, EUCLEN, USTD, STD, RANGE, MIDRANGE, MAXABS, IQR, MAD, ABW, AHUBER, AWAVE, AGK, SPACING, and L. Table 33.9 lists available methods of standardization as well as their corresponding location and scale measures.

Table 33.9 Available Standardization Methods

Method	**Scale**	**Location**
MEAN	1	mean
MEDIAN	1	median
SUM	sum	0
EUCLEN	Euclidean length	0
USTD	standard deviation about origin	0
STD	standard deviation	mean
RANGE	range	minimum
MIDRANGE	range/2	midrange
MAXABS	maximum absolute value	0
IQR	interval quartile range	median
MAD	median absolute deviation from median	median
ABW(c)	biweight A-estimate	biweight 1-step M-estimate
AHUBER(c)	Huber A-estimate	Huber 1-step M-estimate
AWAVE(c)	Wave 1-step M-estimate	Wave A-estimate
AGK(p)	AGK estimate (ACECLUS)	mean
SPACING(p)	minimum spacing	mid minimum-spacing
L(p)	L_p	L_p

These standardization methods are further documented in the section on the METHOD= option in the PROC STDIZE statement of the STDIZE procedure (see the section "Standardization Methods" on page 7124 in Chapter 84, "The STDIZE Procedure").

Standardization is not required if there is only one level of measurement, or if only asymmetric nominal and nominal levels are specified; otherwise, standardization is mandatory. When standardization is mandatory, a default method is provided when the STD= option is not specified. You can suppress the mandatory standardization by using the NOSTD option in the PROC DISTANCE statement. See the NOSTD option in the section "PROC DISTANCE Statement" on page 2064 and the section "Mandatory Standardization" on page 2058 for details.

The default method is STD for standardizing interval variables and MAXABS for standardizing ratio variables unless METHOD=GOWER or METHOD=DGOWER is specified. If METHOD=GOWER is specified, interval variables are standardized by the RANGE method, and whatever is specified in the STD= option is ignored; if METHOD=DGOWER is specified, the RANGE method is the default standardization method for interval variables. The MAXABS method is the default standardization method for ratio variables for both the GOWER and DGOWER methods.

Notice that a ratio variable should always be positive.

Table 33.10 lists standardization methods and the levels of measurement that can be accepted by each method. For example, the SUM method can be used to standardize ratio variables but not interval or ordinal variables. Also, the AGK and SPACING methods should not be used to standardize ordinal variables. If you apply AGK and SPACING to ranks, the results are degenerate because all the spacings of a given order are equal.

Table 33.10 Legitimate Levels of Measurements for Each Method

Standardization Method	Legitimate Levels of Measurement
MEAN	ratio, interval, ordinal
MEDIAN	ratio, interval, ordinal
SUM	ratio
EUCLEN	ratio
USTD	ratio
STD	ratio, interval, ordinal
RANGE	ratio, interval, ordinal
MIDRANGE	ratio, interval, ordinal
MAXABS	ratio
IQR	ratio, interval, ordinal
MAD	ratio, interval, ordinal
ABW(*c*)	ratio, interval, ordinal
AHUBER(*c*)	ratio, interval, ordinal
AWAVE(*c*)	ratio, interval, ordinal
AGK(*p*)	ratio, interval
SPACING(*p*)	ratio, interval
L(*p*)	ratio, interval, ordinal

ABSENT=*numner* **|** *qs*

specifies the value to be used as an absence value in an irrelevant absent-absent match for asymmetric nominal variables. The absence value specified here overwrites the absence value specified through the ABSENT= option in the PROC DISTANCE statement for those variables in the current variable list.

An absence value for a variable can be either a numeric value or a quoted string consisting of combinations of characters. For instance, ., –999, “NA” are legal values for the ABSENT= option.

The default for an absence value for a character variable is “NONE” (notice that a blank value is considered a missing value), and the default for an absence value for a numeric variable is 0.

MISSING=*miss-method* **|** *value*

specifies the method or a numeric value for replacing missing values. If you omit the MISSING= option, the REPLACE option replaces missing values with the location measure given by the STD= option. Specify the MISSING= option when you want to replace missing values with a different value. You can specify any method that is valid in the STD= option. The corresponding location measure is used to replace missing values.

If a numeric value is given, the value replaces missing values after standardizing the data. However, when standardization is not mandatory, you can specify the REPONLY option with the MISSING= option to suppress standardization for cases in which you want only to replace missing values.

If the NOSTD option is specified, there is no standardization, but missing values are replaced by the corresponding location measures or by the numeric value of the MISSING= option. See the section "Missing Values" on page 2085 for details about missing values replacement with and without standardization.

ORDER=ASCENDING | ASC

ORDER=DESCENDING | DESC

ORDER=ASCFORMATTED | ASCFMT

ORDER=DESFORMATTED | DESFMT

ORDER=DSORDER | DATA

specifies the order for assigning score to ordinal variables. The value for the ORDER= option can be one of the following:

ASCENDING scores are assigned in lowest-to-highest order of unformatted values.

DESCENDING scores are assigned in highest-to-lowest order of unformatted values.

ASCFORMATTED scores are assigned in ascending order by their formatted values. This option can be applied to character variables only, since unformatted values are always used for numeric variables.

DESFORMATTED scores are assigned in descending order by their formatted values. This option can be applied to character variables only, since unformatted values are always used for numeric variables.

DSORDER scores are assigned according to the order of their appearance in the input data set.

The default value is ASCENDING.

WEIGHTS=*weight-list*

specifies a list of values for weighting individual variables while computing the proximity. Values in this list can be separated by blanks or commas. You can include one or more items of the form *start* TO *stop* BY *increment*. This list should contain at least one weight. The maximum number of weights you can list is equal to the number of variables. If the number of weights is less than the number of variables, the last value in the *weight-list* is used for the rest of the variables; conversely, if the number of weights is greater than the number of variables, the trailing weights are discarded.

The default value is 1.

ID Statement

ID *variable* **;**

The ID statement specifies a single variable to be copied to the OUT= data set and used to generate names for the distance variables. The ID variable must be character.

Typically, each ID value occurs only once in the input data set or, if you use a BY statement, only once within a BY group.

If you specify both the ID and BY statements, the ID variable must have the same values in the same order in each BY group.

COPY Statement

COPY *variables* **;**

The COPY statement specifies a list of additional variables to be copied to the OUT= data set.

BY Statement

BY *variables* **;**

You can specify a BY statement with PROC DISTANCE to obtain separate analyses on observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the DISTANCE procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

For more information about BY-group processing, see the discussion in *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the discussion in the *Base SAS Procedures Guide*.

FREQ Statement

FREQ | **FREQUENCY** *variable* **;**

The frequency variable is used for either standardizing variables or assigning rank scores to the ordinal variables. It has no direct effect on computing the distances.

For standardizing variables and assigning rank scores, PROC DISTANCE treats the data set as if each observation appeared n times, where n is the value of the FREQ variable for the observation. Nonintegral values of the FREQ variable are truncated to the largest integer less than the FREQ value. If the FREQ variable has a value that is less than 1 or is missing, the observation is not used in the analysis.

WEIGHT Statement

WGT | **WEIGHT** *variable* **;**

The WEIGHT statement specifies a numeric variable in the input data set with values that are used to weight each observation. This weight variable is used for standardizing variables rather than computing the distances. Only one variable can be specified.

The WEIGHT variable values can be nonintegers. An observation is used in the analysis only if the value of the WEIGHT variable is greater than zero. The WEIGHT variable applies to variables that are standardized by the following options: STD=MEAN, STD=SUM, STD=EUCLEN, STD=USTD, STD=STD, STD=AGK, or STD=L.

PROC DISTANCE uses the value of the WEIGHT variable w_i to compute the sample mean, uncorrected sample variances, and sample variances as follows:

$$\overline{x}_w = \sum_i w_i x_i / \sum_i w_i$$

$$u_w^2 = \sum_i w_i x_i^2 / d$$

$$s_w^2 = \sum_i w_i (x_i - \overline{x}_w)^2 / d$$

w_i is the weight value of the ith observation, x_i is the value of the ith observation, and d is the divisor controlled by the VARDEF= option (see the VARDEF= option in the PROC DISTANCE statement for details).

