SAS/STAT® 14.2 User's Guide

The MDS Procedure

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Chapter 75
The MDS Procedure

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Overview: MDS Procedure

Multidimensional scaling (MDS) refers to a class of methods. These methods estimate coordinates for a set of objects in a space of specified dimensionality. The input data are measurements of distances between pairs of objects. A variety of models can be used that include different ways of computing distances and various functions relating the distances to the actual data. The MDS procedure fits two- and three-way, metric and nonmetric multidimensional scaling models.

The data for the MDS procedure consist of one or more square symmetric or asymmetric matrices of similarities or dissimilarities between *objects* or *stimuli* (Kruskal and Wish 1978, pp. 7–11). Such data are also called *proximity* data. In psychometric applications, each matrix typically corresponds to a *subject*, and models that fit different parameters for each subject are called *individual difference* models.

Missing values are permitted. In particular, if the data are all missing except within some off-diagonal rectangle, the analysis is called *unfolding*. There are, however, many difficulties intrinsic to unfolding models (Heiser 1981). PROC MDS does not perform external unfolding; for analyses requiring external unfolding, use the TRANSREG procedure instead.

The MDS procedure estimates the following parameters by nonlinear least squares:

configuration	the coordinates of each object in a Euclidean (Kruskal and Wish 1978, pp. 17–19) or weighted Euclidean space (Kruskal and Wish 1978, pp. 61–63) of one or more dimensions
dimension coefficients	for each data matrix, the coefficients that multiply each coordinate of the *common* or *group* weighted Euclidean space to yield the *individual* unweighted Euclidean space. These coefficients are the square roots of the *subject weights* (Kruskal and Wish 1978, pp. 61–63). A plot of the dimension coefficients is directly interpretable in that it shows how a unit square in the group space is transformed to a rectangle in each individual space. A plot of subject weights has no such simple interpretation. The weighted Euclidean model is related to the INDSCAL model (Carroll and Chang 1970).
transformation parameters	intercept, slope, or exponent in a linear, affine, or power transformation relating the distances to the data (Kruskal and Wish 1978, pp. 19–22). For a nonmetric analysis, monotone transformations involving no explicit parameters are used (Kruskal and Wish 1978, pp. 22–25). For a discussion of metric versus nonmetric transformations, see Kruskal and Wish (1978, pp. 76–78).

Depending on the LEVEL= option, PROC MDS fits either a regression model of the form

$$\mathit{fit}(datum) = \mathit{fit}(trans(distance)) + error$$

or a measurement model of the form

$$\mathit{fit}(trans(datum)) = \mathit{fit}(distance) + error$$

where

fit is a predetermined power or logarithmic transformation specified by the FIT= option.

trans is an estimated ("optimal") linear, affine, power, or monotone transformation specified by the LEVEL= option.

datum is a measure of the similarity or dissimilarity of two objects or stimuli.

distance is a distance computed from the estimated coordinates of the two objects and estimated dimension coefficients in a space of one or more dimensions. If there are no dimension coefficients (COEF=IDENTITY), this is an unweighted Euclidean distance. If dimension coefficients are used (COEF=DIAGONAL), this is a weighted Euclidean distance where the weights are the squares of the dimension coefficients; alternatively, you can multiply each dimension by its coefficient and compute an unweighted Euclidean distance.

error is an error term assumed to have an approximately normal distribution and to be independently and identically distributed for all data. Under these assumptions, least-squares estimation is statistically appropriate.

For an introduction to multidimensional scaling, see Kruskal and Wish (1978) and Arabie, Carroll, and DeSarbo (1987). A more advanced treatment is given by Young (1987). Many practical issues of data collection and analysis are discussed in Schiffman, Reynolds, and Young (1981). The fundamentals of psychological measurement, including both unidimensional and multidimensional scaling, are expounded by Torgerson (1958). Nonlinear least-squares estimation of PROC MDS models is discussed in Null and Sarle (1982).

Getting Started: MDS Procedure

The simplest application of PROC MDS is to reconstruct a map from a table of distances between points on the map (Kruskal and Wish 1978, pp. 7–9). A data set containing a table of flying mileages between 10 U.S. cities is available in the Sashelp library.

Since the flying mileages are very good approximations to Euclidean distance, no transformation is needed to convert distances from the model to data. The analysis can therefore be done at the absolute level of measurement, as displayed in the following PROC MDS step (LEVEL=ABSOLUTE). The following statements produce Figure 75.1 and Figure 75.2:

```
title 'Analysis of Flying Mileages between Ten U.S. Cities';

ods graphics on;

proc mds data=sashelp.mileages level=absolute;
   id city;
run;
```

PROC MDS first displays the iteration history. In this example, only one iteration is required. The badness-of-fit criterion 0.001689 indicates that the data fit the model extremely well. You can also see that the fit is excellent in the fit plot in Figure 75.2.

Figure 75.1 Iteration History from PROC MDS

Analysis of Flying Mileages between Ten U.S. Cities

Multidimensional Scaling: Data=SASHELP.MILEAGES.DATA Shape=TRIANGLE Condition=MATRIX Level=ABSOLUTE Coef=IDENTITY Dimension=2 Formula=1 Fit=1

Gconverge=0.01 Maxiter=100 Over=1 Ridge=0.0001

Iteration	Type	Badness-of-Fit Criterion	Change in Criterion	Convergence Measure
0	Initial	0.003273	.	0.8562
1	Lev-Mar	0.001689	0.001584	0.005128

Convergence criterion is satisfied.

While PROC MDS can recover the relative positions of the cities, it cannot determine absolute location or orientation. In this case, north is toward the bottom of the plot. (See the first plot in Figure 75.2.)

Figure 75.2 Plot of Estimated Configuration and Fit

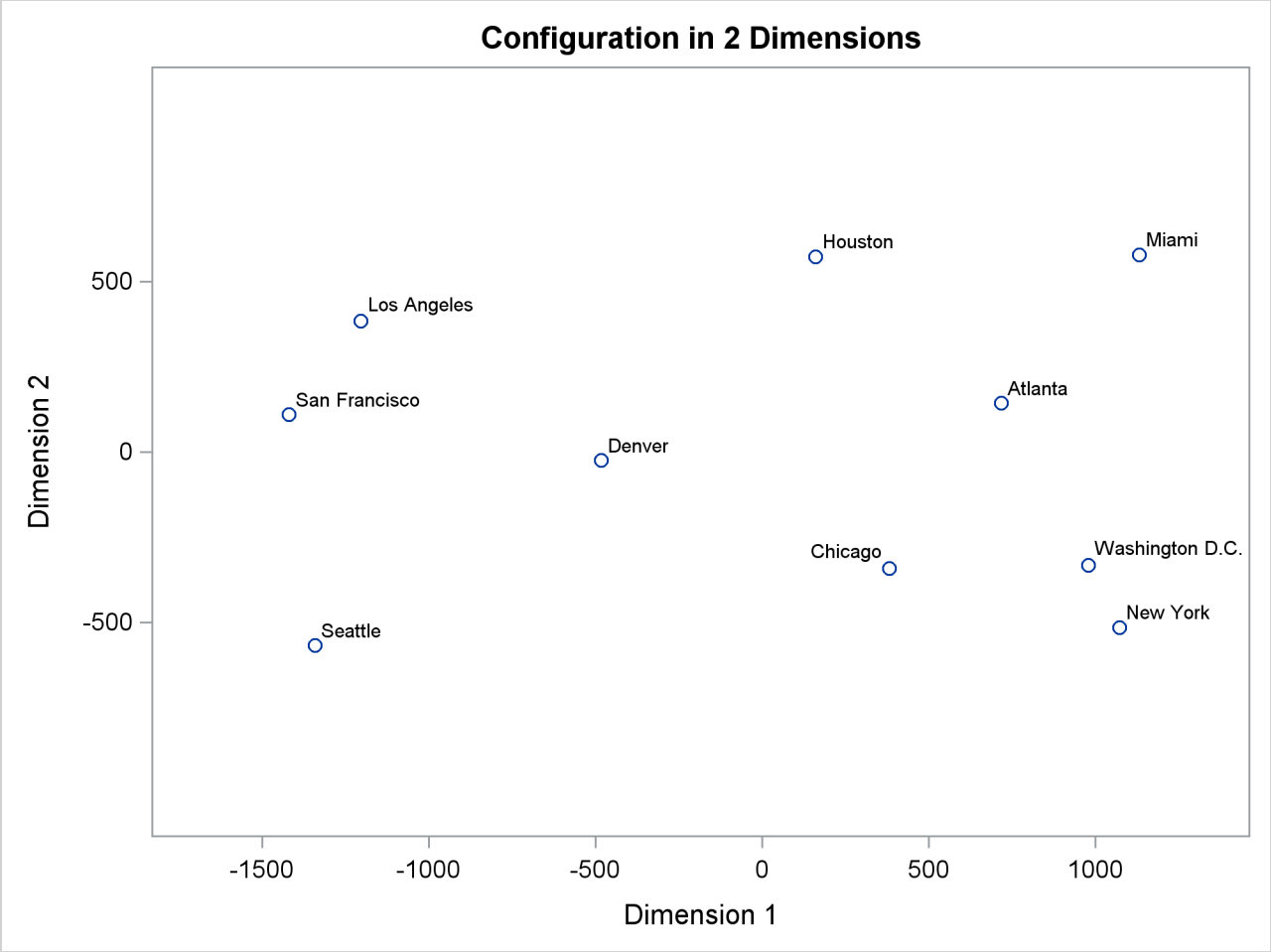
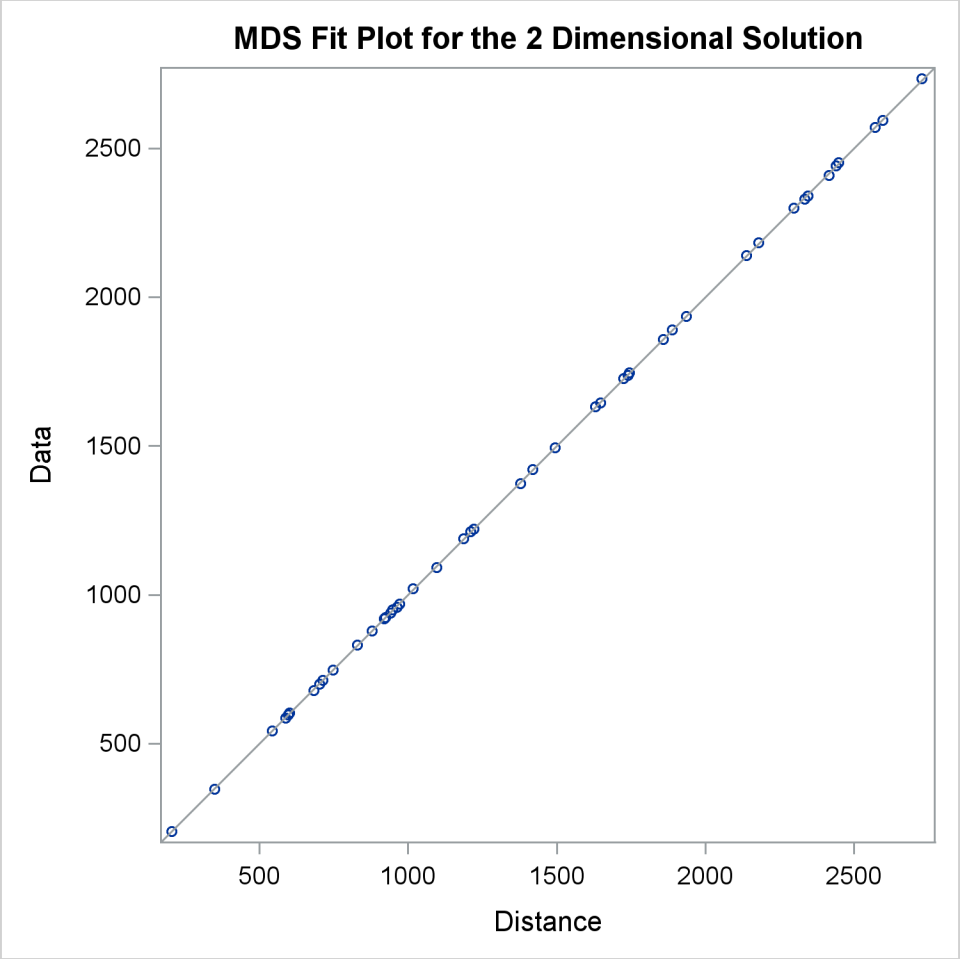


