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# **SAS/STAT<sup>®</sup> 12.1 User's Guide The LOGISTIC Procedure (Chapter)**



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# Chapter 54

## The LOGISTIC Procedure

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## Overview: LOGISTIC Procedure

Binary responses (for example, success and failure), ordinal responses (for example, normal, mild, and severe), and nominal responses (for example, major TV networks viewed at a certain hour) arise in many fields of study. Logistic regression analysis is often used to investigate the relationship between these discrete responses and a set of explanatory variables. Texts that discuss logistic regression include Agresti (2002); Allison (1999); Collett (2003); Cox and Snell (1989); Hosmer and Lemeshow (2000); Stokes, Davis, and Koch (2012).

For binary response models, the response,  $Y$ , of an individual or an experimental unit can take on one of two possible values, denoted for convenience by 1 and 2 (for example,  $Y = 1$  if a disease is present, otherwise  $Y = 2$ ). Suppose  $\mathbf{x}$  is a vector of explanatory variables and  $\pi = \Pr(Y = 1 \mid \mathbf{x})$  is the response probability to be modeled. The linear logistic model has the form

$$\text{logit}(\pi) \equiv \log\left(\frac{\pi}{1 - \pi}\right) = \alpha + \boldsymbol{\beta}'\mathbf{x}$$

where  $\alpha$  is the intercept parameter and  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_s)'$  is the vector of  $s$  slope parameters. Notice that the LOGISTIC procedure, by default, models the probability of the *lower* response levels.

The logistic model shares a common feature with a more general class of linear models: a function  $g = g(\mu)$  of the mean of the response variable is assumed to be linearly related to the explanatory variables. Since the mean  $\mu$  implicitly depends on the stochastic behavior of the response, and the explanatory variables are assumed to be fixed, the function  $g$  provides the link between the random (stochastic) component and the systematic (deterministic) component of the response variable  $Y$ . For this reason, Nelder and Wedderburn (1972) refer to  $g(\mu)$  as a link function. One advantage of the logit function over other link functions is that differences on the logistic scale are interpretable regardless of whether the data are sampled prospectively or retrospectively (McCullagh and Nelder 1989, Chapter 4). Other link functions that are widely used in practice are the probit function and the complementary log-log function. The LOGISTIC procedure enables you to choose one of these link functions, resulting in fitting a broader class of binary response models of the form

$$g(\pi) = \alpha + \boldsymbol{\beta}'\mathbf{x}$$

For ordinal response models, the response,  $Y$ , of an individual or an experimental unit might be restricted to one of a (usually small) number of ordinal values, denoted for convenience by  $1, \dots, k, k + 1$ . For example, the severity of coronary disease can be classified into three response categories as 1=no disease, 2=angina pectoris, and 3=myocardial infarction. The LOGISTIC procedure fits a common slopes cumulative model, which is a parallel lines regression model based on the cumulative probabilities of the response categories rather than on their individual probabilities. The cumulative model has the form

$$g(\Pr(Y \leq i \mid \mathbf{x})) = \alpha_i + \boldsymbol{\beta}'\mathbf{x}, \quad i = 1, \dots, k$$

where  $\alpha_1, \dots, \alpha_k$  are  $k$  intercept parameters, and  $\boldsymbol{\beta}$  is the vector of slope parameters. This model has been considered by many researchers. Aitchison and Silvey (1957) and Ashford (1959) employ a probit scale and provide a maximum likelihood analysis; Walker and Duncan (1967) and Cox and Snell (1989) discuss the use of the log odds scale. For the log odds scale, the cumulative logit model is often referred to as the *proportional odds* model.

For nominal response logistic models, where the  $k + 1$  possible responses have no natural ordering, the logit model can also be extended to a *multinomial* model known as a *generalized* or *baseline-category* logit model, which has the form

$$\log \left( \frac{\Pr(Y = i \mid \mathbf{x})}{\Pr(Y = k + 1 \mid \mathbf{x})} \right) = \alpha_i + \boldsymbol{\beta}_i' \mathbf{x}, \quad i = 1, \dots, k$$

where the  $\alpha_1, \dots, \alpha_k$  are  $k$  intercept parameters, and the  $\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_k$  are  $k$  vectors of slope parameters. These models are a special case of the *discrete choice* or *conditional logit* models introduced by McFadden (1974).

The LOGISTIC procedure fits linear logistic regression models for discrete response data by the method of maximum likelihood. It can also perform conditional logistic regression for binary response data and exact logistic regression for binary and nominal response data. The maximum likelihood estimation is carried out with either the Fisher scoring algorithm or the Newton-Raphson algorithm, and you can perform the bias-reducing penalized likelihood optimization as discussed by Firth (1993) and Heinze and Schemper (2002). You can specify starting values for the parameter estimates. The logit link function in the logistic regression models can be replaced by the probit function, the complementary log-log function, or the generalized logit function.

Any term specified in the model is referred to as an *effect*. The LOGISTIC procedure enables you to specify categorical variables (also known as *classification* or *CLASS variables*) and continuous variables as explanatory effects. You can also specify more complex model terms such as interactions and nested terms in the same way as in the GLM procedure. You can create complex *constructed effects* with the **EFFECT** statement. An effect in the model that is not an interaction or a nested term or a constructed effect is referred to as a *main effect*.

The LOGISTIC procedure allows either a full-rank parameterization or a less-than-full-rank parameterization of the CLASS variables. The full-rank parameterization offers eight coding methods: effect, reference, ordinal, polynomial, and orthogonalizations of these. The effect coding is the same method that is used in the CATMOD procedure. The less-than-full-rank parameterization, often called *dummy coding*, is the same coding as that used in the GLM procedure.

The LOGISTIC procedure provides four effect selection methods: forward selection, backward elimination, stepwise selection, and best subset selection. The best subset selection is based on the likelihood score statistic. This method identifies a specified number of best models containing one, two, three effects, and so on, up to a single model containing effects for all the explanatory variables.

The LOGISTIC procedure has some additional options to control how to move effects in and out of a model with the forward selection, backward elimination, or stepwise selection model-building strategies. When there are no interaction terms, a main effect can enter or leave a model in a single step based on the  $p$ -value of the score or Wald statistic. When there are interaction terms, the selection process also depends on whether you want to preserve model hierarchy. These additional options enable you to specify whether model hierarchy is to be preserved, how model hierarchy is applied, and whether a single effect or multiple effects can be moved in a single step.

Odds ratio estimates are displayed along with parameter estimates. You can also specify the change in the continuous explanatory main effects for which odds ratio estimates are desired. Confidence intervals for the regression parameters and odds ratios can be computed based either on the profile-likelihood function or on the asymptotic normality of the parameter estimators. You can also produce odds ratios for effects that are involved in interactions or nestings, and for any type of parameterization of the CLASS variables.

Various methods to correct for overdispersion are provided, including Williams' method for grouped binary response data. The adequacy of the fitted model can be evaluated by various goodness-of-fit tests, including the Hosmer-Lemeshow test for binary response data.

Like many procedures in SAS/STAT software that enable the specification of CLASS variables, the LOGISTIC procedure provides a **CONTRAST** statement for specifying customized hypothesis tests concerning the model parameters. The **CONTRAST** statement also provides estimation of individual rows of contrasts, which is particularly useful for obtaining odds ratio estimates for various levels of the CLASS variables. The LOGISTIC procedure also provides testing capability through the **ESTIMATE** and **TEST** statements. Analyses of LS-means are enabled with the **LSMEANS**, **LSMESTIMATE**, and **SLICE** statements.

You can perform a conditional logistic regression on binary response data by specifying the **STRATA** statement. This enables you to perform matched-set and case-control analyses. The number of events and nonevents can vary across the strata. Many of the features available with the unconditional analysis are also available with a conditional analysis.

The LOGISTIC procedure enables you to perform exact logistic regression, also known as exact conditional logistic regression, by specifying one or more **EXACT** statements. You can test individual parameters or conduct a joint test for several parameters. The procedure computes two exact tests: the exact conditional score test and the exact conditional probability test. You can request exact estimation of specific parameters and corresponding odds ratios where appropriate. Point estimates, standard errors, and confidence intervals are provided. You can perform stratified exact logistic regression by specifying the **STRATA** statement.

Further features of the LOGISTIC procedure enable you to do the following:

- control the ordering of the response categories
- compute a generalized R Square measure for the fitted model
- reclassify binary response observations according to their predicted response probabilities
- test linear hypotheses about the regression parameters
- create a data set for producing a receiver operating characteristic curve for each fitted model
- specify contrasts to compare several receiver operating characteristic curves
- create a data set containing the estimated response probabilities, residuals, and influence diagnostics
- score a data set by using a previously fitted model

The LOGISTIC procedure uses ODS Graphics to create graphs as part of its output. For general information about ODS Graphics, see Chapter 21, “[Statistical Graphics Using ODS](#).” For more information about the plots implemented in PROC LOGISTIC, see the section “[ODS Graphics](#)” on page 4294.

The remaining sections of this chapter describe how to use PROC LOGISTIC and discuss the underlying statistical methodology. The section “[Getting Started: LOGISTIC Procedure](#)” on page 4166 introduces PROC LOGISTIC with an example for binary response data. The section “[Syntax: LOGISTIC Procedure](#)” on page 4174 describes the syntax of the procedure. The section “[Details: LOGISTIC Procedure](#)” on page 4237 summarizes the statistical technique employed by PROC LOGISTIC. The section “[Examples: LOGISTIC Procedure](#)” on page 4295 illustrates the use of the LOGISTIC procedure.

For more examples and discussion on the use of PROC LOGISTIC, see Stokes, Davis, and Koch (2012); Allison (1999); SAS Institute Inc. (1995).

## Getting Started: LOGISTIC Procedure

The LOGISTIC procedure is similar in use to the other regression procedures in the SAS System. To demonstrate the similarity, suppose the response variable  $y$  is binary or ordinal, and  $x_1$  and  $x_2$  are two explanatory variables of interest. To fit a logistic regression model, you can specify a MODEL statement similar to that used in the REG procedure. For example:

```
proc logistic;
  model y=x1 x2;
run;
```

The response variable  $y$  can be either character or numeric. PROC LOGISTIC enumerates the total number of response categories and orders the response levels according to the response variable option **ORDER=** in the **MODEL** statement.

You can also input binary response data that are grouped. In the following statements,  $n$  represents the number of trials and  $r$  represents the number of events:

```
proc logistic;
  model r/n=x1 x2;
run;
```

The following example illustrates the use of PROC LOGISTIC. The data, taken from Cox and Snell (1989, pp. 10–11), consist of the number,  $r$ , of ingots not ready for rolling, out of  $n$  tested, for a number of combinations of heating time and soaking time.

```
data ingots;
  input Heat Soak r n @@;
  datalines;
7 1.0 0 10 14 1.0 0 31 27 1.0 1 56 51 1.0 3 13
7 1.7 0 17 14 1.7 0 43 27 1.7 4 44 51 1.7 0 1
7 2.2 0 7 14 2.2 2 33 27 2.2 0 21 51 2.2 0 1
7 2.8 0 12 14 2.8 0 31 27 2.8 1 22 51 4.0 0 1
7 4.0 0 9 14 4.0 0 19 27 4.0 1 16
;
```

The following invocation of PROC LOGISTIC fits the binary logit model to the grouped data. The continuous covariates Heat and Soak are specified as predictors, and the bar notation (“|”) includes their interaction, Heat\*Soak. The **ODDSRATIO** statement produces odds ratios in the presence of interactions, and a graphical display of the requested odds ratios is produced when ODS Graphics is enabled.

```
ods graphics on;
proc logistic data=ingots;
  model r/n = Heat | Soak;
  oddsratio Heat / at (Soak=1 2 3 4);
run;
ods graphics off;
```

The results of this analysis are shown in the following figures. PROC LOGISTIC first lists background information in [Figure 54.1](#) about the fitting of the model. Included are the name of the input data set, the response variable(s) used, the number of observations used, and the link function used.

**Figure 54.1** Binary Logit Model

| The LOGISTIC Procedure      |                  |
|-----------------------------|------------------|
| Model Information           |                  |
| Data Set                    | WORK.INGOTS      |
| Response Variable (Events)  | r                |
| Response Variable (Trials)  | n                |
| Model                       | binary logit     |
| Optimization Technique      | Fisher's scoring |
|                             |                  |
| Number of Observations Read | 19               |
| Number of Observations Used | 19               |
| Sum of Frequencies Read     | 387              |
| Sum of Frequencies Used     | 387              |

The “Response Profile” table (Figure 54.2) lists the response categories (which are Event and Nonevent when grouped data are input), their ordered values, and their total frequencies for the given data.

**Figure 54.2** Response Profile with Events/Trials Syntax

| Response Profile                              |                |                 |
|-----------------------------------------------|----------------|-----------------|
| Ordered Value                                 | Binary Outcome | Total Frequency |
| 1                                             | Event          | 12              |
| 2                                             | Nonevent       | 375             |
|                                               |                |                 |
| Model Convergence Status                      |                |                 |
| Convergence criterion (GCONV=1E-8) satisfied. |                |                 |

The “Model Fit Statistics” table (Figure 54.3) contains Akaike’s information criterion (AIC), the Schwarz criterion (SC), and the negative of twice the log likelihood ( $-2 \text{ Log } L$ ) for the intercept-only model and the fitted model. AIC and SC can be used to compare different models, and the ones with smaller values are preferred. Results of the likelihood ratio test and the efficient score test for testing the joint significance of the explanatory variables (Soak, Heat, and their interaction) are included in the “Testing Global Null Hypothesis: BETA=0” table (Figure 54.3); the small  $p$ -values reject the hypothesis that all slope parameters are equal to zero.

**Figure 54.3** Fit Statistics and Hypothesis Tests

| Model Fit Statistics |           |                          |                     |
|----------------------|-----------|--------------------------|---------------------|
| Criterion            | Intercept | Intercept and Covariates |                     |
|                      | Only      | Log Likelihood           | Full Log Likelihood |
| AIC                  | 108.988   | 103.222                  | 35.957              |
| SC                   | 112.947   | 119.056                  | 51.791              |
| -2 Log L             | 106.988   | 95.222                   | 27.957              |

| Testing Global Null Hypothesis: BETA=0 |            |    |            |
|----------------------------------------|------------|----|------------|
| Test                                   | Chi-Square | DF | Pr > ChiSq |
| Likelihood Ratio                       | 11.7663    | 3  | 0.0082     |
| Score                                  | 16.5417    | 3  | 0.0009     |
| Wald                                   | 13.4588    | 3  | 0.0037     |

The “Analysis of Maximum Likelihood Estimates” table in [Figure 54.4](#) lists the parameter estimates, their standard errors, and the results of the Wald test for individual parameters. Note that the Heat\*Soak parameter is not significantly different from zero ( $p=0.727$ ), nor is the Soak variable ( $p=0.6916$ ).

**Figure 54.4** Parameter Estimates

| Analysis of Maximum Likelihood Estimates |    |          |                |                 |            |
|------------------------------------------|----|----------|----------------|-----------------|------------|
| Parameter                                | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                | 1  | -5.9901  | 1.6666         | 12.9182         | 0.0003     |
| Heat                                     | 1  | 0.0963   | 0.0471         | 4.1895          | 0.0407     |
| Soak                                     | 1  | 0.2996   | 0.7551         | 0.1574          | 0.6916     |
| Heat*Soak                                | 1  | -0.00884 | 0.0253         | 0.1219          | 0.7270     |

The “Association of Predicted Probabilities and Observed Responses” table ([Figure 54.5](#)) contains four measures of association for assessing the predictive ability of a model. They are based on the number of pairs of observations with different response values, the number of concordant pairs, and the number of discordant pairs, which are also displayed. Formulas for these statistics are given in the section “[Rank Correlation of Observed Responses and Predicted Probabilities](#)” on page 4253.

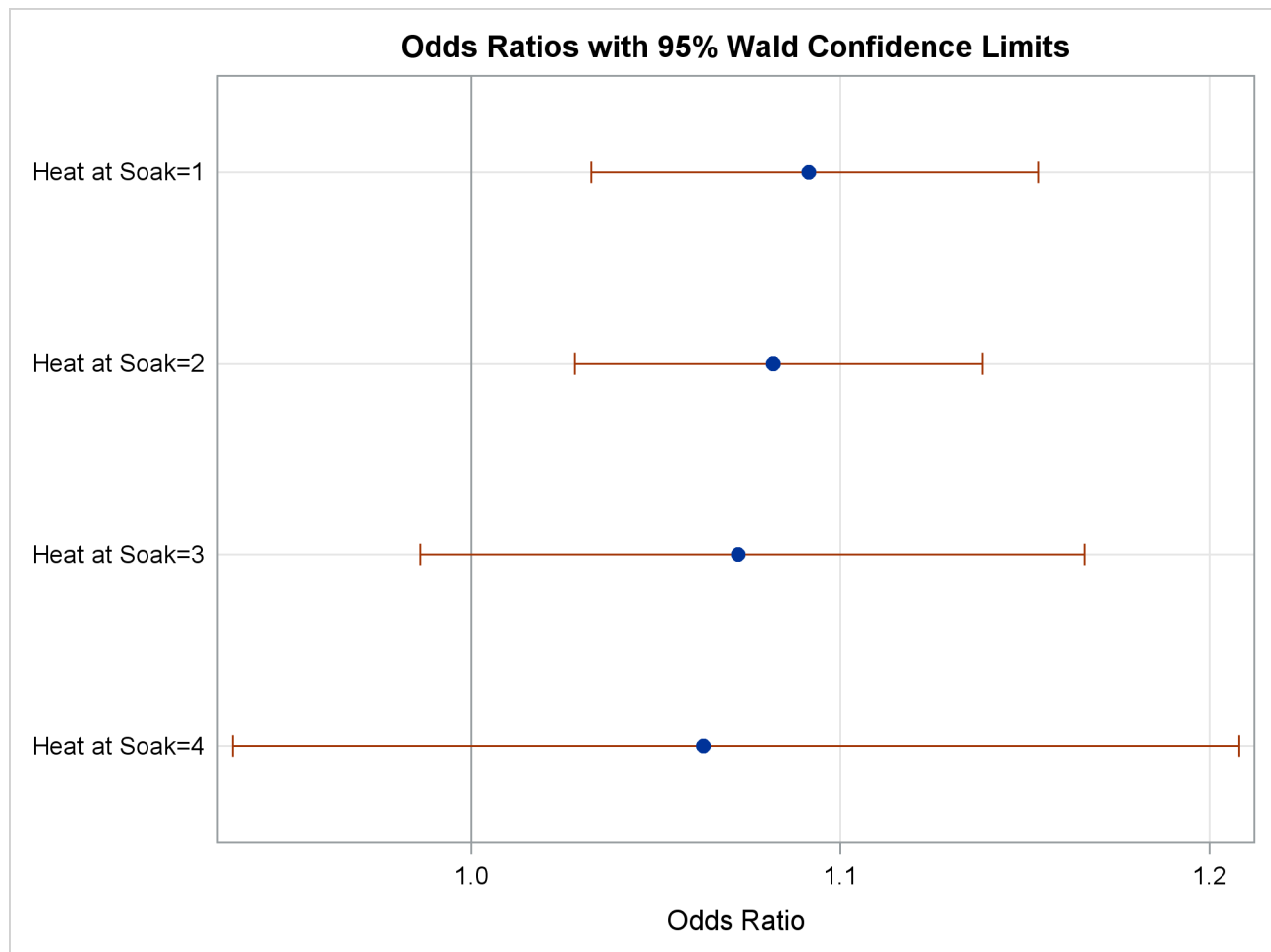
**Figure 54.5** Association Table

| Association of Predicted Probabilities and Observed Responses |      |           |       |
|---------------------------------------------------------------|------|-----------|-------|
| Percent Concordant                                            | 70.9 | Somers' D | 0.537 |
| Percent Discordant                                            | 17.3 | Gamma     | 0.608 |
| Percent Tied                                                  | 11.8 | Tau-a     | 0.032 |
| Pairs                                                         | 4500 | c         | 0.768 |

The **ODDSRATIO** statement produces the “Odds Ratio Estimates and Wald Confidence Intervals” table (Figure 54.6), and a graphical display of these estimates is shown in Figure 54.7. The differences between the odds ratios are small compared to the variability shown by their confidence intervals, which confirms the previous conclusion that the Heat\*Soak parameter is not significantly different from zero.

**Figure 54.6** Odds Ratios of Heat at Several Values of Soak

| Odds Ratio Estimates and Wald Confidence Intervals |          |                       |       |
|----------------------------------------------------|----------|-----------------------|-------|
| Label                                              | Estimate | 95% Confidence Limits |       |
| Heat at Soak=1                                     | 1.091    | 1.032                 | 1.154 |
| Heat at Soak=2                                     | 1.082    | 1.028                 | 1.139 |
| Heat at Soak=3                                     | 1.072    | 0.986                 | 1.166 |
| Heat at Soak=4                                     | 1.063    | 0.935                 | 1.208 |

**Figure 54.7** Plot of Odds Ratios of Heat at Several Values of Soak

Since the Heat\*Soak interaction is nonsignificant, the following statements fit a main-effects model:

```
proc logistic data=ingots;
  model r/n = Heat Soak;
run;
```

The results of this analysis are shown in the following figures. The model information and response profiles are the same as those in Figure 54.1 and Figure 54.2 for the saturated model. The “Model Fit Statistics” table in Figure 54.8 shows that the AIC and SC for the main-effects model are smaller than for the saturated model, indicating that the main-effects model might be the preferred model. As in the preceding model, the “Testing Global Null Hypothesis: BETA=0” table indicates that the parameters are significantly different from zero.



**Figure 54.8** Fit Statistics and Hypothesis Tests

| The LOGISTIC Procedure                 |                   |                                               |                        |
|----------------------------------------|-------------------|-----------------------------------------------|------------------------|
| Model Fit Statistics                   |                   |                                               |                        |
| Criterion                              | Intercept<br>Only | Intercept and Covariates<br>Log<br>Likelihood | Full Log<br>Likelihood |
| AIC                                    | 108.988           | 101.346                                       | 34.080                 |
| SC                                     | 112.947           | 113.221                                       | 45.956                 |
| -2 Log L                               | 106.988           | 95.346                                        | 28.080                 |
| Testing Global Null Hypothesis: BETA=0 |                   |                                               |                        |
| Test                                   | Chi-Square        | DF                                            | Pr > ChiSq             |
| Likelihood Ratio                       | 11.6428           | 2                                             | 0.0030                 |
| Score                                  | 15.1091           | 2                                             | 0.0005                 |
| Wald                                   | 13.0315           | 2                                             | 0.0015                 |

The “Analysis of Maximum Likelihood Estimates” table in [Figure 54.9](#) again shows that the Soak parameter is not significantly different from zero ( $p=0.8639$ ). The odds ratio for each effect parameter, estimated by exponentiating the corresponding parameter estimate, is shown in the “Odds Ratios Estimates” table ([Figure 54.9](#)), along with 95% Wald confidence intervals. The confidence interval for the Soak parameter contains the value 1, which also indicates that this effect is not significant.

**Figure 54.9** Parameter Estimates and Odds Ratios

| Analysis of Maximum Likelihood Estimates |                   |                               |                   |                    |            |
|------------------------------------------|-------------------|-------------------------------|-------------------|--------------------|------------|
| Parameter                                | DF                | Estimate                      | Standard<br>Error | Wald<br>Chi-Square | Pr > ChiSq |
| Intercept                                | 1                 | -5.5592                       | 1.1197            | 24.6503            | <.0001     |
| Heat                                     | 1                 | 0.0820                        | 0.0237            | 11.9454            | 0.0005     |
| Soak                                     | 1                 | 0.0568                        | 0.3312            | 0.0294             | 0.8639     |
| Odds Ratio Estimates                     |                   |                               |                   |                    |            |
| Effect                                   | Point<br>Estimate | 95% Wald<br>Confidence Limits |                   |                    |            |
| Heat                                     | 1.085             | 1.036                         | 1.137             |                    |            |
| Soak                                     | 1.058             | 0.553                         | 2.026             |                    |            |

Figure 54.9 continued

| Association of Predicted Probabilities and Observed Responses |      |           |       |
|---------------------------------------------------------------|------|-----------|-------|
| Percent Concordant                                            | 64.4 | Somers' D | 0.460 |
| Percent Discordant                                            | 18.4 | Gamma     | 0.555 |
| Percent Tied                                                  | 17.2 | Tau-a     | 0.028 |
| Pairs                                                         | 4500 | c         | 0.730 |

Using these parameter estimates, you can calculate the estimated logit of  $\pi$  as

$$-5.5592 + 0.082 \times \text{Heat} + 0.0568 \times \text{Soak}$$

For example, if Heat=7 and Soak=1, then  $\text{logit}(\hat{\pi}) = -4.9284$ . Using this logit estimate, you can calculate  $\hat{\pi}$  as follows:

$$\hat{\pi} = 1/(1 + e^{4.9284}) = 0.0072$$

This gives the predicted probability of the event (ingot not ready for rolling) for Heat=7 and Soak=1. Note that PROC LOGISTIC can calculate these statistics for you; use the **OUTPUT** statement with the **PREDICTED=** option, or use the **SCORE** statement.

To illustrate the use of an alternative form of input data, the following program creates the **ingots** data set with the new variables **NotReady** and **Freq** instead of **n** and **r**. The variable **NotReady** represents the response of individual units; it has a value of 1 for units not ready for rolling (event) and a value of 0 for units ready for rolling (nonevent). The variable **Freq** represents the frequency of occurrence of each combination of Heat, Soak, and NotReady. Note that, compared to the previous data set, NotReady=1 implies Freq=r, and NotReady=0 implies Freq=n-r.

```
data ingots;
  input Heat Soak NotReady Freq @@;
  datalines;
7 1.0 0 10 14 1.0 0 31 14 4.0 0 19 27 2.2 0 21 51 1.0 1 3
7 1.7 0 17 14 1.7 0 43 27 1.0 1 1 27 2.8 1 1 51 1.0 0 10
7 2.2 0 7 14 2.2 1 2 27 1.0 0 55 27 2.8 0 21 51 1.7 0 1
7 2.8 0 12 14 2.2 0 31 27 1.7 1 4 27 4.0 1 1 51 2.2 0 1
7 4.0 0 9 14 2.8 0 31 27 1.7 0 40 27 4.0 0 15 51 4.0 0 1
;
```

The following statements invoke PROC LOGISTIC to fit the main-effects model by using the alternative form of the input data set:

```
proc logistic data=ingots;
  model NotReady(event='1') = Heat Soak;
  freq Freq;
run;
```

Results of this analysis are the same as the preceding single-trial main-effects analysis. The displayed output for the two runs are identical except for the background information of the model fit and the “Response Profile” table shown in [Figure 54.10](#).

**Figure 54.10** Response Profile with Single-Trial Syntax

| The LOGISTIC Procedure             |          |                    |
|------------------------------------|----------|--------------------|
| Response Profile                   |          |                    |
| Ordered<br>Value                   | NotReady | Total<br>Frequency |
| 1                                  | 0        | 375                |
| 2                                  | 1        | 12                 |
| Probability modeled is NotReady=1. |          |                    |

By default, Ordered Values are assigned to the sorted response values in ascending order, and PROC LOGISTIC models the probability of the response level that corresponds to the Ordered Value 1. There are several methods to change these defaults; the preceding statements specify the response variable option `EVENT=` to model the probability of NotReady=1 as displayed in [Figure 54.10](#). See the section “[Response Level Ordering](#)” on page 4237 for more details.

## Syntax: LOGISTIC Procedure

The following statements are available in the LOGISTIC procedure:

```

PROC LOGISTIC < options > ;
  BY variables ;
  CLASS variable < (options) > < variable < (options) > ... > < / options > ;
  CODE < options > ;
  CONTRAST 'label' effect values<, effect values, ... > < / options > ;
  EFFECT name=effect-type(variables < / options >) ;
  EFFECTPLOT < plot-type < (plot-definition-options) > > < / options > ;
  ESTIMATE < 'label' > estimate-specification < / options > ;
  EXACT < 'label' > < INTERCEPT > < effects > < / options > ;
  EXACTOPTIONS options ;
  FREQ variable ;
  ID variables ;
  LSMEANS < model-effects > < / options > ;
  LSMESTIMATE model-effect lsestimate-specification < / options > ;
  < label: > MODEL variable < (variable_options) > = < effects > < / options > ;
  < label: > MODEL events/trials = < effects > < / options > ;
  NLOPTIONS options ;
  ODDSRATIO < 'label' > variable < / options > ;
  OUTPUT < OUT=SAS-data-set > < keyword=name < keyword=name ... > > < / option > ;
  ROC < 'label' > < specification > < / options > ;
  ROCCONTRAST < 'label' > < contrast > < / options > ;
  SCORE < options > ;
  SLICE model-effect < / options > ;
  STORE < OUT= > item-store-name < / LABEL= 'label' > ;
  STRATA effects < / options > ;
  < label: > TEST equation1 <, equation2, ... > < / option > ;
  UNITS < independent1=list1 < independent2=list2 ... > > < / option > ;
  WEIGHT variable < / option > ;

```

The PROC LOGISTIC and MODEL statements are required. The CLASS and EFFECT statements (if specified) must precede the MODEL statement, and the CONTRAST, EXACT, and ROC statements (if specified) must follow the MODEL statement.

The PROC LOGISTIC, MODEL, and ROCCONTRAST statements can be specified at most once. If a FREQ or WEIGHT statement is specified more than once, the variable specified in the first instance is used. If a BY, OUTPUT, or UNITS statement is specified more than once, the last instance is used.

The rest of this section provides detailed syntax information for each of the preceding statements, beginning with the PROC LOGISTIC statement. The remaining statements are covered in alphabetical order. The CODE, EFFECT, EFFECTPLOT, ESTIMATE, LSMEANS, LSMESTIMATE, SLICE, and STORE statements are also available in many other procedures. Summary descriptions of functionality and syntax for these statements are provided, but you can find full documentation on them in the corresponding sections of Chapter 19, “Shared Concepts and Topics.”

## PROC LOGISTIC Statement

**PROC LOGISTIC** < options > ;

The PROC LOGISTIC statement invokes the LOGISTIC procedure. Optionally, it identifies input and output data sets, suppresses the display of results, and controls the ordering of the response levels. Table 54.1 summarizes the options available in the PROC LOGISTIC statement.

**Table 54.1** PROC LOGISTIC Statement Options

| Option                                     | Description                                                      |
|--------------------------------------------|------------------------------------------------------------------|
| <b>Input/Output Data Set Options</b>       |                                                                  |
| COVOUT                                     | Displays the estimated covariance matrix in the OUTEST= data set |
| DATA=                                      | Names the input SAS data set                                     |
| INEST=                                     | Specifies the initial estimates SAS data set                     |
| INMODEL=                                   | Specifies the model information SAS data set                     |
| NOCOVAR                                    | Does not save covariance matrix in the OUTMODEL= data set        |
| OUTDESIGN=                                 | Specifies the design matrix output SAS data set                  |
| OUTDESIGNONLY                              | Outputs the design matrix only                                   |
| OUTEST=                                    | Specifies the parameter estimates output SAS data set            |
| OUTMODEL=                                  | Specifies the model output data set for scoring                  |
| <b>Response and CLASS Variable Options</b> |                                                                  |
| DESCENDING                                 | Reverses the sort order of the response variable                 |
| NAMELEN=                                   | Specifies the maximum length of effect names                     |
| ORDER=                                     | Specifies the sort order of the response variable                |
| TRUNCATE                                   | Truncates class level names                                      |
| <b>Displayed Output Options</b>            |                                                                  |
| ALPHA=                                     | Specifies the significance level for confidence intervals        |
| NOPRINT                                    | Suppresses all displayed output                                  |
| PLOTS                                      | Specifies options for plots                                      |
| SIMPLE                                     | Displays descriptive statistics                                  |
| <b>Large Data Set Option</b>               |                                                                  |
| MULTIPASS                                  | Does not copy the input SAS data set for internal computations   |
| <b>Control of Other Statement Options</b>  |                                                                  |
| EXACTONLY                                  | Performs exact analysis only                                     |
| EXACTOPTIONS                               | Specifies global options for EXACT statements                    |
| ROCOPTIONS                                 | Specifies global options for ROC statements                      |

### **ALPHA=number**

specifies the level of significance  $\alpha$  for  $100(1 - \alpha)\%$  confidence intervals. The value *number* must be between 0 and 1; the default value is 0.05, which results in 95% intervals. This value is used as the default confidence level for limits computed by the following options:

| Statement     | Options                    |
|---------------|----------------------------|
| CONTRAST      | ESTIMATE=                  |
| EXACT         | ESTIMATE=                  |
| MODEL         | CLODDS= CLPARM=            |
| ODDSRATIO     | CL=                        |
| OUTPUT        | LOWER= UPPER=              |
| PROC LOGISTIC | PLOTS=EFFECT(CLBAR CLBAND) |
| ROCONTRAST    | ESTIMATE=                  |
| SCORE         | CLM                        |

You can override the default in most of these cases by specifying the ALPHA= option in the separate statements.

### COVOUT

adds the estimated covariance matrix to the OUTEST= data set. For the COVOUT option to have an effect, the OUTEST= option must be specified. See the section “OUTEST= Output Data Set” on page 4279 for more information.

### DATA=SAS-data-set

names the SAS data set containing the data to be analyzed. If you omit the DATA= option, the procedure uses the most recently created SAS data set. The INMODEL= option cannot be specified with this option.

### DESCENDING

#### DESC

reverses the sort order for the levels of the response variable. If both the DESCENDING and ORDER= options are specified, PROC LOGISTIC orders the levels according to the ORDER= option and then reverses that order. This option has the same effect as the response variable option DESCENDING in the MODEL statement. See the section “Response Level Ordering” on page 4237 for more detail.

### EXACTONLY

requests only the exact analyses. The asymptotic analysis that PROC LOGISTIC usually performs is suppressed.

### EXACTOPTIONS (options)

specifies options that apply to every EXACT statement in the program. The available options are summarized here, and full descriptions are available in the EXACTOPTIONS statement.

| Option       | Description                                                              |
|--------------|--------------------------------------------------------------------------|
| ADDTOBS      | Adds the observed sufficient statistic to the sampled exact distribution |
| BUILDSUBSETS | Builds every distribution for sampling                                   |
| EPSILON=     | Specifies the comparison fuzz for partial sums of sufficient statistics  |
| MAXTIME=     | Specifies the maximum time allowed in seconds                            |
| METHOD=      | Specifies the DIRECT, NETWORK, or NETWORKMC algorithm                    |
| N=           | Specifies the number of Monte Carlo samples                              |
| ONDISK       | Uses disk space                                                          |
| SEED=        | Specifies the initial seed for sampling                                  |
| STATUSN=     | Specifies the sampling interval for printing a status line               |
| STATUSTIME=  | Specifies the time interval for printing a status line                   |

**INEST=SAS-data-set**

names the SAS data set that contains initial estimates for all the parameters in the model. If BY-group processing is used, it must be accommodated in setting up the INEST= data set. See the section “[INEST= Input Data Set](#)” on page 4280 for more information.

**INMODEL=SAS-data-set**

specifies the name of the SAS data set that contains the model information needed for scoring new data. This INMODEL= data set is the [OUTMODEL=](#) data set saved in a previous PROC LOGISTIC call. The OUTMODEL= data set should not be modified before its use as an INMODEL= data set.

The [DATA=](#) option cannot be specified with this option; instead, specify the data sets to be scored in the [SCORE](#) statements. [FORMAT](#) statements are not allowed when the INMODEL= data set is specified; variables in the [DATA=](#) and [PRIOR=](#) data sets in the [SCORE](#) statement should be formatted within the data sets.

You can specify the [BY](#) statement provided that the INMODEL= data set is created under the same BY-group processing.

The [CLASS](#), [EFFECT](#), [EFFECTPLOT](#), [ESTIMATE](#), [EXACT](#), [LSMEANS](#), [LSMESTIMATE](#), [MODEL](#), [OUTPUT](#), [ROC](#), [ROCONTRAST](#), [SLICE](#), [STORE](#), [TEST](#), and [UNIT](#) statements are not available with the INMODEL= option.

**MULTIPASS**

forces the procedure to reread the [DATA=](#) data set as needed rather than require its storage in memory or in a temporary file on disk. By default, the data set is cleaned up and stored in memory or in a temporary file. This option can be useful for large data sets. All exact analyses are ignored in the presence of the MULTIPASS option. If a [STRATA](#) statement is specified, then the data set must first be grouped or sorted by the strata variables.

**NAMELEN=n**

specifies the maximum length of effect names in tables and output data sets to be *n* characters, where *n* is a value between 20 and 200. The default length is 20 characters.

**NOCOV**

specifies that the covariance matrix not be saved in the [OUTMODEL=](#) data set. The covariance matrix is needed for computing the confidence intervals for the posterior probabilities in the [OUT=](#) data set in the [SCORE](#) statement. Specifying this option will reduce the size of the [OUTMODEL=](#) data set.

**NOPRINT**

suppresses all displayed output. Note that this option temporarily disables the Output Delivery System (ODS); see Chapter 20, “[Using the Output Delivery System](#),” for more information.

**ORDER=DATA | FORMATTED | FREQ | INTERNAL****RORDER=DATA | FORMATTED | INTERNAL**

specifies the sort order for the levels of the response variable. See the response variable option [ORDER=](#) in the [MODEL](#) statement for more information. For ordering of [CLASS](#) variable levels, see the [ORDER=](#) option in the [CLASS](#) statement.

**OUTDESIGN=SAS-data-set**

specifies the name of the data set that contains the design matrix for the model. The data set contains the same number of observations as the corresponding [DATA=](#) data set and includes the response

variable (with the same format as in the DATA= data set), the **FREQ** variable, the **WEIGHT** variable, the **OFFSET=** variable, and the design variables for the covariates, including the Intercept variable of constant value 1 unless the **NOINT** option in the **MODEL** statement is specified.

### **OUTDESIGNONLY**

suppresses the model fitting and creates only the **OUTDESIGN=** data set. This option is ignored if the **OUTDESIGN=** option is not specified.

### **OUTEST=SAS-data-set**

creates an output SAS data set that contains the final parameter estimates and, optionally, their estimated covariances (see the preceding **COVOUT** option). The output data set also includes a variable named **\_LNLIKE\_**, which contains the log likelihood. See the section “**OUTEST= Output Data Set**” on page 4279 for more information.

### **OUTMODEL=SAS-data-set**

specifies the name of the SAS data set that contains the information about the fitted model. This data set contains sufficient information to score new data without having to refit the model. It is solely used as the input to the **INMODEL=** option in a subsequent PROC LOGISTIC call. The **OUTMODEL=** option is not available with the **STRATA** statement. Information in this data set is stored in a very compact form, so you should not modify it manually.

**NOTE:** The **STORE** statement can also be used to save your model. See the section “**STORE Statement**” on page 4233 for more information.

**PLOTS** < (*global-plot-options*) > < =*plot-request* < (*options*) > >

**PLOTS** < (*global-plot-options*) > =(*plot-request* < (*options*) > < ... *plot-request* < (*options*) > >)

controls the plots produced through ODS Graphics. When you specify only one *plot-request*, you can omit the parentheses from around the *plot-request*. For example:

```
PLOTS = ALL
PLOTS = (ROC EFFECT INFLUENCE (UNPACK) )
PLOTS (ONLY) = EFFECT (CLBAR SHOWOBS)
```

ODS Graphics must be enabled before plots can be requested. For example:

```
ods graphics on;
proc logistic plots=all;
    model y=x;
run;
ods graphics off;
```

For more information about enabling and disabling ODS Graphics, see the section “**Enabling and Disabling ODS Graphics**” on page 600 in Chapter 21, “**Statistical Graphics Using ODS**.”

If the **PLOTS** option is not specified or is specified with no *plot-requests*, then graphics are produced by default in the following situations:

- If the **INFLUENCE** or **IPLOTS** option is specified in the **MODEL** statement, then the line-printer plots are suppressed, and the **INFLUENCE** plots are produced unless the **MAXPOINTS=** cutoff is exceeded.



- If you specify the **OUTROC=** option in the **MODEL** statement, then ROC curves are produced. If you also specify a **SELECTION=** method, then an overlaid plot of all the ROC curves for each step of the selection process is displayed.
- If the **OUTROC=** option is specified in a **SCORE** statement, then the ROC curve for the scored data set is displayed.
- If you specify **ROC** statements, then an overlaid plot of the ROC curves for the model (or the selected model if a **SELECTION=** method is specified) and for all the ROC statement models is displayed.
- If an odds ratio table is produced, then a plot of the odds ratios and their confidence limits is displayed. These plots correspond to the default odds ratio table and the tables produced by the **CLODDS=** option in the **MODEL** statement and the tables produced by the **ODDSRATIO** statement.

For general information about ODS Graphics, see Chapter 21, “Statistical Graphics Using ODS.”

The following *global-plot-options* are available:

#### **LABEL**

displays a label on diagnostic plots to aid in identifying the outlying observations. This option enhances the plots produced by the **DFBETAS**, **DPC**, **INFLUENCE**, **LEVERAGE**, and **PHAT** options. If an **ID** statement is specified, then the plots are labeled with the ID variables. Otherwise, the observation number is displayed.

#### **MAXPOINTS=NONE** | *number*

suppresses the plots produced by the **DFBETAS**, **DPC**, **INFLUENCE**, **LEVERAGE**, and **PHAT** options if there are more than *number* observations. Also, observations are not displayed on the **EFFECT** plots when the cutoff is exceeded. The default is **MAXPOINTS=5000**. The cutoff is ignored if you specify **MAXPOINTS=NONE**.

#### **ONLY**

specifically requested *plot-requests* are displayed.

#### **UNPACKPANELS** | **UNPACK**

suppresses paneling. By default, multiple plots can appear in some output *panels*. Specify **UNPACKPANEL** to display each plot separately.

The following *plot-requests* are available:

#### **ALL**

produces all appropriate plots. You can specify other options with **ALL**. For example, to display all plots and unpack the **DFBETAS** plots you can specify **plots=(all dfbetas(unpack))**.

#### **DFBETAS <(UNPACK)>**

displays plots of **DFBETAS** versus the case (observation) number. This displays the statistics generated by the **DFBETAS=\_ALL\_** option in the **OUTPUT** statement. The **UNPACK** option displays the plots separately. See [Output 54.6.5](#) for an example of this plot.

**DPC<(UNPACK)>**

displays plots of **DIFCHISQ** and **DIFDEV** versus the predicted event probability, and colors the markers according to the value of the confidence interval displacement **C**. The **UNPACK** option displays the plots separately. See [Output 54.6.8](#) for an example of this plot.

**EFFECT<(effect-options)>**

displays and enhances the effect plots for the model. For more information about effect plots and the available *effect-options*, see the section “**PLOTS=EFFECT Plots**” on page 4182.

**NOTE:** The **EFFECTPLOT** statement provides you with much of the same functionality and more options for creating effect plots. See [Outputs 54.2.11, 54.3.5, 54.4.8, 54.7.4, and 54.15.4](#) for examples of effect plots.

**INFLUENCE<(UNPACK | STDRES)>**

displays index plots of **RESCHI**, **RESDEV**, leverage, confidence interval displacements **C** and **CBar**, **DIFCHISQ**, and **DIFDEV**. These plots are produced by default when any *plot-request* is specified and the **MAXPOINTS=** cutoff is not exceeded. The **UNPACK** option displays the plots separately. The **STDRES** option also displays index plots of **STDRESCHI**, **STDRESDEV**, and **RESLIK**. See [Outputs 54.6.3 and 54.6.4](#) for examples of these plots.

