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SAS/STAT® 14.2 User's Guide
The STDRATE Procedure

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Chapter 109
The STDRATE Procedure

Contents

Overview: STDRATE Procedure

Epidemiology is the study of the occurrence and distribution of health-related states or events in specified populations. Epidemiology also includes the study of the determinants that influence these states, and the application of this knowledge to control health problems (Porta 2008). It is a discipline that describes, quantifies, and postulates causal mechanisms for health phenomena in populations (Friss and Sellers 2009).

A common goal is to establish relationships between various factors (such as exposure to a specific chemical) and the event outcomes (such as incidence of disease). But the measure of an association between an exposure and an event outcome can be biased due to confounding. That is, the association of the exposure to some other variables, such as age, influences the occurrence of the event outcome. With confounding, the usual effect between an exposure and an event outcome can be biased because some of the effect might be accounted for by other variables. For example, with an event rate discrepancy among different age groups of a population, the overall crude rate might not provide a useful summary statistic to compare populations.

One strategy to control confounding is stratification. In stratification, a population is divided into several subpopulations according to specific criteria for the confounding variables, such as age and sex groups. The effect of the exposure on the event outcome is estimated within each stratum, and then these stratum-specific effect estimates are combined into an overall estimate.

Two commonly used event frequency measures are rate and risk:

- A rate is a measure of the frequency with which an event occurs in a defined population in a specified period of time. It measures the change in one quantity per unit of another quantity. For example, an event rate measures how fast the events are occurring. That is, an event rate of a population over a specified time period can be defined as the number of new events divided by population-time (Kleinbaum, Kupper, and Morgenstern 1982, p. 100) over the same time period.
- A risk is the probability that an event occurs in a specified time period. It is assumed that only one event can occur in the time period for each subject or item. The overall crude risk of a population over a specified time period is the number of new events in the time period divided by the population size at the beginning of the time period.

Standardized overall rate and risk estimates based on stratum-specific estimates can be derived with the effects of confounding variables removed. These estimates provide useful summary statistics and allow valid comparison of the populations. There are two types of standardization:

- Direct standardization computes the weighted average of stratum-specific estimates in the study population, using the weights from a standard or reference population. This standardization is applicable when the study population is large enough to provide stable stratum-specific estimates. The directly standardized estimate is the overall crude rate in the study population if it has the same strata distribution as the reference population. When standardized estimates for different populations are derived by using the same reference population, the resulting estimates can also be compared by using the estimated difference and estimated ratio statistics.
- Indirect standardization computes the weighted average of stratum-specific estimates in the reference population, using the weights from the study population. The ratio of the overall crude rate or risk in the study population and the corresponding weighted estimate in the reference population is the standardized morbidity ratio (SMR). This ratio is also the standardized mortality ratio if the event is death. SMR is used to compare rates or risks in the study and reference populations. With SMR, the indirectly standardized estimate is then computed as the product of the SMR and the overall crude estimate for the reference population. SMR and indirect standardization are applicable even when the study population is so small that the resulting stratum-specific rates are not stable.

Assuming that an effect, such as the rate difference between two populations, is homogeneous across strata, each stratum provides an estimate of the same effect. A pooled estimate of the effect can then be derived from these stratum-specific effect estimates. One way to estimate a homogeneous effect is the Mantel-Haenszel method (Greenland and Rothman 2008, p. 271). For a homogeneous rate difference effect between two populations, the Mantel-Haenszel estimate is identical to the difference between two directly standardized rates, but with weights derived from the two populations instead of from an explicitly specified reference population. The Mantel-Haenszel method can also be applied to other homogeneous effects between populations, such as the rate ratio, risk difference, and risk ratio.

The STDRATE procedure computes directly standardized rates and risks for study populations. For two study populations with the same reference population, PROC STDRATE compares directly standardized rates or risks from these two populations. For homogeneous effects across strata, PROC STDRATE computes Mantel-Haenszel estimates. The STDRATE procedure also computes indirectly standardized rates and risks, including SMR.

The attributable fraction measures the excess event rate or risk fraction in the exposed population that can be attributed to the exposure. The rate or risk ratio statistic is required in the attributable fraction computation, and the STDRATE procedure estimates the ratio by using either SMR or the rate ratio statistic in the Mantel-Haenszel estimates.

Although the STDRATE procedure provides useful summary standardized statistics, standardization is not a substitute for individual comparisons of stratum-specific estimates. PROC STDRATE provides summary statistics, such as rate and risk estimates and their confidence limits, in each stratum. In addition, PROC STDRATE also displays these stratum-specific statistics by using ODS Graphics.

Note that the term standardization has different meanings in other statistical applications. For example, the STDIZE procedure standardizes numeric variables in a SAS data set by subtracting a location measure and dividing by a scale measure.

Getting Started: STDRATE Procedure

This example illustrates indirect standardization and uses the standardized mortality ratio to compare the death rate from skin cancer between people who live in the state of Florida and people who live in the United States as a whole.

The Florida_C43 data set contains the stratum-specific mortality information for skin cancer in year 2000 for the state of Florida (Florida Department of Health 2000, 2013). The variable Age is a grouping variable that forms the strata in the standardization, and the variables Event and PYear identify the number of events and total person-years, respectively. The COMMA9. format is specified in the DATA step to input numerical values that contain commas in PYear.

```
data Florida_C43;
   input Age $1-5 Event PYear:comma9.;
   datalines;
00-04    0    953,785
05-14    0  1,997,935
15-24    4  1,885,014
25-34   14  1,957,573
35-44   43  2,356,649
45-54   72  2,088,000
55-64   70  1,548,371
65-74  126  1,447,432
75-84  136  1,087,524
85+     73    335,944
;
```

The US_C43 data set contains the corresponding stratum-specific mortality information for the United States in year 2000 (Miniño et al. 2002; US Bureau of the Census 2011). The variable Age is the grouping variable, and the variables Event and PYear identify the number of events and the total person-years, respectively.

```
data US_C43;
   input Age $1-5 Event:comma5. PYear:comma10.;
   datalines;
00-04      0  19,175,798
05-14      1  41,077,577
15-24     41  39,183,891
25-34    186  39,892,024
35-44    626  45,148,527
45-54  1,199  37,677,952
55-64  1,303  24,274,684
65-74  1,637  18,390,986
75-84  1,624  12,361,180
85+      803   4,239,587
;
```

The following statements invoke the STDRATE procedure and request indirect standardization to compare death rates between the state of Florida and the United States:

```
ods graphics on;
proc stdrate data=Florida_C43 refdata=US_C43
             method=indirect
             stat=rate(mult=100000)
             plots=all
             ;
   population event=Event total=PYear;
   reference  event=Event total=PYear;
   strata Age / stats smr;
run;
```

The DATA= and REFDATA= options name the study data set and reference data set, respectively. The METHOD=INDIRECT option requests indirect standardization. The STAT=RATE option specifies the rate as the frequency measure for standardization, and the MULT=100000 suboption (which is the default) displays the rates per 100,000 person-years in the table output and graphics output. The PLOTS=ALL option requests all appropriate plots with indirect standardization.

The POPULATION statement specifies the options that are related to the study population, and the EVENT= and TOTAL= options specify variables for the number of events and person-years in the study population, respectively.

The REFERENCE statement specifies the options related to the reference population, and the EVENT= and TOTAL= options specify variables for the number of events and person-years in the reference population, respectively.

The STRATA statement lists the variable Age that forms the strata. The STATS option requests a strata information table that contains stratum-specific statistics such as rates, and the SMR option requests a table of stratum-specific SMR estimates.

The "Standardization Information" table in Figure 109.1 displays the standardization information.

Figure 109.1 Standardization Information

The STDRATE Procedure

Standardization Information	
Data Set	WORK.FLORIDA_C43
Reference Data Set	WORK.US_C43
Method	Indirect Standardization
Statistic	Rate
Number of Strata	10
Rate Multiplier	100000

The STATS option in the STRATA statement requests that the "Indirectly Standardized Strata Statistics" table in Figure 109.2 display the strata information and expected number of events at each stratum. The MULT=100000 suboption in the STAT=RATE option requests that crude rates per $100,000$ person-years be displayed. The Expected Events column displays the expected number of events when the stratum-specific rates in the reference data set are applied to the corresponding person-years in the study data set.

Figure 109.2 Strata Information (Indirect Standardization)

The STDRATE Procedure

Indirectly Standardized Strata Statistics
Rate Multiplier = 100000

			Study Population							Reference Population		
			Population-Time							Population-Time		
Stratum Index	Age	Observed Events	Value	Proportion	Crude Rate	Standard Error	95% Normal Confidence Limits			Value	Proportion	Crude Rate
1	00-04	0	953785	0.0609	0.0000	0.00000	0.0000	0.0000		19175798	0.0681	0.0000
2	05-14	0	1997935	0.1276	0.0000	0.00000	0.0000	0.0000		41077577	0.1460	0.0024
3	15-24	4	1885014	0.1204	0.2122	0.10610	0.0042	0.4202		39183891	0.1392	0.1046
4	25-34	14	1957573	0.1250	0.7152	0.19114	0.3405	1.0898		39892024	0.1418	0.4663
5	35-44	43	2356649	0.1505	1.8246	0.27825	1.2793	2.3700		45148527	0.1604	1.3865
6	45-54	72	2088000	0.1333	3.4483	0.40638	2.6518	4.2448		37677952	0.1339	3.1822
7	55-64	70	1548371	0.0989	4.5209	0.54035	3.4618	5.5799		24274684	0.0863	5.3677
8	65-74	126	1447432	0.0924	8.7051	0.77551	7.1851	10.2250		18390986	0.0654	8.9011
9	75-84	136	1087524	0.0695	12.5055	1.07234	10.4037	14.6072		12361180	0.0439	13.1379
10	85+	73	335944	0.0215	21.7298	2.54328	16.7451	26.7146		4239587	0.0151	18.9405

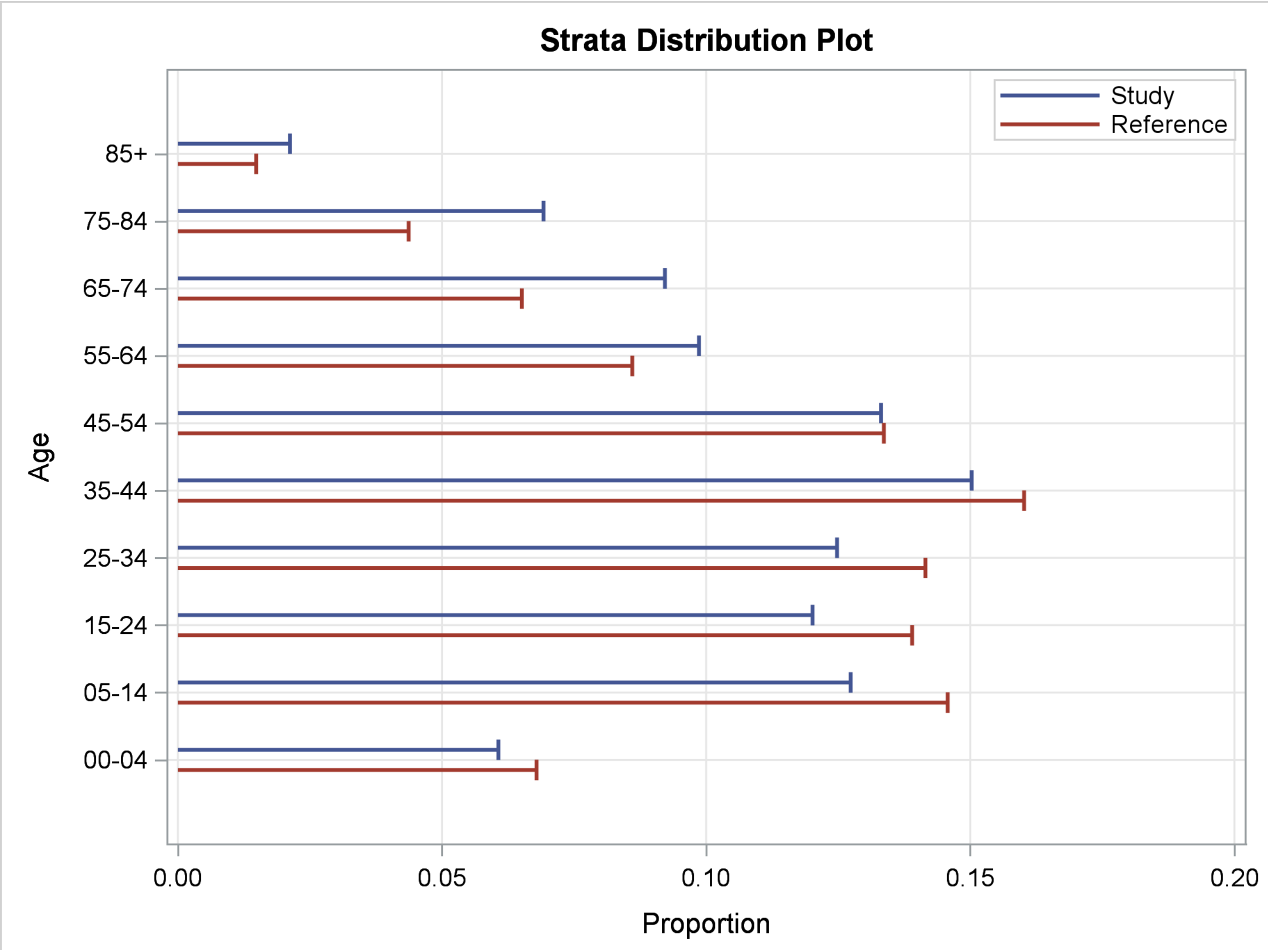
Indirectly Standardized Strata Statistics Rate Multiplier = 100000

Stratum Index	Expected Events
1	0.000
2	0.049
3	1.972
4	9.127
5	32.676
6	66.445
7	83.112
8	128.837
9	142.878
10	63.630

With ODS Graphics enabled, the PLOTS=ALL option displays all appropriate plots. With indirect standardization and a rate statistic, these plots include the strata distribution plot, the strata rate plot, and the strata SMR plot. By default, strata levels are displayed on the vertical axis for these plots.

The strata distribution plot displays proportions for stratum-specific person-years in the study and reference populations, as shown in Figure 109.3.

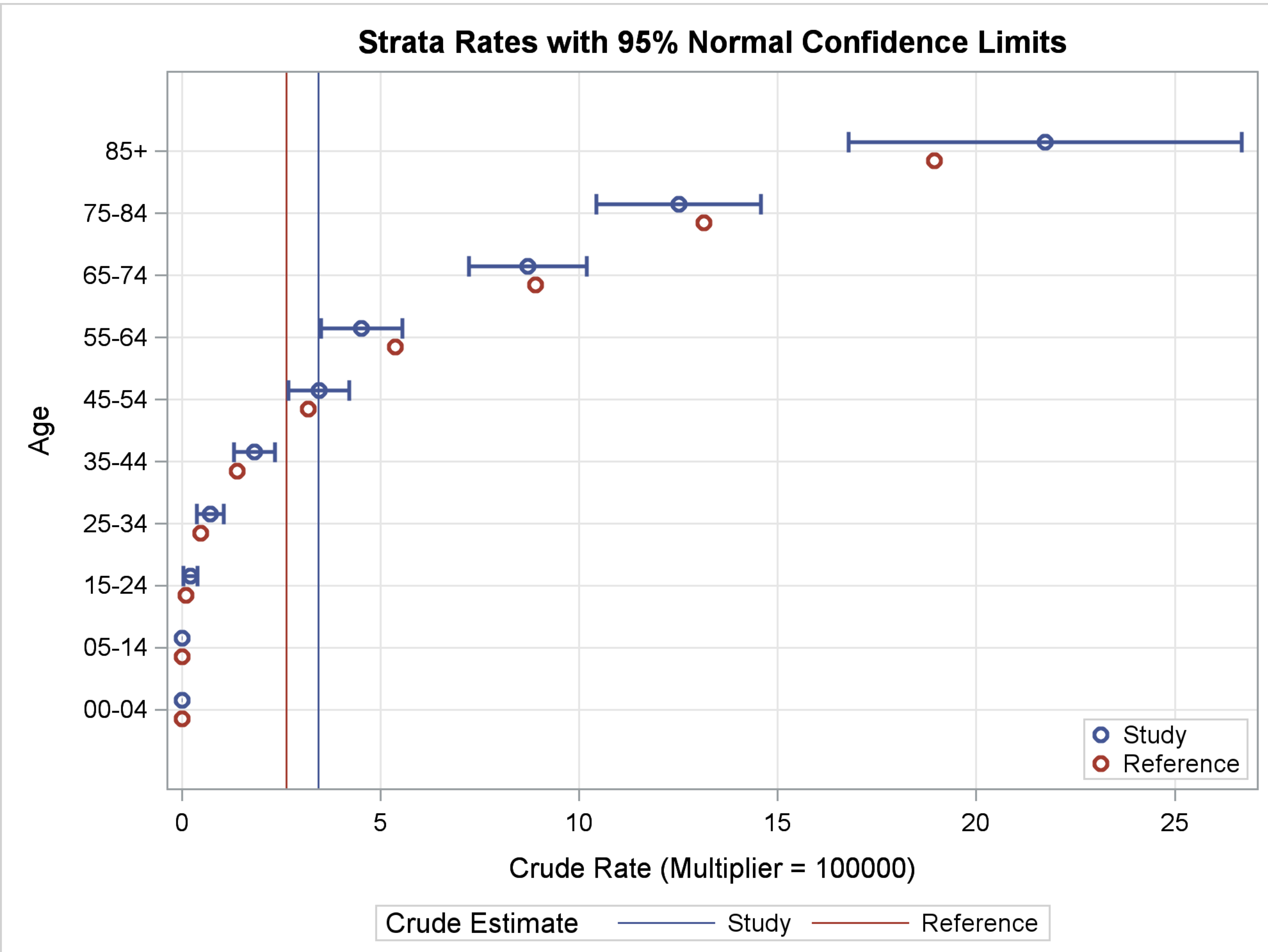
Figure 109.3 Strata Distribution Plot



The strata distribution plot displays the proportions in the "Indirectly Standardized Strata Statistics" table in Figure 109.2. In the plot, the proportions of the study population are identified by the blue lines, and the proportions of the reference population are identified by the red lines. The plot shows that the study population has higher proportions in older age groups and lower proportions in younger age groups than the reference population.

The strata rate plot displays stratum-specific rate estimates in the study and reference populations, as shown in Figure 109.4. This plot displays the rate estimates in the "Indirectly Standardized Strata Statistics" table in Figure 109.2. In addition, the plot displays the confidence limits for the rate estimates in the study population and the overall crude rates for the two populations.

Figure 109.4 Strata Rate Plot



The SMR option in the STRATA statement requests that the "Strata SMR Estimates" table in Figure 109.5 display the strata SMR at each stratum. The MULT=100000 suboption in the STAT=RATE option requests that the reference rates per $100,000$ person-years be displayed.

