SAS/STAT® 14.2 User's Guide
The CORRESP Procedure

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Chapter 35
The CORRESP Procedure

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Overview: CORRESP Procedure

The CORRESP procedure performs simple correspondence analysis and multiple correspondence analysis (MCA). You can use correspondence analysis to find a low-dimensional graphical representation of the rows and columns of a crosstabulation or contingency table. Each row and column is represented by a point in a plot determined from the cell frequencies. PROC CORRESP can also compute coordinates for supplementary rows and columns.

PROC CORRESP can read two kinds of input: raw categorical responses on two or more classification variables or a two-way contingency table. The correspondence analysis plot is displayed with ODS Graphics. For more information about ODS Graphics, see the section "ODS Graphics" on page 2243.

Background

Correspondence analysis is a popular data analysis method in France and Japan. In France, correspondence analysis was developed under the influence of Jean-Paul Benzécri; in Japan, it was developed under Chikio Hayashi. The name *correspondence analysis* is a translation of the French *analyse des correspondances*. The technique apparently has many independent beginnings (for example, Richardson and Kuder 1933; Hirshfield 1935; Horst 1935; Fisher 1940; Guttman 1941; Burt 1950; Hayashi 1950). It has had many other names, including optimal scaling, reciprocal averaging, optimal scoring, and appropriate scoring in the United States; quantification method in Japan; homogeneity analysis in the Netherlands; dual scaling in Canada; and scalogram analysis in Israel.

Correspondence analysis is described in more detail in French in Benzécri (1973) and Lebart, Morineau, and Tabard (1977). In Japanese, the subject is described in Komazawa (1982); Nishisato (1982); Kobayashi (1981). In English, correspondence analysis is described in Lebart, Morineau, and Warwick (1984); Greenacre (1984); Nishisato (1980); Tenenhaus and Young (1985); Gifi (1990); Greenacre and Hastie (1987), and many other sources. Hoffman and Franke (1986) offer a short, introductory treatment that uses examples from the field of market research.

Getting Started: CORRESP Procedure

Data are available containing the numbers of Ph.D.'s awarded in the United States during the years 1973 through 1978 (US Bureau of the Census 1979). The table has six rows, one for each of six academic disciplines, and six columns for the six years.

The following DATA step reads the complete table into a SAS data set, and PROC CORRESP displays correspondence analysis results including the inertia decomposition and coordinates:

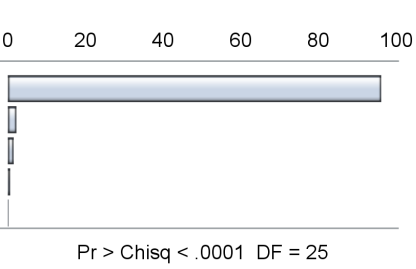
```
title "Number of Ph.D.'s Awarded from 1973 to 1978";

data PhD;
   input Science $ 1-19 y1973-y1978;
   label y1973 = '1973'
         y1974 = '1974'
         y1975 = '1975'
         y1976 = '1976'
         y1977 = '1977'
         y1978 = '1978';
   datalines;
Life Sciences       4489 4303 4402 4350 4266 4361
Physical Sciences   4101 3800 3749 3572 3410 3234
Social Sciences     3354 3286 3344 3278 3137 3008
Behavioral Sciences 2444 2587 2749 2878 2960 3049
Engineering         3338 3144 2959 2791 2641 2432
Mathematics         1222 1196 1149 1003  959  959
;

ods graphics on;

proc corresp data=PhD out=Results short chi2p;
   var y1973-y1978;
   id Science;
run;
```

The results are displayed in Figure 35.1 and Figure 35.2.

Figure 35.1 Inertia and Chi-Square Decomposition

Inertia and Chi-Square Decomposition

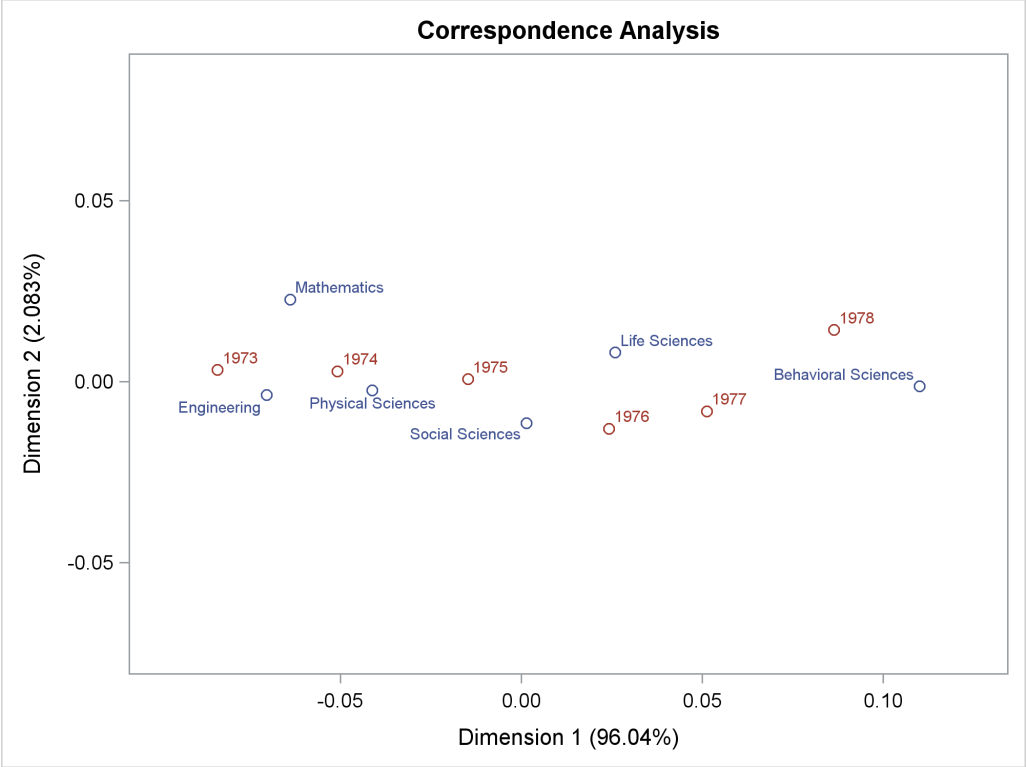
Singular Value	Principal Inertia	Chi-Square	Percent	Cumulative Percent
0.05845	0.00342	368.653	96.04	96.04
0.00861	0.00007	7.995	2.08	98.12
0.00694	0.00005	5.197	1.35	99.48
0.00414	0.00002	1.852	0.48	99.96
0.00122	0.00000	0.160	0.04	100.00
	0.00356	383.856	100.00	



The concept of *inertia* in correspondence analysis is analogous to the concept of variance in principal component analysis, and it is proportional to the chi-square information.

In Figure 35.1, the total chi-square statistic, which is a measure of the association between the rows and columns in the full five dimensions of the (centered) table, is 383.856. The chi-square p-value, < 0.001, is displayed when you specify the CHI2P option. The maximum number of dimensions (or axes) is the minimum of the number of rows and columns, minus one. More than 96% of the total chi-square and inertia is explained by the first dimension, indicating that the association between the row and column categories is essentially one-dimensional. The plot in Figure 35.2 shows how the number of doctorates in the different disciplines changes over time. The plot shows that the number of doctorates in the behavioral sciences is associated with later years, and the number of doctorates in mathematics and engineering is associated with earlier years. This is consistent with the data that show that the number of doctorates in the behavioral sciences is increasing, the number of doctorates in every other discipline is decreasing, and the rate of decrease is greatest for mathematics and engineering.

Figure 35.2 Correspondence Analysis of Ph.D. Data



Syntax: CORRESP Procedure

The following statements are available in the CORRESP procedure.

PROC CORRESP < *options* > ;
  TABLES < *row-variables*, > *column-variables* ;
  VAR *variables* ;
  BY *variables* ;
  ID *variable* ;
  SUPPLEMENTARY *variables* ;
  WEIGHT *variable* ;

There are two separate forms of input to PROC CORRESP. One form is specified in the TABLES statement, the other in the VAR statement. You must specify either the TABLES or the VAR statement, but not both, each time you run PROC CORRESP.

Specify the TABLES statement if you are using raw, categorical data, the levels of which define the rows and columns of a table.

Specify the VAR statement if your data are already in tabular form. PROC CORRESP is generally more efficient with VAR statement input than with TABLES statement input.

The other statements are optional. All of the statements are explained, in alphabetical order, following the PROC CORRESP statement. All of the options in PROC CORRESP can be abbreviated to their first three letters, except for the OUTF= option. This is a special feature of PROC CORRESP and is not generally true of SAS/STAT procedures.

PROC CORRESP Statement

PROC CORRESP < *options* > ;

The PROC CORRESP statement invokes the CORRESP procedure. Table 35.1 summarizes the *options* available in the PROC CORRESP statement. These *options* are described following the table.

Table 35.1 Summary of PROC CORRESP Statement Options

Option	Description
Data Set Options	
DATA=	Specifies input SAS data set
OUTC=	Specifies output coordinate SAS data set
OUTF=	Specifies output frequency SAS data set
Row and Column Coordinates	
DIMENS=	Specifies the number of dimensions or axes
MCA	Performs multiple correspondence analysis
PROFILE=	Standardizes the row and column coordinates
Table Construction	
BINARY	Specifies binary table

Table 35.1 *continued*

Option	Description
CROSS=	Specifies cross levels of TABLES variables
FREQOUT	Specifies input data in PROC FREQ output
MISSING	Includes observations with missing values
Control Displayed Output	
ALL	Displays all output
BENZECRI	Displays inertias adjusted by Benzécri's method
CELLCHI2	Displays cell contributions to chi-square
CHI2P	Displays the chi-square *p*-value
CP	Displays column profile matrix
DEVIATION	Displays observed minus expected values
EXPECTED	Displays chi-square expected values
GREENACRE	Displays inertias adjusted by Greenacre's method
INERTIATABLE	Displays the inertia and chi-square decomposition in tabular form
NOCOLUMN=	Suppresses the display of column coordinates
NOPRINT	Suppresses the display of all output
NOROW=	Suppresses the display of row coordinates
OBSERVED	Displays contingency table of observed frequencies
PLOTS=	Specifies ODS Graphics details
PRINT=	Displays percentages or frequencies
RP	Displays row profile matrix
SHORT	Suppresses all point and coordinate statistics
UNADJUSTED	Displays unadjusted inertias
Other Options	
COLUMN=	Specifies esoteric column coordinate standardizations
MININERTIA=	Specifies minimum inertia
NVARS=	Specifies number of classification variables
ROW=	Specifies esoteric row coordinate standardizations
SINGULAR=	Specifies effective zero
SOURCE	Includes level source in the OUTC= data set

The display options control the amount of displayed output. The CELLCHI2, EXPECTED, DEVIATION, and CHI2P options display additional chi-square information. See the "Details: CORRESP Procedure" section for more information. The unit of the matrices displayed by the CELLCHI2, CP, DEVIATION, EXPECTED, OBSERVED, and RP options depends on the value of the PRINT= option. The table construction options control the construction of the contingency table; these options are valid only when you also specify a TABLES statement.

You can specify the following *options* in the PROC CORRESP statement. They are listed in alphabetical order.

ALL

is equivalent to specifying the OBSERVED, RP, CP, CELLCHI2, EXPECTED, and DEVIATION options. Specifying the ALL option does not affect the PRINT= option. Therefore, only frequencies (not percentages) for these options are displayed unless you specify otherwise with the PRINT= option.

BENZECRI

BEN

displays adjusted inertias when you are performing multiple correspondence analysis. By default, unadjusted inertias (the usual inertias from multiple correspondence analysis) are displayed. However, adjusted inertias that use a method proposed by Benzécri (1979) and described by Greenacre (1984, p. 145) can be displayed by specifying the BENZECRI option. Specify the UNADJUSTED option to output the usual table of unadjusted inertias as well. For more information, see the section "MCA Adjusted Inertias" on page 2236.

BINARY

enables you to create binary tables easily. When you specify the BINARY option, specify only column variables in the TABLES statement. Each input data set observation forms a single row in the constructed table.

CELLCHI2

CEL

displays the contribution to the total chi-square test statistic for each cell. See also the descriptions of the DEVIATION, EXPECTED, and OBSERVED options.

CHI2P

CHI

displays the chi-square *p*-value in the inertia and chi-square decomposition table. The chi-square *p*-value is not displayed by default because in many cases the table being analyzed is not a true two-way contingency table.

COLUMN=B | BD | DB | DBD | DBD1/2 | DBID1/2

COL=B | BD | DB | DBD | DBD1/2 | DBID1/2

provides other standardizations of the column coordinates. The COLUMN= option is rarely needed. Typically, you should use the PROFILE= option instead (see the section "The PROFILE=, ROW=, and COLUMN= Options" on page 2233). By default, COLUMN=DBD.

CP

displays the column profile matrix. Column profiles contain the observed conditional probabilities of row membership given column membership. See also the RP option.

CROSS=BOTH | COLUMN | NONE | ROW

CRO=BOT | COL | NON | ROW

specifies the method of crossing (factorially combining) the levels of the TABLES variables. The default is CROSS=NONE.

NONE causes each level of every row variable to become a row label and each level of every column variable to become a column label.

ROW causes each combination of levels for all row variables to become a row label, whereas each level of every column variable becomes a column label.

COLUMN causes each combination of levels for all column variables to become a column label, whereas each level of every row variable becomes a row label.

BOTH causes each combination of levels for all row variables to become a row label and each combination of levels for all column variables to become a column label.

The section "TABLES Statement" on page 2213 provides a more detailed description of this option.

DATA=*SAS-data-set*

specifies the SAS data set to be used by PROC CORRESP. If you do not specify the DATA= option, PROC CORRESP uses the most recently created SAS data set.

DEVIATION

DEV

displays the matrix of deviations between the observed frequency matrix and the product of its row marginals and column marginals divided by its grand frequency. For ordinary two-way contingency tables, these are the observed minus expected frequencies under the hypothesis of row and column independence and are components of the chi-square test statistic. See also the CELLCHI2, EXPECTED, and OBSERVED options.

DIMENS=*n*

DIM=*n*

specifies the number of dimensions or axes to use. The default is DIMENS=2. The maximum value of the DIMENS= option in an $(n_r \times n_c)$ table is $n_r - 1$ or $n_c - 1$, whichever is smaller. For example, in a table with 4 rows and 5 columns, the maximum specification is DIMENS=3. If your table has 2 rows or 2 columns, specify DIMENS=1.

EXPECTED

EXP

displays the product of the row marginals and the column marginals divided by the grand frequency of the observed frequency table. For ordinary two-way contingency tables, these are the expected frequencies under the hypothesis of row and column independence and are components of the chi-square test statistic. In other situations, this interpretation is not strictly valid. See also the CELLCHI2, DEVIATION, and OBSERVED options.

FREQOUT

FRE

indicates that the PROC CORRESP input data set has the same form as an output data set from the FREQ procedure, even if it was not directly produced by PROC FREQ. The FREQOUT option enables PROC CORRESP to take shortcuts in constructing the contingency table.

When you specify the FREQOUT option, you must also specify a WEIGHT statement. The cell frequencies in a PROC FREQ output data set are contained in a variable called COUNT, so specify COUNT in a WEIGHT statement with PROC CORRESP. The FREQOUT option might produce unexpected results if the DATA= data set is structured incorrectly. Each of the two variable lists specified in the TABLES statement must consist of a single variable, and observations must be grouped by the levels of the row variable and then by the levels of the column variable. It is not required that the observations be sorted by the row variable and column variable, but they must be grouped consistently. There must be as many observations in the input data set (or BY group) as there are cells in the completed contingency table. Zero cells must be specified with zero weights. When you use PROC FREQ to create the PROC CORRESP input data set, you must specify the SPARSE option in the FREQ procedure's TABLES statement so that the zero cells are written to the output data set.

GREENACRE

GRE

displays adjusted inertias when you are performing multiple correspondence analysis. By default, unadjusted inertias (the usual inertias from multiple correspondence analysis) are displayed. However, adjusted inertias that use a method proposed by Greenacre (1984, p. 156) can be displayed by specifying the GREENACRE option. Specify the UNADJUSTED option to output the usual table of unadjusted inertias as well. For more information, see the section "MCA Adjusted Inertias" on page 2236.