**LEVERAGE<(UNPACK)>**

displays plots of **DIFCHISQ**, **DIFDEV**, confidence interval displacement **C**, and the predicted probability versus the leverage. The **UNPACK** option displays the plots separately. See [Output 54.6.7](#) for an example of this plot.

**NONE**

suppresses all plots.

**ODDSRATIO <(oddsratio-options)>**

displays and enhances the odds ratio plots for the model. For more information about odds ratio plots and the available *oddsratio-options*, see the section “**Odds Ratio Plots**” on page 4185. See [Outputs 54.7, 54.2.9, 54.3.3, and 54.4.5](#) for examples of this plot.

**PHAT<(UNPACK)>**

displays plots of **DIFCHISQ**, **DIFDEV**, confidence interval displacement **C**, and leverage versus the predicted event probability. The **UNPACK** option displays the plots separately. See [Output 54.6.6](#) for an example of this plot.

**ROC<(ID=<keyword>)>**

displays the ROC curve. If you also specify a **SELECTION=** method, then an overlaid plot of all the ROC curves for each step of the selection process is displayed. If you specify **ROC** statements, then an overlaid plot of the model (or the selected model if a **SELECTION=** method is specified) and the ROC statement models is displayed. If the **OUTROC=** option is specified in a **SCORE** statement, then the ROC curve for the scored data set is displayed.

The **ID=** option labels certain points on the ROC curve. Typically, the labeled points are closest to the upper-left corner of the plot, and points directly below or to the right of a labeled point are suppressed. This option is identical to, and has the same *keywords* as, the **ID=** suboption of the **ROCOPTIONS** option.

See [Output 54.7.3](#) and [Example 54.8](#) for examples of these ROC plots.

**ROCOPTIONS** (*options*)

specifies options that apply to every model specified in a **ROC** statement. Some of these options also apply to the **SCORE** statement. The following *options* are available:

**ALPHA=number**

sets the significance level for creating confidence limits of the areas and the pairwise differences. The **ALPHA=** value specified in the PROC LOGISTIC statement is the default. If neither **ALPHA=** value is specified, then **ALPHA=0.05** by default.

**EPS=value**

is an alias for the **ROCEPS=** option in the MODEL statement. This value is used to determine which predicted probabilities are equal. The default value is the square root of the machine epsilon, which is about 1E-8.

**ID<=keyword>**

displays labels on certain points on the individual ROC curves and also on the **SCORE** statement's ROC curve. This option overrides the **ID=** suboption of the **PLOTS=ROC** option. If several observations lie at the same place on the ROC curve, the value for the last observation is displayed. If you specify the **ID** option with no *keyword*, any variables that are listed in the **ID** statement are used. If no **ID** statement is specified, the observation number is displayed. The following *keywords* are available:

|                 |                                                                  |
|-----------------|------------------------------------------------------------------|
| <b>PROB</b>     | displays the model predicted probability.                        |
| <b>OBS</b>      | displays the (last) observation number.                          |
| <b>SENSIT</b>   | displays the true positive fraction (sensitivity).               |
| <b>1MSPEC</b>   | displays the false positive fraction (1-specificity).            |
| <b>FALPOS</b>   | displays the fraction of nonevents that are predicted as events. |
| <b>FALNEG</b>   | displays the fraction of events that are predicted as nonevents. |
| <b>POSPRED</b>  | displays the positive predictive value (1-FALPOS).               |
| <b>NEGPRED</b>  | displays the negative predictive value (1-FALNEG).               |
| <b>MISCLASS</b> | displays the misclassification rate.                             |
| <b>ID</b>       | displays the ID variables.                                       |

The **SENSIT**, **1MSPEC**, **FALPOS**, and **FALNEG** statistics are defined in the section “[Receiver Operating Characteristic Curves](#)” on page 4260. The misclassification rate is the number of events that are predicted as nonevents and the number of nonevents that are predicted as events as calculated by using the given cutpoint (predicted probability) divided by the number of observations. If the **PEVENT=** option is also specified, then **FALPOS** and **FALNEG** are computed using the first **PEVENT=** value and Bayes' theorem, as discussed in the section “[Predicted Probability of an Event for Classification](#)” on page 4255.

**NODETAILS**

suppresses the display of the model fitting information for the models specified in the **ROC** statements.

**OUT=SAS-data-set-name**

is an alias for the **OUTROC=** option in the **MODEL** statement.

**WEIGHTED**

uses frequency $\times$ weight in the ROC computations (Izrael et al. 2002) instead of just frequency. Typically, weights are considered in the fit of the model only, and hence are accounted for in the parameter estimates. The “Association of Predicted Probabilities and Observed Responses” table uses frequency (unless the **BINWIDTH=0** option is also specified on the **MODEL** statement), and is suppressed when ROC comparisons are performed. This option also affects **SCORE** statement ROC and AUC computations.

**SIMPLE**

displays simple descriptive statistics (mean, standard deviation, minimum and maximum) for each continuous explanatory variable. For each **CLASS** variable involved in the modeling, the frequency counts of the classification levels are displayed. The **SIMPLE** option generates a breakdown of the simple descriptive statistics or frequency counts for the entire data set and also for individual response categories.

**TRUNCATE**

determines class levels by using no more than the first 16 characters of the formatted values of **CLASS**, response, and strata variables. When formatted values are longer than 16 characters, you can use this option to revert to the levels as determined in releases previous to SAS 9.0. This option invokes the same option in the **CLASS** statement.

**PLOTS=EFFECT Plots**

Only one **PLOTS=EFFECT** plot is produced by default; you must specify other *effect-options* to produce multiple plots. For binary response models, the following plots are produced when an **EFFECT** option is specified with no *effect-options*:

- If you only have continuous covariates in the model, then a plot of the predicted probability versus the first continuous covariate fixing all other continuous covariates at their means is displayed. See [Output 54.7.4](#) for an example with one continuous covariate.
- If you only have classification covariates in the model, then a plot of the predicted probability versus the first **CLASS** covariate at each level of the second **CLASS** covariate, if any, holding all other **CLASS** covariates at their reference levels is displayed.
- If you have **CLASS** and continuous covariates, then a plot of the predicted probability versus the first continuous covariate at up to 10 cross-classifications of the **CLASS** covariate levels, while fixing all other continuous covariates at their means and all other **CLASS** covariates at their reference levels, is displayed. For example, if your model has four binary covariates, there are 16 cross-classifications of the **CLASS** covariate levels. The plot displays the 8 cross-classifications of the levels of the first three covariates while the fourth covariate is fixed at its reference level.

For polytomous response models, similar plots are produced by default, except that the response levels are used in place of the **CLASS** covariate levels. Plots for polytomous response models involving **OFFSET=** variables with multiple values are not available.

The following *effect-options* specify the type of graphic to produce:

**AT(variable=value-list | ALL<...variable=value-list | ALL>)**

specifies fixed values for a covariate. For continuous covariates, you can specify one or more numbers in the *value-list*. For classification covariates, you can specify one or more formatted levels of the covariate enclosed in single quotes (for example, **A='cat' 'dog'**), or you can specify the keyword **ALL** to select all levels of the classification variable. You can specify a variable at most once in the **AT** option. By default, continuous covariates are set to their means when they are not used on an axis, while classification covariates are set to their reference level when they are not used as an **X=**, **SLICEBY=**, or **PLOTBY=** effect. For example, for a model that includes a classification variable **A={cat,dog}** and a continuous covariate **X**, specifying **AT (A='cat' X=7 9)** will set **A** to 'cat' when **A** does not appear in the plot. When **X** does not define an axis it first produces plots setting **X = 7** and then produces plots setting **X = 9**. Note in this example that specifying **AT ( A=ALL )** is the same as specifying the **PLOTBY=A** option.

**FITOBSONLY**

computes the predicted values only at the observed data. If the **FITOBSONLY** option is omitted and the X-axis variable is continuous, the predicted values are computed at a grid of points extending slightly beyond the range of the data (see the **EXTEND=** option for more information). If the **FITOBSONLY** option is omitted and the X-axis effect is categorical, the predicted values are computed at all possible categories.

**INDIVIDUAL**

displays the individual probabilities instead of the cumulative probabilities. This option is available only with cumulative models, and it is not available with the **LINK** option.

**LINK**

displays the linear predictors instead of the probabilities on the Y axis. For example, for a binary logistic regression, the Y axis will be displayed on the logit scale. The **INDIVIDUAL** and **POLYBAR** options are not available with the **LINK** option.

**PLOTBY=effect**

displays an effect plot at each unique level of the **PLOTBY=** effect. You can specify *effect* as one **CLASS** variable or as an interaction of classification covariates. For polytomous-response models, you can also specify the response variable as the lone **PLOTBY=** effect. For nonsingular parameterizations, the complete cross-classification of the **CLASS** variables specified in the effect define the different **PLOTBY=** levels. When the **GLM** parameterization is used, the **PLOTBY=** levels can depend on the model and the data.

**SLICEBY=effect**

displays predicted probabilities at each unique level of the **SLICEBY=** effect. You can specify *effect* as one **CLASS** variable or as an interaction of classification covariates. For polytomous-response models, you can also specify the response variable as the lone **SLICEBY=** effect. For nonsingular parameterizations, the complete cross-classification of the **CLASS** variables specified in the effect define the different **SLICEBY=** levels. When the **GLM** parameterization is used, the **SLICEBY=** levels can depend on the model and the data.

**X=effect****X=(effect...effect)**

specifies effects to be used on the X axis of the effect plots. You can specify several different X axes: continuous variables must be specified as main effects, while **CLASS** variables can be crossed. For nonsingular parameterizations, the complete cross-classification of the **CLASS** variables specified in

the effect define the axes. When the **GLM** parameterization is used, the X= levels can depend on the model and the data. The response variable is not allowed as an *effect*.

**NOTE:** Any variable not specified in a **SLICEBY=** or **PLOTBY=** option is available to be displayed on the X axis. A variable can be specified in at most one of the **SLICEBY=**, **PLOTBY=**, and **X=** options.

The following *effect-options* enhance the graphical output:

**ALPHA=number**

specifies the size of the confidence limits. The **ALPHA=** value specified in the PROC LOGISTIC statement is the default. If neither **ALPHA=** value is specified, then **ALPHA=0.05** by default.

**CLBAND<=YES | NO>**

displays confidence limits on the plots. This option is not available with the **INDIVIDUAL** option. If you have **CLASS** covariates on the X axis, then error bars are displayed (see the **CLBAR** option) unless you also specify the **CONNECT** option.

**CLBAR**

displays the error bars on the plots when you have **CLASS** covariates on the X axis; if the X axis is continuous, then this invokes the **CLBAND** option. For polytomous-response models with **CLASS** covariates only and with the **POLYBAR** option specified, the stacked bar charts are replaced by side-by-side bar charts with error bars.

**CLUSTER<=percent>**

displays the levels of the **SLICEBY=** effect in a side-by-side fashion instead of stacking them. This option is available when you have **CLASS** covariates on the X axis. You can specify *percent* as a percentage of half the distance between X levels. The *percent* value must be between 0.1 and 1; the default *percent* depends on the number of X levels and the number of **SLICEBY=** levels. Default clustering can be removed by specifying the **NOCLUSTER** option.

**CONNECT<=YES | NO>**

**JOIN<=YES | NO>**

connects the predicted values with a line. This option is available when you have **CLASS** covariates on the X axis. Default connecting lines can be suppressed by specifying the **NOCONNECT** option.

**EXTEND=value**

extends continuous X axes by a factor of *value*/2 in each direction. By default, **EXTEND=0.2**.

**MAXATLEN=length**

specifies the maximum number of characters used to display the levels of all the fixed variables. If the text is too long, it is truncated and ellipses (“...”) are appended. By default, *length* is equal to its maximum allowed value, 256.

**NOCLUSTER**

prevents clustering of the levels of the **SLICEBY=** effect. This option is available when you have **CLASS** covariates on the X axis.

**NOCONNECT**

removes the line that connects the predicted values. This option is available when you have **CLASS** covariates on the X axis.

**POLYBAR**

replaces scatter plots of polytomous response models with bar charts. This option has no effect on binary-response models, and it is overridden by the **CONNECT** option. By default, the X axis is chosen to be a crossing of available classification variables so that there are no more than 16 levels; if no such crossing is possible then the first available classification variable is used. You can override this default by specifying the **X=** option.

**SHOWOBS<=YES | NO>**

displays observations on the plot when the **MAXPOINTS=** cutoff is not exceeded. For events/trials notation, the observed proportions are displayed; for single-trial binary-response models, the observed events are displayed at  $\hat{p} = 1$  and the observed nonevents are displayed at  $\hat{p} = 0$ . For polytomous response models the predicted probabilities at the observed values of the covariate are computed and displayed.

**YRANGE=(*< min>* , *< max>*)**

displays the Y axis as [*min*, *max*]. Note that the axis might extend beyond your specified values. By default, the entire Y axis, [0,1], is displayed for the predicted probabilities. This option is useful if your predicted probabilities are all contained in some subset of this range.

**Odds Ratio Plots**

The odds ratios and confidence limits from the default Odds Ratio Estimates table and from the tables produced by the **CLODDS=** option or the **ODDSRATIO** statement can be displayed in a graphic. If you have many odds ratios, you can produce multiple graphics, or *panels*, by displaying subsets of the odds ratios. Odds ratios with duplicate labels are not displayed. See Outputs 54.2.9 and 54.3.3 for examples of odds ratio plots.

The following *oddsratio-options* modify the default odds ratio plot:

**CLDISPLAY=SERIF | LINE | BAR< width>**

controls the look of the confidence limit error bars. The default **CLDISPLAY=SERIF** displays the confidence limits as lines with serifs, **CLDISPLAY=LINE** removes the serifs from the error bars, and **CLDISPLAY=BAR < width>** displays the limits with a bar of width equal to the size of the marker. You can control the width of the bars and the size of the marker by specifying the *width* value as a percentage of the distance between the bars,  $0 < \textit{width} \leq 1$ .

**NOTE:** Your bar might disappear with small values of *width*.

**DOTPLOT**

displays dotted gridlines on the plot.

**GROUP**

displays the odds ratios in panels defined by the **ODDSRATIO** statements. The **NPANELPOS=** option is ignored when this option is specified.

**LOGBASE=2 | E | 10**

displays the odds ratio axis on the specified log scale.



**NPANELPOS=*n***

breaks the plot into multiple graphics having at most  $|n|$  odds ratios per graphic. If  $n$  is positive, then the number of odds ratios per graphic is balanced; but if  $n$  is negative, then no balancing of the number of odds ratios takes place. By default,  $n = 0$  and all odds ratios are displayed in a single plot. For example, suppose you want to display 21 odds ratios. Then specifying **NPANELPOS=20** displays two plots, the first with 11 odds ratios and the second with 10; but specifying **NPANELPOS=-20** displays 20 odds ratios in the first plot and only 1 odds ratio in the second.

**ORDER=ASCENDING | DESCENDING**

displays the odds ratios in sorted order. By default the odds ratios are displayed in the order in which they appear in the corresponding table.

**RANGE=(*< min >* ,*max >*) | CLIP**

specifies the range of the displayed odds ratio axis. The RANGE=CLIP option has the same effect as specifying the minimum odds ratio as *min* and the maximum odds ratio as *max*. By default, all odds ratio confidence intervals are displayed.

**TYPE=HORIZONTAL | HORIZONTALSTAT | VERTICAL | VERTICALBLOCK**

controls the look of the graphic. The default TYPE=HORIZONTAL option places the odds ratio values on the X axis, while the TYPE=HORIZONTALSTAT option also displays the values of the odds ratios and their confidence limits on the right side of the graphic. The TYPE=VERTICAL option places the odds ratio values on the Y axis, while the TYPE=VERTICALBLOCK option (available only with the **CLODDS=** option) places the odds ratio values on the Y axis and puts boxes around the labels.

---

## BY Statement

**BY** *variables* ;

You can specify a BY statement with PROC LOGISTIC to obtain separate analyses of observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the LOGISTIC procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

If a **SCORE** statement is specified, then define the *training data set* to be the **DATA=** data set or the **IN-MODEL=** data set in the PROC LOGISTIC statement, and define the *scoring data set* to be the **DATA=** data set and **PRIOR=** data set in the SCORE statement. The training data set contains all of the BY variables, and



the scoring data set must contain either all of them or none of them. If the scoring data set contains all the BY variables, matching is carried out between the training and scoring data sets. If the scoring data set does not contain any of the BY variables, the entire scoring data set is used for every BY group in the training data set and the BY variables are added to the output data sets that are specified in the [SCORE](#) statement.

**CAUTION:** The order of the levels in the response and classification variables is determined from all the data regardless of BY groups. However, different sets of levels might appear in different BY groups. This might affect the value of the reference level for these variables, and hence your interpretation of the model and the parameters.

For more information about BY-group processing, see the discussion in *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the discussion in the *Base SAS Procedures Guide*.

---

## CLASS Statement

**CLASS** *variable* <(*options*)> ... <*variable* <(*options*)>> </*global-options*> ;

The CLASS statement names the classification variables to be used as explanatory variables in the analysis. Response variables do not need to be specified in the CLASS statement.

The CLASS statement must precede the [MODEL](#) statement. Most options can be specified either as individual variable *options* or as *global-options*. You can specify *options* for each variable by enclosing the options in parentheses after the variable name. You can also specify *global-options* for the CLASS statement by placing them after a slash (/). *Global-options* are applied to all the variables specified in the CLASS statement. If you specify more than one CLASS statement, the *global-options* specified in any one CLASS statement apply to all CLASS statements. However, individual CLASS variable *options* override the *global-options*. You can specify the following values for either an *option* or a *global-option*:

### CPREFIX=*n*

specifies that, at most, the first *n* characters of a CLASS variable name be used in creating names for the corresponding design variables. The default is  $32 - \min(32, \max(2, f))$ , where *f* is the formatted length of the CLASS variable.

### DESCENDING

#### DESC

reverses the sort order of the classification variable. If both the DESCENDING and [ORDER=](#) options are specified, PROC LOGISTIC orders the categories according to the ORDER= option and then reverses that order.

### LPREFIX=*n*

specifies that, at most, the first *n* characters of a CLASS variable label be used in creating labels for the corresponding design variables. The default is  $256 - \min(256, \max(2, f))$ , where *f* is the formatted length of the CLASS variable.

### MISSING

treats missing values ('.', '\_', 'A', ..., 'Z' for numeric variables and blanks for character variables) as valid values for the CLASS variable.

**ORDER=DATA | FORMATTED | FREQ | INTERNAL**

specifies the sort order for the levels of classification variables. This ordering determines which parameters in the model correspond to each level in the data, so the ORDER= option can be useful when you use the CONTRAST statement. By default, ORDER=FORMATTED. For ORDER=FORMATTED and ORDER=INTERNAL, the sort order is machine-dependent. When ORDER=FORMATTED is in effect for numeric variables for which you have supplied no explicit format, the levels are ordered by their internal values.

The following table shows how PROC LOGISTIC interprets values of the ORDER= option.

| Value of ORDER= | Levels Sorted By                                                                                                                         |
|-----------------|------------------------------------------------------------------------------------------------------------------------------------------|
| DATA            | Order of appearance in the input data set                                                                                                |
| FORMATTED       | External formatted values, except for numeric variables with no explicit format, which are sorted by their unformatted (internal) values |
| FREQ            | Descending frequency count; levels with more observations come earlier in the order                                                      |
| INTERNAL        | Unformatted value                                                                                                                        |

For more information about sort order, see the chapter on the SORT procedure in the *Base SAS Procedures Guide* and the discussion of BY-group processing in *SAS Language Reference: Concepts*.

**PARAM=keyword**

specifies the parameterization method for the classification variable or variables. You can specify any of the *keywords* shown in the following table; the default is PARAM=EFFECT. Design matrix columns are created from CLASS variables according to the corresponding coding schemes:

| Value of PARAM=           | Coding                                                                                              |
|---------------------------|-----------------------------------------------------------------------------------------------------|
| EFFECT                    | Effect coding                                                                                       |
| GLM                       | Less-than-full-rank reference cell coding (this <i>keyword</i> can be used only in a global option) |
| ORDINAL<br>THERMOMETER    | Cumulative parameterization for an ordinal CLASS variable                                           |
| POLYNOMIAL<br>POLY        | Polynomial coding                                                                                   |
| REFERENCE<br>REF          | Reference cell coding                                                                               |
| ORTHEFFECT                | Orthogonalizes PARAM=EFFECT coding                                                                  |
| ORTHORDINAL<br>ORTHOTHERM | Orthogonalizes PARAM=ORDINAL coding                                                                 |
| ORTHPOLY                  | Orthogonalizes PARAM=POLYNOMIAL coding                                                              |
| ORTHREF                   | Orthogonalizes PARAM=REFERENCE coding                                                               |

All parameterizations are full rank, except for the GLM parameterization. The [REF=](#) option in the

CLASS statement determines the reference level for EFFECT and REFERENCE coding and for their orthogonal parameterizations. It also indirectly determines the reference level for a singular GLM parameterization through the order of levels.

If PARAM=ORTHOPOLY or PARAM=POLY and the classification variable is numeric, then the ORDER= option in the CLASS statement is ignored, and the internal unformatted values are used. See the section “Other Parameterizations” on page 387 of Chapter 19, “Shared Concepts and Topics,” for further details.

**REF=** *'level' | keyword*

specifies the reference level for PARAM=EFFECT, PARAM=REFERENCE, and their orthogonalizations. For PARAM=GLM, the REF= option specifies a level of the classification variable to be put at the end of the list of levels. This level thus corresponds to the reference level in the usual interpretation of the linear estimates with a singular parameterization.

For an individual variable REF= option (but not for a global REF= option), you can specify the *level* of the variable to use as the reference level. Specify the formatted value of the variable if a format is assigned. For a global or individual variable REF= option, you can use one of the following *keywords*. The default is REF=LAST.

**FIRST**     designates the first ordered level as reference.

**LAST**      designates the last ordered level as reference.

**TRUNCATE** < =*n* >

specifies the length *n* of CLASS variable values to use in determining CLASS variable levels. The default is to use the full formatted length of the CLASS variable. If you specify TRUNCATE without the length *n*, the first 16 characters of the formatted values are used. When formatted values are longer than 16 characters, you can use this option to revert to the levels as determined in releases before SAS 9. The TRUNCATE option is available only as a global option.

## Class Variable Naming Convention

Parameter names for a CLASS predictor variable are constructed by concatenating the CLASS variable name with the CLASS levels. However, for the POLYNOMIAL and orthogonal parameterizations, parameter names are formed by concatenating the CLASS variable name and keywords that reflect the parameterization. See the section “Other Parameterizations” on page 387 in Chapter 19, “Shared Concepts and Topics,” for examples and further details.

## Class Variable Parameterization with Unbalanced Designs

PROC LOGISTIC initially parameterizes the CLASS variables by looking at the levels of the variables across the complete data set. If you have an *unbalanced* replication of levels across variables or BY groups, then the design matrix and the parameter interpretation might be different from what you expect. For instance, suppose you have a model with one CLASS variable A with three levels (1, 2, and 3), and another CLASS variable B with two levels (1 and 2). If the third level of A occurs only with the first level of B, if you use the EFFECT parameterization, and if your model contains the effect A(B) and an intercept, then the design for A within the second level of B is not a differential effect. In particular, the design looks like the following:

| B | A | Design Matrix |    |        |    |
|---|---|---------------|----|--------|----|
|   |   | A(B=1)        |    | A(B=2) |    |
|   |   | A1            | A2 | A1     | A2 |
| 1 | 1 | 1             | 0  | 0      | 0  |
| 1 | 2 | 0             | 1  | 0      | 0  |
| 1 | 3 | -1            | -1 | 0      | 0  |
| 2 | 1 | 0             | 0  | 1      | 0  |
| 2 | 2 | 0             | 0  | 0      | 1  |

PROC LOGISTIC detects linear dependency among the last two design variables and sets the parameter for A2(B=2) to zero, resulting in an interpretation of these parameters as if they were reference- or dummy-coded. The REFERENCE or GLM parameterization might be more appropriate for such problems.

## CODE Statement

**CODE** < options > ;

The CODE statement enables you to write SAS DATA step code for computing predicted values of the fitted model either to a file or to a catalog entry. This code can then be included in a DATA step to score new data.

Table 54.2 summarizes the *options* available in the CODE statement.

**Table 54.2** CODE Statement Options

| Option    | Description                                                                  |
|-----------|------------------------------------------------------------------------------|
| CATALOG=  | Names the catalog entry where the generated code is saved                    |
| DUMMIES   | Retains the dummy variables in the data set                                  |
| ERROR     | Computes the error function                                                  |
| FILE=     | Names the file where the generated code is saved                             |
| FORMAT=   | Specifies the numeric format for the regression coefficients                 |
| GROUP=    | Specifies the group identifier for array names and statement labels          |
| IMPUTE    | Imputes predicted values for observations with missing or invalid covariates |
| LINESIZE= | Specifies the line size of the generated code                                |
| LOOKUP=   | Specifies the algorithm for looking up CLASS levels                          |
| RESIDUAL  | Computes residuals                                                           |

For details about the syntax of the CODE statement, see the section “CODE Statement” on page 390 in Chapter 19, “Shared Concepts and Topics.”

## CONTRAST Statement

**CONTRAST** 'label' row-description< , . . . , row-description > < / options > ;

where a *row-description* is defined as follows:

*effect values*<, ..., *effect values*>

The CONTRAST statement provides a mechanism for obtaining customized hypothesis tests. It is similar to the CONTRAST and ESTIMATE statements in other modeling procedures.

The CONTRAST statement enables you to specify a matrix,  $\mathbf{L}$ , for testing the hypothesis  $\mathbf{L}\boldsymbol{\beta} = \mathbf{0}$ , where  $\boldsymbol{\beta}$  is the vector of intercept and slope parameters. You must be familiar with the details of the model parameterization that PROC LOGISTIC uses (for more information, see the [PARAM=](#) option in the section “[CLASS Statement](#)” on page 4187). Optionally, the CONTRAST statement enables you to estimate each row,  $\mathbf{l}_i'\boldsymbol{\beta}$ , of  $\mathbf{L}\boldsymbol{\beta}$  and test the hypothesis  $\mathbf{l}_i'\boldsymbol{\beta} = 0$ . Computed statistics are based on the asymptotic chi-square distribution of the Wald statistic.

There is no limit to the number of CONTRAST statements that you can specify, but they must appear after the [MODEL](#) statement.

The following parameters are specified in the CONTRAST statement:

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>label</i>  | identifies the contrast in the displayed output. A label is required for every contrast specified, and it must be enclosed in quotes.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <i>effect</i> | identifies an effect that appears in the <a href="#">MODEL</a> statement. The name INTERCEPT can be used as an effect when one or more intercepts are included in the model. You do not need to include all effects that are included in the <a href="#">MODEL</a> statement.                                                                                                                                                                                                                                                                                                                                                         |
| <i>values</i> | are constants that are elements of the $\mathbf{L}$ matrix associated with the effect. To correctly specify your contrast, it is crucial to know the ordering of parameters within each effect and the variable levels associated with any parameter. The “Class Level Information” table shows the ordering of levels within variables. The <a href="#">E</a> option, described later in this section, enables you to verify the proper correspondence of <i>values</i> to parameters. If too many values are specified for an effect, the extra ones are ignored. If too few values are specified, the remaining ones are set to 0. |

Multiple degree-of-freedom hypotheses can be tested by specifying multiple *row-descriptions*; the rows of  $\mathbf{L}$  are specified in order and are separated by commas. The degrees of freedom is the number of linearly independent constraints implied by the CONTRAST statement—that is, the rank of  $\mathbf{L}$ .

More details for specifying contrasts involving effects with full-rank parameterizations are given in the section “[Full-Rank Parameterized Effects](#)” on page 4192, while details for less-than-full-rank parameterized effects are given in the section “[Less-Than-Full-Rank Parameterized Effects](#)” on page 4193.

You can specify the following options after a slash (/):

**ALPHA=number**

specifies the level of significance  $\alpha$  for the  $100(1-\alpha)\%$  confidence interval for each contrast when the ESTIMATE option is specified. The value of *number* must be between 0 and 1. By default, *number* is equal to the value of the [ALPHA=](#) option in the PROC LOGISTIC statement, or 0.05 if that option is not specified.

**E**

displays the  $\mathbf{L}$  matrix.

**ESTIMATE=keyword**

estimates and tests each individual contrast (that is, each row,  $l'_i\beta$ , of  $\mathbf{L}\beta$ ), exponentiated contrast ( $e^{l'_i\beta}$ ), or predicted probability for the contrast ( $g^{-1}(l'_i\beta)$ ). PROC LOGISTIC displays the point estimate, its standard error, a Wald confidence interval, and a Wald chi-square test. The significance level of the confidence interval is controlled by the **ALPHA=** option. You can estimate the individual contrast, the exponentiated contrast, or the predicted probability for the contrast by specifying one of the following *keywords*:

**PARM**

estimates the individual contrast.

**EXP**

estimates the exponentiated contrast.

**BOTH**

estimates both the individual contrast and the exponentiated contrast.

**PROB**

estimates the predicted probability of the contrast.

**ALL**

estimates the individual contrast, the exponentiated contrast, and the predicted probability of the contrast.

For details about the computations of the standard errors and confidence limits, see the section “[Linear Predictor, Predicted Probability, and Confidence Limits](#)” on page 4253.

**SINGULAR=number**

tunes the estimability check. This option is ignored when a full-rank parameterization is specified. If  $\mathbf{v}$  is a vector, define  $\text{ABS}(\mathbf{v})$  to be the largest absolute value of the elements of  $\mathbf{v}$ . For a row vector  $\mathbf{l}'$  of the contrast matrix  $\mathbf{L}$ , define  $c = \text{ABS}(\mathbf{l})$  if  $\text{ABS}(\mathbf{l})$  is greater than 0; otherwise,  $c = 1$ . If  $\text{ABS}(\mathbf{l}' - \mathbf{l}'\mathbf{T})$  is greater than  $c*\text{number}$ , then  $\mathbf{l}$  is declared nonestimable. The  $\mathbf{T}$  matrix is the Hermite form matrix  $\mathbf{I}_0^{-1}\mathbf{I}_0$ , where  $\mathbf{I}_0^{-1}$  represents a generalized inverse of the (observed or expected) information matrix  $\mathbf{I}_0$  of the null model. The value for *number* must be between 0 and 1; the default value is 1E-4.

**Full-Rank Parameterized Effects**

If an effect involving a CLASS variable with a full-rank parameterization does not appear in the CONTRAST statement, then all of its coefficients in the  $\mathbf{L}$  matrix are set to 0.

If you use effect coding by default or by specifying **PARAM=EFFECT** in the **CLASS** statement, then all parameters are directly estimable and involve no other parameters. For example, suppose an effect-coded CLASS variable A has four levels. Then there are three parameters ( $\beta_1, \beta_2, \beta_3$ ) representing the first three levels, and the fourth parameter is represented by

$$-\beta_1 - \beta_2 - \beta_3$$

To test the first versus the fourth level of A, you would test

$$\beta_1 = -\beta_1 - \beta_2 - \beta_3$$

or, equivalently,

$$2\beta_1 + \beta_2 + \beta_3 = 0$$

which, in the form  $\mathbf{L}\boldsymbol{\beta} = 0$ , is

$$\begin{bmatrix} 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = 0$$

Therefore, you would use the following CONTRAST statement:

```
contrast '1 vs. 4' A 2 1 1;
```

To contrast the third level with the average of the first two levels, you would test

$$\frac{\beta_1 + \beta_2}{2} = \beta_3$$

or, equivalently,

$$\beta_1 + \beta_2 - 2\beta_3 = 0$$

Therefore, you would use the following CONTRAST statement:

```
contrast '1&2 vs. 3' A 1 1 -2;
```

Other CONTRAST statements are constructed similarly. For example:

```
contrast '1 vs. 2' A 1 -1 0;
contrast '1&2 vs. 4' A 3 3 2;
contrast '1&2 vs. 3&4' A 2 2 0;
contrast 'Main Effect' A 1 0 0,
                        A 0 1 0,
                        A 0 0 1;
```

## Less-Than-Full-Rank Parameterized Effects

When you use the less-than-full-rank parameterization (by specifying `PARAM=GLM` in the `CLASS` statement), each row is checked for estimability; see the section “[Estimable Functions](#)” on page 59 in Chapter 3, “[Introduction to Statistical Modeling with SAS/STAT Software](#),” for more information. If PROC LOGISTIC finds a contrast to be nonestimable, it displays missing values in corresponding rows in the results. PROC LOGISTIC handles missing level combinations of classification variables in the same manner as PROC GLM: parameters corresponding to missing level combinations are not included in the model. This convention can affect the way in which you specify the  $\mathbf{L}$  matrix in your CONTRAST statement. If the elements of  $\mathbf{L}$  are not specified for an effect that contains a specified effect, then the elements of the specified effect are distributed over the levels of the higher-order effect just as the GLM procedure does for its CONTRAST and ESTIMATE statements. For example, suppose that the model contains effects A and B and their interaction A\*B. If you specify a CONTRAST statement involving A alone, the  $\mathbf{L}$  matrix contains nonzero terms for both A and A\*B, since A\*B contains A. See rule 4 in the section “[Construction of Least Squares Means](#)” on page 3363 in Chapter 42, “[The GLM Procedure](#),” for more details.

## EFFECT Statement

**EFFECT** *name=effect-type (variables < / options>)* ;

The EFFECT statement enables you to construct special collections of columns for design matrices. These collections are referred to as *constructed effects* to distinguish them from the usual model effects that are formed from continuous or classification variables, as discussed in the section “GLM Parameterization of Classification Variables and Effects” on page 383 in Chapter 19, “Shared Concepts and Topics.”

You can specify the following *effect-types*:

|                          |                                                                                                                                                                                                           |
|--------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>COLLECTION</b>        | is a collection effect that defines one or more variables as a single effect with multiple degrees of freedom. The variables in a collection are considered as a unit for estimation and inference.       |
| <b>LAG</b>               | is a classification effect in which the level that is used for a given period corresponds to the level in the preceding period.                                                                           |
| <b>MULTIMEMBER   MM</b>  | is a multimember classification effect whose levels are determined by one or more variables that appear in a CLASS statement.                                                                             |
| <b>POLYNOMIAL   POLY</b> | is a multivariate polynomial effect in the specified numeric variables.                                                                                                                                   |
| <b>SPLINE</b>            | is a regression spline effect whose columns are univariate spline expansions of one or more variables. A spline expansion replaces the original variable with an expanded or larger set of new variables. |

Table 54.3 summarizes the *options* available in the EFFECT statement.

**Table 54.3** EFFECT Statement Options

| Option                             | Description                                                                                                                                          |
|------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Collection Effects Options</b>  |                                                                                                                                                      |
| DETAILS                            | Displays the constituents of the collection effect                                                                                                   |
| <b>Lag Effects Options</b>         |                                                                                                                                                      |
| DESIGNROLE=                        | Names a variable that controls to which lag design an observation is assigned                                                                        |
| DETAILS                            | Displays the lag design of the lag effect                                                                                                            |
| NLAG=                              | Specifies the number of periods in the lag                                                                                                           |
| PERIOD=                            | Names the variable that defines the period                                                                                                           |
| WITHIN=                            | Names the variable or variables that define the group within which each period is defined                                                            |
| <b>Multimember Effects Options</b> |                                                                                                                                                      |
| NOEFFECT                           | Specifies that observations with all missing levels for the multimember variables should have zero values in the corresponding design matrix columns |
| WEIGHT=                            | Specifies the weight variable for the contributions of each of the classification effects                                                            |



Table 54.3 *continued*

| Option                            | Description                                                                                             |
|-----------------------------------|---------------------------------------------------------------------------------------------------------|
| <b>Polynomial Effects Options</b> |                                                                                                         |
| DEGREE=                           | Specifies the degree of the polynomial                                                                  |
| MDEGREE=                          | Specifies the maximum degree of any variable in a term of the polynomial                                |
| STANDARDIZE=                      | Specifies centering and scaling suboptions for the variables that define the polynomial                 |
| <b>Spline Effects Options</b>     |                                                                                                         |
| BASIS=                            | Specifies the type of basis (B-spline basis or truncated power function basis) for the spline expansion |
| DEGREE=                           | Specifies the degree of the spline transformation                                                       |
| KNOTMETHOD=                       | Specifies how to construct the knots for spline effects                                                 |

For more information about the syntax of these *effect-types* and how columns of constructed effects are computed, see the section “EFFECT Statement” on page 393 in Chapter 19, “Shared Concepts and Topics.”

## EFFECTPLOT Statement

**EFFECTPLOT** < *plot-type* < (*plot-definition-options*) > > < / *options* > ;

The EFFECTPLOT statement produces a display of the fitted model and provides options for changing and enhancing the displays. Table 54.4 describes the available *plot-types* and their *plot-definition-options*.

Table 54.4 *Plot-Types and Plot-Definition-Options*

| Plot-Type and Description                                                                                                                                                                                                      | Plot-Definition-Options                                                                                   |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| <b>BOX</b><br>Displays a box plot of continuous response data at each level of a CLASS effect, with predicted values superimposed and connected by a line. This is an alternative to the <b>INTERACTION</b> <i>plot-type</i> . | <b>PLOTBY=</b> variable or CLASS effect<br><b>X=</b> CLASS variable or effect                             |
| <b>CONTOUR</b><br>Displays a contour plot of predicted values against two continuous covariates.                                                                                                                               | <b>PLOTBY=</b> variable or CLASS effect<br><b>X=</b> continuous variable<br><b>Y=</b> continuous variable |
| <b>FIT</b><br>Displays a curve of predicted values versus a continuous variable.                                                                                                                                               | <b>PLOTBY=</b> variable or CLASS effect<br><b>X=</b> continuous variable                                  |

Table 54.4 continued

| Plot-Type and Description                                                                                                                                                                                                         | Plot-Definition-Options                                                                                                   |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------|
| <b>INTERACTION</b><br>Displays a plot of predicted values (possibly with error bars) versus the levels of a CLASS effect. The predicted values are connected with lines and can be grouped by the levels of another CLASS effect. | <b>PLOTBY=</b> variable or CLASS effect<br><b>SLICEBY=</b> variable or CLASS effect<br><b>X=</b> CLASS variable or effect |
| <b>SLICEFIT</b><br>Displays a curve of predicted values versus a continuous variable grouped by the levels of a CLASS effect.                                                                                                     | <b>PLOTBY=</b> variable or CLASS effect<br><b>SLICEBY=</b> variable or CLASS effect<br><b>X=</b> continuous variable      |

For full details about the syntax and options of the EFFECTPLOT statement, see the section “[EFFECTPLOT Statement](#)” on page 411 in Chapter 19, “[Shared Concepts and Topics](#).”

See Outputs [54.2.11](#), [54.2.12](#), [54.3.5](#), [54.4.8](#), [54.7.4](#), and [54.15.4](#) for examples of plots produced by this statement.

## ESTIMATE Statement

```
ESTIMATE <'label'> estimate-specification <(divisor=n)>
      <,...<'label'> estimate-specification <(divisor=n)>>
      </options>;
```

The ESTIMATE statement provides a mechanism for obtaining custom hypothesis tests. Estimates are formed as linear estimable functions of the form  $\mathbf{L}\boldsymbol{\beta}$ . You can perform hypothesis tests for the estimable functions, construct confidence limits, and obtain specific nonlinear transformations.

Table 54.5 summarizes the options available in the ESTIMATE statement.

Table 54.5 ESTIMATE Statement Options

| Option                                                     | Description                                                               |
|------------------------------------------------------------|---------------------------------------------------------------------------|
| <b>Construction and Computation of Estimable Functions</b> |                                                                           |
| <b>DIVISOR=</b>                                            | Specifies a list of values to divide the coefficients                     |
| <b>NOFILL</b>                                              | Suppresses the automatic fill-in of coefficients for higher-order effects |
| <b>SINGULAR=</b>                                           | Tunes the estimability checking difference                                |

Table 54.5 *continued*

| Option                                              | Description                                                                     |
|-----------------------------------------------------|---------------------------------------------------------------------------------|
| <b>Degrees of Freedom and <math>p</math>-values</b> |                                                                                 |
| ADJUST=                                             | Determines the method for multiple comparison adjustment of estimates           |
| ALPHA= $\alpha$                                     | Determines the confidence level $(1 - \alpha)$                                  |
| LOWER                                               | Performs one-sided, lower-tailed inference                                      |
| STEPDOWN                                            | Adjusts multiplicity-corrected $p$ -values further in a step-down fashion       |
| TESTVALUE=                                          | Specifies values under the null hypothesis for tests                            |
| UPPER                                               | Performs one-sided, upper-tailed inference                                      |
| <b>Statistical Output</b>                           |                                                                                 |
| CL                                                  | Constructs confidence limits                                                    |
| CORR                                                | Displays the correlation matrix of estimates                                    |
| COV                                                 | Displays the covariance matrix of estimates                                     |
| E                                                   | Prints the <b>L</b> matrix                                                      |
| JOINT                                               | Produces a joint $F$ or chi-square test for the estimable functions             |
| SEED=                                               | Specifies the seed for computations that depend on random numbers               |
| <b>Generalized Linear Modeling</b>                  |                                                                                 |
| CATEGORY=                                           | Specifies how to construct estimable functions with multinomial data            |
| EXP                                                 | Exponentiates and displays estimates                                            |
| ILINK                                               | Computes and displays estimates and standard errors on the inverse linked scale |

For details about the syntax of the ESTIMATE statement, see the section “[ESTIMATE Statement](#)” on page 437 in Chapter 19, “[Shared Concepts and Topics](#).”

## EXACT Statement

**EXACT** < *label* > < **INTERCEPT** > < *effects* > < / *options* > ;

The EXACT statement performs exact tests of the parameters for the specified *effects* and optionally estimates the parameters and outputs the exact conditional distributions. You can specify the keyword **INTERCEPT** and any effects in the [MODEL](#) statement. Inference on the parameters of the specified effects is performed by conditioning on the sufficient statistics of all the other model parameters (possibly including the intercept).

You can specify several EXACT statements, but they must follow the [MODEL](#) statement. Each statement can optionally include an identifying *label*. If several EXACT statements are specified, any statement without a label is assigned a label of the form “Exact $n$ ,” where  $n$  indicates the  $n$ th EXACT statement. The label is included in the headers of the displayed exact analysis tables.

If a **STRATA** statement is also specified, then a stratified exact logistic regression is performed. The model contains a different intercept for each stratum, and these intercepts are conditioned out of the model along with any other nuisance parameters (parameters for effects specified in the **MODEL** statement that are not in the **EXACT** statement).

If the **LINK=GLOGIT** option is specified in the **MODEL** statement, then the **METHOD=DIRECT** option is invoked in the **EXACTOPTIONS** statement by default and a generalized logit model is fit. Since each effect specified in the **MODEL** statement adds  $k$  parameters to the model (where  $k + 1$  is the number of response levels), exact analysis of the generalized logit model by using this method is limited to rather small problems.

The **CONTRAST**, **ESTIMATE**, **LSMEANS**, **LSMESTIMATE**, **ODDSRATIO**, **OUTPUT**, **ROC**, **ROCCONTRAST**, **SCORE**, **SLICE**, **STORE**, **TEST**, and **UNITS** statements are not available with an exact analysis; results from these statements are based on the asymptotic results. Exact analyses are not performed when you specify a **WEIGHT** statement, a link other than **LINK=LOGIT** or **LINK=GLOGIT**, an offset variable, the **NOFIT** option, or a model selection method. Exact estimation is not available for ordinal response models.

For classification variables, use of the reference parameterization is recommended.