Figure 109.5 Strata SMR Information

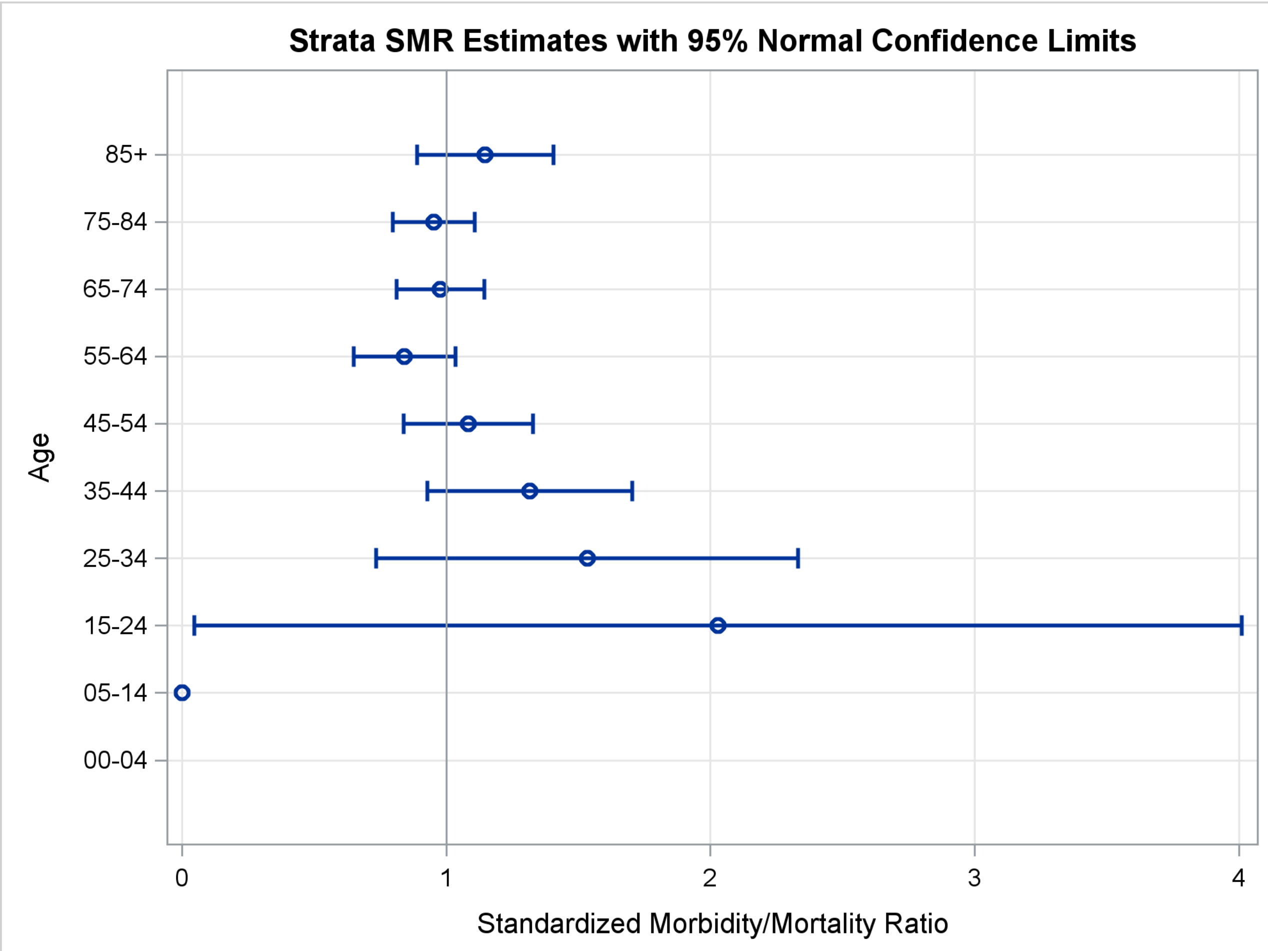
Strata SMR Estimates
Rate Multiplier = 100000

		Study Population							
Stratum Index	Age	Observed Events	Population-Time	Reference Crude Rate	Expected Events	SMR	Standard Error	95% Normal Confidence Limits	
1	00-04	0	953785	0.0000	0.000	.	.	.	.
2	05-14	0	1997935	0.0024	0.049	0.0000	.	.	.
3	15-24	4	1885014	0.1046	1.972	2.0280	1.0140	0.0406	4.0154
4	25-34	14	1957573	0.4663	9.127	1.5339	0.4099	0.7304	2.3373
5	35-44	43	2356649	1.3865	32.676	1.3160	0.2007	0.9226	1.7093
6	45-54	72	2088000	3.1822	66.445	1.0836	0.1277	0.8333	1.3339
7	55-64	70	1548371	5.3677	83.112	0.8422	0.1007	0.6449	1.0395
8	65-74	126	1447432	8.9011	128.837	0.9780	0.0871	0.8072	1.1487
9	75-84	136	1087524	13.1379	142.878	0.9519	0.0816	0.7919	1.1118
10	85+	73	335944	18.9405	63.630	1.1473	0.1343	0.8841	1.4104

The "Strata SMR Estimates" table shows that although SMR is less than 1 only at three age strata (55–64, 65–74, and 75–84), these three strata contain about 60% of the total events.

The strata SMR plot displays stratum-specific SMR estimates with confidence limits, as shown in Figure 109.6. The plot displays the SMR estimates in the "Strata SMR Estimates" table in Figure 109.5.

Figure 109.6 Strata SMR Plot



The METHOD=INDIRECT option requests that the "Standardized Morbidity/Mortality Ratio" table in Figure 109.7 be displayed. The table displays the SMR, its confidence limits, and the test for the null hypothesis H_0 : SMR $=$ 1. The default ALPHA=0.05 option requests that 95% confidence limits be constructed.

Figure 109.7 Standardized Morbidity/Mortality Ratio

Standardized Morbidity/Mortality Ratio

Observed Events	Expected Events	SMR	Standard Error	95% Normal Confidence Limits		Z	Pr > |Z|
538	528.726	1.0175	0.0439	0.9316	1.1035	0.40	0.6893

The 95% normal confidence limits contain the null hypothesis value SMR $=$ 1, and the hypothesis of SMR $=$ 1 is not rejected at the $\alpha = 0.05$ level from the normal test.

The “Indirectly Standardized Rate Estimates” table in Figure 109.8 displays the indirectly standardized rate and related statistics.

Figure 109.8 Standardized Rate Estimates (Indirect Standardization)

Indirectly Standardized Rate Estimates
Rate Multiplier = 100000

Study Population						Standardized Rate			
Observed Events	Population-Time	Crude Rate	Reference Crude Rate	Expected Events	SMR	Estimate	Standard Error	95% Normal Confidence Limits	
538	15658227	3.4359	2.6366	528.726	1.0175	2.6829	0.1157	2.4562	2.9096

The indirectly standardized rate estimate is the product of the SMR and the crude rate estimate for the reference population. The table shows that although the crude rate in the state of Florida (3.4359) is 30% higher than the crude rate in the U.S. (2.6366), the resulting standardized rate (2.6829) is close to the crude rate in the U.S.

Syntax: STDRATE Procedure

The following statements are available in PROC STDRATE:

PROC STDRATE < *options* > **;**
  BY *variables* **;**
  POPULATION *options* **;**
  REFERENCE *options* **;**
  STRATA *variables* < / *option* > **;**

The PROC STDRATE statement invokes the procedure, names the data sets, specifies the standardization method, and identifies the statistic for standardization. The BY statement requests separate analyses of groups defined by the BY variables. The required POPULATION statement specifies the rate or risk information in study populations, and the REFERENCE statement specifies the rate or risk information in the reference population. The STRATA statement lists the variables that form the strata.

The following sections describe the PROC STDRATE statement and then describe the other statements in alphabetical order.

PROC STDRATE Statement

PROC STDRATE < *options* > **;**

Table 109.1 summarizes the *options* in the PROC STDRATE statement.

Table 109.1 Summary of PROC STDRATE Options

Option	Description
Input Data Sets	
DATA=	Names the SAS data set that contains the study populations
REFDATA=	Names the SAS data set that contains the reference population
Standardization Methods	
METHOD=	Specifies the method for standardization
STAT=	Specifies the statistic for standardization
EFFECT	Specifies the test to compare study populations for direct standardization and Mantel-Haenszel estimation
Displayed Output	
ALPHA=	Specifies the significance level for confidence intervals
CL=	Requests the confidence limits for the standardized estimates
PLOTS	Requests stratum-specific plots

You can specify the following *options* in the PROC STDRATE statement to compute standardized rates and risks in the procedure. They are listed in alphabetical order.

ALPHA=α

requests that confidence limits be constructed with confidence level $100(1-\alpha)\%$, where $0 < \alpha < 1$. The default is ALPHA=0.05. These confidence limits include confidence limits for the stratum-specific rates or risks, standardized rate and risk, standardized morbidity/mortality ratio, and population attributable rate and risk.

CL=GAMMA < (TYPE=AVERAGE | CONSERVATIVE) > | LOGNORMAL | NONE | NORMAL | POISSON

specifies the method to construct confidence limits for SMR and standardized rate and risk. You can specify the following values for this option:

GAMMA

requests confidence limits based on a gamma distribution for METHOD=DIRECT and METHOD=MH. This value applies only when STAT=RATE. You can specify the TYPE=CONSERVATIVE suboption to request conservative confidence limits that are based on a gamma distribution and were developed by Fay and Feuer (1997), or you can use the default TYPE=AVERAGE suboption to request modified confidence limits proposed by Tiwari, Clegg, and Zou (2006).

LOGNORMAL
requests confidence limits based on a lognormal distribution.

NONE
suppresses construction of confidence limits.

NORMAL
requests confidence limits based on a normal distribution.

POISSON
requests confidence limits based on a Poisson distribution. This value applies only when METHOD=INDIRECT.

The default is CL=NORMAL.

DATA=*SAS-data-set*
names the required SAS data set that contains the event information in the study populations.

EFFECT < =DIFF | RATIO >
displays a table of the effect estimate and associated confidence limits. This option applies only when METHOD=DIRECT with two study populations and when METHOD=MH, where two study populations are required.

EFFECT and EFFECT=RATIO display a test on the ratio effect of estimates between the study populations, and the EFFECT=DIFF option displays a test on the difference effect.

METHOD= DIRECT | INDIRECT < (AF) > | MH < (AF) >

M= DIRECT | INDIRECT < (AF) > | MH < (AF) >
specifies the required method for standardization. The AF suboption (available only for METHOD=INDIRECT or METHOD=MH) requests the attributable fraction, which measures how much of the excess event rate or risk fraction in the exposed population is attributable to the exposure. This suboption also requests the population attributable fraction, which measures how much of the excess event rate or risk fraction in the total population is attributable to the exposure.

You can specify the following values:

DIRECT requests direct standardization.

INDIRECT requests indirect standardization. If you specify the AF suboption, the study population is treated as the exposed population and the reference population is treated as the unexposed population.

MH requests Mandel-Haenszel estimation. The order of the two study populations is indicated by the ORDER= suboption in the GROUP option in the POPULATION statement. If you specify the AF suboption, the exposed population is identified by the EXPOSED= suboption in the GROUP option in the POPULATION statement. If the EXPOSED= suboption is not specified, then the first study population is treated as the exposed population and the second study population is treated as the unexposed population.

PLOTS < (*global-options*) > < = *plot-request* >

PLOTS < (*global-options*) > < = (*plot-request* < ... *plot-request* >) >

specifies options that control the details of the plots. The default is PLOTS=RATE for STAT=RATE and PLOTS=RISK for STAT=RISK.

You can specify the following *global-options*:

DISPLAY=INDEX | LEVEL

specifies tick mark values for the strata axis. DISPLAY=LEVEL displays strata levels on the strata axis, and DISPLAY=INDEX displays strata indices of sequential strata identification numbers on the strata axis. The default is DISPLAY=LEVEL.

ONLY

suppresses the default plots and displays only plots that are specifically requested.

STRATUM=HORIZONTAL | VERTICAL

controls the orientation of the plots. STRATUM=VERTICAL places the strata information on the vertical axis, and STRATUM=HORIZONTAL places the strata information on the horizontal axis. The default is STRATUM=VERTICAL.

You can specify the following *plot-requests*:

ALL

produces all appropriate plots.

DIST | DISTRIBUTION

displays a plot of the proportions for stratum-specific exposed time or sample size.

EFFECT

displays a plot of the stratum-specific effect estimates and associated confidence limits. This option applies only when METHOD=DIRECT with two study populations and when METHOD=MH, where two study populations are required. If the EFFECT=DIFF option is specified, the stratum-specific rate or risk difference effects are displayed. Otherwise, the stratum-specific rate or risk ratio effects are displayed.

NONE

suppresses all plots.

RATE

displays a plot of the stratum-specific rates and associated confidence limits. This option applies only when STAT=RATE. If a confidence limits method is specified in the STATS(CL=) option in the STRATA statement, that method is used to compute the confidence limits. Otherwise, the normal approximation is used.

RISK

displays a plot of the stratum-specific risks and associated confidence limits. This option applies only when STAT=RISK. If a confidence limits method is specified in the STATS(CL=) option in the STRATA statement, that method is used to compute the confidence limits. Otherwise, the normal approximation is used.

SMR

displays a plot of the stratum-specific SMR estimates and associated confidence limits. This option applies only when METHOD=INDIRECT. If a method is specified in the SMR(CL=) option in the STRATA statement, that method is used to compute the confidence limits. Otherwise, the normal approximation is used.

REFDATA=*SAS-data-set*

names the required SAS data set that contains the event information in the reference population.

STAT=RATE < (MULT =*c* **) >**

STAT=RISK

specifies the statistic for standardization. STAT=RATE computes standardized rates, and STAT=RISK computes standardized risks. The default is STAT=RATE.

The MULT= suboption in the STAT=RATE option specifies a power of 10 constant *c*, and requests that rates per *c* population-time units be displayed in the output tables and graphics. The default is MULT=100000, which specifies rates per 100,000 population-time units.

BY Statement

BY *variables* **;**

You can specify a BY statement with PROC STDRATE to obtain separate analyses of observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the STDRATE procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

For more information about BY-group processing, see the discussion in *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the discussion in the *Base SAS Procedures Guide*.

POPULATION Statement

POPULATION < *options* > **;**

The required POPULATION statement specifies the information in the study data set. You can specify the following *options* in the POPULATION statement:

EVENT=*variable*

specifies the *variable* for the number of events in the study data set.

GROUP < (*group-options* **) > =***variable*

specifies the *variable* whose values identify the various populations. The GROUP= option is required when METHOD=MH and also applies when METHOD=DIRECT in the PROC STDRATE statement.

You can specify the following *group-options*:

EXPOSED='*group***'**

identifies the exposed group in the derivation of the attributable fraction. This option applies only when you specify METHOD=MH(AF). If you do not specify the EXPOSED= option, the first study population, as indicated by the ORDER= option, is treated as the exposed population.

ORDER=DATA | FORMATTED | INTERNAL

specifies the order in which the values of the *variable* are to be displayed. You can specify the following values for the ORDER= suboption:

DATA	sorts by the order in which the values appear in the input data set.
FORMATTED	sorts by their external formatted values.
INTERNAL	sorts by the unformatted values, which yields the same order that the SORT procedure does.

By default, ORDER=INTERNAL. For ORDER=FORMATTED and ORDER=INTERNAL, the sort order is machine-dependent.

POPEVENT=*number*

specifies the total number of events in the study data set. This option applies only when METHOD=INDIRECT is specified in the PROC STDRATE statement and the total number of events is not available in the study data set.

RATE < (MULT=*c* **) > =** *variable*

specifies the *variable* for the observed rate in the study data set. This option applies only when STAT=RATE is specified in the PROC STDRATE statement. The MULT=*c* suboption specifies a power of 10 constant *c* and requests that the rates per *c* population-time units be read from the data set. The default is the value of the MULT= suboption used in the STAT=RATE option in the PROC STDRATE statement.

RISK=*variable*

specifies the *variable* for the observed risk in the study data set. This option applies only when STAT=RISK is specified in the PROC STDRATE statement.

TOTAL=*variable*
: specifies the *variable* for either the population-time (STAT=RATE) or the number of observations (STAT=RISK) in the study data set.

REFERENCE Statement

REFERENCE < *options* > **;**

The REFERENCE statement specifies the information in the reference data set. This statement is required when METHOD=DIRECT or METHOD=INDIRECT is specified in the PROC STDRATE statement.

You can specify the following *options* in the REFERENCE statement:

EVENT=*variable*
: specifies the *variable* for the number of events in the reference data set.

RATE < (MULT=*c* **) > =** *variable*
: specifies the *variable* for the observed rate in the reference data set. This option applies only when STAT=RATE is specified in the PROC STDRATE statement. The MULT=*c* suboption specifies a power of 10 constant *c* and requests that the rates per *c* population-time units be read from the data set. The default is the value of the MULT= suboption used in the STAT=RATE option in the PROC STDRATE statement.

RISK=*variable*
: specifies the *variable* for the observed risk in the reference data set. This option applies only when STAT=RISK is specified in the PROC STDRATE statement.

TOTAL=*variable*
: specifies the *variable* for either the population-time (STAT=RATE) or the number of observations (STAT=RISK) in the reference data set.

When METHOD=INDIRECT is specified in the PROC STDRATE statement, the overall reference population rate and risk are needed to compute indirect standardized rate and risk, respectively. If the information is not available in the reference data set, you can specify the following *options* for overall reference population rate and risk.

REFEVENT=*number*
: specifies the total *number* of events in the reference data set.

REFRATE < (MULT=*c* **) > =** *number*
: specifies the crude rate in the reference data set. This option applies only when STAT=RATE is specified in the PROC STDRATE statement. The MULT=*c* suboption specifies a power of 10 constant *c*, and the *number* is the crude rate per *c* population-time units in the data set. The default is the value of the MULT= suboption in the STAT=RATE option in the PROC STDRATE statement.

REFRISK=*number*
: specifies the crude risk in the reference data set. This option applies only when STAT=RISK is specified in the PROC STDRATE statement.

REFTOTAL=*number*

specifies either the total population-time (STAT=RATE) or the total number of observations (STAT=RISK) in the reference data set.

When STAT=RATE, the REFRATE= option specifies the crude reference rate for the indirect standardized rate. If the REFRATE= option is not specified, the REFEVENT= and REFTOTAL options can be used to compute the crude reference rate. Similarly, when STAT=RISK, the REFRISK= option specifies the crude reference risk for the indirect standardized rate. If the REFRISK= option is not specified, the REFEVENT= and REFTOTAL options can be used to compute the crude reference risk.

STRATA Statement

STRATA *variables* < */ options* > **;**

The STRATA statement names *variables* that form the strata in the standardization. The combinations of categories of STRATA variables define the strata in the population.

The STRATA variables are one or more variables in all input data sets. These variables can be either character or numeric. The formatted values of the STRATA variables determine the levels. Thus, you can use formats to group values into levels. See the FORMAT procedure in the *Base SAS Procedures Guide* and the FORMAT statement and SAS formats in *SAS Language Reference: Dictionary* for more information.

When the STRATA statement is not specified or the statement is specified without variables, all observations in a data set are treated as though they are from a single stratum.

You can specify the following *options* in the STRATA statement after a slash (/):

EFFECT

displays a table of the stratum-specific effect estimates and associated confidence limits. This option applies only when METHOD=DIRECT with two study populations and when METHOD=MH, where two study populations are required. If the EFFECT=DIFF option in the PROC STDRATE statement is specified, the stratum-specific rate or risk difference effects are displayed. Otherwise, the stratum-specific rate or risk ratio effects are displayed.

MISSING

treats missing values as a valid (nonmissing) category for all STRATA variables. When PROC STDRATE determines levels of a STRATA variable, an observation with missing values for that STRATA variable is excluded, unless the MISSING option is specified.

ORDER=DATA | FORMATTED | INTERNAL

specifies the order in which the values of the categorical variables are to be displayed. You can specify the following values for the ORDER= option:

DATA	sorts by the order in which the values appear in the input data set.
FORMATTED	sorts by their external formatted values.
INTERNAL	sorts by the unformatted values, which yields the same order that the SORT procedure does.

By default, ORDER=INTERNAL. For ORDER=FORMATTED and ORDER=INTERNAL, the sort order is machine-dependent.

STATS < (CL=LOGNORMAL | NONE | NORMAL | POISSON) >

displays tables for stratum-specific statistics such as stratum-specific rates and risks. You can specify the following values of the CL= suboption to request confidence limits for the rate or risk estimate in each stratum:

LOGNORMAL

requests confidence limits based on a lognormal approximation.

NONE

suppresses confidence limits.

NORMAL

requests confidence limits based on a normal approximation and also displays the standard error for the rate estimate in each stratum.

POISSON

requests confidence limits based on a Poisson distribution for stratum-specific rates. This values applies only when STAT=RATE in the PROC STDRATE statement.

The default is CL=NORMAL.

SMR < (CL=LOGNORMAL | NONE | NORMAL | POISSON) >

displays tables for stratum-specific SMR estimates. This option applies only when METHOD=INDIRECT is specified in the PROC STDRATE statement. You can specify the following values of the CL= suboption to request confidence limits for the SMR estimate in each stratum:

LOGNORMAL

requests confidence limits based on a lognormal approximation.

NONE

suppresses confidence limits.

NORMAL

requests confidence limits based on a normal approximation and also displays the standard error for the SMR estimate in each stratum.

POISSON

requests confidence limits based on a Poisson distribution for stratum-specific SMR estimates. This values applies only when STAT=RATE in the PROC STDRATE statement.

The default is CL=NORMAL.

Details: STDRATE Procedure

Rate

A major task in epidemiology is to compare event frequencies for groups of people. Both rate and risk are commonly used to measure event frequency in the comparison. Rate is a measure of change in one quantity per unit of another quantity. An event rate measures how fast the events are occurring. In contrast, an event risk is the probability that an event occurs over a specified follow-up time period.

An event rate of a population over a specified time period can be defined as the number of new events divided by the population-time of the population over the same time period,

$$\hat{\lambda} = \frac{d}{\mathcal{T}}$$

where d is the number of events and $\mathcal{T}$ is the population-time that is computed by adding up the time contributed by each subject in the population over the specified time period.