INERTIATABLE

INE

displays the inertia and chi-square decomposition table in addition to the inertia and chi-square decomposition chart when ODS Graphics is enabled. This table is produced by default when ODS Graphics is not enabled or when the chart is not produced. When ODS Graphics is enabled:

- By default, the chart is produced and the table is not produced.
- Specify the PLOTS(ONLY)=CONFIGURATION option to produce the table but not the chart.
- Specify the INERTIATABLE option if you want to see the table in addition to the chart.[1]

MCA

requests a multiple correspondence analysis. This option requires that the input table be a Burt table, which is a symmetric matrix of crosstabulations among several categorical variables. If you specify the MCA option and a VAR statement, you must also specify the NVARS= option, which gives the number of categorical variables that were used to create the table. With raw categorical data, if you want results for the individuals as well as the categories, use the BINARY option instead.

MININERTIA=*n*

MIN=*n*

specifies the minimum inertia ($0 \leq n \leq 1$) used to create the "best" tables—the indicator of which points best explain the inertia of each dimension. By default, MININERTIA=0.8. For more information, see the section "Algorithm and Notation" on page 2231.

MISSING

MIS

specifies that observations with missing values for the TABLES statement variables are included in the analysis. Missing values are treated as a distinct level of each categorical variable. By default, observations with missing values are excluded from the analysis.

NOCOLUMN < = BOTH | DATA | PRINT >

NOC < = BOT | DAT | PRI >

suppresses the display of the column coordinates and statistics and omits them from the output coordinate data set.

[1] The INERTIATABLE option controls whether the inertia table is displayed with the inertia chart, whereas the UNADJUSTED option controls whether the unadjusted inertia table or chart is displayed with the BENZECRI or GREENACRE adjusted table or chart.

BOTH suppresses all column information from both the SAS listing and the output data set. The NOCOLUMN option is equivalent to the option NOCOLUMN=BOTH.

DATA suppresses all column information from the output data set.

PRINT suppresses all column information from the SAS listing.

NOPRINT

NOP

suppresses the display of all output. This option is useful when you need only an output data set. This option disables the Output Delivery System (ODS), including ODS Graphics, for the duration of the PROC. For more information, see Chapter 20, "Using the Output Delivery System."

NOROW < = BOTH | DATA | PRINT >

NOR < = BOT | DAT | PRI >

suppresses the display of the row coordinates and statistics and omits them from the output coordinate data set.

BOTH suppresses all row information from both the SAS listing and the output data set. The NOROW option is equivalent to the option NOROW=BOTH.

DATA suppresses all row information from the output data set.

PRINT suppresses all row information from the SAS listing.

The NOROW option can be useful when the rows of the contingency table are replications.

NVARS=*n*

NVA=*n*

specifies the number of classification variables that were used to create the Burt table. For example, suppose the Burt table was originally created with the following statement:

```
tables a b c;
```

You must specify NVARS=3 to read the table with a VAR statement.

The NVARS= option is required when you specify both the MCA option and a VAR statement. (See the section "VAR Statement" on page 2214 for an example.)

OBSERVED

OBS

displays the contingency table of observed frequencies and its row, column, and grand totals. If you do not specify the OBSERVED or ALL option, the contingency table is not displayed.

OUTC=*SAS-data-set*

OUT=*SAS-data-set*

creates an output coordinate SAS data set to contain the row, column, supplementary observation, and supplementary variable coordinates. This data set also contains the masses, squared cosines, quality of each point's representation in the DIMENS=*n* dimensional display, relative inertias, partial contributions to inertia, and best indicators.

OUTF=*SAS-data-set*
creates an output frequency SAS data set to contain the contingency table, row, and column profiles, the expected values, and the observed minus expected values and contributions to the chi-square statistic.

PLOTS <(*global-plot-options***)> <=***plot-request* **<(***options***)>>**

PLOTS <(*global-plot-options***)> <=(***plot-request* **<(***options***)>** **<...** *plot-request* **<(***options***)>>)>**
specifies options that control the details of the plots. When you specify only one plot request, you can omit the parentheses around the plot request.

ODS Graphics must be enabled before plots can be requested. For example:

```
ods graphics on;

proc corresp;
   tables Marital, Origin;
run;

ods graphics off;
```

For more information about enabling and disabling ODS Graphics, see the section "Enabling and Disabling ODS Graphics" on page 607 in Chapter 21, "Statistical Graphics Using ODS."

By default, for simple correspondence analysis, PROC CORRESP prints the configuration of points consisting of the row coordinates and column coordinates. With MCA, only column coordinates are printed. The default plots ($y * x$) are Dim2 * Dim1, Dim3 * Dim1, Dim3 * Dim2, and so on. When you specify PLOTS(FLIP), the plots are Dim1 * Dim2, Dim1 * Dim3, Dim2 * Dim3, and so on.

The *global-plot-options* are as follows:

FLIP

FLI
flips or interchanges the X-axis and Y-axis dimensions.

ONLY

ONL
suppresses the default plots. Only plots that are specifically requested are displayed.

SOURCE

SOU
displays the levels that correspond to each TABLES statement variable in the same color and shows the source of each group of levels. This option is most useful with multiple correspondence analysis. For example, if Sex and Age are TABLES statement variables, then when you specify SOURCE, `'Male'` and `'Female'` are displayed in one color, and `'Old'` and `'Young'` are displayed in a different color. By default, color groups correspond to rows, supplementary rows, columns, and supplementary columns.

The *plot-requests* include the following:

ALL
produces all appropriate plots.

CONFIGURATION
CONFIG
CON
produces the configuration plot. This plot is produced when ODS Graphics is enabled unless you specify PLOTS(ONLY)=INERTIA.

INERTIA < (*inertia-options*) >
INE< (*inertia-options*) >
requests an inertia decomposition chart and specifies *inertia-options*. An inertia decomposition chart is created when ODS Graphics is enabled unless you specify PLOTS(ONLY)=CONFIGURATION.

Unlike most graphs, the height of the inertia decomposition chart can vary as a function of the number of dimensions that appear in the chart. You can specify the following *inertia-options* to control the height of the inertia decomposition chart:

COMPUTEHEIGHT=*a b < max >*
CH=*a b < max >*
specifies the constants for computing the height of the inertia decomposition chart. For n dimensions, intercept *a*, slope *b*, and maximum height *max*, the height is min($a + b\ (n + 1)$, *max*). By default, COMPUTEHEIGHT=130 15 1200. Thus, the default height in pixels is min($130 + 15(n + 1)$, 1200). The default unit is pixels, and you can use the UNIT= *inertia-option* to change the unit to inches or centimeters.

SETHEIGHT=*height*
SH=*height*
specifies the height of the inertia decomposition chart. By default, the height is based on the COMPUTEHEIGHT= option. The default unit is pixels, and you can use the UNIT= *inertia-option* to change the unit to inches or centimeters.

UNIT=PX | IN | CM
specifies the unit (pixels, inches, or centimeters) for the SETHEIGHT= and COMPUTEHEIGHT= *inertia-options*. Inches equals pixels divided by 96, and centimeters equals inches times 2.54. By default, UNIT=PX.

NONE
NON
suppresses all plots.

PRINT=BOTH | FREQ | PERCENT
PRI=BOT | FRE | PER
affects the OBSERVED, RP, CP, CELLCHI2, EXPECTED, and DEVIATION options. The default is PRINT=FREQ.

- The PRINT=FREQ option displays output in the appropriate raw or natural units. (That is, PROC CORRESP displays raw frequencies for the OBSERVED option, relative frequencies with row marginals of 1.0 for the RP option, and so on.)

- The PRINT=PERCENT option scales results to percentages for the display of the output. (All elements in the OBSERVED matrix sum to 100.0, the row marginals are 100.0 for the RP option, and so on.)
- The PRINT=BOTH option displays both percentages and frequencies.

PROFILE=BOTH | COLUMN | NONE | ROW

PRO=BOT | COL | NON | ROW

specifies the standardization for the row and column coordinates. The default is PROFILE=BOTH.

BOTH specifies a standard correspondence analysis, which jointly displays the principal row and column coordinates. Row coordinates are computed from the row profile matrix, and column coordinates are computed from the column profile matrix.

ROW specifies a correspondence analysis of the row profile matrix. The row coordinates are weighted centroids of the column coordinates.

COLUMN specifies a correspondence analysis of the column profile matrix. The column coordinates are weighted centroids of the row coordinates.

NONE is rarely needed. Row and column coordinates are the generalized singular vectors, without the customary standardizations.

ROW=A | AD | DA | DAD | DAD1/2 | DAID1/2

provides other standardizations of the row coordinates. The ROW= option is rarely needed. Typically, you should use the PROFILE= option instead (see the section "The PROFILE=, ROW=, and COLUMN= Options" on page 2233). By default, ROW=DAD.

RP

displays the row profile matrix. Row profiles contain the observed conditional probabilities of column membership given row membership. See also the CP option.

SHORT

SHO

suppresses the display of all point and coordinate statistics except the coordinates. The following information is suppressed: each point's mass, relative contribution to the total inertia, and quality of representation in the DIMENS=*n* dimensional display; the squared cosines of the angles between each axis and a vector from the origin to the point; the partial contributions of each point to the inertia of each dimension; and the best indicators.

SINGULAR=*n*

SIN=*n*

specifies the largest value that is considered to be within rounding error of zero. The default value is 1E–8. This parameter is used in checking for zero rows and columns, in checking Burt table diagonal sums for equality, in checking denominators before dividing, and so on. Typically, you should not assign a value outside the range 1E–6 to 1E–12.

SOURCE

SOU

adds the variable _VAR_, which contains the name or label of the variable corresponding to the current level, to the OUTC= and OUTF= data sets.

UNADJUSTED

UNA

displays unadjusted inertias when you are performing multiple correspondence analysis. By default, unadjusted inertias (the usual inertias from multiple correspondence analysis) are displayed. However, if adjusted inertias are requested by either the GREENACRE option or the BENZECRI option, then the unadjusted inertia table is not displayed unless the UNADJUSTED option is specified.[2] For more information, see the section "MCA Adjusted Inertias" on page 2236.

BY Statement

BY *variables* **;**

You can specify a BY statement with PROC CORRESP to obtain separate analyses of observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the CORRESP procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

For more information about BY-group processing, see the discussion in *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the discussion in the *Base SAS Procedures Guide*.

ID Statement

ID *variable* **;**

You specify the ID statement only in conjunction with the VAR statement. You cannot specify the ID statement when you use the TABLES statement or the MCA option. When you specify an ID variable, PROC CORRESP labels the rows of the tables with the ID values and places the ID variable in the output data set.

[2] The UNADJUSTED option controls whether the unadjusted inertia table or chart is displayed with the BENZECRI or GREENACRE adjusted table or chart, whereas the INERTIATABLE option controls whether the inertia table is displayed with the inertia chart.

SUPPLEMENTARY Statement

SUPPLEMENTARY *variables* **;**

SUP *variables* **;**

The SUPPLEMENTARY statement specifies variables that are to be represented as points in the joint row and column space but that are not used in determining the locations of the other, active row and column points of the contingency table. Supplementary observations on supplementary variables are ignored in simple correspondence analysis but are needed to compute the squared cosines for multiple correspondence analysis. Variables that are specified in the SUPPLEMENTARY statement must also be specified in the TABLES or VAR statement.

When you specify a VAR statement, each SUPPLEMENTARY variable indicates one supplementary column of the table. Supplementary variables must be numeric with VAR statement input.

When you specify a TABLES statement, each SUPPLEMENTARY variable indicates a set of rows or columns of the table that is supplementary. Supplementary variables can be either character or numeric with TABLES statement input.

TABLES Statement

TABLES < *row-variables*, > *column-variables* **;**

The TABLES statement instructs PROC CORRESP to create a contingency table, Burt table, or binary table from the values of two or more categorical variables. The TABLES statement specifies classification variables that are used to construct the rows and columns of the contingency table. The variables can be either numeric or character. The variable lists in the TABLES statement and the CROSS= option together determine the row and column labels of the contingency table.

You can specify both row variables and column variables separated by a comma, or you can specify only column variables and no comma. If you do not specify row variables (that is, if you list variables but do not use the comma as a delimiter), then you should specify either the MCA or the BINARY option. With the MCA option, PROC CORRESP creates a Burt table, which is a crosstabulation of each variable with itself and every other variable. The Burt table is symmetric. With the BINARY option, PROC CORRESP creates a binary table, which consists of one row for each input data set observation and one column for each category of each TABLES statement variable. If the binary matrix is $\mathbf{Z}$, then the Burt table is $\mathbf{Z}'\mathbf{Z}$. Specifying the BINARY option with the NOROWS option produces the same results as specifying the MCA option (except for the chi-square statistics).

See Figure 35.6 for an example or see the section "The MCA Option" on page 2235 for a detailed description of Burt tables.

You can use the WEIGHT statement with the TABLES statement to read category frequencies. Specify the SUPPLEMENTARY statement to name variables with categories that are supplementary rows or columns. You cannot specify the ID or VAR statement with the TABLES statement. See the section "Using the TABLES Statement" on page 2215 for an example.

VAR Statement

VAR *variables* **;**

You should specify the VAR statement when your data are in tabular form. The VAR variables must be numeric. The VAR statement instructs PROC CORRESP to read an existing contingency table, binary indicator matrix, fuzzy-coded indicator matrix, or Burt table, rather than raw data. See the section "Algorithm and Notation" on page 2231 for a description of a binary indicator matrix and a fuzzy-coded indicator matrix.

You can specify the WEIGHT statement with the VAR statement to read category frequencies and designate supplementary rows. Specify the SUPPLEMENTARY statement to name supplementary variables. You cannot specify the TABLES statement with the VAR statement.

WEIGHT Statement

WEIGHT *variable* **;**

The WEIGHT statement specifies weights for each observation and indicates supplementary observations for simple correspondence analyses with VAR statement input. You can include only one WEIGHT statement, and the weight variable must be numeric.

If you omit the WEIGHT statement, each observation contributes a value of 1 to the frequency count for its category. That is, each observation represents one subject. When you specify a WEIGHT statement, each observation contributes the value of the weighting variable for that observation. For example, a weight of 3 means that the observation represents three subjects. Weight values are not required to be integers.

You can specify the WEIGHT statement with a TABLES statement to indicate category frequencies, as in the following example:

```
proc freq;
   tables a*b / out=outfreq sparse;
run;

proc corresp freqout;
   tables a, b;
   weight count;
run;
```

If you specify a VAR statement, you can specify the WEIGHT statement to indicate supplementary observations and to weight some rows of the table more heavily than others. When the value of the WEIGHT variable is negative, the observation is treated as supplementary, and the absolute value of the weight is used as the weighting value.

You cannot specify a WEIGHT statement with a VAR statement and the MCA option, because the table must be symmetric. Supplementary variables are indicated with the SUPPLEMENTARY statement, so differential weighting of rows is inappropriate.

Details: CORRESP Procedure

Input Data Set

PROC CORRESP can read two kinds of input:

- raw category responses on two or more classification variables with the TABLES statement
- a two-way contingency table with the VAR statement

You can use output from PROC FREQ as input for PROC CORRESP.

The classification variables referred to by the TABLES statement can be either numeric or character variables. Normally, all observations for a given variable that have the same formatted value are placed in the same level, and observations with different values are placed in different levels.

The variables in the VAR statement must be numeric. The values of the observations specify the cell frequencies. These values are not required to be integers, but only those observations with all nonnegative, nonmissing values are used in the correspondence analysis. Observations with one or more negative values are removed from the analysis.

The WEIGHT variable must be numeric. Observations with negative weights are treated as supplementary observations. The absolute values of the weights are used to weight the observations.

Using the TABLES Statement

This section explains some of the choices for the correspondence analysis input data table and illustrates some table-construction capabilities of PROC CORRESP. The SAS data set Neighbor, which follows, will be used throughout this section to illustrate various ways in which PROC CORRESP can read and process data. This data set consists of one observation for each resident in a fictitious neighborhood along with some personal information.

```
title 'PROC CORRESP Table Construction';

data Neighbor;
   input Name $ 1-10 Age $ 12-18 Sex $ 19-25
         Height $ 26-30 Hair $ 32-37;
   datalines;
Jones      Old    Male   Short White
Smith      Young  Female Tall  Brown
Kasavitz   Old    Male   Short Brown
Ernst      Old    Female Tall  White
Zannoria   Old    Female Short Brown
Spangel    Young  Male   Tall  Blond
```

```
Myers      Young  Male   Tall  Brown
Kasinski   Old    Male   Short Blond
Colman     Young  Female Short Blond
Delafave   Old    Male   Tall  Brown
Singer     Young  Male   Tall  Brown
Igor       Old           Short
;
```

This first step creates a simple contingency table or crosstabulation. In the TABLES statement, each variable list consists of a single variable. The following statements produce the table in Figure 35.3.

```
proc corresp data=Neighbor dimens=1 observed short;
   title2 'Simple Crosstabulation';
   ods select observed;
   tables Sex, Age;
run;
```

These statements create a contingency table with two rows (**Female** and **Male**) and two columns (**Old** and **Young**) and show the neighbors categorized by age and sex. The DIMENS=1 option specifies the number of dimensions in the correspondence analysis. Typically, you do not have to specify this option, because typically your tables will be larger than two by two. The default is DIMENS=2, which is too large for a table with a two-level factor. The OBSERVED option displays the contingency table. The SHORT option limits the displayed output. Because it contains missing values, the observation where Name='Igor' is omitted from the analysis. The table is shown in Figure 35.3.