Figure 75.2 *continued*



Syntax: MDS Procedure

The following statements are available in the MDS procedure:

PROC MDS < *options* > **;**
VAR *variables* **;**
INVAR *variables* **;**
ID | **OBJECT** *variable* **;**
MATRIX | **SUBJECT** *variable* **;**
WEIGHT *variables* **;**
BY *variables* **;**

The PROC MDS statement is required. All other statements are optional.

PROC MDS Statement

PROC MDS < *options* > **;**

The PROC MDS statement invokes the MDS procedure. PROC MDS produces an iteration history by default. Graphical displays are produced when ODS Graphics is enabled. Additional displayed output is controlled by the interaction of the PCONFIG, PCOEF, PTRANS, PFIT, and PFITROW options with the PININ, PINIT, PITER, and PFINAL options. The PCONFIG, PCOEF, PTRANS, PFIT, and PFITROW options specify *which* estimates and fit statistics are to be displayed. The PININ, PINIT, PITER, and PFINAL options specify *when* the estimates and fit statistics are to be displayed. If you specify at least one of the PCONFIG, PCOEF, PTRANS, PFIT, and PFITROW options but none of the PININ, PINIT, PITER, and PFINAL options, the final results (PFINAL) are displayed. If you specify at least one of the PININ, PINIT, PITER, and PFINAL options but none of the PCONFIG, PCOEF, PTRANS, PFIT, and PFITROW options, all estimates (PCONFIG, PCOEF, PTRANS) and the fit statistics for each matrix and for the entire sample (PFIT) are displayed. If you do not specify any of these nine options, no estimates or fit statistics are displayed (except the badness-of-fit criterion in the iteration history).

The types of estimates written to the OUT= data set are determined by the OCONFIG, OCOEF, OTRANS, and OCRIT options. If you do not specify any of these four options, the estimates of all the parameters of the PROC MDS model and the value of the badness-of-fit criterion appear in the OUT= data set. If you specify one or more of these *options*, only the information requested by the specified options appears in the OUT= data set. Also, the OITER option causes these statistics to be written to the OUT= data set after initialization and on each iteration, as well as after the iterations have terminated.

Table 75.2 summarizes the *options* available in the PROC MDS statement.

Table 75.2 Summary of PROC MDS Statement Options

Option	Description
Data Set Options	
DATA=	Specifies the input SAS data set
INITIAL=	Specifies the input SAS data set containing initial values
OUT=	Specifies the output data set
OUTFIT=	Specifies the output fit data set
OUTRES=	Specifies the output residual data set
Input Control	
CUTOFF=	Replaces data values with missing values
SHAPE=	Specifies the shape of the input data matrices
SIMILAR=	Specifies that the data are similarity measurements
Model	
COEF=	Specifies the type of matrix for the coefficients
CONDITION=	Specifies the conditionality of the data
DIMENSION=	Specifies the number of dimensions
LEVEL=	Specifies the measurement level
NEGATIVE	Permits slopes or powers to be negative
UNTIE	Permits tied data to be untied

Table 75.2 *continued*

Option	Description
Initialization	
INAV=	Affects the computation of initial coordinates
NOULB	Specifies the missing data initialization
RANDOM=	Specifies initial random coordinates
Estimation	
ALTERNATE=	Specifies the alternating-least-squares algorithm
CONVERGE=	Specifies the convergence criterion
EPSILON=	Specifies the amount added to squared distances
FIT=	Specifies a predetermined transformation
FORMULA=	Specifies the badness-of-fit formula
GCONVERGE=	Specifies the gradient convergence criterion
MAXITER=	Specifies the maximum number of iterations
MCONVERGE=	Specifies the monotone convergence criterion
MINCRIT=	Specifies the minimum badness-of-fit criterion
NONORM	Suppresses normalization of the initial and final estimates
OVER=	Specifies the maximum overrelaxation factor
RIDGE=	Specifies the initial ridge value
SINGULAR=	Specifies the singularity criterion
Control Output Data Set Contents	
OCOEF	Writes the dimension coefficients to the OUT= data set
OCONFIG	Writes the coordinates of the objects to the OUT= data set
OCRIT	Writes the badness-of-fit criterion to the OUT= data set
OITER	Writes current values after initialization and on every iteration
OTRANS	Writes the transformation parameter estimates to the OUT= data set
Control Displayed Output	
DECIMALS=	Specifies how many decimal places to use
NOPHIST	Suppresses the iteration history
PCOEF	Displays the estimated dimension coefficients
PCONFIG	Displays the estimated coordinates
PDATA	Displays each data matrix
PFINAL	Displays final estimates
PFIT	Displays the badness-of-fit criterion
PFITROW	Displays the badness-of-fit criterion for each row
PINAVDATA	Displays INAV= data set information
PINEIGVAL	Displays the initial eigenvalues
PINEIGVEC	Displays the initial eigenvectors
PININ	Displays values read from the INITIAL= data set
PINIT	Displays initial values
PITER	Displays estimates on each iteration
PLOTS=	Controls the graphical displays
PTRANS	Displays the estimated transformation parameters

ALTERNATE=NONE | NO | N

ALTERNATE=MATRIX | MAT | M | SUBJECT | SUB | S

ALTERNATE=ROW | R < *=n* **>**

ALT=N | M| S | R < *=n* **>**

determines what form of alternating-least-squares algorithm is used. The default depends on the amount of memory available. The following ALTERNATE= options are listed in order of decreasing memory requirements:

NONE causes all parameters to be adjusted simultaneously on each iteration. This option is usually best for a small number of subjects and objects.

MATRIX adjusts all the parameters for the first subject, then all the parameters for the second subject, and so on, and finally adjusts all parameters that do not correspond to a subject, such as coordinates and unconditional transformations. This option usually works best for a large number of subjects with a small number of objects.

ROW treats subject parameters the same way as the ALTERNATE=MATRIX option but also includes separate stages for unconditional parameters and for subsets of the objects. The ALT=ROW option usually works best for a large number of objects. Specifying ALT=ROW=*n* divides the objects into subsets of *n* objects each, except possibly for one subset when *n* does not divide the number of objects evenly. If you omit =*n*, the number of objects in the subsets is determined from the amount of memory available. The smaller the value of *n*, the less memory is required.

When you specify the LEVEL=ORDINAL option, the monotone transformation is always computed in a separate stage and is listed as a separate iteration in the iteration history. In this case, estimation is done by iteratively reweighted least squares. The weights are recomputed according to the FORMULA= option on each monotone iteration; hence, it is possible for the badness-of-fit criterion to increase after a monotone iteration.

COEF=IDENTITY | IDEN | I

COEF=DIAGONAL | DIAG | D

specifies the type of matrix for the dimension coefficients.

IDENTITY is the default, which yields Euclidean distances.

DIAGONAL produces weighted Euclidean distances, in which each subject can have different weights for the dimensions. The dimension coefficients that PROC MDS outputs are related to the square roots of what are called subject weights in PROC ALSCAL; the normalization in PROC MDS also differs from that in PROC ALSCAL. The weighted Euclidean model is related to the INDSCAL model (Carroll and Chang 1970).

CONDITION=UN | U

CONDITION=MATRIX | MAT | M | SUBJECT | SUB | S

CONDITION=ROW | R

COND=U | M | S | R

specifies the conditionality of the data (Young 1987, pp. 60–63). The data are divided into disjoint subsets called *partitions*. Within each partition, a separate transformation is applied, as specified by the LEVEL= option. The three types of conditionality are as follows:

UN (unconditional) puts all the data into a single partition.

MATRIX (matrix conditional) makes each data matrix a partition.

ROW (row conditional) makes each row of each data matrix a partition.

The default is CONDITION=MATRIX. The CONDITION= option also determines the default value for the SHAPE= option. If you specify the CONDITION=ROW option and omit the SHAPE= option, each data matrix is stored as a square and possibly asymmetric matrix. If you specify the CONDITION=UN or CONDITION=MATRIX option and omit the SHAPE= option, only one triangle is stored. See the SHAPE= option for details.

CONVERGE | CONV=*p*

sets both the gradient convergence criterion and the monotone convergence criterion to *p*, where $0 \leq p \leq 1$. The default is CONVERGE=0.01; smaller values might greatly increase the number of iterations required. Values less than 0.0001 might be impossible to satisfy because of the limits of machine precision. (See the GCONVERGE= and MCONVERGE= options.)

CUTOFF=*n*

replaces data values less than *n* with missing values. The default is CUTOFF=0.

DATA=*SAS-data-set*

specifies the SAS data set containing one or more square matrices to be analyzed. In typical psychometric data, each matrix contains judgments from one subject, so there is a one-to-one correspondence between data matrices and subjects.

The data matrices contain similarity or dissimilarity measurements to be modeled and, optionally, weights for these data. The data are generally assumed to be dissimilarities unless you use the SIMILAR option. However, if there are nonmissing diagonal values and these values are predominantly larger than the off-diagonal values, the data are assumed to be similarities and are treated as if the SIMILAR option is specified. The diagonal elements are not otherwise used in fitting the model.

Each matrix must have exactly the same number of observations as the number of variables specified by the VAR statement or determined by defaults. This number is the number of objects or stimuli.

The first observation and variable are assumed to contain data for the first object, the second observation and variable are assumed to contain data for the second object, and so on.

When there are two or more matrices, the observations in each matrix must correspond to the same objects in the same order as in the first matrix.

The matrices can be symmetric or asymmetric, as specified by the SHAPE= option.

DECIMALS | DEC=*n*

specifies how many decimal places to use when displaying the parameter estimates and fit statistics. The default is DECIMALS=2, which is generally reasonable except in conjunction with the LEVEL=ABSOLUTE option and very large or very small data.