For a general population, the subsets (strata) might not be homogeneous enough to have a similar rate. Thus, the rate for each stratum should be computed separately to reflect this discrepancy. For a population that consists of K homogeneous strata (such as different age groups), the stratum-specific rate for the jth stratum in a population is computed as

$$\hat{\lambda}_j = \frac{d_j}{\mathcal{T}_j}$$

where d_j is the number of events and $\mathcal{T}_j$ is the population-time for subjects in the jth stratum of the population.

Assuming the number of events in the jth stratum, d_j, has a Poisson distribution, the variance of $\hat{\lambda}_j$ is

$$V(\hat{\lambda}_j) = V(\frac{d_j}{\mathcal{T}_j}) = \frac{1}{{\mathcal{T}_j}^2} V(d_j) = \frac{d_j}{\mathcal{T}_j^2} = \frac{\hat{\lambda}_j}{\mathcal{T}_j}$$

By using the method of statistical differentials (Elandt-Johnson and Johnson 1980, pp. 70–71), the variance of the logarithm of rate can be estimated by

$$V(\log(\hat{\lambda}_j)) = \frac{1}{\hat{\lambda}_j^2} V(\hat{\lambda}_j) = \frac{1}{\hat{\lambda}_j^2} \frac{\hat{\lambda}_j}{\mathcal{T}_j} = \frac{1}{\hat{\lambda}_j \, \mathcal{T}_j} = \frac{1}{d_j}$$

Because the rate value can be very small, especially for rare events, it is sometimes expressed in terms of the product of a multiplier and the rate itself. For example, a rate can be expressed as the number of events per 100,000 person-years.

Normal Distribution Confidence Interval for Rate

A $(1-\alpha)$ confidence interval for $\hat{\lambda}_j$ based on a normal distribution is given by

$$\left(\hat{\lambda}_j - z \sqrt{V(\hat{\lambda}_j)}, \ \ \hat{\lambda}_j + z \sqrt{V(\hat{\lambda}_j)} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution.

Lognormal Distribution Confidence Interval for Rate

A $(1-\alpha)$ confidence interval for $\log(\hat{\lambda}_j)$ based on a normal distribution is given by

$$\left(\log(\hat{\lambda}_j) - z \sqrt{V(\log(\hat{\lambda}_j))}, \ \ \log(\hat{\lambda}_j) + z \sqrt{V(\log(\hat{\lambda}_j))} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution and the variance $V(\log(\hat{\lambda}_j)) = 1/d_j$.

Thus, a $(1-\alpha)$ confidence interval for $\hat{\lambda}_j$ based on a lognormal distribution is given by

$$\left(\hat{\lambda}_j \, e^{-\frac{z}{\sqrt{d_j}}}, \ \ \hat{\lambda}_j \, e^{\frac{z}{\sqrt{d_j}}} \right)$$

Poisson Distribution Confidence Interval for Rate

Denote the $(\alpha/2)$ quantile for the χ^2 distribution with $2\,d_j$ degrees of freedom by

$$q_{lj} = (\chi^2_{2\,d_j})^{-1}\,(\alpha/2)$$

Denote the $(1-\alpha/2)$ quantiles for the χ^2 distribution with $2(d_j+1)$ degrees of freedom by

$$q_{uj} = (\chi^2_{2\,(d_j+1)})^{-1}\,(1-\alpha/2)$$

Then a $(1-\alpha)$ confidence interval for $\hat{\lambda}_j$ based on the χ^2 distribution is given by

$$\left(\frac{q_{lj}}{2\,\mathcal{T}_j}, \ \ \frac{q_{uj}}{2\,\mathcal{T}_j} \right)$$

Confidence Interval for Rate Difference Statistic

For rate estimates from two independent samples, $\hat{\lambda}_{1j}$ and $\hat{\lambda}_{2j}$, a $(1-\alpha)$ confidence interval for the rate difference $\hat{\lambda}_{dj} = \hat{\lambda}_{1j} - \hat{\lambda}_{2j}$ is

$$\left(\hat{\lambda}_{dj} - z \sqrt{V(\hat{\lambda}_{dj})}, \ \ \hat{\lambda}_{dj} + z \sqrt{V(\hat{\lambda}_{dj})} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution and the variance

$$V(\hat{\lambda}_{dj}) = V(\hat{\lambda}_{1j}) + V(\hat{\lambda}_{2j})$$

Confidence Interval for Rate Ratio Statistic

For rate estimates from two independent samples, $\hat{\lambda}_{1j}$ and $\hat{\lambda}_{2j}$, a $(1-\alpha)$ confidence interval for the log rate ratio statistic $\log(\hat{\lambda}_{rj}) = \log(\hat{\lambda}_{1j}/\hat{\lambda}_{2j})$ is

$$\left(\log(\hat{\lambda}_{rj}) - z\,\sqrt{V(\log(\hat{\lambda}_{rj}))}\,,\;\; \log(\hat{\lambda}_{rj}) + z\,\sqrt{V(\log(\hat{\lambda}_{rj}))} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution and the variance

$$V(\log(\hat{\lambda}_{rj})) = V(\log(\hat{\lambda}_{1j})) + V(\log(\hat{\lambda}_{2j}))$$

Thus, a $(1-\alpha)$ confidence interval for the rate ratio statistic $\hat{\lambda}_{rj}$ is given by

$$\left(\frac{\hat{\lambda}_{1j}}{\hat{\lambda}_{2j}}\; e^{-z\sqrt{V(\log(\hat{\lambda}_{rj}))}}\,,\;\; \frac{\hat{\lambda}_{1j}}{\hat{\lambda}_{2j}}\; e^{z\sqrt{V(\log(\hat{\lambda}_{rj}))}} \right)$$

Confidence Interval for Rate SMR

At stratum j, a stratum-specific standardized morbidity/mortality ratio is

$$\mathcal{R}_j = \frac{d_j}{\mathcal{E}_j}$$

where $\mathcal{E}_j$ is the expected number of events.

With the rate

$$\hat{\lambda}_j = \frac{d_j}{\mathcal{T}_j}$$

SMR can be expressed as

$$\mathcal{R}_j = \frac{\mathcal{T}_j}{\mathcal{E}_j}\,\hat{\lambda}_j$$

Thus, a $(1-\alpha)$ confidence interval for $\mathcal{R}_j$ is given by

$$\left(\frac{\mathcal{T}_j}{\mathcal{E}_j}\,\hat{\lambda}_{jl}\,,\;\; \frac{\mathcal{T}_j}{\mathcal{E}_j}\,\hat{\lambda}_{ju} \right)$$

where $(\,\hat{\lambda}_{jl}\,,\,\hat{\lambda}_{ju}\,)$ is a $(1-\alpha)$ confidence interval for the rate $\hat{\lambda}_j$.

Risk

An event risk of a population over a specified time period can be defined as the number of new events in the follow-up time period divided by the event-free population size at the beginning of the time period,

$$\hat{\gamma} = \frac{d}{\mathcal{N}}$$

where $\mathcal{N}$ is the population size.

For a general population, the subsets (strata) might not be homogeneous enough to have a similar risk. Thus, the risk for each stratum should be computed separately to reflect this discrepancy. For a population that consists of K homogeneous strata (such as different age groups), the stratum-specific risk for the jth stratum in a population is computed as

$$\hat{\gamma}_j = \frac{d_j}{\mathcal{N}_j}$$

where $\mathcal{N}_j$ is the population size in the jth stratum of the population.

Assuming the number of events, d_j, has a binomial distribution, then a variance estimate of $\hat{\gamma}_j$ is

$$V(\hat{\gamma}_j) = \frac{\hat{\gamma}_j(1-\hat{\gamma}_j)}{\mathcal{N}_j}$$

By using the method of statistical differentials (Elandt-Johnson and Johnson 1980, pp. 70–71), the variance of the logarithm of risk can be estimated by

$$V(\log(\hat{\gamma}_j)) = \frac{1}{\hat{\gamma}_j^2}\, V(\hat{\gamma}_j) = \frac{1}{\hat{\gamma}_j^2}\, \frac{\hat{\gamma}_j\,(1-\hat{\gamma}_j)}{\mathcal{N}_j} = \frac{1-\hat{\gamma}_j}{\hat{\gamma}_j\, \mathcal{N}_j} = \frac{1}{d_j} - \frac{1}{\mathcal{N}_j}$$

Normal Distribution Confidence Interval for Risk

A $(1-\alpha)$ confidence interval for $\hat{\gamma}_j$ based on a normal distribution is given by

$$\left(\hat{\gamma}_j - z\, \sqrt{V(\hat{\gamma}_j)}\,,\;\; \hat{\gamma}_j + z\, \sqrt{V(\hat{\gamma}_j)} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution.

Lognormal Distribution Confidence Interval for Risk

A $(1-\alpha)$ confidence interval for $\log(\hat{\gamma}_j)$ based on a normal distribution is given by

$$\left(\log(\hat{\gamma}_j) - z\, \sqrt{V(\log(\hat{\gamma}_j))}\,,\;\; \log(\hat{\gamma}_j) + z\, \sqrt{V(\log(\hat{\gamma}_j))} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution and the variance $V(\log(\hat{\gamma}_j)) = 1/d_j - 1/\mathcal{N}_j$.

Thus, a $(1-\alpha)$ confidence interval for $\hat{\gamma}_j$ based on a lognormal distribution is given by

$$\left(\hat{\gamma}_j\; e^{-z\sqrt{\frac{1}{d_j} - \frac{1}{\mathcal{N}_j}}}\,,\;\; \hat{\gamma}_j\; e^{z\sqrt{\frac{1}{d_j} - \frac{1}{\mathcal{N}_j}}} \right)$$

Confidence Interval for Risk Difference Statistic

For rate estimates from two independent samples, $\hat{\gamma}_{1j}$ and $\hat{\gamma}_{2j}$, a $(1-\alpha)$ confidence interval for the risk difference $\hat{\gamma}_{dj} = \hat{\gamma}_{1j} - \hat{\gamma}_{2j}$ is

$$\left(\hat{\gamma}_{dj} - z\, \sqrt{V(\hat{\gamma}_{dj})}\,,\;\; \hat{\gamma}_{dj} + z\, \sqrt{V(\hat{\gamma}_{dj})} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution and the variance

$$V(\hat{\gamma}_{dj}) = V(\hat{\gamma}_{1j}) + V(\hat{\gamma}_{2j})$$

Confidence Interval for Risk Ratio Statistic

For rate estimates from two independent samples, $\hat{\gamma}_{1j}$ and $\hat{\gamma}_{2j}$, a $(1-\alpha)$ confidence interval for the log risk ratio statistic $\log(\hat{\gamma}_{rj}) = \log(\hat{\gamma}_{1j}/\hat{\gamma}_{2j})$ is

$$\left(\log(\hat{\gamma}_{rj}) - z \sqrt{V(\log(\hat{\gamma}_{rj}))} \, , \;\; \log(\hat{\gamma}_{rj}) + z \sqrt{V(\log(\hat{\gamma}_{rj}))} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution and the variance

$$V(\log(\hat{\gamma}_{rj}) = V(\log(\hat{\gamma}_{1j})) + V(\log(\hat{\gamma}_{2j}))$$

Thus, a $(1-\alpha)$ confidence interval for the risk ratio statistic $\hat{\gamma}_{rj}$ is given by

$$\left(\frac{\hat{\gamma}_{1j}}{\hat{\gamma}_{2j}} \, e^{-z\sqrt{V(\log(\hat{\gamma}_{rj}))}} \, , \;\; \frac{\hat{\gamma}_{1j}}{\hat{\gamma}_{2j}} \, e^{z\sqrt{V(\log(\hat{\gamma}_{rj}))}} \right)$$

Confidence Interval for Risk SMR

At stratum j, a stratum-specific standardized morbidity/mortality ratio is

$$\mathcal{R}_j = \frac{d_j}{\mathcal{E}_j}$$

where $\mathcal{E}_j$ is the expected number of events.

With the risk

$$\hat{\gamma}_j = \frac{d_j}{\mathcal{N}_j}$$

SMR can be expressed as

$$\mathcal{R}_j = \frac{\mathcal{N}_j}{\mathcal{E}_j} \, \hat{\gamma}_j$$

Thus, a $(1-\alpha)$ confidence interval for $\mathcal{R}_j$ is given by

$$\left(\frac{\mathcal{N}_j}{\mathcal{E}_j} \, \hat{\gamma}_{jl} \, , \;\; \frac{\mathcal{N}_j}{\mathcal{E}_j} \, \hat{\gamma}_{ju} \right)$$

where $(\, \hat{\gamma}_{jl} \, , \, \hat{\gamma}_{ju} \,)$ is a $(1-\alpha)$ confidence interval for the risk $\hat{\gamma}_j$.

Direct Standardization

Direct standardization uses the weights from a reference population to compute the standardized rate of a study group as the weighted average of stratum-specific rates in the study population. The standardized rate is computed as

$$\hat{\lambda}_{ds} = \frac{\sum_j \mathcal{T}_{rj} \, \hat{\lambda}_{sj}}{\mathcal{T}_r}$$

where $\hat{\lambda}_{sj}$ is the rate in the jth stratum of the study population, $\mathcal{T}_{rj}$ is the population-time in the jth stratum of the reference population, and $\mathcal{T}_r = \sum_k \mathcal{T}_{rk}$ is the population-time in the reference population.

Similarly, direct standardization uses the weights from a reference population to compute the standardized risk of a study group as the weighted average of stratum-specific risks in the study population. The standardized risk is computed as

$$\hat{\gamma}_{ds} = \frac{\sum_j \mathcal{N}_{rj} \, \hat{\gamma}_{sj}}{\mathcal{N}_r}$$

where $\hat{\gamma}_{sj}$ is the risk in the jth stratum of the study population, $\mathcal{N}_{rj}$ is the number of observations in the jth stratum of the reference population, and $\mathcal{N}_r = \sum_k \mathcal{N}_{rk}$ is the total number of observations in the reference population.

That is, the directly standardized rate and risk of a study population are weighted averages of the stratum-specific rates and risks, respectively, where the weights are the corresponding strata population sizes in the reference population. The direct standardization can be used when the study population is large enough to provide stable stratum-specific rates or risks. When the same reference population is used for multiple study populations, directly standardized rates and risks provide valid comparisons between study populations.

The variances of the directly standardized rate and risk are

$$V(\hat{\lambda}_{ds}) = V\left(\frac{\sum_j \mathcal{T}_{rj} \, \hat{\lambda}_{sj}}{\mathcal{T}_r}\right) = \frac{\sum_j \mathcal{T}_{rj}^2 \, V(\hat{\lambda}_{sj})}{\mathcal{T}_r^2}$$

$$V(\hat{\gamma}_{ds}) = V\left(\frac{\sum_j \mathcal{N}_{rj} \, \hat{\gamma}_{sj}}{\mathcal{N}_r}\right) = \frac{\sum_j \mathcal{N}_{rj}^2 \, V(\hat{\gamma}_{sj})}{\mathcal{N}_r^2}$$

By using the method of statistical differentials (Elandt-Johnson and Johnson 1980, pp. 70–71), the variance of the logarithm of directly standardized rate and risk can be estimated by

$$V(\log(\hat{\lambda}_{ds})) = \frac{1}{\hat{\lambda}_{ds}^2} \, V(\hat{\lambda}_{ds})$$

$$V(\log(\hat{\gamma}_{ds})) = \frac{1}{\hat{\gamma}_{ds}^2} \, V(\hat{\gamma}_{ds})$$

The confidence intervals for $\hat{\lambda}_{ds}$ and $\hat{\gamma}_{ds}$ can be constructed based on normal and lognormal distributions. A gamma distribution confidence interval can also be constructed for $\hat{\lambda}_{ds}$.

In the next four subsections, $\beta = \lambda$ denotes the rate statistic and $\beta = \gamma$ denotes the risk statistic.

Normal Distribution Confidence Intervals for Standardized Rate and Risk

A $(1-\alpha)$ confidence interval for $\hat{\beta}_{ds}$ based on a normal distribution is then given by

$$\left(\hat{\beta}_{ds} - z \sqrt{V(\hat{\beta}_{ds})}\,, \;\; \hat{\beta}_{ds} + z \sqrt{V(\hat{\beta}_{ds})} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution.

Lognormal Distribution Confidence Intervals for Standardized Rate and Risk

A $(1-\alpha)$ confidence interval for $\log(\hat{\beta}_{ds})$ based on a normal distribution is given by

$$\left(\log(\hat{\beta}_{ds}) - z \sqrt{V(\log(\hat{\beta}_{ds}))} \, , \;\; \log(\hat{\beta}_{ds}) + z \sqrt{V(\log(\hat{\beta}_{ds}))} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution.

Thus, a $(1-\alpha)$ confidence interval for $\hat{\beta}_{ds}$ based on a lognormal distribution is given by

$$\left(\hat{\beta}_{ds} \; e^{-z\sqrt{V(\log(\hat{\beta}_{ds}))}} \, , \;\; \hat{\beta}_{ds} \; e^{z\sqrt{V(\log(\hat{\beta}_{ds}))}} \right)$$

Gamma Distribution Confidence Interval for Standardized Rate

Fay and Feuer (1997) use the relationship between the Poisson and gamma distributions to derive approximate confidence intervals for the standardized rate $\hat{\lambda}_{ds}$ based on the gamma distribution. As in the construction of the asymptotic normal confidence intervals, it is assumed that the number of events has a Poisson distribution, and the standardized rate is a weighted sum of independent Poisson random variables. A confidence interval for $\hat{\lambda}_{ds}$ is then given by

$$\left(\frac{v}{2\hat{\lambda}_{ds}} \; (\chi^2)^{-1}_{\frac{2\hat{\lambda}^2_{ds}}{v}} \left(\frac{\alpha}{2} \right) \, , \;\; \frac{v + w_x^2}{2(\hat{\lambda}_{ds} + w_x)} \; (\chi^2)^{-1}_{\frac{2(\hat{\lambda}_{ds}+w_x)^2}{v+w_x^2}} \left(1 - \frac{\alpha}{2} \right) \right)$$

where

$$v = \sum_j w_j^2 \, \frac{\hat{\lambda}_{sj}}{\mathcal{T}_{sj}}$$

$$w_j = \frac{\mathcal{T}_{rj}}{\mathcal{T}_r} \, \frac{1}{\mathcal{T}_{sj}}$$

and w_x is the maximum w_j.

Tiwari, Clegg, and Zou (2006) propose a less conservative confidence interval for $\hat{\lambda}_{ds}$ with a different upper confidence limit,

$$\left(\frac{v}{2\hat{\lambda}_{ds}} \; (\chi^2)^{-1}_{\frac{2\hat{\lambda}^2_{ds}}{v}} \left(\frac{\alpha}{2} \right) \, , \;\; \frac{v + w_{2m}}{2(\hat{\lambda}_{ds} + w_m)} \; (\chi^2)^{-1}_{\frac{2(\hat{\lambda}_{ds}+w_m)^2}{v+w_{2m}}} \left(1 - \frac{\alpha}{2} \right) \right)$$

where w_m is the average w_j and w_{2m} is the average w_j^2.

Comparing Standardized Rates and Comparing Standardized Risks

By using the same reference population, two directly standardized rates or risks from different populations can be compared. Both the difference and ratio statistics can be used in the comparison. Assume that $\hat{\beta}_1$ and $\hat{\beta}_2$ are directly standardized rates or risks for two populations with variances $V(\hat{\beta}_1)$ and $V(\hat{\beta}_2)$, respectively. The difference test assumes that the difference statistic

$$\hat{\beta}_1 - \hat{\beta}_2$$

has a normal distribution with mean 0 under the null hypothesis $H_0 : \beta_1 = \beta_2$. The variance is given by

$$V(\hat{\beta}_1 - \hat{\beta}_2) = V(\hat{\beta}_1) + V(\hat{\beta}_2)$$

The ratio test assumes that the log ratio statistic,

$$\log \left(\frac{\hat{\beta}_1}{\hat{\beta}_2} \right)$$

has a normal distribution with mean 0 under the null hypothesis $H_0 : \beta_1 = \beta_2$, or equivalently, $\log(\beta_1/\beta_2) = 0$. An estimated variance is given by

$$V \left(\log \left(\frac{\hat{\beta}_1}{\hat{\beta}_2} \right) \right) = V(\log(\hat{\beta}_1)) + V(\log(\hat{\beta}_2)) = \frac{1}{\hat{\beta}_1^2} \, V(\hat{\beta}_1) \; + \; \frac{1}{\hat{\beta}_2^2} \, V(\hat{\beta}_2)$$

Mantel-Haenszel Effect Estimation

In direct standardization, the derived standardized rates and risks in a study population are the weighted average of the stratum-specific rates and risks in the population, respectively, where the weights are given by the population-time for standardized rate and the number of observations for standardized risk in a reference population.