Figure 35.3 Contingency Table for Sex, Age

PROC CORRESP Table Construction
Simple Crosstabulation

The CORRESP Procedure

Contingency Table			
	Old	Young	Sum
Female	2	2	4
Male	4	3	7
Sum	6	5	11

The preceding example showed how to make a two-way contingency table based on the levels of two categorical variables, which, if it were larger, would be a very typical form of data for a correspondence analysis. However, many other types of tables, $\mathbf{N}$, can be used as input to a correspondence analysis, and all tables can be defined based on a binary matrix, $\mathbf{Z}$. The BINARY option enables you to directly compute and display this matrix. The TABLES statement consists of a single list of all the categorical variables. The following statements produce Figure 35.4.

```
proc corresp data=neighbor observed short binary;
   title2 'Binary Coding';
   ods select binary;
   tables Hair Height Sex Age;
run;
```

Figure 35.4 Binary Table Using the BINARY Option

PROC CORRESP Table Construction
Binary Coding

The CORRESP Procedure

Binary Table

	Blond	Brown	White	Short	Tall	Female	Male	Old	Young
1	0	0	1	1	0	0	1	1	0
2	0	1	0	0	1	1	0	0	1
3	0	1	0	1	0	0	1	1	0
4	0	0	1	0	1	1	0	1	0
5	0	1	0	1	0	1	0	1	0
6	1	0	0	0	1	0	1	0	1
7	0	1	0	0	1	0	1	0	1
8	1	0	0	1	0	0	1	1	0
9	1	0	0	1	0	1	0	0	1
10	0	1	0	0	1	0	1	1	0
11	0	1	0	0	1	0	1	0	1

In this case, $\mathbf{N} = \mathbf{Z}$ is directly analyzed. The binary matrix has one row for each individual or case and one column for each category. A binary table constructed from *m* categorical variables has *m* partitions. This binary table has four partitions, one for each of the four categorical variables. Each partition has a 1 in each row, and each row contains exactly four 1s because there are four categorical variables. More generally, the binary design matrix has exactly *m* 1s in each row. The 1s indicate the categories to which the observation applies. For example, the categorical variable Sex, with two levels (**Female** and **Male**), is coded using two indicator variables. For the variable Sex, a male would be coded Female=0 and Male=1, and a female would be coded Female=1 and Male=0. This is the same kind of coding that procedures like GLM and TRANSREG use for CLASS variables.

Implicitly, the binary table has an automatic row variable that is equal to the observation number. Alternatively, when there is a row ID variable, as there is in this case, you can use it as a row variable in the TABLES statement, and the resulting ordinary observed frequency table is the binary table. This example uses two variable lists: Name for the row variable, and Hair Height Sex Age for the column variables. Because two lists were provided, the BINARY option was not specified. The following statements produce Figure 35.5.

```
proc corresp data=neighbor observed short;
   title2 'Binary Coding';
   ods select observed;
   tables Name, Hair Height Sex Age;
run;
```

Figure 35.5 Binary Table Using a Row Variable

PROC CORRESP Table Construction
Binary Coding

The CORRESP Procedure

Contingency Table

	Blond	Brown	White	Short	Tall	Female	Male	Old	Young	Sum
Colman	1	0	0	1	0	1	0	0	1	4
Delafave	0	1	0	0	1	0	1	1	0	4
Ernst	0	0	1	0	1	1	0	1	0	4
Jones	0	0	1	1	0	0	1	1	0	4
Kasavitz	0	1	0	1	0	0	1	1	0	4
Kasinski	1	0	0	1	0	0	1	1	0	4
Myers	0	1	0	0	1	0	1	0	1	4
Singer	0	1	0	0	1	0	1	0	1	4
Smith	0	1	0	0	1	1	0	0	1	4
Spangel	1	0	0	0	1	0	1	0	1	4
Zannoria	0	1	0	1	0	1	0	1	0	4
Sum	3	6	2	5	6	4	7	6	5	44

With the MCA option, the Burt table ($\mathbf{Z}'\mathbf{Z}$) is analyzed. A Burt table is a partitioned symmetric matrix containing all pairs of crosstabulations among a set of categorical variables. Each diagonal partition is a diagonal matrix containing marginal frequencies (a crosstabulation of a variable with itself). Each off-diagonal partition is an ordinary contingency table. The following statements produce Figure 35.6.

```
proc corresp data=neighbor observed short mca;
   title2 'MCA Burt Table';
   ods select burt;
   tables Hair Height Sex Age;
run;
```

Note that there is a single variable list in the TABLES statement, because the row and column variable lists are the same.

Figure 35.6 MCA Burt Table

PROC CORRESP Table Construction
MCA Burt Table

The CORRESP Procedure

Burt Table									
	Blond	**Brown**	**White**	**Short**	**Tall**	**Female**	**Male**	**Old**	**Young**
Blond	3	0	0	2	1	1	2	1	2
Brown	0	6	0	2	4	2	4	3	3
White	0	0	2	1	1	1	1	2	0
Short	2	2	1	5	0	2	3	4	1
Tall	1	4	1	0	6	2	4	2	4
Female	1	2	1	2	2	4	0	2	2
Male	2	4	1	3	4	0	7	4	3
Old	1	3	2	4	2	2	4	6	0
Young	2	3	0	1	4	2	3	0	5

This Burt table is composed of all pairs of crosstabulations among the variables Hair, Height, Sex, and Age. It is composed of sixteen individual subtables—the number of variables squared. Both the rows and the columns have the same nine categories (in this case Blond, Brown, White, Short, Tall, Female, Male, Old, and Young). Below the diagonal (from left to right, top to bottom) are the following crosstabulations: Height * Hair, Sex * Hair, Sex * Height, Age * Hair, Age * Height, and Age * Sex. Each crosstabulation below the diagonal has a transposed counterpart above the diagonal. The diagonal contains the crosstabulations: Hair * Hair, Height * Height, Sex * Sex, and Age * Age. The diagonal elements of the diagonal partitions contain marginal frequencies of the off-diagonal partitions. The table Hair * Height, for example, has three rows for Hair and two columns for Height. The values of the Hair * Height table, summed across rows, sum to the diagonal values of the Height * Height table, as displayed in the following results. The following statements produce Figure 35.7.

```
proc corresp data=neighbor observed short dimens=1;
   title2 'Part of the Burt Table';
   ods output observed=o;
   tables Hair Height, Height;
run;

proc print data=o(drop=sum) label noobs;
   where label ne 'Sum';
   label label = '00'x;
run;
```

Figure 35.7 Part of the Burt Table

PROC CORRESP Table Construction
Part of the Burt Table

	Short	Tall
Blond	2	1
Brown	2	4
White	1	1
Short	5	0
Tall	0	6

A simple crosstabulation of Hair $\times$ Height is $\mathbf{N} = \mathbf{Z}_{\text{Hair}}{}'\mathbf{Z}_{\text{Height}}$. Tables such as ($\mathbf{N} = \mathbf{Z}_{\text{Hair}}{}'\mathbf{Z}_{\text{Height,Sex}}$), made up of several crosstabulations, can also be analyzed in simple correspondence analysis. The following statements produce Figure 35.8.

```
proc corresp data=neighbor observed short dimens=1;
   title2 'Multiple Crosstabulations';
   ods select observed;
   tables Hair, Height Sex;
run;
```

Figure 35.8 Hair $\times$ (Height Sex) Crosstabulation

PROC CORRESP Table Construction
Multiple Crosstabulations

The CORRESP Procedure

Contingency Table					
	Short	Tall	Female	Male	Sum
Blond	2	1	1	2	6
Brown	2	4	2	4	12
White	1	1	1	1	4
Sum	5	6	4	7	22

The following statements create a table with six rows (**Blond*Short**, **Blond*Tall**, **Brown*Short**, **Brown*Tall**, **White*Short**, and **White*Tall**) and four columns (**Female**, **Male**, **Old**, and **Young**). The levels of the row variables are crossed by the CROSS=ROW option, forming mutually exclusive categories. Hence each individual fits into exactly one row category, but two column categories. The following statements produce Figure 35.9.

```
proc corresp data=Neighbor cross=row observed short;
   title2 'Multiple Crosstabulations with Crossed Rows';
   ods select observed;
   tables Hair Height, Sex Age;
run;
```

Figure 35.9 Contingency Table for Hair * Height, Sex Age

PROC CORRESP Table Construction
Multiple Crosstabulations with Crossed Rows

The CORRESP Procedure

Contingency Table

	Female	Male	Old	Young	Sum
Blond * Short	1	1	1	1	4
Blond * Tall	0	1	0	1	2
Brown * Short	1	1	2	0	4
Brown * Tall	1	3	1	3	8
White * Short	0	1	1	0	2
White * Tall	1	0	1	0	2
Sum	4	7	6	5	22

You can enter supplementary variables with TABLES input by including a SUPPLEMENTARY statement. Variables named in the SUPPLEMENTARY statement indicate TABLES variables with categories that are supplementary. In other words, the categories of the variable Age are represented in the row and column space, but they are not used in determining the scores of the categories of the variables Hair, Height, and Sex. The variable used in the SUPPLEMENTARY statement must be listed in the TABLES statement as well. For example, the following statements create a Burt table with seven active rows and columns (**Blond**, **Brown**, **White**, **Short**, **Tall**, **Female**, **Male**) and two supplementary rows and columns (**Old** and **Young**). The following statements produce Figure 35.10.

```
proc corresp data=Neighbor observed short mca;
   title2 'MCA with Supplementary Variables';
   ods select burt supcols;
   tables Hair Height Sex Age;
   supplementary Age;
run;
```

Figure 35.10 Burt Table from PROC CORRESP with Supplementary Variables

PROC CORRESP Table Construction
MCA with Supplementary Variables

The CORRESP Procedure

Burt Table

	Blond	Brown	White	Short	Tall	Female	Male
Blond	3	0	0	2	1	1	2
Brown	0	6	0	2	4	2	4
White	0	0	2	1	1	1	1
Short	2	2	1	5	0	2	3
Tall	1	4	1	0	6	2	4
Female	1	2	1	2	2	4	0
Male	2	4	1	3	4	0	7

Figure 35.10 *continued*

Supplementary Columns		
	Old	Young
Blond	1	2
Brown	3	3
White	2	0
Short	4	1
Tall	2	4
Female	2	2
Male	4	3

The following statements create a binary table with 7 active columns (**Blond**, **Brown**, **White**, **Short**, **Tall**, **Female**, **Male**), 2 supplementary columns (**Old** and **Young**), and 11 rows for the 11 observations with nonmissing values. The following statements produce Figure 35.11.

```
proc corresp data=Neighbor observed short binary;
   title2 'Supplementary Binary Variables';
   ods select binary supcols;
   tables Hair Height Sex Age;
   supplementary Age;
run;
```

Figure 35.11 Binary Table from PROC CORRESP with Supplementary Variables

PROC CORRESP Table Construction
Supplementary Binary Variables

The CORRESP Procedure

	Binary Table						
	Blond	Brown	White	Short	Tall	Female	Male
1	0	0	1	1	0	0	1
2	0	1	0	0	1	1	0
3	0	1	0	1	0	0	1
4	0	0	1	0	1	1	0
5	0	1	0	1	0	1	0
6	1	0	0	0	1	0	1
7	0	1	0	0	1	0	1
8	1	0	0	1	0	0	1
9	1	0	0	1	0	1	0
10	0	1	0	0	1	0	1
11	0	1	0	0	1	0	1

Figure 35.11 *continued*

	Supplementary Columns	
	Old	Young
1	1	0
2	0	1
3	1	0
4	1	0
5	1	0
6	0	1
7	0	1
8	1	0
9	0	1
10	1	0
11	0	1

Using the VAR Statement

With VAR statement input, the rows of the contingency table correspond to the observations of the input data set, and the columns correspond to the VAR statement variables. The values of the variables typically contain the table frequencies. The table in Figure 35.3 could be created with VAR statement input by using the following statements:

```
data Ages;
   input Sex $ Old Young;
   datalines;
Female  2 2
Male    4 3
;

proc corresp data=Ages dimens=1 observed short;
   var Old Young;
   id Sex;
run;
```

Only nonnegative values are accepted. Negative values are treated as missing, causing the observation to be excluded from the analysis. The values are not required to be integers. Row labels for the table are specified with an ID variable. Column labels are constructed from the variable name or variable label if one is specified. When you specify multiple correspondence analysis (MCA), the row and column labels are the same and are constructed from the variable names or labels, so you cannot include an ID statement. With MCA, the VAR statement must list the variables in the order in which the rows occur. An example is the table in Figure 35.6, which was created with the following TABLES statement.

```
tables Hair Height Sex Age;
```

This table could have been created with a VAR statement as follows:

```
proc corresp data=table nvars=4 mca;
   var Blond Brown White Short Tall Female Male Old Young;
run;
```

You must specify the NVARS= option in order to specify the number of original categorical variables with the MCA option. The option NVARS=*n* is needed to find boundaries between the subtables of the Burt table. If f is the sum of all elements in the Burt table $\mathbf{Z}'\mathbf{Z}$, then fn^{-2} is the number of rows in the binary matrix $\mathbf{Z}$. The sum of all elements in each diagonal subtable of the Burt table must be fn^{-2}.

To enter supplementary observations, include a WEIGHT statement with negative weights for those observations. Specify the SUPPLEMENTARY statement to include supplementary variables. You must list supplementary variables in both the VAR and SUPPLEMENTARY statements.

Missing and Invalid Data

With VAR statement input, observations with missing or negative frequencies are excluded from the analysis. Supplementary variables and supplementary observations with missing or negative frequencies are also excluded. Negative weights are valid with VAR statement input.

With TABLES statement input, observations with negative weights are excluded from the analysis. With this form of input, missing cell frequencies cannot occur. Observations with missing values on the categorical variables are excluded unless you specify the MISSING option. If you specify the MISSING option, ordinary missing values and special missing values are treated as additional levels of a categorical variable. In all cases, if any row or column of the constructed table contains only zeros, that row or column is excluded from the analysis.

Observations with missing weights are excluded from the analysis.

Coding, Fuzzy Coding, and Doubling

Sometimes, binary data such as Yes/No data are available—for example, 1 means “Yes, I have bought this brand in the last month” and 0 means “No, I have not bought this brand in the last month”. The following statements read a data set with Yes/No purchase data for three hypothetical brands.

```
title 'Doubling Yes/No Data';

proc format;
   value yn 0 = 'No '  1 = 'Yes';
run;

data BrandChoice;
   input a b c;
   label a = 'Brand A' b = 'Brand B' c = 'Brand B';
```

```
   format a b c yn.;
   datalines;
0 0 1
1 1 0
0 1 1
0 1 0
1 0 0
;
```

Data such as these cannot be analyzed directly because the raw data do not consist of partitions, each with one column per level and exactly one 1 in each row. (See the section "Using the TABLES Statement" on page 2215.) The data must be *doubled* so that both Yes and No are represented by a column in the data matrix. The TRANSREG procedure provides one way of doubling. In the following statements, the DESIGN option specifies that PROC TRANSREG is being used only for coding, not analysis. The option SEPARATORS=': ' specifies that labels for the coded columns are constructed from input variable labels, followed by a colon and space, followed by the formatted value. The variables are designated in the MODEL statement as CLASS variables, and the ZERO=NONE option creates binary variables for all levels. The OUTPUT statement specifies the output data set and drops the _NAME_, _TYPE_, and Intercept variables. PROC TRANSREG stores a list of coded variable names in a macro variable &_TRGIND, which in this case has the value "aNo aYes bNo bYes cNo cYes". This macro variable can be used directly in the VAR statement in PROC CORRESP. The following statements produce Figure 35.12. Only the input table is displayed.

```
proc transreg data=BrandChoice design separators=': ';
   model class(a b c / zero=none);
   output out=Doubled(drop=_: Intercept);
run;

proc print label;
run;

proc corresp data=Doubled norow short;
   var &_trgind;
run;
```

Figure 35.12 Doubling Yes/No Data

Doubling Yes/No Data

Obs	Brand A: No	Brand A: Yes	Brand B: No	Brand B: Yes	Brand B: No	Brand B: Yes	Brand A	Brand B	Brand B
1	1	0	1	0	0	1	No	No	Yes
2	0	1	0	1	1	0	Yes	Yes	No
3	1	0	0	1	0	1	No	Yes	Yes
4	1	0	0	1	1	0	No	Yes	No
5	0	1	1	0	1	0	Yes	No	No

A fuzzy-coded indicator also sums to 1.0 across levels of the categorical variable, but it is coded with fractions rather than with 0 and 1. The fractions represent the distribution of the attribute across several levels of the categorical variable.