PROC DISTANCE uses the value of the WEIGHT variable to calculate the following statistics for standardization:

MEAN the weighted mean, $\overline{x}_w$

SUM the weighted sum, $\sum_i w_i x_i$

USTD the weighted uncorrected standard deviation, $\sqrt{u_w^2}$

STD the weighted standard deviation, $\sqrt{s_w^2}$

EUCLEN the weighted Euclidean length, computed as the square root of the weighted uncorrected sum of squares:

$$\sqrt{\sum_i w_i x_i^2}$$

AGK the AGK estimate. This estimate is documented further in the ACECLUS procedure as the METHOD=COUNT option. See the discussion of the WEIGHT statement in Chapter 23, "The ACECLUS Procedure," for information about how the WEIGHT variable is applied to the AGK estimate.

L the L_p estimate. This estimate is documented further in the FASTCLUS procedure as the LEAST= option. See the discussion of the WEIGHT statement in Chapter 35, "The FASTCLUS Procedure," for information about how the WEIGHT variable is used to compute weighted cluster means. Note that the number of clusters is always 1.

Details: DISTANCE Procedure

Proximity Measures

The following notation is used in this section:

v the number of variables or the dimensionality

x_j data for observation x and the jth variable, where $j = 1$ to v

y_j data for observation y and the jth variable, where $j = 1$ to v

w_j weight for the jth variable from the WEIGHTS= option in the VAR statement. $w_j = 0$ when either x_j or y_j is missing.

W the sum of total weights. No matter if the observation is missing or not, its weight is added to this metric.

$\bar{x}$ mean for observation x

$\bar{x} = \sum_{j=1}^{v} w_j x_j / \sum_{j=1}^{v} w_j$

$\bar{y}$ mean for observation y

$\bar{y} = \sum_{j=1}^{v} w_j y_j / \sum_{j=1}^{v} w_j$

$d(x, y)$ the distance or dissimilarity between observations x and y

$s(x, y)$ the similarity between observations x and y

The factor $W/\sum_{j=1}^{v} w_j$ is used to adjust some of the proximity measures for missing values.

Methods That Accept All Measurement Levels

GOWER Gower's similarity

$s_1(x, y) = \sum_{j=1}^{v} w_j \delta_{x,y}^{j} d_{x,y}^{j} / \sum_{j=1}^{v} w_j \delta_{x,y}^{j}$

$\delta_{x,y}^{j}$ is computed as follows:

For nominal, ordinal, interval, or ratio variable,

$$\delta_{x,y}^{j} = 1$$

For asymmetric nominal variable,

$$\begin{aligned} \delta_{x,y}^{j} &= 1, \text{ if either } x_j \text{ or } y_j \text{ is present} \\ \delta_{x,y}^{j} &= 0, \text{ if both } x_j \text{ and } y_j \text{ are absent} \end{aligned}$$

For nominal or asymmetric nominal variable,

$$\begin{aligned} d_{x,y}^{j} &= 1, \text{ if } x_j = y_j \\ d_{x,y}^{j} &= 0, \text{ if } x_j \neq y_j \end{aligned}$$

For ordinal, interval, or ratio variable,

$$d_{x,y}^{j} = 1 - |x_j - y_j|$$

DGOWER 1 minus Gower

$d_2(x, y) = 1 - s_1(x, y)$

Methods That Accept Ratio, Interval, and Ordinal Variables

EUCLID Euclidean distance

$d_3(x, y) = \sqrt{(\sum_{j=1}^{v} w_j (x_j - y_j)^2) W / (\sum_{j=1}^{v} w_j)}$

SQEUCLID squared Euclidean distance

$d_4(x, y) = (\sum_{j=1}^{v} w_j (x_j - y_j)^2) W / (\sum_{j=1}^{v} w_j)$

SIZE size distance

$d_5(x, y) = |\sum_{j=1}^{v} w_j (x_j - y_j)| \sqrt{W} / (\sum_{j=1}^{v} w_j)$

SHAPE

shape distance

$d_6(x,y) = \sqrt{(\sum_{j=1}^{v} w_j[(x_j - \bar{x}) - (y_j - \bar{y})]^2)W/(\sum_{j=1}^{v} w_j)}$

NOTE: squared shape distance plus squared size distance equals squared Euclidean distance.

COV

covariance similarity coefficient

$s_7(x,y) = \sum_{j=1}^{v} w_j(x_j - \bar{x})(y_j - \bar{y})/vardiv$, where

$$\begin{aligned} vardiv &= v \text{ if VARDEF} = \text{N} \\ &= v-1 \text{ if VARDEF} = \text{DF} \\ &= \textstyle\sum_{j=1}^{v} w_j \text{ if VARDEF} = \text{WEIGHT} \\ &= \textstyle\sum_{j=1}^{v} w_j - 1 \text{ if VARDEF} = \text{WDF} \end{aligned}$$

CORR

correlation similarity coefficient

$s_8(x,y) = \frac{\sum_{j=1}^{v} w_j(x_j-\bar{x})(y_j-\bar{y})}{\sqrt{\sum_{j=1}^{v} w_j(x_j-\bar{x})^2 \sum_{j=1}^{v} w_j(y_j-\bar{y})^2}}$

DCORR

correlation transformed to Euclidean distance as sqrt(1–CORR)

$d_9(x,y) = \sqrt{1 - s_8(x,y)}$

SQCORR

squared correlation

$s_{10}(x,y) = \frac{[\sum_{j=1}^{v} w_j(x_j-\bar{x})(y_j-\bar{y})]^2}{\sum_{j=1}^{v} w_j(x_j-\bar{x})^2 \sum_{j=1}^{v} w_j(y_j-\bar{y})^2}$

DSQCORR

squared correlation transformed to squared Euclidean distance as (1–SQCORR)

$d_{11}(x,y) = 1 - s_{10}(x,y)$

L(*p*)

Minkowski (L_p) distance, where *p* is a positive numeric value

$d_{12}(x,y) = [(\sum_{j=1}^{v} w_j|x_j - y_j|^p)W/(\sum_{j=1}^{v} w_j)]^{1/p}$

CITYBLOCK

L_1

$d_{13}(x,y) = (\sum_{j=1}^{v} w_j|x_j - y_j|)W/(\sum_{j=1}^{v} w_j)$

CHEBYCHEV

L_∞

$d_{14}(x,y) = \max_{j=1}^{v} w_j|x_j - y_j|$

POWER(*p*,*r*)

generalized Euclidean distance, where *p* is a nonnegative numeric value and *r* is a positive numeric value. The distance between two observations is the *r*th root of sum of the absolute differences to the *p*th power between the values for the observations:

$d_{15}(x,y) = [(\sum_{j=1}^{v} w_j|x_j - y_j|^p)W/(\sum_{j=1}^{v} w_j)]^{1/r}$

Methods That Accept Ratio Variables

SIMRATIO similarity ratio
$s_{16}(x,y) = \frac{\sum_j^v w_j(x_j y_j)}{\sum_{j=1}^v w_j(x_j y_j) + \sum_j^v w_j(x_j - y_j)^2}$

DISRATIO one minus similarity ratio
$d_{17}(x,y) = 1 - s_{16}(x,y)$

NONMETRIC Lance-Williams nonmetric coefficient
$d_{18}(x,y) = \frac{\sum_{j=1}^v w_j|x_j - y_j|}{\sum_{j=1}^v w_j(x_j + y_j)}$

CANBERRA Canberra metric coefficient. See Sneath and Sokal (1973, pp. 125–126)
$d_{19}(x,y) = \sum_{j=1}^v \frac{w_j|x_j - y_j|}{w_j(x_j + y_j)}$

COSINE cosine coefficient
$s_{20}(x,y) = \frac{\sum_{j=1}^v w_j(x_j y_j)}{\sqrt{\sum_{j=1}^v w_j {x_j}^2 \sum_{j=1}^v w_j {y_j}^2}}$

DOT dot (inner) product coefficient
$s_{21}(x,y) = [\sum_{j=1}^v w_j(x_j y_j)] / \sum_{j=1}^v w_j$

OVERLAP sum of the minimum values
$s_{22}(x,y) = \sum_{j=1}^v w_j[\min(x_j, y_j)]$