DIMENSION | DIMENS | DIM=*n* **< TO** *m* **< BY=***i* **> >**

specifies the number of dimensions to use in the MDS model, where $1 \leq n, m < \textit{number of objects}$. The parameter *i* can be either positive or negative but not zero. If you specify different values for *n* and *m*, a separate model is fitted for each requested dimension. If you specify only DIMENSION=*n*, then only *n* dimensions are fitted. The default is DIMENSION=2 if there are three or more objects; otherwise, DIMENSION=1 is the only valid specification. The analyses for each number of dimensions are done independently. For information about choosing the dimensionality, see Kruskal and Wish (1978 , pp. 48–60).

EPSILON | EPS=*n*

specifies a number *n*, $0 < n < 1$, that determines the amount added to squared distances computed from the model to avoid numerical problems such as division by 0. This amount is computed as ϵ equal to *n* times the mean squared distance in the initial configuration. The distance in the MDS model is thus computed as

$$distance = \sqrt{sqdist + \epsilon}$$

where *sqdist* is the squared Euclidean distance or the weighted squared Euclidean distance.

The default is EPSILON=1E–12, which is small enough to have no practical effect on the estimates unless the FIT= value is nonpositive and there are dissimilarities that are very close to 0. Hence, when the FIT= value is nonpositive, dissimilarities less than *n* times 100 times the maximum dissimilarity are not permitted.

FIT=DISTANCE | DIS | D

FIT=SQUARED | SQU | S

FIT=LOG | L

FIT=*n*

specifies a predetermined (not estimated) transformation to apply to both sides of the MDS model before the error term is added.

The default is FIT=DISTANCE or, equivalently, FIT=1, which fits data to distances.

The option FIT=SQUARED or FIT=2 fits squared data to squared distances. This gives greater importance to large data and distances and lesser importance to small data and distances in fitting the model.

The FIT=LOG or FIT=0 option fits log data to log distances. This gives lesser importance to large data and distances and greater importance to small data and distances in fitting the model.

In general, the FIT=*n* option fits *n*th-power data to *n*th-power distances. Values of *n* that are large in absolute value can cause floating-point overflows.

If the FIT= value is 0 or negative, the data must be strictly positive (see the EPSILON= option). Negative data might produce strange results with any value other than FIT=1.

FORMULA=*0* | OLS | O

FORMULA=*1* | USS | U

FORMULA=*2* | CSS | C

FOR=O | U | C

determines how the badness-of-fit criterion is standardized in correspondence with stress formulas 1 and 2 (Kruskal and Wish 1978, pp. 24–26). The default is FORMULA=1 unless you specify FIT=LOG, in which case the default is FORMULA=2. Data partitions are defined by the CONDITION= option.

0 fits a regression model by ordinary least squares (Null and Sarle 1982) without standardizing the partitions; this option cannot be used with the LEVEL=ORDINAL option. The badness-of-fit criterion is the square root of the error sum of squares.

1 standardizes each partition by the uncorrected sum of squares of the (possibly transformed) data; this option should not be used with the FIT=LOG option. With the FIT=DISTANCE and LEVEL=ORDINAL options, this is equivalent to Kruskal's stress formula 1 or an obvious generalization thereof. With the FIT=SQUARED and LEVEL=ORDINAL options, this is equivalent to Young's s-stress formula 1 or an obvious generalization thereof. The badness-of-fit criterion is analogous to $\sqrt{1 - \mathrm{R}^2}$, where R is a multiple correlation about the origin.

2 standardizes each partition by the corrected sum of squares of the (possibly transformed) data; this option is the recommended method for unfolding. With the FIT=DISTANCE and LEVEL=ORDINAL options, this is equivalent to Kruskal's stress formula 2 or an obvious generalization thereof. With the FIT=SQUARED and LEVEL=ORDINAL options, this is equivalent to Young's s-stress formula 2 or an obvious generalization thereof. The badness-of-fit criterion is analogous to $\sqrt{1 - \mathrm{R}^2}$, where R is a multiple correlation computed with a denominator corrected for the mean.

GCONVERGE | GCONV=*p*

sets the gradient convergence criterion to *p*, where $0 \leq p \leq 1$. The default is GCONVERGE=0.01; smaller values might greatly increase the number of iterations required. Values less than 0.0001 might be impossible to satisfy because of the limits of machine precision.

The gradient convergence measure is the multiple correlation of the Jacobian matrix with the residual vector, uncorrected for the mean. (See the CONVERGE= and MCONVERGE= options.)

INAV=DATA | D

INAV=SSCP | S

affects the computation of initial coordinates. The default is INAV=DATA.

DATA computes a weighted average of the data matrices. Its value is estimated only if an element is missing from every data matrix. The weighted average of the data matrices with missing values filled in is then converted to a scalar products matrix (or what would be a scalar products matrix if the fit were perfect), from which the initial coordinates are computed.

SSCP estimates missing values in each data matrix and converts each data matrix to a scalar products matrix. The initial coordinates are computed from the unweighted average of the scalar products matrices.

INITIAL | IN=*SAS-data-set*

specifies a SAS data set containing initial values for some or all of the parameters of the MDS model. If the INITIAL= option is omitted, the initial values are computed from the data.

LEVEL=ABSOLUTE | ABS | A

LEVEL=RATIO | RAT | R

LEVEL=INTERVAL | INT | I

LEVEL=LOGINTERVAL | LOG | L

LEVEL=ORDINAL | ORD | O

specifies the measurement level of the data and hence the type of estimated (optimal) transformations applied to the data or distances (Young 1987, pp. 57–60; Krantz et al. 1971, pp. 9–12) within each partition as specified by the CONDITION= option. LEVEL=ORDINAL specifies a nonmetric analysis, while all other LEVEL= options specify metric analyses. The default is LEVEL=ORDINAL.

ABSOLUTE permits no optimal transformations. Hence, the distinction between regression and measurement models is irrelevant.

RATIO fits a regression model in which the distances are multiplied by a slope parameter in each partition (a linear transformation). In this case, the regression model is equivalent to the measurement model with the slope parameter reciprocated.

INTERVAL fits a regression model in which the distances are multiplied by a slope parameter and added to an intercept parameter in each partition (an affine transformation). In this case, the regression and measurement models differ if there is more than one partition.

LOGINTERVAL fits a regression model in which the distances are raised to a power and multiplied by a slope parameter in each partition (a power transformation).

ORDINAL fits a measurement model in which a least-squares monotone increasing transformation is applied to the data in each partition. At the ordinal measurement level, the regression and measurement models differ.

MAXITER | ITER=*n*

specifies the maximum number of iterations, where $n \geq 0$. The default is MAXITER=100.

MCONVERGE | MCONV=*p*

sets the monotone convergence criterion to p, where $0 \leq p \leq 1$, for use with the LEVEL=ORDINAL option. The default is MCONVERGE=0.01; if you want greater precision, MCONVERGE=0.001 is usually reasonable, but smaller values might greatly increase the number of iterations required.

The monotone convergence criterion is the Euclidean norm of the change in the optimally scaled data divided by the Euclidean norm of the optimally scaled data, averaged across partitions defined by the CONDITION= option. (See the CONVERGE= and GCONVERGE= options.)

MINCRIT | CRITMIN=*n*

causes iteration to terminate when the badness-of-fit criterion is less than or equal to n, where $n \geq 0$. The default is MINCRIT=1E–6.

NEGATIVE
permits slopes or powers to be negative with the LEVEL=RATIO, INTERVAL, or LOGINTERVAL option.

NONORM
suppresses normalization of the initial and final estimates.

NOPHIST | NOPRINT | NOP
suppresses the output of the iteration history.

NOULB
causes missing data to be estimated during initialization by the average nonmissing value, where the average is computed according to the FIT= option. Otherwise, missing data are estimated by interpolating between the Rabinowitz (1976) upper and lower bounds.

OCOEF
writes the dimension coefficients to the OUT= data set. See the OUT= option for interactions with other options.

OCONFIG
writes the coordinates of the objects to the OUT= data set. See the OUT= option for interactions with other options.

OCRIT
writes the badness-of-fit criterion to the OUT= data set. See the OUT= option for interactions with other options.

OITER | OUTITER
writes current values to the output data sets after initialization and on every iteration. Otherwise, only the final values are written to any output data sets. (See the OUT=, OUTFIT=, and OUTRES= options.)

OTRANS
writes the transformation parameter estimates to the OUT= data set if any such estimates are computed. There are no transformation parameters with the LEVEL=ORDINAL option. See the OUT= option for interactions with other options.

OUT=*SAS-data-set*
creates a SAS data set containing, by default, the estimates of all the parameters of the PROC MDS model and the value of the badness-of-fit criterion. However, if you specify one or more of the OCONFIG, OCOEF, OTRANS, and OCRIT options, only the information requested by the specified options appears in the OUT= data set. (See also the OITER option.)

OUTFIT=*SAS-data-set*
creates a SAS data set containing goodness-of-fit and badness-of-fit measures for each partition as well as for the entire data set. (See also the OITER option.)

OUTRES=*SAS-data-set*
creates a SAS data set containing one observation for each nonmissing data value from the DATA= data set. Each observation contains the original data value, the estimated distance computed from the MDS model, transformed data and distances, and the residual. (See also the OITER option.)

OVER=*n*
specifies the maximum overrelaxation factor, where $n \geq 1$. Values between 1 and 2 are generally reasonable. The default is OVER=2 with the LEVEL=ORDINAL, ALTERNATE=MATRIX, or ALTERNATE=ROW option; otherwise, the default is OVER=1. Use this option only if you have convergence problems.

PCOEF
produces the estimated dimension coefficients.

PCONFIG
produces the estimated coordinates of the objects in the configuration.

PDATA
displays each data matrix.

PFINAL
displays final estimates.

PFIT
displays the badness-of-fit criterion and various types of correlations between the data and fitted values for each data matrix, as well as for the entire sample.

PFITROW
displays the badness-of-fit criterion and various types of correlations between the data and fitted values for each row, as well as for each data matrix and for the entire sample. This option works only with the CONDITION=ROW option.

PINAVDATA
displays the sum of the weights and the weighted average of the data matrices computed during initialization with the INAV=DATA option.

PINEIGVAL
displays the eigenvalues computed during initialization.

PINEIGVEC
displays the eigenvectors computed during initialization.

PININ
displays values read from the INITIAL= data set. Since these values might be incomplete, the PFIT and PFITROW options do not apply.

PINIT
displays initial values.

PITER
displays estimates on each iteration.

PLOTS<(*global-plot-option***)> <=** *plot-request***<(***options***)>>**
PLOTS<(*global-plot-options***)> <=** **(***plot-request***<(***options***)> <...** *plot-request***<(***options***)>>)>**

specifies options that control the details of the plots. When you specify only one plot request, you can omit the parentheses around the plot request.

ODS Graphics must be enabled before plots can be requested. For example:

```
ods graphics on;

proc mds plots(flip);
run;

ods graphics off;
```

For more information about enabling and disabling ODS Graphics, see the section "Enabling and Disabling ODS Graphics" on page 607 in Chapter 21, "Statistical Graphics Using ODS."

The global plot option is as follows:

FLIP
flips or interchanges the X-axis and Y-axis dimensions for configuration and coefficient plots.