The following options can be specified in each **EXACT** statement after a slash (/):

**ALPHA=number**

specifies the level of significance  $\alpha$  for  $100(1 - \alpha)\%$  confidence limits for the parameters or odds ratios. The value of *number* must be between 0 and 1. By default, *number* is equal to the value of the **ALPHA=** option in the **PROC LOGISTIC** statement, or 0.05 if that option is not specified.

**CLTYPE=EXACT | MIDP**

requests either the exact or mid- $p$  confidence intervals for the parameter estimates. By default, the exact intervals are produced. The confidence coefficient can be specified with the **ALPHA=** option. The mid- $p$  interval can be modified with the **MIDPFACTOR=** option. See the section “Exact Conditional Logistic Regression” on page 4274 for details.

**ESTIMATE <=keyword>**

estimates the individual parameters (conditioned on all other parameters) for the effects specified in the **EXACT** statement. For each parameter, a point estimate, a standard error, a confidence interval, and a  $p$ -value for a two-sided test that the parameter is zero are displayed. Note that the two-sided  $p$ -value is twice the one-sided  $p$ -value. You can optionally specify one of the following keywords:

**PARM** specifies that the parameters be estimated. This is the default.

**ODDS** specifies that the odds ratios be estimated. If you have classification variables, then you must also specify the **PARAM=REF** option in the **CLASS** statement.

**BOTH** specifies that both the parameters and odds ratios be estimated.

**JOINT**

performs the joint test that all of the parameters are simultaneously equal to zero, performs individual hypothesis tests for the parameter of each continuous variable, and performs joint tests for the parameters of each classification variable. The joint test is indicated in the “Conditional Exact Tests” table by the label “Joint.”

**JOINTONLY**

performs only the joint test of the parameters. The test is indicated in the “Conditional Exact Tests” table by the label “Joint.” When this option is specified, individual tests for the parameters of each continuous variable and joint tests for the parameters of the classification variables are not performed.

**MIDPFACTOR= $\delta_1$  | ( $\delta_1, \delta_2$ )**

sets the tie factors used to produce the mid- $p$  hypothesis statistics and the mid- $p$  confidence intervals.  $\delta_1$  modifies both the hypothesis tests and confidence intervals, while  $\delta_2$  affects only the hypothesis tests. By default,  $\delta_1 = 0.5$  and  $\delta_2 = 1.0$ . See the section “[Exact Conditional Logistic Regression](#)” on page 4274 for details.

**ONESIDED**

requests one-sided confidence intervals and  $p$ -values for the individual parameter estimates and odds ratios. The one-sided  $p$ -value is the smaller of the left- and right-tail probabilities for the observed sufficient statistic of the parameter under the null hypothesis that the parameter is zero. The two-sided  $p$ -values (default) are twice the one-sided  $p$ -values. See the section “[Exact Conditional Logistic Regression](#)” on page 4274 for more details.

**OUTDIST=*SAS-data-set***

names the SAS data set that contains the exact conditional distributions. This data set contains all of the exact conditional distributions that are required to process the corresponding EXACT statement. This data set contains the possible sufficient statistics for the parameters of the effects specified in the EXACT statement, the counts, and, when hypothesis tests are performed on the parameters, the probability of occurrence and the score value for each sufficient statistic. When you request an OUTDIST= data set, the observed sufficient statistics are displayed in the “Sufficient Statistics” table. See the section “[OUTDIST= Output Data Set](#)” on page 4281 for more information.

**EXACT Statement Examples**

In the following example, two exact tests are computed: one for  $x_1$  and the other for  $x_2$ . The test for  $x_1$  is based on the exact conditional distribution of the sufficient statistic for the  $x_1$  parameter given the observed values of the sufficient statistics for the intercept,  $x_2$ , and  $x_3$  parameters; likewise, the test for  $x_2$  is conditional on the observed sufficient statistics for the intercept,  $x_1$ , and  $x_3$ .

```
proc logistic;
  model y= x1 x2 x3;
  exact x1 x2;
run;
```

PROC LOGISTIC determines, from all the specified EXACT statements, the distinct conditional distributions that need to be evaluated. For example, there is only one exact conditional distribution for the following two EXACT statements:

```
exact 'One' x1 / estimate=parm;
exact 'Two' x1 / estimate=parm onesided;
```

For each EXACT statement, individual tests for the parameters of the specified effects are computed unless the **JOINTONLY** option is specified. Consider the following EXACT statements:

```

exact 'E12' x1 x2 / estimate;
exact 'E1'  x1      / estimate;
exact 'E2'  x2      / estimate;
exact 'J12' x1 x2 / joint;

```

In the E12 statement, the parameters for x1 and x2 are estimated and tested separately. Specifying the E12 statement is equivalent to specifying both the E1 and E2 statements. In the J12 statement, the joint test for the parameters of x1 and x2 is computed in addition to the individual tests for x1 and x2.

---

## EXACTOPTIONS Statement

**EXACTOPTIONS** *options* ;

The EXACTOPTIONS statement specifies options that apply to every **EXACT** statement in the program. The following *options* are available:

### **ABSFCNV=value**

specifies the absolute function convergence criterion. Convergence requires a small change in the log-likelihood function in subsequent iterations,

$$|l_i - l_{i-1}| < \text{value}$$

where  $l_i$  is the value of the log-likelihood function at iteration  $i$ .

By default, ABSFCNV=1E-12. You can also specify the **FCONV=** and **XCONV=** criteria; optimizations are terminated as soon as one criterion is satisfied.

### **ADDTOBS**

adds the observed sufficient statistic to the sampled exact distribution if the statistic was not sampled. This option has no effect unless the **METHOD=NETWORKMC** option is specified and the **ESTIMATE** option is specified in the **EXACT** statement. If the observed statistic has not been sampled, then the parameter estimate does not exist; by specifying this option, you can produce (biased) estimates.

### **BUILDSUBSETS**

builds every distribution for sampling. By default, some exact distributions are created by taking a subset of a previously generated exact distribution. When the **METHOD=NETWORKMC** option is invoked, this subsetting behavior has the effect of using fewer than the desired  $n$  samples; see the **N=option** for more details. Use the **BUILDSUBSETS** option to suppress this subsetting.

### **EPSILON=value**

controls how the partial sums  $\sum_{i=1}^j y_i x_i$  are compared. *value* must be between 0 and 1; by default, *value*=1E-8.

### **FCONV=value**

specifies the relative function convergence criterion. Convergence requires a small relative change in the log-likelihood function in subsequent iterations,

$$\frac{|l_i - l_{i-1}|}{|l_{i-1}| + 1\text{E-}6} < \text{value}$$

where  $l_i$  is the value of the log likelihood at iteration  $i$ .

By default, FCONV=1E-8. You can also specify the **ABSFCNV=** and **XCONV=** criteria; if more than one criterion is specified, then optimizations are terminated as soon as one criterion is satisfied.

**MAXTIME=seconds**

specifies the maximum clock time (in seconds) that PROC LOGISTIC can use to calculate the exact distributions. If the limit is exceeded, the procedure halts all computations and prints a note to the LOG. The default maximum clock time is seven days.

**METHOD=keyword**

specifies which exact conditional algorithm to use for every **EXACT** statement specified. You can specify one of the following *keywords*:

**DIRECT** invokes the multivariate shift algorithm of Hirji, Mehta, and Patel (1987). This method directly builds the exact distribution, but it can require an excessive amount of memory in its intermediate stages. **METHOD=DIRECT** is invoked by default when you are conditioning out at most the intercept, or when the **LINK=GLOGIT** option is specified in the **MODEL** statement.

**NETWORK** invokes an algorithm described in Mehta, Patel, and Senchaudhuri (1992). This method builds a network for each parameter that you are conditioning out, combines the networks, then uses the multivariate shift algorithm to create the exact distribution. The **NETWORK** method can be faster and require less memory than the **DIRECT** method. The **NETWORK** method is invoked by default for most analyses.

**NETWORKMC** invokes the hybrid network and Monte Carlo algorithm of Mehta, Patel, and Senchaudhuri (1992). This method creates a network, then samples from that network; this method does not reject any of the samples at the cost of using a large amount of memory to create the network. **METHOD=NETWORKMC** is most useful for producing parameter estimates for problems that are too large for the **DIRECT** and **NETWORK** methods to handle and for which asymptotic methods are invalid—for example, for sparse data on a large grid.

**N=n**

specifies the number of Monte Carlo samples to take when the **METHOD=NETWORKMC** option is specified. By default,  $n = 10,000$ . If the procedure cannot obtain  $n$  samples due to a lack of memory, then a note is printed in the SAS log (the number of valid samples is also reported in the listing) and the analysis continues.

The number of samples used to produce any particular statistic might be smaller than  $n$ . For example, let  $X1$  and  $X2$  be continuous variables, denote their joint distribution by  $f(X1, X2)$ , and let  $f(X1 | X2 = x2)$  denote the marginal distribution of  $X1$  conditioned on the observed value of  $X2$ . If you request the **JOINT** test of  $X1$  and  $X2$ , then  $n$  samples are used to generate the estimate  $\hat{f}(X1, X2)$  of  $f(X1, X2)$ , from which the test is computed. However, the parameter estimate for  $X1$  is computed from the subset of  $\hat{f}(X1, X2)$  that has  $X2 = x2$ , and this subset need not contain  $n$  samples. Similarly, the distribution for each level of a classification variable is created by extracting the appropriate subset from the joint distribution for the **CLASS** variable.

In some cases, the marginal sample size can be too small to admit accurate estimation of a particular statistic; a note is printed in the SAS log when a marginal sample size is less than 100. Increasing  $n$

increases the number of samples used in a marginal distribution; however, if you want to control the sample size exactly, you can either specify the **BUILDSUBSETS** option or do both of the following:

- Remove the **JOINT** option from the **EXACT** statement.
- Create dummy variables in a DATA step to represent the levels of a **CLASS** variable, and specify them as independent variables in the **MODEL** statement.

### **NOLOGSCALE**

specifies that computations for the exact conditional models be computed by using normal scaling. Log scaling can handle numerically larger problems than normal scaling; however, computations in the log scale are slower than computations in normal scale.

### **ONDISK**

uses disk space instead of random access memory to build the exact conditional distribution. Use this option to handle larger problems at the cost of slower processing.

### **SEED=seed**

specifies the initial seed for the random number generator used to take the Monte Carlo samples when the **METHOD=NETWORKMC** option is specified. The value of the **SEED=** option must be an integer. If you do not specify a seed, or if you specify a value less than or equal to zero, then PROC LOGISTIC uses the time of day from the computer's clock to generate an initial seed.

### **STATUSN=number**

prints a status line in the SAS log after every *number* of Monte Carlo samples when the **METHOD=NETWORKMC** option is specified. The number of samples taken and the current exact *p*-value for testing the significance of the model are displayed. You can use this status line to track the progress of the computation of the exact conditional distributions.

### **STATUSTIME=seconds**

specifies the time interval (in seconds) for printing a status line in the LOG. You can use this status line to track the progress of the computation of the exact conditional distributions. The time interval you specify is approximate; the actual time interval varies. By default, no status reports are produced.

### **XCONV=value**

specifies the relative parameter convergence criterion. Convergence requires a small relative parameter change in subsequent iterations,

$$\max_j |\delta_j^{(i)}| < \text{value}$$

where

$$\delta_j^{(i)} = \begin{cases} \beta_j^{(i)} - \beta_j^{(i-1)} & |\beta_j^{(i-1)}| < 0.01 \\ \frac{\beta_j^{(i)} - \beta_j^{(i-1)}}{\beta_j^{(i-1)}} & \text{otherwise} \end{cases}$$

and  $\beta_j^{(i)}$  is the estimate of the *j*th parameter at iteration *i*.

By default, **XCONV=1E-4**. You can also specify the **ABSFCNV=** and **FCNV=** criteria; if more than one criterion is specified, then optimizations are terminated as soon as one criterion is satisfied.



---

## FREQ Statement

**FREQ** *variable* ;

The FREQ statement identifies a *variable* that contains the frequency of occurrence of each observation. PROC LOGISTIC treats each observation as if it appears  $n$  times, where  $n$  is the value of the FREQ variable for the observation. If it is not an integer, the frequency value is truncated to an integer. If the frequency value is less than 1 or missing, the observation is not used in the model fitting. When the FREQ statement is not specified, each observation is assigned a frequency of 1. If you specify more than one FREQ statement, then the first statement is used.

If a **SCORE** statement is specified, then the FREQ variable is used for computing fit statistics and the ROC curve, but they are not required for scoring. If the **DATA=** data set in the **SCORE** statement does not contain the FREQ variable, the frequency values are assumed to be 1 and a warning message is issued in the LOG. If you fit a model and perform the scoring in the same run, the same FREQ variable is used for fitting and scoring. If you fit a model in a previous run and input it with the **INMODEL=** option in the current run, then the FREQ variable can be different from the one used in the previous run. However, if a FREQ variable was not specified in the previous run, you can still specify a FREQ variable in the current run.

---

## ID Statement

**ID** *variable* < *variable*,... > ;

The ID statement specifies variables in the **DATA=** data set that are used for labeling ROC curves and influence diagnostic plots. If more than one ID variable is specified, then the plots are labeled by concatenating the ID variable values together. See the **PLOTS(LABEL)** and **ROCOPTIONS(ID)** options in the PROC LOGISTIC statement for more details.

---

## LSMEANS Statement

**LSMEANS** < *model-effects* > < / *options* > ;

The LSMEANS statement computes and compares least squares means (LS-means) of fixed effects. LS-means are *predicted population margins*—that is, they estimate the marginal means over a balanced population. In a sense, LS-means are to unbalanced designs as class and subclass arithmetic means are to balanced designs.

Table 54.6 summarizes the options available in the LSMEANS statement.

**Table 54.6** LSMEANS Statement Options

| Option                                              | Description                                                                                                       |
|-----------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| <b>Construction and Computation of LS-Means</b>     |                                                                                                                   |
| AT                                                  | Modifies the covariate value in computing LS-means                                                                |
| BYLEVEL                                             | Computes separate margins                                                                                         |
| DIFF                                                | Requests differences of LS-means                                                                                  |
| OM=                                                 | Specifies the weighting scheme for LS-means computation as determined by the input data set                       |
| SINGULAR=                                           | Tunes estimability checking                                                                                       |
| <b>Degrees of Freedom and <math>p</math>-values</b> |                                                                                                                   |
| ADJUST=                                             | Determines the method for multiple-comparison adjustment of LS-means differences                                  |
| ALPHA= $\alpha$                                     | Determines the confidence level ( $1 - \alpha$ )                                                                  |
| STEPDOWN                                            | Adjusts multiple-comparison $p$ -values further in a step-down fashion                                            |
| <b>Statistical Output</b>                           |                                                                                                                   |
| CL                                                  | Constructs confidence limits for means and mean differences                                                       |
| CORR                                                | Displays the correlation matrix of LS-means                                                                       |
| COV                                                 | Displays the covariance matrix of LS-means                                                                        |
| E                                                   | Prints the <b>L</b> matrix                                                                                        |
| LINES                                               | Produces a “Lines” display for pairwise LS-means differences                                                      |
| MEANS                                               | Prints the LS-means                                                                                               |
| PLOTS=                                              | Requests graphs of means and mean comparisons                                                                     |
| SEED=                                               | Specifies the seed for computations that depend on random numbers                                                 |
| <b>Generalized Linear Modeling</b>                  |                                                                                                                   |
| EXP                                                 | Exponentiates and displays estimates of LS-means or LS-means differences                                          |
| ILINK                                               | Computes and displays estimates and standard errors of LS-means (but not differences) on the inverse linked scale |
| ODDSRATIO                                           | Reports (simple) differences of least squares means in terms of odds ratios if permitted by the link function     |

For details about the syntax of the LSMEANS statement, see the section “[LSMEANS Statement](#)” on page 453 in Chapter 19, “[Shared Concepts and Topics](#).”

**NOTE:** If you have classification variables in your model, then the LSMEANS statement is allowed only if you also specify the [PARAM=GLM](#) option.

## LSMESTIMATE Statement

```
LSMESTIMATE model-effect < 'label' > values < divisor=n >
            < , ... < 'label' > values < divisor=n > >
            < / options > ;
```

The LSMESTIMATE statement provides a mechanism for obtaining custom hypothesis tests among least squares means.

Table 54.7 summarizes the options available in the LSMESTIMATE statement.

**Table 54.7** LSMESTIMATE Statement Options

| Option                                              | Description                                                                         |
|-----------------------------------------------------|-------------------------------------------------------------------------------------|
| <b>Construction and Computation of LS-Means</b>     |                                                                                     |
| AT                                                  | Modifies covariate values in computing LS-means                                     |
| BYLEVEL                                             | Computes separate margins                                                           |
| DIVISOR=                                            | Specifies a list of values to divide the coefficients                               |
| OM=                                                 | Specifies the weighting scheme for LS-means computation as determined by a data set |
| SINGULAR=                                           | Tunes estimability checking                                                         |
| <b>Degrees of Freedom and <math>p</math>-values</b> |                                                                                     |
| ADJUST=                                             | Determines the method for multiple-comparison adjustment of LS-means differences    |
| ALPHA= $\alpha$                                     | Determines the confidence level $(1 - \alpha)$                                      |
| LOWER                                               | Performs one-sided, lower-tailed inference                                          |
| STEPDOWN                                            | Adjusts multiple-comparison $p$ -values further in a step-down fashion              |
| TESTVALUE=                                          | Specifies values under the null hypothesis for tests                                |
| UPPER                                               | Performs one-sided, upper-tailed inference                                          |
| <b>Statistical Output</b>                           |                                                                                     |
| CL                                                  | Constructs confidence limits for means and mean differences                         |
| CORR                                                | Displays the correlation matrix of LS-means                                         |
| COV                                                 | Displays the covariance matrix of LS-means                                          |
| E                                                   | Prints the <b>L</b> matrix                                                          |
| ELSM                                                | Prints the <b>K</b> matrix                                                          |
| JOINT                                               | Produces a joint $F$ or chi-square test for the LS-means and LS-means differences   |
| SEED=                                               | Specifies the seed for computations that depend on random numbers                   |

Table 54.7 continued

| Option                             | Description                                                                                                       |
|------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| <b>Generalized Linear Modeling</b> |                                                                                                                   |
| <b>CATEGORY=</b>                   | Specifies how to construct estimable functions with multinomial data                                              |
| <b>EXP</b>                         | Exponentiates and displays LS-means estimates                                                                     |
| <b>ILINK</b>                       | Computes and displays estimates and standard errors of LS-means (but not differences) on the inverse linked scale |

For details about the syntax of the LSMESTIMATE statement, see the section “[LSMESTIMATE Statement](#)” on page 470 in Chapter 19, “[Shared Concepts and Topics](#).”

**NOTE:** If you have classification variables in your model, then the LSMESTIMATE statement is allowed only if you also specify the [PARAM=GLM](#) option.

## MODEL Statement

```
<label:> MODEL variable<(variable_options)> = <effects> </options> ;
```

```
<label:> MODEL events/trials = <effects> </options> ;
```

The MODEL statement names the response variable and the explanatory effects, including covariates, main effects, interactions, and nested effects; see the section “[Specification of Effects](#)” on page 3324 of Chapter 42, “[The GLM Procedure](#),” for more information. If you omit the explanatory effects, the procedure fits an intercept-only model. You must specify exactly one MODEL statement.

Two forms of the MODEL statement can be specified. The first form, referred to as *single-trial* syntax, is applicable to binary, ordinal, and nominal response data. The second form, referred to as *events/trials* syntax, is restricted to the case of binary response data. The single-trial syntax is used when each observation in the DATA= data set contains information about only a single trial, such as a single subject in an experiment. When each observation contains information about multiple binary-response trials, such as the counts of the number of subjects observed and the number responding, then events/trials syntax can be used.

In the events/trials syntax, you specify two variables that contain count data for a binomial experiment. These two variables are separated by a slash. The value of the first variable, *events*, is the number of positive responses (or events). The value of the second variable, *trials*, is the number of trials. The values of both *events* and (*trials*–*events*) must be nonnegative and the value of *trials* must be positive for the response to be valid.

In the single-trial syntax, you specify one variable (on the left side of the equal sign) as the response variable. This variable can be character or numeric. [Variable\\_options](#) specific to the response variable can be specified immediately after the response variable with parentheses around them.

For both forms of the MODEL statement, explanatory *effects* follow the equal sign. Variables can be either continuous or classification variables. Classification variables can be character or numeric, and they must be declared in the [CLASS](#) statement. When an effect is a classification variable, the procedure inserts a set of coded columns into the design matrix instead of directly entering a single column containing the values of the variable.

## Response Variable Options

### DESCENDING | DESC

reverses the order of the response categories. If both the DESCENDING and **ORDER=** options are specified, PROC LOGISTIC orders the response categories according to the **ORDER=** option and then reverses that order. See the section “[Response Level Ordering](#)” on page 4237 for more detail.

### EVENT='category' | keyword

specifies the event category for the binary response model. PROC LOGISTIC models the probability of the event category. The **EVENT=** option has no effect when there are more than two response categories. You can specify the value (formatted if a format is applied) of the event category in quotes, or you can specify one of the following keywords. The default is **EVENT=FIRST**.

#### FIRST

designates the first ordered category as the event.

#### LAST

designates the last ordered category as the event.

One of the most common sets of response levels is {0,1}, with 1 representing the event for which the probability is to be modeled. Consider the example where Y takes the values 1 and 0 for event and nonevent, respectively, and Exposure is the explanatory variable. To specify the value 1 as the event category, use the following MODEL statement:

```
model Y(event='1') = Exposure;
```

### ORDER= DATA | FORMATTED | FREQ | INTERNAL

specifies the sort order for the levels of the response variable. The following table displays the available **ORDER=** options:

| ORDER=    | Levels Sorted By                                                                                                                       |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------|
| DATA      | order of appearance in the input data set                                                                                              |
| FORMATTED | external formatted value, except for numeric variables with no explicit format, which are sorted by their unformatted (internal) value |
| FREQ      | descending frequency count; levels with the most observations come first in the order                                                  |
| INTERNAL  | unformatted value                                                                                                                      |

By default, **ORDER=FORMATTED**. For **ORDER=FORMATTED** and **ORDER=INTERNAL**, the sort order is machine dependent. When **ORDER=FORMATTED** is in effect for numeric variables for which you have supplied no explicit format, the levels are ordered by their internal values.

For more information about sort order, see the chapter on the SORT procedure in the *Base SAS Procedures Guide* and the discussion of BY-group processing in *SAS Language Reference: Concepts*.

**REFERENCE=**'category' | keyword

**REF=**'category' | keyword

specifies the reference category for the generalized logit model and the binary response model. For the generalized logit model, each logit contrasts a nonreference category with the reference category. For the binary response model, specifying one response category as the reference is the same as specifying the other response category as the event category. You can specify the value (formatted if a format is applied) of the reference category in quotes, or you can specify one of the following keywords:

**FIRST**     designates the first ordered category as the reference.

**LAST**      designates the last ordered category as the reference. This is the default.

## Model Options

Table 54.8 summarizes the options available in the MODEL statement. These options can be specified after a slash (/).

**Table 54.8** Model Statement Options

| Option                                     | Description                                                                                                                                        |
|--------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Model Specification Options</b>         |                                                                                                                                                    |
| <b>LINK=</b>                               | Specifies the link function                                                                                                                        |
| <b>NOFIT</b>                               | Suppresses model fitting                                                                                                                           |
| <b>NOINT</b>                               | Suppresses the intercept                                                                                                                           |
| <b>OFFSET=</b>                             | Specifies the offset variable                                                                                                                      |
| <b>SELECTION=</b>                          | Specifies the effect selection method                                                                                                              |
| <b>UNEQUALSLOPES</b>                       | Specifies cumulative partial proportional odds models                                                                                              |
| <b>Effect Selection Options</b>            |                                                                                                                                                    |
| <b>BEST=</b>                               | Controls the number of models displayed for <b>SCORE</b> selection                                                                                 |
| <b>DETAILS</b>                             | Requests detailed results at each step                                                                                                             |
| <b>FAST</b>                                | Uses the fast elimination method                                                                                                                   |
| <b>HIERARCHY=</b>                          | Specifies whether and how hierarchy is maintained and whether a single effect or multiple effects are allowed to enter or leave the model per step |
| <b>INCLUDE=</b>                            | Specifies the number of effects included in every model                                                                                            |
| <b>MAXSTEP=</b>                            | Specifies the maximum number of steps for <b>STEPWISE</b> selection                                                                                |
| <b>SEQUENTIAL</b>                          | Adds or deletes effects in sequential order                                                                                                        |
| <b>SLENTY=</b>                             | Specifies the significance level for entering effects                                                                                              |
| <b>SLSTAY=</b>                             | Specifies the significance level for removing effects                                                                                              |
| <b>START=</b>                              | Specifies the number of variables in the first model                                                                                               |
| <b>STOP=</b>                               | Specifies the number of variables in the final model                                                                                               |
| <b>STOPRES</b>                             | Adds or deletes variables by the residual chi-square criterion                                                                                     |
| <b>Model-Fitting Specification Options</b> |                                                                                                                                                    |
| <b>ABSFCONV=</b>                           | Specifies the absolute function convergence criterion                                                                                              |
| <b>FCONV=</b>                              | Specifies the relative function convergence criterion                                                                                              |

**Table 54.8** *continued*

| Option                                                 | Description                                                                                                                |
|--------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|
| FIRTH                                                  | Specifies Firth's penalized likelihood method                                                                              |
| GCONV=                                                 | Specifies the relative gradient convergence criterion                                                                      |
| MAXFUNCTION=                                           | Specifies the maximum number of function calls for the conditional analysis                                                |
| MAXITER=                                               | Specifies the maximum number of iterations                                                                                 |
| NOCHECK                                                | Suppresses checking for infinite parameters                                                                                |
| RIDGING=                                               | Specifies the technique used to improve the log-likelihood function when its value is worse than that of the previous step |
| SINGULAR=                                              | Specifies the tolerance for testing singularity                                                                            |
| TECHNIQUE=                                             | Specifies the iterative algorithm for maximization                                                                         |
| XCONV=                                                 | Specifies the relative parameter convergence criterion                                                                     |
| <b>Confidence Interval Options</b>                     |                                                                                                                            |
| ALPHA=                                                 | Specifies $\alpha$ for the $100(1 - \alpha)\%$ confidence intervals                                                        |
| CLODDS=                                                | Computes confidence intervals for odds ratios                                                                              |
| CLPARM=                                                | Computes confidence intervals for parameters                                                                               |
| PLCONV=                                                | Specifies the profile-likelihood convergence criterion                                                                     |
| <b>Classification Options</b>                          |                                                                                                                            |
| CTABLE                                                 | Displays the classification table                                                                                          |
| PEVENT=                                                | Specifies prior event probabilities                                                                                        |
| PPROB=                                                 | Specifies probability cutpoints for classification                                                                         |
| <b>Overdispersion and Goodness-of-Fit Test Options</b> |                                                                                                                            |
| AGGREGATE=                                             | Determines subpopulations for Pearson chi-square and deviance                                                              |
| LACKFIT                                                | Requests the Hosmer and Lemeshow goodness-of-fit test                                                                      |
| SCALE=                                                 | Specifies the method to correct overdispersion                                                                             |
| <b>ROC Curve Options</b>                               |                                                                                                                            |
| OUTROC=                                                | Names the output ROC data set                                                                                              |
| ROCEPS=                                                | Specifies the probability grouping criterion                                                                               |
| <b>Regression Diagnostics Options</b>                  |                                                                                                                            |
| INFLUENCE                                              | Displays influence statistics                                                                                              |
| IPLOTS                                                 | Requests index plots                                                                                                       |
| <b>Display Options</b>                                 |                                                                                                                            |
| CORRB                                                  | Displays the correlation matrix                                                                                            |
| COVB                                                   | Displays the covariance matrix                                                                                             |
| EXPB                                                   | Displays exponentiated values of the estimates                                                                             |
| ITPRINT                                                | Displays the iteration history                                                                                             |
| NODUMMYPRINT                                           | Suppresses the "Class Level Information" table                                                                             |
| PARMLABEL                                              | Displays parameter labels                                                                                                  |
| PCORR                                                  | Displays the partial correlation statistic                                                                                 |
| RSQUARE                                                | Displays the generalized R Square                                                                                          |
| STB                                                    | Displays standardized estimates                                                                                            |

Table 54.8 *continued*

| Option                       | Description                                                  |
|------------------------------|--------------------------------------------------------------|
| <b>Computational Options</b> |                                                              |
| <b>BINWIDTH=</b>             | Specifies the bin size for estimating association statistics |
| <b>NOLOGSCALE</b>            | Performs calculations by using normal scaling                |

The following list describes these options.

**ABSFCNV=*value***

specifies the absolute function convergence criterion. Convergence requires a small change in the log-likelihood function in subsequent iterations,

$$|l_i - l_{i-1}| < \text{value}$$

where  $l_i$  is the value of the log-likelihood function at iteration  $i$ . See the section “[Convergence Criteria](#)” on page 4242 for more information.

**AGGREGATE<=(*variable-list*)>**

specifies the subpopulations on which the Pearson chi-square test statistic and the likelihood ratio chi-square test statistic (deviance) are calculated. Observations with common values in the given list of variables are regarded as coming from the same subpopulation. Variables in the list can be any variables in the input data set. Specifying the AGGREGATE option is equivalent to specifying the AGGREGATE= option with a variable list that includes all explanatory variables in the MODEL statement. The deviance and Pearson goodness-of-fit statistics are calculated only when the [SCALE=](#) option is specified. Thus, the AGGREGATE (or AGGREGATE=) option has no effect if the [SCALE=](#) option is not specified.

See the section “[Rescaling the Covariance Matrix](#)” on page 4257 for more information.

**ALPHA=*number***

sets the level of significance  $\alpha$  for  $100(1 - \alpha)\%$  confidence intervals for regression parameters or odds ratios. The value of *number* must be between 0 and 1. By default, *number* is equal to the value of the [ALPHA=](#) option in the PROC LOGISTIC statement, or 0.05 if the option is not specified. This option has no effect unless confidence limits for the parameters ([CLPARM=](#) option) or odds ratios ([CLODDS=](#) option or [ODDSRATIO](#) statement) are requested.

**BEST=*n***

specifies that  $n$  models with the highest score chi-square statistics are to be displayed for each model size. It is used exclusively with the [SCORE](#) model selection method. If the BEST= option is omitted and there are no more than 10 explanatory variables, then all possible models are listed for each model size. If the option is omitted and there are more than 10 explanatory variables, then the number of models selected for each model size is, at most, equal to the number of explanatory variables listed in the MODEL statement.

**BINWIDTH=*width***

specifies the size of the bins used for estimating the association statistics. See the section “[Rank Correlation of Observed Responses and Predicted Probabilities](#)” on page 4253 for details. Valid values are  $0 \leq \text{width} < 1$  (for polytomous response models,  $0 < \text{width} < 1$ ). The default *width* is 0.002. If the *width* does not evenly divide the unit interval, it is reduced to a valid value and a message



is displayed in the SAS log. The width is also constrained by the amount of memory available on your machine; if you specify a *width* that is too small, it is adjusted to a value for which memory can be allocated and a note is displayed in the SAS log.

If you have a binary response and specify **BINWIDTH=0**, then no binning is performed and the exact values of the statistics are computed; this method is a bit slower and might require more memory than the binning approach.

The BINWIDTH= option is ignored and no binning is performed when a **ROC** statement is specified, when ROC graphics are produced, or when the **SCORE** statement computes an ROC area.

#### **CLODDS=PL | WALD | BOTH**

produces confidence intervals for odds ratios of main effects not involved in interactions or nestings. Computation of these confidence intervals is based on the profile likelihood (CLODDS=PL) or based on individual Wald tests (CLODDS=WALD). By specifying CLODDS=BOTH, the procedure computes two sets of confidence intervals for the odds ratios, one based on the profile likelihood and the other based on the Wald tests. The confidence coefficient can be specified with the **ALPHA=** option. The CLODDS=PL option is not available with the **STRATA** statement. Classification main effects that use parameterizations other than REF, EFFECT, or GLM are ignored. If you need to compute odds ratios for an effect involved in interactions or nestings, or using some other parameterization, then you should specify an **ODDSRATIO** statement for that effect.

#### **CLPARM=PL | WALD | BOTH**

requests confidence intervals for the parameters. Computation of these confidence intervals is based on the profile likelihood (CLPARM=PL) or individual Wald tests (CLPARM=WALD). If you specify CLPARM=BOTH, the procedure computes two sets of confidence intervals for the parameters, one based on the profile likelihood and the other based on individual Wald tests. The confidence coefficient can be specified with the **ALPHA=** option. The CLPARM=PL option is not available with the **STRATA** statement.

See the section “[Confidence Intervals for Parameters](#)” on page 4248 for more information.

#### **CORRB**

displays the correlation matrix of the parameter estimates.

#### **COVB**

displays the covariance matrix of the parameter estimates.

#### **CTABLE**

classifies the input binary response observations according to whether the predicted event probabilities are above or below some cutpoint value  $z$  in the range (0, 1). An observation is predicted as an event if the predicted event probability exceeds or equals  $z$ . You can supply a list of cutpoints other than the default list by specifying the **PPROB=** option (page 4217). Also, false positive and negative rates can be computed as posterior probabilities by using Bayes’ theorem. You can use the **PEVENT=** option to specify prior probabilities for computing these rates. The CTABLE option is ignored if the data have more than two response levels. The CTABLE option is not available with the **STRATA** statement.

For more information, see the section “[Classification Table](#)” on page 4255.

**DETAILS**

produces a summary of computational details for each step of the effect selection process. It produces the “Analysis of Effects Eligible for Entry” table before displaying the effect selected for entry for forward or stepwise selection. For each model fitted, it produces the “Type 3 Analysis of Effects” table if the fitted model involves CLASS variables, the “Analysis of Maximum Likelihood Estimates” table, and measures of association between predicted probabilities and observed responses. For the statistics included in these tables, see the section “[Displayed Output](#)” on page 4286. The DETAILS option has no effect when `SELECTION=NONE`.

**EXPB****EXPEST**

displays the exponentiated values ( $e^{\hat{\beta}_i}$ ) of the parameter estimates  $\hat{\beta}_i$  in the “Analysis of Maximum Likelihood Estimates” table for the logit model. These exponentiated values are the estimated odds ratios for parameters corresponding to the continuous explanatory variables, and for CLASS effects that use reference or GLM parameterizations.

**FAST**

uses a computational algorithm of Lawless and Singhal (1978) to compute a first-order approximation to the remaining slope estimates for each subsequent elimination of a variable from the model. Variables are removed from the model based on these approximate estimates. The FAST option is extremely efficient because the model is not refitted for every variable removed. The FAST option is used when `SELECTION=BACKWARD` and in the backward elimination steps when `SELECTION=STEPWISE`. The FAST option is ignored when `SELECTION=FORWARD` or `SELECTION=NONE`.

**FCONV=value**

specifies the relative function convergence criterion. Convergence requires a small relative change in the log-likelihood function in subsequent iterations,

$$\frac{|l_i - l_{i-1}|}{|l_{i-1}| + 1\text{E-}6} < \text{value}$$

where  $l_i$  is the value of the log likelihood at iteration  $i$ . See the section “[Convergence Criteria](#)” on page 4242 for more information.

**FIRTH**

performs Firth’s penalized maximum likelihood estimation to reduce bias in the parameter estimates (Heinze and Schemper 2002; Firth 1993). This method is useful in cases of separability, as often occurs when the event is rare, and is an alternative to performing an exact logistic regression. See the section “[Firth’s Bias-Reducing Penalized Likelihood](#)” on page 4242 for more information.

**NOTE:** The intercept-only log likelihood is modified by using the full-model Hessian, computed with the slope parameters equal to zero. Therefore, in order to use the likelihood ratio test to compare models, you should use the log likelihoods from the “Model Fit Statistics” tables instead of the Likelihood Ratio statistic that is reported in the “Testing Global Null Hypothesis: BETA=0” table. When fitting a model and scoring a data set in the same PROC LOGISTIC step, the model is fit using Firth’s penalty for parameter estimation purposes, but the penalty is not applied to the scored log likelihood.

**GCONV=value**

specifies the relative gradient convergence criterion. Convergence requires that the normalized prediction function reduction is small,

$$\frac{\mathbf{g}_i' \mathbf{I}_i^{-1} \mathbf{g}_i}{|l_i| + 1\text{E-}6} < \text{value}$$

where  $l_i$  is the value of the log-likelihood function,  $\mathbf{g}_i$  is the gradient vector, and  $\mathbf{I}_i$  is the negative (expected) Hessian matrix, all at iteration  $i$ . This is the default convergence criterion, and the default value is 1E-8. See the section “[Convergence Criteria](#)” on page 4242 for more information.

**HIERARCHY=keyword****HIER=keyword**

specifies whether and how the model hierarchy requirement is applied and whether a single effect or multiple effects are allowed to enter or leave the model in one step. You can specify that only CLASS effects, or both CLASS and interval effects, be subject to the hierarchy requirement. The HIERARCHY= option is ignored unless you also specify one of the following options: [SELECTION=FORWARD](#), [SELECTION=BACKWARD](#), or [SELECTION=STEPWISE](#).

Model hierarchy refers to the requirement that, for any term to be in the model, all effects contained in the term must be present in the model. For example, in order for the interaction A\*B to enter the model, the main effects A and B must be in the model. Likewise, neither effect A nor B can leave the model while the interaction A\*B is in the model.

The keywords you can specify in the HIERARCHY= option are as follows:

- NONE** indicates that the model hierarchy is not maintained. Any single effect can enter or leave the model at any given step of the selection process.
- SINGLE** indicates that only one effect can enter or leave the model at one time, subject to the model hierarchy requirement. For example, suppose that you specify the main effects A and B and the interaction A\*B in the model. In the first step of the selection process, either A or B can enter the model. In the second step, the other main effect can enter the model. The interaction effect can enter the model only when both main effects have already been entered. Also, before A or B can be removed from the model, the A\*B interaction must first be removed. All effects (CLASS and interval) are subject to the hierarchy requirement.
- SINGLECLASS** is the same as HIERARCHY=SINGLE except that only CLASS effects are subject to the hierarchy requirement.
- MULTIPLE** indicates that more than one effect can enter or leave the model at one time, subject to the model hierarchy requirement. In a forward selection step, a single main effect can enter the model, or an interaction can enter the model together with all the effects that are contained in the interaction. In a backward elimination step, an interaction itself, or the interaction together with all the effects that the interaction contains, can be removed. All effects (CLASS and continuous) are subject to the hierarchy requirement.
- MULTIPLECLASS** is the same as HIERARCHY=MULTIPLE except that only CLASS effects are subject to the hierarchy requirement.

The default value is HIERARCHY=SINGLE, which means that model hierarchy is to be maintained for all effects (that is, both CLASS and continuous effects) and that only a single effect can enter or leave the model at each step.

**INCLUDE=*n***

includes the first *n* effects in the MODEL statement in every model. By default, INCLUDE=0. The INCLUDE= option has no effect when SELECTION=NONE.

Note that the INCLUDE= and START= options perform different tasks: the INCLUDE= option includes the first *n* effects variables in every model, whereas the START= option requires only that the first *n* effects appear in the first model.

**INFLUENCE<(STDRES)>**

displays diagnostic measures for identifying influential observations in the case of a binary response model. For each observation, the INFLUENCE option displays the case number (which is the sequence number of the observation), the values of the explanatory variables included in the final model, and the regression diagnostic measures developed by Pregibon (1981). The STDRES option includes standardized and likelihood residuals in the display.

For a discussion of these diagnostic measures, see the section “Regression Diagnostics” on page 4263. When a STRATA statement is specified, the diagnostics are computed following Storer and Crowley (1985); see the section “Regression Diagnostic Details” on page 4272 for details.

**IPLOTS**

produces an index plot for the regression diagnostic statistics developed by Pregibon (1981). An index plot is a scatter plot with the regression diagnostic statistic represented on the Y axis and the case number on the X axis. See Example 54.6 for an illustration.

**ITPRINT**

displays the iteration history of the maximum-likelihood model fitting. The ITPRINT option also displays the last evaluation of the gradient vector and the final change in the  $-2$  Log Likelihood.

**LACKFIT<(n)>**

performs the Hosmer and Lemeshow goodness-of-fit test (Hosmer and Lemeshow 2000) for the case of a binary response model. The subjects are divided into approximately 10 groups of roughly the same size based on the percentiles of the estimated probabilities. The discrepancies between the observed and expected number of observations in these groups are summarized by the Pearson chi-square statistic, which is then compared to a chi-square distribution with *t* degrees of freedom, where *t* is the number of groups minus *n*. By default, *n* = 2. A small *p*-value suggests that the fitted model is not an adequate model. The LACKFIT option is not available with the STRATA statement. See the section “The Hosmer-Lemeshow Goodness-of-Fit Test” on page 4259 for more information.

**LINK=keyword****L=keyword**

specifies the link function linking the response probabilities to the linear predictors. You can specify one of the following keywords. The default is LINK=LOGIT.

**CLOGLOG** is the complementary log-log function. PROC LOGISTIC fits the binary complementary log-log model when there are two response categories and fits the cumulative complementary log-log model when there are more than two response categories. The aliases are CCLOGLOG, CCLL, and CUMCLOGLOG.

- GLOGIT** is the generalized logit function. PROC LOGISTIC fits the generalized logit model where each nonreference category is contrasted with the reference category. You can use the response variable option **REF=** to specify the reference category.
- LOGIT** is the log odds function. PROC LOGISTIC fits the binary logit model when there are two response categories and fits the cumulative logit model when there are more than two response categories. The aliases are CLOGIT and CUMLOGIT.
- PROBIT** is the inverse standard normal distribution function. PROC LOGISTIC fits the binary probit model when there are two response categories and fits the cumulative probit model when there are more than two response categories. The aliases are NORMIT, CPROBIT, and CUMPROBIT.

The **LINK=** option is not available with the **STRATA** statement.

See the section “[Link Functions and the Corresponding Distributions](#)” on page 4238 for more details.

#### **MAXFUNCTION=***number*

specifies the maximum number of function calls to perform when maximizing the conditional likelihood. This option is valid only when a **STRATA** statement or the **UNEQUALSLOPES** option is specified. The default values are as follows:

- 125 when the number of parameters  $p < 40$
- 500 when  $40 \leq p < 400$
- 1000 when  $p \geq 400$

Since the optimization is terminated only after completing a full iteration, the number of function calls that are actually performed can exceed *number*. If convergence is not attained, the displayed output and all output data sets created by the procedure contain results based on the last maximum likelihood iteration.

#### **MAXITER=***number*

specifies the maximum number of iterations to perform. By default, **MAXITER=25**. If convergence is not attained in *number* iterations, the displayed output and all output data sets created by the procedure contain results that are based on the last maximum likelihood iteration.

#### **MAXSTEP=***n*

specifies the maximum number of times any explanatory variable is added to or removed from the model when **SELECTION=STEPWISE**. The default number is twice the number of explanatory variables in the MODEL statement. When the **MAXSTEP=** limit is reached, the stepwise selection process is terminated. All statistics displayed by the procedure (and included in output data sets) are based on the last model fitted. The **MAXSTEP=** option has no effect when **SELECTION=NONE**, **FORWARD**, or **BACKWARD**.

#### **NOCHECK**

disables the checking process to determine whether maximum likelihood estimates of the regression parameters exist. If you are sure that the estimates are finite, this option can reduce the execution time if the estimation takes more than eight iterations. For more information, see the section “[Existence of Maximum Likelihood Estimates](#)” on page 4242.

**NODUMMYPRINT****NODESIGNPRINT****NODP**

suppresses the “Class Level Information” table, which shows how the design matrix columns for the CLASS variables are coded.