Assuming that an effect, such as rate difference, rate ratio, risk difference, and risk ratio between two populations, is homogeneous across strata, the Mantel-Haenszel estimates of this effect can be constructed from directly standardized rates or risks in the two populations, where the weights are constructed from the stratum-specific population-times for rate and number of observations for risk of the two populations.

That is, for population k, k=1 and 2, the standardized rate and risk are

$$\hat{\lambda}_k = \frac{\sum_j w_j \, \hat{\lambda}_{kj}}{\sum_j w_j} \quad \text{and} \quad \hat{\gamma}_k = \frac{\sum_j w_j \, \hat{\gamma}_{kj}}{\sum_j w_j}$$

where the weights are

$$w_j = \frac{\mathcal{T}_{1j} \, \mathcal{T}_{2j}}{\mathcal{T}_{1j} + \mathcal{T}_{2j}}$$

for standardized rate, and

$$w_j = \frac{\mathcal{N}_{1j} \, \mathcal{N}_{2j}}{\mathcal{N}_{1j} + \mathcal{N}_{2j}}$$

for standardized risk.

Rate and Risk Difference Statistics

Denote $\beta = \lambda$ for rate and $\beta = \gamma$ for risk. The variance is

$$V(\hat{\beta}_k) = V \left(\frac{\sum_j w_j \, \hat{\beta}_{kj}}{\sum_j w_j} \right) = \frac{1}{(\sum_j w_j)^2} \; \sum_j w_j^2 \, V(\hat{\beta}_{kj})$$

The Mantel-Haenszel difference statistic is

$$\hat{\beta}_1 - \hat{\beta}_2$$

with variance

$$V(\hat{\beta}_1 - \hat{\beta}_2) = V(\hat{\beta}_1) + V(\hat{\beta}_2)$$

Under the null hypothesis $H_0 : \beta_1 = \beta_2$, the difference statistic $\hat{\beta}_1 - \hat{\beta}_2$ has a normal distribution with mean 0.

Rate Ratio Statistic

The Mantel-Haenszel rate ratio statistic is $\hat{\lambda}_1 / \hat{\lambda}_2$, and the log ratio statistic is

$$\log\left(\frac{\hat{\lambda}_1}{\hat{\lambda}_2}\right)$$

Under the null hypothesis $H_0 : \lambda_1 = \lambda_2$ (or equivalently, $\log(\lambda_1/\lambda_2) = 0$), the log ratio statistic has a normal distribution with mean 0 and variance

$$V\left(\log\left(\frac{\hat{\lambda}_1}{\hat{\lambda}_2}\right)\right) = \frac{\sum_j w_j \, \hat{\lambda}_{pj}}{\left(\sum_j w_j \, \hat{\lambda}_{1j}\right) \left(\sum_j w_j \, \hat{\lambda}_{2j}\right)}$$

where

$$\hat{\lambda}_{pj} = \frac{d_{1j} + d_{2j}}{\mathcal{T}_{1j} + \mathcal{T}_{2j}}$$

is the combined rate estimate in stratum *j* under the null hypothesis of equal rates (Greenland and Robins 1985; Greenland and Rothman 2008, p. 273).

Risk Ratio Statistic

The Mantel-Haenszel risk ratio statistic is $\hat{\gamma}_1 / \hat{\gamma}_2$, and the log ratio statistic is

$$\log\left(\frac{\hat{\gamma}_1}{\hat{\gamma}_2}\right)$$

Under the null hypothesis $H_0 : \gamma_1 = \gamma_2$ (or equivalently, $\log(\gamma_1/\gamma_2) = 0$), the log ratio statistic has a normal distribution with mean 0 and variance

$$V\left(\log\left(\frac{\hat{\gamma}_1}{\hat{\gamma}_2}\right)\right) = \frac{\sum_j w_j \left(\hat{\gamma}_{pj} - \hat{\gamma}_{1j}\hat{\gamma}_{2j}\right)}{\left(\sum_j w_j \, \hat{\gamma}_{1j}\right) \left(\sum_j w_j \, \hat{\gamma}_{2j}\right)}$$

where

$$\hat{\gamma}_{pj} = \frac{d_{1j} + d_{2j}}{\mathcal{N}_{1j} + \mathcal{N}_{2j}}$$

is the combined risk estimate in stratum *j* under the null hypothesis of equal risks (Greenland and Robins 1985; Greenland and Rothman 2008, p. 275).

Indirect Standardization and Standardized Morbidity/Mortality Ratio

Indirect standardization compares the rates of the study and reference populations by applying the stratum-specific rates in the reference population to the study population, where the stratum-specific rates might not be reliable.

The expected number of events in the study population is

$$\mathcal{E} = \sum_j \mathcal{T}_{sj} \; \hat{\lambda}_{rj}$$

where $\mathcal{T}_{sj}$ is the population-time in the jth stratum of the study population and $\hat{\lambda}_{rj}$ is the rate in the jth stratum of the reference population.

With the expected number of events, $\mathcal{E}$, the standardized morbidity ratio or standardized mortality ratio can be expressed as

$$\mathcal{R}_{sm} = \frac{\mathcal{D}}{\mathcal{E}}$$

where $\mathcal{D}$ is the observed number of events (Breslow and Day 1987, p. 65).

The ratio $\mathcal{R}_{sm} > 1$ indicates that the mortality rate or risk in the study population is larger than the estimate in the reference population, and $\mathcal{R}_{sm} < 1$ indicates that the mortality rate or risk in the study population is smaller than the estimate in the reference population.

With the ratio $\mathcal{R}_{sm}$, an indirectly standardized rate for the study population is computed as

$$\hat{\lambda}_{is} = \mathcal{R}_{sm} \; \hat{\lambda}_r$$

where $\hat{\lambda}_r$ is the overall crude rate in the reference population.

Similarly, to compare the risks of the study and reference populations, the stratum-specific risks in the reference population are used to compute the expected number of events in the study population

$$\mathcal{E} = \sum_j \mathcal{N}_{sj} \; \hat{\gamma}_{rj}$$

where $\mathcal{N}_{sj}$ is the number of observations in the jth stratum of the study population and $\hat{\gamma}_{rj}$ is the risk in the jth stratum of the reference population.

Also, with the standardized morbidity ratio $\mathcal{R}_{sm} = \mathcal{D}/\mathcal{E}$, an indirectly standardized risk for the study population is computed as

$$\hat{\gamma}_{is} = \mathcal{R}_{sm} \; \hat{\gamma}_r$$

where $\hat{\gamma}_r$ is the overall crude risk in the reference population.

The observed number of events in the study population is $\mathcal{D} = \sum_j d_{sj}$, where d_{sj} is the number of events in the jth stratum of the population. For the rate estimate, if d_{sj} has a Poisson distribution, then the variance of the standardized mortality ratio $\mathcal{R}_{sm} = \mathcal{D} / \mathcal{E}$ is

$$V(\mathcal{R}_{sm}) = \frac{1}{\mathcal{E}^2} \sum_j V(d_{sj}) = \frac{1}{\mathcal{E}^2} \sum_j d_{sj} = \frac{\mathcal{D}}{\mathcal{E}^2} = \frac{\mathcal{R}_{sm}}{\mathcal{E}}$$

For the risk estimate, if d_{sj} has a binomial distribution, then the variance of $\mathcal{R}_{sm} = \mathcal{D} / \mathcal{E}$ is

$$V(\mathcal{R}_{sm}) = V\left(\frac{1}{\mathcal{E}} \sum_j d_{sj} \right) = \frac{1}{\mathcal{E}^2} \sum_j V(d_{sj}) = \frac{1}{\mathcal{E}^2} \sum_j \mathcal{N}_{sj}^2 V(\hat{\gamma}_{sj})$$

where

$$V(\hat{\gamma}_{sj}) = \frac{\hat{\gamma}_{sj}(1-\hat{\gamma}_{sj})}{\mathcal{N}_{sj}}$$

By using the method of statistical differentials (Elandt-Johnson and Johnson 1980, pp. 70–71), the variance of the logarithm of $\mathcal{R}_{sm}$ can be estimated by

$$V(\log(\mathcal{R}_{sm})) = \frac{1}{\mathcal{R}_{sm}^2} V(\mathcal{R}_{sm})$$

For the rate estimate,

$$V(\log(\mathcal{R}_{sm})) = \frac{1}{\mathcal{R}_{sm}^2} V(\mathcal{R}_{sm}) = \frac{1}{\mathcal{R}_{sm}^2} \frac{\mathcal{R}_{sm}}{\mathcal{E}} = \frac{1}{\mathcal{R}_{sm}} \frac{1}{\mathcal{E}} = \frac{1}{\mathcal{D}}$$

The confidence intervals for $\mathcal{R}_{sm}$ can be constructed based on normal, lognormal, and Poisson distributions.

Normal Distribution Confidence Interval for SMR

A $(1-\alpha)$ confidence interval for $\mathcal{R}_{sm}$ based on a normal distribution is given by

$$(\mathcal{R}_l,\ \mathcal{R}_u) = \left(\mathcal{R}_{sm} - z\, \sqrt{V(\mathcal{R}_{sm})},\ \ \mathcal{R}_{sm} + z\, \sqrt{V(\mathcal{R}_{sm})} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution.

A test statistic for the null hypothesis H_0 : SMR = 1 is then given by

$$\frac{\mathcal{R}_{sm} - 1}{\sqrt{V(\mathcal{R}_{sm})}}$$

The test statistic has an approximate standard normal distribution under H_0.

Lognormal Distribution Confidence Interval for SMR

A $(1-\alpha)$ confidence interval for $\log(\mathcal{R}_{sm})$ based on a normal distribution is given by

$$\left(\log(\mathcal{R}_{sm}) - z\, \sqrt{V(\log(\mathcal{R}_{sm}))},\ \ \log(\mathcal{R}_{sm}) + z\, \sqrt{V(\log(\mathcal{R}_{sm}))} \right)$$

where $z = \Phi^{-1}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the standard normal distribution.

Thus, a $(1-\alpha)$ confidence interval for $\mathcal{R}_{sm}$ based on a lognormal distribution is given by

$$\left(\mathcal{R}_{sm}\ e^{-z\sqrt{V(\log(\mathcal{R}_{sm}))}},\ \ \mathcal{R}_{sm}\ e^{z\sqrt{V(\log(\mathcal{R}_{sm}))}} \right)$$

A test statistic for the null hypothesis H_0 : SMR = 1 is then given by

$$\frac{\log(\mathcal{R}_{sm})}{\sqrt{V(\log(\mathcal{R}_{sm}))}}$$

The test statistic has an approximate standard normal distribution under H_0.

Poisson Distribution Confidence Interval for SMR

Denote the $(\alpha/2)$ quantile for the χ^2 distribution with $2\mathcal{D}$ degrees of freedom by

$$q_l = (\chi^2_{2\mathcal{D}})^{-1}\,(\alpha/2)$$

Denote the $(1-\alpha/2)$ quantiles for the χ^2 distribution with $2(\mathcal{D}+1)$ degrees of freedom by

$$q_u = (\chi^2_{2(\mathcal{D}+1)})^{-1}\,(1-\alpha/2)$$

Then a $(1-\alpha)$ confidence interval for $\mathcal{R}_{sm}$ based on the χ^2 distribution is given by

$$(\mathcal{R}_l,\ \mathcal{R}_u) = \left(\frac{q_l}{2\,\mathcal{E}},\ \frac{q_u}{2\,\mathcal{E}} \right)$$

A p-value for the test of the null hypothesis H_0 : SMR $= 1$ is given by

$$2\min\left(\sum_{k=0}^{\mathcal{D}} \frac{e^{-\mathcal{E}}\mathcal{E}^k}{k!},\ \sum_{k=\mathcal{D}}^{\infty} \frac{e^{-\mathcal{E}}\mathcal{E}^k}{k!} \right)$$

Indirectly Standardized Rate and Its Confidence Interval

With a rate-standardized mortality ratio $\mathcal{R}_{sm}$, an indirectly standardized rate for the study population is computed as

$$\hat{\lambda}_{is} = \mathcal{R}_{sm}\ \hat{\lambda}_r$$

where $\hat{\lambda}_r$ is the overall crude rate in the reference population.

The $(1-\alpha/2)$ confidence intervals for $\hat{\lambda}_{is}$ can be constructed as

$$(\mathcal{R}_l\,\hat{\lambda}_r,\ \mathcal{R}_u\,\hat{\lambda}_r)$$

where $(\mathcal{R}_l,\ \mathcal{R}_u)$ is the confidence interval for $\mathcal{R}_{sm}$.

Indirectly Standardized Risk and Its Confidence Interval

With a risk-standardized mortality ratio $\mathcal{R}_{sm}$, an indirectly standardized risk for the study population is computed as

$$\hat{\gamma}_{is} = \mathcal{R}_{sm}\ \hat{\gamma}_r$$

where $\hat{\gamma}_r$ is the overall crude risk in the reference population.

The $(1-\alpha/2)$ confidence intervals for $\hat{\gamma}_{is}$ can be constructed as

$$(\mathcal{R}_l\,\hat{\gamma}_r,\ \mathcal{R}_u\,\hat{\gamma}_r)$$

where $(\mathcal{R}_l,\ \mathcal{R}_u)$ is the confidence interval for $\mathcal{R}_{sm}$.

Attributable Fraction and Population Attributable Fraction

The attributable fraction measures the excess event rate or risk fraction in the exposed population that is attributable to the exposure. That is, it is the proportion of event rate or risk in the exposed population that would be reduced if the exposure were not present. In contrast, the population attributable fraction measures the excess event rate or risk fraction in the total population that is attributable to the exposure.

In the STDRATE procedure, you can compute the attributable fraction by using either indirect standardization or Mantel-Haenszel estimation.

Indirect Standardization

With indirect standardization, you specify a study population that consists of subjects who are exposed to a factor, such as smoking, and a reference population that consists of subjects who are not exposed to the factor. Denote the numbers of events in the study and reference populations by $\mathcal{D}_s$ and $\mathcal{D}_r$, respectively.

For the rate estimate, denote the population-times in the study and reference populations by $\mathcal{T}_s$ and $\mathcal{T}_r$, respectively. Then the event rates in the two populations can be expressed as the following equations, respectively:

$$\hat{\lambda}_s = \frac{\mathcal{D}_s}{\mathcal{T}_s} \quad \text{and} \quad \hat{\lambda}_r = \frac{\mathcal{D}_r}{\mathcal{T}_r}$$

Similarly, for the risk estimate, denote the numbers of observations in the study and reference populations by $\mathcal{N}_s$ and $\mathcal{N}_r$, respectively. Then the event risks in the two populations can be expressed as the following equations, respectively:

$$\hat{\gamma}_s = \frac{\mathcal{D}_s}{\mathcal{N}_s} \quad \text{and} \quad \hat{\gamma}_r = \frac{\mathcal{D}_r}{\mathcal{N}_r}$$

In the next two subsections, $\beta = \lambda$ denotes the rate statistic and $\beta = \gamma$ denotes the risk statistic.

Attributable Fraction with Indirect Standardization

The attributable fraction is the fraction of event rate or risk in the exposed population that is attributable to exposure:

$$\mathcal{R}_a = \frac{\hat{\beta}_s - \hat{\beta}_r}{\hat{\beta}_s}$$

With a standardized mortality ratio $\mathcal{R}_{sm}$, the attributable fraction is estimated by

$$\mathcal{R}_a = \frac{\mathcal{R}sm - 1}{\mathcal{R}_{sm}}$$

The confidence intervals for the attributable fraction can be computed using the confidence intervals for $\mathcal{R}_{sm}$. That is, with a confidence interval $(\mathcal{R}_l,\ \mathcal{R}_u)$ for $\mathcal{R}_{sm}$, the corresponding $\mathcal{R}_a$ confidence interval is given by

$$\left(\frac{\mathcal{R}_l - 1}{\mathcal{R}_l},\ \frac{\mathcal{R}_u - 1}{\mathcal{R}_u} \right)$$

Population Attributable Fraction with Indirect Standardization

The population attributable fraction for a population is the fraction of event rate or risk in a given time period that is attributable to exposure. The population attributable fraction is

$$\mathcal{R}_{pa} = \frac{\hat{\beta}_0 - \hat{\beta}_r}{\hat{\beta}_0}$$

where

$$\hat{\beta}_0 = \frac{\mathcal{D}_s + \mathcal{D}_r}{\mathcal{T}_s + \mathcal{T}_r}$$

is the combined rate in the total population for the rate statistic and where

$$\hat{\beta}_0 = \frac{\mathcal{D}_s + \mathcal{D}_r}{\mathcal{N}_s + \mathcal{N}_r}$$

is the combined risk in the total population for the risk statistic.

Denote $\rho = \mathcal{D}_s / (\mathcal{D}_s + \mathcal{D}_r)$, the proportion of exposure among events, then $\mathcal{R}_{pa}$ can also be expressed as

$$\mathcal{R}_{pa} = \rho \; \frac{\mathcal{R}_{sm} - 1}{\mathcal{R}_{sm}}$$

where $\mathcal{R}_{sm}$ is the standardized mortality ratio.

An approximate confidence interval for the population attributable rate $\mathcal{R}_{pa}$ can be derived by using the complementary log transformation (Greenland 2008, p. 296). That is, with

$$\mathcal{H} = \log(1 - \mathcal{R}_{pa})$$

a variance estimator for the estimated $\mathcal{H}$ is given by

$$\mathrm{Var}(\hat{\mathcal{H}}) = \frac{\mathcal{R}_{pa}^2}{(1 - \mathcal{R}_{pa})^2} \left(\frac{\hat{V}}{(\mathcal{R}_{sm} - 1)^2} + \frac{2}{\mathcal{D}_s \, (\mathcal{R}_{sm} - 1)} + \frac{\mathcal{D}_r}{\mathcal{D}_s \, (\mathcal{D}_s + \mathcal{D}_r)} \right)$$

where $\hat{V}$ is a variance estimate for $\log(\mathcal{R}_{sm})$.

Mantel-Haenszel Estimation

With Mantel-Haenszel estimation, you specify one study population that consists of subjects who are exposed to a factor and another study population that consists of subjects who are not exposed to the factor. Denote the numbers of events in the exposed and nonexposed study populations by $\mathcal{D}_1$ and $\mathcal{D}_2$, respectively.

For the rate estimate, denote the population-times in the two populations by $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively. Then the event rates in the two populations can be expressed as the following equations, respectively:

$$\hat{\lambda}_1 = \frac{\mathcal{D}_1}{\mathcal{T}_1} \quad \text{and} \quad \hat{\lambda}_2 = \frac{\mathcal{D}_2}{\mathcal{T}_2}$$

Similarly, for the risk estimate, denote the numbers of observations in the two populations by $\mathcal{N}_1$ and $\mathcal{N}_2$, respectively. Then the event risks in the two populations can be expressed as the following equations, respectively:

$$\hat{\gamma}_1 = \frac{\mathcal{D}_1}{\mathcal{N}_1} \quad \text{and} \quad \hat{\gamma}_2 = \frac{\mathcal{D}_2}{\mathcal{N}_2}$$

In the next two subsections, $\beta = \lambda$ denotes the rate statistic and $\beta = \gamma$ denotes the risk statistic.

Attributable Fraction with Mantel-Haenszel Estimation

The attributable fraction is the fraction of event rate or risk in the exposed population that is attributable to exposure:

$$\mathcal{R}_a = \frac{\hat{\beta}_1 - \hat{\beta}_2}{\hat{\beta}_1}$$

Denote the rate or risk ratio by $\mathcal{R} = \hat{\beta}_1 / \hat{\beta}_2$. The attributable fraction is given by

$$\mathcal{R}_a = \frac{\mathcal{R} - 1}{\mathcal{R}}$$

The confidence intervals for the attributable fraction can be computed using the confidence intervals for the rate or risk ratio $\mathcal{R}$. That is, with a confidence interval $(\mathcal{R}_l,\ \mathcal{R}_u)$ for $\mathcal{R}$, the corresponding $\mathcal{R}_a$ confidence interval is given by

$$\left(\frac{\mathcal{R}_l - 1}{\mathcal{R}_l},\ \frac{\mathcal{R}_u - 1}{\mathcal{R}_u} \right)$$

For Mantel-Haenszel estimation, you can use the Mantel-Haenszel rate or risk ratio to estimate $\mathcal{R}$.

Population Attributable Fraction with Mantel-Haenszel Estimation

The population attributable fraction for a population is the fraction of event rate or risk in a given time period that is attributable to exposure. The population attributable fraction is

$$\mathcal{R}_{pa} = \frac{\hat{\beta}_0 - \hat{\beta}_2}{\hat{\beta}_0}$$

where

$$\hat{\beta}_0 = \frac{\mathcal{D}_1 + \mathcal{D}_2}{\mathcal{T}_1 + \mathcal{T}_2}$$

is the combined rate in the total population for the rate statistic and where

$$\hat{\beta}_0 = \frac{\mathcal{D}_1 + \mathcal{D}_2}{\mathcal{N}_1 + \mathcal{N}_2}$$

is the combined risk in the total population for the risk statistic.