DOVERLAP maximum of the sum of the x and the sum of y minus overlap
$d_{23}(x,y) = \max(\sum_{j=1}^v w_j x_j, \sum_{j=1}^v w_j y_j) - s_{22}(x,y)$

CHISQ chi-squared

If the data represent the frequency counts, chi-squared dissimilarity between two sets of frequencies can be computed. A 2-by-v contingency table is illustrated to explain how the chi-squared dissimilarity is computed as follows:

	Variable				Row
Observation	Var 1	Var 2	...	Var v	Sum
X	x_1	x_2	...	x_v	r_x
Y	y_1	y_2	...	y_v	r_y
Column Sum	c_1	c_2	...	c_v	T

where

$$\begin{aligned} r_x &= \sum_{j=1}^v w_j x_j \\ r_y &= \sum_{j=1}^v w_j y_j \\ c_j &= w_j(x_j + y_j) \\ T &= r_x + r_y = \sum_{j=1}^v c_j \end{aligned}$$

The chi-squared measure is computed as follows:

$$d_{24}(x,y) = (\sum_{j=1}^v \frac{(w_j x_j - E(x_j))^2}{E(x_j)} + \sum_{j=1}^v \frac{(w_j y_j - E(y_j))^2}{E(y_j)}) W / (\sum_{j=1}^v w_j)$$

where for j= 1, 2, . . . , v

$$\begin{aligned} E(x_j) &= r_x c_j / T \\ E(y_j) &= r_y c_j / T \end{aligned}$$

CHI squared root of chi-squared
$d_{25}(x, y) = \sqrt{d_{23}(x, y)}$

PHISQ phi-squared
This is the CHISQ dissimilarity normalized by the sum of weights
$d_{26}(x, y) = d_{24}(x, y) / (\sum_{j=1}^{v} w_j)$

PHI squared root of phi-squared
$d_{27}(x, y) = \sqrt{d_{25}(x, y)}$

Methods That Accept Symmetric Nominal Variables

The following notation is used for computing $d_{28}(x, y)$ to $s_{35}(x, y)$. Notice that only the nonmissing pairs are discussed below; all the pairs with at least one missing value will be excluded from any of the computations in the following section because $w_j = 0$, if either x_j or y_j is missing.

M nonmissing matches
$M = \sum_{j=1}^{v} w_j \delta_{x,y}^{j}$, where

$$\begin{aligned} \delta_{x,y}^{j} &= 1, \text{ if } x_j = y_j \\ \delta_{x,y}^{j} &= 0, \text{ otherwise} \end{aligned}$$

X nonmissing mismatches
$X = \sum_{j=1}^{v} w_j \delta_{x,y}^{j}$, where

$$\begin{aligned} \delta_{x,y}^{j} &= 1, \text{ if } x_j \neq y_j \\ \delta_{x,y}^{j} &= 0, \text{ otherwise} \end{aligned}$$

N total nonmissing pairs
$N = \sum_{j=1}^{v} w_j$

HAMMING Hamming distance
$d_{28}(x, y) = X$

MATCH simple matching coefficient
$s_{29}(x, y) = M/N$

DMATCH simple matching coefficient transformed to Euclidean distance
$d_{30}(x, y) = \sqrt{1 - M/N} = \sqrt{(X/N)}$

DSQMATCH
simple matching coefficient transformed to squared Euclidean distance
$d_{31}(x, y) = 1 - M/N = X/N$

HAMANN
Hamann coefficient
$s_{32}(x, y) = (M - X)/N$

RT
Roger and Tanimoto
$s_{33}(x, y) = M/(M + 2X)$

SS1
Sokal and Sneath 1
$s_{34}(x, y) = 2M/(2M + X)$

SS3
Sokal and Sneath 3. The coefficient between an observation and itself is always indeterminate (missing) since there is no mismatch.
$s_{35}(x, y) = M/X$

The following notation is used for computing $s_{36}(x, y)$ to $d_{41}(x, y)$. Notice that only the nonmissing pairs are discussed in the following section; all the pairs with at least one missing value are excluded from any of the computations in the following section because $w_j = 0$, if either x_j or y_j is missing.

Also, the observed nonmissing data of an asymmetric binary variable can have only two possible outcomes: presence or absence. Therefore, the notation, *PX* (present mismatches), always has a value of zero for an asymmetric binary variable.

The following methods distinguish between the presence and absence of attributes.

X
mismatches with at least one present
$X = \sum_{j=1}^{v} w_j \delta_{x,y}^{j}$, where

$$\begin{aligned} \delta_{x,y}^{j} &= 1, \text{ if } x_j \neq y_j \text{ and not both } x_j \text{ and } y_j \text{ are absent} \\ \delta_{x,y}^{j} &= 0, \text{ otherwise} \end{aligned}$$

PM
present matches
$PM = \sum_{j=1}^{v} w_j \delta_{x,y}^{j}$, where

$$\begin{aligned} \delta_{x,y}^{j} &= 1, \text{ if } x_j = y_j \text{ and both } x_j \text{ and } y_j \text{ are present} \\ \delta_{x,y}^{j} &= 0, \text{ otherwise} \end{aligned}$$

PX
present mismatches
$PX = \sum_{j=1}^{v} w_j \delta_{x,y}^{j}$, where

$$\begin{aligned} \delta_{x,y}^{j} &= 1, \text{ if } x_j \neq y_j \text{ and both } x_j \text{ and } y_j \text{ are present} \\ \delta_{x,y}^{j} &= 0, \text{ otherwise} \end{aligned}$$

PP
both present = $PM + PX$

P at least one present = $PM + X$

PAX present-absent mismatches

$PAX = \sum_{j=1}^{v} w_j \delta_{x,y}^{j}$, where

$$\begin{aligned} \delta_{x,y}^{j} &= 1,\ \text{if } x_j \neq y_j \text{ and either } x_j \text{ is present and } y_j \text{ is absent or} \\ & \quad\ x_j \text{ is absent and } y_j \text{ is present} \\ \delta_{x,y}^{j} &= 0 \text{ otherwise} \end{aligned}$$

N total nonmissing pairs

$N = \sum_{j=1}^{v} w_j$

Methods That Accept Asymmetric Nominal and Ratio Variables

JACCARD Jaccard similarity coefficient

The JACCARD method is equivalent to the SIMRATIO method if there are only ratio variables; if there are both ratio and asymmetric nominal variables, the coefficient is computed as sum of the coefficient from the ratio variables (SIMRATIO) and the coefficient from the asymmetric nominal variables.

$s_{36}(x, y) = s_{16}(x, y) + PM/P$

DJACCARD Jaccard dissimilarity coefficient

The DJACCARD method is equivalent to the DISRATIO method if there are only ratio variables; if there are both ratio and asymmetric nominal variables, the coefficient is computed as sum of the coefficient from the ratio variables (DISRATIO) and the coefficient from the asymmetric nominal variables.

$d_{37}(x, y) = d_{17}(x, y) + X/P$

Methods That Accept Asymmetric Nominal Variables

DICE Dice coefficient or Czekanowski/Sorensen similarity coefficient

$s_{38}(x, y) = 2PM/(P + PM)$

RR Russell and Rao. This is the binary equivalent of the dot product coefficient.

$s_{39}(x, y) = PM/N$

BLWNM

BRAYCURTIS Binary Lance and Williams, also known as Bray and Curtis coefficient

$d_{40}(x, y) = X/(PAX + 2PP)$

K1 Kulcynski 1. The coefficient between an observation and itself is always indeterminate (missing) since there is no mismatch.
$d_{41}(x, y) = PM/X$

Missing Values

Standardization versus No Standardization

You can replace the missing values with or without standardization. Missing values are replaced after standardization by specifying either the REPLACE option in the PROC DISTANCE statement or the MISSING= option in the VAR statement.

To replace missing values without standardization, use the following two options:

- the NOSTD option in the PROC DISTANCE statement. The NOSTD option suppresses standardization but still replaces the missing values with the location of the method or the numeric value specified in the MISSING= option in the VAR statement.
- the REPONLY option in the PROC DISTANCE statement. PROC DISTANCE replaces missing values with the location of the standardization method or with the numeric value specified in the MISSING= option in the VAR statement. This approach assumes that standardization is not mandatory (see the section "Standardization" on page 2058).