Ordinal variables, such as survey responses of 1 to 3, can be represented as two fuzzy-coded variables, as shown in Table 35.2.

Table 35.2 Coding an Ordinal Variable

Ordinal Values	Coding	
1	0.25	0.75
2	0.50	0.50
3	0.75	0.25

The values of the coding sum to one across the two coded variables.

These next steps illustrate the use of binary and fuzzy-coded indicator variables. Fuzzy-coded indicators are used to represent missing data. Note that the missing values in the observation Igor are coded with equal proportions. The following statements produce Figure 35.13.

```
title 'Fuzzy Coding of Missing Values';

proc transreg data=Neighbor design cprefix=0;
   model class(Age Sex Height Hair / zero=none);
   output out=Neighbor2(drop=_: Intercept);
   id Name;
run;

data Neighbor3;
   set Neighbor2;
   if Sex = ' ' then do;
      Female = 0.5;
      Male   = 0.5;
   end;
   if Hair = ' ' then do;
      White = 1/3;
      Brown = 1/3;
      Blond = 1/3;
   end;
run;

proc print label noobs data=Neighbor3(drop=age--name);
   format _numeric_ best4.;
run;
```

Figure 35.13 Fuzzy Coding of Missing Values

Fuzzy Coding of Missing Values

Age Old	Age Young	Sex Female	Sex Male	Height Short	Height Tall	Hair Blond	Hair Brown	Hair White
1	0	0	1	1	0	0	0	1
0	1	1	0	0	1	0	1	0
1	0	0	1	1	0	0	1	0
1	0	1	0	0	1	0	0	1
1	0	1	0	1	0	0	1	0
0	1	0	1	0	1	1	0	0
0	1	0	1	0	1	0	1	0
1	0	0	1	1	0	1	0	0
0	1	1	0	1	0	1	0	0
1	0	0	1	0	1	0	1	0
0	1	0	1	0	1	0	1	0
1	0	0.5	0.5	1	0	0.33	0.33	0.33

There is one set of coded variables for each input categorical variable. If observation 12 is excluded, each set is a binary design matrix. Each design matrix has one column for each category and exactly one 1 in each row. Fuzzy coding is shown in the final observation, which corresponds to Igor. The observation for Igor has missing values for the variables Sex and Hair. The design matrix variables are coded with fractions that sum to one within each categorical variable.

An alternative way to represent missing data is to treat missing values as an additional level of the categorical variable. This alternative is available with the MISSING option in the PROC CORRESP statement. This approach yields coordinates for missing responses, allowing the comparison of "missing" along with the other levels of the categorical variables.

Greenacre and Hastie (1987) discuss additional coding schemes, including one for continuous variables. Continuous variables can be coded with PROC TRANSREG by specifying BSPLINE(*variables* / degree=1) in the MODEL statement.

Creating a Data Set Containing the Crosstabulation

The CORRESP procedure can read or create a contingency or Burt table. PROC CORRESP is generally more efficient with VAR statement input than with TABLES statement input. TABLES statement input requires that the table be created from raw categorical variables, whereas the VAR statement is used to read an existing table. For extremely large problems, if PROC CORRESP runs out of memory, it might be possible to use some other method to create the table and then use VAR statement input with PROC CORRESP.

The following example uses the CORRESP, FREQ, and TRANSPOSE procedures to create rectangular tables from a SAS data set WORK.A that contains the categorical variables V1–V5. The Burt table examples assume that no categorical variable has a value found in any of the other categorical variables (that is, that each row and column label is unique).

You can use PROC CORRESP and the ODS OUTPUT statement as follows to create a rectangular two-way contingency table from two categorical variables:

```
proc corresp data=a observed short;
   ods output Observed=Obs(drop=Sum where=(Label ne 'Sum'));
   tables v1, v2;
run;
```

You can use PROC FREQ and PROC TRANSPOSE to create a rectangular two-way contingency table from two categorical variables, as in the following statements:

```
proc freq data=a;
   tables v1 * v2 / sparse noprint out=freqs;
run;

proc transpose data=freqs out=rfreqs(drop=_:);
   id  v2;
   var count;
   by  v1;
run;
```

You can use PROC CORRESP and the ODS OUTPUT statement as follows to create a Burt table from five categorical variables:

```
proc corresp data=a observed short mca;
   ods output Burt=Obs;
   tables v1-v5;
run;
```

You can use a DATA step, PROC FREQ, and PROC TRANSPOSE to create a Burt table from five categorical variables, as in the following statements:

```
data b;
   set a;
   array v[5] $ v1-v5;
   do i = 1 to 5;
      row = v[i];
      do j = 1 to 5;
         column = v[j];
         output;
      end;
   end;
   keep row column;
run;

proc freq data=b;
   tables row * column / sparse  noprint out=freqs;
run;

proc transpose data=freqs out=rfreqs(drop=_:);
   id  column;
   var count;
   by  row;
run;
```

Output Data Sets

PROC CORRESP has two output data sets. The OUTC= data set contains coordinates and the results of the correspondence analysis. The OUTF= data set contains frequencies and other cross-tabulation results.

The OUTC= Data Set

The OUTC= data set contains two or three character variables and $4n + 4$ numeric variables, where n is the number of axes from DIMENS=n (two by default). The OUTC= data set contains one observation for each row, column, supplementary row, and supplementary column point, and one observation for inertias.

The first variable is named _TYPE_ and identifies the type of observation. The values of _TYPE_ are as follows:

- The 'INERTIA' observation contains the total inertia in the INERTIA variable, and each dimension's inertia in the Contr1–Contr$n$ variables.
- The 'OBS' observations contain the coordinates and statistics for the rows of the table.
- The 'SUPOBS' observations contain the coordinates and statistics for the supplementary rows of the table.
- The 'VAR' observations contain the coordinates and statistics for the columns of the table.
- The 'SUPVAR' observations contain the coordinates and statistics for the supplementary columns of the table.

If you specify the SOURCE option, then the data set also contains a variable _VAR_ containing the name or label of the input variable from which that row originates. The name of the next variable is either _NAME_ or (if you specify an ID statement) the name of the ID variable.

For observations with a value of 'OBS' or 'SUPOBS' for the _TYPE_ variable, the values of the second variable are constructed as follows:

- When you use a VAR statement without an ID statement, the values are 'Row1', 'Row2', and so on.
- When you specify a VAR statement with an ID statement, the values are set equal to the values of the ID variable.
- When you specify a TABLES statement, the _NAME_ variable has values formed from the appropriate row variable values.

For observations with a value of 'VAR' or 'SUPVAR' for the _TYPE_ variable, the values of the second variable are equal to the names or labels of the VAR (or SUPPLEMENTARY) variables. When you specify a TABLES statement, the values are formed from the appropriate column variable values.

The third and subsequent variables contain the numerical results of the correspondence analysis.

- Quality contains the quality of each point's representation in the DIMENS=n dimensional display, which is the sum of squared cosines over the first n dimensions.
- Mass contains the masses or marginal sums of the relative frequency matrix.
- Inertia contains each point's relative contribution to the total inertia.

- Dim1–Dim*n* contain the point coordinates.
- Contr1–Contr*n* contain the partial contributions to inertia.
- SqCos1–SqCos*n* contain the squared cosines.
- Best1–Best*n* and Best contain the summaries of the partial contributions to inertia.

The OUTF= Data Set

The OUTF= data set contains frequencies and percentages. It is similar to a PROC FREQ output data set. The OUTF= data set begins with a variable called _TYPE_, which contains the observation type. If the SOURCE option is specified, the data set contains two variables, _ROWVAR_ and _COLVAR_, that contain the names or labels of the row and column input variables from which each cell originates. The next two variables are classification variables that contain the row and column levels. If you use TABLES statement input and each variable list consists of a single variable, the names of the first two variables match the names of the input variables; otherwise, these variables are named Row and Column. The next two variables are Count and Percent, which contain frequencies and percentages.

The _TYPE_ variable can have the following values:

- 'OBSERVED' observations contain the contingency table.
- 'SUPOBS' observations contain the supplementary rows.
- 'SUPVAR' observations contain the supplementary columns.
- 'EXPECTED' observations contain the product of the row marginals and the column marginals divided by the grand frequency of the observed frequency table. For ordinary two-way contingency tables, these are the expected frequency matrix under the hypothesis of row and column independence.
- 'DEVIATION' observations contain the matrix of deviations between the observed frequency matrix and the product of its row marginals and column marginals divided by its grand frequency. For ordinary two-way contingency tables, these are the observed minus expected frequencies under the hypothesis of row and column independence.
- 'CELLCHI2' observations contain contributions to the total chi-square test statistic.
- 'RP' observations contain the row profiles.
- 'SUPRP' observations contain supplementary row profiles.
- 'CP' observations contain the column profiles.
- 'SUPCP' observations contain supplementary column profiles.

Computational Resources

Let

$$
\begin{aligned}
n_r &= \text{number of rows in the table} \\
n_c &= \text{number of columns in the table} \\
n &= \text{number of observations} \\
v &= \text{number of VAR statement variables} \\
t &= \text{number of TABLES statement variables} \\
c &= \max(n_r, n_c) \\
d &= \min(n_r, n_c)
\end{aligned}
$$

For TABLES statement input, more than

$$32(t+1) + 8(\max(2tn, (n_r+3)(n_c+3)))$$

bytes of array space are required.

For VAR statement input, more than

$$16(v+2) + 8(n_r+3)(n_c+3)$$

bytes of array space are required.

Memory

The computational resources formulas are underestimates of the amounts of memory needed to handle most problems. If you use a utility data set, and if memory could be used with perfect efficiency, then roughly the stated amount of memory would be needed. In reality, most problems require at least two or three times the minimum.

PROC CORRESP tries to store the raw data (TABLES input) and the contingency table in memory. If there is not enough memory, a utility data set is used, potentially resulting in a large increase in execution time.

Time

The time required to perform the generalized singular value decomposition is roughly proportional to $2cd^2 + 5d^3$. Overall computation time increases with table size at a rate roughly proportional to $(n_r n_c)^{\frac{3}{2}}$.

Algorithm and Notation

This section is primarily based on the theory of correspondence analysis found in Greenacre (1984). If you are interested in other references, see the section "Background" on page 2200.

Let $\mathbf{N}$ be the contingency table formed from those observations and variables that are not supplementary and from those observations that have no missing values and have a positive weight. This table is an $(n_r \times n_c)$

rank q matrix of nonnegative numbers with nonzero row and column sums. If $\mathbf{Z}_a$ is the binary coding for variable A, and $\mathbf{Z}_b$ is the binary coding for variable B, then $\mathbf{N} = \mathbf{Z}_a'\mathbf{Z}_b$ is a contingency table. Similarly, if $\mathbf{Z}_{b,c}$ contains the binary coding for both variables B and C, then $\mathbf{N} = \mathbf{Z}_a'\mathbf{Z}_{b,c}$ can also be input to a correspondence analysis. With the BINARY option, $\mathbf{N} = \mathbf{Z}$, and the analysis is based on a binary table. In multiple correspondence analysis, the analysis is based on a Burt table, $\mathbf{Z}'\mathbf{Z}$.

Let $\mathbf{1}$ be a vector of 1s of the appropriate order, let $\mathbf{I}$ be an identity matrix, and let diag$(\cdot)$ be a matrix-valued function that creates a diagonal matrix from a vector. Let

$$
\begin{aligned}
f &= \mathbf{1}'\mathbf{N}\mathbf{1} \\
\mathbf{P} &= \frac{1}{f}\mathbf{N} \\
\mathbf{r} &= \mathbf{P}\mathbf{1} \\
\mathbf{c} &= \mathbf{P}'\mathbf{1} \\
\mathbf{D_r} &= \mathrm{diag}(\mathbf{r}) \\
\mathbf{D_c} &= \mathrm{diag}(\mathbf{c}) \\
\mathbf{R} &= \mathbf{D_r^{-1}}\mathbf{P} \\
\mathbf{C}' &= \mathbf{D_c^{-1}}\mathbf{P}'
\end{aligned}
$$

The scalar f is the sum of all elements in $\mathbf{N}$. The matrix $\mathbf{P}$ is a matrix of relative frequencies. The vector $\mathbf{r}$ contains row marginal proportions or row "masses." The vector $\mathbf{c}$ contains column marginal proportions or column masses. The matrices $\mathbf{D_r}$ and $\mathbf{D_c}$ are diagonal matrices of marginals.

The rows of $\mathbf{R}$ contain the "row profiles." The elements of each row of $\mathbf{R}$ sum to one. Each (i, j) element of $\mathbf{R}$ contains the observed probability of being in column j given membership in row i. Similarly, the columns of $\mathbf{C}$ contain the column profiles. The coordinates in correspondence analysis are based on the generalized singular value decomposition of $\mathbf{P}$,

$$\mathbf{P} = \mathbf{A}\mathbf{D_u}\mathbf{B}'$$

where

$$\mathbf{A}'\mathbf{D_r^{-1}}\mathbf{A} = \mathbf{B}'\mathbf{D_c^{-1}}\mathbf{B} = \mathbf{I}$$

In multiple correspondence analysis,

$$\mathbf{P} = \mathbf{B}\mathbf{D_u^2}\mathbf{B}'$$

The matrix $\mathbf{A}$, which is the rectangular matrix of left generalized singular vectors, has n_r rows and q columns; the matrix $\mathbf{D_u}$, which is a diagonal matrix of singular values, has q rows and columns; and the matrix $\mathbf{B}$, which is the rectangular matrix of right generalized singular vectors, has n_c rows and q columns. The columns of $\mathbf{A}$ and $\mathbf{B}$ define the principal axes of the column and row point clouds, respectively.

The generalized singular value decomposition of $\mathbf{P} - \mathbf{r}\mathbf{c}'$, discarding the last singular value (which is zero) and the last left and right singular vectors, is exactly the same as a generalized singular value decomposition of $\mathbf{P}$, discarding the first singular value (which is one), the first left singular vector, $\mathbf{r}$, and the first right singular vector, $\mathbf{c}$. The first (trivial) column of $\mathbf{A}$ and $\mathbf{B}$ and the first singular value in $\mathbf{D_u}$ are discarded before

any results are displayed. You can obtain the generalized singular value decomposition of $\mathbf{P} - \mathbf{rc}'$ from the ordinary singular value decomposition of $\mathbf{D_r^{-1/2}}(\mathbf{P} - \mathbf{rc}')\mathbf{D_c^{-1/2}}$:

$$\mathbf{D_r^{-1/2}}(\mathbf{P} - \mathbf{rc}')\mathbf{D_c^{-1/2}} = \mathbf{U}\mathbf{D_u}\mathbf{V}' = (\mathbf{D_r^{-1/2}}\mathbf{A})\mathbf{D_u}(\mathbf{D_c^{-1/2}}\mathbf{B})'$$

$$\mathbf{P} - \mathbf{rc}' = \mathbf{D_r^{1/2}}\mathbf{U}\mathbf{D_u}\mathbf{V}'\mathbf{D_c^{1/2}} = (\mathbf{D_r^{1/2}}\mathbf{U})\mathbf{D_u}(\mathbf{D_c^{1/2}}\mathbf{V})' = \mathbf{A}\mathbf{D_u}\mathbf{B}'$$

Hence, $\mathbf{A} = \mathbf{D_r^{1/2}}\mathbf{U}$ and $\mathbf{B} = \mathbf{D_c^{1/2}}\mathbf{V}$.