Denote the proportion of exposure among events as $\rho = \mathcal{D}_1 / (\mathcal{D}_1 + \mathcal{D}_2)$. Then $\mathcal{R}_{pa}$ can also be expressed as

$$\mathcal{R}_{pa} = \rho \ \frac{\mathcal{R} - 1}{\mathcal{R}}$$

where $\mathcal{R} = \hat{\beta}_1 / \hat{\beta}_2$ is the rate or risk ratio.

An approximate confidence interval for the population attributable rate $\mathcal{R}_{pa}$ can be derived by using the complementary log transformation (Greenland 2008, p. 296). That is, with

$$\mathcal{H} = \log(1 - \mathcal{R}_{pa})$$

a variance estimator for the estimated $\mathcal{H}$ is given by

$$\mathrm{Var}(\hat{\mathcal{H}}) = \frac{\mathcal{R}_{pa}^2}{(1-\mathcal{R}_{pa})^2} \left(\frac{\hat{V}}{(\mathcal{R}-1)^2} + \frac{2}{\mathcal{D}_1\,(\mathcal{R}-1)} + \frac{\mathcal{D}_2}{\mathcal{D}_1\,(\mathcal{D}_1+\mathcal{D}_2)} \right)$$

where $\hat{V}$ is a variance estimate for $\log(\mathcal{R})$.

For Mantel-Haenszel estimation, you can use the Mantel-Haenszel rate or risk ratio to estimate $\mathcal{R}$.

Applicable Data Sets and Required Variables for Method Specifications

The METHOD= and DATA= options are required in the STDRATE procedure. The METHOD= option specifies the standardization method, and the DATA= and REFDATA= options specify the study populations and reference population, respectively. You can use the GROUP= option in the POPULATION statement to identify various study populations. Table 109.2 lists applicable data sets for each method.

Table 109.2 Applicable Data Sets for Method Specifications

METHOD=	Number of Populations in DATA= Data Set	REFDATA= Data Set
DIRECT	1	X
	2	X
MH	2	
INDIRECT	1	X

Table 109.3 lists the required variables for each method.

Table 109.3 Required Variables for Method Specifications

		DATA= Data Set			REFDATA= Data Set		
METHOD=	STAT=	RATE	RISK	TOTAL	RATE	RISK	TOTAL
DIRECT	RATE	X					X
	RISK		X				X
MH	RATE	X		X			
	RISK		X	X			
INDIRECT	RATE			X	X		
	RISK			X		X	

The symbol "X" indicates that the variable is either explicitly specified or implicitly available from other variables. For example, when STAT=RATE, the variable RATE is available if the corresponding variables EVENT and TOTAL are specified.

Applicable Confidence Limits for Rate and Risk Statistics

In the STDRATE procedure, the METHOD= option specifies the standardization method, and the STAT= option specifies either rate or risk for standardization. Table 109.4 lists applicable confidence limits for different methods with standardized rate, rate SMR, standardized risk, and risk SMR.

Table 109.4 Applicable Confidence Limits for Standardized Rate and Risk Statistics

		Confidence Limits			
Statistic	**METHOD=**	**Normal**	**Lognormal**	**Gamma**	**Poisson**
Rate	DIRECT	X	X	X	
	MH	X	X	X	
	INDIRECT	X	X		X
Rate SMR	INDIRECT	X	X		X
Risk	DIRECT	X	X		
	MH	X	X		
	INDIRECT	X	X		X
Risk SMR	INDIRECT	X	X		X

Table 109.5 lists applicable confidence limits for stratum-specific rate, rate SMR, risk, and risk SMR.

Table 109.5 Applicable Confidence Limits for Strata Rate and Risk Statistics

	Confidence Limits		
Statistic	**Normal**	**Lognormal**	**Poisson**
Rate	X	X	X
Rate SMR	X	X	X
Risk	X	X	
Risk SMR	X	X	

Table Output

The STDRATE procedure displays the "Standardization Information" table by default. In addition, the procedure also displays the "Standardized Rate Estimates" table (with the default STAT=RATE option in the PROC STDRATE statement) and the "Standardized Risk Estimates" table (with the STAT=RISK option) by default. The rest of this section describes the output tables in alphabetical order.

Attributable Fraction Estimates

The "Attributable Fraction Estimates" table displays the following information:

- Parameter: attributable rate and population attributable rate for the rate statistic, and attributable risk and population attributable risk for the risk statistic
- Estimate: estimate of the parameter
- Method: method to construct confidence limits
- Lower and Upper: lower and upper confidence limits

Effect Estimates

The "Effect Estimates" table displays the following information:

- Standardized Rate: directly standardized rates for study populations
- Standardized Risk: directly standardized risks for study populations

When EFFECT=RATIO, the table displays the following:

- Estimate: the rate or risk ratio estimate
- Log Ratio: the logarithm of rate ratio or risk ratio estimate
- Standard Error: standard error of the logarithm of the ratio estimate
- Z: the standard Z statistic
- Pr > |Z|: the p-value for the test

When EFFECT=DIFF, the table displays the following:

- Estimate: the rate or risk difference estimate
- Standard Error: standard error of the difference estimate
- Z: the standard Z statistic
- Pr > |Z|: the p-value for the test

Standardization Information

The "Standardization Information" table displays the input data sets, type of statistic to be standardized, standardization method, and number of strata. The table also displays the variance divisor for the risk estimate, and the rate multiplier for the rate estimate. With a rate multiplier c, the rates per c population-time units are displayed in the output tables.

Standardized Morbidity/Mortality Ratio

The "Standardized Morbidity/Mortality Ratio" table displays the following information:

- SMR: standardized morbidity/mortality ratio
- Standard Error: standard error for SMR
- Lower and Upper: lower and upper confidence limits for SMR
- Test Statistic: SMR-1, for the test of SMR=1
- Estimate: value of test statistic
- Standard Error: standard error of the estimate
- Z: the standard Z statistic
- Pr > |Z|: the p-value for the test

Standardized Rate Estimates

The "Standardized Rate Estimates" table displays the following information:

- Population: study populations, and reference population for indirect standardization
- Number of Events: number of events in population
- Population-Time: total contributed time in population, for the rate statistic
- Crude Rate: event rate in the population
- Expected Number of Events
- SMR: standardized morbidity/mortality ratio, for indirect standardization
- Standardized Rate: for the rate statistic
- Standard Error: standard error of the standardized estimate of rate
- Confidence Limits: lower and upper confidence limits for standardized estimate

Standardized Risk Estimates

The “Standardized Risk Estimates” table displays the following information:

- Population: study populations, and reference population for indirect standardization
- Number of Events: number of events in population
- Number of Observations: number of observations in population
- Crude Risk: event risk in the population
- Expected Number of Events
- SMR: standardized morbidity/mortality ratio, for indirect standardization
- Standardized Risk
- Standard Error: standard error of the standardized estimate of risk
- Confidence Limits: lower and upper confidence limits for standardized estimate

Strata Effect Estimates

The “Strata Effect Estimates” table displays the following information for each stratum:

- Stratum Index: a sequential stratum identification number
- STRATA variables: the levels of STRATA variables
- Rate: rates for the study populations, for the rate statistic
- Risk: risks for the study populations, for the risk statistic

When EFFECT=DIFF, the table displays the following information for each stratum:

- Estimate: rate or risk difference estimate of the study populations
- Standard Error: the standard error of the difference estimate
- Confidence Limits: confidence limits for the difference estimate

When EFFECT=RATIO, the table displays the following information for each stratum:

- Estimate: rate or risk ratio estimate of the study populations
- Confidence Limits: confidence limits for the ratio estimate

Strata Statistics

For each POPULATION statement, the "Strata Information" table displays the following information for each stratum:

- Stratum Index: a sequential stratum identification number
- STRATA variables: the levels of STRATA variables

If the REFERENCE statement is specified, the table displays the following information for each stratum in the reference population:

- Population-Time Value: population-time for the rate statistic for direct standardization
- Population-Time Proportion: proportion for the population-time
- Number of Observations Value: number of observations for the risk statistic for direct standardization
- Number of Observations Proportion: proportion for the number of observations
- Rate: for the rate statistic for indirect standardization
- Risk: for the risk statistic for indirect standardization

For the rate statistic, the table displays the following information for each stratum in the specified study data set:

- Number of Events
- Population-Time Value
- Population-Time Proportion
- Rate Estimate
- Standard Error: standard error for the rate estimate if the CL=NORMAL suboption is specified in the STATS option in the STRATA statement
- Confidence Limits: confidence limits for the risk estimate if the CL suboption is specified in the STATS option in the STRATA statement
- Expected Number of Events: expected number of events that use the reference population population-time for direct standardization, Mantel-Haenszel weight for Mantel-Haenszel estimation, or reference population rate for indirect standardization

For the risk statistic, the table displays the following information for each stratum in the specified study data set:

- Number of Events
- Number of Observations Value
- Number of Observations Proportion
- Risk
- Standard Error: standard error for the risk estimate, if the CL=NORMAL suboption is specified in the STATS option in the STRATA statement
- Confidence Limits: confidence limits for the risk estimate, if the CL suboption is specified in the STATS option in the STRATA statement
- Expected Number of Events: expected number of events that uses the reference population number of observations for direct standardization, Mantel-Haenszel weight for Mantel-Haenszel estimation, or reference population risk for indirect standardization

Strata SMR Estimates

The "Strata SMR Estimates" table displays the following information for each stratum:

- Stratum Index: a sequential stratum identification number
- STRATA variables: the levels of STRATA variables
- Number of Events
- Expected Number of Observations
- SMR Estimate
- Standard Error: standard error for the SMR estimate, if the CL=NORMAL suboption is specified in the SMR option in the STRATA statement
- Confidence Limits: confidence limits for the SMR estimate, if the CL suboption is specified in the SMR option in the STRATA statement

For the rate statistic, the table also displays the following information for each stratum:

- Population-Time
- Reference Rate

For the risk statistic, the table also displays the following information for each stratum:

- Number of Observations
- Reference Risk

ODS Table Names

PROC STDRATE assigns a name to each table it creates. You must use these names to refer to tables when you use the Output Delivery System (ODS). These names are listed in Table 109.6. For more information about ODS, see Chapter 20, "Using the Output Delivery System."

Table 109.6 ODS Tables Produced by PROC STDRATE

ODS Table Name	Description	Statement	Option
AttrFraction	Attributable fraction	PROC STDRATE	METHOD=INDIRECT(AF)
		PROC STDRATE	METHOD=MH(AF)
Effect	Effect estimates	PROC STDRATE	EFFECT
SMR	Standardized morbidity/mortality ratio	PROC STDRATE	METHOD=INDIRECT
StdInfo	Standardization information	PROC STDRATE	
StdRate	Standardized rate estimates	PROC STDRATE	STAT=RATE
StdRisk	Standardized risk estimates	PROC STDRATE	STAT=RISK
StrataEffect	Strata effect estimates	STRATA	EFFECT
StrataStats	Strata statistics	STRATA	STATS
StrataSMR	Strata SMR estimates	STRATA	SMR

Graphics Output

This section describes the use of ODS for creating graphics with the STDRATE procedure. To request these graphs, ODS Graphics must be enabled and you must specify the associated graphics options in the PROC STDRATE statement. For more information about ODS Graphics, see Chapter 21, "Statistical Graphics Using ODS."

Strata Distribution Plot

The PLOTS=DIST option displays the proportion of exposed time or sample size for each stratum in the populations.

Strata Effect Plot

The PLOTS=EFFECT option displays the stratum-specific effect measure of rate difference, rate ratio, risk difference, or risk ratio. In addition, the crude effect measure and confidence limits of these stratum-specific effect estimates are also displayed.

Strata Rate Plot

The PLOTS=RATE option displays the stratum-specific rate estimates and their confidence limits of populations. In addition, the overall crude rates of populations are also displayed.

Strata Risk Plot

The PLOTS=RISK option displays the stratum-specific risk estimates and their confidence limits of populations. In addition, the overall crude risks of populations are also displayed.

Strata SMR Plot

The PLOTS=SMR option displays the SMR for each stratum in the populations.

ODS Graphics

Statistical procedures use ODS Graphics to create graphs as part of their output. ODS Graphics is described in detail in Chapter 21, "Statistical Graphics Using ODS."

Before you create graphs, ODS Graphics must be enabled (for example, by specifying the ODS GRAPHICS ON statement). For more information about enabling and disabling ODS Graphics, see the section "Enabling and Disabling ODS Graphics" on page 607 in Chapter 21, "Statistical Graphics Using ODS."

The overall appearance of graphs is controlled by ODS styles. Styles and other aspects of using ODS Graphics are discussed in the section "A Primer on ODS Statistical Graphics" on page 606 in Chapter 21, "Statistical Graphics Using ODS."

PROC STDRATE assigns a name to each graph it creates. You can use these names to refer to the graphs when you use ODS. To request the graph, ODS Graphics must be enabled and you must use the PLOTS option in the PROC STDRATE statement to specify the *plot-request* indicated in Table 109.7.

Table 109.7 Graphs Produced by PROC STDRATE

ODS Graph Name	Plot Description	*plot-request* in PLOTS Option
StrataDistPlot	Strata proportion of exposed time or sample size	DIST
StrataEffectPlot	Strata effect measure of rate difference, rate ratio, risk difference, or risk ratio	EFFECT
StrataRatePlot	Strata rate estimates	RATE
StrataRiskPlot	Strata risk estimates	RISK
StrataSMRPlot	Strata SMR of rates or risks	SMR

Examples: STDRATE Procedure

Example 109.1: Comparing Directly Standardized Rates

This example computes directly standardized mortality rates for populations in the states of Alaska and Florida, and then compares these two standardized rates with a rate ratio statistic.

The Alaska data set contains the stratum-specific mortality information in a given period of time for the state of Alaska (Alaska Bureau of Vital Statistics 2000a, b). Variables Sex and Age are the grouping variables that form the strata in the standardization, and variables Death and PYear indicate the number of events and person-years, respectively. The COMMA7. format is specified in the DATA step to input numerical values that contain commas in PYear.

```
data Alaska;
   State='Alaska';
   input Sex $ Age $ Death PYear:comma7.;
   datalines;
Male    00-14   37   81,205
Male    15-34   68   93,662
Male    35-54  206  108,615
Male    55-74  369   35,139
Male    75+    556    5,491
Female  00-14   78   77,203
Female  15-34  181   85,412
Female  35-54  395  100,386
Female  55-74  555   32,118
Female  75+    479    7,701
;
```

The Florida data set contains the corresponding stratum-specific mortality information for the state of Florida (Florida Department of Health 2000, 2013). Variables Sex and Age are the grouping variables that form the strata in the standardization, and variables Death and PYear indicate the number of events and person-years, respectively.

```
data Florida;
   State='Florida';
   input Sex $ Age $ Death:comma6. PYear:comma9.;
   datalines;
Male    00-14   1,189  1,505,889
Male    15-34   2,962  1,972,157
Male    35-54  10,279  2,197,912
Male    55-74  26,354  1,383,533
Male    75+    42,443    554,632
Female  00-14     906  1,445,831
Female  15-34   1,234  1,870,430
Female  35-54   5,630  2,246,737
Female  55-74  18,309  1,612,270
Female  75+    53,489    868,838
;
```

The TwoStates data set contains the data sets Alaska and Florida:

```
data TwoStates;
   length State $ 7.;
   set Alaska Florida;
run;
```

The US data set contains the corresponding stratum-specific person-years information for the United States (US Bureau of the Census 2011). Variables Sex and Age are the grouping variables that form the strata in the standardization, and the variable PYear indicates the person-years.

```
data US;
   input Sex $ Age $ PYear:comma10.;
   datalines;
Male    00-14  30,854,207
Male    15-34  40,199,647
Male    35-54  40,945,028
Male    55-74  19,948,630
Male    75+     6,106,351
Female  00-14  29,399,168
Female  15-34  38,876,268
Female  35-54  41,881,451
Female  55-74  22,717,040
Female  75+    10,494,416
;
```

The following statements invoke the STDRATE procedure and compute the direct standardized rates for the states of Florida and Alaska by using the United States as the reference population. The DATA= option names the data set for the study populations, and the REFDATA= option names the data set for the reference population.

```
ods graphics on;
proc stdrate data=TwoStates
             refdata=US
             method=direct
             stat=rate(mult=1000)
             effect
             plots(only)=(dist effect)
             ;
   population group=State event=Death total=PYear;
   reference  total=PYear;
   strata Sex Age / effect;
run;
```

The METHOD=DIRECT option requests direct standardization, and the STAT=RATE option specifies the rate statistic for standardization. With the EFFECT option, the procedure computes the rate effect between the study populations with the default rate ratio statistics.

The "Standardization Information" table in Output 109.1.1 displays the standardization information.

Output 109.1.1 Standardization Information

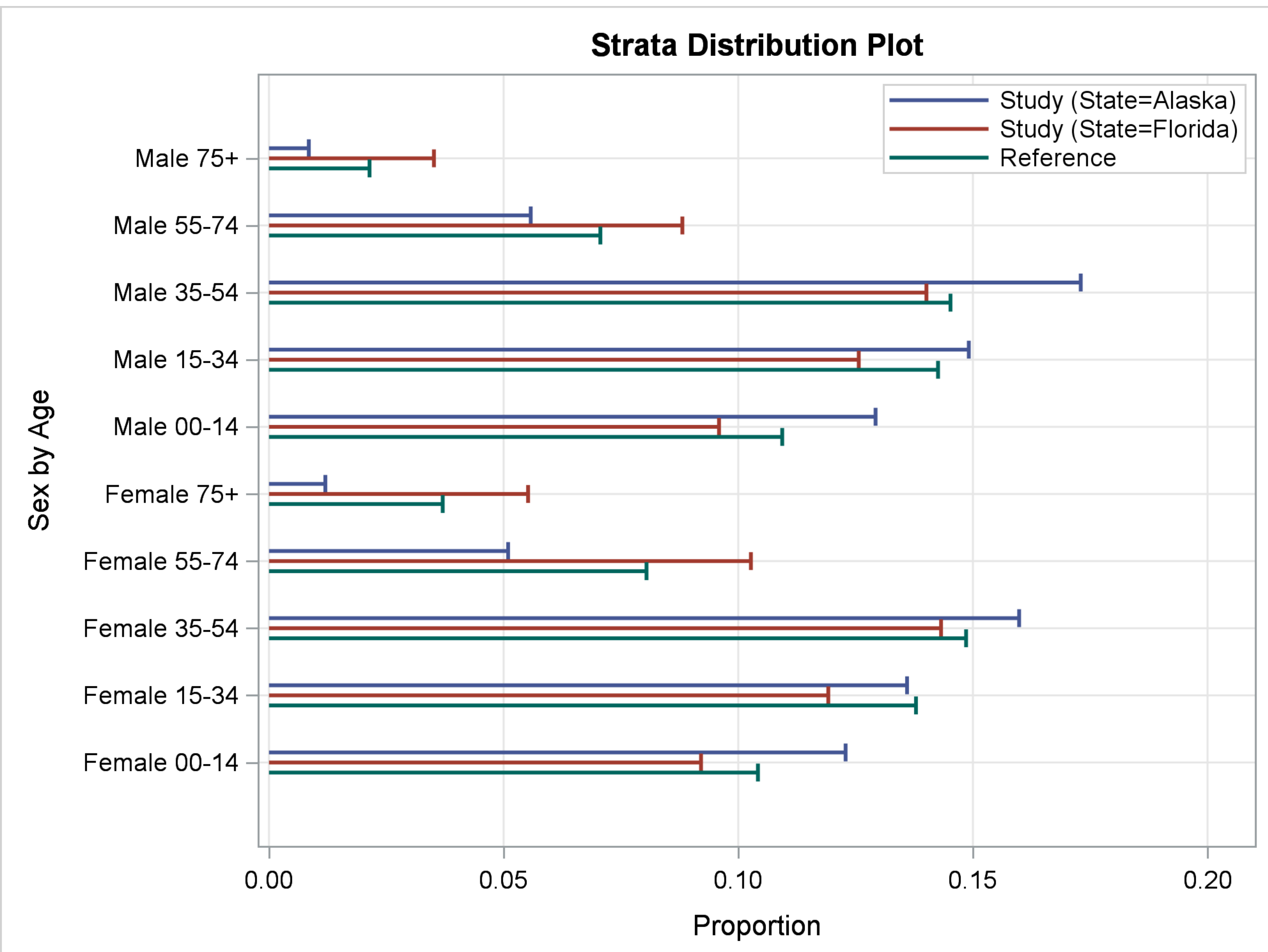
The STDRATE Procedure

Standardization Information	
Data Set	WORK.TWOSTATES
Group Variable	State
Reference Data Set	WORK.US
Method	Direct Standardization
Statistic	Rate
Number of Strata	10
Rate Multiplier	1000

With ODS Graphics enabled, the PLOTS(ONLY)=(DIST EFFECT) option displays the strata distribution plot and the strata effect plot, but does not display the default strata rate plot.