The plot requests include the following:

COEFFICIENTS(ONE)
combines the INDSCAL coefficients panel of plots into a single plot. By default, the display consists of a panel with two plots. The vectors are displayed in the left plot, and the labels are displayed in the right plot. The right plot provides a magnification of the region of the vector endpoints. In contrast, the single display, requested by COEFFICIENTS(ONE), is more compact, but there is less room for vector labels. It is often easier to identify the vectors in the default display.

NONE
suppresses all plots.

By default, a fit plot is produced. When more then one dimension is requested, plots of the configuration are also plotted. For individual differences models with more than one dimension, the subject weights or coefficients are plotted. When more than one value is specified for the DIMENSION= option, the badness-of-fit plot is produced.

PTRANS
displays the estimated transformation parameters if any are computed. There are no transformation parameters with the LEVEL=ORDINAL option.

RANDOM< =*seed* >
causes initial coordinate values to be pseudo-random numbers. In one dimension, the pseudo-random numbers are uniformly distributed on an interval. In two or more dimensions, the pseudo-random numbers are uniformly distributed on the circumference of a circle or the surface of a (hyper)sphere.

RIDGE=*n*

specifies the initial ridge value, where $n \geq 0$. The default is RIDGE=1E–4.

If you get a floating-point overflow in the first few iterations, specify a larger value such as RIDGE=0.01, RIDGE=1, or RIDGE=100.

If you know that the initial estimates are very good, using RIDGE=0 might speed convergence.

SHAPE=TRIANGULAR | TRIANGLE | TRI | T

SHAPE=SQUARE | SQU | S

determines whether the entire data matrix for each subject or only one triangle of the matrix is stored and analyzed. If you specify the CONDITION=ROW option, the default is SHAPE=SQUARE. Otherwise, the default is SHAPE=TRIANGLE.

SQUARE causes the entire matrix to be stored and analyzed. The matrix can be asymmetric.

TRIANGLE causes only one triangle to be stored. However, PROC MDS reads both upper and lower triangles to look for nonmissing values and to symmetrize the data if needed. If corresponding elements in the upper and lower triangles both contain nonmissing values, only the average of the two values is stored and analyzed (Kruskal and Wish 1978 , p. 74). Also, if an OUTRES= data set is requested, only the average of the two corresponding elements is output.

SIMILAR | SIM< =*max* **>**

causes the data to be treated as similarity measurements rather than dissimilarities. If =*max* is not specified, each data value is converted to a dissimilarity by subtracting it from the maximum value in the data set or BY group. If =*max* is specified, each data value is subtracted from the maximum of *max* and the data. The diagonal data are included in computing these maxima.

By default, the data are assumed to be dissimilarities unless there are nonmissing diagonal values and these values are predominantly larger than the off-diagonal values. In this case, the data are assumed to be similarities and are treated as if the SIMILAR option is specified.

SINGULAR=*p*

specifies the singularity criterion p, $0 \leq p \leq 1$. The default is SINGULAR=1E–8.

UNTIE

permits tied data to be assigned different optimally scaled values with the LEVEL=ORDINAL option. Otherwise, tied data are assigned equal optimally scaled values. The UNTIE option has no effect with values of the LEVEL= option other than LEVEL=ORDINAL.

BY Statement

BY *variables* **;**

You can specify a BY statement with PROC MDS to obtain separate analyses of observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the MDS procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

If the INITIAL= data set contains the BY variables, the BY groups must appear in the same order as in the DATA= data set. If the BY variables are not in the INITIAL= data set, the entire data set is used to provide initial values for each BY group in the DATA= data set.

For more information about BY-group processing, see the discussion in *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the discussion in the *Base SAS Procedures Guide*.

ID Statement

ID | OBJECT | OBJ *variable* **;**

The ID statement specifies a variable in the DATA= data set that contains descriptive labels for the objects. The labels are used in the output and are copied to the OUT= data set. If there is more than one data matrix, only the ID values from the observations containing the first data matrix are used.

The ID variable is not used to establish any correspondence between observations and variables.

If the ID statement is omitted, the variable labels or names are used as object labels.

INVAR Statement

INVAR *variables* **;**

The INVAR statement specifies the numeric variables in the INITIAL= data set that contain initial parameter estimates. The first variable corresponds to the first dimension, the second variable to the second dimension, and so on.

If the INVAR statement is omitted, the variables Dim1, . . . , Dim*m* are used, where *m* is the maximum number of dimensions.

MATRIX Statement

MATRIX | MAT | SUBJECT | SUB *variable* **;**

The MATRIX statement specifies a variable in the DATA= data set that contains descriptive labels for the data matrices or subjects. The labels are used in the output and are copied to the OUT= and OUTRES= data sets. Only the first observation from each data matrix is used to obtain the label for that matrix.

If the MATRIX statement is omitted, the matrices are labeled 1, 2, 3, and so on.

VAR Statement

VAR *variables* **;**

The VAR statement specifies the numeric variables in the DATA= data set that contain similarity or dissimilarity measurements on a set of objects or stimuli. Each variable corresponds to one object.

If the VAR statement is omitted, all numeric variables that are not specified in another statement are used.

To analyze a subset of the objects in a data set, you can specify the variable names corresponding to the columns in the subset, but you must also use a DATA step or a WHERE clause to specify the rows in the subset. PROC MDS expects to read one or more square matrices, and you must ensure that the rows in the data set correctly correspond to the columns in number and order.

WEIGHT Statement

WEIGHT *variables* **;**

The WEIGHT statement specifies numeric variables in the DATA= data set that contain weights for each similarity or dissimilarity measurement. These weights are used to compute weighted least-squares estimates. The number of WEIGHT variables must be the same as the number of VAR variables, and the variables in the WEIGHT statement must be in the same order as the corresponding variables in the VAR statement.

If the WEIGHT statement is omitted, all data within a partition are assigned equal weights.

Data with zero or negative weights are ignored in fitting the model but are included in the OUTRES= data set and in monotone transformations.

Details: MDS Procedure

Formulas

The following notation is used:

A_p	intercept for partition p
B_p	slope for partition p
C_p	power for partition p
D_{rcs}	distance computed from the model between objects r and c for subject s
F_{rcs}	data weight for objects r and c for subject s obtained from the cth WEIGHT variable, or 1 if there is no WEIGHT statement
f	value of the FIT= option
N	number of objects
O_{rcs}	observed dissimilarity between objects r and c for subject s
P_{rcs}	partition index for objects r and c for subject s
Q_{rcs}	dissimilarity after applying any applicable estimated transformation for objects r and c for subject s
R_{rcs}	residual for objects r and c for subject s
S_p	standardization factor for partition p
$T_p(\cdot)$	estimated transformation for partition p
V_{sd}	coefficient for subject s on dimension d
X_{nd}	coordinate for object n on dimension d

Summations are taken over nonmissing values.

Distances are computed from the model as

$$D_{rcs} = \begin{cases} \sqrt{\sum_d (X_{rd} - X_{cd})^2} & \text{for COEF=IDENTITY: Euclidean distance} \\ \sqrt{\sum_d V_{sd}^2 (X_{rd} - X_{cd})^2} & \text{for COEF=DIAGONAL: weighted Euclidean distance} \end{cases}$$

Partition indexes are

$$\begin{aligned} P_{rcs} &= 1 && \text{for CONDITION=UN} \\ &= s && \text{for CONDITION=MATRIX} \\ &= (s-1)N + r && \text{for CONDITION=ROW} \end{aligned}$$

The estimated transformation for each partition is

$$\begin{aligned} T_p(d) &= d && \text{for LEVEL=ABSOLUTE} \\ &= B_p d && \text{for LEVEL=RATIO} \\ &= A_p + B_p d && \text{for LEVEL=INTERVAL} \\ &= B_p d^{C_p} && \text{for LEVEL=LOGINTERVAL} \end{aligned}$$

For LEVEL=ORDINAL, $T_p(\cdot)$ is computed as a least-squares monotone transformation.

For LEVEL=ABSOLUTE, RATIO, or INTERVAL, the residuals are computed as

$$\begin{aligned} Q_{rcs} &= O_{rcs} \\ R_{rcs} &= Q_{rcs}^f - [T_{P_{rcs}}(D_{rcs})]^f \end{aligned}$$

For LEVEL=ORDINAL, the residuals are computed as

$$\begin{aligned} Q_{rcs} &= T_{P_{rcs}}(O_{rcs}) \\ R_{rcs} &= Q_{rcs}^f - D_{rcs}^f \end{aligned}$$

If f is 0, then natural logarithms are used in place of the fth powers.

For each partition, let

$$U_p = \frac{\sum_{r,c,s} F_{rcs}}{\sum_{r,c,s|P_{rcs}=p} F_{rcs}}$$

and

$$\overline{Q}_p = \frac{\sum_{r,c,s|P_{rcs}=p} Q_{rcs} F_{rcs}}{\sum_{r,c,s|P_{rcs}=p} F_{rcs}}$$

Then the standardization factor for each partition is

$$\begin{aligned} S_p &= 1 && \text{for FORMULA=0} \\ &= U_p \sum_{r,c,s|P_{rcs}=p} Q_{rcs}^2 F_{rcs} && \text{for FORMULA=1} \\ &= U_p \sum_{r,c,s|P_{rcs}=p} (Q_{rcs} - \overline{Q}_p)^2 F_{rcs} && \text{for FORMULA=2} \end{aligned}$$

The badness-of-fit criterion that the MDS procedure tries to minimize is

$$\sqrt{\sum_{r,c,s} \frac{R_{rcs}^2 F_{rcs}}{S_{P_{rcs}}}}$$

OUT= Data Set

The OUT= data set contains the following variables:

- BY variables, if any
- _ITER_ (if the OUTITER option is specified), a numeric variable containing the iteration number
- _DIMENS_, a numeric variable containing the number of dimensions
- _MATRIX_ or the variable in the MATRIX statement, identifying the data matrix or subject to which the observation pertains. This variable contains a missing value for observations that pertain to the data set as a whole and not to a particular matrix, such as the coordinates (_TYPE_='CONFIG').
- _TYPE_, a character variable of length 10 identifying the type of information in the observation

  The values of _TYPE_ are as follows:

CONFIG	the estimated coordinates of the configuration of objects
DIAGCOEF	the estimated dimension coefficients for COEF=DIAGONAL
INTERCEPT	the estimated intercept parameters
SLOPE	the estimated slope parameters
POWER	the estimated power parameters
CRITERION	the badness-of-fit criterion

- _LABEL_ or the variable in the ID statement, containing the variable label or value of the ID variable of the object to which the observation pertains. This variable contains a missing value for observations that do not pertain to a particular object or dimension.
- _NAME_, a character variable containing the variable name of the object or dimension to which the observation pertains. This variable contains a missing value for observations that do not pertain to a particular object or dimension.
- Dim1, . . . , Dim*m*, where *m* is the maximum number of dimensions

OUTFIT= Data Set

The OUTFIT= data set contains various measures of goodness and badness of fit. There is one observation for the entire sample plus one observation for each matrix. For the CONDITION=ROW option, there is also one observation for each row.