**NOINT**

suppresses the intercept for the binary response model, the first intercept for the ordinal response model (which forces all intercepts to be nonnegative), or all intercepts for the generalized logit model. This can be particularly useful in conditional logistic analysis; see [Example 54.11](#).

**NOFIT**

performs the global score test without fitting the model. The global score test evaluates the joint significance of the effects in the MODEL statement. No further analyses are performed. If the NOFIT option is specified along with other MODEL statement options, NOFIT takes effect and all other options except FIRTH, LINK=, NOINT, OFFSET=, ROC, and TECHNIQUE= are ignored. The NOFIT option is not available with the STRATA statement.

**NOLOGSCALE**

specifies that computations for the conditional and exact logistic regression models should be computed by using normal scaling. Log scaling can handle numerically larger problems than normal scaling; however, computations in the log scale are slower than computations in normal scale.

**OFFSET=name**

names the offset variable. The regression coefficient for this variable will be fixed at 1. For an example that uses this option, see [Example 54.13](#). You can also use the OFFSET= option to restrict parameters to a fixed value. For example, if you want to restrict the parameter for variable X1 to 1 and the parameter for X2 to 2, compute Restrict=  $X1 + 2 * X2$  in a DATA step, specify the option **offset=Restrict**, and leave X1 and X2 out of the model.

**OUTROC=SAS-data-set****OUTR=SAS-data-set**

creates, for binary response models, an output SAS data set that contains the data necessary to produce the receiver operating characteristic (ROC) curve. The OUTROC= option is not available with the STRATA statement. See the section “[OUTROC= Output Data Set](#)” on page 4283 for the list of variables in this data set.

**PARMLABEL**

displays the labels of the parameters in the “Analysis of Maximum Likelihood Estimates” table.

**PCORR**

computes the partial correlation statistic  $sign(\beta_i) \sqrt{\frac{\chi_i^2 - 2}{-2 \log L_0}}$  for each parameter  $i$ , where  $\chi_i^2$  is the Wald chi-square statistic for the parameter and  $\log L_0$  is the log-likelihood of the intercept-only model (Hilbe 2009, p. 101). If  $\chi_i^2 < 2$  then the partial correlation is set to 0. The partial correlation for the intercept terms is set to missing.

**PEVENT=***value*

**PEVENT=**(*list*)

specifies one prior probability or a list of prior probabilities for the event of interest. The false positive and false negative rates are then computed as posterior probabilities by Bayes' theorem. The prior probability is also used in computing the rate of correct prediction. For each prior probability in the given list, a classification table of all observations is computed. By default, the prior probability is the total sample proportion of events. The PEVENT= option is useful for stratified samples. It has no effect if the CTABLE option is not specified. For more information, see the section “[False Positive, False Negative, and Correct Classification Rates Using Bayes' Theorem](#)” on page 4256. Also see the PPROB= option for information about how the *list* is specified.

**PLCL**

is the same as specifying **CLPARM=PL**.

**PLCONV=***value*

controls the convergence criterion for confidence intervals based on the profile-likelihood function. The quantity *value* must be a positive number, with a default value of 1E-4. The PLCONV= option has no effect if profile-likelihood confidence intervals (**CLPARM=PL**) are not requested.

**PLRL**

is the same as specifying **CLODDS=PL**.

**PPROB=***value*

**PPROB=**(*list*)

specifies one critical probability value (or cutpoint) or a list of critical probability values for classifying observations with the **CTABLE** option. Each *value* must be between 0 and 1. A response that has a cross validated predicted probability greater than or equal to the current PPROB= value is classified as an event response. The PPROB= option is ignored if the **CTABLE** option is not specified.

A classification table for each of several cutpoints can be requested by specifying a list. For example, the following statement requests a classification of the observations for each of the cutpoints 0.3, 0.5, 0.6, 0.7, and 0.8:

```
pprob= (0.3, 0.5 to 0.8 by 0.1)
```

If the PPROB= option is not specified, the default is to display the classification for a range of probabilities from the smallest estimated probability (rounded down to the nearest 0.02) to the highest estimated probability (rounded up to the nearest 0.02) with 0.02 increments.

**RIDGING=ABSOLUTE | RELATIVE | NONE**

specifies the technique used to improve the log-likelihood function when its value in the current iteration is less than that in the previous iteration. If you specify the RIDGING=ABSOLUTE option, the diagonal elements of the negative (expected) Hessian are inflated by adding the ridge value. If you specify the RIDGING=RELATIVE option, the diagonal elements are inflated by a factor of 1 plus the ridge value. If you specify the RIDGING=NONE option, the crude line search method of taking half a step is used instead of ridging. By default, RIDGING=RELATIVE.



**RISKLIMITS****RL****WALDRL**

is the same as specifying **CLODDS=WALD**.

**ROCEPS=number**

specifies a criterion for the ROC curve used for grouping estimated event probabilities that are close to each other. In each group, the difference between the largest and the smallest estimated event probabilities does not exceed the given value. The value for *number* must be between 0 and 1; the default value is the square root of the machine epsilon, which is about 1E–8 (in releases prior to 9.2, the default was 1E–4). The smallest estimated probability in each group serves as a cutpoint for predicting an event response. The **ROCEPS=** option has no effect unless the **OUTROC=** option, the **BINWIDTH=0** option, or a **ROC statement** is specified.

**RSQUARE****RSQ**

requests a generalized R Square measure for the fitted model. For more information, see the section “Generalized Coefficient of Determination” on page 4246.

**SCALE=scale**

enables you to supply the value of the dispersion parameter or to specify the method for estimating the dispersion parameter. It also enables you to display the “Deviance and Pearson Goodness-of-Fit Statistics” table. To correct for overdispersion or underdispersion, the covariance matrix is multiplied by the estimate of the dispersion parameter. Valid values for *scale* are as follows:

**D | DEVIANCE** specifies that the dispersion parameter be estimated by the deviance divided by its degrees of freedom.

**P | PEARSON** specifies that the dispersion parameter be estimated by the Pearson chi-square statistic divided by its degrees of freedom.

**WILLIAMS** <(*constant*)> specifies that Williams’ method be used to model overdispersion. This option can be used only with the events/trials syntax. An optional *constant* can be specified as the scale parameter; otherwise, a scale parameter is estimated under the full model. A set of weights is created based on this scale parameter estimate. These weights can then be used in fitting subsequent models of fewer terms than the full model. When fitting these submodels, specify the computed scale parameter as *constant*. See [Example 54.10](#) for an illustration.

**N | NONE** specifies that no correction is needed for the dispersion parameter; that is, the dispersion parameter remains as 1. This specification is used for requesting the deviance and the Pearson chi-square statistic without adjusting for overdispersion.

*constant* sets the estimate of the dispersion parameter to be the square of the given *constant*. For example, **SCALE=2** sets the dispersion parameter to 4. The value *constant* must be a positive number.

You can use the **AGGREGATE** (or **AGGREGATE=**) option to define the subpopulations for calculating the Pearson chi-square statistic and the deviance. In the absence of the **AGGREGATE** (or **AGGREGATE=**) option, each observation is regarded as coming from a different subpopulation. For the events/trials syntax, each observation consists of *n* Bernoulli trials, where *n* is the value of the



*trials* variable. For single-trial syntax, each observation consists of a single response, and for this setting it is not appropriate to carry out the Pearson or deviance goodness-of-fit analysis. Thus, PROC LOGISTIC ignores specifications SCALE=P, SCALE=D, and SCALE=N when single-trial syntax is specified without the [AGGREGATE](#) (or AGGREGATE=) option.

The “Deviance and Pearson Goodness-of-Fit Statistics” table includes the Pearson chi-square statistic, the deviance, the degrees of freedom, the ratio of each statistic divided by its degrees of freedom, and the corresponding *p*-value. The SCALE= option is not available with the [STRATA](#) statement. For more information, see the section “[Overdispersion](#)” on page 4257.

**SELECTION=BACKWARD | B**  
**| FORWARD | F**  
**| NONE | N**  
**| STEPWISE | S**  
**| SCORE**

specifies the method used to select the variables in the model. BACKWARD requests backward elimination, FORWARD requests forward selection, NONE fits the complete model specified in the MODEL statement, and STEPWISE requests stepwise selection. SCORE requests best subset selection. By default, SELECTION=NONE.

For more information, see the section “[Effect-Selection Methods](#)” on page 4244.

## SEQUENTIAL

### SEQ

forces effects to be added to the model in the order specified in the MODEL statement or eliminated from the model in the reverse order of that specified in the MODEL statement. The model-building process continues until the next effect to be added has an insignificant adjusted chi-square statistic or until the next effect to be deleted has a significant Wald chi-square statistic. The SEQUENTIAL option has no effect when [SELECTION=NONE](#).

### SINGULAR=value

specifies the tolerance for testing the singularity of the Hessian matrix (Newton-Raphson algorithm) or the expected value of the Hessian matrix (Fisher scoring algorithm). The Hessian matrix is the matrix of second partial derivatives of the log-likelihood function. The test requires that a pivot for sweeping this matrix be at least this number times a norm of the matrix. Values of the SINGULAR= option must be numeric. By default, *value* is the machine epsilon times 1E7, which is approximately 1E-9.

### SLENTRY=value

### SLE=value

specifies the significance level of the score chi-square for entering an effect into the model in the FORWARD or STEPWISE method. Values of the SLENTRY= option should be between 0 and 1, inclusive. By default, SLENTRY=0.05. The SLENTRY= option has no effect when [SELECTION=NONE](#), [SELECTION=BACKWARD](#), or [SELECTION=SCORE](#).

**SLSTAY=value****SLS=value**

specifies the significance level of the Wald chi-square for an effect to stay in the model in a backward elimination step. Values of the SLSTAY= option should be between 0 and 1, inclusive. By default, SLSTAY=0.05. The SLSTAY= option has no effect when **SELECTION=NONE**, **SELECTION=FORWARD**, or **SELECTION=SCORE**.

**START=n**

begins the FORWARD, BACKWARD, or STEPWISE effect selection process with the first  $n$  effects listed in the MODEL statement. The value of  $n$  ranges from 0 to  $s$ , where  $s$  is the total number of effects in the MODEL statement. The default value of  $n$  is  $s$  for the BACKWARD method and 0 for the FORWARD and STEPWISE methods. Note that START= $n$  specifies only that the first  $n$  effects appear in the first model, while **INCLUDE= $n$**  requires that the first  $n$  effects be included in every model. For the SCORE method, START= $n$  specifies that the smallest models contain  $n$  effects, where  $n$  ranges from 1 to  $s$ ; the default value is 1. The START= option has no effect when **SELECTION=NONE**.

**STB**

displays the standardized estimates for the parameters in the “Analysis of Maximum Likelihood Estimates” table. The standardized estimate of  $\beta_i$  is given by  $\hat{\beta}_i / (s/s_i)$ , where  $s_i$  is the total sample standard deviation for the  $i$ th explanatory variable and

$$s = \begin{cases} \pi/\sqrt{3} & \text{Logistic} \\ 1 & \text{Normal} \\ \pi/\sqrt{6} & \text{Extreme-value} \end{cases}$$

The sample standard deviations for parameters associated with **CLASS** and **EFFECT** variables are computed using their codings. For the intercept parameters, the standardized estimates are set to missing.

**STOP=n**

specifies the maximum (**SELECTION=FORWARD**) or minimum (**SELECTION=BACKWARD**) number of effects to be included in the final model. The effect selection process is stopped when  $n$  effects are found. The value of  $n$  ranges from 0 to  $s$ , where  $s$  is the total number of effects in the MODEL statement. The default value of  $n$  is  $s$  for the FORWARD method and 0 for the BACKWARD method. For the SCORE method, STOP= $n$  specifies that the largest models contain  $n$  effects, where  $n$  ranges from 1 to  $s$ ; the default value of  $n$  is  $s$ . The STOP= option has no effect when **SELECTION=NONE** or **STEPWISE**.

**STOPRES****SR**

specifies that the removal or entry of effects be based on the value of the residual chi-square. If **SELECTION=FORWARD**, then the STOPRES option adds the effects into the model one at a time until the residual chi-square becomes insignificant (until the  $p$ -value of the residual chi-square exceeds the **SLENTY= value**). If **SELECTION=BACKWARD**, then the STOPRES option removes effects from the model one at a time until the residual chi-square becomes significant (until the  $p$ -value of the residual chi-square becomes less than the **SLSTAY= value**). The STOPRES option has no effect when **SELECTION=NONE** or **SELECTION=STEPWISE**.

**TECHNIQUE=FISHER | NEWTON****TECH=FISHER | NEWTON**

specifies the optimization technique for estimating the regression parameters. NEWTON (or NR) is the Newton-Raphson algorithm and FISHER (or FS) is the Fisher scoring algorithm. Both techniques yield the same estimates, but the estimated covariance matrices are slightly different except for the case when the LOGIT link is specified for binary response data. The default is TECHNIQUE=FISHER. If the [LINK=GLOGIT](#) option is specified, then Newton-Raphson is the default and only available method. The TECHNIQUE= option is not applied to conditional and exact conditional analyses. This option is not available when the [UNEQUALSLOPES](#) option is specified. See the section “[Iterative Algorithms for Model Fitting](#)” on page 4240 for more details.

**UNEQUALSLOPES<=effect | (effect-list)>**

specifies one or more effects in a cumulative response model that have a different set of parameters for each response function. If you specify more than one effect, enclose the effects in parentheses. The effects must be explanatory effects that are specified in the MODEL statement.

If you do not specify this option, the cumulative response models make the parallel lines assumption,  $F(\Pr(Y < i)) = \alpha_i + \mathbf{x}'\boldsymbol{\beta}$ , where each response function has the same slope parameters  $\boldsymbol{\beta}$ . If you specify this option without an *effect* or *effect-list*, all slope parameters vary across the response functions, resulting in the model  $F(\Pr(Y < i)) = \alpha_i + \mathbf{x}'\boldsymbol{\beta}_i$ . Specifying an *effect* or *effect-list* enables you to choose which effects have different parameters across the response functions. If you select the first  $\mathbf{x}_1$  parameters to have equal slopes and the remaining  $\mathbf{x}_2$  parameters to have unequal slopes, the model can be written as  $F(\Pr(Y < i)) = \alpha_i + \mathbf{x}'_1\boldsymbol{\beta}_1 + \mathbf{x}'_2\boldsymbol{\beta}_{2i}$ . A model that uses the [CLOGIT](#) link is called a *partial proportional odds model* (Peterson and Harrell 1990).

For more information, see [Example 54.17](#). The following statements are not available with this option: EFFECTPLOT, ESTIMATE, EXACT, LSMEANS, LSMESTIMATE, ROC, ROCCONTRAST, SLICE, STORE, and STRATA. The following options are not available with this option: FIRTH, RIDGING=, TECHNIQUE=, CTABLE, PEVENT=, PPROB=, OUTROC=, and ROCEPS=.

**WALDCL****CL**

is the same as specifying [CLPARM=WALD](#).

**XCONV=value**

specifies the relative parameter convergence criterion. Convergence requires a small relative parameter change in subsequent iterations,

$$\max_j |\delta_j^{(i)}| < value$$

where

$$\delta_j^{(i)} = \begin{cases} \frac{\beta_j^{(i)} - \beta_j^{(i-1)}}{\beta_j^{(i-1)}} & |\beta_j^{(i-1)}| < 0.01 \\ \beta_j^{(i)} - \beta_j^{(i-1)} & \text{otherwise} \end{cases}$$

and  $\beta_j^{(i)}$  is the estimate of the  $j$ th parameter at iteration  $i$ . See the section “[Convergence Criteria](#)” on page 4242 for more information.

---

## NLOPTIONS Statement

**NLOPTIONS** < options > ;

The NLOPTIONS statement controls the optimization process for conditional analyses (which result from specifying a **STRATA** statement) and for partial parallel slope models (which result from specifying the **UNEQUALSLOPES** option in the MODEL statement). An *option* specified in the NLOPTIONS statement takes precedence over the same option specified in the MODEL statement.

The default optimization techniques are chosen according to the number of parameters,  $p$ , as follows:

- Newton-Raphson with ridging when  $p < 40$
- quasi-Newton when  $40 \leq p < 400$
- conjugate gradient when  $p \geq 400$

The available *options* are described in the section “NLOPTIONS Statement” on page 482 of Chapter 19, “Shared Concepts and Topics.”

---

## ODDSRATIO Statement

**ODDSRATIO** < 'label' > variable < / options > ;

The ODDSRATIO statement produces odds ratios for *variable* even when the variable is involved in interactions with other covariates, and for classification variables that use any parameterization. You can also specify variables on which **constructed effects** are based, in addition to the names of **COLLECTION** or **MULTIMEMBER** effects. You can specify several ODDSRATIO statements.

If *variable* is continuous, then the odds ratios honor any values specified in the **UNITS** statement. If *variable* is a classification variable, then odds ratios comparing each pairwise difference between the levels of *variable* are produced. If *variable* interacts with a continuous variable, then the odds ratios are produced at the mean of the interacting covariate by default. If *variable* interacts with a classification variable, then the odds ratios are produced at each level of the interacting covariate by default. The computed odds ratios are independent of the parameterization of any classification variable.

The odds ratios are uniquely labeled by concatenating the following terms to *variable*:

1. If this is a polytomous response model, then prefix the response variable and the level describing the logit followed by a colon; for example, “Y 0:”.
2. If *variable* is continuous and the **UNITS** statement provides a value that is not equal to 1, then append “Units=value”; otherwise, if *variable* is a classification variable, then append the levels being contrasted; for example, “cat vs dog”.
3. Append all interacting covariates preceded by “At”; for example, “At X=1.2 A=cat”.

If you are also creating odds ratio plots, then this label is displayed on the plots (see the **PLOTS** option for more information). If you specify a ‘label’ in the ODDSRATIO statement, then the odds ratios produced by this statement are also labeled: ‘label’, ‘label 2’, ‘label 3’, ..., and these are the labels used in the plots. If

there are any duplicated labels across all ODDSRATIO statements, then the corresponding odds ratios are not displayed on the plots.

The following *options* are available:

**AT**(*covariate=value-list* | **REF** | **ALL**< ...*covariate=value-list* | **REF** | **ALL**>)

specifies fixed levels of the interacting covariates. If a specified *covariate* does not interact with the *variable*, then its AT list is ignored.

For continuous interacting covariates, you can specify one or more numbers in the *value-list*. For classification covariates, you can specify one or more formatted levels of the covariate enclosed in single quotes (for example, **A='cat' 'dog'**), you can specify the keyword **REF** to select the reference-level, or you can specify the keyword **ALL** to select all levels of the classification variable. By default, continuous covariates are set to their means, while **CLASS** covariates are set to **ALL**. For a model that includes a classification variable **A={cat,dog}** and a continuous covariate **X**, specifying **AT (A='cat' X=7 9)** will set **A** to 'cat', and **X** to 7 and then 9.

**CL=WALD** | **PL** | **BOTH**

specifies whether to create Wald or profile-likelihood confidence limits, or both. By default, Wald confidence limits are produced.

**DIFF=REF** | **ALL**

specifies whether the odds ratios for a classification variable are computed against the reference level, or all pairs of variable are compared. By default, **DIFF=ALL**. The **DIFF=** option is ignored when variable is continuous.

**PLCONV=value**

controls the convergence criterion for confidence intervals based on the profile-likelihood function. The quantity *value* must be a positive number, with a default value of 1E-4. The **PLCONV=** option has no effect if profile-likelihood confidence intervals (**CL=PL**) are not requested.

**PLMAXITER=n**

specifies the maximum number of iterations to perform. By default, **PLMAXITER=25**. If convergence is not attained in *n* iterations, the odds ratio or the confidence limits are set to missing. The **PLMAXITER=** option has no effect if profile-likelihood confidence intervals (**CL=PL**) are not requested.

**PLSINGULAR=value**

specifies the tolerance for testing the singularity of the Hessian matrix (Newton-Raphson algorithm) or the expected value of the Hessian matrix (Fisher scoring algorithm). The test requires that a pivot for sweeping this matrix be at least this number times a norm of the matrix. Values of the **PLSINGULAR=** option must be numeric. By default, *value* is the machine epsilon times 1E7, which is approximately 1E-9. The **PLSINGULAR=** option has no effect if profile-likelihood confidence intervals (**CL=PL**) are not requested.

---

## OUTPUT Statement

**OUTPUT** < **OUT=SAS-data-set** > < *options* > ;

The OUTPUT statement creates a new SAS data set that contains all the variables in the input data set and, optionally, the estimated linear predictors and their standard error estimates, the estimates of the cumulative or individual response probabilities, and the confidence limits for the cumulative probabilities. Regression diagnostic statistics and estimates of cross validated response probabilities are also available for binary response models. If you specify more than one OUTPUT statement, only the last one is used. Formulas for the statistics are given in the sections “[Linear Predictor, Predicted Probability, and Confidence Limits](#)” on page 4253 and “[Regression Diagnostics](#)” on page 4263, and, for conditional logistic regression, in the section “[Conditional Logistic Regression](#)” on page 4271.

If you use the single-trial syntax, the data set also contains a variable named `_LEVEL_`, which indicates the level of the response that the given row of output is referring to. For instance, the value of the cumulative probability variable is the probability that the response variable is as large as the corresponding value of `_LEVEL_`. For details, see the section “[OUT= Output Data Set in the OUTPUT Statement](#)” on page 4280.

The estimated linear predictor, its standard error estimate, all predicted probabilities, and the confidence limits for the cumulative probabilities are computed for all observations in which the explanatory variables have no missing values, even if the response is missing. By adding observations with missing response values to the input data set, you can compute these statistics for new observations or for settings of the explanatory variables not present in the data without affecting the model fit. Alternatively, the [SCORE](#) statement can be used to compute predicted probabilities and confidence intervals for new observations.

Table 54.9 summarizes the options available in the OUTPUT statement. These options can be specified after a slash (/). The statistic and diagnostic options specify the statistics to be included in the output data set and name the new variables that contain the statistics. If a [STRATA](#) statement is specified, only the [PREDICTED=](#), [DFBETAS=](#), and [H=](#) options are available; see the section “[Regression Diagnostic Details](#)” on page 4272 for details.

**Table 54.9** OUTPUT Statement Options

| Option                                        | Description                                                                     |
|-----------------------------------------------|---------------------------------------------------------------------------------|
| <a href="#">ALPHA=</a>                        | Specifies $\alpha$ for the $100(1 - \alpha)\%$ confidence intervals             |
| <a href="#">OUT=</a>                          | Names the output data set                                                       |
| <b>Statistic Options</b>                      |                                                                                 |
| <a href="#">LOWER=</a>                        | Names the lower confidence limit                                                |
| <a href="#">PREDICTED=</a>                    | Names the predicted probabilities                                               |
| <a href="#">PREDPROBS=</a>                    | Requests the individual, cumulative, or cross validated predicted probabilities |
| <a href="#">STDXBETA=</a>                     | Names the standard error estimate of the linear predictor                       |
| <a href="#">UPPER=</a>                        | Names the upper confidence limit                                                |
| <a href="#">XBETA=</a>                        | Names the linear predictor                                                      |
| <b>Diagnostic Options for Binary Response</b> |                                                                                 |
| <a href="#">C=</a>                            | Names the confidence interval displacement                                      |
| <a href="#">CBAR=</a>                         | Names the confidence interval displacement                                      |
| <a href="#">DFBETAS=</a>                      | Names the standardized deletion parameter differences                           |
| <a href="#">DIFCHISQ=</a>                     | Names the deletion chi-square goodness-of-fit change                            |
| <a href="#">DIFDEV=</a>                       | Names the deletion deviance change                                              |
| <a href="#">H=</a>                            | Names the leverage                                                              |
| <a href="#">RESCHI=</a>                       | Names the Pearson chi-square residual                                           |
| <a href="#">RESDEV=</a>                       | Names the deviance residual                                                     |

Table 54.9 *continued*

| Option            | Description                                        |
|-------------------|----------------------------------------------------|
| <b>RESLIK=</b>    | Names the likelihood residual                      |
| <b>STDRESCHI=</b> | Names the standardized Pearson chi-square residual |
| <b>STDRESDEV=</b> | Names the standardized deviance residual           |

The following list describes these options.

**ALPHA=***number*

sets the level of significance  $\alpha$  for  $100(1 - \alpha)\%$  confidence limits for the appropriate response probabilities. The value of *number* must be between 0 and 1. By default, *number* is equal to the value of the **ALPHA=** option in the PROC LOGISTIC statement, or 0.05 if that option is not specified.

**C=***name*

specifies the confidence interval displacement diagnostic that measures the influence of individual observations on the regression estimates.

**CBAR=***name*

specifies the confidence interval displacement diagnostic that measures the overall change in the global regression estimates due to deleting an individual observation.

**DFBETAS=**ALL

**DFBETAS=***var-list*

specifies the standardized differences in the regression estimates for assessing the effects of individual observations on the estimated regression parameters in the fitted model. You can specify a list of up to  $s + 1$  variable names, where  $s$  is the number of explanatory variables in the **MODEL** statement, or you can specify just the keyword ALL. In the former specification, the first variable contains the standardized differences in the intercept estimate, the second variable contains the standardized differences in the parameter estimate for the first explanatory variable in the **MODEL** statement, and so on. In the latter specification, the DFBETAS statistics are named DFBETA\_XXX, where XXX is the name of the regression parameter. For example, if the model contains two variables X1 and X2, the specification **DFBETAS=**ALL produces three DFBETAS statistics: DFBETA\_Intercept, DFBETA\_X1, and DFBETA\_X2. If an explanatory variable is not included in the final model, the corresponding output variable named in **DFBETAS=***var-list* contains missing values.

**DIFCHISQ=***name*

specifies the change in the chi-square goodness-of-fit statistic attributable to deleting the individual observation.

**DIFDEV=***name*

specifies the change in the deviance attributable to deleting the individual observation.

**H=***name*

specifies the diagonal element of the hat matrix for detecting extreme points in the design space.

**LOWER=***name*

**L=***name*

names the variable containing the lower confidence limits for  $\pi$ , where  $\pi$  is the probability of the event response if events/trials syntax or single-trial syntax with binary response is specified; for a cumulative model,  $\pi$  is cumulative probability (that is, the probability that the response is less than



or equal to the value of `_LEVEL_`); for the generalized logit model, it is the individual probability (that is, the probability that the response category is represented by the value of `_LEVEL_`). See the [ALPHA=](#) option to set the confidence level.

**OUT=SAS-data-set**

names the output data set. If you omit the `OUT=` option, the output data set is created and given a default name by using the `DATA` convention.

**PREDICTED=name**

**PRED=name**

**PROB=name**

**P=name**

names the variable containing the predicted probabilities. For the events/trials syntax or single-trial syntax with binary response, it is the predicted event probability. For a cumulative model, it is the predicted cumulative probability (that is, the probability that the response variable is less than or equal to the value of `_LEVEL_`); and for the generalized logit model, it is the predicted individual probability (that is, the probability of the response category represented by the value of `_LEVEL_`).

**PREDPROBS=(keywords)**

requests individual, cumulative, or cross validated predicted probabilities. Descriptions of the *keywords* are as follows.

**INDIVIDUAL | I** requests the predicted probability of each response level. For a response variable `Y` with three levels, 1, 2, and 3, the individual probabilities are  $\Pr(Y=1)$ ,  $\Pr(Y=2)$ , and  $\Pr(Y=3)$ .

**CUMULATIVE | C** requests the cumulative predicted probability of each response level. For a response variable `Y` with three levels, 1, 2, and 3, the cumulative probabilities are  $\Pr(Y \leq 1)$ ,  $\Pr(Y \leq 2)$ , and  $\Pr(Y \leq 3)$ . The cumulative probability for the last response level always has the constant value of 1. For generalized logit models, the cumulative predicted probabilities are not computed and are set to missing.

**CROSSVALIDATE | XVALIDATE | X** requests the cross validated individual predicted probability of each response level. These probabilities are derived from the leave-one-out principle—that is, dropping the data of one subject and reestimating the parameter estimates. PROC LOGISTIC uses a less expensive one-step approximation to compute the parameter estimates. This option is valid only for binary response models; for nominal and ordinal models, the cross validated probabilities are not computed and are set to missing.

See the section “[Details of the PREDPROBS= Option](#)” on page 4227 at the end of this section for further details.

**RESCHI=name**

specifies the Pearson (chi-square) residual for identifying observations that are poorly accounted for by the model.

**RESDEV=name**

specifies the deviance residual for identifying poorly fitted observations.



**RESLIK=*name***

specifies the likelihood residual for identifying poorly fitted observations.

**STDRESCHI=*name***

specifies the standardized Pearson (chi-square) residual for identifying observations that are poorly accounted for by the model.

**STDRESDEV=*name***

specifies the standardized deviance residual for identifying poorly fitted observations.

**STDXBETA=*name***

names the variable containing the standard error estimates of **XBETA**. See the section “[Linear Predictor, Predicted Probability, and Confidence Limits](#)” on page 4253 for details.

**UPPER=*name*****U=*name***

names the variable containing the upper confidence limits for  $\pi$ , where  $\pi$  is the probability of the event response if events/trials syntax or single-trial syntax with binary response is specified; for a cumulative model,  $\pi$  is cumulative probability (that is, the probability that the response is less than or equal to the value of `_LEVEL_`); for the generalized logit model, it is the individual probability (that is, the probability that the response category is represented by the value of `_LEVEL_`). See the [ALPHA=](#) option to set the confidence level.

**XBETA=*name***

names the variable containing the estimates of the linear predictor  $\alpha_i + \beta'x$ , where  $i$  is the corresponding ordered value of `_LEVEL_`.

## Details of the PREDPROBS= Option

You can request any of the three types of predicted probabilities. For example, you can request both the individual predicted probabilities and the cross validated probabilities by specifying `PREDPROBS=(I X)`.

When you specify the `PREDPROBS=` option, two automatic variables, `_FROM_` and `_INTO_`, are included for the single-trial syntax and only one variable, `_INTO_`, is included for the events/trials syntax. The variable `_FROM_` contains the formatted value of the observed response. The variable `_INTO_` contains the formatted value of the response level with the largest individual predicted probability.

If you specify `PREDPROBS=INDIVIDUAL`, the `OUT=` data set contains  $k$  additional variables representing the individual probabilities, one for each response level, where  $k$  is the maximum number of response levels across all BY groups. The names of these variables have the form `IP_xxx`, where `xxx` represents the particular level. The representation depends on the following situations:

- If you specify events/trials syntax, `xxx` is either ‘Event’ or ‘Nonevent’. Thus, the variable containing the event probabilities is named `IP_Event` and the variable containing the nonevent probabilities is named `IP_Nonevent`.
- If you specify the single-trial syntax with more than one BY group, `xxx` is 1 for the first ordered level of the response, 2 for the second ordered level of the response, and so forth, as given in the “Response Profile” table. The variable containing the predicted probabilities  $\Pr(Y=1)$  is named `IP_1`, where  $Y$  is the response variable. Similarly, `IP_2` is the name of the variable containing the predicted probabilities  $\Pr(Y=2)$ , and so on.

- If you specify the single-trial syntax with no BY-group processing, *xxx* is the left-justified formatted value of the response level (the value might be truncated so that *IP\_**xxx* does not exceed 32 characters). For example, if *Y* is the response variable with response levels ‘None’, ‘Mild’, and ‘Severe’, the variables representing individual probabilities  $\Pr(Y=\text{‘None’})$ ,  $\Pr(Y=\text{‘Mild’})$ , and  $\Pr(Y=\text{‘Severe’})$  are named *IP\_None*, *IP\_Mild*, and *IP\_Severe*, respectively.

If you specify *PREDPROBS=CUMULATIVE*, the *OUT=* data set contains *k* additional variables representing the cumulative probabilities, one for each response level, where *k* is the maximum number of response levels across all BY groups. The names of these variables have the form *CP\_**xxx*, where *xxx* represents the particular response level. The naming convention is similar to that given by *PREDPROBS=INDIVIDUAL*. The *PREDPROBS=CUMULATIVE* values are the same as those output by the *PREDICT=* option, but are arranged in variables on each output observation rather than in multiple output observations.

If you specify *PREDPROBS=CROSSVALIDATE*, the *OUT=* data set contains *k* additional variables representing the cross validated predicted probabilities of the *k* response levels, where *k* is the maximum number of response levels across all BY groups. The names of these variables have the form *XP\_**xxx*, where *xxx* represents the particular level. The representation is the same as that given by *PREDPROBS=INDIVIDUAL* except that for the events/trials syntax there are four variables for the cross validated predicted probabilities instead of two:

*XP\_EVENT\_R1E* is the cross validated predicted probability of an event when a single event is removed from the current observation.

*XP\_NONEVENT\_R1E* is the cross validated predicted probability of a nonevent when a single event is removed from the current observation.

*XP\_EVENT\_R1N* is the cross validated predicted probability of an event when a single nonevent is removed from the current observation.

*XP\_NONEVENT\_R1N* is the cross validated predicted probability of a nonevent when a single nonevent is removed from the current observation.

The cross validated predicted probabilities are precisely those used in the *CTABLE* option. See the section “[Predicted Probability of an Event for Classification](#)” on page 4255 for details of the computation.

---

## ROC Statement

**ROC** < *label* > < *specification* > < / *options* > ;

The ROC statements specify models to be used in the ROC comparisons. You can specify more than one ROC statement. ROC statements are identified by their *label*—if you do not specify a *label*, the *i*th ROC statement is labeled “ROC*i*”. Additionally, the specified or selected model is labeled with the *MODEL* statement label or “Model” if the *MODEL* label is not present. The *specification* can be either a list of effects that have previously been specified in the *MODEL* statement, or *PRED=variable*, where the *variable* does not have to be specified in the *MODEL* statement. The *PRED=* option enables you to input a criterion produced outside PROC LOGISTIC; for example, you can fit a random-intercept model by using PROC GLIMMIX or use survey weights in PROC SURVEYLOGISTIC, then use the predicted values from those models to produce an ROC curve for the comparisons. If you do not make a *specification*, then an intercept-only model is fit to the data, resulting in a noninformative ROC curve that can be used for comparing the area under another ROC curve to 0.5.

You can specify a **ROCCONTRAST** statement and a **ROCOPTIONS** option in the PROC LOGISTIC statement to control how the models are compared, while the **PLOTS=ROC** option controls the ODS Graphics displays. See [Example 54.8](#) for an example that uses the ROC statement.

If you specify any *options*, then a “ROC Model Information” table summarizing the new ROC model is displayed. The *options* are ignored for the PRED= specification. The following *options* are available:

#### **NOOFFSET**

does not include an offset variable if the OFFSET= option is specified in the **MODEL** statement. A constant offset has no effect on the ROC curve, although the cutpoints might be different, but a nonconstant offset can affect the parameter estimates and hence the ROC curve.

#### **LINK=keyword**

specifies the link function to be used in the model. The available keywords are LOGIT, NORMIT, and CLOGLOG. The logit link is the default. Note that the **LINK=** option in the MODEL statement is ignored.

---

## **ROCCONTRAST Statement**

**ROCCONTRAST** < 'label' > < contrast > < / options > ;

The ROCCONTRAST statement compares the different ROC models. You can specify only one ROCCONTRAST statement. The **ROCOPTIONS** options in the PROC LOGISTIC statement control how the models are compared. You can specify one of the following *contrast* specifications:

#### **REFERENCE<(MODEL | 'roc-label')>**

produces a contrast matrix of differences between each ROC curve and a reference curve. The MODEL keyword specifies that the reference curve is that produced from the MODEL statement; the *roc-label* specifies the *label* of the ROC curve that is to be used as the reference curve. If neither the MODEL keyword nor the *roc-label* label is specified, then the reference ROC curve is either the curve produced from the MODEL statement, the selected model if a selection method is specified, or the model from the first ROC statement if the **NOFIT** option is specified.

#### **ADJACENTPAIRS**

produces a contrast matrix of each ROC curve minus the succeeding curve.

#### *matrix*

specifies the contrast in the form **row1, row2, . . .**, where each *row* contains the coefficients used to compare the ROC curves. Each *row* must contain the same number of entries as there are ROC curves being compared. The elements of each *row* refer to the ROC statements in the order in which they are specified. However, the first element of each *row* refers either to the fitted model, the selected model if a **SELECTION=** method is specified, or the first specified ROC statement if the **NOFIT** option is specified.

If no *contrast* is specified, then the REFERENCE contrast with the default reference curve is used. See the section “[Comparing ROC Curves](#)” on page 4261 for more information about comparing ROC curves, and see [Example 54.8](#) for an example.

The following *options* are available:

**E**

displays the contrast.

**ESTIMATE <= ROWS | ALLPAIRS >**

produces estimates of each row of the contrast when ESTIMATE or ESTIMATE=ROWS is specified. If the ESTIMATE=ALLPAIRS option is specified, then estimates of every pairwise difference of ROC curves are produced.

The row contrasts are labeled “ModelLabel1 – ModelLabel2”, where the model labels are as described in the [ROC](#) statement; in particular, for the REFERENCE contrast, ModelLabel2 is the reference model label. If you specify your own contrast matrix, then the *i*th contrast row estimate is labeled “Row*i*”.

**COV**

displays covariance matrices used in the computations.

---

## SCORE Statement

**SCORE** < options > ;

The SCORE statement creates a data set that contains all the data in the [DATA=](#) data set together with posterior probabilities and, optionally, prediction confidence intervals. Fit statistics are displayed on request. If you have binary response data, the SCORE statement can be used to create a data set containing data for the ROC curve. You can specify several SCORE statements. [FREQ](#), [WEIGHT](#), and [BY](#) statements can be used with the SCORE statements. The SCORE statement is not available with the [STRATA](#) statement.

If a [SCORE](#) statement is specified in the same run as fitting the model, [FORMAT](#) statements should be specified after the [SCORE](#) statement in order for the formats to apply to all the [DATA=](#) and [PRIOR=](#) data sets in the [SCORE](#) statement.

See the section “[Scoring Data Sets](#)” on page 4266 for more information, and see [Example 54.15](#) for an illustration of how to use this statement.

[Table 54.10](#) summarizes the options available in the SCORE statement.

**Table 54.10** SCORE Statement Options

| Option                      | Description                                                                |
|-----------------------------|----------------------------------------------------------------------------|
| <a href="#">ALPHA=</a>      | Specifies the significance level                                           |
| <a href="#">CLM</a>         | Outputs the Wald-test-based confidence limits                              |
| <a href="#">CUMULATIVE</a>  | Outputs the cumulative predicted probabilities                             |
| <a href="#">DATA=</a>       | Names the SAS data that you want to score                                  |
| <a href="#">FITSTAT</a>     | Displays fit statistics                                                    |
| <a href="#">OUT=</a>        | Names the SAS data set that contains the predicted information             |
| <a href="#">OUTROC=</a>     | Names the SAS data set that contains the ROC curve                         |
| <a href="#">PRIOR=</a>      | Names the SAS data set that contains the priors of the response categories |
| <a href="#">PRIOREVENT=</a> | Specifies the prior event probability                                      |
| <a href="#">ROCEPS=</a>     | Specifies the criterion for grouping estimated event probabilities         |

You can specify the following options:

**ALPHA=number**

specifies the significance level  $\alpha$  for  $100(1 - \alpha)\%$  confidence intervals. By default, the value of *number* is equal to the **ALPHA=** option in the PROC LOGISTIC statement, or 0.05 if that option is not specified. This option has no effect unless the **CLM** option in the SCORE statement is requested.

**CLM**

outputs the Wald-test-based confidence limits for the predicted probabilities. This option is not available when the **INMODEL=** data set is created with the **NOCOV** option.

**CUMULATIVE**

outputs the cumulative predicted probabilities  $\Pr(Y \leq i), i = 1, \dots, k + 1$ , to the **OUT=** data set. This option is valid only when you have more than two response levels; otherwise, the option is ignored and a note is printed in the SAS log. These probabilities are named *CP\_level\_i*, where *level\_i* is the *i*th response level.

If the **CLM** option is also specified in the SCORE statement, then the Wald-based confidence limits for the cumulative predicted probabilities are also output. The confidence limits are named *CLCL\_level\_i* and *CUCL\_level\_i*. In particular, for the lowest response level, the cumulative values (CP, CLCL, CUCL) should be identical to the individual values (P, LCL, UCL), and for the highest response level  $CP=CLCL=CUCL=1$ .

**DATA=SAS-data-set**

names the SAS data set that you want to score. If you omit the **DATA=** option in the SCORE statement, then scoring is performed on the **DATA=** input data set in the PROC LOGISTIC statement, if specified; otherwise, the **DATA=\_LAST\_** data set is used.

It is not necessary for the **DATA=** data set in the SCORE statement to contain the response variable unless you are specifying the **FITSTAT** or **OUTROC=** option.

Only those variables involved in the fitted model effects are required in the **DATA=** data set in the SCORE statement. For example, the following statements use forward selection to select effects:

```
proc logistic data=Neuralgia outmodel=sasuser.Model;
  class Treatment Sex;
  model Pain(event='Yes') = Treatment|Sex Age
    / selection=forward sle=.01;
run;
```

Suppose Treatment and Age are the effects selected for the final model. You can score a data set that does not contain the variable Sex since the effect Sex is not in the model that the scoring is based on. For example, the following statements score the Neuralgia data set after dropping the Sex variable:

```
proc logistic inmodel=sasuser.Model;
  score data=Neuralgia(drop=Sex);
run;
```

**FITSTAT**

displays fit statistics for the data set you are scoring. The data set must contain the response variable. See the section “[Fit Statistics for Scored Data Sets](#)” on page 4266 for details.

**OUT=SAS-data-set**

names the SAS data set that contains the predicted information. If you omit the OUT= option, the output data set is created and given a default name by using the DATA $n$  convention.

**OUTROC=SAS-data-set**

names the SAS data set that contains the ROC curve for the DATA= data set. The ROC curve is computed only for binary response data. See the section “[OUTROC= Output Data Set](#)” on page 4283 for the list of variables in this data set.

**PRIOR=SAS-data-set**

names the SAS data set that contains the priors of the response categories. The priors can be values proportional to the prior probabilities; thus, they do not necessarily sum to one. This data set should include a variable named `_PRIOR_` that contains the prior probabilities. For events/trials MODEL statement syntax, this data set should also include an `_OUTCOME_` variable that contains the values EVENT and NONEVENT; for single-trial syntax, this data set should include the response variable that contains the unformatted response categories. See [Example 54.15](#) for an example.

**PRIOREVENT=value**

specifies the prior event probability for a binary response model. If both **PRIOR=** and **PRIOREVENT=** options are specified, the **PRIOR=** option takes precedence.

**ROCEPS=value**

specifies the criterion for grouping estimated event probabilities that are close to each other for the ROC curve. In each group, the difference between the largest and the smallest estimated event probability does not exceed the given value. The *value* must be between 0 and 1; the default value is the square root of the machine epsilon, which is about 1E–8 (in releases prior to 9.2, the default was 1E–4). The smallest estimated probability in each group serves as a cutpoint for predicting an event response. The ROCEPS= option has no effect if the **OUTROC=** option is not specified in the SCORE statement.

---

## SLICE Statement

**SLICE** *model-effect* </ options> ;

The SLICE statement provides a general mechanism for performing a partitioned analysis of the LS-means for an interaction. This analysis is also known as an analysis of simple effects.

The SLICE statement uses the same options as the **LSMEANS** statement, which are summarized in [Table 19.21](#). For details about the syntax of the SLICE statement, see the section “[SLICE Statement](#)” on page 498 in Chapter 19, “[Shared Concepts and Topics](#).”

**NOTE:** If you have classification variables in your model, then the SLICE statement is allowed only if you also specify the **PARAM=GLM** option.