The strata distribution plot displays proportions for stratum-specific person-years in the study populations and reference population, as shown in Output 109.1.2.

Output 109.1.2 Strata Distribution Plot



The EFFECT option in the STRATA statement and the STAT=RATE option request that the "Strata Rate Effect Estimates" table in Output 109.1.3 display the stratum-specific rate effect statistics between the two study populations. The default EFFECT=RATIO in the PROC STDRATE statement requests that the stratum-specific rate ratio statistics be displayed.

Output 109.1.3 Strata Effect Estimates

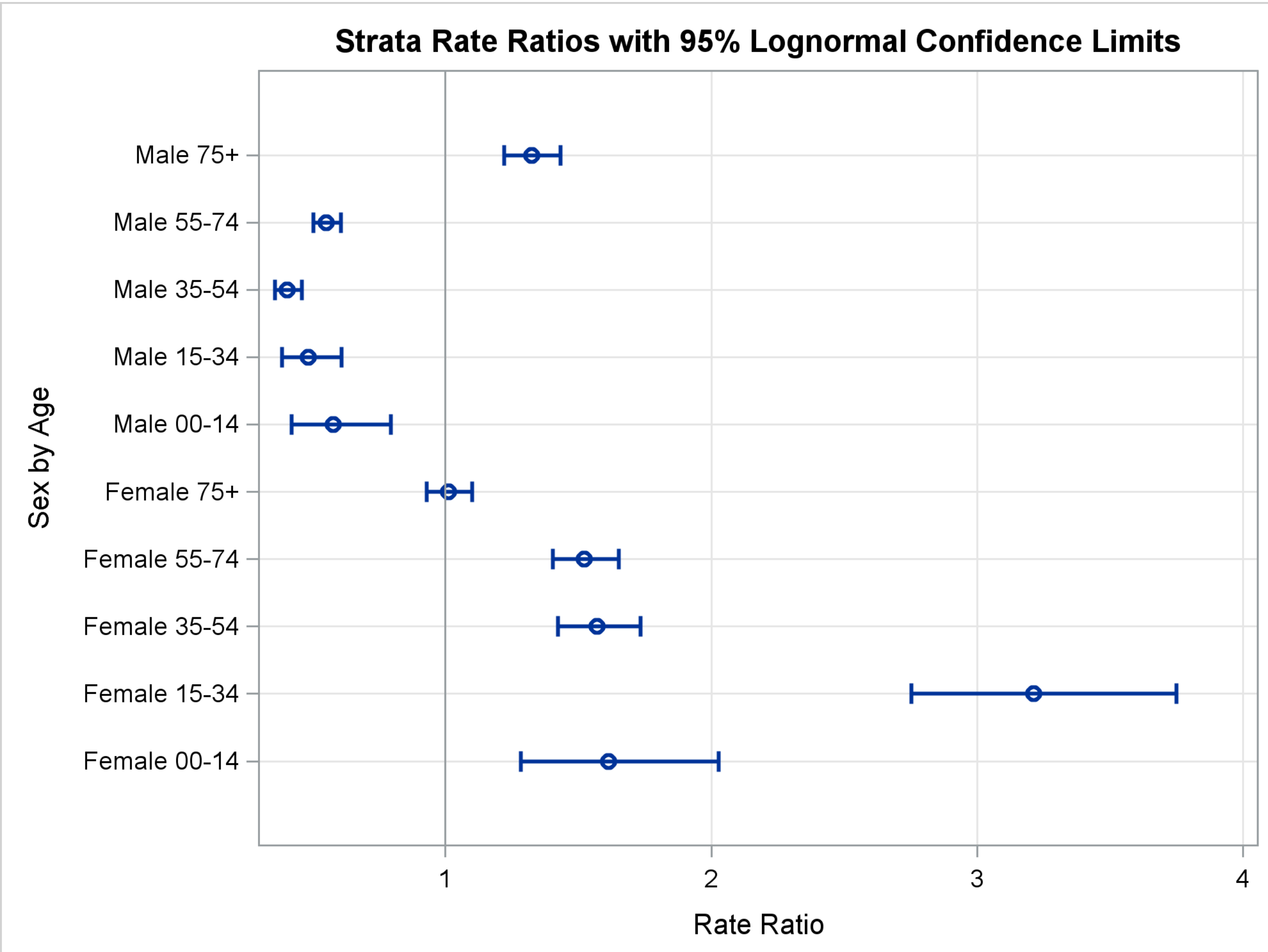
Strata Rate Effect Estimates (Rate Multiplier = 1000)

			State			95% Lognormal Confidence Limits	
Stratum Index	Sex	Age	Alaska	Florida	Rate Ratio		
1	Female	00-14	1.010	0.6266	1.61231	1.27940	2.03185
2	Female	15-34	2.119	0.6597	3.21208	2.74812	3.75437
3	Female	35-54	3.935	2.5059	1.57025	1.41795	1.73889
4	Female	55-74	17.280	11.3560	1.52166	1.39844	1.65574
5	Female	75+	62.200	61.5638	1.01033	0.92341	1.10542
6	Male	00-14	0.456	0.7896	0.57707	0.41604	0.80044
7	Male	15-34	0.726	1.5019	0.48339	0.38010	0.61476
8	Male	35-54	1.897	4.6767	0.40554	0.35330	0.46552
9	Male	55-74	10.501	19.0483	0.55129	0.49746	0.61094
10	Male	75+	101.257	76.5246	1.32319	1.21699	1.43866

The "Strata Rate Effect Estimates" table shows that except for the age group 75+, Alaska has lower mortality rates for male groups and higher mortality rates for female groups than Florida. For age group 75+, Alaska has higher mortality rates than Florida for both male and female groups.

With ODS Graphics enabled and two study populations, the PLOTS=EFFECT option displays the stratum-specific effect measures and their associated confidence limits, as shown in Output 109.1.4. The STAT=RATE option and the default EFFECT=RATIO option request that the strata rate ratios be displayed. By default, confidence limits are generated with 95% confidence level. This plot displays the stratum-specific rate ratios in the "Strata Rate Effect Estimates" table in Output 109.1.3.

Output 109.1.4 Strata Effect Measure Plot



The "Directly Standardized Rate Estimates" table in Output 109.1.5 displays directly standardized rates and related statistics.

Output 109.1.5 Directly Standardized Rate Estimates

Directly Standardized Rate Estimates
Rate Multiplier = 1000

	Study Population			Reference Population		Standardized Rate		
State	Observed Events	Population-Time	Crude Rate	Expected Events	Population-Time	Estimate	Standard Error	95% Normal Confidence Limits	
Alaska	2924	626932	4.6640	2270876	281422206	8.0693	0.1643	7.7472	8.3913
Florida	162795	15658229	10.3968	2176572	281422206	7.7342	0.0195	7.6959	7.7725

The MULT=1000 suboption in the STAT=RATE option requests that rates per $1,000$ person-years be displayed. The table shows that the although the crude rate in the Florida population (10.3968) is higher than the crude rate in the Alaska population (4.664), the resulting standardized rate in the Florida population (7.7342) is lower than the crude rate in the Alaska population (8.0693).

The EFFECT option requests that the "Rate Effect Estimates" table in Output 109.1.6 display the log rate ratio statistics of the two directly standardized rates by default.

Output 109.1.6 Effect Estimates

Rate Effect Estimates (Rate Multiplier = 1000)

State								
Alaska	Florida	Rate Ratio	95% Lognormal Confidence Limits		Log Rate Ratio	Standard Error	Z	Pr > |Z|
8.0693	7.7342	1.0433	1.00220	1.08614	0.0424	0.0205	2.07	0.0387

The table shows that with a log rate ratio statistic 1.0433, the resulting *p*-value is 0.0387, which indicates that the death rate is significantly higher in Alaska than in Florida at the 5% significance level.

Example 109.2: Computing Mantel-Haenszel Risk Estimation

This example uses Mantel-Haenszel method to estimate the effect of household smoking on respiratory symptoms of school children, after adjusting for the effects of the student's grade and household pets.

Suppose that the School data set contains the stratum-specific numbers of cases of respiratory symptoms in a given school year for a school district. Variables Pet and Grade are the grouping variables that form the strata in the standardization, and the variable Smoking identifies students who have smokers in their households. The variables Case and Student indicate the number of cases with respiratory symptoms and the total number of students, respectively.

```
data School;
   input Smoking $ Pet $ Grade $ Case Student;
   datalines;
Yes   Yes   K-1   109    807
Yes   Yes   2-3   106    791
Yes   Yes   4-5   112    868
Yes   No    K-1   168   1329
Yes   No    2-3   162   1337
Yes   No    4-5   183   1594
No    Yes   K-1   284   2403
No    Yes   2-3   266   2237
No    Yes   4-5   273   2279
No    No    K-1   414   3398
No    No    2-3   372   3251
No    No    4-5   382   3270
;
```

The following statements invoke the STDRATE procedure and compute the Mantel-Haenszel rate difference statistic between students with household smokers and students without household smokers:

```
ods graphics on;
proc stdrate data=School
             method=mh
             stat=risk
             effect=diff
             plots=all
             ;
   population group=Smoking event=Case total=Student;
   strata Pet Grade / order=data stats(cl=none) effect;
run;
```

The ORDER=DATA option in the STRATA statement sorts the strata by the order of appearance in the input data set.

The “Standardization Information” table in Output 109.2.1 displays the standardization information.

Output 109.2.1 Standardization Information

The STDRATE Procedure

Standardization Information	
Data Set	WORK.SCHOOL
Group Variable	Smoking
Method	Mantel-Haenszel
Statistic	Risk
Number of Strata	6

The STATS option in the STRATA statement requests that the STDRATE procedure display a “Mantel-Haenszel Standardized Strata Statistics” table for study populations, as shown in Output 109.2.2. The table displays the strata information and the expected number of events in each stratum. The Expected Events column shows the expected number of events when the Mantel-Haenszel weights are applied to the corresponding stratum-specific risks in the study populations. The CL=NONE suboption requests that confidence limits for strata risks not be displayed.

Output 109.2.2 Mantel-Haenszel Standardized Strata Statistics

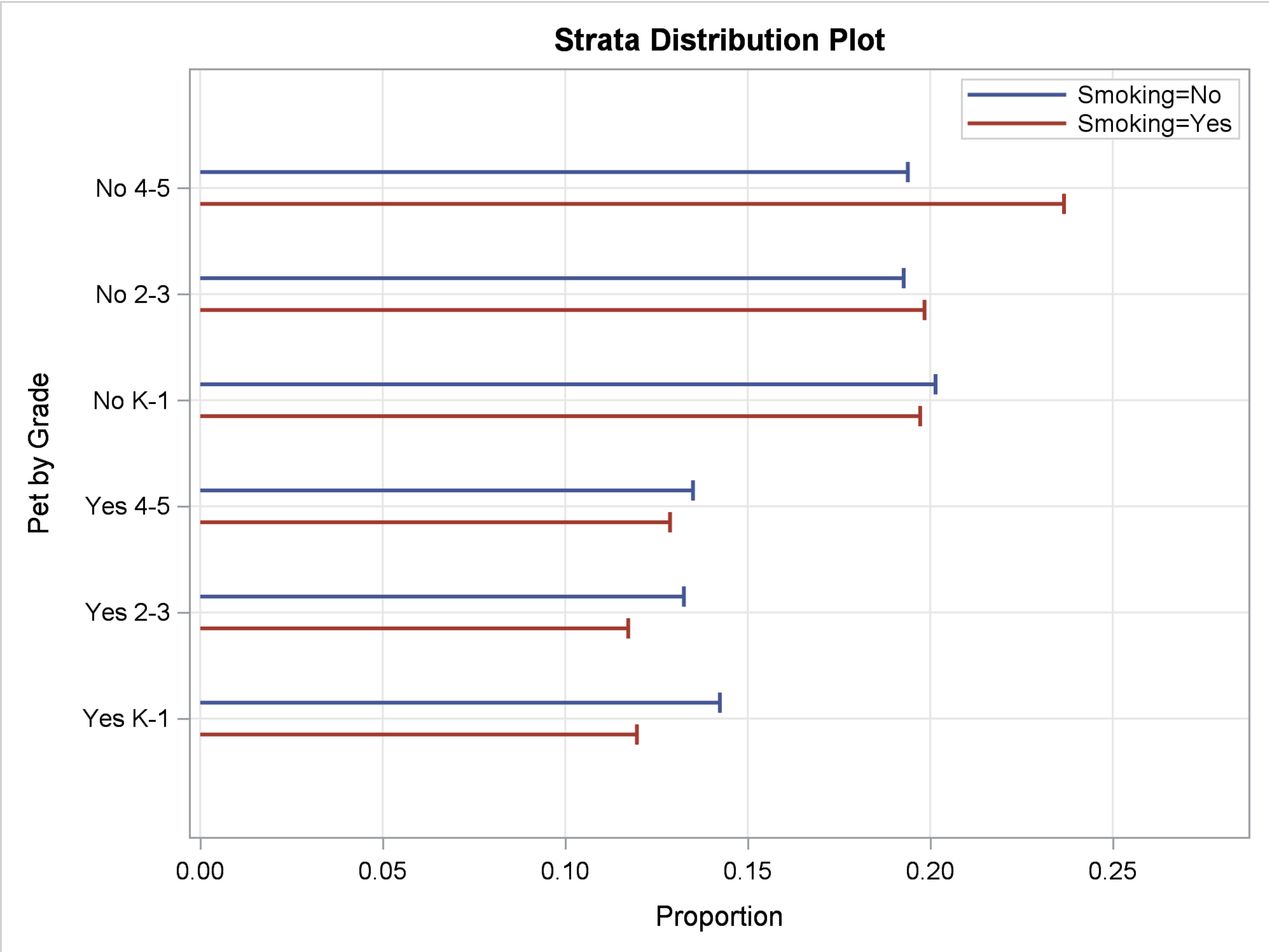
The STDRATE Procedure

Mantel-Haenszel Standardized Strata Statistics									
				Study Population					
				Number of Observations				Mantel-Haenszel	
Smoking	Stratum Index	Pet	Grade	Observed Events	Value	Proportion	Crude Risk	Weight	Expected Events
No	**1**	Yes	K-1	284	2403	0.1427	0.118186	604.12	71.398
No	**2**	Yes	2-3	266	2237	0.1329	0.118909	584.37	69.487
No	**3**	Yes	4-5	273	2279	0.1353	0.119789	628.59	75.298
No	**4**	No	K-1	414	3398	0.2018	0.121836	955.35	116.396
No	**5**	No	2-3	372	3251	0.1931	0.114426	947.38	108.405
No	**6**	No	4-5	382	3270	0.1942	0.116820	1071.62	125.187
Yes	**1**	Yes	K-1	109	807	0.1200	0.135068	604.12	81.597
Yes	**2**	Yes	2-3	106	791	0.1176	0.134008	584.37	78.310
Yes	**3**	Yes	4-5	112	868	0.1291	0.129032	628.59	81.108
Yes	**4**	No	K-1	168	1329	0.1976	0.126411	955.35	120.767
Yes	**5**	No	2-3	162	1337	0.1988	0.121167	947.38	114.791
Yes	**6**	No	4-5	183	1594	0.2370	0.114806	1071.62	123.028

With ODS Graphics enabled, the PLOTS=ALL option displays all appropriate plots. With the METHOD=MH and STAT=RISK options, these plots include the strata distribution plot, strata risk plot, and strata effect plot.

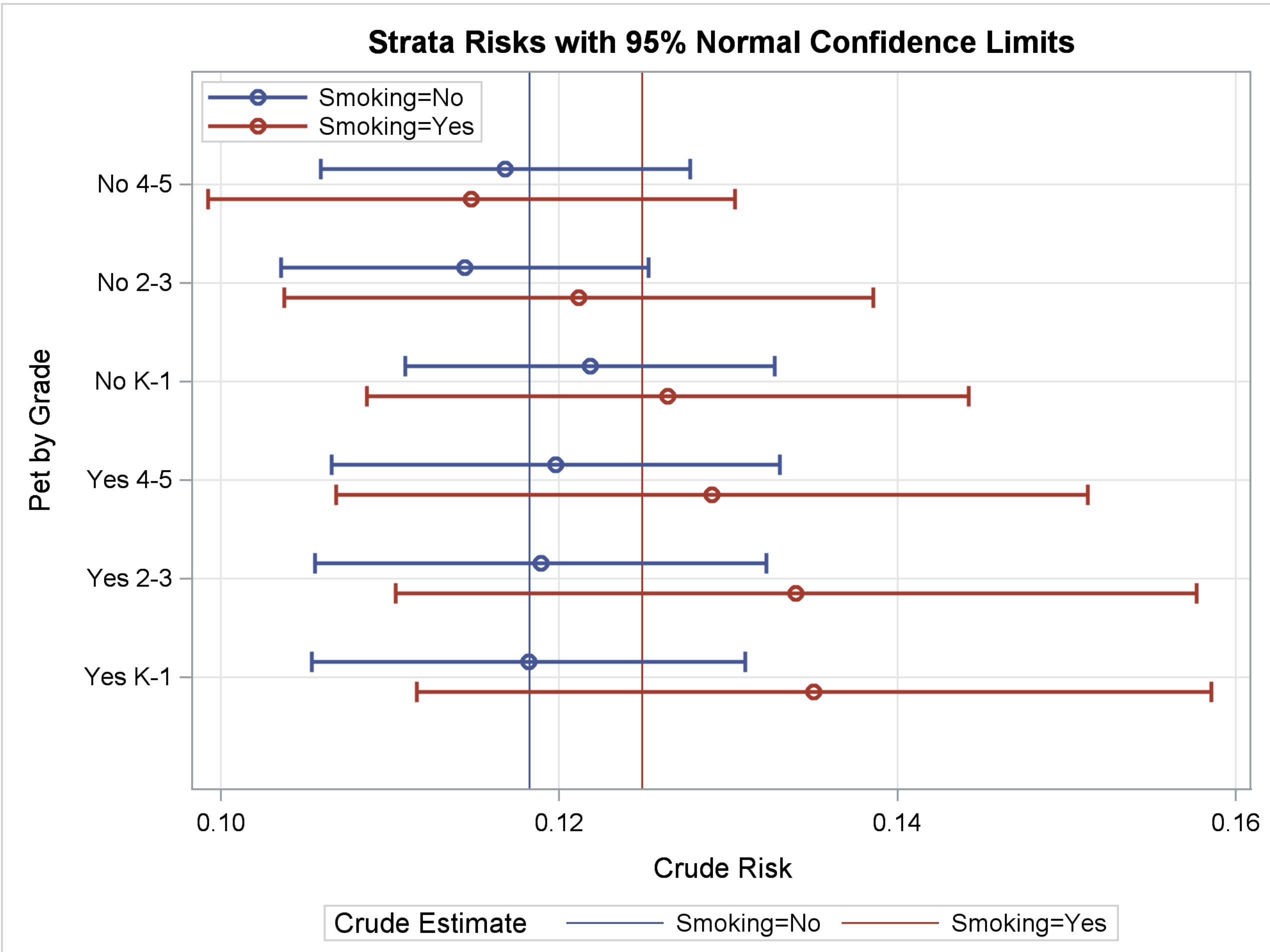
The strata distribution plot displays proportions for stratum-specific numbers of students in the study populations, as shown in Output 109.2.3.

Output 109.2.3 Strata Distribution Plot



The strata risk plot displays stratum-specific risk estimates with confidence limits in the study populations, as shown in Output 109.2.4. This plot displays stratum-specific risk estimates in the "Mantel-Haenszel Standardized Strata Statistics" table in Output 109.2.2. In addition, the overall crude risks for the two study populations are also displayed. By default, strata levels are displayed on the vertical axis.

Output 109.2.4 Strata Risk Plot



The EFFECT option in the STRATA statement requests that the "Strata Risk Effect Estimates" table be displayed, as shown in Output 109.2.5. The EFFECT=DIFF option in the PROC STDRATE statement requests that strata risk differences be displayed.