The OUTFIT= data set contains the following variables:

- BY variables, if any
- _ITER_ (if the OUTITER option is specified), a numeric variable containing the iteration number

- _DIMENS_, a numeric variable containing the number of dimensions
- _MATRIX_ or the variable in the MATRIX statement, identifying the data matrix or subject to which the observation pertains
- _LABEL_ or the variable in the ID statement, containing the variable label or value of the ID variable of the object to which the observation pertains when CONDITION=ROW
- _NAME_, a character variable containing the variable name of the object or dimension to which the observation pertains when CONDITION=ROW
- N, the number of nonmissing data
- WEIGHT, the weight of the partition
- CRITER, the badness-of-fit criterion
- DISCORR, the correlation between the transformed data and the distances for LEVEL=ORDINAL or the correlation between the data and the transformed distances otherwise
- UDISCORR, the correlation uncorrected for the mean between the transformed data and the distances for LEVEL=ORDINAL or the correlation between the data and the transformed distances otherwise
- FITCORR, the correlation between the fit-transformed data and the fit-transformed distances
- UFITCORR, the correlation uncorrected for the mean between the fit-transformed data and the fit-transformed distances

OUTRES= Data Set

The OUTRES= data set has one observation for each nonmissing data value. It contains the following variables:

- BY variables, if any
- _ITER_ (if the OUTITER option is specified), a numeric variable containing the iteration number
- _DIMENS_, a numeric variable containing the number of dimensions
- _MATRIX_ or the variable in the MATRIX statement, identifying the data matrix or subject to which the observation pertains
- _ROW_, containing the variable label or value of the ID variable of the row to which the observation pertains
- _COL_, containing the variable label or value of the ID variable of the column to which the observation pertains
- DATA, the original data value
- TRANDATA, the optimally transformed data value when LEVEL=ORDINAL

- DISTANCE, the distance computed from the PROC MDS model
- TRANSDIST, the optimally transformed distance when the LEVEL= option is not ORDINAL or ABSOLUTE
- FITDATA, the data value further transformed according to the FIT= option
- FITDIST, the distance further transformed according to the FIT= option
- WEIGHT, the combined weight of the data value based on the WEIGHT variable(s), if any, and the standardization specified by the FORMULA= option
- RESIDUAL, FITDATA minus FITDIST

If you assign a nonmissing data value a weight of zero, PROC MDS will ignore it when the model is fit, but the value will still appear in the OUTRES= data set (see the section "WEIGHT Statement" on page 5925).

INITIAL= Data Set

The INITIAL= data set has the same structure as the OUT= data set but is not required to have all of the variables or observations that appear in the OUT= data set. You can use an OUT= data set previously created by PROC MDS (without the OUTITER option) as an INITIAL= data set in a subsequent invocation of the procedure.

The only variables that are required are Dim1, . . . , Dim*m* (where *m* is the maximum number of dimensions) or equivalent variables specified in the INVAR statement. If these are the only variables, then all the observations are assumed to contain coordinates of the configuration; you cannot read dimension coefficients or transformation parameters.

To read initial values for the dimension coefficients or transformation parameters, the INITIAL= data set must contain the _TYPE_ variable and either the variable specified in the ID statement or, if no ID statement is used, the variable _NAME_. In addition, if there is more than one data matrix, either the variable specified in the MATRIX statement or, if no MATRIX statement is used, the variable _MATRIX_ or _MATNUM_ is required.

If the INITIAL= data set contains the variable _DIMENS_, initial values are obtained from observations with the corresponding number of dimensions. If there is no _DIMENS_ variable, the same observations are used for each number of dimensions analyzed.

If you want PROC MDS to read initial values from some but not all of the observations in the INITIAL= data set, use the WHERE= data set option to select the desired observations.

Missing Values

Missing data in the similarity or dissimilarity matrices are ignored in fitting the model and are omitted from the OUTRES= data set. Any matrix that is completely missing is omitted from the analysis.

Missing weights are treated as 0.

Missing values are also permitted in the INITIAL= data set, but a large number of missing values might yield a degenerate initial configuration.

Normalization of the Estimates

In multidimensional scaling models, the parameter estimates are not uniquely determined; the estimates can be transformed in various ways without changing their badness of fit. The initial and final estimates from PROC MDS are, therefore, normalized (unless you specify the NONORM option) to make it easier to compare results from different analyses.

The configuration always has a mean of 0 for each dimension.

With the COEF=IDENTITY option, the configuration is rotated to a principal-axis orientation. Unless you specify the LEVEL=ABSOLUTE option, the entire configuration is scaled so that the root-mean-square element is 1, and the transformations are adjusted to compensate.

With the COEF=DIAGONAL option, each dimension is scaled to a root-mean-square value of 1, and the dimension coefficients are adjusted to compensate. Unless you specify the LEVEL=ABSOLUTE option, the dimension coefficients are normalized as follows. If you specify the CONDITION=UN option, all of the dimension coefficients are scaled to a root-mean-square value of 1. For other values of the CONDITION= option, the dimension coefficients are scaled separately for each subject to a root-mean-square value of 1. In either case, the transformations are adjusted to compensate.

Each dimension is reflected to give a positive rank correlation with the order of the objects in the data set.

For the LEVEL=ORDINAL option, if the intercept, slope, or power parameters are fitted, the transformed data are normalized to eliminate these parameters if possible.

Comparison with Earlier Procedures

PROC MDS shares many of the features of the ALSCAL procedure (Young, Lewyckyj, and Takane 1986; Young 1982), as well as some features of the MLSCALE procedure (Ramsay 1986). Both PROC ALSCAL and PROC MLSCALE are no longer a part of SAS; however, they are described in the *SUGI Supplemental Library User's Guide, Version 5 Edition*. The MDS procedure generally produces results similar to those from the ALSCAL procedure (Young, Lewyckyj, and Takane 1986; Young 1982) if you use the following *options* in PROC MDS:

- FIT=SQUARED
- FORMULA=1 except for unfolding data, which require FORMULA=2
- PFINAL to get output similar to that from PROC ALSCAL

Running the MDS procedure with certain options generally produces results similar to those from using the MLSCALE procedure (Ramsay 1986) with other options. This is illustrated with the following statements:

```
proc mds fit=log level=loginterval ... ;

proc mlscale stvarnce=constant suvarnce=constant ... ;
```

Alternatively, using the FIT=DISTANCE option in the PROC MDS statement produces results similar to those from specifying the NORMAL option in the PROC MLSCALE statement.

Displayed Output

Unless you specify the NOPHIST option, PROC MDS displays the iteration history containing the following:

- Iteration number
- Type of iteration:

Initial	initial configuration
Monotone	monotone transformation
Gau-New	Gauss-Newton step
Lev-Mar	Levenberg-Marquardt step

- Badness-of-Fit Criterion
- Change in Criterion
- Convergence Measures:

Monotone	the Euclidean norm of the change in the optimally scaled data divided by the Euclidean norm of the optimally scaled data, averaged across partitions
Gradient	the multiple correlation of the Jacobian matrix with the residual vector, uncorrected for the mean

Depending on what options are specified, PROC MDS can also display the following tables:

- Data Matrix and possibly Weight Matrix for each subject
- Eigenvalues from the computation of the initial coordinates
- Sum of Data Weights and Pooled Data Matrix computed during initialization with INAV=DATA
- Configuration, the estimated coordinates of the objects
- Dimension Coefficients
- A table of transformation parameters, including one or more of the following:

  Intercept

  Slope

  Power

- A table of fit statistics for each matrix and possibly each row, including the following:

  Number of Nonmissing Data

  Weight of the matrix or row, permitting both observation weights and standardization factors

  Badness-of-Fit Criterion

  Distance Correlation computed between the distances and data with optimal transformation

Uncorrected Distance Correlation not corrected for the mean

Fit Correlation computed after applying the FIT= transformation to both distances and data

Uncorrected Fit Correlation not corrected for the mean

ODS Table Names

PROC MDS assigns a name to each table it creates. You can use these names to reference the table when using the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in Table 75.3. For more information about ODS, see Chapter 20, "Using the Output Delivery System."

Table 75.3 ODS Tables Produced by PROC MDS

ODS Table Name	Description	Option
ConvergenceStatus	Convergence status	default
DimensionCoef	Dimension coefficients	PCOEF with COEF= not IDENTITY
FitMeasures	Measures of fit	PFIT
IterHistory	Iteration history	default
PConfig	Configuration of coordinates	PCONFIG
PData	Data matrices	PDATA
PInAvData	INAV= data set information	PINAVDATA
PInEigval	Initial eigenvalues	PINEIGVAL
PInEigvec	Initial eigenvectors	PINEIGVEC
PInWeight	Initialization weights	PINWEIGHT
Transformations	Transformation parameters	PTRANS with LEVEL=RATIO, INTERVAL, LOGINTERVAL

ODS Graphics

Statistical procedures use ODS Graphics to create graphs as part of their output. ODS Graphics is described in detail in Chapter 21, "Statistical Graphics Using ODS."

Before you create graphs, ODS Graphics must be enabled (for example, by specifying the ODS GRAPHICS ON statement). For more information about enabling and disabling ODS Graphics, see the section "Enabling and Disabling ODS Graphics" on page 607 in Chapter 21, "Statistical Graphics Using ODS."

The overall appearance of graphs is controlled by ODS styles. Styles and other aspects of using ODS Graphics are discussed in the section "A Primer on ODS Statistical Graphics" on page 606 in Chapter 21, "Statistical Graphics Using ODS."

All graphs are produced by default when they are appropriate. You can reference every graph produced through ODS Graphics with a name. The names of the graphs that PROC MDS generates are listed in Table 75.4, along with the required options.

Table 75.4 Graphs Produced by PROC MDS

ODS Graph Name	Plot Description	Option
BadnessPlot	Badness of fit	DIMENSION=*range*
CoefficientsPlot	Individual coefficients	DIMENSION=*n*, *n* > 1 COEF=DIAGONAL
ConfigPlot	Configuration	DIMENSION=*n*, *n* > 1
FitPlot	Fit	default

Example: MDS Procedure

Example 75.1: Jacobowitz Body Parts Data from Children and Adults

Jacobowitz (1975) collected conditional rank-order data regarding perceived similarity of parts of the body from children of ages 6, 8, and 10 years and from college sophomores. The data set includes data from 15 children (6-year-olds) and 15 sophomores. The method of data collection and some results of an analysis are also described by Young (1987, pp. 4–10). The following statements create the input data set:

```
data body;
   title  'Jacobowitz Body Parts Data from 6-Year-Olds and Adults';
   title2 'First 15 Subjects (obs 1-225) Are Children';
   title3 'Second 15 Subjects (obs 226-450) Are Adults';
   input  (Cheek Face Mouth Head Ear Body Arm Elbow Hand
           Palm Finger Leg Knee Foot Toe) (2.);
   if _n_ <= 225 then Subject='C'; else subject='A';
   datalines;
 0  2  1  3  4 10  5  9  6  7  8 11 12 13 14
 2  0 12  1 13  3  8 10 11  9  7  4  5  6 14
 3  2  0  1  4  9  5 11  6  7  8 10 13 12 14
 2  1  3  0  4  9  5  6 11  7  8 10 12 13 14

   ... more lines ...

10 12 11 13  9 14  8  7  4  6  2  3  5  1  0
;
```