---

## STORE Statement

**STORE** <OUT= >*item-store-name* </ LABEL='label'> ;

The STORE statement requests that the procedure save the context and results of the statistical analysis. The resulting item store has a binary file format that cannot be modified. The contents of the item store can be processed with the PLM procedure.

For details about the syntax of the STORE statement, see the section “STORE Statement” on page 501 in Chapter 19, “Shared Concepts and Topics.”

---

## STRATA Statement

**STRATA** *variable* <(option)> ... <*variable* <(option)>> </ options> ;

The STRATA statement names the *variables* that define *strata* or *matched sets* to use in *stratified logistic regression* of binary response data.

Observations that have the same *variable* values are in the same matched set. For a stratified logistic model, you can analyze 1: 1, 1:  $n$ ,  $m$ :  $n$ , and general  $m_i$ :  $n_i$  matched sets where the number of cases and controls varies across strata. At least one variable must be specified to invoke the stratified analysis, and the usual unconditional asymptotic analysis is not performed. The stratified logistic model has the form

$$\text{logit}(\pi_{hi}) = \alpha_h + \mathbf{x}_{hi}'\boldsymbol{\beta}$$

where  $\pi_{hi}$  is the event probability for the  $i$ th observation in stratum  $h$  with covariates  $\mathbf{x}_{hi}$  and where the stratum-specific intercepts  $\alpha_h$  are the nuisance parameters that are to be conditioned out.

STRATA variables can also be specified in the MODEL statement as classification or continuous covariates; however, the effects are nondegenerate only when crossed with a nonstratification variable. Specifying several STRATA statements is the same as specifying one STRATA statement that contains all the strata variables. The STRATA variables can be either character or numeric, and the formatted values of the STRATA variables determine the levels. Thus, you can also use formats to group values into levels; see the discussion of the FORMAT procedure in the *Base SAS Procedures Guide*.

The “Strata Summary” table is displayed by default. For an exact logistic regression, it displays the number of strata that have a specific number of events and non-events. For example, if you are analyzing a 1: 5 matched study, this table enables you to verify that every stratum in the analysis has exactly one event and five non-events. Strata that contain only events or only non-events are reported in this table, but such strata are uninformative and are not used in the analysis.

If an EXACT statement is also specified, then a stratified *exact* logistic regression is performed.

The EFFECTPLOT, SCORE, and WEIGHT statements are not available with a STRATA statement. The following MODEL options are also not supported with a STRATA statement: CLPARM=PL, CLODDS=PL, CTABLE, FIRTH, LACKFIT, LINK=, NOFIT, OUTMODEL=, OUTROC=, ROC, and SCALE=.

The following *option* can be specified for a stratification variable by enclosing the option in parentheses after the variable name, or it can be specified globally for all STRATA variables after a slash (/).



**MISSING**

treats missing values ('.', '\_', 'A', ..., 'Z' for numeric variables and blanks for character variables) as valid STRATA variable values.

The following strata *options* are also available after the slash:

**CHECKDEPENDENCY | CHECK=keyword**

specifies which variables are to be tested for dependency before the analysis is performed. The available *keywords* are as follows:

**NONE** performs no dependence checking. Typically, a message about a singular information matrix is displayed if you have dependent variables. Dependent variables can be identified after the analysis by noting any missing parameter estimates.

**COVARIATES** checks dependence between covariates and an added intercept. Dependent covariates are removed from the analysis. However, covariates that are linear functions of the strata variable might not be removed, which results in a singular information matrix message being displayed in the SAS log. This is the default.

**ALL** checks dependence between all the strata and covariates. This option can adversely affect performance if you have a large number of strata.

**NOSUMMARY**

suppresses the display of the “Strata Summary” table.

**INFO**

displays the “Strata Information” table, which includes the stratum number, levels of the STRATA variables that define the stratum, the number of events, the number of non-events, and the total frequency for each stratum. Since the number of strata can be very large, this table is displayed only by request.

---

## TEST Statement

```
<label> TEST equation1 <, equation2, ... > </option> ;
```

The TEST statement tests linear hypotheses about the regression coefficients. The Wald test is used to perform a joint test of the null hypotheses  $H_0: \mathbf{L}\boldsymbol{\beta} = \mathbf{c}$  specified in a single TEST statement, where  $\boldsymbol{\beta}$  is the vector of intercept and slope parameters. When  $\mathbf{c} = \mathbf{0}$  you should specify a CONTRAST statement instead.

Each *equation* specifies a linear hypothesis (a row of the **L** matrix and the corresponding element of the **c** vector). Multiple *equations* are separated by commas. The *label*, which must be a valid SAS name, is used to identify the resulting output and should always be included. You can submit multiple TEST statements.

The form of an *equation* is as follows:

```
term<± term ...> <= ±term <±term...>>
```

where *term* is a parameter of the model, or a constant, or a constant times a parameter. Intercept and CLASS variable parameter names should be specified as described in the section “[Parameter Names in the OUTEST= Data Set](#)” on page 4279. Note for generalized logit models that this enables you to construct tests of parameters from specific logits. When no equal sign appears, the expression is set to 0. The following statements illustrate possible uses of the TEST statement:



```
proc logistic;
  model y= a1 a2 a3 a4;
  test1: test intercept + .5 * a2 = 0;
  test2: test intercept + .5 * a2;
  test3: test a1=a2=a3;
  test4: test a1=a2, a2=a3;
run;
```

Note that the first and second TEST statements are equivalent, as are the third and fourth TEST statements.

You can specify the following option in the TEST statement after a slash(/):

### PRINT

displays intermediate calculations in the testing of the null hypothesis  $H_0: \mathbf{L}\boldsymbol{\beta} = \mathbf{c}$ . This includes  $\mathbf{L}\hat{\mathbf{V}}(\hat{\boldsymbol{\beta}})\mathbf{L}'$  bordered by  $(\mathbf{L}\hat{\boldsymbol{\beta}} - \mathbf{c})$  and  $[\mathbf{L}\hat{\mathbf{V}}(\hat{\boldsymbol{\beta}})\mathbf{L}']^{-1}$  bordered by  $[\mathbf{L}\hat{\mathbf{V}}(\hat{\boldsymbol{\beta}})\mathbf{L}']^{-1}(\mathbf{L}\hat{\boldsymbol{\beta}} - \mathbf{c})$ , where  $\hat{\boldsymbol{\beta}}$  is the maximum likelihood estimator of  $\boldsymbol{\beta}$  and  $\hat{\mathbf{V}}(\hat{\boldsymbol{\beta}})$  is the estimated covariance matrix of  $\hat{\boldsymbol{\beta}}$ .

For more information, see the section “[Testing Linear Hypotheses about the Regression Coefficients](#)” on page 4262.

---

## UNITS Statement

**UNITS** < *independent1=list1* < *independent2=list2* . . . > < / *option* > ;

The UNITS statement enables you to specify units of change for the continuous explanatory variables so that customized odds ratios can be estimated. If you specify more than one UNITS statement, only the last one is used. An estimate of the corresponding odds ratio is produced for each unit of change specified for an explanatory variable. The UNITS statement is ignored for CLASS variables. Odds ratios are computed only for main effects that are not involved in interactions or nestings, unless an **ODDSRATIO** statement is also specified. If the **CLODDS=** option is specified in the MODEL statement, the corresponding confidence limits for the odds ratios are also displayed, as are odds ratios and confidence limits for any CLASS main effects that are not involved in interactions or nestings. The CLASS effects must use the GLM, reference, or effect coding.

The UNITS statement also enables you to customize the odds ratios for effects specified in **ODDSRATIO** statements, in which case interactions and nestings are allowed, and CLASS variables can be specified with any parameterization.

The term *independent* is the name of an explanatory variable and *list* represents a list of units of change, separated by spaces, that are of interest for that variable. Each unit of change in a list has one of the following forms:

- *number*
- SD or –SD
- *number* \* SD

where *number* is any nonzero number, and SD is the sample standard deviation of the corresponding independent variable. For example,  $X = -2$  requests an odds ratio that represents the change in the odds when

the variable  $X$  is decreased by two units.  $X = 2*SD$  requests an estimate of the change in the odds when  $X$  is increased by two sample standard deviations.

You can specify the following option in the UNITS statement after a slash(/):

**DEFAULT=*list***

gives a list of units of change for all explanatory variables that are not specified in the UNITS statement. Each unit of change can be in any of the forms described previously. If the DEFAULT= option is not specified, PROC LOGISTIC does not produce customized odds ratio estimates for any continuous explanatory variable that is not listed in the UNITS statement.

For more information, see the section “[Odds Ratio Estimation](#)” on page 4250.

---

## WEIGHT Statement

**WEIGHT** *variable* </ option> ;

When a WEIGHT statement appears, each observation in the input data set is weighted by the value of the WEIGHT variable. Unlike a [FREQ](#) variable, the values of the WEIGHT variable can be nonintegral and are not truncated. Observations with negative, zero, or missing values for the WEIGHT variable are not used in the model fitting. When the WEIGHT statement is not specified, each observation is assigned a weight of 1. The WEIGHT statement is not available with the [STRATA](#) statement. If you specify more than one WEIGHT statement, then the first WEIGHT variable is used.

If a [SCORE](#) statement is specified, then the WEIGHT variable is used for computing fit statistics and the ROC curve, but it is not required for scoring. If the [DATA=](#) data set in the [SCORE](#) statement does not contain the WEIGHT variable, the weights are assumed to be 1 and a warning message is issued in the SAS log. If you fit a model and perform the scoring in the same run, the same WEIGHT variable is used for fitting and scoring. If you fit a model in a previous run and input it with the [INMODEL=](#) option in the current run, then the WEIGHT variable can be different from the one used in the previous run; however, if a WEIGHT variable was not specified in the previous run, you can still specify a WEIGHT variable in the current run.

**CAUTION:** PROC LOGISTIC does not compute the proper variance estimators if you are analyzing survey data and specifying the sampling weights through the WEIGHT statement. The [SURVEYLOGISTIC](#) procedure is designed to perform the necessary, and correct, computations.

The following option can be added to the WEIGHT statement after a slash (/):

**NORMALIZE**

**NORM**

causes the weights specified by the WEIGHT variable to be normalized so that they add up to the actual sample size. Weights  $w_i$  are normalized by multiplying them by  $\frac{n}{\sum_{i=1}^n w_i}$ , where  $n$  is the sample size. With this option, the estimated covariance matrix of the parameter estimators is invariant to the scale of the WEIGHT variable.

---

## Details: LOGISTIC Procedure

---

### Missing Values

Any observation with missing values for the response, offset, strata, or explanatory variables is excluded from the analysis; however, missing values are valid for variables specified with the **MISSING** option in the **CLASS** or **STRATA** statement. Observations with a nonpositive or missing weight or with a frequency less than 1 are also excluded. The estimated linear predictor and its standard error estimate, the fitted probabilities and confidence limits, and the regression diagnostic statistics are not computed for any observation with missing offset or explanatory variable values. However, if only the response value is missing, the linear predictor, its standard error, the fitted individual and cumulative probabilities, and confidence limits for the cumulative probabilities can be computed and output to a data set by using the **OUTPUT** statement.

---

### Response Level Ordering

Response level ordering is important because, by default, PROC LOGISTIC models the probability of response levels with *lower Ordered Value*. Ordered Values are assigned to response levels in ascending sorted order (that is, the lowest response level is assigned Ordered Value 1, the next lowest is assigned Ordered Value 2, and so on) and are displayed in the “Response Profiles” table. If your response variable  $Y$  takes values in  $\{1, \dots, k + 1\}$ , then, by default, the functions modeled with the binary or cumulative model are

$$\text{logit}(\Pr(Y \leq i | \mathbf{x})), \quad i = 1, \dots, k$$

and for the generalized logit model the functions modeled are

$$\log \left( \frac{\Pr(Y = i | \mathbf{x})}{\Pr(Y = k + 1 | \mathbf{x})} \right), \quad i = 1, \dots, k$$

where the highest Ordered Value  $Y = k + 1$  is the reference level. You can change which probabilities are modeled by specifying the **EVENT=**, **REF=**, **DESCENDING**, or **ORDER=** response variable options in the **MODEL** statement.

For binary response data with event and nonevent categories, if your event category has a higher Ordered Value, then by default the nonevent is modeled. Since the default response function modeled is

$$\text{logit}(\pi) = \log \left( \frac{\pi}{1 - \pi} \right)$$

where  $\pi$  is the probability of the response level assigned Ordered Value 1, and since

$$\text{logit}(\pi) = -\text{logit}(1 - \pi)$$

the effect of modeling the nonevent is to change the signs of  $\alpha$  and  $\beta$  in the model for the event,  $\text{logit}(\pi) = \alpha + \beta' \mathbf{x}$ .

For example, suppose the binary response variable  $Y$  takes the values 1 and 0 for event and nonevent, respectively, and *Exposure* is the explanatory variable. By default, PROC LOGISTIC assigns Ordered

Value 1 to response level  $Y=0$ , and Ordered Value 2 to response level  $Y=1$ . As a result, PROC LOGISTIC models the probability of the nonevent (Ordered Value=1) category, and your parameter estimates have the opposite sign from those in the model for the event. To model the event without using a DATA step to change the values of the variable  $Y$ , you can control the ordering of the response levels or select the event or reference level, as shown in the following list:

- Explicitly state which response level is to be modeled by using the response variable option **EVENT=** in the **MODEL** statement:

```
model Y(event='1') = Exposure;
```

- Specify the nonevent category for the response variable in the response variable option **REF=** in the **MODEL** statement. This option is most useful for generalized logit models where the **EVENT=** option cannot be used.

```
model Y(ref='0') = Exposure;
```

- Specify the response variable option **DESCENDING** in the **MODEL** statement to assign the lowest Ordered Value to  $Y=1$ :

```
model Y(descending)=Exposure;
```

- Assign a format to  $Y$  such that the first formatted value (when the formatted values are put in sorted order) corresponds to the event. In the following example,  $Y=1$  is assigned the formatted value 'event' and  $Y=0$  is assigned the formatted value 'nonevent'. Since **ORDER=FORMATTED** by default, Ordered Value 1 is assigned to response level  $Y=1$ , so the procedure models the event.

```
proc format;
  value Disease 1='event' 0='nonevent';
run;
proc logistic;
  format Y Disease.;
  model Y=Exposure;
run;
```

---

## Link Functions and the Corresponding Distributions

Four link functions are available in the LOGISTIC procedure. The logit function is the default. To specify a different link function, use the **LINK=** option in the **MODEL** statement. The link functions and the corresponding distributions are as follows:

- The logit function

$$g(p) = \log(p/(1 - p))$$

is the inverse of the cumulative logistic distribution function, which is

$$F(x) = 1/(1 + \exp(-x)) = \exp(x)/(1 + \exp(x))$$

- The probit (or normit) function

$$g(p) = \Phi^{-1}(p)$$

is the inverse of the cumulative standard normal distribution function, which is

$$F(x) = \Phi(x) = (2\pi)^{-1/2} \int_{-\infty}^x \exp(-z^2/2) dz$$

Traditionally, the probit function contains the additive constant 5, but throughout PROC LOGISTIC, the terms probit and normit are used interchangeably.

- The complementary log-log function

$$g(p) = \log(-\log(1 - p))$$

is the inverse of the cumulative extreme-value function (also called the Gompertz distribution), which is

$$F(x) = 1 - \exp(-\exp(x))$$

- The generalized logit function extends the binary logit link to a vector of levels  $(p_1, \dots, p_{k+1})$  by contrasting each level with a fixed level

$$g(p_i) = \log(p_i/p_{k+1}) \quad i = 1, \dots, k$$

The variances of the normal, logistic, and extreme-value distributions are not the same. Their respective means and variances are shown in the following table:

| Distribution  | Mean      | Variance  |
|---------------|-----------|-----------|
| Normal        | 0         | 1         |
| Logistic      | 0         | $\pi^2/3$ |
| Extreme-value | $-\gamma$ | $\pi^2/6$ |

Here  $\gamma$  is the Euler constant. In comparing parameter estimates from different link functions, you need to take into account the different scalings of the corresponding distributions and, for the complementary log-log function, a possible shift in location. For example, if the fitted probabilities are in the neighborhood of 0.1 to 0.9, then the parameter estimates from the logit link function should be about  $\pi/\sqrt{3}$  larger than the estimates from the probit link function.

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## Determining Observations for Likelihood Contributions

If you use events/trials MODEL statement syntax, split each observation into two observations. One has response value 1 with a frequency equal to the frequency of the original observation (which is 1 if the FREQ statement is not used) times the value of the *events* variable. The other observation has response value 2 and a frequency equal to the frequency of the original observation times the value of (*trials*–*events*). These two observations will have the same explanatory variable values and the same FREQ and WEIGHT values as the original observation.

For either single-trial or events/trials syntax, let  $j$  index all observations. In other words, for single-trial syntax,  $j$  indexes the actual observations. And, for events/trials syntax,  $j$  indexes the observations after splitting (as described in the preceding paragraph). If your data set has 30 observations and you use single-trial syntax,  $j$  has values from 1 to 30; if you use events/trials syntax,  $j$  has values from 1 to 60.

Suppose the response variable in a cumulative response model can take on the ordered values  $1, \dots, k, k+1$ , where  $k$  is an integer  $\geq 1$ . The likelihood for the  $j$ th observation with ordered response value  $y_j$  and explanatory variables vector  $\mathbf{x}_j$  is given by

$$L_j = \begin{cases} F(\alpha_1 + \boldsymbol{\beta}'\mathbf{x}_j) & y_j = 1 \\ F(\alpha_i + \boldsymbol{\beta}'\mathbf{x}_j) - F(\alpha_{i-1} + \boldsymbol{\beta}'\mathbf{x}_j) & 1 < y_j = i \leq k \\ 1 - F(\alpha_k + \boldsymbol{\beta}'\mathbf{x}_j) & y_j = k + 1 \end{cases}$$

where  $F(\cdot)$  is the logistic, normal, or extreme-value distribution function,  $\alpha_1, \dots, \alpha_k$  are ordered intercept parameters, and  $\boldsymbol{\beta}$  is the common slope parameter vector.

For the generalized logit model, letting the  $k+1$ st level be the reference level, the intercepts  $\alpha_1, \dots, \alpha_k$  are unordered and the slope vector  $\boldsymbol{\beta}_i$  varies with each logit. The likelihood for the  $j$ th observation with response value  $y_j$  and explanatory variables vector  $\mathbf{x}_j$  is given by

$$L_j = \Pr(Y = y_j | \mathbf{x}_j) = \begin{cases} \frac{e^{\alpha_i + \mathbf{x}_j' \boldsymbol{\beta}_i}}{1 + \sum_{m=1}^k e^{\alpha_m + \mathbf{x}_j' \boldsymbol{\beta}_m}} & 1 \leq y_j = i \leq k \\ \frac{1}{1 + \sum_{m=1}^k e^{\alpha_m + \mathbf{x}_j' \boldsymbol{\beta}_m}} & y_j = k + 1 \end{cases}$$

## Iterative Algorithms for Model Fitting

Two iterative maximum likelihood algorithms are available in PROC LOGISTIC for fitting an unconditional logistic regression, and these two methods are discussed in this section. For conditional logistic regression and models with the [UNEQUALSLOPES](#) specification, see the section “[NLOPTIONS Statement](#)” on page 4222 for details about available optimization techniques. Exact logistic regression uses a special algorithm described in the section “[Exact Conditional Logistic Regression](#)” on page 4274.

The default maximum likelihood algorithm is the Fisher scoring method, which is equivalent to fitting by iteratively reweighted least squares. The alternative algorithm is the Newton-Raphson method. Both algorithms give the same parameter estimates; however, the estimated covariance matrix of the parameter estimators can differ slightly. This is due to the fact that Fisher scoring is based on the expected information matrix while the Newton-Raphson method is based on the observed information matrix. In the case of a binary logit model, the observed and expected information matrices are identical, resulting in identical estimated covariance matrices for both algorithms. You can specify the [TECHNIQUE=](#) option to select a fitting algorithm, and specify the [FIRTH](#) option to perform a bias-reducing penalized maximum likelihood fit. Note for generalized logit models that only the Newton-Raphson technique is available.

### Iteratively Reweighted Least Squares Algorithm (Fisher Scoring)

Consider the multinomial variable  $\mathbf{Z}_j = (Z_{1j}, \dots, Z_{k+1,j})'$  such that

$$Z_{ij} = \begin{cases} 1 & \text{if } Y_j = i \\ 0 & \text{otherwise} \end{cases}$$

With  $\pi_{ij}$  denoting the probability that the  $j$ th observation has response value  $i$ , the expected value of  $\mathbf{Z}_j$  is  $\boldsymbol{\pi}_j = (\pi_{1j}, \dots, \pi_{k+1,j})'$  where  $\pi_{k+1,j} = 1 - \sum_{i=1}^k \pi_{ij}$ . The covariance matrix of  $\mathbf{Z}_j$  is  $\mathbf{V}_j$ , which is the covariance matrix of a multinomial random variable for one trial with parameter vector  $\boldsymbol{\pi}_j$ . Let  $\boldsymbol{\beta}$  be the vector of regression parameters; in other words,  $\boldsymbol{\beta} = (\alpha_1, \dots, \alpha_k, \beta_1, \dots, \beta_s)'$ . Let  $\mathbf{D}_j$  be the matrix of partial derivatives of  $\boldsymbol{\pi}_j$  with respect to  $\boldsymbol{\beta}$ . The estimating equation for the regression parameters is

$$\sum_j \mathbf{D}_j' \mathbf{W}_j (\mathbf{Z}_j - \boldsymbol{\pi}_j) = \mathbf{0}$$

where  $\mathbf{W}_j = w_j f_j \mathbf{V}_j^{-}$ ,  $w_j$  and  $f_j$  are the weight and frequency of the  $j$ th observation, and  $\mathbf{V}_j^{-}$  is a generalized inverse of  $\mathbf{V}_j$ . PROC LOGISTIC chooses  $\mathbf{V}_j^{-}$  as the inverse of the diagonal matrix with  $\boldsymbol{\pi}_j$  as the diagonal.

With a starting value of  $\boldsymbol{\beta}^{(0)}$ , the maximum likelihood estimate of  $\boldsymbol{\beta}$  is obtained iteratively as

$$\boldsymbol{\beta}^{(m+1)} = \boldsymbol{\beta}^{(m)} + \left( \sum_j \mathbf{D}_j' \mathbf{W}_j \mathbf{D}_j \right)^{-1} \sum_j \mathbf{D}_j' \mathbf{W}_j (\mathbf{Z}_j - \boldsymbol{\pi}_j)$$

where  $\mathbf{D}_j$ ,  $\mathbf{W}_j$ , and  $\boldsymbol{\pi}_j$  are evaluated at  $\boldsymbol{\beta}^{(m)}$ . The expression after the plus sign is the step size. If the likelihood evaluated at  $\boldsymbol{\beta}^{(m+1)}$  is less than that evaluated at  $\boldsymbol{\beta}^{(m)}$ , then  $\boldsymbol{\beta}^{(m+1)}$  is recomputed by step-halving or ridging as determined by the value of the **RIDGING=** option. The iterative scheme continues until convergence is obtained—that is, until  $\boldsymbol{\beta}^{(m+1)}$  is sufficiently close to  $\boldsymbol{\beta}^{(m)}$ . Then the maximum likelihood estimate of  $\boldsymbol{\beta}$  is  $\hat{\boldsymbol{\beta}} = \boldsymbol{\beta}^{(m+1)}$ .

The covariance matrix of  $\hat{\boldsymbol{\beta}}$  is estimated by

$$\widehat{\text{Cov}}(\hat{\boldsymbol{\beta}}) = \left( \sum_j \hat{\mathbf{D}}_j' \hat{\mathbf{W}}_j \hat{\mathbf{D}}_j \right)^{-1} = \hat{\mathbf{I}}^{-1}$$

where  $\hat{\mathbf{D}}_j$  and  $\hat{\mathbf{W}}_j$  are, respectively,  $\mathbf{D}_j$  and  $\mathbf{W}_j$  evaluated at  $\hat{\boldsymbol{\beta}}$ .  $\hat{\mathbf{I}}$  is the information matrix, or the negative expected Hessian matrix, evaluated at  $\hat{\boldsymbol{\beta}}$ .

By default, starting values are zero for the slope parameters, and for the intercept parameters, starting values are the observed cumulative logits (that is, logits of the observed cumulative proportions of response). Alternatively, the starting values can be specified with the **INEST=** option.

## Newton-Raphson Algorithm

For cumulative models, let the parameter vector be  $\boldsymbol{\beta} = (\alpha_1, \dots, \alpha_k, \beta_1, \dots, \beta_s)'$ , and for the generalized logit model let  $\boldsymbol{\beta} = (\alpha_1, \dots, \alpha_k, \beta_1', \dots, \beta_k')'$ . The gradient vector and the Hessian matrix are given, respectively, by

$$\begin{aligned} \mathbf{g} &= \sum_j w_j f_j \frac{\partial l_j}{\partial \boldsymbol{\beta}} \\ \mathbf{H} &= \sum_j w_j f_j \frac{\partial^2 l_j}{\partial \boldsymbol{\beta}^2} \end{aligned}$$

where  $l_j = \log L_j$  is the log likelihood for the  $j$ th observation. With a starting value of  $\boldsymbol{\beta}^{(0)}$ , the maximum likelihood estimate  $\hat{\boldsymbol{\beta}}$  of  $\boldsymbol{\beta}$  is obtained iteratively until convergence is obtained:

$$\boldsymbol{\beta}^{(m+1)} = \boldsymbol{\beta}^{(m)} - \mathbf{H}^{-1} \mathbf{g}$$

where  $\mathbf{H}$  and  $\mathbf{g}$  are evaluated at  $\boldsymbol{\beta}^{(m)}$ . If the likelihood evaluated at  $\boldsymbol{\beta}^{(m+1)}$  is less than that evaluated at  $\boldsymbol{\beta}^{(m)}$ , then  $\boldsymbol{\beta}^{(m+1)}$  is recomputed by step-halving or ridging.

The covariance matrix of  $\hat{\boldsymbol{\beta}}$  is estimated by

$$\widehat{\text{Cov}}(\hat{\boldsymbol{\beta}}) = \hat{\mathbf{I}}^{-1}$$

where the observed information matrix  $\hat{\mathbf{I}} = -\hat{\mathbf{H}}$  is computed by evaluating  $\mathbf{H}$  at  $\hat{\boldsymbol{\beta}}$ .

### Firth's Bias-Reducing Penalized Likelihood

Firth's method is currently available only for binary logistic models. It replaces the usual score (gradient) equation

$$g(\beta_j) = \sum_{i=1}^n (y_i - \pi_i) x_{ij} = 0 \quad (j = 1, \dots, p)$$

where  $p$  is the number of parameters in the model, with the modified score equation

$$g(\beta_j)^* = \sum_{i=1}^n \{y_i - \pi_i + h_i(0.5 - \pi_i)\} x_{ij} = 0 \quad (j = 1, \dots, p)$$

where the  $h_i$ s are the  $i$ th diagonal elements of the hat matrix  $\mathbf{W}^{1/2} \mathbf{X}(\mathbf{X}' \mathbf{W} \mathbf{X})^{-1} \mathbf{X}' \mathbf{W}^{1/2}$  and  $\mathbf{W} = \text{diag}\{\pi_i(1 - \pi_i)\}$ . The Hessian matrix is not modified by this penalty, and the optimization method is performed in the usual manner.

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## Convergence Criteria

Four convergence criteria are available: **ABSFCNV=**, **FCONV=**, **GCONV=**, and **XCONV=**. If you specify more than one convergence criterion, the optimization is terminated as soon as one of the criteria is satisfied. If none of the criteria is specified, the default is **GCONV=1E-8**.

If you specify a **STRATA** statement or the **UNEQUALSLOPES** option in the **MODEL** statement, all unspecified (or nondefault) criteria are also compared to zero. For example, specifying only the criterion **XCONV=1E-8** but attaining **FCONV=0** terminates the optimization even if the **XCONV=** criterion is not satisfied, because the log likelihood has reached its maximum. More convergence criteria are also available; see the section “**NLOPTIONS Statement**” on page 4222 for details.

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## Existence of Maximum Likelihood Estimates

The likelihood equation for a logistic regression model does not always have a finite solution. Sometimes there is a nonunique maximum on the boundary of the parameter space, at infinity. The existence, finiteness, and uniqueness of maximum likelihood estimates for the logistic regression model depend on the patterns of data points in the observation space (Albert and Anderson 1984; Santner and Duffy 1986). Existence checks are not performed for conditional logistic regression.



Consider a binary response model. Let  $Y_j$  be the response of the  $j$ th subject, and let  $\mathbf{x}_j$  be the vector of explanatory variables (including the constant 1 associated with the intercept). There are three mutually exclusive and exhaustive types of data configurations: complete separation, quasi-complete separation, and overlap.

**Complete Separation** There is a complete separation of data points if there exists a vector  $\mathbf{b}$  that correctly allocates all observations to their response groups; that is,

$$\begin{cases} \mathbf{b}'\mathbf{x}_j > 0 & Y_j = 1 \\ \mathbf{b}'\mathbf{x}_j < 0 & Y_j = 2 \end{cases}$$

This configuration gives nonunique infinite estimates. If the iterative process of maximizing the likelihood function is allowed to continue, the log likelihood diminishes to zero, and the dispersion matrix becomes unbounded.

**Quasi-complete Separation** The data are not completely separable, but there is a vector  $\mathbf{b}$  such that

$$\begin{cases} \mathbf{b}'\mathbf{x}_j \geq 0 & Y_j = 1 \\ \mathbf{b}'\mathbf{x}_j \leq 0 & Y_j = 2 \end{cases}$$

and equality holds for at least one subject in each response group. This configuration also yields nonunique infinite estimates. If the iterative process of maximizing the likelihood function is allowed to continue, the dispersion matrix becomes unbounded and the log likelihood diminishes to a nonzero constant.

**Overlap** If neither complete nor quasi-complete separation exists in the sample points, there is an overlap of sample points. In this configuration, the maximum likelihood estimates exist and are unique.

Complete separation and quasi-complete separation are problems typically encountered with small data sets. Although complete separation can occur with any type of data, quasi-complete separation is not likely with truly continuous explanatory variables.

The LOGISTIC procedure uses a simple empirical approach to recognize the data configurations that lead to infinite parameter estimates. The basis of this approach is that any convergence method of maximizing the log likelihood must yield a solution giving complete separation, if such a solution exists. In maximizing the log likelihood, there is no checking for complete or quasi-complete separation if convergence is attained in eight or fewer iterations. Subsequent to the eighth iteration, the probability of the observed response is computed for each observation. If the predicted response equals the observed response for every observation, there is a complete separation of data points and the iteration process is stopped. If the complete separation of data has not been determined and an observation is identified to have an extremely large probability ( $\geq 0.95$ ) of predicting the observed response, there are two possible situations. First, there is overlap in the data set, and the observation is an atypical observation of its own group. The iterative process, if allowed to continue, will stop when a maximum is reached. Second, there is quasi-complete separation in the data set, and the asymptotic dispersion matrix is unbounded. If any of the diagonal elements of the dispersion matrix for the standardized observations vectors (all explanatory variables standardized to zero mean and unit variance) exceeds 5000, quasi-complete separation is declared and the iterative process is stopped. If either complete separation or quasi-complete separation is detected, a warning message is displayed in the procedure output.

Checking for quasi-complete separation is less foolproof than checking for complete separation. The **NOCHECK** option in the **MODEL** statement turns off the process of checking for infinite parameter estimates. In cases of complete or quasi-complete separation, turning off the checking process typically results

in the procedure failing to converge. The presence of a **WEIGHT** statement also turns off the checking process.

To address the separation issue, you can change your model, specify the **FIRTH** option to use Firth's penalized likelihood method, or for small data sets specify an **EXACT** statement to perform an exact logistic regression.

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## Effect-Selection Methods

Five effect-selection methods are available by specifying the **SELECTION=** option in the **MODEL** statement. The simplest method (and the default) is **SELECTION=NONE**, for which PROC LOGISTIC fits the complete model as specified in the **MODEL** statement. The other four methods are **FORWARD** for forward selection, **BACKWARD** for backward elimination, **STEPWISE** for stepwise selection, and **SCORE** for best subsets selection. Intercept parameters are forced to stay in the model unless the **NOINT** option is specified.

When **SELECTION=FORWARD**, PROC LOGISTIC first estimates parameters for effects forced into the model. These effects are the intercepts and the first  $n$  explanatory effects in the **MODEL** statement, where  $n$  is the number specified by the **START=** or **INCLUDE=** option in the **MODEL** statement ( $n$  is zero by default). Next, the procedure computes the score chi-square statistic for each effect not in the model and examines the largest of these statistics. If it is significant at the **SLENTRY=** level, the corresponding effect is added to the model. Once an effect is entered in the model, it is never removed from the model. The process is repeated until none of the remaining effects meet the specified level for entry or until the **STOP=** value is reached.

When **SELECTION=BACKWARD**, parameters for the complete model as specified in the **MODEL** statement are estimated unless the **START=** option is specified. In that case, only the parameters for the intercepts and the first  $n$  explanatory effects in the **MODEL** statement are estimated, where  $n$  is the number specified by the **START=** option. Results of the Wald test for individual parameters are examined. The least significant effect that does not meet the **SLSTAY=** level for staying in the model is removed. Once an effect is removed from the model, it remains excluded. The process is repeated until no other effect in the model meets the specified level for removal or until the **STOP=** value is reached. Backward selection is often less successful than forward or stepwise selection because the full model fit in the first step is the model most likely to result in a complete or quasi-complete separation of response values as described in the section “Existence of Maximum Likelihood Estimates” on page 4242.

The **SELECTION=STEPWISE** option is similar to the **SELECTION=FORWARD** option except that effects already in the model do not necessarily remain. Effects are entered into and removed from the model in such a way that each forward selection step can be followed by one or more backward elimination steps. The stepwise selection process terminates if no further effect can be added to the model or if the current model is identical to a previously visited model.

For **SELECTION=SCORE**, PROC LOGISTIC uses the branch-and-bound algorithm of Furnival and Wilson (1974) to find a specified number of models with the highest likelihood score (chi-square) statistic for all possible model sizes, from 1, 2, 3 effect models, and so on, up to the single model containing all of the explanatory effects. The number of models displayed for each model size is controlled by the **BEST=** option. You can use the **START=** option to impose a minimum model size, and you can use the **STOP=** option to impose a maximum model size. For instance, with **BEST=3**, **START=2**, and **STOP=5**, the **SCORE** selection method displays the best three models (that is, the three models with the highest score chi-squares)

containing 2, 3, 4, and 5 effects. The **SELECTION=SCORE** option is not available for models with CLASS variables.

The options **FAST**, **SEQUENTIAL**, and **STOPRES** can alter the default criteria for entering or removing effects from the model when they are used with the **FORWARD**, **BACKWARD**, or **STEPWISE** selection method.

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## Model Fitting Information

For the  $j$ th observation, let  $\hat{\pi}_j$  be the estimated probability of the observed response. The three criteria displayed by the LOGISTIC procedure are calculated as follows:

- $-2 \log$  likelihood:

$$-2 \text{ Log L} = -2 \sum_j \frac{w_j}{\sigma^2} f_j \log(\hat{\pi}_j)$$

where  $w_j$  and  $f_j$  are the weight and frequency values of the  $j$ th observation, and  $\sigma^2$  is the dispersion parameter, which equals 1 unless the **SCALE=** option is specified. For binary response models that use events/trials MODEL statement syntax, this is

$$-2 \text{ Log L} = -2 \sum_j \frac{w_j}{\sigma^2} f_j \left[ \log \binom{n_j}{r_j} + r_j \log(\hat{\pi}_j) + (n_j - r_j) \log(1 - \hat{\pi}_j) \right]$$

where  $r_j$  is the number of events,  $n_j$  is the number of trials,  $\hat{\pi}_j$  is the estimated event probability, and the statistic is reported both with and without the constant term.

- Akaike's information criterion:

$$\text{AIC} = -2 \text{ Log L} + 2p$$

where  $p$  is the number of parameters in the model. For cumulative response models,  $p = k + s$ , where  $k$  is the total number of response levels minus one and  $s$  is the number of explanatory effects. For the generalized logit model,  $p = k(s + 1)$ .

- Schwarz (Bayesian information) criterion:

$$\text{SC} = -2 \text{ Log L} + p \log \left( \sum_j f_j n_j \right)$$

where  $p$  is the number of parameters in the model,  $n_j$  is the number of trials when events/trials syntax is specified, and  $n_j = 1$  with single-trial syntax.

The AIC and SC statistics give two different ways of adjusting the  $-2 \text{ Log L}$  statistic for the number of terms in the model and the number of observations used. These statistics can be used when comparing different models for the same data (for example, when you use the **SELECTION=STEPWISE** option in the **MODEL** statement). The models being compared do not have to be nested; lower values of the statistics indicate a more desirable model.

The difference in the  $-2 \log L$  statistics between the intercepts-only model and the specified model has a  $p - k$  degree-of-freedom chi-square distribution under the null hypothesis that all the explanatory effects in the model are zero, where  $p$  is the number of parameters in the specified model and  $k$  is the number of intercepts. The likelihood ratio test in the “Testing Global Null Hypothesis: BETA=0” table displays this difference and the associated  $p$ -value for this statistic. The score and Wald tests in that table test the same hypothesis and are asymptotically equivalent; see the sections “Residual Chi-Square” on page 4247 and “Testing Linear Hypotheses about the Regression Coefficients” on page 4262 for details.

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## Generalized Coefficient of Determination

Cox and Snell (1989, pp. 208–209) propose the following generalization of the coefficient of determination to a more general linear model:

$$R^2 = 1 - \left\{ \frac{L(\mathbf{0})}{L(\hat{\boldsymbol{\beta}})} \right\}^{\frac{2}{n}}$$

where  $L(\mathbf{0})$  is the likelihood of the intercept-only model,  $L(\hat{\boldsymbol{\beta}})$  is the likelihood of the specified model,  $n = \sum_j f_j n_j$  is the sample size,  $f_j$  is the frequency of the  $j$ th observation, and  $n_j$  is the number of trials when events/trials syntax is specified or  $n_j = 1$  with single-trial syntax.

The quantity  $R^2$  achieves a maximum of less than one for discrete models, where the maximum is given by

$$R_{\max}^2 = 1 - \{L(\mathbf{0})\}^{\frac{2}{n}}$$

Nagelkerke (1991) proposes the following adjusted coefficient, which can achieve a maximum value of one:

$$\tilde{R}^2 = \frac{R^2}{R_{\max}^2}$$

Specifying the **NORMALIZE** option in the **WEIGHT** statement makes these coefficients invariant to the scale of the weights.

Like the AIC and SC statistics described in the section “Model Fitting Information” on page 4245,  $R^2$  and  $\tilde{R}^2$  are most useful for comparing competing models that are not necessarily nested—larger values indicate better models. More properties and interpretation of  $R^2$  and  $\tilde{R}^2$  are provided in Nagelkerke (1991). In the “Testing Global Null Hypothesis: BETA=0” table,  $R^2$  is labeled as “RSquare” and  $\tilde{R}^2$  is labeled as “Max-rescaled RSquare.” Use the **RSQUARE** option to request  $R^2$  and  $\tilde{R}^2$ .

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## Score Statistics and Tests

To understand the general form of the score statistics, let  $\mathbf{g}(\boldsymbol{\beta})$  be the vector of first partial derivatives of the log likelihood with respect to the parameter vector  $\boldsymbol{\beta}$ , and let  $\mathbf{H}(\boldsymbol{\beta})$  be the matrix of second partial derivatives of the log likelihood with respect to  $\boldsymbol{\beta}$ . That is,  $\mathbf{g}(\boldsymbol{\beta})$  is the gradient vector, and  $\mathbf{H}(\boldsymbol{\beta})$  is the Hessian matrix. Let  $\mathbf{I}(\boldsymbol{\beta})$  be either  $-\mathbf{H}(\boldsymbol{\beta})$  or the expected value of  $-\mathbf{H}(\boldsymbol{\beta})$ . Consider a null hypothesis  $H_0$ . Let  $\hat{\boldsymbol{\beta}}_{H_0}$  be the MLE of  $\boldsymbol{\beta}$  under  $H_0$ . The chi-square score statistic for testing  $H_0$  is defined by

$$\mathbf{g}'(\hat{\boldsymbol{\beta}}_{H_0}) \mathbf{I}^{-1}(\hat{\boldsymbol{\beta}}_{H_0}) \mathbf{g}(\hat{\boldsymbol{\beta}}_{H_0})$$

and it has an asymptotic  $\chi^2$  distribution with  $r$  degrees of freedom under  $H_0$ , where  $r$  is the number of restrictions imposed on  $\boldsymbol{\beta}$  by  $H_0$ .

## Residual Chi-Square

When you use **SELECTION=FORWARD**, **BACKWARD**, or **STEPWISE**, the procedure calculates a residual chi-square score statistic and reports the statistic, its degrees of freedom, and the  $p$ -value. This section describes how the statistic is calculated.

Suppose there are  $s$  explanatory effects of interest. The full cumulative response model has a parameter vector

$$\boldsymbol{\beta} = (\alpha_1, \dots, \alpha_k, \beta_1, \dots, \beta_s)'$$

where  $\alpha_1, \dots, \alpha_k$  are intercept parameters, and  $\beta_1, \dots, \beta_s$  are the common slope parameters for the  $s$  explanatory effects. The full generalized logit model has a parameter vector

$$\begin{aligned}\boldsymbol{\beta} &= (\alpha_1, \dots, \alpha_k, \boldsymbol{\beta}'_1, \dots, \boldsymbol{\beta}'_k)' \quad \text{with} \\ \boldsymbol{\beta}'_i &= (\beta_{i1}, \dots, \beta_{is}), \quad i = 1, \dots, k\end{aligned}$$

where  $\beta_{ij}$  is the slope parameter for the  $j$ th effect in the  $i$ th logit.

Consider the null hypothesis  $H_0: \beta_{t+1} = \dots = \beta_s = 0$ , where  $t < s$  for the cumulative response model, and  $H_0: \beta_{i,t+1} = \dots = \beta_{is} = 0, t < s, i = 1, \dots, k$ , for the generalized logit model. For the reduced model with  $t$  explanatory effects, let  $\hat{\alpha}_1, \dots, \hat{\alpha}_k$  be the MLEs of the unknown intercept parameters, let  $\hat{\beta}_1, \dots, \hat{\beta}_t$  be the MLEs of the unknown slope parameters, and let  $\hat{\boldsymbol{\beta}}'_{i(t)} = (\hat{\beta}_{i1}, \dots, \hat{\beta}_{it}), i = 1, \dots, k$ , be those for the generalized logit model. The residual chi-square is the chi-square score statistic testing the null hypothesis  $H_0$ ; that is, the residual chi-square is

$$\mathbf{g}'(\hat{\boldsymbol{\beta}}_{H_0})\mathbf{I}^{-1}(\hat{\boldsymbol{\beta}}_{H_0})\mathbf{g}(\hat{\boldsymbol{\beta}}_{H_0})$$

where for the cumulative response model  $\hat{\boldsymbol{\beta}}_{H_0} = (\hat{\alpha}_1, \dots, \hat{\alpha}_k, \hat{\beta}_1, \dots, \hat{\beta}_t, 0, \dots, 0)'$ , and for the generalized logit model  $\hat{\boldsymbol{\beta}}_{H_0} = (\hat{\alpha}_1, \dots, \hat{\alpha}_k, \hat{\boldsymbol{\beta}}'_{1(t)}, \mathbf{0}'_{(s-t)}, \dots, \hat{\boldsymbol{\beta}}'_{k(t)}, \mathbf{0}'_{(s-t)})'$ , where  $\mathbf{0}_{(s-t)}$  denotes a vector of  $s - t$  zeros.