The default row coordinates are $\mathbf{D_r^{-1}}\mathbf{A}\mathbf{D_u}$, and the default column coordinates are $\mathbf{D_c^{-1}}\mathbf{B}\mathbf{D_u}$. Typically the first two columns of $\mathbf{D_r^{-1}}\mathbf{A}\mathbf{D_u}$ and $\mathbf{D_c^{-1}}\mathbf{B}\mathbf{D_u}$ are plotted to display graphically associations between the row and column categories. The plot consists of two overlaid plots, one for rows and one for columns. The row points are row profiles, and the column points are column profiles, both rescaled so that distances between profiles can be displayed as ordinary Euclidean distances, then orthogonally rotated to a principal axes orientation. Distances between row points and other row points have meaning, as do distances between column points and other column points. However, distances between column points and row points are not interpretable.

The PROFILE=, ROW=, and COLUMN= Options

The PROFILE=, ROW=, and COLUMN= options standardize the coordinates before they are displayed and placed in the output data set. The options PROFILE=BOTH, PROFILE=ROW, and PROFILE=COLUMN provide the standardizations that are typically used in correspondence analysis. There are six choices each for row and column coordinates (see Table 35.3). However, most of the combinations of the ROW= and COLUMN= options are not useful. The ROW= and COLUMN= options are provided for completeness, but they are not intended for general use.

Table 35.3 Coordinates

ROW=	Matrix Formula
A	$\mathbf{A}$
AD	$\mathbf{A}\mathbf{D_u}$
DA	$\mathbf{D_r^{-1}}\mathbf{A}$
DAD	$\mathbf{D_r^{-1}}\mathbf{A}\mathbf{D_u}$
DAD1/2	$\mathbf{D_r^{-1}}\mathbf{A}\mathbf{D_u^{1/2}}$
DAID1/2	$\mathbf{D_r^{-1}}\mathbf{A}(\mathbf{I} + \mathbf{D_u})^{1/2}$
COLUMN=	**Matrix Formula**
B	$\mathbf{B}$
BD	$\mathbf{B}\mathbf{D_u}$
DB	$\mathbf{D_c^{-1}}\mathbf{B}$
DBD	$\mathbf{D_c^{-1}}\mathbf{B}\mathbf{D_u}$
DBD1/2	$\mathbf{D_c^{-1}}\mathbf{B}\mathbf{D_u^{1/2}}$
DBID1/2	$\mathbf{D_c^{-1}}\mathbf{B}(\mathbf{I} + \mathbf{D_u})^{1/2}$

When PROFILE=ROW (ROW=DAD and COLUMN=DB), the row coordinates $\mathbf{D_r^{-1}AD_u}$ and column coordinates $\mathbf{D_c^{-1}B}$ provide a correspondence analysis based on the row profile matrix. The row profile (conditional probability) matrix is defined as $\mathbf{R} = \mathbf{D_r^{-1}P} = \mathbf{D_r^{-1}AD_uB'}$. The elements of each row of $\mathbf{R}$ sum to one. Each (i, j) element of $\mathbf{R}$ contains the observed probability of being in column j given membership in row i. The "principal" row coordinates $\mathbf{D_r^{-1}AD_u}$ and "standard" column coordinates $\mathbf{D_c^{-1}B}$ provide a decomposition of $\mathbf{D_r^{-1}AD_uB'D_c^{-1}} = \mathbf{D_r^{-1}PD_c^{-1}} = \mathbf{RD_c^{-1}}$. Because $\mathbf{D_r^{-1}AD_u} = \mathbf{RD_c^{-1}B}$, the row coordinates are weighted centroids of the column coordinates. Each column point, with coordinates scaled to standard coordinates, defines a vertex in $(n_c - 1)$-dimensional space. All of the principal row coordinates are located in the space defined by the standard column coordinates. Distances among row points have meaning, but distances among column points and distances between row and column points are not interpretable.

The option PROFILE=COLUMN can be described as applying the PROFILE=ROW formulas to the transpose of the contingency table. When PROFILE=COLUMN (ROW=DA and COLUMN=DBD), the principal column coordinates $\mathbf{D_c^{-1}BD_u}$ are weighted centroids of the standard row coordinates $\mathbf{D_r^{-1}A}$. Each row point, with coordinates scaled to standard coordinates, defines a vertex in $(n_r - 1)$-dimensional space. All of the principal column coordinates are located in the space defined by the standard row coordinates. Distances among column points have meaning, but distances among row points and distances between row and column points are not interpretable.

The usual sets of coordinates are given by the default PROFILE=BOTH (ROW=DAD and COLUMN=DBD). All of the summary statistics, such as the squared cosines and contributions to inertia, apply to these two sets of points. One advantage to using these coordinates is that both sets ($\mathbf{D_r^{-1}AD_u}$ and $\mathbf{D_c^{-1}BD_u}$) are postmultiplied by the diagonal matrix $\mathbf{D_u}$, which has diagonal values that are all less than or equal to one. When $\mathbf{D_u}$ is a part of the definition of only one set of coordinates, that set forms a tight cluster near the centroid, whereas the other set of points is more widely dispersed. Including $\mathbf{D_u}$ in both sets makes a better graphical display. However, care must be taken in interpreting such a plot. No correct interpretation of distances between row points and column points can be made.

Another property of this choice of coordinates concerns the geometry of distances between points within each set. The default row coordinates can be decomposed into $\mathbf{D_r^{-1}AD_u} = \mathbf{D_r^{-1}AD_uB'D_c^{-1}B} = (\mathbf{D_r^{-1}P})(\mathbf{D_c^{-1/2}})(\mathbf{D_c^{-1/2}B})$. The row coordinates are row profiles ($\mathbf{D_r^{-1}P}$), rescaled by $\mathbf{D_c^{-1/2}}$ (rescaled so that distances between profiles are transformed from a chi-square metric to a Euclidean metric), then orthogonally rotated (with $\mathbf{D_c^{-1/2}B}$) to a principal axes orientation. Similarly, the column coordinates are column profiles rescaled to a Euclidean metric and orthogonally rotated to a principal axes orientation.

The rationale for computing distances between row profiles by using the non-Euclidean chi-square metric is as follows. Each row of the contingency table can be viewed as a realization of a multinomial distribution conditional on its row marginal frequency. The null hypothesis of row and column independence is equivalent to the hypothesis of homogeneity of the row profiles. A significant chi-square statistic is geometrically interpreted as a significant deviation of the row profiles from their centroid, $\mathbf{c}'$. The chi-square metric is the Mahalanobis metric between row profiles based on their estimated covariance matrix under the homogeneity assumption (Greenacre and Hastie 1987). A parallel argument can be made for the column profiles.

When ROW=DAD1/2 and COLUMN=DBD1/2 (Gifi 1990; Van der Heijden and de Leeuw 1985), the row coordinates $\mathbf{D_r^{-1}AD_u^{1/2}}$ and column coordinates $\mathbf{D_c^{-1}BD_u^{1/2}}$ are a decomposition of $\mathbf{D_r^{-1}PD_c^{-1}}$.

In all of the preceding pairs, distances between row and column points are not meaningful. This prompted Carroll, Green, and Schaffer (1986) to propose that row coordinates $\mathbf{D_r^{-1}A(I+D_u)^{1/2}}$ and column coordinates

$\mathbf{D_c^{-1}B(I + D_u)^{1/2}}$ be used. These coordinates are (except for a constant scaling) the coordinates from a multiple correspondence analysis of a Burt table created from two categorical variables. This standardization is available with ROW=DAID1/2 and COLUMN=DBID1/2. However, this approach has been criticized on both theoretical and empirical grounds by Greenacre (1989). The Carroll, Green, and Schaffer standardization relies on the assumption that the chi-square metric is an appropriate metric for measuring the distance between the columns of a bivariate indicator matrix. See the section "Using the TABLES Statement" on page 2215 for a description of indicator matrices. Greenacre (1989) showed that this assumption cannot be justified.

The MCA Option

The MCA option performs a multiple correspondence analysis (MCA). This option requires a Burt table. You can specify the MCA option with a table created from a design matrix with fuzzy coding schemes as long as every row of every partition of the design matrix has the same marginal sum. For example, each row of each partition could contain the probabilities that the observation is a member of each level. Then the Burt table constructed from this matrix no longer contains all integers, and the diagonal partitions are no longer diagonal matrices, but MCA is still valid.

A TABLES statement with a single variable list creates a Burt table. Thus, you can always specify the MCA option with this type of input. If you use the MCA option when reading an existing table with a VAR statement, you must ensure that the table is a Burt table.

If you perform MCA on a table that is not a Burt table, the results of the analysis are invalid. If the table is not symmetric, or if the sums of all elements in each diagonal partition are not equal, PROC CORRESP displays an error message and quits.

A subset of the columns of a Burt table is not necessarily a Burt table, so in MCA it is not appropriate to designate arbitrary columns as supplementary. You can, however, designate all columns from one or more categorical variables as supplementary.

The results of a multiple correspondence analysis of a Burt table $\mathbf{Z}'\mathbf{Z}$ are the same as the column results from a simple correspondence analysis of the binary (or fuzzy) matrix $\mathbf{Z}$. Multiple correspondence analysis is not a simple correspondence analysis of the Burt table. It is not appropriate to perform a simple correspondence analysis of a Burt table. The MCA option is based on $\mathbf{P} = \mathbf{BD_u^2B'}$, whereas a simple correspondence analysis of the Burt table would be based on $\mathbf{P} = \mathbf{BD_uB'}$.

Because the rows and columns of the Burt table are the same, no row information is displayed or written to the output data sets. The resulting inertias and the default (COLUMN=DBD) column coordinates are the appropriate inertias and coordinates for an MCA. The supplementary column coordinates, cosines, and quality of representation formulas for MCA differ from the simple correspondence analysis formulas because the design matrix column profiles and left singular vectors are not available.

The following statements create a Burt table and perform a multiple correspondence analysis:

```
proc corresp data=Neighbor observed short mca;
   tables Hair Height Sex Age;
run;
```

Both the rows and the columns have the same nine categories (Blond, Brown, White, Short, Tall, Female, Male, Old, and Young).

MCA Adjusted Inertias

The usual principal inertias of a Burt table constructed from m categorical variables in MCA are the eigenvalues u_k from $\mathbf{D_u^2}$. The problem with these inertias is that they provide a pessimistic indication of fit. Benzécri (1979) proposed the following inertia adjustment, which is also described by Greenacre (1984, p. 145):

$$\left(\frac{m}{m-1}\right)^2 \times \left(u_k - \frac{1}{m}\right)^2 \qquad \text{for } u_k > \frac{1}{m}$$

This adjustment computes the percent of adjusted inertia relative to the sum of the adjusted inertias for all inertias greater than $\frac{1}{m}$. The Benzécri adjustment is available with the BENZECRI option.

Greenacre (1994, p. 156) argues that the Benzécri adjustment overestimates the quality of fit. Greenacre proposes instead to compute the percentage of adjusted inertia relative to

$$\frac{m}{m-1}\left(\text{trace}(D_u^4) - \frac{n_c - m}{m^2}\right)$$

for all inertias greater than $\frac{1}{m}$, where trace(D_u^4) is the sum of squared inertias. The Greenacre adjustment is available with the GREENACRE option.

Ordinary unadjusted inertias are printed by default with MCA when neither the BENZECRI nor the GREENACRE option is specified. However, the unadjusted inertias are not printed by default when either the BENZECRI or the GREENACRE option is specified. To display both adjusted and unadjusted inertias, specify the UNADJUSTED option in addition to the relevant adjusted inertia option (BENZECRI, GREENACRE, or both).

Supplementary Rows and Columns

Supplementary rows and columns are represented as points in the joint row and column space, but they are not used in determining the locations of the other active rows and columns of the table. The formulas that are used to compute coordinates for the supplementary rows and columns depend on the PROFILE= option or the ROW= and COLUMN= options. Let $\mathbf{S_o}$ be a matrix with rows that contain the supplementary observations, and let $\mathbf{S_v}$ be a matrix with rows that contain the supplementary variables. Note that $\mathbf{S_v}$ is defined to be the transpose of the supplementary variable partition of the table. Let $\mathbf{R_s} = \text{diag}(\mathbf{S_o 1})^{-1}\mathbf{S_o}$ be the supplementary observation profile matrix, and let $\mathbf{C_s} = \text{diag}(\mathbf{S_v 1})^{-1}\mathbf{S_v}$ be the supplementary variable profile matrix. Note that the notation diag$(\cdot)^{-1}$ means to convert the vector to a diagonal matrix, then invert the diagonal matrix. The coordinates for the supplementary observations and variables are shown in Table 35.4.

Table 35.4 Coordinates for Supplementary Observations

ROW=	**Matrix Formula**
A	$\frac{1}{f}\mathbf{S_o D_c^{-1} B D_u^{-1}}$
AD	$\frac{1}{f}\mathbf{S_o D_c^{-1} B}$
DA	$\mathbf{R_s D_c^{-1} B D_u^{-1}}$
DAD	$\mathbf{R_s D_c^{-1} B}$
DAD1/2	$\mathbf{R_s D_c^{-1} B D_u^{-1/2}}$
DAID1/2	$\mathbf{R_s D_c^{-1} B D_u^{-1}(I + D_u)^{1/2}}$
COLUMN=	**Matrix Formula**
B	$\frac{1}{f}\mathbf{S_v D_r^{-1} A D_u^{-1}}$
BD	$\frac{1}{f}\mathbf{S_v D_r^{-1} A}$
DB	$\mathbf{C_s D_r^{-1} A D_u^{-1}}$
DBD	$\mathbf{C_s D_r^{-1} A}$
DBD1/2	$\mathbf{C_s D_r^{-1} A D_u^{-1/2}}$
DBID1/2	$\mathbf{C_s D_r^{-1} A D_u^{-1}(I + D_u)^{1/2}}$
MCA COLUMN=	**Matrix Formula**
B	not allowed
BD	not allowed
DB	$\mathbf{C_s D_r^{-1} B D_u^{-2}}$
DBD	$\mathbf{C_s D_r^{-1} B D_u^{-1}}$
DBD1/2	$\mathbf{C_s D_r^{-1} B D_u^{-3/2}}$
DBID1/2	$\mathbf{C_s D_r^{-1} B D_u^{-2}(I + D_u)^{1/2}}$

Statistics That Aid Interpretation

The partial contributions to inertia, squared cosines, quality of representation, inertia, and mass provide additional information about the coordinates. These statistics are displayed by default. Include the SHORT or NOPRINT option in the PROC CORRESP statement to avoid having these statistics displayed.

These statistics pertain to the default PROFILE=BOTH coordinates, no matter what values you specify for the ROW=, COLUMN=, or PROFILE= option. Let $\mathrm{sq}(\cdot)$ be a matrix-valued function denoting element-wise squaring of the argument matrix. Let t be the total inertia (the sum of the elements in $\mathbf{D_u^2}$).

In MCA, let $\mathbf{D_s}$ be the Burt table partition containing the intersection of the supplementary columns and the supplementary rows. The matrix $\mathbf{D_s}$ is a diagonal matrix of marginal frequencies of the supplemental columns of the binary matrix $\mathbf{Z}$. Let p be the number of rows in this design matrix. The statistics are defined in Table 35.5.

Table 35.5 Statistics That Aid Interpretation

Statistic	Matrix Formula
Row partial contributions to inertia	$\mathbf{D_r^{-1}}\mathrm{sq}(\mathbf{A})$
Column partial contributions to inertia	$\mathbf{D_c^{-1}}\mathrm{sq}(\mathbf{B})$
Row squared cosines	$\mathrm{diag}(\mathrm{sq}(\mathbf{AD_u})\mathbf{1})^{\mathbf{-1}}\mathrm{sq}(\mathbf{AD_u})$
Column squared cosines	$\mathrm{diag}(\mathrm{sq}(\mathbf{BD_u})\mathbf{1})^{\mathbf{-1}}\mathrm{sq}(\mathbf{BD_u})$
Row mass	$\mathbf{r}$
Column mass	$\mathbf{c}$
Row inertia	$\frac{1}{t}\mathbf{D_r^{-1}}\mathrm{sq}(\mathbf{AD_u})\mathbf{1}$
Column inertia	$\frac{1}{t}\mathbf{D_c^{-1}}\mathrm{sq}(\mathbf{BD_u})\mathbf{1}$
Supplementary row squared cosines	$\mathrm{diag}(\mathrm{sq}(\mathbf{R_s} - \mathbf{1c'})\mathbf{D_c^{-1}1})^{\mathbf{-1}}\mathrm{sq}(\mathbf{R_s D_c^{-1} B})$
Supplementary column squared cosines	$\mathrm{diag}(\mathrm{sq}(\mathbf{C_s} - \mathbf{1r'})\mathbf{D_r^{-1}1})^{\mathbf{-1}}\mathrm{sq}(\mathbf{C_s D_r^{-1} A})$
MCA supplementary column squared cosines	$\mathbf{D_s}(p\mathbf{I} - \mathbf{D_s})^{\mathbf{-1}}\mathrm{sq}(\mathbf{C_s D_r^{-1} B D_u^{-1}})$

The quality of representation in the DIMENS=*n* dimensional display of any point is the sum of its squared cosines over only the *n* dimensions. Inertia and mass are not defined for supplementary points.