Eliminating Observations with Missing Values

If you specify the NOMISS option, PROC DISTANCE omits observations with any missing values in the analyzed variables from computation of the location and scale measures.

Distance Measures

If you specify the NOMISS option, PROC DISTANCE generates missing distance for observations with missing values. If the NOMISS option is not specified, the sum of total weights, no matter if an observation is missing or not, is incorporated into the computation of some of the proximity measures. See the section "Details: DISTANCE Procedure" on page 2078 for the formulas and descriptions.

Formatted versus Unformatted Values

PROC DISTANCE uses the formatted values from a character variable, if the variable has a format—for example, one assigned by a format statement. PROC DISTANCE uses the unformatted values from a numeric variable, even if it has a format.

Output Data Sets

OUT= Data Set

The DISTANCE procedure always produces an output data set, regardless of whether you specify the OUT= option in the PROC DISTANCE statement. PROC DISTANCE displays no output. Use PROC PRINT to display the output data set.

The output data set contains the following variables:

- the ID variable, if any
- the BY variables, if any
- the COPY variables, if any
- the FREQ variable, if any
- the WEIGHT variable, if any
- the new distance variables, named from PREFIX= options along with the ID values, or from the default values

The output data set is of type TYPE=DISTANCE or TYPE=SIMILAR. See the METHOD= option for more information about the output data set types. Data set types do not persist when you copy or modify a data set. You must specify the TYPE= data set option for the new data set, as in the following example:

```
data dist2(type=distance);
   set dist;
run;
```

See the OUT= option for more information about data set type persistence.

OUTSDZ= Data Set

The output data set is a copy of the DATA= data set except that the analyzed variables have been standardized. Analyzed variables are those listed in the VAR statement.

Examples: DISTANCE Procedure

Example 33.1: Divorce Grounds – the Jaccard Coefficient

A wide variety of distance and similarity measures are used in cluster analysis (Anderberg 1973; Sneath and Sokal 1973). If your data are in coordinate form and you want to use a non-Euclidean distance for clustering, you can compute a distance matrix by using the DISTANCE procedure.

Similarity measures must be converted to dissimilarities before being used in PROC CLUSTER. Such conversion can be done in a variety of ways, such as taking reciprocals or subtracting from a large value. The choice of conversion method depends on the application and the similarity measure. If applicable, PROC DISTANCE provides a corresponding dissimilarity measure for each similarity measure.

In the following example, the observations are states. Binary-valued variables correspond to various grounds for divorce and indicate whether the grounds for divorce apply in each of the U.S. states. A value of “1” indicates that the ground for divorce applies, and a value of “0” indicates the opposite. The 0-0 matches are treated as totally irrelevant; therefore, each variable has an asymmetric nominal level of measurement. The absence value is 0.

The DISTANCE procedure is used to compute the Jaccard coefficient (Anderberg 1973, pp. 89, 115, and 117) between each pair of states. The Jaccard coefficient is defined as the number of variables that are coded as 1 for both states divided by the number of variables that are coded as 1 for either or both states. Since dissimilarity measures are required by PROC CLUSTER, the DJACCARD coefficient is selected. Output 33.1.1 displays the distance matrix between the first 10 states.

The CENTROID method is used to perform the cluster analysis, and the resulting tree diagram from PROC CLUSTER is saved into the tree output data set. Output 33.1.2 displays the cluster history.

The TREE procedure generates nine clusters in the output data set out. After being sorted by the state, the out data set is then merged with the input data set divorce. After being sorted by the state, the merged data set is printed to display the cluster membership as shown in Output 33.1.3.

The following statements produce Output 33.1.1 through Output 33.1.3:

```
data divorce;
   input State $15.
         (Incompatibility Cruelty Desertion Non_Support Alcohol
          Felony Impotence Insanity Separation) (1.) @@;
   if mod(_n_,2) then input +4 @@; else input;
   datalines;
Alabama        111111111    Alaska         111011110
Arizona        100000000    Arkansas       011111111
California     100000010    Colorado       100000000

   ... more lines ...

Wisconsin      100000001    Wyoming        100000011
;
```

```
title 'Grounds for Divorce';
proc distance data=divorce method=djaccard absent=0 out=distjacc;
   var anominal(Incompatibility--Separation);
   id state;
run;
proc print data=distjacc(obs=10);
   id state; var alabama--georgia;
   title2 'First 10 States';
run;
title2;

proc cluster data=distjacc method=centroid
             pseudo outtree=tree;
   id state;
   var alabama--wyoming;
run;

proc tree data=tree noprint n=9 out=out;
   id state;
run;

proc sort;
   by state;
run;

data clus;
   merge divorce out;
   by state;
run;

proc sort;
   by cluster;
run;
proc print;
   id state;
   var Incompatibility--Separation;
   by cluster;
run;
```

Output 33.1.1 Distance Matrix Based on the Jaccard Coefficient

```
                              Grounds for Divorce
                                First 10 States

State          Alabama    Alaska   Arizona   Arkansas   California   Colorado

Alabama        0.00000      .         .          .           .           .
Alaska         0.22222   0.00000      .          .           .           .
Arizona        0.88889   0.85714   0.00000       .           .           .
Arkansas       0.11111   0.33333   1.00000    0.00000        .           .
California     0.77778   0.71429   0.50000    0.88889     0.00000        .
Colorado       0.88889   0.85714   0.00000    1.00000     0.50000     0.00000
Connecticut    0.11111   0.33333   0.87500    0.22222     0.75000     0.87500
Delaware       0.77778   0.87500   0.50000    0.88889     0.66667     0.50000
Florida        0.77778   0.71429   0.50000    0.88889     0.00000     0.50000
Georgia        0.22222   0.00000   0.85714    0.33333     0.71429     0.85714

State          Connecticut    Delaware    Florida    Georgia

Alabama            .              .           .          .
Alaska             .              .           .          .
Arizona            .              .           .          .
Arkansas           .              .           .          .
California         .              .           .          .
Colorado           .              .           .          .
Connecticut      0.00000          .           .          .
Delaware         0.75000       0.00000        .          .
Florida          0.75000       0.66667     0.00000       .
Georgia          0.33333       0.87500     0.71429       0
```

Output 33.1.2 Clustering History

```
                              Grounds for Divorce

                             The CLUSTER Procedure
                      Centroid Hierarchical Cluster Analysis

            Root-Mean-Square Distance Between Observations    0.694873
```

Output 33.1.2 *continued*

```
                          Cluster History
                                                                Norm T
                                                                Cent i
NCL -------Clusters Joined-------     Freq     Ps F     PsT2    Dist e

 49 Arizona        Colorado              2        .        .       0 T
 48 California     Florida               2        .        .       0 T
 47 Alaska         Georgia               2        .        .       0 T
 46 Delaware       Hawaii                2        .        .       0 T
 45 Connecticut    Idaho                 2        .        .       0 T
 44 CL49           Iowa                  3        .        .       0 T
 43 CL47           Kansas                3        .        .       0 T
 42 CL44           Kentucky              4        .        .       0 T
 41 CL42           Michigan              5        .        .       0 T
 40 CL41           Minnesota             6        .        .       0 T
 39 CL43           Mississippi           4        .        .       0 T
 38 CL40           Missouri              7        .        .       0 T
 37 CL38           Montana               8        .        .       0 T
 36 CL37           Nebraska              9        .        .       0 T
 35 North Dakota   Oklahoma              2        .        .       0 T
 34 CL36           Oregon               10        .        .       0 T
 33 Massachusetts  Rhode Island          2        .        .       0 T
 32 New Hampshire  Tennessee             2        .        .       0 T
 31 CL46           Washington            3        .        .       0 T
 30 CL31           Wisconsin             4        .        .       0 T
 29 Nevada         Wyoming               2        .        .       0
 28 Alabama        Arkansas              2     1561        .  0.1599 T
 27 CL33           CL32                  4      479        .  0.1799 T
 26 CL39           CL35                  6      265        .  0.1799 T
 25 CL45           West Virginia         3      231        .  0.1799
 24 Maryland       Pennsylvania          2      199        .  0.2399
 23 CL28           Utah                  3      167      3.2  0.2468
 22 CL27           Ohio                  5      136      5.4  0.2698
 21 CL26           Maine                 7      111      8.9  0.2998
 20 CL23           CL21                 10     75.2      8.7  0.3004
 19 CL25           New Jersey            4     71.8      6.5  0.3053 T
 18 CL19           Texas                 5     69.1      2.5  0.3077
 17 CL20           CL22                 15     48.7      9.9  0.3219
 16 New York       Virginia              2     50.1        .  0.3598
 15 CL18           Vermont               6     49.4      2.9  0.3797
 14 CL17           Illinois             16     47.0      3.2  0.4425
 13 CL14           CL15                 22     29.2     15.3  0.4722
 12 CL48           CL29                  4     29.5        .  0.4797 T
 11 CL13           CL24                 24     27.6      4.5  0.5042
 10 CL11           South Dakota         25     28.4      2.4  0.5449
  9 Louisiana      CL16                  3     30.3      3.5  0.5844
  8 CL34           CL30                 14     23.3        .  0.7196
  7 CL8            CL12                 18     19.3     15.0  0.7175
  6 CL10           South Carolina       26     21.4      4.2  0.7384
  5 CL6            New Mexico           27     24.0      4.7  0.8303
  4 CL5            Indiana              28     28.9      4.1  0.8343
  3 CL4            CL9                  31     31.7     10.9  0.8472
  2 CL3            North Carolina       32     55.1      4.1  1.0017
  1 CL2            CL7                  50        .     55.1  1.0663
```