The residual chi-square has an asymptotic chi-square distribution with  $s - t$  degrees of freedom ( $k(s - t)$  for the generalized logit model). A special case is the global score chi-square, where the reduced model consists of the  $k$  intercepts and no explanatory effects. The global score statistic is displayed in the “Testing Global Null Hypothesis: BETA=0” table. The table is not produced when the **NOFIT** option is used, but the global score statistic is displayed.

## Testing Individual Effects Not in the Model

These tests are performed when you specify **SELECTION=FORWARD** or **STEPWISE**, and are displayed when the **DETAILS** option is specified. In the displayed output, the tests are labeled “Score Chi-Square” in the “Analysis of Effects Not in the Model” table and in the “Summary of Stepwise (Forward) Selection” table. This section describes how the tests are calculated.

Suppose that  $k$  intercepts and  $t$  explanatory variables (say  $v_1, \dots, v_t$ ) have been fit to a model and that  $v_{t+1}$  is another explanatory variable of interest. Consider a full model with the  $k$  intercepts and  $t + 1$  explanatory variables ( $v_1, \dots, v_t, v_{t+1}$ ) and a reduced model with  $v_{t+1}$  excluded. The significance of  $v_{t+1}$  adjusted for  $v_1, \dots, v_t$  can be determined by comparing the corresponding residual chi-square with a chi-square distribution with one degree of freedom ( $k$  degrees of freedom for the generalized logit model).

## Testing the Parallel Lines Assumption

For an ordinal response, PROC LOGISTIC performs a test of the parallel lines assumption. In the displayed output, this test is labeled “Score Test for the Equal Slopes Assumption” when the `LINK=` option is `NORMIT` or `CLOGLOG`. When `LINK=LOGIT`, the test is labeled as “Score Test for the Proportional Odds Assumption” in the output. For small sample sizes, this test might be too liberal (Stokes, Davis, and Koch 2000, p. 249). This section describes the methods used to calculate the test.

For this test the number of response levels,  $k + 1$ , is assumed to be strictly greater than 2. Let  $Y$  be the response variable taking values  $1, \dots, k, k + 1$ . Suppose there are  $s$  explanatory variables. Consider the general cumulative model without making the parallel lines assumption

$$g(\Pr(Y \leq i \mid \mathbf{x})) = (1, \mathbf{x}')\boldsymbol{\beta}_i, \quad 1 \leq i \leq k$$

where  $g(\cdot)$  is the link function, and  $\boldsymbol{\beta}_i = (\alpha_i, \beta_{i1}, \dots, \beta_{is})'$  is a vector of unknown parameters consisting of an intercept  $\alpha_i$  and  $s$  slope parameters  $\beta_{i1}, \dots, \beta_{is}$ . The parameter vector for this general cumulative model is

$$\boldsymbol{\beta} = (\boldsymbol{\beta}'_1, \dots, \boldsymbol{\beta}'_k)'$$

Under the null hypothesis of parallelism  $H_0: \beta_{1m} = \beta_{2m} = \dots = \beta_{km}, 1 \leq m \leq s$ , there is a single common slope parameter for each of the  $s$  explanatory variables. Let  $\beta_1, \dots, \beta_s$  be the common slope parameters. Let  $\hat{\alpha}_1, \dots, \hat{\alpha}_k$  and  $\hat{\beta}_1, \dots, \hat{\beta}_s$  be the MLEs of the intercept parameters and the common slope parameters. Then, under  $H_0$ , the MLE of  $\boldsymbol{\beta}$  is

$$\hat{\boldsymbol{\beta}}_{H_0} = (\hat{\boldsymbol{\beta}}'_1, \dots, \hat{\boldsymbol{\beta}}'_k)' \quad \text{with} \quad \hat{\boldsymbol{\beta}}_i = (\hat{\alpha}_i, \hat{\beta}_1, \dots, \hat{\beta}_s)' \quad 1 \leq i \leq k$$

and the chi-square score statistic  $\mathbf{g}'(\hat{\boldsymbol{\beta}}_{H_0})\mathbf{I}^{-1}(\hat{\boldsymbol{\beta}}_{H_0})\mathbf{g}(\hat{\boldsymbol{\beta}}_{H_0})$  has an asymptotic chi-square distribution with  $s(k - 1)$  degrees of freedom. This tests the parallel lines assumption by testing the equality of separate slope parameters simultaneously for all explanatory variables.

---

## Confidence Intervals for Parameters

There are two methods of computing confidence intervals for the regression parameters. One is based on the profile-likelihood function, and the other is based on the asymptotic normality of the parameter estimators. The latter is not as time-consuming as the former, since it does not involve an iterative scheme; however, it is not thought to be as accurate as the former, especially with small sample size. You use the `CLPARM=` option to request confidence intervals for the parameters.

## Likelihood Ratio-Based Confidence Intervals

The likelihood ratio-based confidence interval is also known as the profile-likelihood confidence interval. The construction of this interval is derived from the asymptotic  $\chi^2$  distribution of the generalized likelihood ratio test (Venzon and Moolgavkar 1988). Suppose that the parameter vector is  $\boldsymbol{\beta} = (\beta_0, \beta_1, \dots, \beta_s)'$  and you want to compute a confidence interval for  $\beta_j$ . The profile-likelihood function for  $\beta_j = \gamma$  is defined as

$$l_j^*(\gamma) = \max_{\boldsymbol{\beta} \in \mathcal{B}_j(\gamma)} l(\boldsymbol{\beta})$$

where  $\mathcal{B}_j(\gamma)$  is the set of all  $\boldsymbol{\beta}$  with the  $j$ th element fixed at  $\gamma$ , and  $l(\boldsymbol{\beta})$  is the log-likelihood function for  $\boldsymbol{\beta}$ . If  $l_{\max} = l(\hat{\boldsymbol{\beta}})$  is the log likelihood evaluated at the maximum likelihood estimate  $\hat{\boldsymbol{\beta}}$ , then  $2(l_{\max} - l_j^*(\beta_j))$  has a limiting chi-square distribution with one degree of freedom if  $\beta_j$  is the true parameter value. Let  $l_0 = l_{\max} - 0.5\chi_1^2(1 - \alpha)$ , where  $\chi_1^2(1 - \alpha)$  is the  $100(1 - \alpha)$  percentile of the chi-square distribution with one degree of freedom. A  $100(1 - \alpha)\%$  confidence interval for  $\beta_j$  is

$$\{\gamma : l_j^*(\gamma) \geq l_0\}$$

The endpoints of the confidence interval are found by solving numerically for values of  $\beta_j$  that satisfy equality in the preceding relation. To obtain an iterative algorithm for computing the confidence limits, the log-likelihood function in a neighborhood of  $\boldsymbol{\beta}$  is approximated by the quadratic function

$$\tilde{l}(\boldsymbol{\beta} + \boldsymbol{\delta}) = l(\boldsymbol{\beta}) + \boldsymbol{\delta}'\mathbf{g} + \frac{1}{2}\boldsymbol{\delta}'\mathbf{V}\boldsymbol{\delta}$$

where  $\mathbf{g} = \mathbf{g}(\boldsymbol{\beta})$  is the gradient vector and  $\mathbf{V} = \mathbf{V}(\boldsymbol{\beta})$  is the Hessian matrix. The increment  $\boldsymbol{\delta}$  for the next iteration is obtained by solving the likelihood equations

$$\frac{d}{d\boldsymbol{\delta}} \{\tilde{l}(\boldsymbol{\beta} + \boldsymbol{\delta}) + \lambda(\mathbf{e}_j'\boldsymbol{\delta} - \gamma)\} = \mathbf{0}$$

where  $\lambda$  is the Lagrange multiplier,  $\mathbf{e}_j$  is the  $j$ th unit vector, and  $\gamma$  is an unknown constant. The solution is

$$\boldsymbol{\delta} = -\mathbf{V}^{-1}(\mathbf{g} + \lambda\mathbf{e}_j)$$

By substituting this  $\boldsymbol{\delta}$  into the equation  $\tilde{l}(\boldsymbol{\beta} + \boldsymbol{\delta}) = l_0$ , you can estimate  $\lambda$  as

$$\lambda = \pm \left( \frac{2(l_0 - l(\boldsymbol{\beta}) + \frac{1}{2}\mathbf{g}'\mathbf{V}^{-1}\mathbf{g})}{\mathbf{e}_j'\mathbf{V}^{-1}\mathbf{e}_j} \right)^{\frac{1}{2}}$$

The upper confidence limit for  $\beta_j$  is computed by starting at the maximum likelihood estimate of  $\boldsymbol{\beta}$  and iterating with positive values of  $\lambda$  until convergence is attained. The process is repeated for the lower confidence limit by using negative values of  $\lambda$ .

Convergence is controlled by the value  $\epsilon$  specified with the **PLCONV=** option in the **MODEL** statement (the default value of  $\epsilon$  is 1E-4). Convergence is declared on the current iteration if the following two conditions are satisfied:

$$|l(\boldsymbol{\beta}) - l_0| \leq \epsilon$$

and

$$(\mathbf{g} + \lambda\mathbf{e}_j)'\mathbf{V}^{-1}(\mathbf{g} + \lambda\mathbf{e}_j) \leq \epsilon$$



## Wald Confidence Intervals

Wald confidence intervals are sometimes called the normal confidence intervals. They are based on the asymptotic normality of the parameter estimators. The  $100(1 - \alpha)\%$  Wald confidence interval for  $\beta_j$  is given by

$$\hat{\beta}_j \pm z_{1-\alpha/2} \hat{\sigma}_j$$

where  $z_p$  is the  $100p$  percentile of the standard normal distribution,  $\hat{\beta}_j$  is the maximum likelihood estimate of  $\beta_j$ , and  $\hat{\sigma}_j$  is the standard error estimate of  $\hat{\beta}_j$ .

---

## Odds Ratio Estimation

Consider a dichotomous response variable with outcomes *event* and *nonevent*. Consider a dichotomous risk factor variable  $X$  that takes the value 1 if the risk factor is present and 0 if the risk factor is absent. According to the logistic model, the log odds function,  $\text{logit}(X)$ , is given by

$$\text{logit}(X) \equiv \log\left(\frac{\Pr(\text{event} | X)}{\Pr(\text{nonevent} | X)}\right) = \alpha + X\beta$$

The odds ratio  $\psi$  is defined as the ratio of the odds for those with the risk factor ( $X = 1$ ) to the odds for those without the risk factor ( $X = 0$ ). The log of the odds ratio is given by

$$\log(\psi) \equiv \log(\psi(X = 1, X = 0)) = \text{logit}(X = 1) - \text{logit}(X = 0) = (\alpha + 1 \times \beta) - (\alpha + 0 \times \beta) = \beta$$

In general, the odds ratio can be computed by exponentiating the difference of the logits between any two population profiles. This is the approach taken by the **ODDSRATIO** statement, so the computations are available regardless of parameterization, interactions, and nestings. However, as shown in the preceding equation for  $\log(\psi)$ , odds ratios of main effects can be computed as functions of the parameter estimates, and the remainder of this section is concerned with this methodology.

The parameter,  $\beta$ , associated with  $X$  represents the change in the log odds from  $X = 0$  to  $X = 1$ . So the odds ratio is obtained by simply exponentiating the value of the parameter associated with the risk factor. The odds ratio indicates how the odds of the event change as you change  $X$  from 0 to 1. For instance,  $\psi = 2$  means that the odds of an event when  $X = 1$  are twice the odds of an event when  $X = 0$ . You can also express this as follows: the percent change in the odds of an event from  $X = 0$  to  $X = 1$  is  $(\psi - 1)100\% = 100\%$ .

Suppose the values of the dichotomous risk factor are coded as constants  $a$  and  $b$  instead of 0 and 1. The odds when  $X = a$  become  $\exp(\alpha + a\beta)$ , and the odds when  $X = b$  become  $\exp(\alpha + b\beta)$ . The odds ratio corresponding to an increase in  $X$  from  $a$  to  $b$  is

$$\psi = \exp[(b - a)\beta] = [\exp(\beta)]^{b-a} \equiv [\exp(\beta)]^c$$

Note that for any  $a$  and  $b$  such that  $c = b - a = 1$ ,  $\psi = \exp(\beta)$ . So the odds ratio can be interpreted as the change in the odds for any increase of one unit in the corresponding risk factor. However, the change in odds for some amount other than one unit is often of greater interest. For example, a change of one pound in body weight might be too small to be considered important, while a change of 10 pounds might be more meaningful. The odds ratio for a change in  $X$  from  $a$  to  $b$  is estimated by raising the odds ratio estimate for a unit change in  $X$  to the power of  $c = b - a$  as shown previously.



For a polytomous risk factor, the computation of odds ratios depends on how the risk factor is parameterized. For illustration, suppose that **Race** is a risk factor with four categories: White, Black, Hispanic, and Other.

For the effect parameterization scheme (**PARAM=EFFECT**) with White as the reference group (**REF='White'**), the design variables for **Race** are as follows:

| <b>Race</b> | <b>Design Variables</b> |       |       |
|-------------|-------------------------|-------|-------|
|             | $X_1$                   | $X_2$ | $X_3$ |
| Black       | 1                       | 0     | 0     |
| Hispanic    | 0                       | 1     | 0     |
| Other       | 0                       | 0     | 1     |
| White       | -1                      | -1    | -1    |

The log odds for Black is

$$\begin{aligned}\text{logit(Black)} &= \alpha + (X_1 = 1)\beta_1 + (X_2 = 0)\beta_2 + (X_3 = 0)\beta_3 \\ &= \alpha + \beta_1\end{aligned}$$

The log odds for White is

$$\begin{aligned}\text{logit(White)} &= \alpha + (X_1 = -1)\beta_1 + (X_2 = -1)\beta_2 + (X_3 = -1)\beta_3 \\ &= \alpha - \beta_1 - \beta_2 - \beta_3\end{aligned}$$

Therefore, the log odds ratio of Black versus White becomes

$$\begin{aligned}\log(\psi(\text{Black, White})) &= \text{logit(Black)} - \text{logit(White)} \\ &= 2\beta_1 + \beta_2 + \beta_3\end{aligned}$$

For the reference cell parameterization scheme (**PARAM=REF**) with White as the reference cell, the design variables for race are as follows:

| <b>Race</b> | <b>Design Variables</b> |       |       |
|-------------|-------------------------|-------|-------|
|             | $X_1$                   | $X_2$ | $X_3$ |
| Black       | 1                       | 0     | 0     |
| Hispanic    | 0                       | 1     | 0     |
| Other       | 0                       | 0     | 1     |
| White       | 0                       | 0     | 0     |

The log odds ratio of Black versus White is given by

$$\begin{aligned}\log(\psi(\text{Black, White})) &= \text{logit(Black)} - \text{logit(White)} \\ &= (\alpha + (X_1 = 1)\beta_1 + (X_2 = 0)\beta_2 + (X_3 = 0)\beta_3) - \\ &\quad (\alpha + (X_1 = 0)\beta_1 + (X_2 = 0)\beta_2 + (X_3 = 0)\beta_3) \\ &= \beta_1\end{aligned}$$

For the GLM parameterization scheme (**PARAM=GLM**), the design variables are as follows:

| Race     | Design Variables |       |       |       |
|----------|------------------|-------|-------|-------|
|          | $X_1$            | $X_2$ | $X_3$ | $X_4$ |
| Black    | 1                | 0     | 0     | 0     |
| Hispanic | 0                | 1     | 0     | 0     |
| Other    | 0                | 0     | 1     | 0     |
| White    | 0                | 0     | 0     | 1     |

The log odds ratio of Black versus White is

$$\begin{aligned}
 \log(\psi(\text{Black}, \text{White})) &= \text{logit}(\text{Black}) - \text{logit}(\text{White}) \\
 &= (\alpha + (X_1 = 1)\beta_1 + (X_2 = 0)\beta_2 + (X_3 = 0)\beta_3 + (X_4 = 0)\beta_4) - \\
 &\quad (\alpha + (X_1 = 0)\beta_1 + (X_2 = 0)\beta_2 + (X_3 = 0)\beta_3 + (X_4 = 1)\beta_4) \\
 &= \beta_1 - \beta_4
 \end{aligned}$$

Consider the hypothetical example of heart disease among race in Hosmer and Lemeshow (2000, p. 56). The entries in the following contingency table represent counts:

| Disease Status | Race  |       |          |       |
|----------------|-------|-------|----------|-------|
|                | White | Black | Hispanic | Other |
| Present        | 5     | 20    | 15       | 10    |
| Absent         | 20    | 10    | 10       | 10    |

The computation of odds ratio of Black versus White for various parameterization schemes is tabulated in Table 54.11.

**Table 54.11** Odds Ratio of Heart Disease Comparing Black to White

| PARAM= | Parameter Estimates |                 |                 |                 | Odds Ratio Estimates                          |
|--------|---------------------|-----------------|-----------------|-----------------|-----------------------------------------------|
|        | $\hat{\beta}_1$     | $\hat{\beta}_2$ | $\hat{\beta}_3$ | $\hat{\beta}_4$ |                                               |
| EFFECT | 0.7651              | 0.4774          | 0.0719          |                 | $\exp(2 \times 0.7651 + 0.4774 + 0.0719) = 8$ |
| REF    | 2.0794              | 1.7917          | 1.3863          |                 | $\exp(2.0794) = 8$                            |
| GLM    | 2.0794              | 1.7917          | 1.3863          | 0.0000          | $\exp(2.0794) = 8$                            |

Since the log odds ratio ( $\log(\psi)$ ) is a linear function of the parameters, the Wald confidence interval for  $\log(\psi)$  can be derived from the parameter estimates and the estimated covariance matrix. Confidence intervals for the odds ratios are obtained by exponentiating the corresponding confidence limits for the log odds ratios. In the displayed output of PROC LOGISTIC, the “Odds Ratio Estimates” table contains the odds ratio estimates and the corresponding 95% Wald confidence intervals. For continuous explanatory variables, these odds ratios correspond to a unit increase in the risk factors.

To customize odds ratios for specific units of change for a continuous risk factor, you can use the **UNITS** statement to specify a list of relevant units for each explanatory variable in the model. Estimates of these customized odds ratios are given in a separate table. Let  $(L_j, U_j)$  be a confidence interval for  $\log(\psi)$ . The corresponding lower and upper confidence limits for the customized odds ratio  $\exp(c\beta_j)$  are  $\exp(cL_j)$  and  $\exp(cU_j)$ , respectively (for  $c > 0$ ), or  $\exp(cU_j)$  and  $\exp(cL_j)$ , respectively (for  $c < 0$ ). You use the **CLODDS=** option or **ODDSRATIO** statement to request the confidence intervals for the odds ratios.

For a generalized logit model, odds ratios are computed similarly, except  $k$  odds ratios are computed for each effect, corresponding to the  $k$  logits in the model.

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## Rank Correlation of Observed Responses and Predicted Probabilities

The predicted mean score of an observation is the sum of the Ordered Values (shown in the “Response Profile” table) minus one, weighted by the corresponding predicted probabilities for that observation; that is, the predicted means score =  $\sum_{i=1}^{k+1} (i-1)\hat{\pi}_i$ , where  $k+1$  is the number of response levels and  $\hat{\pi}_i$  is the predicted probability of the  $i$ th (ordered) response.

A pair of observations with different observed responses is said to be *concordant* if the observation with the lower ordered response value has a lower predicted mean score than the observation with the higher ordered response value. If the observation with the lower ordered response value has a higher predicted mean score than the observation with the higher ordered response value, then the pair is *discordant*. If the pair is neither concordant nor discordant, it is a *tie*. Enumeration of the total numbers of concordant and discordant pairs is carried out by categorizing the predicted mean score into intervals of length  $k/500$  and accumulating the corresponding frequencies of observations. Note that the length of these intervals can be modified by specification of the `BINWIDTH=` option in the `MODEL` statement.

Let  $N$  be the sum of observation frequencies in the data. Suppose there are a total of  $t$  pairs with different responses:  $n_c$  of them are concordant,  $n_d$  of them are discordant, and  $t - n_c - n_d$  of them are tied. PROC LOGISTIC computes the following four indices of rank correlation for assessing the predictive ability of a model:

$$\begin{aligned} c &= (n_c + 0.5(t - n_c - n_d))/t \\ \text{Somers' } D \text{ (Gini coefficient)} &= (n_c - n_d)/t \\ \text{Goodman-Kruskal Gamma} &= (n_c - n_d)/(n_c + n_d) \\ \text{Kendall's Tau-}a &= (n_c - n_d)/(0.5N(N - 1)) \end{aligned}$$

If there are no ties, then Somers'  $D$  (Gini's coefficient) =  $2c - 1$ . Note that the concordance index,  $c$ , also gives an estimate of the area under the receiver operating characteristic (ROC) curve when the response is binary (Hanley and McNeil 1982). See the section “[ROC Computations](#)” on page 4261 for more information about this area.

For binary responses, the predicted mean score is equal to the predicted probability for Ordered Value 2. As such, the preceding definition of concordance is consistent with the definition used in previous releases for the binary response model.

These statistics are not available when the `STRATA` statement is specified.

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## Linear Predictor, Predicted Probability, and Confidence Limits

This section describes how predicted probabilities and confidence limits are calculated by using the maximum likelihood estimates (MLEs) obtained from PROC LOGISTIC. For a specific example, see the section “[Getting Started: LOGISTIC Procedure](#)” on page 4166. Predicted probabilities and confidence limits can be output to a data set with the `OUTPUT` statement.

## Binary and Cumulative Response Models

For a vector of explanatory variables  $\mathbf{x}$ , the linear predictor

$$\eta_i = g(\Pr(Y \leq i | \mathbf{x})) = \alpha_i + \mathbf{x}'\boldsymbol{\beta} \quad 1 \leq i \leq k$$

is estimated by

$$\hat{\eta}_i = \hat{\alpha}_i + \mathbf{x}'\hat{\boldsymbol{\beta}}$$

where  $\hat{\alpha}_i$  and  $\hat{\boldsymbol{\beta}}$  are the MLEs of  $\alpha_i$  and  $\boldsymbol{\beta}$ . The estimated standard error of  $\eta_i$  is  $\hat{\sigma}(\hat{\eta}_i)$ , which can be computed as the square root of the quadratic form  $(1, \mathbf{x}')\hat{\mathbf{V}}_{\mathbf{b}}(1, \mathbf{x})'$ , where  $\hat{\mathbf{V}}_{\mathbf{b}}$  is the estimated covariance matrix of the parameter estimates. The asymptotic  $100(1 - \alpha)\%$  confidence interval for  $\eta_i$  is given by

$$\hat{\eta}_i \pm z_{\alpha/2}\hat{\sigma}(\hat{\eta}_i)$$

where  $z_{\alpha/2}$  is the  $100(1 - \alpha/2)$  percentile point of a standard normal distribution.

The predicted probability and the  $100(1 - \alpha)\%$  confidence limits for  $\pi_i = \Pr(Y \leq i | \mathbf{x})$  are obtained by back-transforming the corresponding measures for the linear predictor, as shown in the following table:

| Link    | Predicted Probability           | 100(1- $\alpha$ )% Confidence Limits                                       |
|---------|---------------------------------|----------------------------------------------------------------------------|
| LOGIT   | $1/(1 + \exp(-\hat{\eta}_i))$   | $1/(1 + \exp(-\hat{\eta}_i \pm z_{\alpha/2}\hat{\sigma}(\hat{\eta}_i)))$   |
| PROBIT  | $\Phi(\hat{\eta}_i)$            | $\Phi(\hat{\eta}_i \pm z_{\alpha/2}\hat{\sigma}(\hat{\eta}_i))$            |
| CLOGLOG | $1 - \exp(-\exp(\hat{\eta}_i))$ | $1 - \exp(-\exp(\hat{\eta}_i \pm z_{\alpha/2}\hat{\sigma}(\hat{\eta}_i)))$ |

The **CONTRAST** statement also enables you to estimate the exponentiated contrast,  $e^{\hat{\eta}_i}$ . The corresponding standard error is  $e^{\hat{\eta}_i}\hat{\sigma}(\hat{\eta}_i)$ , and the confidence limits are computed by exponentiating those for the linear predictor:  $\exp\{\hat{\eta}_i \pm z_{\alpha/2}\hat{\sigma}(\hat{\eta}_i)\}$ .

## Generalized Logit Model

For a vector of explanatory variables  $\mathbf{x}$ , define the linear predictors  $\eta_i = \alpha_i + \mathbf{x}'\boldsymbol{\beta}_i$ , and let  $\pi_i$  denote the probability of obtaining the response value  $i$ :

$$\pi_i = \begin{cases} \pi_{k+1}e^{\eta_i} & 1 \leq i \leq k \\ \frac{1}{1 + \sum_{j=1}^k e^{\eta_j}} & i = k + 1 \end{cases}$$

By the *delta method*,

$$\sigma^2(\pi_i) = \left(\frac{\partial \pi_i}{\partial \boldsymbol{\beta}}\right)' \mathbf{V}(\boldsymbol{\beta}) \frac{\partial \pi_i}{\partial \boldsymbol{\beta}}$$

A  $100(1 - \alpha)\%$  confidence level for  $\pi_i$  is given by

$$\hat{\pi}_i \pm z_{\alpha/2}\hat{\sigma}(\hat{\pi}_i)$$

where  $\hat{\pi}_i$  is the estimated expected probability of response  $i$ , and  $\hat{\sigma}(\hat{\pi}_i)$  is obtained by evaluating  $\sigma(\pi_i)$  at  $\boldsymbol{\beta} = \hat{\boldsymbol{\beta}}$ .

Note that the contrast  $\hat{\eta}_i$  and exponentiated contrast  $e^{\hat{\eta}_i}$ , their standard errors, and their confidence intervals are computed in the same fashion as for the cumulative response models, replacing  $\boldsymbol{\beta}$  with  $\boldsymbol{\beta}_i$ .

## Classification Table

For binary response data, the response is either an *event* or a *nonevent*. In PROC LOGISTIC, the response with Ordered Value 1 is regarded as the event, and the response with Ordered Value 2 is the nonevent. PROC LOGISTIC models the probability of the event. From the fitted model, a predicted event probability can be computed for each observation. A method to compute a reduced-bias estimate of the predicted probability is given in the section “[Predicted Probability of an Event for Classification](#)” on page 4255. If the predicted event probability exceeds or equals some cutpoint value  $z \in [0, 1]$ , the observation is predicted to be an event observation; otherwise, it is predicted as a nonevent. A  $2 \times 2$  frequency table can be obtained by cross-classifying the observed and predicted responses. The [CTABLE](#) option produces this table, and the [PPROB=](#) option selects one or more cutpoints. Each cutpoint generates a classification table. If the [PEVENT=](#) option is also specified, a classification table is produced for each combination of [PEVENT=](#) and [PPROB=](#) values.

The accuracy of the classification is measured by its *sensitivity* (the ability to predict an event correctly) and *specificity* (the ability to predict a nonevent correctly). *Sensitivity* is the proportion of event responses that were predicted to be events. *Specificity* is the proportion of nonevent responses that were predicted to be nonevents. PROC LOGISTIC also computes three other conditional probabilities: *false positive rate*, *false negative rate*, and *rate of correct classification*. The *false positive rate* is the proportion of predicted event responses that were observed as nonevents. The *false negative rate* is the proportion of predicted nonevent responses that were observed as events. Given prior probabilities specified with the [PEVENT=](#) option, these conditional probabilities can be computed as posterior probabilities by using Bayes’ theorem.

### Predicted Probability of an Event for Classification

When you classify a set of binary data, if the same observations used to fit the model are also used to estimate the classification error, the resulting error-count estimate is biased. One way of reducing the bias is to remove the binary observation to be classified from the data, reestimate the parameters of the model, and then classify the observation based on the new parameter estimates. However, it would be costly to fit the model by leaving out each observation one at a time. The LOGISTIC procedure provides a less expensive one-step approximation to the preceding parameter estimates. Let  $\hat{\beta}$  be the MLE of the parameter vector  $(\alpha, \beta_1, \dots, \beta_s)'$  based on all observations. Let  $\hat{\beta}_{(j)}$  denote the MLE computed without the  $j$ th observation. The one-step estimate of  $\hat{\beta}_{(j)}$  is given by

$$\hat{\beta}_{(j)}^1 = \hat{\beta} - \frac{w_j(y_j - \hat{\pi}_j)}{1 - h_j} \hat{\mathbf{V}}(\hat{\beta}) \begin{pmatrix} 1 \\ \mathbf{x}_j \end{pmatrix}$$

where

- $y_j$  is 1 for an observed event response and 0 otherwise
- $w_j$  is the weight of the observation
- $\hat{\pi}_j$  is the predicted event probability based on  $\hat{\beta}$
- $h_j$  is the [hat diagonal element](#) (defined on page 4263) with  $n_j = 1$  and  $r_j = y_j$
- $\hat{\mathbf{V}}(\hat{\beta})$  is the estimated covariance matrix of  $\hat{\beta}$

### False Positive, False Negative, and Correct Classification Rates Using Bayes' Theorem

Suppose  $n_1$  of  $n$  individuals experience an event, such as a disease. Let this group be denoted by  $\mathcal{C}_1$ , and let the group of the remaining  $n_2 = n - n_1$  individuals who do not have the disease be denoted by  $\mathcal{C}_2$ . The  $j$ th individual is classified as giving a positive response if the predicted probability of disease ( $\hat{\pi}_{(j)}^*$ ) is large. The probability  $\hat{\pi}_{(j)}^*$  is the reduced-bias estimate based on the one-step approximation given in the preceding section. For a given cutpoint  $z$ , the  $j$ th individual is predicted to give a positive response if  $\hat{\pi}_{(j)}^* \geq z$ .

Let  $B$  denote the event that a subject has the disease, and let  $\bar{B}$  denote the event of not having the disease. Let  $A$  denote the event that the subject responds positively, and let  $\bar{A}$  denote the event of responding negatively. Results of the classification are represented by two conditional probabilities,  $\Pr(A|B)$  and  $\Pr(A|\bar{B})$ , where  $\Pr(A|B)$  is the sensitivity and  $\Pr(A|\bar{B})$  is one minus the specificity.

These probabilities are given by

$$\Pr(A|B) = \frac{\sum_{j \in \mathcal{C}_1} I(\hat{\pi}_{(j)}^* \geq z)}{n_1}$$

$$\Pr(A|\bar{B}) = \frac{\sum_{j \in \mathcal{C}_2} I(\hat{\pi}_{(j)}^* \geq z)}{n_2}$$

where  $I(\cdot)$  is the indicator function.

Bayes' theorem is used to compute several rates of the classification. For a given prior probability  $\Pr(B)$  of the disease, the false positive rate  $P_{F+}$ , the false negative rate  $P_{F-}$ , and the correct classification rate  $P_C$  are given by Fleiss (1981, pp. 4–5) as follows:

$$P_{F+} = \Pr(\bar{B}|A) = \frac{\Pr(A|\bar{B})[1 - \Pr(B)]}{\Pr(A|\bar{B}) + \Pr(B)[\Pr(A|B) - \Pr(A|\bar{B})]}$$

$$P_{F-} = \Pr(B|\bar{A}) = \frac{[1 - \Pr(A|B)]\Pr(B)}{1 - \Pr(A|\bar{B}) - \Pr(B)[\Pr(A|B) - \Pr(A|\bar{B})]}$$

$$P_C = \Pr(B|A) + \Pr(\bar{B}|\bar{A}) = \frac{\Pr(A|B)\Pr(B) + \Pr(\bar{A}|\bar{B})[1 - \Pr(B)]}{1 - \Pr(A|\bar{B}) - \Pr(B)[\Pr(A|B) - \Pr(A|\bar{B})]}$$

The prior probability  $\Pr(B)$  can be specified by the **PEVENT=** option. If the **PEVENT=** option is not specified, the sample proportion of diseased individuals is used; that is,  $\Pr(B) = n_1/n$ . In such a case, the false positive rate and the false negative rate reduce to

$$P_{F+} = \frac{\sum_{j \in \mathcal{C}_2} I(\hat{\pi}_{(j)}^* \geq z)}{\sum_{j \in \mathcal{C}_1} I(\hat{\pi}_{(j)}^* \geq z) + \sum_{j \in \mathcal{C}_2} I(\hat{\pi}_{(j)}^* \geq z)}$$

$$P_{F-} = \frac{\sum_{j \in \mathcal{C}_1} I(\hat{\pi}_{(j)}^* < z)}{\sum_{j \in \mathcal{C}_1} I(\hat{\pi}_{(j)}^* < z) + \sum_{j \in \mathcal{C}_2} I(\hat{\pi}_{(j)}^* < z)}$$

$$P_C = \frac{\sum_{j \in \mathcal{C}_1} I(\hat{\pi}_{(j)}^* \geq z) + \sum_{j \in \mathcal{C}_2} I(\hat{\pi}_{(j)}^* < z)}{n}$$

Note that for a stratified sampling situation in which  $n_1$  and  $n_2$  are chosen a priori,  $n_1/n$  is not a desirable estimate of  $\Pr(B)$ . For such situations, the **PEVENT=** option should be specified.

## Overdispersion

For a correctly specified model, the Pearson chi-square statistic and the deviance, divided by their degrees of freedom, should be approximately equal to one. When their values are much larger than one, the assumption of binomial variability might not be valid and the data are said to exhibit overdispersion. Underdispersion, which results in the ratios being less than one, occurs less often in practice.

When fitting a model, there are several problems that can cause the goodness-of-fit statistics to exceed their degrees of freedom. Among these are such problems as outliers in the data, using the wrong link function, omitting important terms from the model, and needing to transform some predictors. These problems should be eliminated before proceeding to use the following methods to correct for overdispersion.

### Rescaling the Covariance Matrix

One way of correcting overdispersion is to multiply the covariance matrix by a dispersion parameter. This method assumes that the sample sizes in each subpopulation are approximately equal. You can supply the value of the dispersion parameter directly, or you can estimate the dispersion parameter based on either the Pearson chi-square statistic or the deviance for the fitted model.

The Pearson chi-square statistic  $\chi_P^2$  and the deviance  $\chi_D^2$  are given by

$$\chi_P^2 = \sum_{i=1}^m \sum_{j=1}^{k+1} \frac{(r_{ij} - n_i \hat{\pi}_{ij})^2}{n_i \hat{\pi}_{ij}}$$

$$\chi_D^2 = 2 \sum_{i=1}^m \sum_{j=1}^{k+1} r_{ij} \log \left( \frac{r_{ij}}{n_i \hat{\pi}_{ij}} \right)$$

where  $m$  is the number of subpopulation profiles,  $k + 1$  is the number of response levels,  $r_{ij}$  is the total weight (sum of the product of the frequencies and the weights) associated with  $j$ th level responses in the  $i$ th profile,  $n_i = \sum_{j=1}^{k+1} r_{ij}$ , and  $\hat{\pi}_{ij}$  is the fitted probability for the  $j$ th level at the  $i$ th profile. Each of these chi-square statistics has  $mk - p$  degrees of freedom, where  $p$  is the number of parameters estimated. The dispersion parameter is estimated by

$$\hat{\sigma}^2 = \begin{cases} \chi_P^2 / (mk - p) & \text{SCALE=PEARSON} \\ \chi_D^2 / (mk - p) & \text{SCALE=DEVIANC} \\ (\text{constant})^2 & \text{SCALE=constant} \end{cases}$$

In order for the Pearson statistic and the deviance to be distributed as chi-square, there must be sufficient replication within the subpopulations. When this is not true, the data are sparse, and the  $p$ -values for these statistics are not valid and should be ignored. Similarly, these statistics, divided by their degrees of freedom, cannot serve as indicators of overdispersion. A large difference between the Pearson statistic and the deviance provides some evidence that the data are too sparse to use either statistic.

You can use the **AGGREGATE** (or **AGGREGATE=**) option to define the subpopulation profiles. If you do not specify this option, each observation is regarded as coming from a separate subpopulation. For events/trials syntax, each observation represents  $n$  Bernoulli trials, where  $n$  is the value of the *trials* variable; for single-trial syntax, each observation represents a single trial. Without the **AGGREGATE**

(or AGGREGATE=) option, the Pearson chi-square statistic and the deviance are calculated only for events/trials syntax.

Note that the parameter estimates are not changed by this method. However, their standard errors are adjusted for overdispersion, affecting their significance tests.

### Williams' Method

Suppose that the data consist of  $n$  binomial observations. For the  $i$ th observation, let  $r_i/n_i$  be the observed proportion and let  $\mathbf{x}_i$  be the associated vector of explanatory variables. Suppose that the response probability for the  $i$ th observation is a random variable  $P_i$  with mean and variance

$$E(P_i) = \pi_i \quad \text{and} \quad V(P_i) = \phi \pi_i(1 - \pi_i)$$

where  $\pi_i$  is the probability of the event, and  $\phi$  is a nonnegative but otherwise unknown scale parameter. Then the mean and variance of  $r_i$  are

$$E(r_i) = n_i \pi_i \quad \text{and} \quad V(r_i) = n_i \pi_i(1 - \pi_i)[1 + (n_i - 1)\phi]$$

Williams (1982) estimates the unknown parameter  $\phi$  by equating the value of Pearson's chi-square statistic for the full model to its approximate expected value. Suppose  $w_i^*$  is the weight associated with the  $i$ th observation. The Pearson chi-square statistic is given by

$$\chi^2 = \sum_{i=1}^n \frac{w_i^* (r_i - n_i \hat{\pi}_i)^2}{n_i \hat{\pi}_i (1 - \hat{\pi}_i)}$$

Let  $g'(\cdot)$  be the first derivative of the link function  $g(\cdot)$ . The approximate expected value of  $\chi^2$  is

$$E_{\chi^2} = \sum_{i=1}^n w_i^* (1 - w_i^* v_i d_i) [1 + \phi(n_i - 1)]$$

where  $v_i = n_i / (\pi_i(1 - \pi_i)[g'(\pi_i)]^2)$  and  $d_i$  is the variance of the linear predictor  $\hat{\alpha}_i + \mathbf{x}_i' \hat{\boldsymbol{\beta}}$ . The scale parameter  $\phi$  is estimated by the following iterative procedure.

At the start, let  $w_i^* = 1$  and let  $\pi_i$  be approximated by  $r_i/n_i$ ,  $i = 1, 2, \dots, n$ . If you apply these weights and approximated probabilities to  $\chi^2$  and  $E_{\chi^2}$  and then equate them, an initial estimate of  $\phi$  is

$$\hat{\phi}_0 = \frac{\chi^2 - (n - p)}{\sum_i (n_i - 1)(1 - v_i d_i)}$$

where  $p$  is the total number of parameters. The initial estimates of the weights become  $\hat{w}_{i0}^* = [1 + (n_i - 1)\hat{\phi}_0]^{-1}$ . After a weighted fit of the model, the  $\hat{\alpha}_i$  and  $\hat{\boldsymbol{\beta}}$  are recalculated, and so is  $\chi^2$ . Then a revised estimate of  $\phi$  is given by

$$\hat{\phi}_1 = \frac{\chi^2 - \sum_i w_i^* (1 - w_i^* v_i d_i)}{w_i^* (n_i - 1)(1 - w_i^* v_i d_i)}$$

The iterative procedure is repeated until  $\chi^2$  is very close to its degrees of freedom.

Once  $\phi$  has been estimated by  $\hat{\phi}$  under the full model, weights of  $(1 + (n_i - 1)\hat{\phi})^{-1}$  can be used to fit models that have fewer terms than the full model. See [Example 54.10](#) for an illustration.

**NOTE:** If the **WEIGHT** statement is specified with the **NORMALIZE** option, then the initial  $w_i^*$  values are set to the normalized weights, and the weights resulting from Williams' method will not add up to the actual sample size. However, the estimated covariance matrix of the parameter estimates remains invariant to the scale of the **WEIGHT** variable.



## The Hosmer-Lemeshow Goodness-of-Fit Test

Sufficient replication within subpopulations is required to make the Pearson and deviance goodness-of-fit tests valid. When there are one or more continuous predictors in the model, the data are often too sparse to use these statistics. Hosmer and Lemeshow (2000) proposed a statistic that they show, through simulation, is distributed as chi-square when there is no replication in any of the subpopulations. This test is available only for binary response models.

First, the observations are sorted in increasing order of their estimated event probability. The event is the response level specified in the response variable option **EVENT=**, or the response level that is not specified in the **REF=** option, or, if neither of these options was specified, then the event is the response level identified in the “Response Profiles” table as “Ordered Value 1”. The observations are then divided into approximately 10 groups according to the following scheme. Let  $N$  be the total number of subjects. Let  $M$  be the target number of subjects for each group given by

$$M = [0.1 \times N + 0.5]$$

where  $[x]$  represents the integral value of  $x$ . If the single-trial syntax is used, blocks of subjects are formed of observations with identical values of the explanatory variables. Blocks of subjects are not divided when being placed into groups.

Suppose there are  $n_1$  subjects in the first block and  $n_2$  subjects in the second block. The first block of subjects is placed in the first group. Subjects in the second block are added to the first group if

$$n_1 < M \quad \text{and} \quad n_1 + [0.5 \times n_2] \leq M$$

Otherwise, they are placed in the second group. In general, suppose subjects of the  $(j - 1)$  block have been placed in the  $k$ th group. Let  $c$  be the total number of subjects currently in the  $k$ th group. Subjects for the  $j$ th block (containing  $n_j$  subjects) are also placed in the  $k$ th group if

$$c < M \quad \text{and} \quad c + [0.5 \times n_j] \leq M$$

Otherwise, the  $n_j$  subjects are put into the next group. In addition, if the number of subjects in the last group does not exceed  $[0.05 \times N]$  (half the target group size), the last two groups are collapsed to form only one group.

Note that the number of groups,  $g$ , can be smaller than 10 if there are fewer than 10 patterns of explanatory variables. There must be at least three groups in order for the Hosmer-Lemeshow statistic to be computed.

The Hosmer-Lemeshow goodness-of-fit statistic is obtained by calculating the Pearson chi-square statistic from the  $2 \times g$  table of observed and expected frequencies, where  $g$  is the number of groups. The statistic is written

$$\chi_{HL}^2 = \sum_{i=1}^g \frac{(O_i - N_i \bar{\pi}_i)^2}{N_i \bar{\pi}_i (1 - \bar{\pi}_i)}$$

where  $N_i$  is the total frequency of subjects in the  $i$ th group,  $O_i$  is the total frequency of event outcomes in the  $i$ th group, and  $\bar{\pi}_i$  is the average estimated predicted probability of an event outcome for the  $i$ th group. (Note that the predicted probabilities are computed as shown in the section “[Linear Predictor, Predicted Probability, and Confidence Limits](#)” on page 4253 and are not the cross validated estimates discussed in the section “[Classification Table](#)” on page 4255.) The Hosmer-Lemeshow statistic is then compared to a chi-square distribution with  $(g - n)$  degrees of freedom, where the value of  $n$  can be specified in the **LACKFIT** option in the **MODEL** statement. The default is  $n = 2$ . Large values of  $\chi_{HL}^2$  (and small  $p$ -values) indicate a lack of fit of the model.

## Receiver Operating Characteristic Curves

ROC curves are used to evaluate and compare the performance of diagnostic tests; they can also be used to evaluate model fit. An ROC curve is just a plot of the proportion of true positives (events predicted to be events) versus the proportion of false positives (nonevents predicted to be events).