A table that summarizes the partial contributions to inertia table is also computed. The points that best explain the inertia of each dimension and the dimension to which each point contributes the most inertia are indicated. The output data set variable names for this table are Best1–Best*n* (where DIMENS=*n*) and Best. The Best column contains the dimension number of the largest partial contribution to inertia for each point (the index of the maximum value in each row of $\mathbf{D_r^{-1}}\mathrm{sq}(\mathbf{A})$ or $\mathbf{D_c^{-1}}\mathrm{sq}(\mathbf{B})$).

For each row, the Best1–Best*n* columns contain either the corresponding value of Best, if the point is one of the biggest contributors to the dimension's inertia, or 0 if it is not. Specifically, Best1 contains the value of Best for the point with the largest contribution to dimension one's inertia. A cumulative proportion sum is initialized to this point's partial contribution to the inertia of dimension one. If this sum is less than the value for the MININERTIA= option, then Best1 contains the value of Best for the point with the second-largest contribution to dimension one's inertia. Otherwise, this point's Best1 is 0. This point's partial contribution to inertia is added to the sum. This process continues for the point with the third-largest partial contribution, and so on, until adding a point's contribution to the sum increases the sum beyond the value of the MININERTIA= option. This same algorithm is then used for Best2, and so on.

For example, the following table contains contributions to inertia and the corresponding Best variables. The contribution to inertia variables are proportions that sum to 1 within each column. The first point makes its greatest contribution to the inertia of dimension two, so Best for point one is set to 2, and Best1–Best3 for point one must all be 0 or 2. The second point also makes its greatest contribution to the inertia of dimension two, so Best for point two is set to 2, and Best1–Best3 for point two must all be 0 or 2, and so on.

Assume MININERTIA=0.8, the default. Table 35.6 shows some contributions to inertia. In dimension one, the largest contribution is 0.41302 for the fourth point, so Best1 is set to 1, the value of Best for the fourth point. Because this value is less than 0.8, the second-largest value (0.36456 for point five) is found and its Best1 is set to its Best's value of 1. Because $0.41302 + 0.36456 = 0.77758$ is less than 0.8, the third point (0.0882 at point eight) is found and Best1 is set to 3, because the contribution to dimension three for that point is greater than the contribution to dimension one. This increases the sum of the partial contributions to greater than 0.8, so the remaining Best1 values are all 0.

Table 35.6 Best Statistics

Contr1	Contr2	Contr3	Best1	Best2	Best3	Best
0.01593	0.32178	0.07565	0	2	2	2
0.03014	0.24826	0.07715	0	2	2	2
0.00592	0.02892	0.02698	0	0	0	2
0.41302	0.05191	0.05773	1	0	0	1
0.36456	0.00344	0.15565	1	0	1	1
0.03902	0.30966	0.11717	0	2	2	2
0.00019	0.01840	0.00734	0	0	0	2
0.08820	0.00527	0.16555	3	0	3	3
0.01447	0.00024	0.03851	0	0	0	3
0.02855	0.01213	0.27827	0	0	3	3

Displayed Output

The display options control the amount of displayed output. By default, the following information is displayed:

- an inertia and chi-square decomposition table including the total inertia, the principal inertias of each dimension (eigenvalues), the singular values (square roots of the eigenvalues), each dimension's percentage of inertia, a horizontal bar chart of the percentages, and the total chi-square with its degrees of freedom and decomposition. The chi-square statistics and degrees of freedom are valid only when the constructed table is an ordinary two-way contingency table.
- the coordinates of the rows and columns on the dimensions
- the mass, relative contribution to the total inertia, and quality of representation in the DIMENS=*n* dimensional display of each row and column
- the squared cosines of the angles between each axis and a vector from the origin to the point
- the partial contributions of each point to each dimension's inertia
- the table of indicators of which points best explain the inertia of each dimension

Specific display options and combinations of options display output as follows.

If you specify the OBSERVED or ALL option and you do not specify PRINT=PERCENT, PROC CORRESP displays the following:

- the contingency table, including the row and column marginal frequencies; or with BINARY, the binary table; or the Burt table in MCA
- the supplementary rows
- the supplementary columns

If you specify the OBSERVED or ALL option, with the PRINT=PERCENT or PRINT=BOTH option, PROC CORRESP displays the following:

- the contingency table or Burt table in MCA, scaled to percentages, including the row and column marginal percentages
- the supplementary rows, scaled to percentages
- the supplementary columns, scaled to percentages

If you specify the EXPECTED or ALL option and you do not specify PRINT=PERCENT, PROC CORRESP displays the product of the row marginals and the column marginals divided by the grand frequency of the observed frequency table. For ordinary two-way contingency tables, these are the expected frequencies under the hypothesis of row and column independence.

If you specify the EXPECTED or ALL option with the PRINT=PERCENT or PRINT=BOTH option, PROC CORRESP displays the product of the row marginals and the column marginals divided by the grand frequency of the observed percentages table. For ordinary two-way contingency tables, these are the expected percentages under the hypothesis of row and column independence.

If you specify the DEVIATION or ALL option and you do not specify PRINT=PERCENT, PROC CORRESP displays the observed minus expected frequencies. For ordinary two-way contingency tables, these are the expected frequencies under the hypothesis of row and column independence.

If you specify the DEVIATION or ALL option with the PRINT=PERCENT or PRINT=BOTH option, PROC CORRESP displays the observed minus expected percentages. For ordinary two-way contingency tables, these are the expected percentages under the hypothesis of row and column independence.

If you specify the CELLCHI2 or ALL option and you do not specify PRINT=PERCENT, PROC CORRESP displays contributions to the total chi-square test statistic, including the row and column marginals. The intersection of the marginals contains the total chi-square statistic.

If you specify the CELLCHI2 or ALL option with the PRINT=PERCENT or the PRINT=BOTH option, PROC CORRESP displays contributions to the total chi-square, scaled to percentages, including the row and column marginals.

If you specify the RP or ALL option and you do not specify PRINT=PERCENT, PROC CORRESP displays the row profiles and the supplementary row profiles.

If you specify the RP or ALL option with the PRINT=PERCENT or the PRINT=BOTH option, PROC CORRESP displays the row profiles (scaled to percentages) and the supplementary row profiles (scaled to percentages).

If you specify the CP or ALL option and you do not specify PRINT=PERCENT, PROC CORRESP displays the column profiles and the supplementary column profiles.

If you specify the CP or ALL option with the PRINT=PERCENT or PRINT=BOTH option, PROC CORRESP displays the column profiles (scaled to percentages) and the supplementary column profiles (scaled to percentages).

If you do not specify the NOPRINT option, PROC CORRESP displays the inertia and chi-square decomposition table. This includes the nonzero singular values of the contingency table (or, in MCA, the binary matrix **Z** used to create the Burt table), the nonzero principal inertias (or eigenvalues) for each dimension, the total inertia, the total chi-square, the decomposition of chi-square, the chi-square degrees of freedom (appropriate only when the table is an ordinary two-way contingency table), the percentage of the total chi-square and inertia for each dimension, and a bar chart of the percentages.

If you specify the MCA option and you do not specify the NOPRINT option, PROC CORRESP displays the adjusted inertias. This includes the nonzero adjusted inertias, percentages, cumulative percentages, and a bar chart of the percentages.

If you do not specify the NOROW, NOPRINT, or MCA option, PROC CORRESP displays the row coordinates and the supplementary row coordinates (displayed when there are supplementary row points).

If you do not specify the NOROW, NOPRINT, MCA, or SHORT option, PROC CORRESP displays the following:

- the summary statistics for the row points, including the quality of representation of the row points in the *n*-dimensional display, the mass, and the relative contributions to inertia
- the quality of representation of the supplementary row points in the *n*-dimensional display (displayed when there are supplementary row points)
- the partial contributions to inertia for the row points
- the table of indicators of which row points best explain the inertia of each dimension
- the squared cosines for the row points
- the squared cosines for the supplementary row points (displayed when there are supplementary row points)

If you do not specify the NOCOLUMN or NOPRINT option, PROC CORRESP displays the column coordinates and the supplementary column coordinates (displayed when there are supplementary column points).

If you do not specify the NOCOLUMN, NOPRINT, or SHORT option, PROC CORRESP displays the following:

- the summary statistics for the column points, including the quality of representation of the column points in the *n*-dimensional display, the mass, and the relative contributions to inertia for the supplementary column points
- the quality of representation of the supplementary column points in the *n*-dimensional display (displayed when there are supplementary column points)
- the partial contributions to inertia for the column points

- the table of indicators of which column points best explain the inertia of each dimension
- the squared cosines for the column points
- the squared cosines for the supplementary column points

ODS Table Names

PROC CORRESP assigns a name to each table it creates. You can use these names to reference the table when using the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in Table 35.7 along with the PROC CORRESP statement options needed to produce the table. For more information about ODS, see Chapter 20, "Using the Output Delivery System."

Table 35.7 ODS Tables Produced by PROC CORRESP

ODS Table Name	Description	Option
AdjInGreenacre	Greenacre Inertia Adjustment	GREENACRE
AdjInBenzecri	Benzécri Inertia Adjustment	BENZECRI
Binary	Binary table	OBSERVED, BINARY
BinaryPct	Binary table percentages	OBSERVED, BINARY*
Burt	Burt table	OBSERVED, MCA
BurtPct	Burt table percentages	OBSERVED, MCA*
CellChiSq	Contributions to chi-square	CELLCHI2
CellChiSqPct	Contributions, percentages	CELLCHI2*
ColBest	Col best indicators	default
ColContr	Col contributions to inertia	default
ColCoors	Col coordinates	default
ColProfiles	Col profiles	CP
ColProfilesPct	Col profiles, percentages	CP*
ColQualMassIn	Col quality, mass, inertia	default
ColSqCos	Col squared cosines	default
DF	DF, chi-square (not displayed)	default
Deviations	Observed - expected freqs	DEVIATIONS
DeviationsPct	Observed - expected percentages	DEVIATIONS*
Expected	Expected frequencies	EXPECTED
ExpectedPct	Expected percentages	EXPECTED*
Inertias	Inertia decomposition table	default
Observed	Observed frequencies	OBSERVED
ObservedPct	Observed percentages	OBSERVED*
RowBest	Row best indicators	default
RowContr	Row contributions to inertia	default
RowCoors	Row coordinates	default
RowProfiles	Row profiles	RP
RowProfilesPct	Row profiles, percentages	RP*
RowQualMassIn	Row quality, mass, inertia	default
RowSqCos	Row squared cosines	default
SupColCoors	Supp col coordinates	default

Table 35.7 *continued*

ODS Table Name	Description	Option
SupColProfiles	Supp col profiles	CP
SupColProfilesPct	Supp col profiles, percentages	CP*
SupColQuality	Supp col quality	default
SupCols	Supplementary col freq	OBSERVED
SupColsPct	Supplementary col percentages	OBSERVED*
SupColSqCos	Supp col squared cosines	default
SupRows	Supplementary row freqs	OBSERVED
SupRowCoors	Supp row coordinates	default
SupRowProfiles	Supp row profiles	RP
SupRowProfilesPct	Supp row profiles, percentages	RP*
SupRowQuality	Supp row quality	default
SupRowsPct	Supplementary row percentages	OBSERVED*
SupRowSqCos	Supp row squared cosines	default

*Percentages are displayed when you specify the PRINT=PERCENT or PRINT=BOTH option.

ODS Graphics

Statistical procedures use ODS Graphics to create graphs as part of their output. ODS Graphics is described in detail in Chapter 21, "Statistical Graphics Using ODS."

Before you create graphs, ODS Graphics must be enabled (for example, by specifying the ODS GRAPHICS ON statement). For more information about enabling and disabling ODS Graphics, see the section "Enabling and Disabling ODS Graphics" on page 607 in Chapter 21, "Statistical Graphics Using ODS."

The overall appearance of graphs is controlled by ODS styles. Styles and other aspects of using ODS Graphics are discussed in the section "A Primer on ODS Statistical Graphics" on page 606 in Chapter 21, "Statistical Graphics Using ODS."

You can reference every graph produced through ODS Graphics with a name. The names of the graphs that PROC CORRESP generates are listed in Table 35.8. Graphs are displayed by default when ODS graphics is enabled.

Table 35.8 Graphs Produced by PROC CORRESP

ODS Graph Name	Plot Description
ConfigPlot	Correspondence analysis plot
InertiaChart	Inertia and chi-square decomposition

Examples: CORRESP Procedure

Example 35.1: Simple and Multiple Correspondence Analysis of Automobiles and Their Owners

In this example, PROC CORRESP creates a contingency table from categorical data and performs a simple correspondence analysis. The data are from a sample of individuals who were asked to provide information about themselves and their automobiles. The questions included origin of the automobile (American, Japanese, or European) and family status (single, married, single living with children, or married living with children).

The first steps read the input data and assign formats:

```
title1 'Automobile Owners and Auto Attributes';
title2 'Simple Correspondence Analysis';

proc format;
   value Origin  1 = 'American' 2 = 'Japanese' 3 = 'European';
   value Size    1 = 'Small'    2 = 'Medium'   3 = 'Large';
   value Type    1 = 'Family'   2 = 'Sporty'   3 = 'Work';
   value Home    1 = 'Own'      2 = 'Rent';
   value Sex     1 = 'Male'     2 = 'Female';
   value Income  1 = '1 Income' 2 = '2 Incomes';
   value Marital 1 = 'Single with Kids' 2 = 'Married with Kids'
                 3 = 'Single'           4 = 'Married';
run;

data Cars;
   missing a;
   input (Origin Size Type Home Income Marital Kids Sex) (1.) @@;
   * Check for End of Line;
   if n(of Origin -- Sex) eq 0 then do; input; return; end;
   marital = 2 * (kids le 0) + marital;
   format Origin Origin. Size Size. Type Type. Home Home.
          Sex Sex. Income Income. Marital Marital.;
   output;
   datalines;
13111221212111012111220113121101121122112211212113112212321122221221220 1
12112202312122123221110112212202212111012211210213111221112111011231110 1
21111211321122312112220222112211131112313121110232112222322122022122110 1
12212202212122021121220122112202112211013211220221311211133122612222110 1

   ... more lines ...

21212201121112213122110112121102221222021212110 1
;
```

PROC CORRESP is used to perform the simple correspondence analysis. The ALL option displays all tables, including the contingency table, chi-square information, profiles, and all results of the correspondence analysis. The CHI2P option displays the chi-square *p*-value. The TABLES statement specifies the row and column categorical variables. The results are displayed with ODS Graphics.

The following statements produce Output 35.1.1:

```
ods graphics on;

* Perform Simple Correspondence Analysis;
proc corresp data=Cars all chi2p;
   tables Marital, Origin;
run;
```

Correspondence analysis locates all the categories in a Euclidean space. The first two dimensions of this space are plotted to examine the associations among the categories.[3] Because the smallest dimension of this table is three, there is no loss of information when only two dimensions are plotted. The plot should be thought of as two different overlaid plots, one for each categorical variable. Distances between points within a variable have meaning, but distances between points from different variables do not.

Output 35.1.1 Simple Correspondence Analysis

Automobile Owners and Auto Attributes
Simple Correspondence Analysis

The CORRESP Procedure

Contingency Table

	American	European	Japanese	Sum
Married	37	14	51	102
Married with Kids	52	15	44	111
Single	33	15	63	111
Single with Kids	6	1	8	15
Sum	128	45	166	339

Chi-Square Statistic Expected Values

	American	European	Japanese
Married	38.5133	13.5398	49.9469
Married with Kids	41.9115	14.7345	54.3540
Single	41.9115	14.7345	54.3540
Single with Kids	5.6637	1.9912	7.3451

Observed Minus Expected Values

	American	European	Japanese
Married	-1.5133	0.4602	1.0531
Married with Kids	10.0885	0.2655	-10.3540
Single	-8.9115	0.2655	8.6460
Single with Kids	0.3363	-0.9912	0.6549

[3] In this analysis, the chi-square statistic is not significantly different from 0. Hence, you would not reject the null hypothesis that the rows and columns are independent. If your goal is hypothesis testing, you might stop at this point and not proceed to interpret the graphical and tabular results. If your goal is exploratory data analysis, you might proceed and interpret the results. This example will proceed, because it is intended to be a small, simple teaching example.