The data are analyzed as row conditional (CONDITION=ROW) at the ordinal level of measurement (LEVEL=ORDINAL) by using the weighted Euclidean model (COEF=DIAGONAL) in three dimensions (DIMENSION=3). The final estimates are displayed (PFINAL). The estimates (OUT=OUT) and fitted values (OUTRES=RES) are saved in output data sets. The following statements produce Output 75.1.1:

```
ods graphics on;

proc mds data=body condition=row level=ordinal coef=diagonal
         dimension=3 pfinal out=out outres=res;
   subject subject;
   title5 'Nonmetric Weighted MDS';
run;
```

Output 75.1.1 Analysis of Body Parts Data

Jacobowitz Body Parts Data from 6-Year-Olds and Adults
First 15 Subjects (obs 1-225) Are Children
Second 15 Subjects (obs 226-450) Are Adults

Nonmetric Weighted MDS

Multidimensional Scaling: Data=WORK.BODY.DATA
Shape=SQUARE Condition=ROW Level=ORDINAL
Coef=DIAGONAL Dimension=3 Formula=1 Fit=1

Mconverge=0.01 Gconverge=0.01 Maxiter=100 Over=2 Ridge=0.0001 Alternate=MATRIX

				Convergence Measures	
Iteration	Type	Badness-of-Fit Criterion	Change in Criterion	Monotone	Gradient
0	Initial	0.5938	.	.	.
1	Monotone	0.2344	0.3594	0.4693	0.4028
2	Gau-New	0.2080	0.0264	.	.
3	Monotone	0.1963	0.0118	0.0556	0.2630
4	Gau-New	0.1927	0.003592	.	.
5	Monotone	0.1797	0.0130	0.0463	0.1544
6	Gau-New	0.1779	0.001809	.	.
7	Monotone	0.1744	0.003430	0.0225	0.1210
8	Gau-New	0.1736	0.000807	.	.
9	Monotone	0.1717	0.001929	0.0161	0.1128
10	Gau-New	0.1712	0.000474	.	.
11	Monotone	0.1698	0.001413	0.0135	0.1119
12	Gau-New	0.1696	0.000188	.	.
13	Monotone	0.1684	0.001261	0.0121	0.1121
14	Gau-New	0.1683	0.000117	.	.
15	Monotone	0.1672	0.001096	0.0111	0.1064
16	Gau-New	0.1670	0.000131	.	.
17	Monotone	0.1661	0.000902	0.0103	0.0965
18	Gau-New	0.1660	0.000160	.	.
19	Monotone	0.1652	0.000736	0.009740	0.0980
20	Gau-New	0.1651	0.000169	.	0.1062
21	Gau-New	0.1645	0.000542	.	0.0161
22	Gau-New	0.1645	4.2645E-6	.	0.009969

Convergence criteria are satisfied.

Output 75.1.1 *continued*

	Configuration		
	Dim1	**Dim2**	**Dim3**
Cheek	0.25	1.47	2.06
Face	0.36	0.32	0.33
Mouth	0.78	0.08	1.08
Head	2.10	1.07	-0.01
Ear	0.26	0.72	-0.34
Body	1.51	-0.61	-0.68
Arm	-0.74	2.07	-0.59
Elbow	-0.51	-0.21	0.01
Hand	0.46	-1.50	-0.60
Palm	-1.44	-0.81	1.48
Finger	-0.24	-0.24	-0.81
Leg	-1.68	0.26	-0.05
Knee	-1.19	-1.19	-1.36
Foot	0.16	-0.03	-1.56
Toe	-0.10	-1.39	1.02

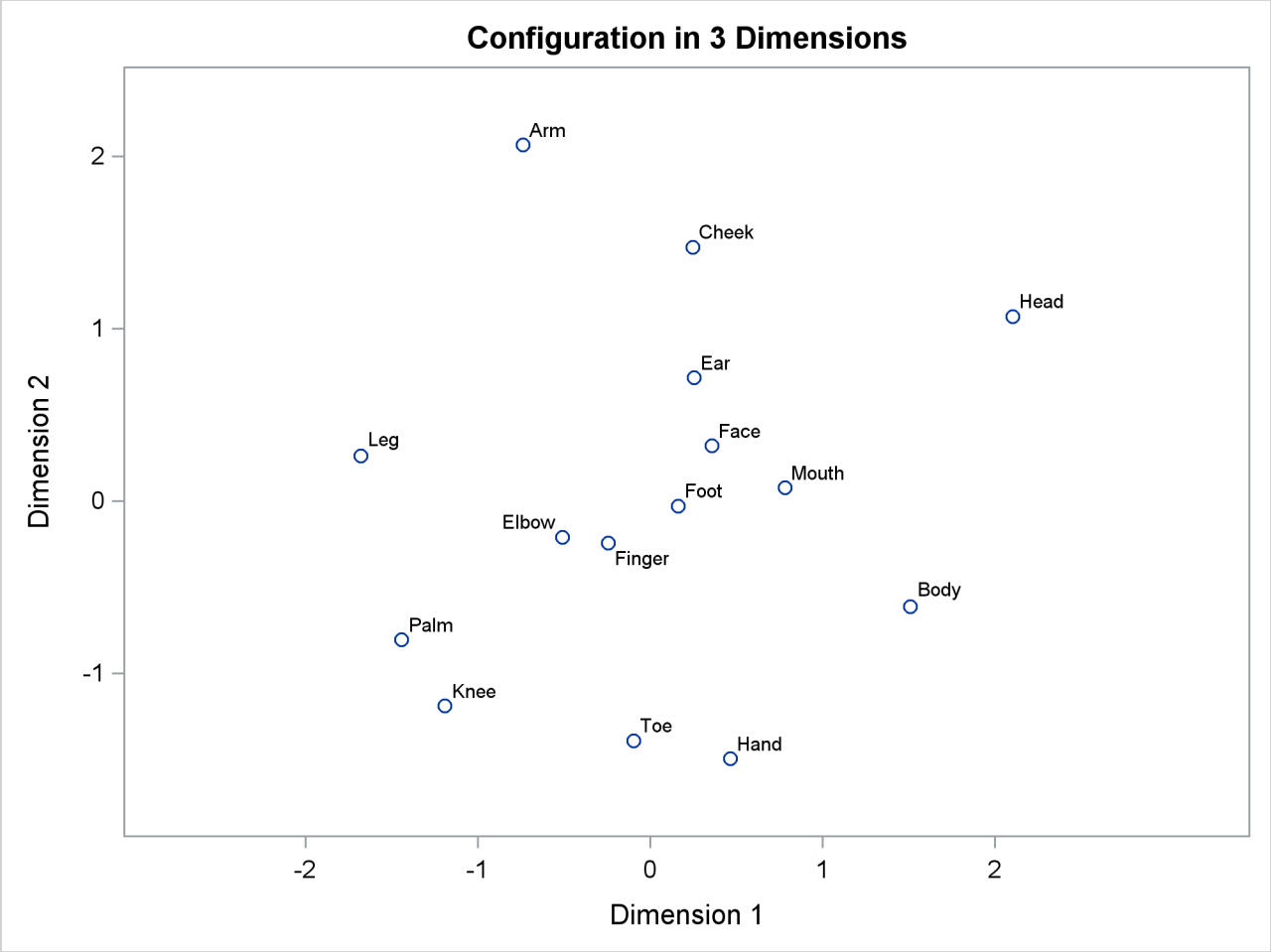
Output 75.1.1 *continued*

	Dimension Coefficients		
Subject	**1**	**2**	**3**
C	1.00	1.12	0.86
C	0.96	1.02	1.01
C	0.98	1.05	0.98
C	1.02	1.08	0.89
C	0.95	1.04	1.01
C	0.99	1.12	0.89
C	1.07	1.00	0.93
C	1.04	1.02	0.94
C	0.99	1.15	0.83
C	0.89	1.11	0.99
C	1.04	1.03	0.92
C	1.06	1.01	0.93
C	0.92	1.24	0.78
C	0.97	0.98	1.05
C	1.03	1.00	0.97
A	0.93	1.17	0.88
A	0.89	1.12	0.97
A	0.88	1.17	0.94
A	0.81	1.14	1.02
A	0.90	1.11	0.98
A	0.90	1.17	0.91
A	0.92	1.17	0.88
A	0.97	1.19	0.80
A	0.95	1.16	0.87
A	1.08	1.07	0.83
A	0.95	1.20	0.81
A	1.00	0.97	1.02
A	0.89	1.18	0.91
A	0.97	1.15	0.86
A	0.93	1.21	0.82