In a sample of  $n$  individuals, suppose  $n_1$  individuals are observed to have a certain condition or event. Let this group be denoted by  $\mathcal{C}_1$ , and let the group of the remaining  $n_2 = n - n_1$  individuals who do not have the condition be denoted by  $\mathcal{C}_2$ . Risk factors are identified for the sample, and a logistic regression model is fitted to the data. For the  $j$ th individual, an estimated probability  $\hat{\pi}_j$  of the event of interest is calculated. Note that the  $\hat{\pi}_j$  are computed as shown in the section “[Linear Predictor, Predicted Probability, and Confidence Limits](#)” on page 4253 and are not the cross validated estimates discussed in the section “[Classification Table](#)” on page 4255.

Suppose the  $n$  individuals undergo a test for predicting the event and the test is based on the estimated probability of the event. Higher values of this estimated probability are assumed to be associated with the event. A receiver operating characteristic (ROC) curve can be constructed by varying the cutpoint that determines which estimated event probabilities are considered to predict the event. For each cutpoint  $z$ , the following measures can be output to a data set by specifying the `OUTROC=` option in the `MODEL` statement or the `OUTROC=` option in the `SCORE` statement:

$$\begin{aligned} \_POS\_ (z) &= \sum_{i \in \mathcal{C}_1} I(\hat{\pi}_i \geq z) \\ \_NEG\_ (z) &= \sum_{i \in \mathcal{C}_2} I(\hat{\pi}_i < z) \\ \_FALPOS\_ (z) &= \sum_{i \in \mathcal{C}_2} I(\hat{\pi}_i \geq z) \\ \_FALNEG\_ (z) &= \sum_{i \in \mathcal{C}_1} I(\hat{\pi}_i < z) \\ \_SENSIT\_ (z) &= \frac{\_POS\_ (z)}{n_1} \\ \_1MSPEC\_ (z) &= \frac{\_FALPOS\_ (z)}{n_2} \end{aligned}$$

where  $I(\cdot)$  is the indicator function.

Note that  $\_POS\_ (z)$  is the number of correctly predicted event responses,  $\_NEG\_ (z)$  is the number of correctly predicted nonevent responses,  $\_FALPOS\_ (z)$  is the number of falsely predicted event responses,  $\_FALNEG\_ (z)$  is the number of falsely predicted nonevent responses,  $\_SENSIT\_ (z)$  is the sensitivity of the test, and  $\_1MSPEC\_ (z)$  is one minus the specificity of the test.

The ROC curve is a plot of sensitivity ( $\_SENSIT\_$ ) against 1–specificity ( $\_1MSPEC\_$ ). The plot can be produced by using the `PLOTS` option or by using the `GPLOT` or `SGPLOT` procedure with the `OUTROC=` data set. See [Example 54.7](#) for an illustration. The area under the ROC curve, as determined by the trapezoidal rule, is estimated by the concordance index,  $c$ , in the “Association of Predicted Probabilities and Observed Responses” table.

## Comparing ROC Curves

ROC curves can be created from each model fit in a selection routine, from the specified model in the **MODEL** statement, from specified models in ROC statements, or from input variables which act as  $\hat{\pi}$  in the preceding discussion. Association statistics are computed for these models, and the models are compared when the **ROCCONTRAST** statement is specified. The ROC comparisons are performed by using a contrast matrix to take differences of the areas under the empirical ROC curves (DeLong, DeLong, and Clarke-Pearson 1988). For example, if you have three curves and the second curve is the reference, the contrast used for the overall test is

$$\mathbf{L}_1 = \begin{pmatrix} l'_1 \\ l'_2 \end{pmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

and you can optionally estimate and test each row of this contrast, in order to test the difference between the reference curve and each of the other curves. If you do not want to use a reference curve, the global test optionally uses the following contrast:

$$\mathbf{L}_2 = \begin{pmatrix} l'_1 \\ l'_2 \end{pmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

You can also specify your own contrast matrix. Instead of estimating the rows of these contrasts, you can request that the difference between every pair of ROC curves be estimated and tested.

By default for the reference contrast, the specified or selected model is used as the reference unless the **NOFIT** option is specified in the **MODEL** statement, in which case the first ROC model is the reference.

In order to label the contrasts, a name is attached to every model. The name for the specified or selected model is the **MODEL** statement label, or “Model” if the **MODEL** label is not present. The ROC statement models are named with their labels, or as “ROC*i*” for the *i*th ROC statement if a label is not specified. The contrast  $\mathbf{L}_1$  is labeled as “Reference = ModelName”, where ModelName is the reference model name, while  $\mathbf{L}_2$  is labeled “Adjacent Pairwise Differences”. The estimated rows of the contrast matrix are labeled “ModelName1 – ModelName2”. In particular, for the rows of  $\mathbf{L}_1$ , ModelName2 is the reference model name. If you specify your own contrast matrix, then the contrast is labeled “Specified” and the *i*th contrast row estimates are labeled “Row*i*”.

If ODS Graphics is enabled, then all ROC curves are displayed individually and are also overlaid in a final display. If a selection method is specified, then the curves produced in each step of the model selection process are overlaid onto a single plot and are labeled “Step*i*”, and the selected model is displayed on a separate plot and on a plot with curves from specified ROC statements. See [Example 54.8](#) for an example.

## ROC Computations

The trapezoidal area under an empirical ROC curve is equal to the Mann-Whitney two-sample rank measure of association statistic (a generalized *U*-statistic) applied to two samples,  $\{X_i\}, i = 1, \dots, n_1$ , in  $\mathcal{C}_1$  and  $\{Y_i\}, i = 1, \dots, n_2$ , in  $\mathcal{C}_2$ . PROC LOGISTIC uses the predicted probabilities in place of **X** and **Y**; however, in general any criterion could be used. Denote the frequency of observation *i* in  $\mathcal{C}_k$  as  $f_{ki}$ , and denote the total frequency in  $\mathcal{C}_k$  as  $F_k$ . The **WEIGHTED** option replaces  $f_{ki}$  with  $f_{ki}w_{ki}$ , where  $w_{ki}$  is the weight of

observation  $i$  in group  $C_k$ . The trapezoidal area under the curve is computed as

$$\hat{c} = \frac{1}{F_1 F_2} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \psi(X_i, Y_j) f_{1i} f_{2j}$$

$$\psi(X, Y) = \begin{cases} 1 & Y < X \\ \frac{1}{2} & Y = X \\ 0 & Y > X \end{cases}$$

so that  $E(\hat{c}) = \Pr(Y < X) + \frac{1}{2} \Pr(Y = X)$ . Note that the concordance index,  $c$ , in the “Association of Predicted Probabilities and Observed Responses” table does not use weights unless both the **WEIGHTED** and **BINWIDTH=0** options are specified. Also, in this table,  $c$  is computed by creating 500 bins and binning the  $X_i$  and  $Y_j$ ; this results in more ties than the preceding method (unless the **BINWIDTH=0** or **ROCEPS=0** option is specified), so  $c$  is not necessarily equal to  $E(\hat{c})$ .

To compare  $K$  empirical ROC curves, first compute the trapezoidal areas. Asymptotic normality of the estimated area follows from  $U$ -statistic theory, and a covariance matrix  $\mathbf{S}$  can be computed; see DeLong, DeLong, and Clarke-Pearson (1988) for details. A Wald confidence interval for the  $r$ th area,  $1 \leq r \leq K$ , can be constructed as

$$\hat{c}_r \pm z_{1-\frac{\alpha}{2}} s_{r,r}$$

where  $s_{r,r}$  is the  $r$ th diagonal of  $\mathbf{S}$ .

For a contrast of ROC curve areas,  $\mathbf{Lc}$ , the statistic

$$(\hat{\mathbf{c}} - \mathbf{c})' \mathbf{L}' [\mathbf{LSL}']^{-1} \mathbf{L}(\hat{\mathbf{c}} - \mathbf{c})$$

has a chi-square distribution with  $\text{df} = \text{rank}(\mathbf{LSL}')$ . For a row of the contrast,  $\mathbf{l}'\mathbf{c}$ ,

$$\frac{\mathbf{l}'\hat{\mathbf{c}} - \mathbf{l}'\mathbf{c}}{[\mathbf{l}'\mathbf{S}\mathbf{l}]^{1/2}}$$

has a standard normal distribution. The corresponding confidence interval is

$$\mathbf{l}'\hat{\mathbf{c}} \pm z_{1-\frac{\alpha}{2}} [\mathbf{l}'\mathbf{S}\mathbf{l}]^{1/2}$$

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## Testing Linear Hypotheses about the Regression Coefficients

Linear hypotheses for  $\boldsymbol{\beta}$  are expressed in matrix form as

$$H_0: \mathbf{L}\boldsymbol{\beta} = \mathbf{c}$$

where  $\mathbf{L}$  is a matrix of coefficients for the linear hypotheses, and  $\mathbf{c}$  is a vector of constants. The vector of regression coefficients  $\boldsymbol{\beta}$  includes slope parameters as well as intercept parameters. The Wald chi-square statistic for testing  $H_0$  is computed as

$$\chi_W^2 = (\mathbf{L}\hat{\boldsymbol{\beta}} - \mathbf{c})' [\mathbf{L}\hat{\mathbf{V}}(\hat{\boldsymbol{\beta}})\mathbf{L}']^{-1} (\mathbf{L}\hat{\boldsymbol{\beta}} - \mathbf{c})$$

where  $\hat{\mathbf{V}}(\hat{\boldsymbol{\beta}})$  is the estimated covariance matrix. Under  $H_0$ ,  $\chi_W^2$  has an asymptotic chi-square distribution with  $r$  degrees of freedom, where  $r$  is the rank of  $\mathbf{L}$ .

## Regression Diagnostics

For binary response data, regression diagnostics developed by Pregibon (1981) can be requested by specifying the **INFLUENCE** option. For diagnostics available with conditional logistic regression, see the section “**Regression Diagnostic Details**” on page 4272. These diagnostics can also be obtained from the **OUTPUT** statement.

This section uses the following notation:

- $r_j, n_j$       $r_j$  is the number of event responses out of  $n_j$  trials for the  $j$ th observation. If events/trials syntax is used,  $r_j$  is the value of *events* and  $n_j$  is the value of *trials*. For single-trial syntax,  $n_j = 1$ , and  $r_j = 1$  if the ordered response is 1, and  $r_j = 0$  if the ordered response is 2.
- $w_j$      is the weight of the  $j$ th observation.
- $\pi_j$      is the probability of an event response for the  $j$ th observation given by  $\pi_j = F(\alpha + \beta'x_j)$ , where  $F(\cdot)$  is the **inverse link function** defined on page 4238.
- $\hat{\beta}$      is the maximum likelihood estimate (MLE) of  $(\alpha, \beta_1, \dots, \beta_s)'$ .
- $\hat{V}(\hat{\beta})$      is the estimated covariance matrix of  $\hat{\beta}$ .
- $\hat{p}_j, \hat{q}_j$       $\hat{p}_j$  is the estimate of  $\pi_j$  evaluated at  $\hat{\beta}$ , and  $\hat{q}_j = 1 - \hat{p}_j$ .

Pregibon (1981) suggests using the index plots of several diagnostic statistics to identify influential observations and to quantify the effects on various aspects of the maximum likelihood fit. In an index plot, the diagnostic statistic is plotted against the observation number. In general, the distributions of these diagnostic statistics are not known, so cutoff values cannot be given for determining when the values are large. However, the **IPLOTS** and **INFLUENCE** options in the **MODEL** statement and the **PLOTS** option in the **PROC LOGISTIC** statement provide displays of the diagnostic values, allowing visual inspection and comparison of the values across observations. In these plots, if the model is correctly specified and fits all observations well, then no extreme points should appear.

The next five sections give formulas for these diagnostic statistics.

### Hat Matrix Diagonal (Leverage)

The diagonal elements of the hat matrix are useful in detecting extreme points in the design space where they tend to have larger values. The  $j$ th diagonal element is

$$h_j = \begin{cases} \tilde{w}_j(1, \mathbf{x}'_j)\hat{V}(\hat{\beta})(1, \mathbf{x}'_j)' & \text{Fisher scoring} \\ \hat{w}_j(1, \mathbf{x}'_j)\hat{V}(\hat{\beta})(1, \mathbf{x}'_j)' & \text{Newton-Raphson} \end{cases}$$

where

$$\begin{aligned} \tilde{w}_j &= \frac{w_j n_j}{\hat{p}_j \hat{q}_j [g'(\hat{p}_j)]^2} \\ \hat{w}_j &= \tilde{w}_j + \frac{w_j(r_j - n_j \hat{p}_j)[\hat{p}_j \hat{q}_j g''(\hat{p}_j) + (\hat{q}_j - \hat{p}_j)g'(\hat{p}_j)]}{(\hat{p}_j \hat{q}_j)^2 [g'(\hat{p}_j)]^3} \end{aligned}$$

and  $g'(\cdot)$  and  $g''(\cdot)$  are the first and second derivatives of the link function  $g(\cdot)$ , respectively.

For a binary response logit model, the hat matrix diagonal elements are

$$h_j = w_j n_j \hat{p}_j \hat{q}_j (1, \mathbf{x}'_j) \hat{\mathbf{V}}(\hat{\boldsymbol{\beta}}) \begin{pmatrix} 1 \\ \mathbf{x}_j \end{pmatrix}$$

If the estimated probability is extreme (less than 0.1 and greater than 0.9, approximately), then the hat diagonal might be greatly reduced in value. Consequently, when an observation has a very large or very small estimated probability, its hat diagonal value is not a good indicator of the observation's distance from the design space (Hosmer and Lemeshow 2000, p. 171).

## Residuals

Residuals are useful in identifying observations that are not explained well by the model. Pearson residuals are components of the Pearson chi-square statistic and deviance residuals are components of the deviance. The Pearson residual for the  $j$ th observation is

$$\chi_j = \frac{\sqrt{w_j}(r_j - n_j \hat{p}_j)}{\sqrt{n_j \hat{p}_j \hat{q}_j}}$$

The Pearson chi-square statistic is the sum of squares of the Pearson residuals.

The deviance residual for the  $j$ th observation is

$$d_j = \begin{cases} -\sqrt{-2w_j n_j \log(\hat{q}_j)} & \text{if } r_j = 0 \\ \pm \sqrt{2w_j [r_j \log(\frac{r_j}{n_j \hat{p}_j}) + (n_j - r_j) \log(\frac{n_j - r_j}{n_j \hat{q}_j})]} & \text{if } 0 < r_j < n_j \\ \sqrt{-2w_j n_j \log(\hat{p}_j)} & \text{if } r_j = n_j \end{cases}$$

where the plus (minus) in  $\pm$  is used if  $r_j/n_j$  is greater (less) than  $\hat{p}_j$ . The deviance is the sum of squares of the deviance residuals.

The STDRES option in the **INFLUENCE** and **PLOTS=INFLUENCE** options computes three more residuals (Collett 2003). The Pearson and deviance residuals are standardized to have approximately unit variance:

$$e_{pj} = \frac{\chi_j}{\sqrt{1 - h_j}}$$

$$e_{dj} = \frac{d_j}{\sqrt{1 - h_j}}$$

The likelihood residuals, which estimate components of a likelihood ratio test of deleting an individual observation, are a weighted combination of the standardized Pearson and deviance residuals

$$e_{lj} = \text{sign}(r_j - n_j \hat{p}_j) \sqrt{h_j e_{pj}^2 + (1 - h_j) e_{dj}^2}$$

## DFBETAS

For each parameter estimate, the procedure calculates a DFBETAS diagnostic for each observation. The DFBETAS diagnostic for an observation is the standardized difference in the parameter estimate due to deleting the observation, and it can be used to assess the effect of an individual observation on each estimated parameter of the fitted model. Instead of reestimating the parameter every time an observation is deleted, PROC LOGISTIC uses the one-step estimate. See the section “[Predicted Probability of an Event for Classification](#)” on page 4255. For the  $j$ th observation, the DFBETAS are given by

$$\text{DFBETAS}_{ij} = \Delta_i \hat{\beta}_j^1 / \hat{\sigma}_i$$

where  $i = 0, 1, \dots, s$ ,  $\hat{\sigma}_i$  is the standard error of the  $i$ th component of  $\hat{\beta}$ , and  $\Delta_i \hat{\beta}_j^1$  is the  $i$ th component of the one-step difference

$$\Delta \hat{\beta}_j^1 = \frac{w_j(r_j - n_j \hat{p}_j)}{1 - h_j} \hat{\mathbf{V}}(\hat{\beta}) \begin{pmatrix} 1 \\ \mathbf{x}_j \end{pmatrix}$$

$\Delta \hat{\beta}_j^1$  is the approximate change ( $\hat{\beta} - \hat{\beta}_j^1$ ) in the vector of parameter estimates due to the omission of the  $j$ th observation. The DFBETAS are useful in detecting observations that are causing instability in the selected coefficients.

## C and CBAR

C and CBAR are confidence interval displacement diagnostics that provide scalar measures of the influence of individual observations on  $\hat{\beta}$ . These diagnostics are based on the same idea as the Cook distance in linear regression theory (Cook and Weisberg 1982), but use the one-step estimate. C and CBAR for the  $j$ th observation are computed as

$$C_j = \chi_j^2 h_j / (1 - h_j)^2$$

and

$$\overline{C}_j = \chi_j^2 h_j / (1 - h_j)$$

respectively.

Typically, to use these statistics, you plot them against an index and look for outliers.

## DIFDEV and DIFCHISQ

DIFDEV and DIFCHISQ are diagnostics for detecting ill-fitted observations; in other words, observations that contribute heavily to the disagreement between the data and the predicted values of the fitted model. DIFDEV is the change in the deviance due to deleting an individual observation while DIFCHISQ is the change in the Pearson chi-square statistic for the same deletion. By using the one-step estimate, DIFDEV and DIFCHISQ for the  $j$ th observation are computed as

$$\text{DIFDEV} = d_j^2 + \overline{C}_j$$

and

$$\text{DIFCHISQ} = \overline{C}_j / h_j$$

---

## Scoring Data Sets

*Scoring a data set*, which is especially important for predictive modeling, means applying a previously fitted model to a new data set in order to compute the conditional, or *posterior*, probabilities of each response category given the values of the explanatory variables in each observation.

The **SCORE** statement enables you to score new data sets and output the scored values and, optionally, the corresponding confidence limits into a SAS data set. If the response variable is included in the new data set, then you can request **fit statistics** for the data, which is especially useful for test or validation data. If the response is binary, you can also create a SAS data set containing the *receiver operating characteristic* (ROC) curve. You can specify multiple **SCORE** statements in the same invocation of PROC LOGISTIC.

By default, the posterior probabilities are based on implicit prior probabilities that are proportional to the frequencies of the response categories in the *training data* (the data used to fit the model). Explicit prior probabilities should be specified with the **PRIOR=** or **PRIOREVENT=** option when the sample proportions of the response categories in the training data differ substantially from the operational data to be scored. For example, to detect a rare category, it is common practice to use a training set in which the rare categories are overrepresented; without prior probabilities that reflect the true incidence rate, the predicted posterior probabilities for the rare category will be too high. By specifying the correct priors, the posterior probabilities are adjusted appropriately.

The model fit to the **DATA=** data set in the PROC LOGISTIC statement is the default model used for the scoring. Alternatively, you can save a model fit in one run of PROC LOGISTIC and use it to score new data in a subsequent run. The **OUTMODEL=** option in the PROC LOGISTIC statement saves the model information in a SAS data set. Specifying this data set in the **INMODEL=** option of a new PROC LOGISTIC run will score the **DATA=** data set in the **SCORE** statement without refitting the model.

The **STORE** statement can also be used to save your model. The PLM procedure can use this model to score new data sets; see Chapter 69, “[The PLM Procedure](#),” for more information. You cannot specify priors in PROC PLM.

### Fit Statistics for Scored Data Sets

Specifying the **FITSTAT** option displays the following fit statistics when the data set being scored includes the response variable:



| Statistic                                | Description                                                                        |
|------------------------------------------|------------------------------------------------------------------------------------|
| Total frequency                          | $F = \sum_i f_i n_i$                                                               |
| Total weight                             | $W = \sum_i f_i w_i n_i$                                                           |
| Log likelihood                           | $\log L = \sum_i f_i w_i \log(\hat{\pi}_i)$                                        |
| Full log likelihood                      | $\log L_f = \text{constant} + \log L$                                              |
| Misclassification (error) rate           | $\frac{\sum_i 1\{F\_Y_i \neq I\_Y_i\} f_i n_i}{F}$                                 |
| AIC                                      | $-2 \log L_f + 2p$                                                                 |
| AICC                                     | $-2 \log L_f + \frac{2pn}{n - p - 1}$                                              |
| BIC                                      | $-2 \log L_f + p \log(n)$                                                          |
| SC                                       | $-2 \log L_f + p \log(F)$                                                          |
| R-square                                 | $R^2 = 1 - \left(\frac{L_0}{L}\right)^{2/F}$                                       |
| Maximum-rescaled R-square                | $\frac{R^2}{1 - L_0^{2/F}}$                                                        |
| AUC                                      | Area under the ROC curve                                                           |
| Brier score (polytomous response)        | $\frac{1}{W} \sum_i f_i w_i \sum_j (y_{ij} - \hat{\pi}_{ij})^2$                    |
| Brier score (binary response)            | $\frac{1}{W} \sum_i f_i w_i (r_i (1 - \hat{\pi}_i)^2 + (n_i - r_i) \hat{\pi}_i^2)$ |
| Brier reliability (events/trials syntax) | $\frac{1}{W} \sum_i f_i w_i (r_i / n_i - \hat{\pi}_i)^2$                           |

In the preceding table,  $f_i$  is the frequency of the  $i$ th observation in the data set being scored,  $w_i$  is the weight of the observation, and  $n = \sum_i f_i$ . The number of trials when events/trials syntax is specified is  $n_i$ , and with single-trial syntax  $n_i = 1$ . The values  $F\_Y_i$  and  $I\_Y_i$  are described in the section “[OUT= Output Data Set in a SCORE Statement](#)” on page 4281. The indicator function  $1\{A\}$  is 1 if  $A$  is true and 0 otherwise. The likelihood of the model is  $L$ , and  $L_0$  denotes the likelihood of the intercept-only model. For polytomous response models,  $y_i$  is the observed polytomous response level,  $\hat{\pi}_{ij}$  is the predicted probability of the  $j$ th response level for observation  $i$ , and  $y_{ij} = 1\{y_i = j\}$ . For binary response models,  $\hat{\pi}_i$  is the predicted probability of the observation,  $r_i$  is the number of events when you specify events/trials syntax, and  $r_i = y_i$  when you specify single-trial syntax.

The log likelihood, Akaike’s information criterion (AIC), and Schwarz criterion (SC) are described in the section “[Model Fitting Information](#)” on page 4245. The full log likelihood is displayed for models specified with events/trials syntax, and the constant term is described in the section “[Model Fitting Information](#)” on page 4245. The AICC is a small-sample bias-corrected version of the AIC (Hurvich and Tsai 1993; Burnham and Anderson 1998). The Bayesian information criterion (BIC) is the same as the SC except when events/trials syntax is specified. The area under the ROC curve for binary response models is defined in the section “[ROC Computations](#)” on page 4261. The R-square and maximum-rescaled R-square statistics, defined in “[Generalized Coefficient of Determination](#)” on page 4246, are not computed when you specify both an **OFFSET=** variable and the **INMODEL=** data set. The Brier score (Brier 1950) is the weighted squared difference between the predicted probabilities and their observed response levels. For events/trials syntax, the Brier reliability is the weighted squared difference between the predicted probabilities and the observed proportions (Murphy 1973).

## Posterior Probabilities and Confidence Limits

Let  $F$  be the inverse link function. That is,

$$F(t) = \begin{cases} \frac{1}{1+\exp(-t)} & \text{logistic} \\ \Phi(t) & \text{normal} \\ 1 - \exp(-\exp(t)) & \text{complementary log-log} \end{cases}$$

The first derivative of  $F$  is given by

$$F'(t) = \begin{cases} \frac{\exp(-t)}{(1+\exp(-t))^2} & \text{logistic} \\ \phi(t) & \text{normal} \\ \exp(t) \exp(-\exp(t)) & \text{complementary log-log} \end{cases}$$

Suppose there are  $k + 1$  response categories. Let  $Y$  be the response variable with levels  $1, \dots, k + 1$ . Let  $\mathbf{x} = (x_0, x_1, \dots, x_s)'$  be a  $(s + 1)$ -vector of covariates, with  $x_0 \equiv 1$ . Let  $\boldsymbol{\beta}$  be the vector of intercept and slope regression parameters.

Posterior probabilities are given by

$$p(Y = i | \mathbf{x}) = \frac{p_o(Y = i | \mathbf{x}) \frac{\tilde{p}(Y=i)}{p_o(Y=i)}}{\sum_j p_o(Y = j | \mathbf{x}) \frac{\tilde{p}(Y=j)}{p_o(Y=j)}} \quad i = 1, \dots, k + 1$$

where the old posterior probabilities ( $p_o(Y = i | \mathbf{x}), i = 1, \dots, k + 1$ ) are the conditional probabilities of the response categories given  $\mathbf{x}$ , the old priors ( $p_o(Y = i), i = 1, \dots, k + 1$ ) are the sample proportions of response categories of the training data, and the new priors ( $\tilde{p}(Y = i), i = 1, \dots, k + 1$ ) are specified in the **PRIOR=** or **PRIOREVENT=** option. To simplify notation, absorb the old priors into the new priors; that is

$$p(Y = i) = \frac{\tilde{p}(Y = i)}{p_o(Y = i)} \quad i = 1, \dots, k + 1$$

Note if the **PRIOR=** and **PRIOREVENT=** options are not specified, then  $p(Y = i) = 1$ .

The posterior probabilities are functions of  $\boldsymbol{\beta}$  and their estimates are obtained by substituting  $\boldsymbol{\beta}$  by its MLE  $\hat{\boldsymbol{\beta}}$ . The variances of the estimated posterior probabilities are given by the *delta method* as follows:

$$\text{Var}(\hat{p}(Y = i | \mathbf{x})) = \left[ \frac{\partial p(Y = i | \mathbf{x})}{\partial \boldsymbol{\beta}} \right]' \text{Var}(\hat{\boldsymbol{\beta}}) \left[ \frac{\partial p(Y = i | \mathbf{x})}{\partial \boldsymbol{\beta}} \right]$$

where

$$\frac{\partial p(Y = i | \mathbf{x})}{\partial \boldsymbol{\beta}} = \frac{\frac{\partial p_o(Y=i|\mathbf{x})}{\partial \boldsymbol{\beta}} p(Y = i)}{\sum_j p_o(Y = j | \mathbf{x}) p(Y = j)} - \frac{p_o(Y = i | \mathbf{x}) p(Y = i) \sum_j \frac{\partial p_o(Y=j|\mathbf{x})}{\partial \boldsymbol{\beta}} p(Y = j)}{[\sum_j p_o(Y = j | \mathbf{x}) p(Y = j)]^2}$$

and the old posterior probabilities  $p_o(Y = i | \mathbf{x})$  are described in the following sections.

A  $100(1 - \alpha)\%$  confidence interval for  $p(Y = i | \mathbf{x})$  is

$$\hat{p}(Y = i | \mathbf{x}) \pm z_{1-\alpha/2} \sqrt{\widehat{\text{Var}}(\hat{p}(Y = i | \mathbf{x}))}$$

where  $z_\tau$  is the upper  $100\tau$  percentile of the standard normal distribution.

**Binary and Cumulative Response Models**

Let  $\alpha_1, \dots, \alpha_k$  be the intercept parameters and let  $\beta_s$  be the vector of slope parameters. Denote  $\beta = (\alpha_1, \dots, \alpha_k, \beta_s')'$ . Let

$$\eta_i = \eta_i(\beta) = \alpha_i + \mathbf{x}'\beta_s, i = 1, \dots, k$$

Estimates of  $\eta_1, \dots, \eta_k$  are obtained by substituting the maximum likelihood estimate  $\hat{\beta}$  for  $\beta$ .

The predicted probabilities of the responses are

$$\widehat{p}_o(Y = i|\mathbf{x}) = \widehat{\Pr}(Y = i) = \begin{cases} F(\hat{\eta}_1) & i = 1 \\ F(\hat{\eta}_i) - F(\hat{\eta}_{i-1}) & i = 2, \dots, k \\ 1 - F(\hat{\eta}_k) & i = k + 1 \end{cases}$$

For  $i = 1, \dots, k$ , let  $\delta_i(\mathbf{x})$  be a  $(k + 1)$  column vector with  $i$ th entry equal to 1,  $k + 1$  entry equal to  $\mathbf{x}$ , and all other entries 0. The derivative of  $p_o(Y = i|\mathbf{x})$  with respect to  $\beta$  are

$$\frac{\partial p_o(Y = i|\mathbf{x})}{\partial \beta} = \begin{cases} F'(\alpha_1 + \mathbf{x}'\beta_s)\delta_1(\mathbf{x}) & i = 1 \\ F'(\alpha_i + \mathbf{x}'\beta_s)\delta_i(\mathbf{x}) - F'(\alpha_{i-1} + \mathbf{x}'\beta_s)\delta_{i-1}(\mathbf{x}) & i = 2, \dots, k \\ -F'(\alpha_k + \mathbf{x}'\beta_s)\delta_k(\mathbf{x}) & i = k + 1 \end{cases}$$

The cumulative posterior probabilities are

$$p(Y \leq i|\mathbf{x}) = \frac{\sum_{j=1}^i p_o(Y = j|\mathbf{x})p(Y = j)}{\sum_{j=1}^{k+1} p_o(Y = j|\mathbf{x})p(Y = j)} = \sum_{j=1}^i p(Y = j|\mathbf{x}) \quad i = 1, \dots, k + 1$$

Their derivatives are

$$\frac{\partial p(Y \leq i|\mathbf{x})}{\partial \beta} = \sum_{j=1}^i \frac{\partial p(Y = j|\mathbf{x})}{\partial \beta} \quad i = 1, \dots, k + 1$$

In the delta-method equation for the variance, replace  $p(Y = \cdot|\mathbf{x})$  with  $p(Y \leq \cdot|\mathbf{x})$ .

Finally, for the cumulative response model, use

$$\begin{aligned} \widehat{p}_o(Y \leq i|\mathbf{x}) &= F(\hat{\eta}_i) \quad i = 1, \dots, k \\ \widehat{p}_o(Y \leq k + 1|\mathbf{x}) &= 1 \\ \frac{\partial p_o(Y \leq i|\mathbf{x})}{\partial \beta} &= F'(\alpha_i + \mathbf{x}'\beta_s)\delta_i(\mathbf{x}) \quad i = 1, \dots, k \\ \frac{\partial p_o(Y \leq k + 1|\mathbf{x})}{\partial \beta} &= 0 \end{aligned}$$

**Generalized Logit Model**

Consider the last response level ( $Y=k+1$ ) as the reference. Let  $\beta_1, \dots, \beta_k$  be the (intercept and slope) parameter vectors for the first  $k$  logits, respectively. Denote  $\beta = (\beta'_1, \dots, \beta'_k)'$ . Let  $\eta = (\eta_1, \dots, \eta_k)'$  with

$$\eta_i = \eta_i(\beta) = \mathbf{x}'\beta_i \quad i = 1, \dots, k$$

Estimates of  $\eta_1, \dots, \eta_k$  are obtained by substituting the maximum likelihood estimate  $\hat{\beta}$  for  $\beta$ .

The predicted probabilities are

$$\begin{aligned} \widehat{p}_o(Y = k + 1 | \mathbf{x}) &\equiv \Pr(Y = k + 1 | \mathbf{x}) = \frac{1}{1 + \sum_{l=1}^k \exp(\hat{\eta}_l)} \\ \widehat{p}_o(Y = i | \mathbf{x}) &\equiv \Pr(Y = i | \mathbf{x}) = \widehat{p}_o(Y = k + 1 | \mathbf{x}) \exp(\eta_i), i = 1, \dots, k \end{aligned}$$

The derivative of  $p_o(Y = i | \mathbf{x})$  with respect to  $\beta$  are

$$\begin{aligned} \frac{\partial p_o(Y = i | \mathbf{x})}{\partial \beta} &= \frac{\partial \eta}{\partial \beta} \frac{\partial p_o(Y = i | \mathbf{x})}{\partial \eta} \\ &= (I_k \otimes \mathbf{x}) \left( \frac{\partial p_o(Y = i | \mathbf{x})}{\partial \eta_1}, \dots, \frac{\partial p_o(Y = i | \mathbf{x})}{\partial \eta_k} \right)' \end{aligned}$$

where

$$\frac{\partial p_o(Y = i | \mathbf{x})}{\partial \eta_j} = \begin{cases} p_o(Y = i | \mathbf{x})(1 - p_o(Y = i | \mathbf{x})) & j = i \\ -p_o(Y = i | \mathbf{x})p_o(Y = j | \mathbf{x}) & \text{otherwise} \end{cases}$$

**Special Case of Binary Response Model with No Priors**

Let  $\beta$  be the vector of regression parameters. Let

$$\eta = \eta(\beta) = \mathbf{x}'\beta$$

The variance of  $\hat{\eta}$  is given by

$$\text{Var}(\hat{\eta}) = \mathbf{x}'\text{Var}(\hat{\beta})\mathbf{x}$$

A  $100(1 - \alpha)$  percent confidence interval for  $\eta$  is

$$\hat{\eta} \pm z_{1-\alpha/2} \sqrt{\widehat{\text{Var}}(\hat{\eta})}$$

Estimates of  $p_o(Y = 1 | \mathbf{x})$  and confidence intervals for the  $p_o(Y = 1 | \mathbf{x})$  are obtained by back-transforming  $\hat{\eta}$  and the confidence intervals for  $\eta$ , respectively. That is,

$$\widehat{p}_o(Y = 1 | \mathbf{x}) = F(\hat{\eta})$$

and the confidence intervals are

$$F \left( \hat{\eta} \pm z_{1-\alpha/2} \sqrt{\widehat{\text{Var}}(\hat{\eta})} \right)$$

## Conditional Logistic Regression

The method of maximum likelihood described in the preceding sections relies on large-sample asymptotic normality for the validity of estimates and especially of their standard errors. When you do not have a large sample size compared to the number of parameters, this approach might be inappropriate and might result in biased inferences. This situation typically arises when your data are stratified and you fit intercepts to each stratum so that the number of parameters is of the same order as the sample size. For example, in a 1:1 matched pairs study with  $n$  pairs and  $p$  covariates, you would estimate  $n - 1$  intercept parameters and  $p$  slope parameters. Taking the stratification into account by “conditioning out” (and not estimating) the stratum-specific intercepts gives consistent and asymptotically normal MLEs for the slope coefficients. See Breslow and Day (1980) and Stokes, Davis, and Koch (2012) for more information. If your nuisance parameters are not just stratum-specific intercepts, you can perform an [exact conditional logistic regression](#).

### Computational Details

For each stratum  $h$ ,  $h = 1, \dots, H$ , number the observations as  $i = 1, \dots, n_h$  so that  $hi$  indexes the  $i$ th observation in stratum  $h$ . Denote the  $p$  covariates for the  $hi$ th observation as  $\mathbf{x}_{hi}$  and its binary response as  $y_{hi}$ , and let  $\mathbf{y} = (y_{11}, \dots, y_{1n_1}, \dots, y_{H1}, \dots, y_{Hn_H})'$ ,  $\mathbf{X}_h = (\mathbf{x}_{h1} \dots \mathbf{x}_{hn_h})'$ , and  $\mathbf{X} = (\mathbf{X}'_1 \dots \mathbf{X}'_H)'$ . Let the dummy variables  $z_h$ ,  $h = 1, \dots, H$ , be indicator functions for the strata ( $z_h = 1$  if the observation is in stratum  $h$ ), and denote  $\mathbf{z}_{hi} = (z_1, \dots, z_H)$  for the  $hi$ th observation,  $\mathbf{Z}_h = (\mathbf{z}_{h1} \dots \mathbf{z}_{hn_h})'$ , and  $\mathbf{Z} = (\mathbf{Z}'_1 \dots \mathbf{Z}'_H)'$ . Denote  $\mathbf{X}^* = (\mathbf{Z}|\mathbf{X})$  and  $\mathbf{x}^*_{hi} = (\mathbf{z}'_{hi}|\mathbf{x}'_{hi})'$ . Arrange the observations in each stratum  $h$  so that  $y_{hi} = 1$  for  $i = 1, \dots, m_h$ , and  $y_{hi} = 0$  for  $i = m_h + 1, \dots, n_h$ . Suppose all observations have unit frequency.

Consider the [binary logistic regression model](#) on page 4163 written as

$$\text{logit}(\pi) = \mathbf{X}^* \boldsymbol{\theta}$$

where the parameter vector  $\boldsymbol{\theta} = (\boldsymbol{\alpha}', \boldsymbol{\beta}')'$  consists of  $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_H)'$ ,  $\alpha_h$  is the intercept for stratum  $h$ ,  $h = 1, \dots, H$ , and  $\boldsymbol{\beta}$  is the parameter vector for the  $p$  covariates.

From the section “[Determining Observations for Likelihood Contributions](#)” on page 4239, you can write the likelihood contribution of observation  $hi$ ,  $i = 1, \dots, n_h$ ,  $h = 1, \dots, H$ , as

$$L_{hi}(\boldsymbol{\theta}) = \frac{e^{y_{hi}\mathbf{x}^*_{hi}'\boldsymbol{\theta}}}{1 + e^{\mathbf{x}^*_{hi}'\boldsymbol{\theta}}}$$

where  $y_{hi} = 1$  when the response takes Ordered Value 1, and  $y_{hi} = 0$  otherwise.

The full likelihood is

$$L(\boldsymbol{\theta}) = \prod_{h=1}^H \prod_{i=1}^{n_h} L_{hi}(\boldsymbol{\theta}) = \frac{e^{\mathbf{y}'\mathbf{X}^*\boldsymbol{\theta}}}{\prod_{h=1}^H \prod_{i=1}^{n_h} (1 + e^{\mathbf{x}^*_{hi}'\boldsymbol{\theta}})}$$

Unconditional likelihood inference is based on maximizing this likelihood function.

When your nuisance parameters are the stratum-specific intercepts  $(\alpha_1, \dots, \alpha_H)'$ , and the slopes  $\boldsymbol{\beta}$  are your parameters of interest, “conditioning out” the nuisance parameters produces the conditional likelihood (Lachin 2000)

$$L(\boldsymbol{\beta}) = \prod_{h=1}^H L_h(\boldsymbol{\beta}) = \prod_{h=1}^H \frac{\prod_{i=1}^{m_h} \exp(\mathbf{x}'_{hi}\boldsymbol{\beta})}{\sum \prod_{j=j_1}^{j_{m_h}} \exp(\mathbf{x}'_{hj}\boldsymbol{\beta})}$$

where the summation is over all  $\binom{n_h}{m_h}$  subsets  $\{j_1, \dots, j_{m_h}\}$  of  $m_h$  observations chosen from the  $n_h$  observations in stratum  $h$ . Note that the nuisance parameters have been factored out of this equation.

For conditional asymptotic inference, maximum likelihood estimates  $\hat{\beta}$  of the regression parameters are obtained by maximizing the conditional likelihood, and asymptotic results are applied to the conditional likelihood function and the maximum likelihood estimators. A relatively fast method of computing this conditional likelihood and its derivatives is given by Gail, Lubin, and Rubinstein (1981) and Howard (1972). The optimization techniques can be controlled by specifying the **NLOPTIONS** statement.

Sometimes the log likelihood converges but the estimates diverge. This condition is flagged by having inordinately large standard errors for some of your parameter estimates, and can be monitored by specifying the **ITPRINT** option. Unfortunately, broad existence criteria such as those discussed in the section “[Existence of Maximum Likelihood Estimates](#)” on page 4242 do not exist for this model. It might be possible to circumvent such a problem by standardizing your independent variables before fitting the model.

### Regression Diagnostic Details

Diagnostics are used to indicate observations that might have undue influence on the model fit or that might be outliers. Further investigation should be performed before removing such an observation from the data set.

The derivations in this section use an augmentation method described by Storer and Crowley (1985), which provides an estimate of the “one-step” DFBETAS estimates advocated by Pregibon (1984). The method also provides estimates of conditional stratum-specific predicted values, residuals, and leverage for each observation. The augmentation method can take a lot of time and memory.

Following Storer and Crowley (1985), the log-likelihood contribution can be written as

$$l_h = \log(L_h) = \mathbf{y}_h' \boldsymbol{\gamma}_h - a(\boldsymbol{\gamma}_h) \quad \text{where}$$

$$a(\boldsymbol{\gamma}_h) = \log \left[ \sum_{j=j_1}^{j_{m_h}} \prod \exp(\boldsymbol{\gamma}_{hj}) \right]$$

and the  $h$  subscript on matrices indicates the submatrix for the stratum,  $\boldsymbol{\gamma}_h = (\gamma_{h1}, \dots, \gamma_{hn_h})'$ , and  $\gamma_{hi} = \mathbf{x}_{hi}' \boldsymbol{\beta}$ . Then the gradient and information matrix are

$$\mathbf{g}(\boldsymbol{\beta}) = \left\{ \frac{\partial l_h}{\partial \boldsymbol{\beta}} \right\}_{h=1}^H = \mathbf{X}'(\mathbf{y} - \boldsymbol{\pi})$$

$$\boldsymbol{\Lambda}(\boldsymbol{\beta}) = \left\{ \frac{\partial^2 l_h}{\partial \boldsymbol{\beta}^2} \right\}_{h=1}^H = \mathbf{X}' \text{diag}(\mathbf{U}_1, \dots, \mathbf{U}_H) \mathbf{X}$$

where

$$\begin{aligned}\pi_{hi} &= \frac{\partial a(\boldsymbol{\gamma}_h)}{\partial \gamma_{hi}} = \frac{\sum_{j(i)} \prod_{j=j_1}^{j_{m_h}} \exp(\gamma_{hj})}{\sum \prod_{j=j_1}^{j_{m_h}} \exp(\gamma_{hj})} \\ \boldsymbol{\pi}_h &= (\pi_{h1}, \dots, \pi_{hn_h}) \\ \mathbf{U}_h &= \frac{\partial^2 a(\boldsymbol{\gamma}_h)}{\partial \boldsymbol{\gamma}_h^2} = \left\{ \frac{\partial^2 a(\boldsymbol{\gamma}_h)}{\partial \gamma_{hi} \partial \gamma_{hj}} \right\} = \{a_{ij}\} \\ a_{ij} &= \frac{\sum_{k(i,j)} \prod_{k=k_1}^{k_{m_h}} \exp(\gamma_{hk})}{\sum \prod_{k=k_1}^{k_{m_h}} \exp(\gamma_{hk})} - \frac{\partial a(\boldsymbol{\gamma}_h)}{\partial \gamma_{hi}} \frac{\partial a(\boldsymbol{\gamma}_h)}{\partial \gamma_{hj}} = \pi_{hij} - \pi_{hi} \pi_{hj}\end{aligned}$$

and where  $\pi_{hi}$  is the conditional stratum-specific probability that subject  $i$  in stratum  $h$  is a case, the summation on  $j(i)$  is over all subsets from  $\{1, \dots, n_h\}$  of size  $m_h$  that contain the index  $i$ , and the summation on  $k(i, j)$  is over all subsets from  $\{1, \dots, n_h\}$  of size  $m_h$  that contain the indices  $i$  and  $j$ .

To produce the true one-step estimate  $\boldsymbol{\beta}_{hi}^1$ , start at the MLE  $\hat{\boldsymbol{\beta}}$ , delete the  $h$ th observation, and use this reduced data set to compute the next Newton-Raphson step. Note that if there is only one event or one nonevent in a stratum, deletion of that single observation is equivalent to deletion of the entire stratum. The augmentation method does not take this into account.