Output 35.1.1 *continued*

Contributions to the Total Chi-Square Statistic				
	American	European	Japanese	Sum
Married	0.05946	0.01564	0.02220	0.09730
Married with Kids	2.42840	0.00478	1.97235	4.40553
Single	1.89482	0.00478	1.37531	3.27492
Single with Kids	0.01997	0.49337	0.05839	0.57173
Sum	4.40265	0.51858	3.42825	8.34947

Row Profiles			
	American	European	Japanese
Married	0.362745	0.137255	0.500000
Married with Kids	0.468468	0.135135	0.396396
Single	0.297297	0.135135	0.567568
Single with Kids	0.400000	0.066667	0.533333

Column Profiles			
	American	European	Japanese
Married	0.289063	0.311111	0.307229
Married with Kids	0.406250	0.333333	0.265060
Single	0.257813	0.333333	0.379518
Single with Kids	0.046875	0.022222	0.048193

Inertia and Chi-Square Decomposition

Singular Value	Principal Inertia	Chi-Square	Percent	Cumulative Percent	0 20 40 60 80
0.15122	0.02287	7.7516	92.84	92.84	
0.04200	0.00176	0.5979	7.16	100.00	
	0.02463	8.3495	100.00		

Pr > Chisq = .2136 DF = 6

Row Coordinates		
	Dim1	Dim2
Married	-0.0278	0.0134
Married with Kids	0.1991	0.0064
Single	-0.1716	0.0076
Single with Kids	-0.0144	-0.1947

Summary Statistics for the Row Points			
	Quality	Mass	Inertia
Married	1.0000	0.3009	0.0117
Married with Kids	1.0000	0.3274	0.5276
Single	1.0000	0.3274	0.3922
Single with Kids	1.0000	0.0442	0.0685

Partial Contributions to Inertia for the Row Points

	Dim1	Dim2
Married	0.0102	0.0306
Married with Kids	0.5678	0.0076
Single	0.4217	0.0108
Single with Kids	0.0004	0.9511

Indices of the Coordinates That Contribute Most to Inertia for the Row Points

	Dim1	Dim2	Best
Married	0	0	2
Married with Kids	1	0	1
Single	1	0	1
Single with Kids	0	2	2

Squared Cosines for the Row Points

	Dim1	Dim2
Married	0.8121	0.1879
Married with Kids	0.9990	0.0010
Single	0.9980	0.0020
Single with Kids	0.0054	0.9946

Column Coordinates

	Dim1	Dim2
American	0.1847	-0.0166
European	0.0013	0.1073
Japanese	-0.1428	-0.0163

Summary Statistics for the Column Points

	Quality	Mass	Inertia
American	1.0000	0.3776	0.5273
European	1.0000	0.1327	0.0621
Japanese	1.0000	0.4897	0.4106

Partial Contributions to Inertia for the Column Points

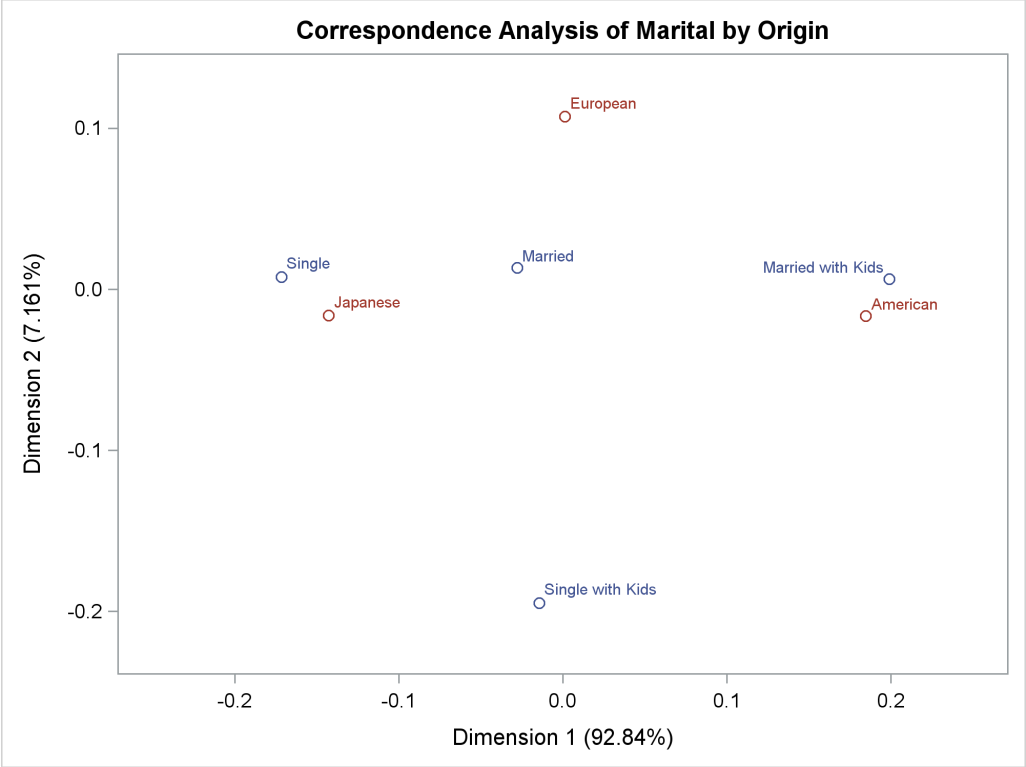
	Dim1	Dim2
American	0.5634	0.0590
European	0.0000	0.8672
Japanese	0.4366	0.0737

Indices of the Coordinates That Contribute Most to Inertia for the Column Points

	Dim1	Dim2	Best
American	1	0	1
European	0	2	2
Japanese	1	0	1

Squared Cosines for the Column Points

	Dim1	Dim2
American	0.9920	0.0080
European	0.0001	0.9999
Japanese	0.9871	0.0129



To interpret the plot, start by interpreting the row points separately from the column points. The European point is near and to the left of the centroid, so it makes a relatively small contribution to the chi-square statistic (because it is near the centroid), it contributes almost nothing to the inertia of dimension one (because its coordinate on dimension one has a small absolute value relative to the other column points), and it makes a relatively large contribution to the inertia of dimension two (because its coordinate on dimension two has a large absolute value relative to the other column points). Its squared cosines for dimension one and two, approximately 0 and 1, respectively, indicate that its position is almost completely determined by its location on dimension two. Its quality of display is 1.0, indicating perfect quality, because the table is two-dimensional after the centering. The American and Japanese points are far from the centroid, and they lie along dimension one. They make relatively large contributions to the chi-square statistic and the inertia of dimension one. The horizontal dimension seems to be largely determined by Japanese versus American automobile ownership.

In the row points, the Married point is near the centroid, and the Single with Kids point has a small coordinate on dimension one that is near zero. The horizontal dimension seems to be largely determined by the Single versus the Married with Kids points. The two interpretations of dimension one show the association with being Married with Kids and owning an American auto, and being single and owning a Japanese auto. The fact that the Married with Kids point is close to the American point and the fact that the Japanese point is near the Single point should be ignored. Distances between row and column points are not defined. The plot shows that more people who are married with kids than you would expect if the rows and columns were independent drive an American auto, and more people who are single than you would expect if the rows and columns were independent drive a Japanese auto.

In the second part of this example, PROC CORRESP creates a Burt table from categorical data and performs a multiple correspondence analysis. The variables used in this example are Origin, Size, Type, Income, Home, Marital, and Sex. MCA specifies multiple correspondence analysis, OBSERVED displays the Burt table. The TABLES statement with only a single variable list and no comma creates the Burt table.

The following statements produce Output 35.1.2:

```
title2 'Multiple Correspondence Analysis';

* Perform Multiple Correspondence Analysis;
proc corresp mca observed data=Cars;
   tables Origin Size Type Income Home Marital Sex;
run;
```

Output 35.1.2 Multiple Correspondence Analysis

Automobile Owners and Auto Attributes
Multiple Correspondence Analysis

The CORRESP Procedure

Burt Table

	American	European	Japanese	Large	Medium	Small	Family	Sporty	Work
American	125	0	0	36	60	29	81	24	20
European	0	44	0	4	20	20	17	23	4
Japanese	0	0	165	2	61	102	76	59	30
Large	36	4	2	42	0	0	30	1	11
Medium	60	20	61	0	141	0	89	39	13
Small	29	20	102	0	0	151	55	66	30
Family	81	17	76	30	89	55	174	0	0
Sporty	24	23	59	1	39	66	0	106	0
Work	20	4	30	11	13	30	0	0	54
1 Income	58	18	74	20	57	73	69	55	26
2 Incomes	67	26	91	22	84	78	105	51	28
Own	93	38	111	35	106	101	130	71	41
Rent	32	6	54	7	35	50	44	35	13
Married	37	13	51	9	42	50	50	35	16
Married with Kids	50	15	44	21	51	37	79	12	18
Single	32	15	62	11	40	58	35	57	17
Single with Kids	6	1	8	1	8	6	10	2	3
Female	58	21	70	17	70	62	83	44	22
Male	67	23	95	25	71	89	91	62	32

Output 35.1.2 *continued*

Automobile Owners and Auto Attributes
Multiple Correspondence Analysis

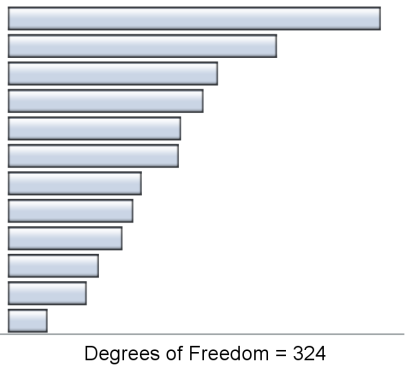
The CORRESP Procedure

Burt Table

	1 Income	2 Incomes	Own	Rent	Married	Married with Kids	Single	Single with Kids	Female	Male
American	58	67	93	32	37	50	32	6	58	67
European	18	26	38	6	13	15	15	1	21	23
Japanese	74	91	111	54	51	44	62	8	70	95
Large	20	22	35	7	9	21	11	1	17	25
Medium	57	84	106	35	42	51	40	8	70	71
Small	73	78	101	50	50	37	58	6	62	89
Family	69	105	130	44	50	79	35	10	83	91
Sporty	55	51	71	35	35	12	57	2	44	62
Work	26	28	41	13	16	18	17	3	22	32
1 Income	150	0	80	70	10	27	99	14	47	103
2 Incomes	0	184	162	22	91	82	10	1	102	82
Own	80	162	242	0	76	106	52	8	114	128
Rent	70	22	0	92	25	3	57	7	35	57
Married	10	91	76	25	101	0	0	0	53	48
Married with Kids	27	82	106	3	0	109	0	0	48	61
Single	99	10	52	57	0	0	109	0	35	74
Single with Kids	14	1	8	7	0	0	0	15	13	2
Female	47	102	114	35	53	48	35	13	149	0
Male	103	82	128	57	48	61	74	2	0	185

Inertia and Chi-Square Decomposition

Singular Value	Principal Inertia	Chi-Square	Percent	Cumulative Percent
0.56934	0.32415	970.77	18.91	18.91
0.48352	0.23380	700.17	13.64	32.55
0.42716	0.18247	546.45	10.64	43.19
0.41215	0.16987	508.73	9.91	53.10
0.38773	0.15033	450.22	8.77	61.87
0.38520	0.14838	444.35	8.66	70.52
0.34066	0.11605	347.55	6.77	77.29
0.32983	0.10879	325.79	6.35	83.64
0.31517	0.09933	297.47	5.79	89.43
0.28069	0.07879	235.95	4.60	94.03
0.26115	0.06820	204.24	3.98	98.01
0.18477	0.03414	102.24	1.99	100.00
	1.71429	5133.92	100.00	



Column Coordinates		
	Dim1	Dim2
American	-0.4035	0.8129
European	-0.0568	-0.5552
Japanese	0.3208	-0.4678
Large	-0.6949	1.5666
Medium	-0.2562	0.0965
Small	0.4326	-0.5258
Family	-0.4201	0.3602
Sporty	0.6604	-0.6696
Work	0.0575	0.1539
1 Income	0.8251	0.5472
2 Incomes	-0.6727	-0.4461
Own	-0.3887	-0.0943
Rent	1.0225	0.2480
Married	-0.4169	-0.7954
Married with Kids	-0.8200	0.3237
Single	1.1461	0.2930
Single with Kids	0.4373	0.8736
Female	-0.3365	-0.2057
Male	0.2710	0.1656

Summary Statistics for the Column Points			
	Quality	Mass	Inertia
American	0.4925	0.0535	0.0521
European	0.0473	0.0188	0.0724
Japanese	0.3141	0.0706	0.0422
Large	0.4224	0.0180	0.0729
Medium	0.0548	0.0603	0.0482
Small	0.3825	0.0646	0.0457
Family	0.3330	0.0744	0.0399
Sporty	0.4112	0.0453	0.0569
Work	0.0052	0.0231	0.0699
1 Income	0.7991	0.0642	0.0459
2 Incomes	0.7991	0.0787	0.0374
Own	0.4208	0.1035	0.0230
Rent	0.4208	0.0393	0.0604
Married	0.3496	0.0432	0.0581
Married with Kids	0.3765	0.0466	0.0561
Single	0.6780	0.0466	0.0561
Single with Kids	0.0449	0.0064	0.0796
Female	0.1253	0.0637	0.0462
Male	0.1253	0.0791	0.0372

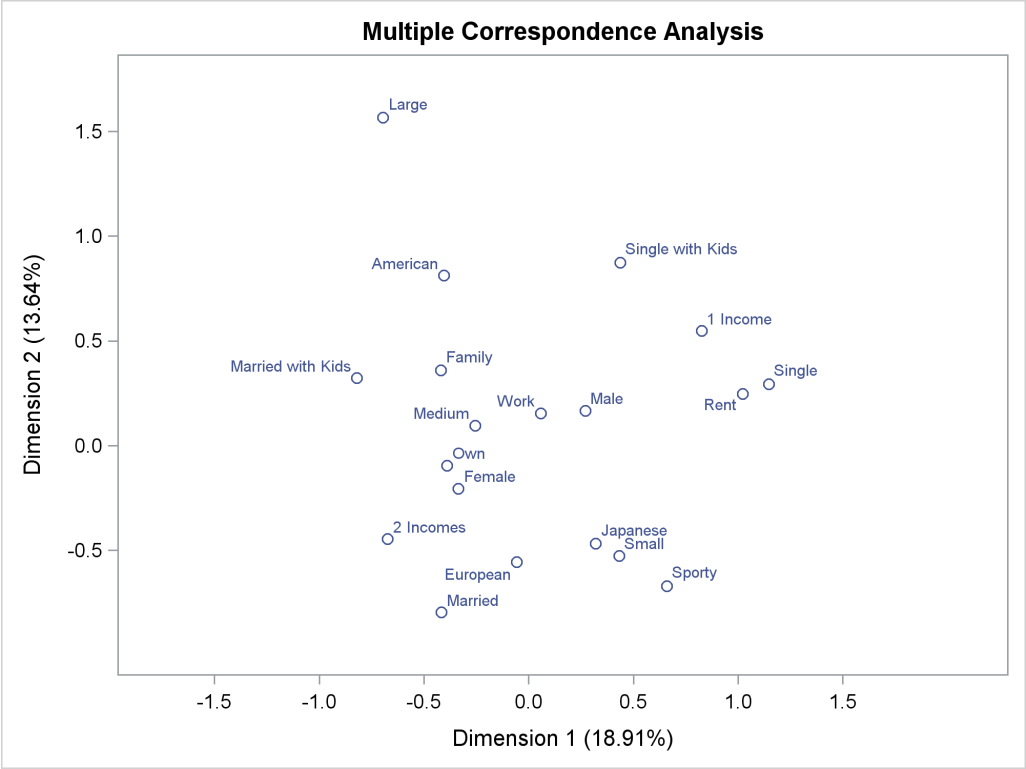
Partial Contributions to Inertia for the Column Points

	Dim1	Dim2
American	0.0268	0.1511
European	0.0002	0.0248
Japanese	0.0224	0.0660
Large	0.0268	0.1886
Medium	0.0122	0.0024
Small	0.0373	0.0764
Family	0.0405	0.0413
Sporty	0.0610	0.0870
Work	0.0002	0.0023
1 Income	0.1348	0.0822
2 Incomes	0.1099	0.0670
Own	0.0482	0.0039
Rent	0.1269	0.0103
Married	0.0232	0.1169
Married with Kids	0.0967	0.0209
Single	0.1889	0.0171
Single with Kids	0.0038	0.0209
Female	0.0223	0.0115
Male	0.0179	0.0093

Indices of the Coordinates That Contribute Most to Inertia for the Column Points

	Dim1	Dim2	Best
American	0	2	2
European	0	0	2
Japanese	0	2	2
Large	0	2	2
Medium	0	0	1
Small	0	2	2
Family	2	0	2
Sporty	2	2	2
Work	0	0	2
1 Income	1	1	1
2 Incomes	1	1	1
Own	1	0	1
Rent	1	0	1
Married	0	2	2
Married with Kids	1	0	1
Single	1	0	1
Single with Kids	0	0	2
Female	0	0	1
Male	0	0	1

Squared Cosines for the Column Points		
	Dim1	**Dim2**
American	0.0974	0.3952
European	0.0005	0.0468
Japanese	0.1005	0.2136
Large	0.0695	0.3530
Medium	0.0480	0.0068
Small	0.1544	0.2281
Family	0.1919	0.1411
Sporty	0.2027	0.2085
Work	0.0006	0.0046
1 Income	0.5550	0.2441
2 Incomes	0.5550	0.2441
Own	0.3975	0.0234
Rent	0.3975	0.0234
Married	0.0753	0.2742
Married with Kids	0.3258	0.0508
Single	0.6364	0.0416
Single with Kids	0.0090	0.0359
Female	0.0912	0.0341
Male	0.0912	0.0341



Multiple correspondence analysis locates all the categories in a Euclidean space. The first two dimensions of this space are plotted to examine the associations among the categories. The top-right quadrant of the plot

shows that the categories Single, Single with Kids, 1 Income, and Rent are associated. Proceeding clockwise, the categories Sporty, Small, and Japanese are associated. The bottom-left quadrant shows the association between being married, owning your own home, and having two incomes. Having children is associated with owning a large American family auto. Such information could be used in market research to identify target audiences for advertisements.