Output 109.2.5 Strata Effect Estimates

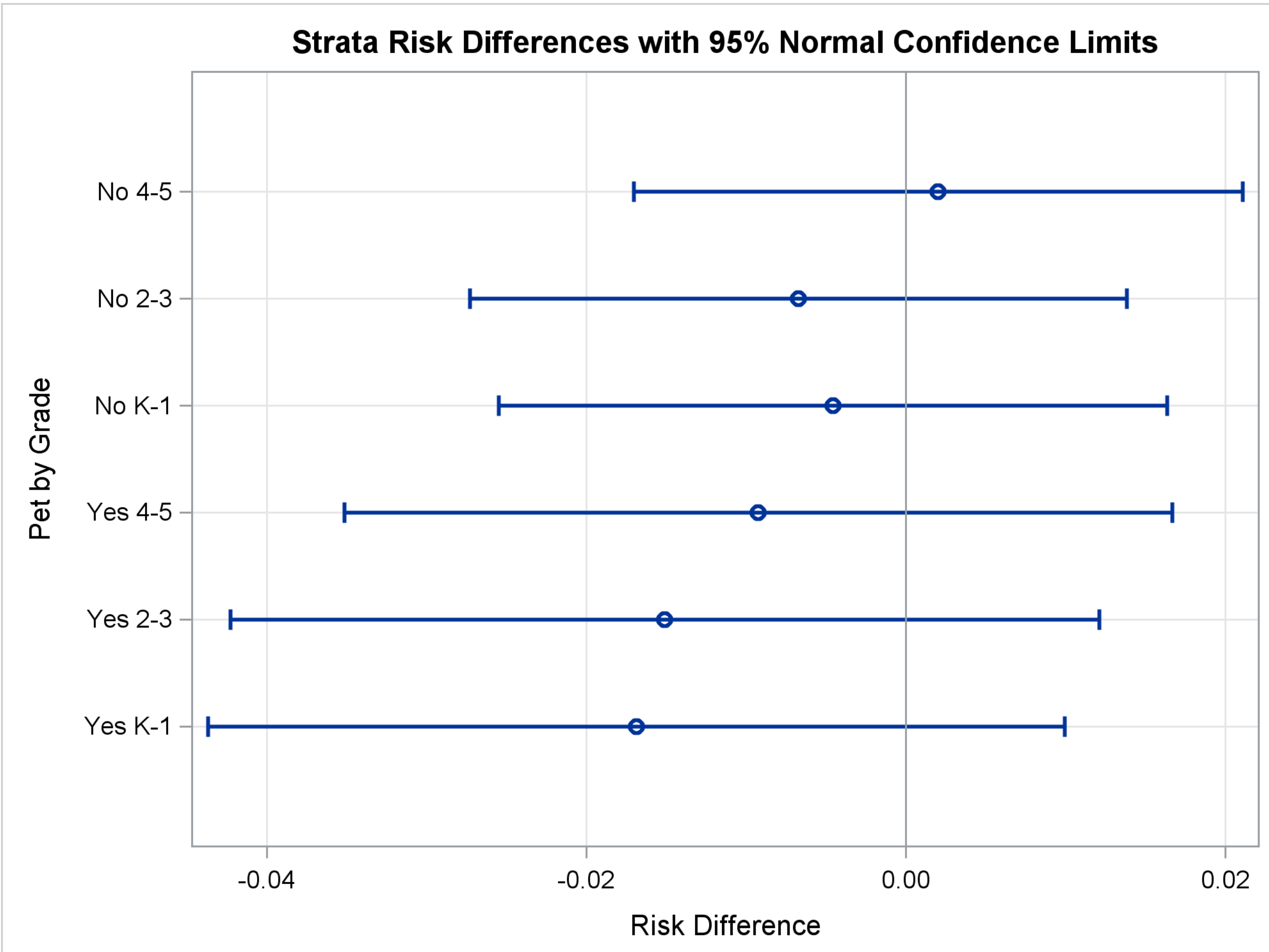
Strata Risk Effect Estimates

Stratum Index	Pet	Grade	Smoking No	Smoking Yes	Risk Difference	Standard Error	95% Normal Confidence Limits	
1	Yes	K-1	0.11819	0.13507	-.016883	0.013716	-.043766	0.010001
2	Yes	2-3	0.11891	0.13401	-.015098	0.013912	-.042366	0.012169
3	Yes	4-5	0.11979	0.12903	-.009243	0.013257	-.035225	0.016740
4	No	K-1	0.12184	0.12641	-.004574	0.010704	-.025554	0.016405
5	No	2-3	0.11443	0.12117	-.006740	0.010527	-.027373	0.013892
6	No	4-5	0.11682	0.11481	0.002014	0.009762	-.017120	0.021148

The "Strata Risk Effect Estimates" table shows that for the stratum of students without household pets in Grade 4–5, the risk is higher for students without household smokers than for students with household smokers. For all other strata, the risk is lower for students without household smokers than for students with household smokers. The difference is not significant in each stratum because the null value 0 is between the lower and upper confidence limits.

With ODS Graphics enabled, the PLOTS=EFFECT option displays the plot with the stratum-specific risk effect measures and their associated confidence limits, as shown in Output 109.2.6. The EFFECT=DIFF option requests that the risk difference be displayed. By default, confidence limits are generated with 95% confidence level. This plot displays the stratum-specific risk differences in the "Strata Risk Effect Estimates" table in Output 109.2.5.

Output 109.2.6 Strata Risk Plot



The "Mantel-Haenszel Standardized Risk Estimates" table in Output 109.2.7 displays the Mantel-Haenszel standardized risks and related statistics.

Output 109.2.7 Standardized Risk Estimates (Mantel-Haenszel Estimation)

Mantel-Haenszel Standardized Risk Estimates

	Study Population			Mantel-Haenszel		Standardized Risk			
Smoking	Observed Events	Number of Observations	Crude Risk	Expected Events	Weight	Estimate	Standard Error	95% Normal Confidence Limits	
No	1991	16838	0.1182	566.172	4791.43	0.1182	0.00250	0.1133	0.1231
Yes	840	6726	0.1249	599.602	4791.43	0.1251	0.00404	0.1172	0.1331

The EFFECT=DIFF option requests that the "Risk Effect Estimates" table display the risk difference statistic for the two directly standardized risks, as shown in Output 109.2.8.

Output 109.2.8 Mantel-Haenszel Effect Estimates

Risk Effect Estimates

Smoking							
No	Yes	Risk Difference	95% Normal Confidence Limits		Standard Error	Z	Pr > |Z|
0.1182	0.1251	-0.00698	-.016284	0.002330	0.00475	-1.47	0.1418

The table shows that although the standardized risk for students without household smokes is lower than the standardized risk for students with household smokes, the difference (–0.00698) is not significant at the 5% significance level, (p-value 0.1418).

Example 109.3: Computing Attributable Fraction Estimates

This example computes the excess event risk fraction that is attributable to a specific chemical exposure for workers in a factory.

Suppose that the Factory data set contains the stratum-specific event information for exposure to a specific chemical agent. The variable Age is the grouping variable that forms the strata. The variables Event_E and Count_E indicate the number of events and number of workers for workers with the specific chemical exposure, respectively. The variables Event_NE and Count_NE indicate the number of events and number of workers for workers without the specific chemical exposure, respectively.

```
data Factory;
   input Age $ Event_E Count_E Event_NE Count_NE;
   datalines;
20-29   31  352  143  2626
30-39   57  486  392  4124
40-49   62  538  459  4662
50-59   50  455  337  3622
60-69   38  322  199  2155
70+      9   68   35   414
;
```

The following statements invoke the STDRATE procedure and compute the attributable risk and population attributable risk for the chemical exposure:

```
ods graphics on;
proc stdrate data=Factory
             refdata=Factory
             method=indirect(af)
             stat=risk
             plots(stratum=horizontal)
             ;
   population event=Event_E  total=Count_E;
   reference  event=Event_NE total=Count_NE;
   strata Age / stats;
run;
```

The “Standardization Information” table in Output 109.3.1 displays the standardization information.

Output 109.3.1 Standardization Information

The STDRATE Procedure

Standardization Information	
Data Set	WORK.FACTORY
Reference Data Set	WORK.FACTORY
Method	Indirect Standardization
Statistic	Risk
Number of Strata	6

The STATS option in the STRATA statement requests that the "Indirectly Standardized Strata Statistics" table in Output 109.3.2 display the strata information and the expected number of events at each stratum. The Expected Events column shows the expected numbers of events when the stratum-specific risks in the reference data set are applied to the corresponding numbers of workers in the study data set.

Output 109.3.2 Strata Information (Indirect Standardization)

The STDRATE Procedure

Indirectly Standardized Strata Statistics

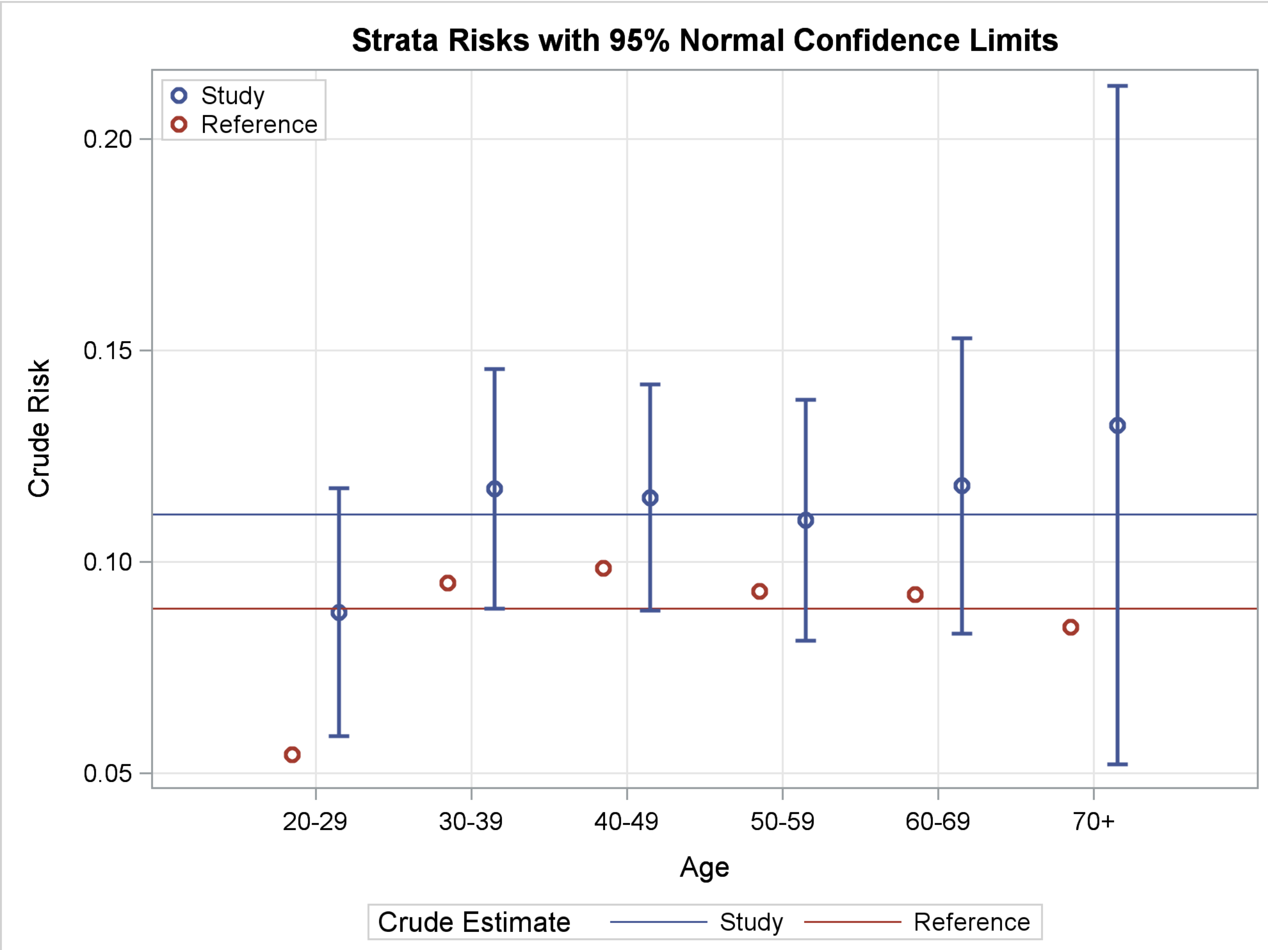
			Study Population						Reference Population		
			Number of Observations						Number of Observations		
Stratum Index	Age	Observed Events	Value	Proportion	Crude Risk	Standard Error	95% Normal Confidence Limits		Value	Proportion	Crude Risk
1	20-29	31	352	0.1585	0.088068	0.015105	0.058463	0.117673	2626	0.1492	0.05446
2	30-39	57	486	0.2188	0.117284	0.014595	0.088678	0.145890	4124	0.2343	0.09505
3	40-49	62	538	0.2422	0.115242	0.013767	0.088260	0.142224	4662	0.2648	0.09846
4	50-59	50	455	0.2049	0.109890	0.014662	0.081153	0.138627	3622	0.2058	0.09304
5	60-69	38	322	0.1450	0.118012	0.017979	0.082774	0.153251	2155	0.1224	0.09234
6	70+	9	68	0.0306	0.132353	0.041095	0.051809	0.212897	414	0.0235	0.08454

Indirectly Standardized Strata Statistics

Stratum Index	Expected Events
1	19.1683
2	46.1959
3	52.9691
4	42.3343
5	29.7346
6	5.7488

With ODS Graphics enabled and the specified STAT=RISK option, the default PLOTS=RISK option displays the stratum-specific risk estimates in the study and reference populations, as shown in Output 109.3.3. The STRATUM=HORIZONTAL global option in the PLOTS option displays the strata information on the horizontal axis. The plot displays the stratum-specific risk estimates in the "Indirect Standardized Strata Statistics" table in Output 109.3.2. In addition, confidence limits for the risk estimates in the study population and the overall crude risks for the two populations are also displayed

Output 109.3.3 Strata Risk Plot



The METHOD=INDIRECT option requests that the "Standardized Morbidity/Mortality Ratio" table in Output 109.3.4 display the SMR, its 95% confidence limits, and the test for the null hypothesis H_0 : SMR = 1.

Output 109.3.4 Standardized Morbidity/Mortality Ratio

Standardized Morbidity/Mortality Ratio

Observed Events	Expected Events	SMR	Standard Error	95% Normal Confidence Limits		Z	Pr > |Z|
247	196.151	1.2592	0.0755	1.1113	1.4072	3.43	0.0006

The "Standardized Morbidity/Mortality Ratio" table shows that SMR=1.259, the 95% confidence limits do not contain the null value SMR=1, and the null hypothesis of SMR=1 is rejected at $\alpha = 0.05$ level from the normal test.

The “Indirectly Standardized Risk Estimates” table in Output 109.3.5 displays the standardized risks and related statistics.

Output 109.3.5 Standardized Risks (Indirect Standardization)

Indirectly Standardized Risk Estimates

Study Population						Standardized Risk			
Observed Events	Number of Observations	Crude Risk	Reference Crude Risk	Expected Events	SMR	Estimate	Standard Error	95% Normal Confidence Limits	
247	2221	0.1112	0.0889	196.151	1.2592	0.1120	0.00671	0.0988	0.1251

The AF suboption in the METHOD=INDIRECT option requests that the “Attributable Fraction Estimates” table display the attributable risk and population attributable risk, as shown in Output 109.3.6

Output 109.3.6 Attributable Fraction Estimates

Attributable Fraction Estimates

Parameter	Estimate	95% Confidence Limits	
Attributable Risk	0.20587	0.10013	0.28937
Population Attributable Risk	0.02806	0.01159	0.04426

The attributable risk fraction 0.206 indicates that 20.6% of all events in the chemical exposure group are attributed to the chemical exposure, and the population attributable risk fraction 0.028 indicates that about 2.8% of all events in the total population are attributed to the chemical exposure.

The Attributable fraction can also be computed by using Mantel-Haenszel method.

Suppose that the Factory1 data set contains the stratum-specific event information for exposure to a specific chemical agent. The variable Age is the grouping variable that forms the strata, and the variable Exposure identifies workers with chemical exposure. The variables Event and Count indicate the number of events and number of workers, respectively.

```
data Factory1;
   input Exposure $ Age $ Event Count;
   datalines;
Yes  20-29   31   352
Yes  30-39   57   486
Yes  40-49   62   538
Yes  50-59   50   455
Yes  60-69   38   322
Yes  70+      9    68
No   20-29  143  2626
No   30-39  392  4124
No   40-49  459  4662
No   50-59  337  3622
No   60-69  199  2155
No   70+     35   414
;
```

The following statements invoke the STDRATE procedure and compute the attributable risk and population attributable risk for the chemical exposure:

```
proc stdrate data=Factory1
             method=mh(af)
             stat=risk
             effect
             ;
   population group(order=data exposed='Yes')=Exposure
              event=Event total=Count;
   strata Age;
run;
```

The GROUP=EXPOSURE option specifies the variable Exposure, whose values identify the various populations. The ORDER= suboption specifies the order in which the values of Exposure are to be displayed, and the EXPOSED= option identifies the exposed group in the derivation of the attributable fraction.

The "Standardization Information" table in Output 109.3.7 displays the standardization information.

Output 109.3.7 Standardization Information

The STDRATE Procedure

Standardization Information	
Data Set	WORK.FACTORY1
Group Variable	Exposure
Method	Mantel-Haenszel
Statistic	Risk
Number of Strata	6

The "Mantel-Haenszel Standardized Risk Estimates" table in Output 109.3.8 displays the Mantel-Haenszel standardized risks and related statistics.

Output 109.3.8 Standardized Risk Estimates (Mantel-Haenszel Estimation)

Mantel-Haenszel Standardized Risk Estimates

	Study Population			Mantel-Haenszel		Standardized Risk			
Exposure	Observed Events	Number of Observations	Crude Risk	Expected Events	Weight	Estimate	Standard Error	95% Normal Confidence Limits	
Yes	247	2221	0.1112	219.122	1970.26	0.1112	0.00667	0.0981	0.1243
No	1565	17603	0.0889	174.134	1970.26	0.0884	0.00214	0.0842	0.0926

The EFFECT option requests that the "Risk Effect Estimates" table display the risk ratio statistic for the two directly standardized risks, as shown in Output 109.3.9.

Output 109.3.9 Mantel-Haenszel Effect Estimates

Risk Effect Estimates

Exposure								
Yes	No	Risk Ratio	95% Lognormal Confidence Limits		Log Risk Ratio	Standard Error	Z	Pr > |Z|
0.1112	0.0884	1.2584	1.10851	1.42845	0.2298	0.0647	3.55	0.0004

The AF suboption in the METHOD=MH option requests that the "Attributable Fraction Estimates" table display the attributable risk and population attributable risk, as shown in Output 109.3.10

Output 109.3.10 Attributable Fraction Estimates

Attributable Fraction Estimates
Exposed = Yes

Parameter	Estimate	95% Confidence Limits	
Attributable Risk	0.20531	0.09789	0.29994
Population Attributable Risk	0.02799	0.01070	0.04497

Similar to the results of using the SMR estimates, the attributable risk fraction (0.205) indicates that 20.5% of all events in the chemical exposure group are attributed to the chemical exposure, and the population attributable risk fraction (0.028) indicates that about 2.8% of all events in the total population are attributed to the chemical exposure.

Example 109.4: Displaying SMR Results from BY Groups

This example illustrates the use of ODS OUTPUT statement to save standardized mortality ratios for different causes and to display these statistics together in a table and in a plot.