Output 33.1.3 Cluster Membership

```
                              Grounds for Divorce

---------------------------------- CLUSTER=1 -----------------------------------

                          I
                          n
                          c
                          o
                          m              N
                          p              o                        S
                          a         D    n              I         e
                          t         e    _              m    I    p
                          i    C    s    S    A         p    n    a
                          b    r    e    u    l    F    o    s    r
                 S        i    u    r    p    c    e    t    a    a
                 t        l    e    t    p    o    l    e    n    t
                 a        i    l    i    o    h    o    n    i    i
                 t        t    t    o    r    o    n    c    t    o
                 e        y    y    n    t    l    y    e    y    n

             Arizona      1    0    0    0    0    0    0    0    0
             Colorado     1    0    0    0    0    0    0    0    0
             Iowa         1    0    0    0    0    0    0    0    0
             Kentucky     1    0    0    0    0    0    0    0    0
             Michigan     1    0    0    0    0    0    0    0    0
             Minnesota    1    0    0    0    0    0    0    0    0
             Missouri     1    0    0    0    0    0    0    0    0
             Montana      1    0    0    0    0    0    0    0    0
             Nebraska     1    0    0    0    0    0    0    0    0
             Oregon       1    0    0    0    0    0    0    0    0
```

Output 33.1.3 *continued*

```
                              Grounds for Divorce

---------------------------------- CLUSTER=2 -----------------------------------

                          I
                          n
                          c
                          o
                          m              N
                          p              o                        S
                          a         D    n              I         e
                          t         e    _              m    I    p
                          i    C    s    S    A         p    n    a
                          b    r    e    u    l    F    o    s    r
                S         i    u    r    p    c    e    t    a    a
                t         l    e    t    p    o    l    e    n    t
                a         i    l    i    o    h    o    n    i    i
                t         t    t    o    r    o    n    c    t    o
                e         y    y    n    t    l    y    e    y    n

            California    1    0    0    0    0    0    0    1    0
            Florida       1    0    0    0    0    0    0    1    0
            Nevada        1    0    0    0    0    0    0    1    1
            Wyoming       1    0    0    0    0    0    0    1    1


---------------------------------- CLUSTER=3 -----------------------------------

                           I
                           n
                           c
                           o
                           m              N
                           p              o                        S
                           a         D    n              I         e
                           t         e    _              m    I    p
                           i    C    s    S    A         p    n    a
                           b    r    e    u    l    F    o    s    r
                 S         i    u    r    p    c    e    t    a    a
                 t         l    e    t    p    o    l    e    n    t
                 a         i    l    i    o    h    o    n    i    i
                 t         t    t    o    r    o    n    c    t    o
                 e         y    y    n    t    l    y    e    y    n

            Alabama        1    1    1    1    1    1    1    1    1
            Alaska         1    1    1    0    1    1    1    1    0
            Arkansas       0    1    1    1    1    1    1    1    1
            Connecticut    1    1    1    1    1    1    0    1    1
            Georgia        1    1    1    0    1    1    1    1    0
```

Output 33.1.3 *continued*

```
                              Grounds for Divorce

---------------------------------- CLUSTER=3 -----------------------------------

                                  (continued)

                            I
                            n
                            c
                            o
                            m              N
                            p              o                        S
                            a         D    n              I         e
                            t         e    _              m    I    p
                            i    C    s    S    A         p    n    a
                            b    r    e    u    l    F    o    s    r
                 S          i    u    r    p    c    e    t    a    a
                 t          l    e    t    p    o    l    e    n    t
                 a          i    l    i    o    h    o    n    i    i
                 t          t    t    o    r    o    n    c    t    o
                 e          y    y    n    t    l    y    e    y    n

           Idaho            1    1    1    1    1    1    0    1    1
           Illinois         0    1    1    0    1    1    1    0    0
           Kansas           1    1    1    0    1    1    1    1    0
           Maine            1    1    1    1    1    0    1    1    0
           Maryland         0    1    1    0    0    1    1    1    1
           Massachusetts    1    1    1    1    1    1    1    0    1
           Mississippi      1    1    1    0    1    1    1    1    0
           New Hampshire    1    1    1    1    1    1    1    0    0
           New Jersey       0    1    1    0    1    1    0    1    1
           North Dakota     1    1    1    1    1    1    1    1    0
           Ohio             1    1    1    0    1    1    1    0    1
           Oklahoma         1    1    1    1    1    1    1    1    0
           Pennsylvania     0    1    1    0    0    1    1    1    0
           Rhode Island     1    1    1    1    1    1    1    0    1
           South Dakota     0    1    1    1    1    1    0    0    0
           Tennessee        1    1    1    1    1    1    1    0    0
           Texas            1    1    1    0    0    1    0    1    1
           Utah             0    1    1    1    1    1    1    1    0
           Vermont          0    1    1    1    0    1    0    1    1
           West Virginia    1    1    1    0    1    1    0    1    1
```

Output 33.1.3 *continued*

```
                              Grounds for Divorce

---------------------------------- CLUSTER=4 -----------------------------------

                          I
                          n
                          c
                          o
                          m              N
                          p              o                        S
                          a         D    n              I         e
                          t         e    _              m    I    p
                          i    C    s    S    A         p    n    a
                          b    r    e    u    l    F    o    s    r
                S         i    u    r    p    c    e    t    a    a
                t         l    e    t    p    o    l    e    n    t
                a         i    l    i    o    h    o    n    i    i
                t         t    t    o    r    o    n    c    t    o
                e         y    y    n    t    l    y    e    y    n

            Delaware      1    0    0    0    0    0    0    0    1
            Hawaii        1    0    0    0    0    0    0    0    1
            Washington    1    0    0    0    0    0    0    0    1
            Wisconsin     1    0    0    0    0    0    0    0    1


---------------------------------- CLUSTER=5 -----------------------------------

                          I
                          n
                          c
                          o
                          m              N
                          p              o                        S
                          a         D    n              I         e
                          t         e    _              m    I    p
                          i    C    s    S    A         p    n    a
                          b    r    e    u    l    F    o    s    r
                 S        i    u    r    p    c    e    t    a    a
                 t        l    e    t    p    o    l    e    n    t
                 a        i    l    i    o    h    o    n    i    i
                 t        t    t    o    r    o    n    c    t    o
                 e        y    y    n    t    l    y    e    y    n

             Louisiana    0    0    0    0    0    1    0    0    1
             New York     0    1    1    0    0    1    0    0    1
             Virginia     0    1    0    0    0    1    0    0    1
```