This interpretation is based on points found in approximately the same direction from the origin and in approximately the same region of the space. Distances between points do not have a straightforward interpretation in multiple correspondence analysis. The geometry of multiple correspondence analysis is not a simple generalization of the geometry of simple correspondence analysis (Greenacre and Hastie 1987; Greenacre 1988).

If you want to perform a multiple correspondence analysis and get scores for the individuals, you can specify the BINARY option to analyze the binary table, as in the following statements. In the interest of space, only the first 10 rows of coordinates are printed in Output 35.1.3.

```
title2 'Binary Table';

* Perform Multiple Correspondence Analysis;
proc corresp data=Cars binary;
   ods select RowCoors;
   tables Origin Size Type Income Home Marital Sex;
run;
```

Output 35.1.3 Correspondence Analysis of a Binary Table

Automobile Owners and Auto Attributes
Binary Table

The CORRESP Procedure

Row Coordinates

	Dim1	Dim2
1	-0.4093	1.0878
2	0.8198	-0.2221
3	-0.2193	-0.5328
4	0.4382	1.1799
5	-0.6750	0.3600
6	-0.1778	0.1441
7	-0.9375	0.6846
8	-0.7405	-0.1539
9	-0.3027	-0.2749
10	-0.7263	-0.0803

Example 35.2: Simple Correspondence Analysis of U.S. Population

In this example, PROC CORRESP reads an existing contingency table with supplementary observations and performs a simple correspondence analysis. The data are populations of the 50 U.S. states, grouped into regions, for each of the census years from 1920 to 1970 (US Bureau of the Census 1979). Alaska and Hawaii

are treated as supplementary regions, because they were not states during this entire period and are not physically connected to the other 48 states. Consequently, it is reasonable to expect that population changes in these two states operate differently from population changes in the other states. The correspondence analysis is performed giving the supplementary points negative weight, and then the coordinates for the supplementary points are computed in the solution defined by the other points.

The initial DATA step reads the table, provides labels for the years, flags the supplementary rows with negative weights, and specifies absolute weights of 1000 for all observations because the data were originally reported in units of 1000 people.

In the PROC CORRESP statement, PRINT=PERCENT and the display options display the table of cell percentages (OBSERVED), cell contributions to the total chi-square scaled to sum to 100 (CELLCHI2), row profile rows that sum to 100 (RP), and column profile columns that sum to 100 (CP). The SHORT option specifies that the correspondence analysis summary statistics, contributions to inertia, and squared cosines should not be displayed. Because the data are already in table form, a VAR statement is used to read the table. Row labels are specified with the ID statement, and column labels come from the variable labels. The WEIGHT statement flags the supplementary observations and restores the table values to populations.

The following statements produce Output 35.2.1:

```
title 'United States Population, 1920-1970';

data USPop;

   * Regions:
   * New England     - ME, NH, VT, MA, RI, CT.
   * Great Lakes     - OH, IN, IL, MI, WI.
   * South Atlantic  - DE, MD, DC, VA, WV, NC, SC, GA, FL.
   * Mountain        - MT, ID, WY, CO, NM, AZ, UT, NV.
   * Pacific         - WA, OR, CA.
   *
   * Note: Multiply data values by 1000 to get populations.;

   input Region $14. y1920 y1930 y1940 y1950 y1960 y1970;

   label y1920 = '1920'    y1930 = '1930'    y1940 = '1940'
         y1950 = '1950'    y1960 = '1960'    y1970 = '1970';

   if region = 'Hawaii' or region = 'Alaska'
      then w = -1000;       /* Flag Supplementary Observations */
      else w =  1000;

   datalines;
New England        7401  8166  8437  9314 10509 11842
NY, NJ, PA        22261 26261 27539 30146 34168 37199
Great Lakes       21476 25297 26626 30399 36225 40252
Midwest           12544 13297 13517 14061 15394 16319
South Atlantic    13990 15794 17823 21182 25972 30671
KY, TN, AL, MS     8893  9887 10778 11447 12050 12803
AR, LA, OK, TX    10242 12177 13065 14538 16951 19321
Mountain           3336  3702  4150  5075  6855  8282
Pacific            5567  8195  9733 14486 20339 25454
Alaska               55    59    73   129   226   300
Hawaii              256   368   423   500   633   769
;
```

```
ods graphics on;

* Perform Simple Correspondence Analysis;
proc corresp data=uspop print=percent observed cellchi2 rp cp chi2p
     short plot(flip);
   var y1920 -- y1970;
   id Region;
   weight w;
run;
```

The contingency table shows that the population of all regions increased over this time period. The row profiles show that population increased at a different rate for the different regions. There was a small increase in population in the Midwest, for example, but the population more than quadrupled in the Pacific region over the same period. The column profiles show that in 1920, the U.S. population was concentrated in the NY, NJ, PA, Great Lakes, Midwest, and South Atlantic regions. With time, the population shifted more to the South Atlantic, Mountain, and Pacific regions. This is also clear from the correspondence analysis. The inertia and chi-square decomposition table shows that there are five nontrivial dimensions in the table, but the association between the rows and columns is almost entirely one-dimensional.

Output 35.2.1 United States Population, 1920–1970

United States Population, 1920-1970

The CORRESP Procedure

Contingency Table

Percents	1920	1930	1940	1950	1960	1970	Sum
New England	0.830	0.916	0.946	1.045	1.179	1.328	6.245
NY, NJ, PA	2.497	2.946	3.089	3.382	3.833	4.173	19.921
Great Lakes	2.409	2.838	2.987	3.410	4.064	4.516	20.224
Midwest	1.407	1.492	1.516	1.577	1.727	1.831	9.550
South Atlantic	1.569	1.772	1.999	2.376	2.914	3.441	14.071
KY, TN, AL, MS	0.998	1.109	1.209	1.284	1.352	1.436	7.388
AR, LA, OK, TX	1.149	1.366	1.466	1.631	1.902	2.167	9.681
Mountain	0.374	0.415	0.466	0.569	0.769	0.929	3.523
Pacific	0.625	0.919	1.092	1.625	2.282	2.855	9.398
Sum	11.859	13.773	14.771	16.900	20.020	22.677	100.000

Supplementary Rows

Percents	1920	1930	1940	1950	1960	1970
Alaska	0.006170	0.006619	0.008189	0.014471	0.025353	0.033655
Hawaii	0.028719	0.041283	0.047453	0.056091	0.071011	0.086268

Output 35.2.1 *continued*

Contributions to the Total Chi-Square Statistic

Percents	1920	1930	1940	1950	1960	1970	Sum
New England	0.937	0.314	0.054	0.009	0.352	0.469	2.135
NY, NJ, PA	0.665	1.287	0.633	0.006	0.521	2.265	5.378
Great Lakes	0.004	0.085	0.000	0.001	0.005	0.094	0.189
Midwest	5.749	2.039	0.684	0.072	1.546	4.472	14.563
South Atlantic	0.509	1.231	0.259	0.000	0.285	1.688	3.973
KY, TN, AL, MS	1.454	0.711	1.098	0.087	0.946	2.945	7.242
AR, LA, OK, TX	0.000	0.069	0.077	0.001	0.059	0.030	0.238
Mountain	0.391	0.868	0.497	0.098	0.498	1.834	4.187
Pacific	18.591	9.380	5.458	0.074	7.346	21.248	62.096
Sum	28.302	15.986	8.761	0.349	11.558	35.046	100.000

Row Profiles

Percents	1920	1930	1940	1950	1960	1970
New England	13.2947	14.6688	15.1557	16.7310	18.8777	21.2722
NY, NJ, PA	12.5362	14.7888	15.5085	16.9766	19.2416	20.9484
Great Lakes	11.9129	14.0325	14.7697	16.8626	20.0943	22.3281
Midwest	14.7348	15.6193	15.8777	16.5167	18.0825	19.1691
South Atlantic	11.1535	12.5917	14.2093	16.8872	20.7060	24.4523
KY, TN, AL, MS	13.5033	15.0126	16.3655	17.3813	18.2969	19.4403
AR, LA, OK, TX	11.8687	14.1111	15.1401	16.8471	19.6433	22.3897
Mountain	10.6242	11.7898	13.2166	16.1624	21.8312	26.3758
Pacific	6.6453	9.7823	11.6182	17.2918	24.2784	30.3841

Supplementary Row Profiles

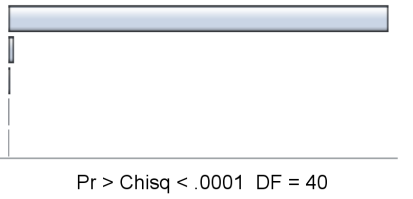
Percents	1920	1930	1940	1950	1960	1970
Alaska	6.5321	7.0071	8.6698	15.3207	26.8409	35.6295
Hawaii	8.6809	12.4788	14.3438	16.9549	21.4649	26.0766

Column Profiles

Percents	1920	1930	1940	1950	1960	1970
New England	7.0012	6.6511	6.4078	6.1826	5.8886	5.8582
NY, NJ, PA	21.0586	21.3894	20.9155	20.0109	19.1457	18.4023
Great Lakes	20.3160	20.6042	20.2221	20.1788	20.2983	19.9126
Midwest	11.8664	10.8303	10.2660	9.3337	8.6259	8.0730
South Atlantic	13.2343	12.8641	13.5363	14.0606	14.5532	15.1729
KY, TN, AL, MS	8.4126	8.0529	8.1857	7.5985	6.7521	6.3336
AR, LA, OK, TX	9.6888	9.9181	9.9227	9.6503	9.4983	9.5581
Mountain	3.1558	3.0152	3.1519	3.3688	3.8411	4.0971
Pacific	5.2663	6.6748	7.3921	9.6158	11.3968	12.5921

Inertia and Chi-Square Decomposition

Singular Value	Principal Inertia	Chi-Square	Percent	Cumulative Percent
0.10664	0.01137	10137829	98.16	98.16
0.01238	0.00015	136586	1.32	99.48
0.00658	0.00004	38540	0.37	99.85
0.00333	0.00001	9897	0.10	99.95
0.00244	0.00001	5310	0.05	100.00
	0.01159	10328162	100.00	



Row Coordinates

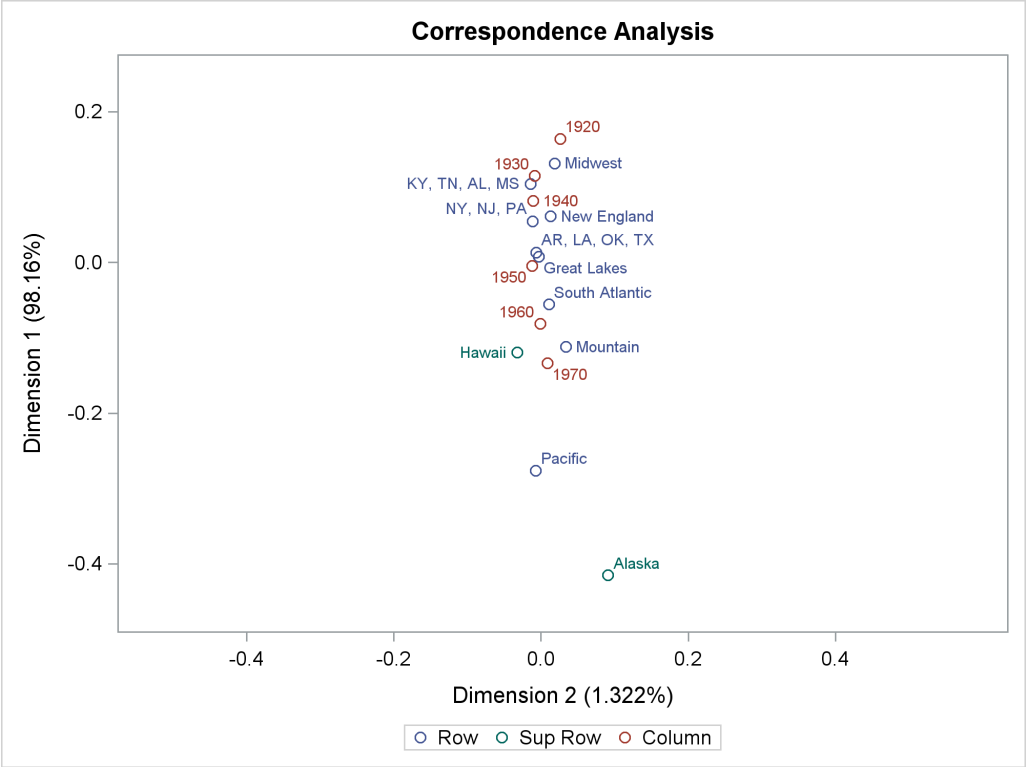
	Dim1	Dim2
New England	0.0611	0.0132
NY, NJ, PA	0.0546	-0.0117
Great Lakes	0.0074	-0.0028
Midwest	0.1315	0.0186
South Atlantic	-0.0553	0.0105
KY, TN, AL, MS	0.1044	-0.0144
AR, LA, OK, TX	0.0131	-0.0067
Mountain	-0.1121	0.0338
Pacific	-0.2766	-0.0070

Supplementary Row Coordinates

	Dim1	Dim2
Alaska	-0.4152	0.0912
Hawaii	-0.1198	-0.0321

Column Coordinates

	Dim1	Dim2
1920	0.1642	0.0263
1930	0.1149	-0.0089
1940	0.0816	-0.0108
1950	-0.0046	-0.0125
1960	-0.0815	-0.0007
1970	-0.1335	0.0086



ODS Graphics is used to plot the results. The results are essentially one-dimensional. For results such as these, it is better to plot the first dimension vertically, as opposed to the default, which is horizontally. The vertical orientation has fewer opportunities for label collisions. Specifying PLOTS(FLIP) on the PROC CORRESP statement switches the vertical and horizontal axes to improve the graphical display.

The plot shows that the first dimension correctly orders the years. There is nothing in the correspondence analysis that forces this to happen; the analysis has no information about the inherent ordering of the column categories. The ordering of the regions and the ordering of the years reflect the shift over time of the U.S. population from the Northeast quadrant of the country to the South and to the West. The results show that the West and Southeast grew faster than the rest of the contiguous 48 states during this period.

The plot also shows that the growth pattern for Hawaii was similar to the growth pattern for the mountain states and that Alaska's growth was even more extreme than the Pacific states' growth. The row profiles confirm this interpretation.

The Pacific region is farther from the origin than all other active points. The Midwest is the extreme region in the other direction. The table of contributions to the total chi-square shows that 62% of the total chi-square statistic is contributed by the Pacific region, which is followed by the Midwest at over 14%. Similarly the two extreme years, 1920 and 1970, together contribute over 63% to the total chi-square, whereas the years nearer the origin of the plot contribute less.

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