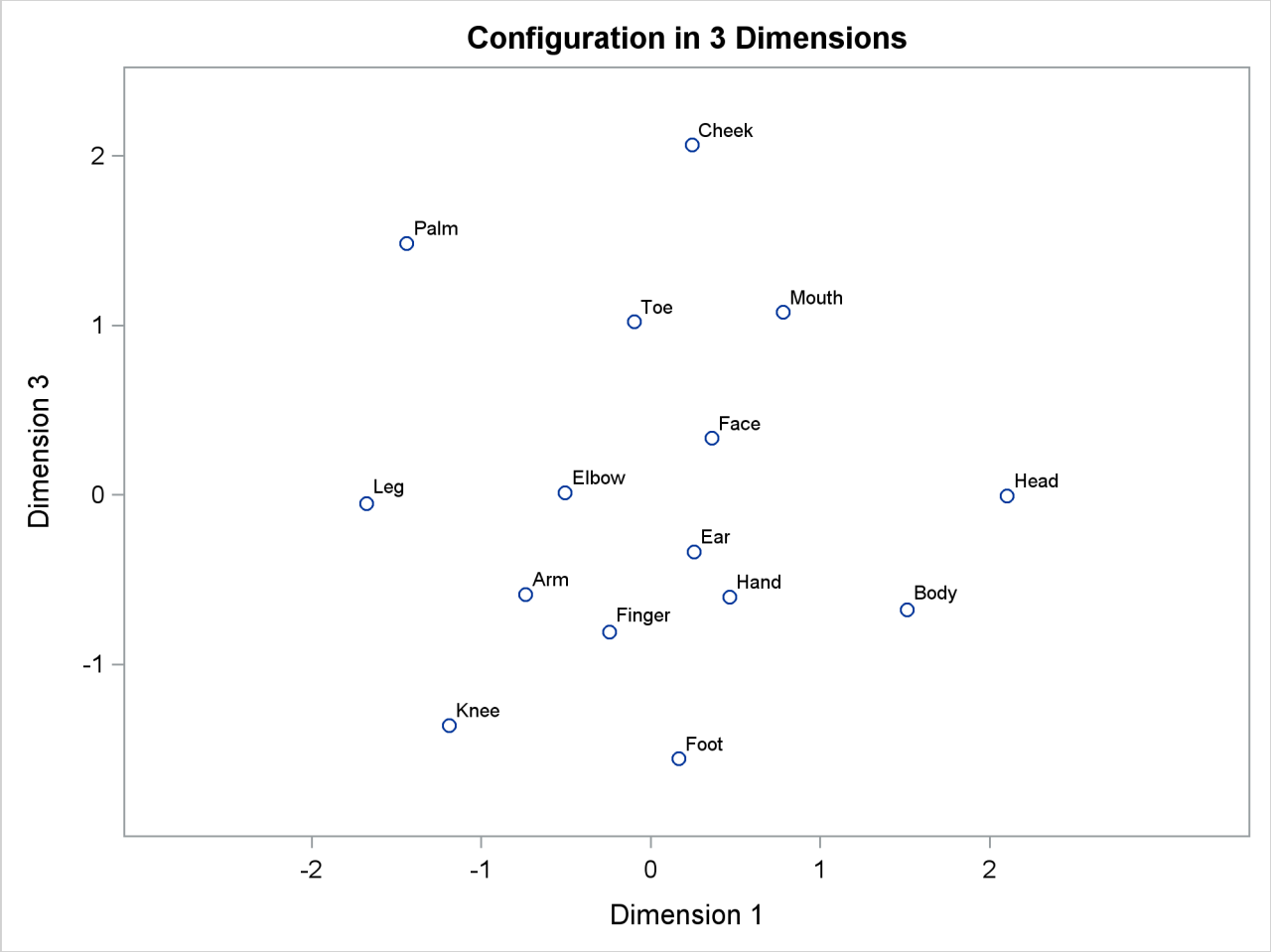
Output 75.1.1 *continued*

Subject	Number of Nonmissing Data	Weight	Badness-of-Fit Criterion	Distance Correlation	Uncorrected Distance Correlation
C	160	0.03	0.15	0.86	0.99
C	163	0.03	0.19	0.78	0.98
C	166	0.03	0.20	0.79	0.98
C	158	0.03	0.16	0.84	0.99
C	173	0.03	0.18	0.83	0.98
C	164	0.03	0.14	0.90	0.99
C	158	0.03	0.20	0.77	0.98
C	170	0.03	0.18	0.83	0.98
C	156	0.03	0.15	0.88	0.99
C	165	0.03	0.18	0.79	0.98
C	153	0.03	0.19	0.79	0.98
C	162	0.03	0.17	0.83	0.98
C	161	0.03	0.14	0.90	0.99
C	164	0.03	0.17	0.83	0.99
C	161	0.03	0.18	0.81	0.98
A	163	0.03	0.15	0.87	0.99
A	174	0.04	0.17	0.85	0.99
A	172	0.03	0.15	0.89	0.99
A	175	0.04	0.17	0.85	0.98
A	171	0.03	0.15	0.87	0.99
A	163	0.03	0.16	0.86	0.99
A	173	0.03	0.14	0.90	0.99
A	160	0.03	0.14	0.89	0.99
A	164	0.03	0.14	0.90	0.99
A	158	0.03	0.16	0.86	0.99
A	165	0.03	0.16	0.87	0.99
A	168	0.03	0.18	0.82	0.98
A	175	0.04	0.15	0.89	0.99
A	172	0.03	0.16	0.88	0.99
A	175	0.04	0.15	0.90	0.99
- All -	4962	1.00	0.16	0.85	0.99