Output 33.1.3 *continued*

```
                              Grounds for Divorce

---------------------------------- CLUSTER=6 -----------------------------------

                            I
                            n
                            c
                            o
                            m              N
                            p              o                        S
                            a         D    n              I         e
                            t         e    _              m    I    p
                            i    C    s    S    A         p    n    a
                            b    r    e    u    l    F    o    s    r
                S           i    u    r    p    c    e    t    a    a
                t           l    e    t    p    o    l    e    n    t
                a           i    l    i    o    h    o    n    i    i
                t           t    t    o    r    o    n    c    t    o
                e           y    y    n    t    l    y    e    y    n

          South Carolina    0    1    1    0    1    0    0    0    1


---------------------------------- CLUSTER=7 -----------------------------------

                          I
                          n
                          c
                          o
                          m              N
                          p              o                        S
                          a         D    n              I         e
                          t         e    _              m    I    p
                          i    C    s    S    A         p    n    a
                          b    r    e    u    l    F    o    s    r
                S         i    u    r    p    c    e    t    a    a
                t         l    e    t    p    o    l    e    n    t
                a         i    l    i    o    h    o    n    i    i
                t         t    t    o    r    o    n    c    t    o
                e         y    y    n    t    l    y    e    y    n

            New Mexico    1    1    1    0    0    0    0    0    0
```

Output 33.1.3 *continued*

```
                              Grounds for Divorce

---------------------------------- CLUSTER=8 -----------------------------------

                         I
                         n
                         c
                         o
                         m              N
                         p              o                        S
                         a         D    n              I         e
                         t         e    _              m    I    p
                         i    C    s    S    A         p    n    a
                         b    r    e    u    l    F    o    s    r
                 S       i    u    r    p    c    e    t    a    a
                 t       l    e    t    p    o    l    e    n    t
                 a       i    l    i    o    h    o    n    i    i
                 t       t    t    o    r    o    n    c    t    o
                 e       y    y    n    t    l    y    e    y    n

              Indiana    1    0    0    0    0    1    1    1    0


---------------------------------- CLUSTER=9 -----------------------------------

                            I
                            n
                            c
                            o
                            m              N
                            p              o                        S
                            a         D    n              I         e
                            t         e    _              m    I    p
                            i    C    s    S    A         p    n    a
                            b    r    e    u    l    F    o    s    r
                S           i    u    r    p    c    e    t    a    a
                t           l    e    t    p    o    l    e    n    t
                a           i    l    i    o    h    o    n    i    i
                t           t    t    o    r    o    n    c    t    o
                e           y    y    n    t    l    y    e    y    n

          North Carolina    0    0    0    0    0    0    1    1    1
```

Example 33.2: Financial Data – Stock Dividends

The following data set contains the average dividend yields for 15 utility stocks in the United States. The observations are names of the companies, and the variables correspond to the annual dividend yields for the period 1986–1990. The objective is to group similar stocks into clusters.

Before the cluster analysis is performed, the correlation similarity is chosen for measuring the closeness between each observation. Since distance type of measures are required by PROC CLUSTER, METHOD=DCORR is used in the PROC DISTANCE statement to transform the correlation measures to the distance measures. Notice that in Output 33.2.1, all the values in the distance matrix are between 0 and 2.

PROC CLUSTER performs hierarchical clustering by using agglomerative methods based on the distance data created from the previous PROC DISTANCE statement. Since the cubic clustering criterion is not suitable for distance data, only the pseudo F statistic is requested to identify the number of clusters.

The two clustering methods are Ward's and the average linkage methods. Since the results of the pseudo t^2 statistic from both Ward's and the average linkage methods contain many missing values, only the plot of the pseudo F statistic versus the number of clusters is requested along with the dendrogram by specifying PLOTS(ONLY)=(PSF DENDROGRAM) in the PROC CLUSTER statement.

Both Output 33.2.2 and Output 33.2.3 suggest four clusters. Both methods produce the same clustering result, as shown in Output 33.2.4 and Output 33.2.5. The four clusters are as follows:

- Cincinnati G&E and Detroit Edison
- Texas Utilities and Pennsylvania Power & Light
- Union Electric, Iowa-Ill Gas & Electric, Oklahoma Gas & Electric, and Wisconsin Energy
- Orange & Rockland Utilities, Kentucky Utilities, Kansas Power & Light, Allegheny Power, Green Mountain Power, Dominion Resources, and Minnesota Power & Light

```
title 'Stock Dividends';

data stock;
   input Company $27.  Div_1986 Div_1987 Div_1988 Div_1989 Div_1990;
   datalines;
Cincinnati G&E              8.4    8.2    8.4    8.1    8.0
Texas Utilities             7.9    8.9   10.4    8.9    8.3
Detroit Edison              9.7   10.7   11.4    7.8    6.5
Orange & Rockland Utilities 6.5    7.2    7.3    7.7    7.9
Kentucky Utilities          6.5    6.9    7.0    7.2    7.5
Kansas Power & Light        5.9    6.4    6.9    7.4    8.0
Union Electric              7.1    7.5    8.4    7.8    7.7
Dominion Resources          6.7    6.9    7.0    7.0    7.4
Allegheny Power             6.7    7.3    7.8    7.9    8.3
Minnesota Power & Light     5.6    6.1    7.2    7.0    7.5
Iowa-Ill Gas & Electric     7.1    7.5    8.5    7.8    8.0
Pennsylvania Power & Light  7.2    7.6    7.7    7.4    7.1
Oklahoma Gas & Electric     6.1    6.7    7.4    6.7    6.8
Wisconsin Energy            5.1    5.7    6.0    5.7    5.9
Green Mountain Power        7.1    7.4    7.8    7.8    8.3
;

proc distance data=stock method=dcorr out=distdcorr;
   var interval(div_1986 div_1987 div_1988 div_1989 div_1990);
   id company;
run;
proc print data=distdcorr;
   id company;
   title2 'Distance Matrix for 15 Utility Stocks';
run;
title2;

ods graphics on;

/* compute pseudo statistic versus number of clusters and create plot */
proc cluster data=distdcorr method=ward pseudo plots(only)=(psf dendrogram);
   id company;
run;

/* compute pseudo statistic versus number of clusters and create plot */
proc cluster data=distdcorr method=average pseudo plots(only)=(psf dendrogram);
   id company;
run;

ods graphics off;
```

Output 33.2.1 Distance Matrix Based on the DCORR Coefficient

```
                               Stock Dividends
                    Distance Matrix for 15 Utility Stocks

                                                                    Orange___
                            Cincinnati_      Texas_     Detroit_    Rockland_
Company                         G_E        Utilities     Edison     Utilities

Cincinnati G&E                0.00000        .            .           .
Texas Utilities               0.82056       0.00000       .           .
Detroit Edison                0.40511       0.65453      0.00000      .
Orange & Rockland Utilities   1.35380       0.88583      1.27306     0.00000
Kentucky Utilities            1.35581       0.92539      1.29382     0.12268
Kansas Power & Light          1.34227       0.94371      1.31696     0.19905
Union Electric                0.98516       0.29043      0.89048     0.68798

                                       Kansas_
                            Kentucky_ Power___  Union_  Dominion_ Allegheny_
Company                     Utilities   Light  Electric Resources     Power

Cincinnati G&E                .         .        .        .          .
Texas Utilities               .         .        .        .          .
Detroit Edison                .         .        .        .          .
Orange & Rockland Utilities   .         .        .        .          .
Kentucky Utilities           0.00000    .        .        .          .
Kansas Power & Light         0.12874   0.00000   .        .          .
Union Electric               0.71824   0.72082  0.00000   .          .

                            Minnesota_   Iowa_Ill_                   Oklahoma_
                             Power___      Gas___   Pennsylvania_     Gas___
Company                        Light      Electric  Power___Light    Electric

Cincinnati G&E                 .           .             .             .
Texas Utilities                .           .             .             .
Detroit Edison                 .           .             .             .
Orange & Rockland Utilities    .           .             .             .
Kentucky Utilities             .           .             .             .
Kansas Power & Light           .           .             .             .
Union Electric                 .           .             .             .

                                            Green_
                            Wisconsin_    Mountain_
Company                       Energy        Power

Cincinnati G&E                 .              .
Texas Utilities                .              .
Detroit Edison                 .              .
Orange & Rockland Utilities    .              .
Kentucky Utilities             .              .
Kansas Power & Light           .              .
Union Electric                 .              .
```