The Florida_Cs data set contains the stratum-specific mortality information for stomach cancer and skin cancer in year 2000 for the state of Florida (Florida Department of Health 2000, 2013). The variable Age is the grouping variable that forms the strata in the standardization. The variables Event_C16, Event_C43, and PYear identify the number of events for stomach cancer, the number of events for skin cancer, and the person-years, respectively. The COMMA9. format is specified in the DATA step to input numerical values that contain commas in PYear.

```
data Florida_Cs;
   input Age $1-5 Event_C16 Event_C43 PYear:comma9.;
   datalines;
00-04    0    0    953,785
05-14    0    0  1,997,935
15-24    0    4  1,885,014
25-34    1   14  1,957,573
35-44   19   43  2,356,649
45-54   64   72  2,088,000
55-64  114   70  1,548,371
65-74  201  126  1,447,432
75-84  294  136  1,087,524
85+    136   73    335,944
;
```

The following statements construct and list the mortality information by cancer cause:

```
data Florida_Cs;
   set Florida_Cs;
   Cause='Stomach';  Event=Event_C16;  output;
   Cause='Skin';     Event=Event_C43;  output;
   drop Event_C16 Event_C43;
run;

proc sort data=Florida_Cs;
   by Cause;
run;

proc print data=Florida_Cs;
   var Cause Age Event PYear;
run;
```

Output 109.4.1 Florida Data

Obs	Cause	Age	Event	PYear
1	Skin	00-04	0	953785
2	Skin	05-14	0	1997935
3	Skin	15-24	4	1885014
4	Skin	25-34	14	1957573
5	Skin	35-44	43	2356649
6	Skin	45-54	72	2088000
7	Skin	55-64	70	1548371
8	Skin	65-74	126	1447432
9	Skin	75-84	136	1087524
10	Skin	85+	73	335944
11	Stomach	00-04	0	953785
12	Stomach	05-14	0	1997935
13	Stomach	15-24	0	1885014
14	Stomach	25-34	1	1957573
15	Stomach	35-44	19	2356649
16	Stomach	45-54	64	2088000
17	Stomach	55-64	114	1548371
18	Stomach	65-74	201	1447432
19	Stomach	75-84	294	1087524
20	Stomach	85+	136	335944

The US_Cs data set contains the corresponding stratum-specific mortality information for the United States (Miniño et al. 2002; US Bureau of the Census 2011). The variable Age is the grouping variable that forms the strata in the standardization. The variables Event_C16, Event_C43, and PYear identify the number of events for stomach cancer, the number of events for skin cancer, and the person-years, respectively.

```
data US_Cs;
   input Age $1-5 Event_C16 Event_C43 PYear:comma10.;
   datalines;
00-04     0     0  19,175,798
05-14     1     1  41,077,577
15-24    14    41  39,183,891
25-34   124   186  39,892,024
35-44   484   626  45,148,527
45-54  1097  1199  37,677,952
55-64  1804  1303  24,274,684
65-74  3054  1637  18,390,986
75-84  3833  1624  12,361,180
85+    2234   803   4,239,587
;
```

The following statements construct and list the mortality information by cancer cause:

```
data US_Cs;
   set US_Cs;
   Cause='Stomach';  Event=Event_C16;  output;
   Cause='Skin';     Event=Event_C43;  output;
   drop Event_C16 Event_C43;
run;
```

```
proc sort data=US_Cs;
   by Cause;
run;

proc print data=US_Cs;
   var Cause Age Event PYear;
run;
```

Output 109.4.2 lists the mortality information by cancer cause.

Output 109.4.2 Florida Data

Obs	Cause	Age	Event	PYear
1	Skin	00-04	0	19175798
2	Skin	05-14	1	41077577
3	Skin	15-24	41	39183891
4	Skin	25-34	186	39892024
5	Skin	35-44	626	45148527
6	Skin	45-54	1199	37677952
7	Skin	55-64	1303	24274684
8	Skin	65-74	1637	18390986
9	Skin	75-84	1624	12361180
10	Skin	85+	803	4239587
11	Stomach	00-04	0	19175798
12	Stomach	05-14	1	41077577
13	Stomach	15-24	14	39183891
14	Stomach	25-34	124	39892024
15	Stomach	35-44	484	45148527
16	Stomach	45-54	1097	37677952
17	Stomach	55-64	1804	24274684
18	Stomach	65-74	3054	18390986
19	Stomach	75-84	3833	12361180
20	Stomach	85+	2234	4239587

The following statements invoke the STDRATE procedure and request indirect standardization to compute the skin and stomach SMR estimates for the state of Florida. The BY statement requests separate analyses of causes that are defined by the Cause variable.

```
ods graphics on;
ods select StdInfo StrataSmrPlot Smr;
proc stdrate data=Florida_Cs refdata=US_Cs
             stat=rate
             method=indirect
             plots=smr
             ;
   population event=Event total=PYear;
   reference  event=Event total=PYear;
   strata Age;
   by Cause;
ods output smr=Smr_Cs;
run;
```

Only the tables and plots that are specified in the ODS SELECT statement are displayed.

The STDINFO option in the ODS SELECT statement requests that the "Standardization Information" table display the standardization information for the first BY group, skin cancer, as shown in Output 109.4.3

Output 109.4.3 Standardization Information

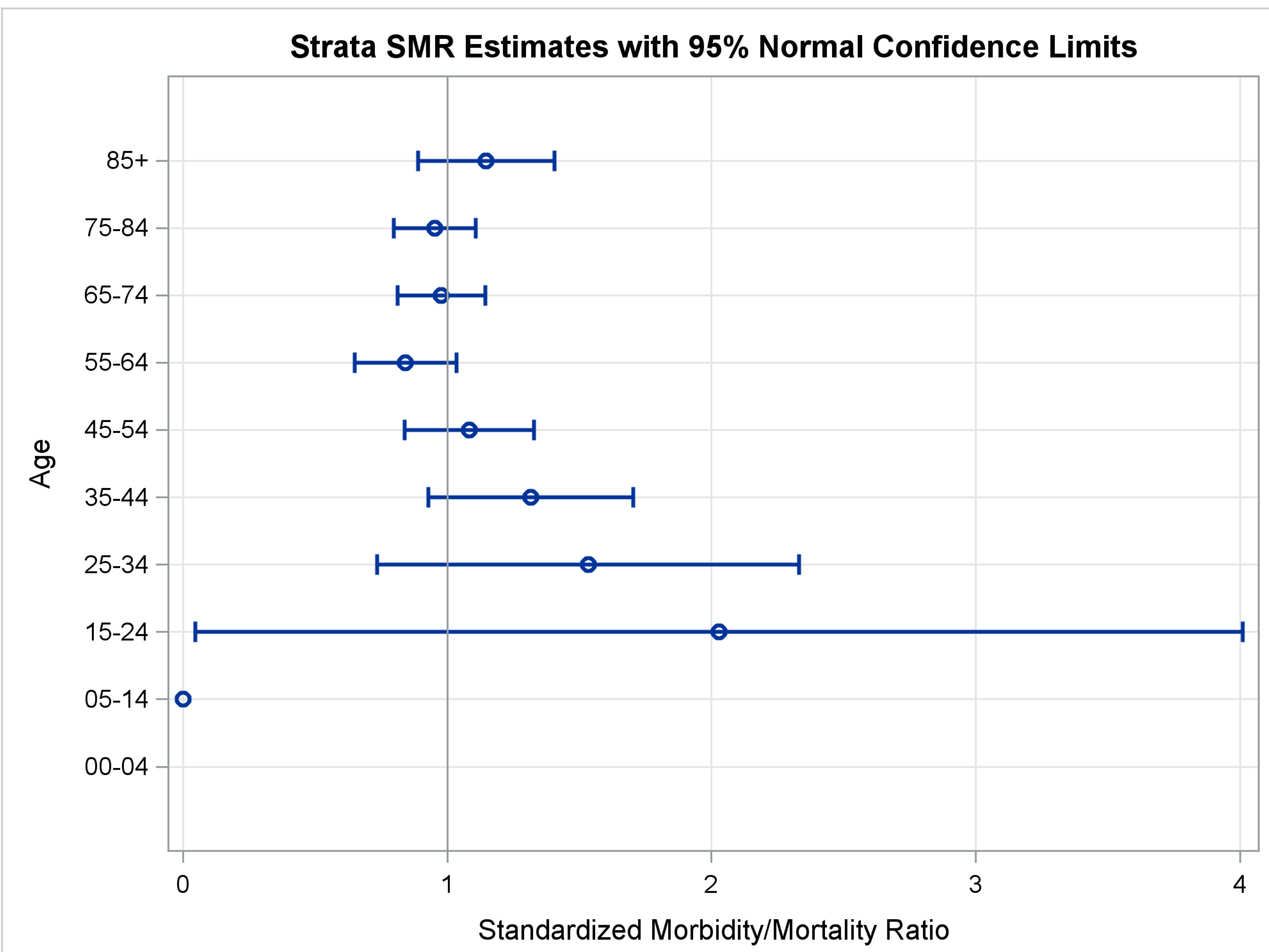
The STDRATE Procedure

Cause=Skin

Standardization Information	
Data Set	WORK.FLORIDA_CS
Reference Data Set	WORK.US_CS
Method	Indirect Standardization
Statistic	Rate
Number of Strata	10
Rate Multiplier	100000

The STRATASMRPLOT option in the ODS SELECT statement requests that the strata SMR plot display stratum-specific SMR estimates for skin cancer with confidence limits, as shown in Output 109.4.4.

Output 109.4.4 Strata SMR Plot



The SMR option in the ODS SELECT statement requests that the "Standardized Morbidity/Mortality Ratio" table display the SMR, its confidence limits, and the test for the null hypothesis H_0 : SMR $=$ 1 for skin cancer, as shown in Output 109.4.5. With the default ALPHA=0.05, 95% confidence limits are constructed.

Output 109.4.5 Standardized Morbidity/Mortality Ratio

Cause=Skin

Standardized Morbidity/Mortality Ratio							
Observed Events	**Expected Events**	**SMR**	**Standard Error**	**95% Normal Confidence Limits**		**Z**	**Pr > |Z|**
538	528.726	1.0175	0.0439	0.9316	1.1035	0.40	0.6893

Similarly, the "Standardization Information" table in Output 109.4.6 displays the standardization information for the second BY group, stomach cancer.

Output 109.4.6 Standardization Information

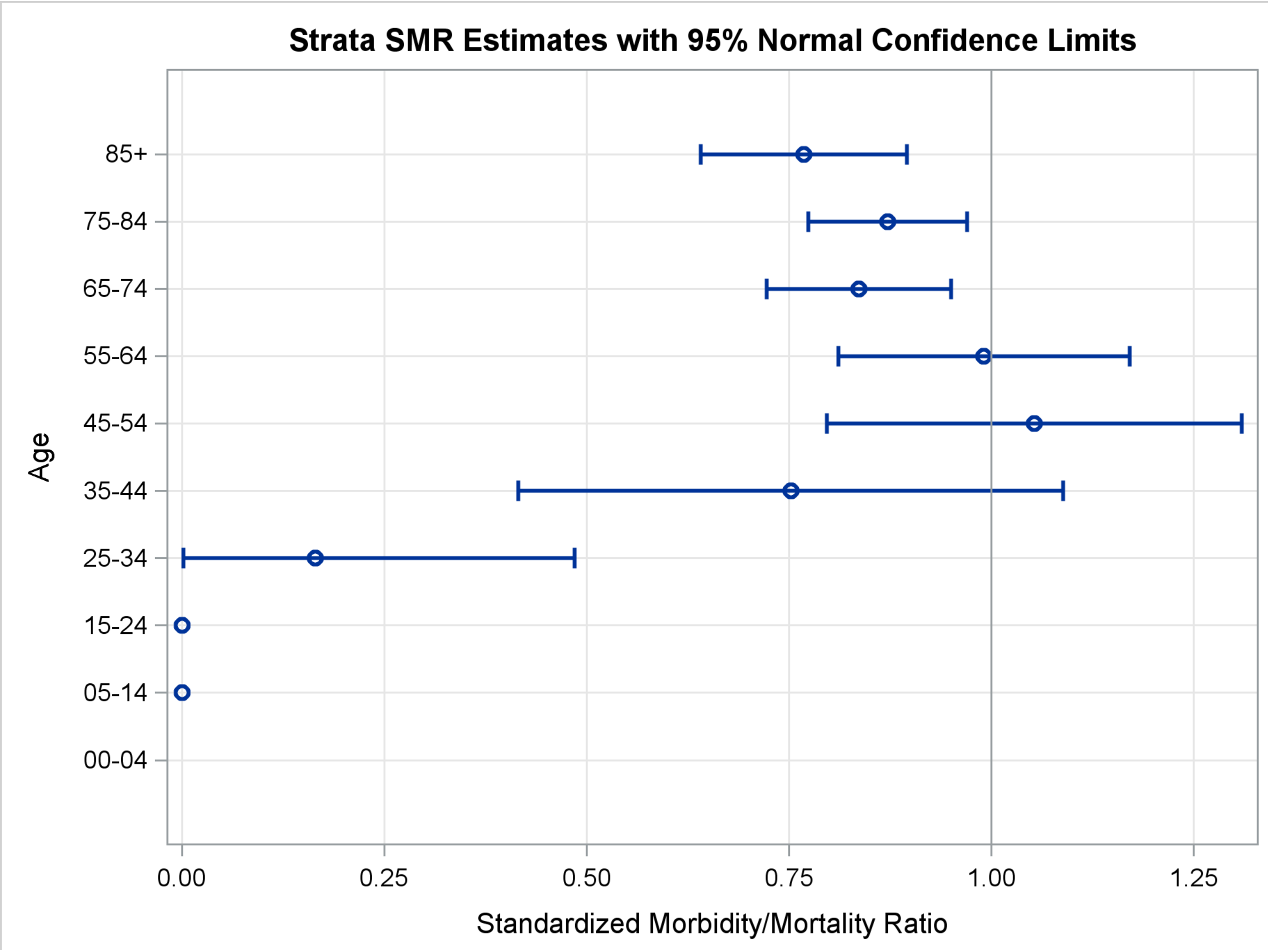
The STDRATE Procedure

Cause=Stomach

Standardization Information	
Data Set	WORK.FLORIDA_CS
Reference Data Set	WORK.US_CS
Method	Indirect Standardization
Statistic	Rate
Number of Strata	10
Rate Multiplier	100000

The "Strata SMR Plot" displays stratum-specific SMR estimates with confidence limits for stomach cancer, as shown in Output 109.4.7.

Output 109.4.7 Strata SMR Plot



The "Standardized Morbidity/Mortality Ratio" table displays the SMR, its confidence limits, and the test for the null hypothesis H_0 : SMR $=$ 1 for stomach cancer, as shown in Output 109.4.8.

Output 109.4.8 Standardized Morbidity/Mortality Ratio

Cause=Stomach

Standardized Morbidity/Mortality Ratio

Observed Events	Expected Events	SMR	Standard Error	95% Normal Confidence Limits		Z	Pr > |Z|
829	962.537	0.8613	0.0299	0.8026	0.9199	-4.64	<.0001

The ODS OUTPUT SMR=SMR_CS statement requests that the “Standardized Morbidity/Mortality Ratio” tables for the two cancer causes be saved in the data set Smr_Cs. The following statements display the selected output variables for the data set:

```
proc print data=Smr_Cs;
   var Cause ObservedEvents ExpectedEvents Smr SmrLcl SmrUcl;
run;
```

Output 109.4.9 SMR Results from BY Groups

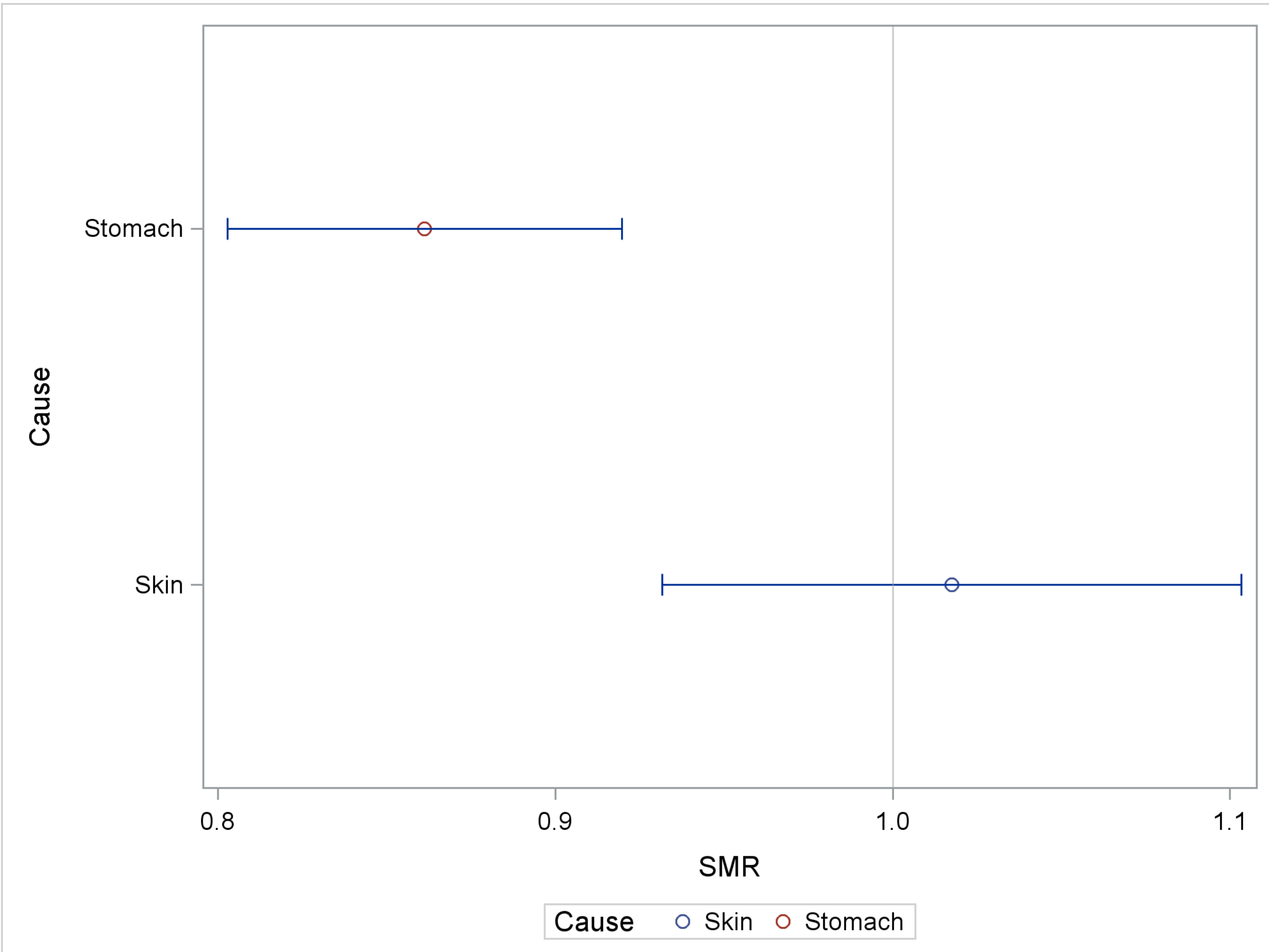
Obs	Cause	ObservedEvents	ExpectedEvents	Smr	SmrLcl	SmrUcl
1	Skin	538	528.726	1.0175	0.9316	1.1035
2	Stomach	829	962.537	0.8613	0.8026	0.9199

The table in Output 109.4.9 shows that the study population (state of Florida) has a higher skin cancer rate and a lower stomach cancer rate than the reference population (United States), but only the lower stomach cancer rate is significant because its corresponding SMR upper confidence limit (0.9199) is less than 1.

The following statements display the standardized morbidity/mortality ratios for the two causes in a plot:

```
proc sgplot data=Smr_Cs;
   scatter y=Cause x=Smr / group=Cause;
   highlow y=Cause high=SmrUcl low=SmrLcl / highcap=serif lowcap=serif;
   yaxis type=discrete;
   xaxis label="SMR";
   refline 1 / axis=x transparency=0.5;
run;
```

Output 109.4.10 SMR Results



Alternatively, you can also use the following statements to obtain separate analyses for the two cancer causes, and then to display these standardized mortality ratios together in a table and in a plot:

```
/*----- Perform Separate Analyses for Different Causes ------*/
ods graphics on;
ods select StdInfo StrataSmrPlot Smr;
proc stdrate data=Florida_Cs refdata=US_Cs
             stat=rate
             method=indirect
             plots=smr
             ;
   population event=Event_C43 total=PYear;
   reference  event=Event_C43 total=PYear;
   strata Age;
ods output smr=Smr_c43;
run;
```

```
ods select StdInfo StrataSmrPlot Smr;
proc stdrate data=Florida_Cs refdata=US_Cs
             stat=rate
             method=indirect
             plots=smr
             ;
   population event=Event_C16 total=PYear;
   reference  event=Event_C16 total=PYear;
   strata Age;
ods output smr=Smr_c16;
run;

/*------------ Combine SMRs -----------*/
data Smr_C43;
   set Smr_C43;
   length Cause $ 7.;
   Cause='Skin';
run;

data Smr_C16;
   set Smr_C16;
   length Cause $ 7.;
   Cause='Stomach';
run;

data Smr_Cs;
   set Smr_C43 Smr_C16;
run;

/*------- Display the Cause-Specific SMRs --------*/
proc print data=Smr_Cs;
   var Cause ObservedEvents ExpectedEvents Smr SmrLcl SmrUcl;
run;

proc sgplot data=Smr_Cs;
   scatter y=Cause x=Smr / group=Cause;
   highlow y=Cause high=SmrUcl low=SmrLcl / highcap=serif lowcap=serif;
   yaxis type=discrete;
   xaxis label="SMR";
   refline 1 / axis=x transparency=0.5;
run;
```

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