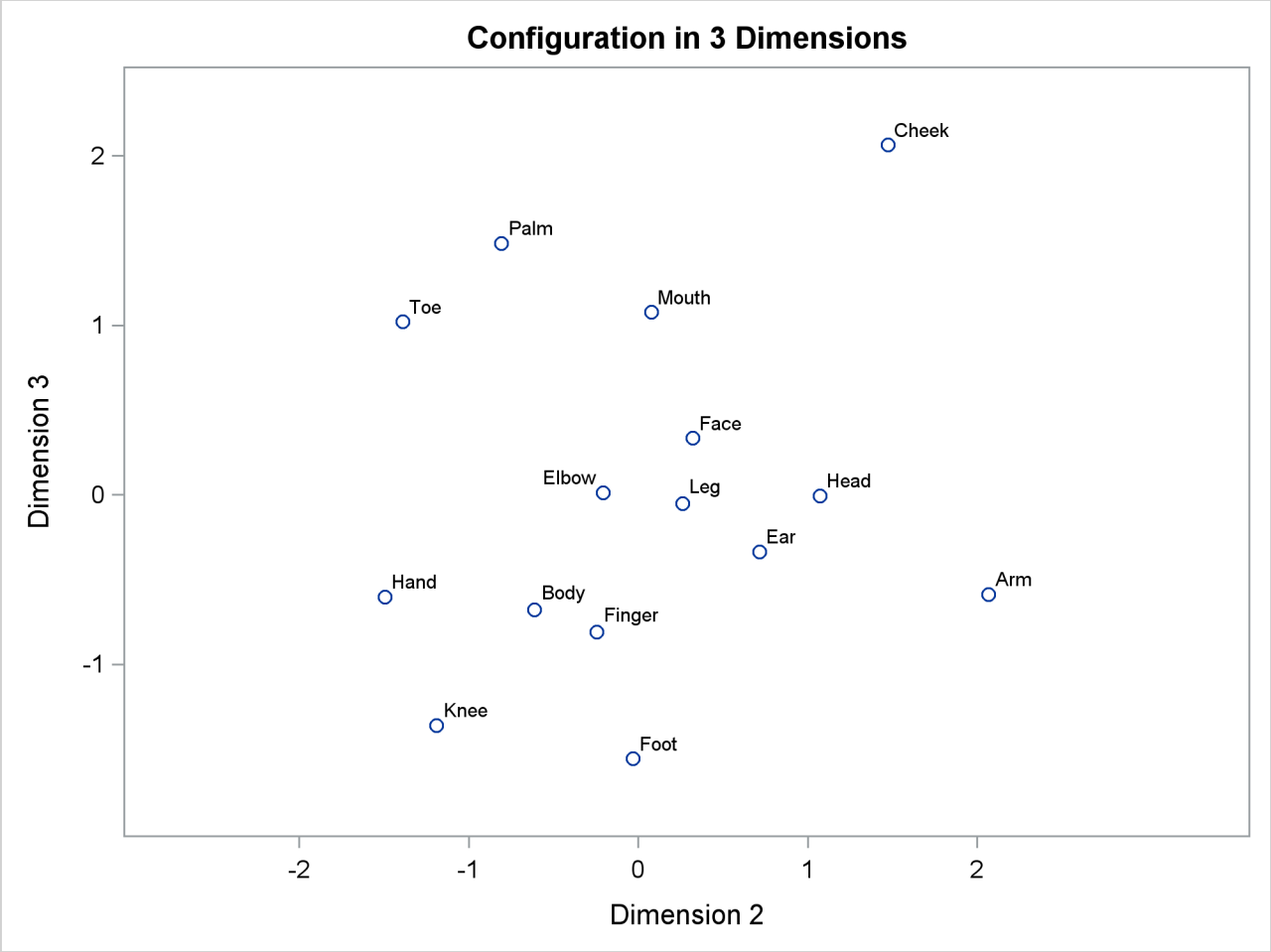
Output 75.1.1 *continued*



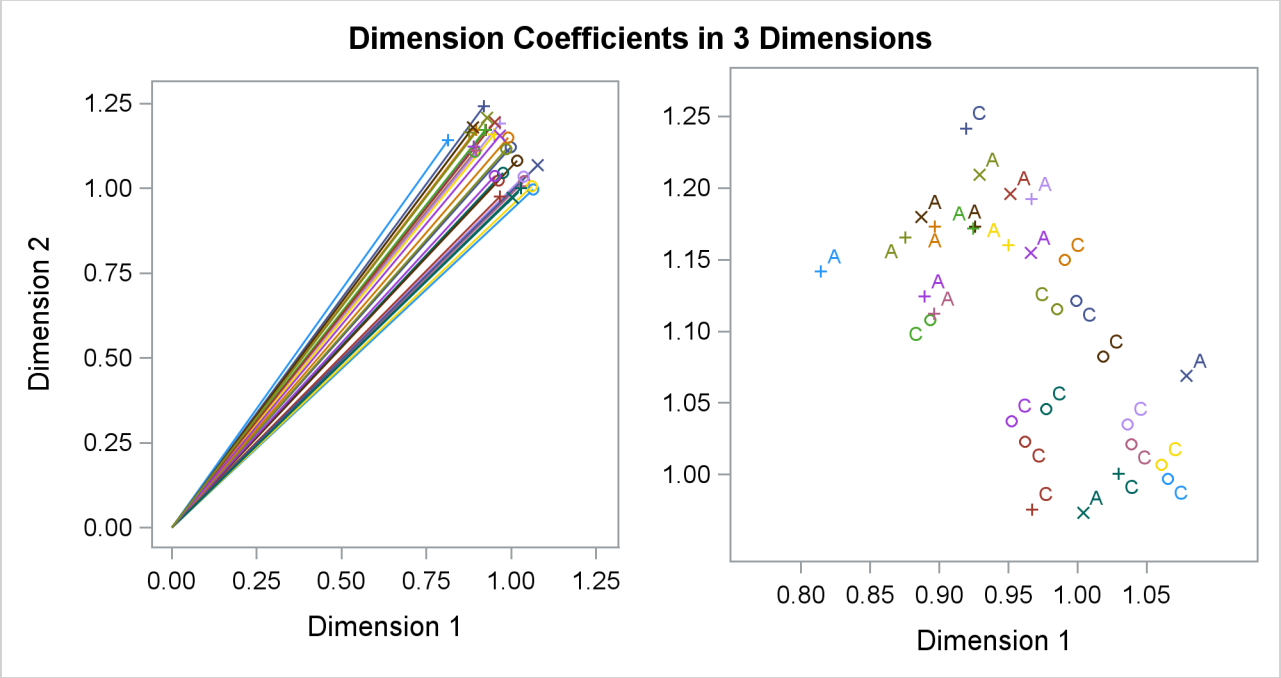
Output 75.1.1 *continued*

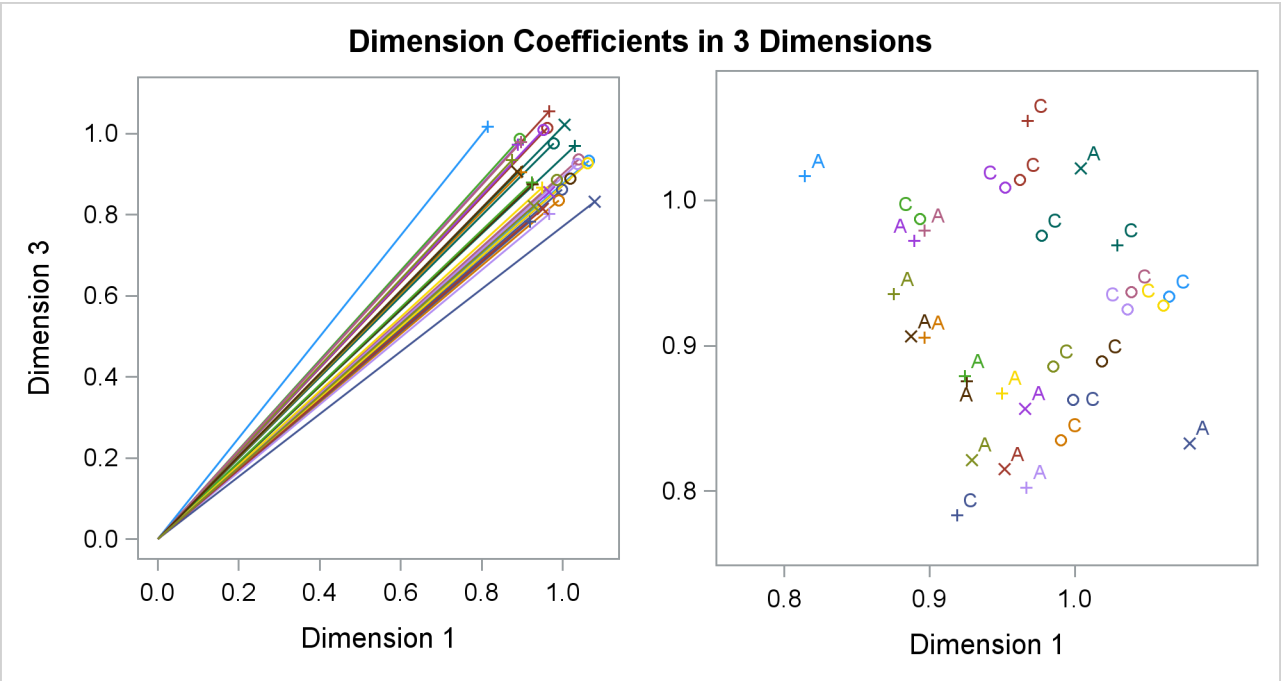


Output 75.1.1 *continued*

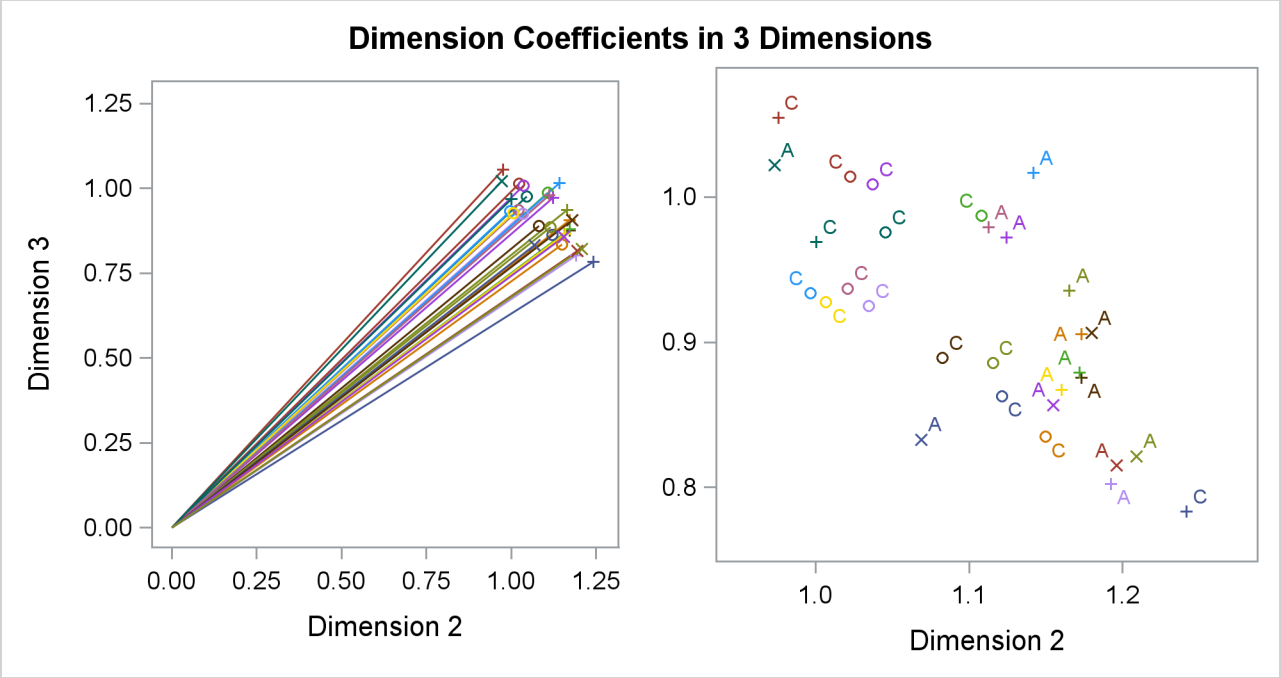


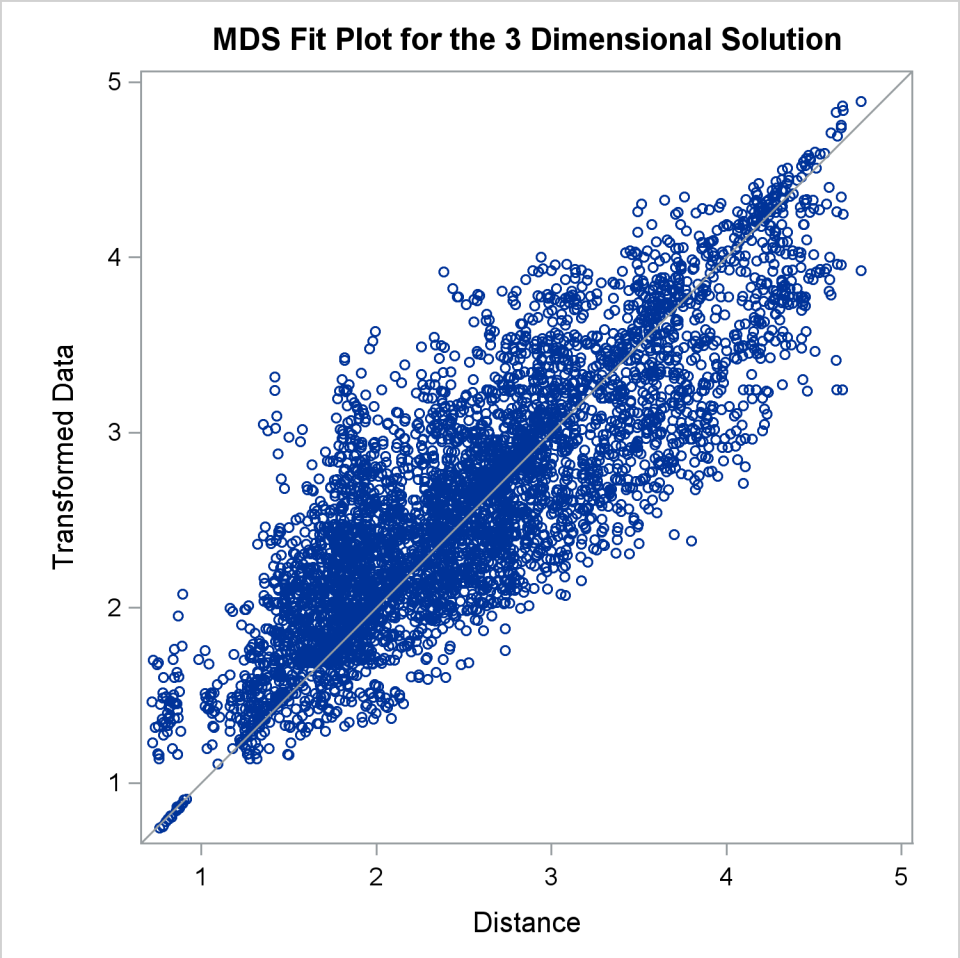
Output 75.1.1 *continued*





Output 75.1.1 *continued*





Often the output of greatest interest in an MDS analysis is the graphical output. The first plots show two-dimensional view of the three-dimensional configuration. Next, the coefficients are plotted. The last plot is the fit plot.

In the fit plot, the transformed data are plotted on the vertical axis, and the distances from the model are plotted on the horizontal axis. If the model fits perfectly, all points lie on a diagonal line from lower left to upper right. The vertical departure of each point from this diagonal line represents the residual of the corresponding observation.

The configuration has a tripodal shape with Body at the apex. The three legs of the tripod can be distinguished in the plot of dimension 2 by dimension 1, which shows three distinct clusters with Body in the center. Dimension 1 separates head parts from arm and leg parts. Dimension 2 separates arm parts from leg parts. The plot of dimension 3 by dimension 1 shows the tripod from the side. Dimension 3 distinguishes the more inclusive body parts (at the top) from the less inclusive body parts (at the bottom).

The plots of dimension coefficients show that children differ from adults primarily in the emphasis given to dimension 2. Children give about the same weight (approximately 1) to each dimension. Adults are much more variable than children, but all have coefficients less than 1.0 for dimension 2, with an average of about 0.7. Referring back to the configuration plot, you can see that adults consider arm parts to be more similar to leg parts than children do. Many adults also give a high weight to dimension 1, indicating that they consider head parts to be more dissimilar from arm and leg parts than children do. Dimension 3 shows considerable variability for both children and adults.

References

Arabie, P., Carroll, J. D., and DeSarbo, W. S. (1987). *Three-Way Scaling and Clustering.* Vol. 07-065 of Sage University Paper Series on Quantitative Applications in the Social Sciences. Beverly Hills, CA: Sage Publications.

Carroll, J. D., and Chang, J. J. (1970). "Analysis of Individual Differences in Multidimensional Scaling via an N-Way Generalization of the 'Eckart-Young' Decomposition." *Psychometrika* 35:283–319.

Heiser, W. J. (1981). *Unfolding Analysis of Proximity Data*. Leiden, Netherlands: Department of Data Theory, University of Leiden.

Jacobowitz, D. (1975). "The Acquisition of Semantic Structures." Ph.D. diss., University of North Carolina at Chapel Hill.

Krantz, D. H., Luce, R. D., Suppes, P., and Tversky, A. (1971). *Foundations of Measurement*. New York: Academic Press.

Kruskal, J. B., and Wish, M. (1978). *Multidimensional Scaling.* Vol. 07-011 of Sage University Paper Series on Quantitative Applications in the Social Sciences. Beverly Hills, CA: Sage Publications.

Null, C. H., and Sarle, W. S. (1982). "Multidimensional Scaling by Least Squares." In *Proceedings of the Seventh Annual SAS Users Group International Conference*, 643–650. Cary, NC: SAS Institute Inc. http://www.sascommunity.org/sugi/SUGI82/Sugi-82-120%20Null%20Sarle.pdf.

Rabinowitz, G. (1976). "A Procedure for Ordering Object Pairs Consistent with the Multidimensional Unfolding Model." *Psychometrika* 41:349–373.

Ramsay, J. O. (1986). "The MLSCALE Procedure." In *SUGI Supplemental Library User's Guide, Version 5 Edition*. Cary, NC: SAS Institute Inc.

Schiffman, S. S., Reynolds, M. L., and Young, F. W. (1981). *Introduction to Multidimensional Scaling*. New York: Academic Press.

Torgerson, W. S. (1958). *Theory and Methods of Scaling*. New York: John Wiley & Sons.

Young, F. W. (1982). "Enhancements in ALSCAL-82." In *Proceedings of the Seventh Annual SAS Users Group International Conference*, 633–642. Cary, NC: SAS Institute Inc. http://www.sascommunity.org/sugi/SUGI82/Sugi-82-119%20Young.pdf.

Young, F. W. (1987). *Multidimensional Scaling: History, Theory, and Applications.* Edited by R. M. Hamer. Hillsdale, NJ: Lawrence Erlbaum Associates.

Young, F. W., Lewyckyj, R., and Takane, Y. (1986). "The ALSCAL Procedure." In *SUGI Supplemental Library User's Guide, Version 5 Edition*. Cary, NC: SAS Institute Inc.

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