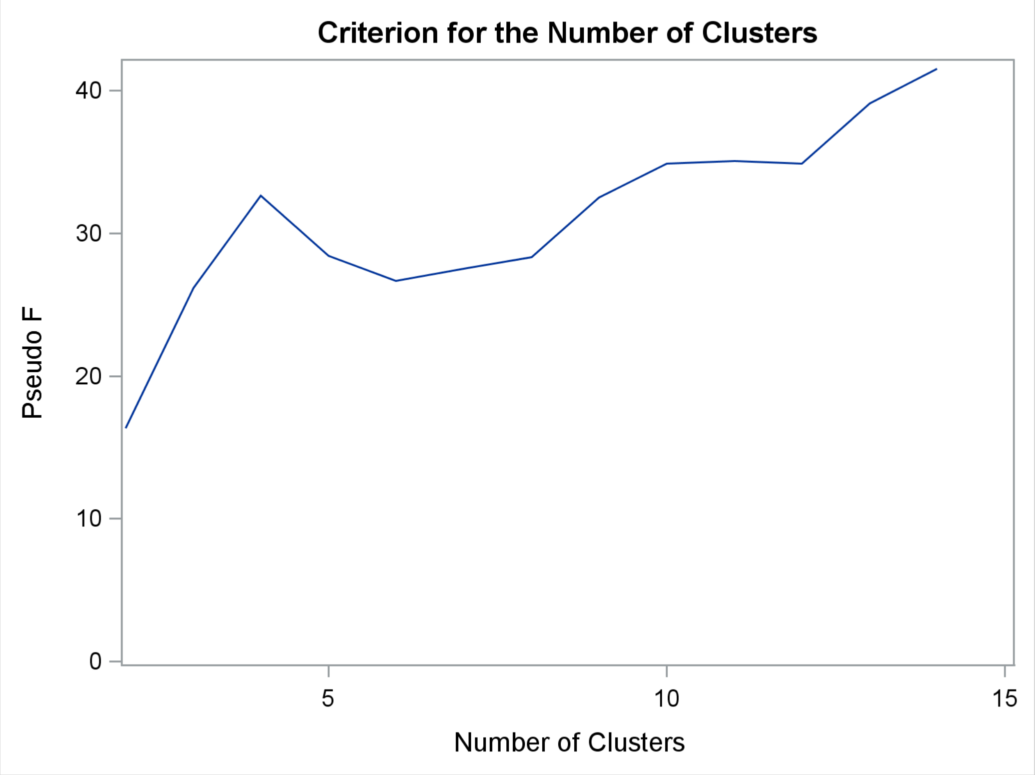
Output 33.2.1 *continued*

```
                               Stock Dividends
                    Distance Matrix for 15 Utility Stocks

                                                                    Orange___
                            Cincinnati_      Texas_     Detroit_    Rockland_
Company                         G_E        Utilities     Edison     Utilities

Dominion Resources            1.32945       0.96853      1.29016     0.33290
Allegheny Power               1.30492       0.81666      1.24565     0.17844
Minnesota Power & Light       1.24069       0.74082      1.20432     0.32581
Iowa-Ill Gas & Electric       1.04924       0.43100      0.97616     0.61166
Pennsylvania Power & Light    0.74931       0.37821      0.44256     1.03566
Oklahoma Gas & Electric       1.00604       0.30141      0.86200     0.68021
Wisconsin Energy              1.17988       0.54830      1.03081     0.45013

                                       Kansas_
                            Kentucky_ Power___  Union_  Dominion_ Allegheny_
Company                     Utilities   Light  Electric Resources    Power

Dominion Resources           0.21510   0.24189  0.76587  0.00000     .
Allegheny Power              0.15759   0.17029  0.58452  0.27819    0.00000
Minnesota Power & Light      0.30462   0.27231  0.48372  0.35733    0.15615
Iowa-Ill Gas & Electric      0.61760   0.61736  0.16923  0.63545    0.47900
Pennsylvania Power & Light   1.08878   1.12876  0.63285  1.14354    1.02358
Oklahoma Gas & Electric      0.70259   0.73158  0.17122  0.72977    0.58391
Wisconsin Energy             0.47184   0.53381  0.37405  0.51969    0.37522

                            Minnesota_   Iowa_Ill_                   Oklahoma_
                             Power___      Gas___    Pennsylvania_     Gas___
Company                         Light      Electric   Power___Light    Electric

Dominion Resources              .           .             .             .
Allegheny Power                 .           .             .             .
Minnesota Power & Light       0.00000       .             .             .
Iowa-Ill Gas & Electric       0.36368     0.00000         .             .
Pennsylvania Power & Light    0.99384     0.75596       0.00000         .
Oklahoma Gas & Electric       0.50744     0.19673       0.60216       0.00000
Wisconsin Energy              0.36319     0.30259       0.76085       0.28070

                                            Green_
                            Wisconsin_    Mountain_
Company                       Energy        Power

Dominion Resources              .              .
Allegheny Power                 .              .
Minnesota Power & Light         .              .
Iowa-Ill Gas & Electric         .              .
Pennsylvania Power & Light      .              .
Oklahoma Gas & Electric         .              .
Wisconsin Energy              0.00000          .
```

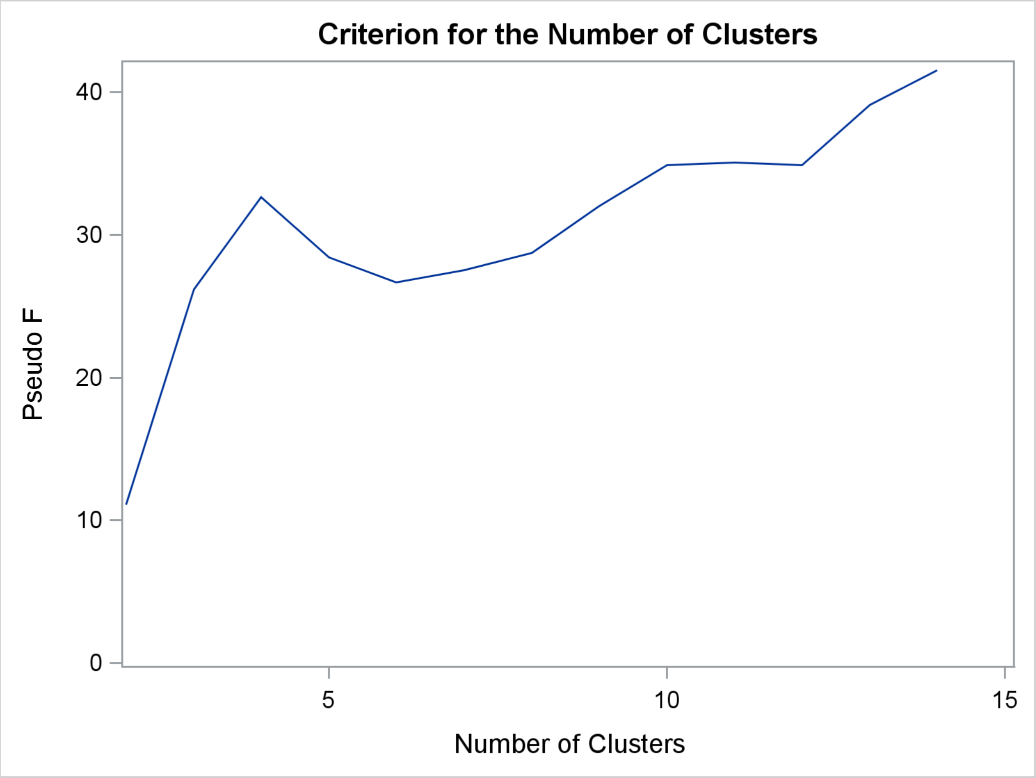
Output 33.2.1 *continued*

```
                               Stock Dividends
                    Distance Matrix for 15 Utility Stocks

                                                                    Orange___
                            Cincinnati_      Texas_     Detroit_    Rockland_
Company                         G_E        Utilities     Edison     Utilities

Green Mountain Power          1.30397       0.88063      1.27176     0.26948

                                       Kansas_
                            Kentucky_ Power___  Union_  Dominion_ Allegheny_
Company                     Utilities   Light  Electric Resources    Power

Green Mountain Power         0.17909   0.15377  0.64869  0.17360    0.13958

                            Minnesota_   Iowa_Ill_                   Oklahoma_
                             Power___      Gas___    Pennsylvania_     Gas___
Company                        Light      Electric   Power___Light    Electric

Green Mountain Power          0.19370     0.52083       1.09269       0.64175

                                            Green_
                            Wisconsin_    Mountain_
Company                       Energy        Power

Green Mountain Power          0.44814         0
```

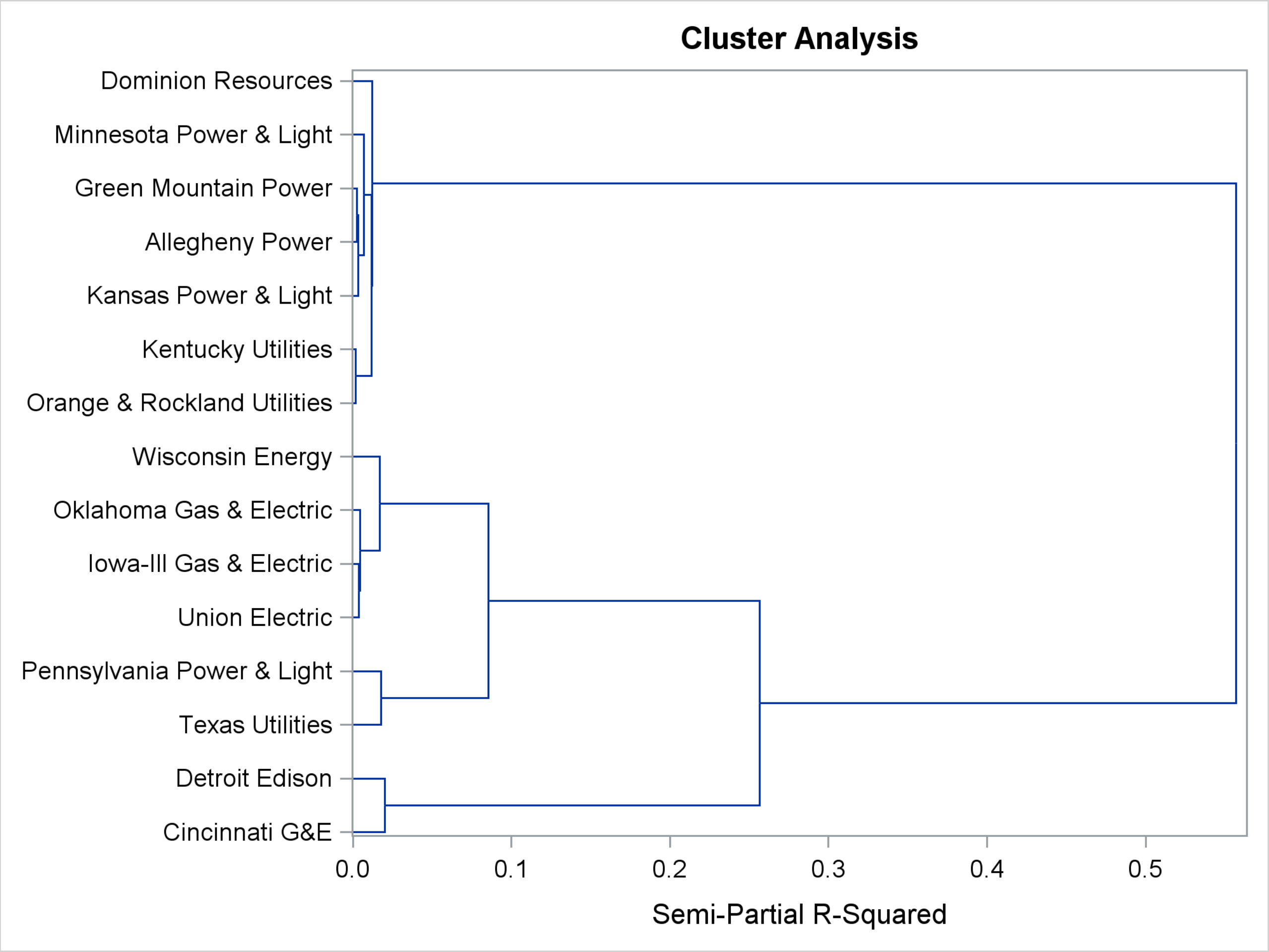
Output 33.2.2 Pseudo *F* versus Number of Clusters When METHOD=WARD



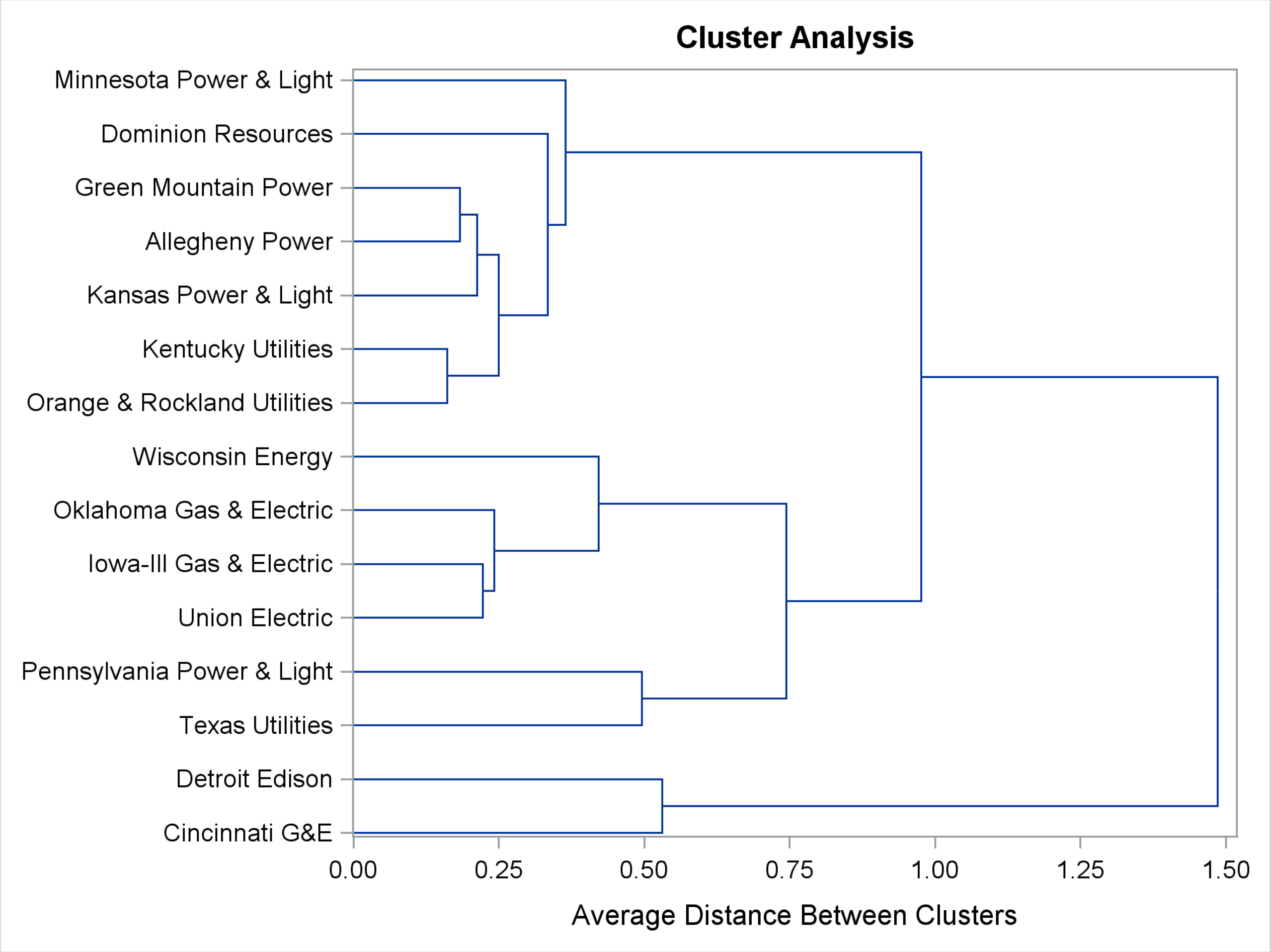
Output 33.2.3 Pseudo *F* versus Number of Clusters When METHOD=AVERAGE



Output 33.2.4 Dendrogram of Semipartial R-Square Values When METHOD=WARD



Output 33.2.5 Dendrogram of Average Distance between Clusters When METHOD=AVERAGE



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Syntax Index

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