The augmented model is

$$\text{logit}(\Pr(y_{hi} = 1 | \mathbf{x}_{hi})) = \mathbf{x}'_{hi} \boldsymbol{\beta} + \mathbf{z}'_{hi} \gamma$$

where  $\mathbf{z}_{hi} = (0, \dots, 0, 1, 0, \dots, 0)'$  has a 1 in the  $h$ th coordinate, and use  $\boldsymbol{\beta}^0 = (\hat{\boldsymbol{\beta}}', 0)'$  as the initial estimate for  $(\boldsymbol{\beta}', \gamma)'$ . The gradient and information matrix before the step are

$$\begin{aligned}\mathbf{g}(\boldsymbol{\beta}^0) &= \begin{bmatrix} \mathbf{X}' \\ \mathbf{z}'_{hi} \end{bmatrix} (\mathbf{y} - \boldsymbol{\pi}) = \begin{bmatrix} \mathbf{0} \\ y_{hi} - \pi_{hi} \end{bmatrix} \\ \boldsymbol{\Lambda}(\boldsymbol{\beta}^0) &= \begin{bmatrix} \mathbf{X}' \\ \mathbf{z}'_{hi} \end{bmatrix} \mathbf{U} [\mathbf{X} \quad \mathbf{z}_{hi}] = \begin{bmatrix} \boldsymbol{\Lambda}(\boldsymbol{\beta}) & \mathbf{X}' \mathbf{U} \mathbf{z}_{hi} \\ \mathbf{z}'_{hi} \mathbf{U} \mathbf{X} & \mathbf{z}'_{hi} \mathbf{U} \mathbf{z}_{hi} \end{bmatrix}\end{aligned}$$

Inserting the  $\boldsymbol{\beta}^0$  and  $(\mathbf{X}', \mathbf{z}'_{hi})'$  into the Gail, Lubin, and Rubinstein (1981) algorithm provides the appropriate estimates of  $\mathbf{g}(\boldsymbol{\beta}^0)$  and  $\boldsymbol{\Lambda}(\boldsymbol{\beta}^0)$ . Indicate these estimates with  $\hat{\boldsymbol{\pi}} = \boldsymbol{\pi}(\hat{\boldsymbol{\beta}})$ ,  $\hat{\mathbf{U}} = \mathbf{U}(\hat{\boldsymbol{\beta}})$ ,  $\hat{\mathbf{g}}$ , and  $\hat{\boldsymbol{\Lambda}}$ .

DFBETA is computed from the information matrix as

$$\begin{aligned}\Delta_{hi} \boldsymbol{\beta} &= \boldsymbol{\beta}^0 - \boldsymbol{\beta}_{hi}^1 \\ &= -\hat{\boldsymbol{\Lambda}}^{-1}(\boldsymbol{\beta}^0) \hat{\mathbf{g}}(\boldsymbol{\beta}^0) \\ &= -\hat{\boldsymbol{\Lambda}}^{-1}(\hat{\boldsymbol{\beta}}) (\mathbf{X}' \hat{\mathbf{U}} \mathbf{z}_{hi}) \mathbf{M}^{-1} \mathbf{z}'_{hi} (\mathbf{y} - \hat{\boldsymbol{\pi}})\end{aligned}$$

where

$$\mathbf{M} = (\mathbf{z}'_{hi} \hat{\mathbf{U}} \mathbf{z}_{hi}) - (\mathbf{z}'_{hi} \hat{\mathbf{U}} \mathbf{X}) \hat{\boldsymbol{\Lambda}}^{-1}(\hat{\boldsymbol{\beta}}) (\mathbf{X}' \hat{\mathbf{U}} \mathbf{z}_{hi})$$

For each observation in the data set, a DFBETA statistic is computed for each parameter  $\beta_j$ ,  $1 \leq j \leq p$ , and standardized by the standard error of  $\beta_j$  from the full data set to produce the estimate of DFBETAS.

The estimated leverage is defined as

$$h_{hi} = \frac{\text{trace}\{(\mathbf{z}'_{hi}\hat{\mathbf{U}}\mathbf{X})\hat{\mathbf{\Lambda}}^{-1}(\hat{\boldsymbol{\beta}})(\mathbf{X}'\hat{\mathbf{U}}\mathbf{z}_{hi})\}}{\text{trace}\{\mathbf{z}'_{hi}\hat{\mathbf{U}}\mathbf{z}_{hi}\}}$$

This definition of leverage produces different values from those defined by Pregibon (1984); Moolgavkar, Lustbader, and Venzon (1985); Hosmer and Lemeshow (2000); however, it has the advantage that no extra computations beyond those for the DFBETAS are required.

The estimated residuals  $e_{hi} = y_{hi} - \hat{\pi}_{hi}$  are obtained from  $\hat{\mathbf{g}}(\boldsymbol{\beta}^0)$ , and the weights, or predicted probabilities, are then  $\hat{\pi}_{hi} = y_{hi} - e_{hi}$ . The residuals are standardized and reported as (estimated) Pearson residuals:

$$\frac{r_{hi} - n_{hi}\hat{\pi}_{hi}}{\sqrt{n_{hi}\hat{\pi}_{hi}(1 - \hat{\pi}_{hi})}}$$

where  $r_{hi}$  is the number of events in the observation and  $n_{hi}$  is the number of trials.

The STDRES option in the INFLUENCE and PLOTS=INFLUENCE options computes the standardized Pearson residual:

$$e_{s,hi} = \frac{e_{hi}}{\sqrt{1 - h_{hi}}}$$

For events/trials MODEL statement syntax, treat each observation as two observations (the first for the nonevents and the second for the events) with frequencies  $f_{h,2i-1} = n_{hi} - r_{hi}$  and  $f_{h,2i} = r_{hi}$ , and augment the model with a matrix  $\mathbf{Z}_{hi} = [\mathbf{z}_{h,2i-1} \mathbf{z}_{h,2i}]$  instead of a single  $\mathbf{z}_{hi}$  vector. Writing  $\gamma_{hi} = \mathbf{x}'_{hi}\boldsymbol{\beta} f_{hi}$  in the preceding section results in the following gradient and information matrix:

$$\begin{aligned} \mathbf{g}(\boldsymbol{\beta}^0) &= \begin{bmatrix} \mathbf{0} \\ f_{h,2i-1}(y_{h,2i-1} - \pi_{h,2i-1}) \\ f_{h,2i}(y_{h,2i} - \pi_{h,2i}) \end{bmatrix} \\ \mathbf{\Lambda}(\boldsymbol{\beta}^0) &= \begin{bmatrix} \mathbf{\Lambda}(\boldsymbol{\beta}) & \mathbf{X}'\text{diag}(\mathbf{f})\text{Udiag}(\mathbf{f})\mathbf{Z}_{hi} \\ \mathbf{Z}'_{hi}\text{diag}(\mathbf{f})\text{Udiag}(\mathbf{f})\mathbf{X} & \mathbf{Z}'_{hi}\text{diag}(\mathbf{f})\text{Udiag}(\mathbf{f})\mathbf{Z}_{hi} \end{bmatrix} \end{aligned}$$

The predicted probabilities are then  $\hat{\pi}_{hi} = y_{h,2i} - e_{h,2i}/r_{h,2i}$ , while the leverage and the DFBETAS are produced from  $\mathbf{\Lambda}(\boldsymbol{\beta}^0)$  in a fashion similar to that for the preceding single-trial equations.

## Exact Conditional Logistic Regression

The theory of exact logistic regression, also known as exact conditional logistic regression, was originally laid out by Cox (1970), and the computational methods employed in PROC LOGISTIC are described in Hirji, Mehta, and Patel (1987); Hirji (1992); Mehta, Patel, and Senchaudhuri (1992). Other useful references for the derivations include Cox and Snell (1989); Agresti (1990); Mehta and Patel (1995).

Exact conditional inference is based on generating the conditional distribution for the sufficient statistics of the parameters of interest. This distribution is called the *permutation* or *exact conditional* distribution. Using the notation in the section “Computational Details” on page 4271, follow Mehta and Patel (1995) and



first note that the sufficient statistics  $\mathbf{T} = (T_1, \dots, T_p)$  for the parameter vector of intercepts and slopes,  $\boldsymbol{\beta}$ , are

$$T_j = \sum_{i=1}^n y_i x_{ij}, \quad j = 1, \dots, p$$

Denote a vector of observable sufficient statistics as  $\mathbf{t} = (t_1, \dots, t_p)'$ .

The probability density function (PDF) for  $\mathbf{T}$  can be created by summing over all binary sequences  $\mathbf{y}$  that generate an observable  $\mathbf{t}$  and letting  $C(\mathbf{t}) = ||\{\mathbf{y} : \mathbf{y}'\mathbf{X} = \mathbf{t}'\}||$  denote the number of sequences  $\mathbf{y}$  that generate  $\mathbf{t}$

$$\Pr(\mathbf{T} = \mathbf{t}) = \frac{C(\mathbf{t}) \exp(\mathbf{t}'\boldsymbol{\beta})}{\prod_{i=1}^n [1 + \exp(\mathbf{x}_i'\boldsymbol{\beta})]}$$

In order to condition out the nuisance parameters, partition the parameter vector  $\boldsymbol{\beta} = (\boldsymbol{\beta}'_N, \boldsymbol{\beta}'_I)'$ , where  $\boldsymbol{\beta}_N$  is a  $p_N \times 1$  vector of the nuisance parameters, and  $\boldsymbol{\beta}_I$  is the parameter vector for the remaining  $p_I = p - p_N$  parameters of interest. Likewise, partition  $\mathbf{X}$  into  $\mathbf{X}_N$  and  $\mathbf{X}_I$ ,  $\mathbf{T}$  into  $\mathbf{T}_N$  and  $\mathbf{T}_I$ , and  $\mathbf{t}$  into  $\mathbf{t}_N$  and  $\mathbf{t}_I$ . The nuisance parameters can be removed from the analysis by conditioning on their sufficient statistics to create the conditional likelihood of  $\mathbf{T}_I$  given  $\mathbf{T}_N = \mathbf{t}_N$ ,

$$\begin{aligned} \Pr(\mathbf{T}_I = \mathbf{t}_I | \mathbf{T}_N = \mathbf{t}_N) &= \frac{\Pr(\mathbf{T} = \mathbf{t})}{\Pr(\mathbf{T}_N = \mathbf{t}_N)} \\ &= f_{\boldsymbol{\beta}_I}(\mathbf{t}_I | \mathbf{t}_N) = \frac{C(\mathbf{t}_N, \mathbf{t}_I) \exp(\mathbf{t}_I'\boldsymbol{\beta}_I)}{\sum_{\mathbf{u}} C(\mathbf{t}_N, \mathbf{u}) \exp(\mathbf{u}'\boldsymbol{\beta}_I)} \end{aligned}$$

where  $C(\mathbf{t}_N, \mathbf{u})$  is the number of vectors  $\mathbf{y}$  such that  $\mathbf{y}'\mathbf{X}_N = \mathbf{t}_N$  and  $\mathbf{y}'\mathbf{X}_I = \mathbf{u}$ . Note that the nuisance parameters have factored out of this equation, and that  $C(\mathbf{t}_N, \mathbf{t}_I)$  is a constant.

The goal of the exact conditional analysis is to determine how likely the observed response  $\mathbf{y}_0$  is with respect to all  $2^n$  possible responses  $\mathbf{y} = (y_1, \dots, y_n)'$ . One way to proceed is to generate every  $\mathbf{y}$  vector for which  $\mathbf{y}'\mathbf{X}_N = \mathbf{t}_N$ , and count the number of vectors  $\mathbf{y}$  for which  $\mathbf{y}'\mathbf{X}_I$  is equal to each unique  $\mathbf{t}_I$ . Generating the conditional distribution from complete enumeration of the joint distribution is conceptually simple; however, this method becomes computationally infeasible very quickly. For example, if you had only 30 observations, you would have to scan through  $2^{30}$  different  $\mathbf{y}$  vectors.

Several algorithms are available in PROC LOGISTIC to generate the exact distribution. All of the algorithms are based on the following observation. Given any  $\mathbf{y} = (y_1, \dots, y_n)'$  and a design  $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_n)'$ , let  $\mathbf{y}_{(i)} = (y_1, \dots, y_i)'$  and  $\mathbf{X}_{(i)} = (\mathbf{x}_1, \dots, \mathbf{x}_i)'$  be the first  $i$  rows of each matrix. Write the sufficient statistic based on these  $i$  rows as  $\mathbf{t}_{(i)} = \mathbf{y}_{(i)}'\mathbf{X}_{(i)}$ . A recursion relation results:  $\mathbf{t}_{(i+1)} = \mathbf{t}_{(i)} + y_{i+1}\mathbf{x}_{i+1}$ .

The following methods are available:

- The *multivariate shift algorithm* developed by Hirji, Mehta, and Patel (1987), which steps through the recursion relation by adding one observation at a time and building an intermediate distribution at each step. If it determines that  $\mathbf{t}_{(i)}$  for the nuisance parameters could eventually equal  $\mathbf{t}$ , then  $\mathbf{t}_{(i)}$  is added to the intermediate distribution.
- An extension of the multivariate shift algorithm to generalized logit models by Hirji (1992). Since the generalized logit model fits a new set of parameters to each logit, the number of parameters in the

model can easily get too large for this algorithm to handle. Note for these models that the hypothesis tests for each effect are computed across the logit functions, while individual parameters are estimated for each logit function.

- A network algorithm described in Mehta, Patel, and Senchaudhuri (1992), which builds a network for each parameter that you are conditioning out in order to identify feasible  $y_i$  for the  $\mathbf{y}$  vector. These networks are combined and the set of feasible  $y_i$  is further reduced, and then the multivariate shift algorithm uses this knowledge to build the exact distribution without adding as many intermediate  $t_{(i+1)}$  as the multivariate shift algorithm does.
- A hybrid Monte Carlo and network algorithm described by Mehta, Patel, and Senchaudhuri (2000), which extends their 1992 algorithm by sampling from the combined network to build the exact distribution.

The bulk of the computation time and memory for these algorithms is consumed by the creation of the networks and the exact joint distribution. After the joint distribution for a set of effects is created, the computational effort required to produce hypothesis tests and parameter estimates for any subset of the effects is (relatively) trivial. See the section “[Computational Resources for Exact Logistic Regression](#)” on page 4284 for more computational notes about exact analyses.

**NOTE:** An alternative to using these exact conditional methods is to perform Firth’s bias-reducing penalized likelihood method (see the [FIRTH](#) option in the [MODEL](#) statement); this method has the advantage of being much faster and less memory intensive than exact algorithms, but it might not converge to a solution.

## Hypothesis Tests

Consider testing the null hypothesis  $H_0: \beta_I = \mathbf{0}$  against the alternative  $H_A: \beta_I \neq \mathbf{0}$ , conditional on  $\mathbf{T}_N = \mathbf{t}_N$ . Under the null hypothesis, the test statistic for the *exact probability test* is just  $f_{\beta_I=\mathbf{0}}(\mathbf{t}_I|\mathbf{t}_N)$ , while the corresponding  $p$ -value is the probability of getting a less likely (more extreme) statistic,

$$p(\mathbf{t}_I|\mathbf{t}_N) = \sum_{\mathbf{u} \in \Omega_p} f_0(\mathbf{u}|\mathbf{t}_N)$$

where  $\Omega_p = \{\mathbf{u}: \text{there exist } \mathbf{y} \text{ with } \mathbf{y}'\mathbf{X}_I = \mathbf{u}, \mathbf{y}'\mathbf{X}_N = \mathbf{t}_N, \text{ and } f_0(\mathbf{u}|\mathbf{t}_N) \leq f_0(\mathbf{t}_I|\mathbf{t}_N)\}$ .

For the *exact conditional scores test*, the conditional mean  $\mu_I$  and variance matrix  $\Sigma_I$  of the  $\mathbf{T}_I$  (conditional on  $\mathbf{T}_N = \mathbf{t}_N$ ) are calculated, and the score statistic for the observed value,

$$s = (\mathbf{t}_I - \mu_I)' \Sigma_I^{-1} (\mathbf{t}_I - \mu_I)$$

is compared to the score for each member of the distribution

$$S(\mathbf{T}_I) = (\mathbf{T}_I - \mu_I)' \Sigma_I^{-1} (\mathbf{T}_I - \mu_I)$$

The resulting  $p$ -value is

$$p(\mathbf{t}_I|\mathbf{t}_N) = \Pr(S \geq s) = \sum_{\mathbf{u} \in \Omega_s} f_0(\mathbf{u}|\mathbf{t}_N)$$

where  $\Omega_s = \{\mathbf{u}: \text{there exist } \mathbf{y} \text{ with } \mathbf{y}'\mathbf{X}_I = \mathbf{u}, \mathbf{y}'\mathbf{X}_N = \mathbf{t}_N, \text{ and } S(\mathbf{u}) \geq s\}$ .

The mid- $p$  statistic, defined as

$$p(\mathbf{t}_I|\mathbf{t}_N) - \frac{1}{2}f_0(\mathbf{t}_I|\mathbf{t}_N)$$

was proposed by Lancaster (1961) to compensate for the discreteness of a distribution. See Agresti (1992) for more information. However, to allow for more flexibility in handling ties, you can write the mid- $p$  statistic as (based on a suggestion by Lamotte (2002) and generalizing Vollset, Hirji, and Afifi (1991))

$$\sum_{\mathbf{u} \in \Omega_{<}} f_0(\mathbf{u}|\mathbf{t}_N) + \delta_1 f_0(\mathbf{t}_I|\mathbf{t}_N) + \delta_2 \sum_{\mathbf{u} \in \Omega_{=}} f_0(\mathbf{u}|\mathbf{t}_N)$$

where, for  $i \in \{p, s\}$ ,  $\Omega_{<}$  is  $\Omega_i$  using strict inequalities, and  $\Omega_{=}$  is  $\Omega_i$  using equalities with the added restriction that  $\mathbf{u} \neq \mathbf{t}_I$ . Letting  $(\delta_1, \delta_2) = (0.5, 1.0)$  yields Lancaster's mid- $p$ .

**CAUTION:** When the exact distribution has ties and **METHOD=NETWORKMCMC** is specified, the Monte Carlo algorithm estimates  $p(\mathbf{t}|\mathbf{t}_N)$  with error, and hence it cannot determine precisely which values contribute to the reported  $p$ -values. For example, if the exact distribution has densities  $\{0.2, 0.2, 0.2, 0.4\}$  and if the observed statistic has probability 0.2, then the exact probability  $p$ -value is exactly 0.6. Under Monte Carlo sampling, if the densities after  $N$  samples are  $\{0.18, 0.21, 0.23, 0.38\}$  and the observed probability is 0.21, then the resulting  $p$ -value is 0.39. Therefore, the exact probability test  $p$ -value for this example fluctuates between 0.2, 0.4, and 0.6, and the reported  $p$ -values are actually lower bounds for the true  $p$ -values. If you need more precise values, you can specify the **OUTDIST=** option, determine appropriate cutoff values for the observed probability and score, and then construct the true  $p$ -value estimates from the **OUTDIST=** data set and display them in the SAS log by using the following statements:

```
data _null_;
  set outdist end=end;
  retain pvalueProb 0 pvalueScore 0;
  if prob < ProbCutOff then pvalueProb+prob;
  if score > ScoreCutOff then pvalueScore+prob;
  if end then put pvalueProb= pvalueScore=;
run;
```

## Inference for a Single Parameter

Exact parameter estimates are derived for a single parameter  $\beta_i$  by regarding all the other parameters  $\beta_N = (\beta_1, \dots, \beta_{i-1}, \beta_{i+1}, \dots, \beta_{p_N+p_I})'$  as nuisance parameters. The appropriate sufficient statistics are  $\mathbf{T}_I = T_i$  and  $\mathbf{T}_N = (T_1, \dots, T_{i-1}, T_{i+1}, \dots, T_{p_N+p_I})'$ , with their observed values denoted by the lowercase  $t$ . Hence, the conditional PDF used to create the parameter estimate for  $\beta_i$  is

$$f_{\beta_i}(t_i|\mathbf{t}_N) = \frac{C(\mathbf{t}_N, t_i) \exp(t_i \beta_i)}{\sum_{\mathbf{u} \in \Omega} C(\mathbf{t}_N, u) \exp(u \beta_i)}$$

for  $\Omega = \{u: \text{there exist } \mathbf{y} \text{ with } T_i = u \text{ and } \mathbf{T}_N = \mathbf{t}_N\}$ .

The maximum exact conditional likelihood estimate is the quantity  $\hat{\beta}_i$ , which maximizes the conditional PDF. A Newton-Raphson algorithm is used to perform this search. However, if the observed  $t_i$  attains either its maximum or minimum value in the exact distribution (that is, either  $t_i = \min\{u : u \in \Omega\}$  or  $t_i = \max\{u : u \in \Omega\}$ ), then the conditional PDF is monotonically increasing in  $\beta_i$  and cannot be maximized. In this case, a median unbiased estimate (Hirji, Tsiatis, and Mehta 1989)  $\hat{\beta}_i$  is produced that satisfies  $\hat{f}_{\hat{\beta}_i}(t_i|\mathbf{t}_N) = 0.5$ , and a Newton-Raphson algorithm is used to perform the search.

The standard error of the exact conditional likelihood estimate is just the negative of the inverse of the second derivative of the exact conditional log likelihood (Agresti 2002).

Likelihood ratio tests based on the conditional PDF are used to test the null  $H_0: \beta_i = 0$  against the alternative  $H_A: \beta_i > 0$ . The critical region for this UMP test consists of the upper tail of values for  $T_i$  in the exact distribution. Thus, the one-sided significance level  $p_+(t_i; 0)$  is

$$p_+(t_i; 0) = \sum_{u \geq t_i} f_0(u | \mathbf{t}_N)$$

Similarly, the one-sided significance level  $p_-(t_i; 0)$  against  $H_A: \beta_i < 0$  is

$$p_-(t_i; 0) = \sum_{u \leq t_i} f_0(u | \mathbf{t}_N)$$

The two-sided significance level  $p(t_i; 0)$  against  $H_A: \beta_i \neq 0$  is calculated as

$$p(t_i; 0) = 2 \min[p_-(t_i; 0), p_+(t_i; 0)]$$

An upper  $100(1 - 2\epsilon)\%$  exact confidence limit for  $\hat{\beta}_i$  corresponding to the observed  $t_i$  is the solution  $\beta_U(t_i)$  of  $\epsilon = p_-(t_i, \beta_U(t_i))$ , while the lower exact confidence limit is the solution  $\beta_L(t_i)$  of  $\epsilon = p_+(t_i, \beta_L(t_i))$ . Again, a Newton-Raphson procedure is used to search for the solutions. Note that one of the confidence limits for a median unbiased estimate is set to infinity, but the other is still computed at  $\epsilon$ . This results in the display of a one-sided  $100(1 - \epsilon)\%$  confidence interval; if you want the  $2\epsilon$  limit instead, you can specify the **ONESIDED** option.

Specifying the **ONESIDED** option displays only one  $p$ -value and one confidence interval, because small values of  $p_+(t_i; 0)$  and  $p_-(t_i; 0)$  support different alternative hypotheses and only one of these  $p$ -values can be less than 0.50.

The mid- $p$  confidence limits are the solutions to  $\min\{p_-(t_i, \beta(t_i)), p_+(t_i, \beta(t_i))\} - (1 - \delta_1) f_{\beta(t_i)}(t_i | \mathbf{t}_N) = \epsilon$  for  $\epsilon = \alpha/2, 1 - \alpha/2$  (Vollset, Hirji, and Afifi 1991).  $\delta_1 = 1$  produces the usual exact (or *max-p*) confidence interval,  $\delta_1 = 0.5$  yields the mid- $p$  interval, and  $\delta_1 = 0$  gives the *min-p* interval. The mean of the endpoints of the *max-p* and *min-p* intervals provides the *mean-p* interval as defined by Hirji, Mehta, and Patel (1988).

Estimates and confidence intervals for the odds ratios are produced by exponentiating the estimates and interval endpoints for the parameters.

### Notes about Exact $p$ -Values

In the “Conditional Exact Tests” table, the exact probability test is not necessarily a sum of tail areas and can be inflated if the distribution is skewed. The more robust exact conditional scores test is a sum of tail areas and is generally preferred over the exact probability test.

The  $p$ -value reported for a single parameter in the “Exact Parameter Estimates” table is twice the one-sided tail area of a likelihood ratio test against the null hypothesis of the parameter equaling zero.

## Input and Output Data Sets

### OUTEST= Output Data Set

The **OUTEST=** data set contains one observation for each BY group containing the maximum likelihood estimates of the regression coefficients. If you also use the **COVOUT** option in the PROC LOGISTIC statement, there are additional observations containing the rows of the estimated covariance matrix. If you specify **SELECTION=FORWARD**, **BACKWARD**, or **STEPWISE**, only the estimates of the parameters and covariance matrix for the final model are output to the OUTEST= data set.

#### *Variables in the OUTEST= Data Set*

The OUTEST= data set contains the following variables:

- any BY variables specified
- **\_LINK\_**, a character variable of length 8 with four possible values: CLOGLOG for the complementary log-log function, LOGIT for the logit function, NORMIT for the probit (alias normit) function, and GLOGIT for the generalized logit function
- **\_TYPE\_**, a character variable of length 8 with two possible values: PARMS for parameter estimates or COV for covariance estimates. If an EXACT statement is also specified, then two other values are possible: EPARMMLE for the exact maximum likelihood estimates and EPARMMUE for the exact median unbiased estimates.
- **\_NAME\_**, a character variable containing the name of the response variable when **\_TYPE\_=PARMS**, EPARMMLE, and EPARMMUE, or the name of a model parameter when **\_TYPE\_=COV**
- **\_STATUS\_**, a character variable that indicates whether the estimates have converged
- one variable for each intercept parameter
- one variable for each slope parameter and one variable for the offset variable if the **OFFSET=** option is specified. If an effect is not included in the final model in a model building process, the corresponding parameter estimates and covariances are set to missing values.
- **\_LNLIKE\_**, the log likelihood

#### *Parameter Names in the OUTEST= Data Set*

If there are only two response categories in the entire data set, the intercept parameter is named Intercept. If there are more than two response categories in the entire data set, the intercept parameters are named Intercept\_xxx, where xxx is the value (formatted if a format is applied) of the corresponding response category.

For continuous explanatory variables, the names of the parameters are the same as the corresponding variables. For CLASS variables, the parameter names are obtained by concatenating the corresponding CLASS variable name with the CLASS category; see the section “[Class Variable Naming Convention](#)” on page 4189 for more details. For interaction and nested effects, the parameter names are created by concatenating the names of each effect.

For the generalized logit model, names of parameters corresponding to each nonreference category contain \_xxx as the suffix, where xxx is the value (formatted if a format is applied) of the corresponding nonreference

category. For example, suppose the variable Net3 represents the television network (ABC, CBS, and NBC) viewed at a certain time. The following statements fit a generalized logit model with Age and Gender (a CLASS variable with values Female and Male) as explanatory variables:

```
proc logistic;
  class Gender;
  model Net3 = Age Gender / link=glogit;
run;
```

There are two logit functions, one contrasting ABC with NBC and the other contrasting CBS with NBC. For each logit, there are three parameters: an intercept parameter, a slope parameter for Age, and a slope parameter for Gender (since there are only two gender levels and the EFFECT parameterization is used by default). The names of the parameters and their descriptions are as follows:

|                  |                                                                      |
|------------------|----------------------------------------------------------------------|
| Intercept_ABC    | intercept parameter for the logit contrasting ABC with NBC           |
| Intercept_CBS    | intercept parameter for the logit contrasting CBS with NBC           |
| Age_ABC          | Age slope parameter for the logit contrasting ABC with NBC           |
| Age_CBS          | Age slope parameter for the logit contrasting CBS with NBC           |
| GenderFemale_ABC | Gender=Female slope parameter for the logit contrasting ABC with NBC |
| GenderFemale_CBS | Gender=Female slope parameter for the logit contrasting CBS with NBC |

### INEST= Input Data Set

You can specify starting values for the iterative algorithm in the **INEST=** data set. The **INEST=** data set has the same structure as the **OUTEST=** data set but is not required to have all the variables or observations that appear in the **OUTEST=** data set. A previous **OUTEST=** data set can be used as, or modified for use as, an **INEST=** data set.

The **INEST=** data set must contain the intercept variables (named Intercept for binary response models and Intercept, Intercept\_2, Intercept\_3, and so forth, for ordinal and nominal response models) and all explanatory variables in the **MODEL** statement. If BY processing is used, the **INEST=** data set should also include the BY variables, and there must be one observation for each BY group. If the **INEST=** data set also contains the **\_TYPE\_** variable, only observations with **\_TYPE\_** value 'PARMS' are used as starting values.

### OUT= Output Data Set in the OUTPUT Statement

The **OUT=** data set in the **OUTPUT** statement contains all the variables in the input data set along with statistics you request by specifying *keyword=name* options or the **PREDPROBS=** option in the **OUTPUT** statement. In addition, if you use the single-trial syntax and you request any of the **XBETA=**, **STDXBETA=**, **PREDICTED=**, **LCL=**, and **UCL=** options, the **OUT=** data set contains the automatic variable **\_LEVEL\_**. The value of **\_LEVEL\_** identifies the response category upon which the computed values of **XBETA=**, **STDXBETA=**, **PREDICTED=**, **LCL=**, and **UCL=** are based.

When there are more than two response levels, only variables named by the **XBETA=**, **STDXBETA=**, **PREDICTED=**, **LOWER=**, and **UPPER=** options and the variables given by **PREDPROBS=(INDIVIDUAL CUMULATIVE)** have their values computed; the other variables have missing values. If you fit a generalized logit model, the cumulative predicted probabilities are not computed.

When there are only two response categories, each input observation produces one observation in the OUT= data set.

If there are more than two response categories and you specify only the PREDPROBS= option, then each input observation produces one observation in the OUT= data set. However, if you fit an ordinal (cumulative) model and specify options other than the PREDPROBS= options, each input observation generates as many output observations as one fewer than the number of response levels, and the predicted probabilities and their confidence limits correspond to the cumulative predicted probabilities. If you fit a generalized logit model and specify options other than the PREDPROBS= options, each input observation generates as many output observations as the number of response categories; the predicted probabilities and their confidence limits correspond to the probabilities of individual response categories.

For observations in which only the response variable is missing, values of the XBETA=, STDXBETA=, PREDICTED=, UPPER=, LOWER=, and the PREDPROBS= options are computed even though these observations do not affect the model fit. This enables, for instance, predicted probabilities to be computed for new observations.

### OUT= Output Data Set in a SCORE Statement

The OUT= data set in a SCORE statement contains all the variables in the data set being scored. The data set being scored can be either the input DATA= data set in the PROC LOGISTIC statement or the DATA= data set in the SCORE statement. The DATA= data set in the SCORE statement does not need to contain the response variable.

If the data set being scored contains the response variable, then denote the *normalized* levels (left-justified, formatted values of 16 characters or less) of your response variable Y by  $Y_1, \dots, Y_{k+1}$ . For each response level, the OUT= data set also contains the following:

- F\_Y, the normalized levels of the response variable Y in the data set being scored. If the events/trials syntax is used, the F\_Y variable is not created.
- I\_Y, the normalized levels that the observations are classified into. Note that an observation is classified into the level with the largest probability. If the events/trials syntax is used, the \_INTO\_ variable is created instead, and it contains the values EVENT and NONEVENT.
- P\_Y<sub>i</sub>, the posterior probabilities of the normalized response level Y<sub>i</sub>
- If the CLM option is specified in the SCORE statement, the OUT= data set also includes the following:
  - LCL\_Y<sub>i</sub>, the lower 100(1 –  $\alpha$ )% confidence limits for P\_Y<sub>i</sub>
  - UCL\_Y<sub>i</sub>, the upper 100(1 –  $\alpha$ )% confidence limits for P\_Y<sub>i</sub>

### OUTDIST= Output Data Set

The OUTDIST= data set contains every exact conditional distribution necessary to process the corresponding EXACT statement. For example, the following statements create one distribution for the x1 parameter and another for the x2 parameters, and produce the data set dist shown in Table 54.12:

```

data test;
  input y x1 x2 count;
  datalines;
0 0 0 1
1 0 0 1
0 1 1 2
1 1 1 1
1 0 2 3
1 1 2 1
1 2 0 3
1 2 1 2
1 2 2 1
;

proc logistic data=test exactonly;
  class x2 / param=ref;
  model y=x1 x2;
  exact x1 x2/ outdist=dist;
run;
proc print data=dist;
run;

```

**Table 54.12** OUTDIST= Data Set

| Obs | x1 | x20 | x21 | Count | Score   | Prob    |
|-----|----|-----|-----|-------|---------|---------|
| 1   | .  | 0   | 0   | 3     | 5.81151 | 0.03333 |
| 2   | .  | 0   | 1   | 15    | 1.66031 | 0.16667 |
| 3   | .  | 0   | 2   | 9     | 3.12728 | 0.10000 |
| 4   | .  | 1   | 0   | 15    | 1.46523 | 0.16667 |
| 5   | .  | 1   | 1   | 18    | 0.21675 | 0.20000 |
| 6   | .  | 1   | 2   | 6     | 4.58644 | 0.06667 |
| 7   | .  | 2   | 0   | 19    | 1.61869 | 0.21111 |
| 8   | .  | 2   | 1   | 2     | 3.27293 | 0.02222 |
| 9   | .  | 3   | 0   | 3     | 6.27189 | 0.03333 |
| 10  | 2  | .   | .   | 6     | 3.03030 | 0.12000 |
| 11  | 3  | .   | .   | 12    | 0.75758 | 0.24000 |
| 12  | 4  | .   | .   | 11    | 0.00000 | 0.22000 |
| 13  | 5  | .   | .   | 18    | 0.75758 | 0.36000 |
| 14  | 6  | .   | .   | 3     | 3.03030 | 0.06000 |

The first nine observations in the dist data set contain an exact distribution for the parameters of the x2 effect (hence the values for the x1 parameter are missing), and the remaining five observations are for the x1 parameter. If a joint distribution was created, there would be observations with values for both the x1 and x2 parameters. For **CLASS** variables, the corresponding parameters in the dist data set are identified by concatenating the variable name with the appropriate classification level.

The data set contains the possible sufficient statistics of the parameters for the effects specified in the **EXACT** statement, and the Count variable contains the number of different responses that yield these statistics. In particular, there are six possible response vectors  $y$  for which the dot product  $y'x_1$  was equal to 2, and



for which  $y'x_{20}$ ,  $y'x_{21}$ , and  $y'1$  were equal to their actual observed values (displayed in the “Sufficient Statistics” table).

When hypothesis tests are performed on the parameters, the **Prob** variable contains the probability of obtaining that statistic (which is just the count divided by the total count), and the **Score** variable contains the score for that statistic.

The **OUTDIST=** data set can contain a different exact conditional distribution for each specified **EXACT** statement. For example, consider the following **EXACT** statements:

```
exact 'O1'    x1    /          outdist=o1;
exact 'OJ12' x1 x2 / jointonly outdist=oj12;
exact 'OA12' x1 x2 / joint     outdist=oa12;
exact 'OE12' x1 x2 / estimate  outdist=oe12;
```

The **O1** statement outputs a single exact conditional distribution. The **OJ12** statement outputs only the joint distribution for  $x_1$  and  $x_2$ . The **OA12** statement outputs three conditional distributions: one for  $x_1$ , one for  $x_2$ , and one jointly for  $x_1$  and  $x_2$ . The **OE12** statement outputs two conditional distributions: one for  $x_1$  and the other for  $x_2$ . Data set **oe12** contains both the  $x_1$  and  $x_2$  variables; the distribution for  $x_1$  has missing values in the  $x_2$  column while the distribution for  $x_2$  has missing values in the  $x_1$  column.

## OUTROC= Output Data Set

The **OUTROC=** data set contains data necessary for producing the ROC curve, and can be created by specifying the **OUTROC=** option in the **MODEL** statement or the **OUTROC=** option in the **SCORE** statement: It has the following variables:

- any **BY** variables specified
- **\_STEP\_**, the model step number. This variable is not included if model selection is not requested.
- **\_PROB\_**, the estimated probability of an event. These estimated probabilities serve as cutpoints for predicting the response. Any observation with an estimated event probability that exceeds or equals **\_PROB\_** is predicted to be an event; otherwise, it is predicted to be a nonevent. Predicted probabilities that are close to each other are grouped together, with the maximum allowable difference between the largest and smallest values less than a constant that is specified by the **ROCEPS=** option. The smallest estimated probability is used to represent the group.
- **\_POS\_**, the number of correctly predicted event responses
- **\_NEG\_**, the number of correctly predicted nonevent responses
- **\_FALPOS\_**, the number of falsely predicted event responses
- **\_FALNEG\_**, the number of falsely predicted nonevent responses
- **\_SENSIT\_**, the sensitivity, which is the proportion of event observations that were predicted to have an event response
- **\_1MSPEC\_**, one minus specificity, which is the proportion of nonevent observations that were predicted to have an event response

Note that none of these statistics are affected by the bias-correction method discussed in the section “[Classification Table](#)” on page 4255. An ROC curve is obtained by plotting `_SENSIT_` against `_1MSPEC_`.

For more information, see the section “[Receiver Operating Characteristic Curves](#)” on page 4260.

## Computational Resources

The memory needed to fit an unconditional model is approximately  $8n(p + 2) + 24(p + 2)^2$  bytes, where  $p$  is the number of parameters estimated and  $n$  is the number of observations in the data set. For cumulative response models with more than two response levels, a test of the parallel lines assumption requires an additional memory of approximately  $4k^2(m + 1)^2 + 24(m + 2)^2$  bytes, where  $k$  is the number of response levels and  $m$  is the number of slope parameters. However, if this additional memory is not available, the procedure skips the test and finishes the other computations. You might need more memory if you use the `SELECTION=` option for model building.

The data that consist of relevant variables (including the design variables for model effects) and observations for fitting the model are stored in a temporary utility file. If sufficient memory is available, such data will also be kept in memory; otherwise, the data are reread from the utility file for each evaluation of the likelihood function and its derivatives, with the resulting execution time of the procedure substantially increased. Specifying the `MULTIPASS` option in the `MODEL` statement avoids creating this utility file and also does not store the data in memory; instead, the `DATA=` data set is reread when needed. This saves approximately  $8n(p + 2)$  bytes of memory but increases the execution time.

If a conditional logistic regression is performed, then approximately  $4(m^2 + m + 4) \max_h(m_h) + (8s_H + 36)H + 12s_H$  additional bytes of memory are needed, where  $m_h$  is the number of events in stratum  $h$ ,  $H$  is the total number of strata, and  $s_H$  is the number of variables used to define the strata. If the `CHECK-DEPENDENCY=ALL` option is specified in the `STRATA` statement, then an extra  $4(m + H)(m + H + 1)$  bytes are required, and the resulting execution time of the procedure might be substantially increased.

## Computational Resources for Exact Logistic Regression

Many problems require a prohibitive amount of time and memory for exact computations, depending on the speed and memory available on your computer. For such problems, consider whether exact methods are really necessary. Stokes, Davis, and Koch (2012) suggest looking at exact  $p$ -values when the sample size is small and the approximate  $p$ -values from the unconditional analysis are less than 0.10, and they provide *rules of thumb* for determining when various models are valid.

A formula does not exist that can predict the amount of time and memory necessary to generate the exact conditional distributions for a particular problem. The time and memory required depends on several factors, including the total sample size, the number of parameters of interest, the number of nuisance parameters, and the order in which the parameters are processed. To provide a feel for how these factors affect performance, 19 data sets containing  $\text{Nobs} \in \{10, \dots, 500\}$  observations consisting of up to 10 independent uniform binary covariates ( $X_1, \dots, X_N$ ) and a binary response variable ( $Y$ ), are generated, and the following statements create exact conditional distributions for  $X_1$  conditional on the other covariates by using the default `METHOD=NETWORK`. [Figure 54.11](#) displays results obtained on a 400Mhz PC with 768MB RAM running Microsoft Windows NT.

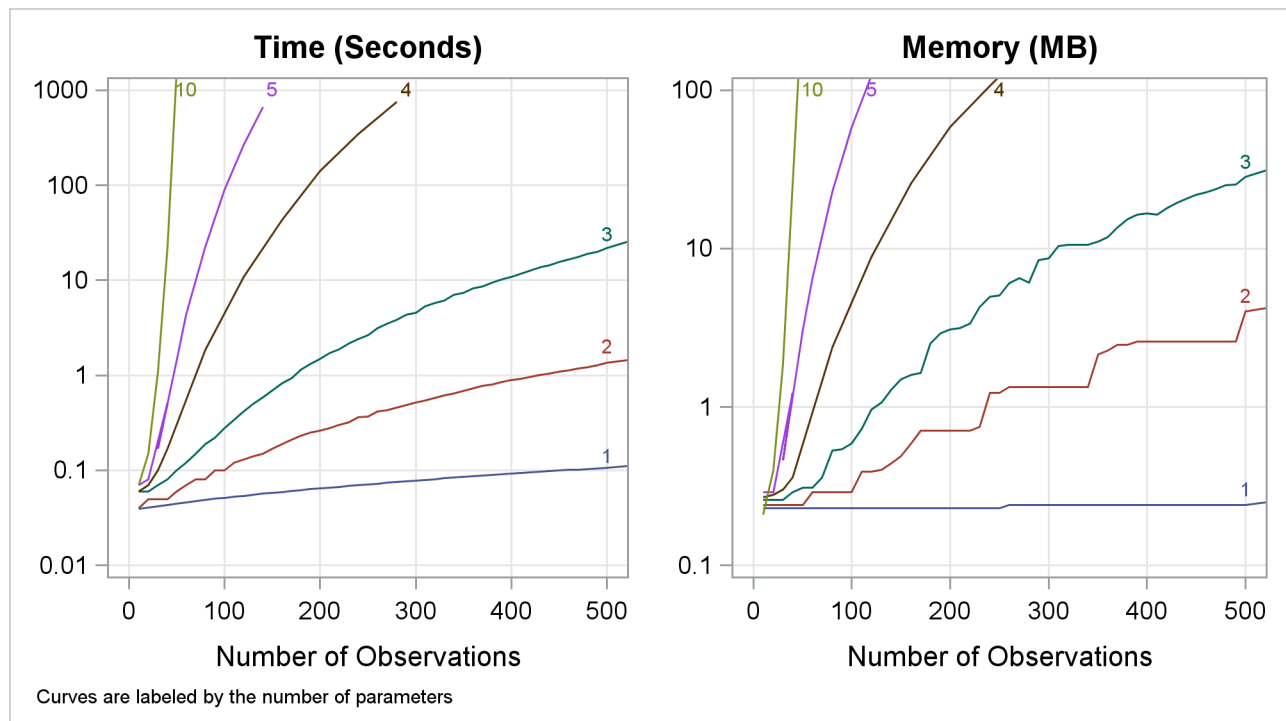
```
data one;
  do obs=1 to HalfNobs;
```

```

do Y=0 to 1;
  X1=round(ranuni(0));
  ...
  XN=round(ranuni(0));
  output;
end;
end;
options fullstimer;
proc logistic exactonly;
  exactoptions method=network maxtime=1200;
  class X1...XN / param=ref;
  model Y=X1...XN;
  exact X1 / outdist=dist;
run;

```

Figure 54.11 Mean Time and Memory Required



At any time while PROC LOGISTIC is deriving the distributions, you can terminate the computations by pressing the system interrupt key sequence (see the SAS Companion for your system) and choosing to stop computations. If you run out of memory, see the SAS Companion for your system to see how to allocate more.

You can use the [EXACTOPTIONS](#) option [MAXTIME=](#) to limit the total amount of time PROC LOGISTIC uses to derive all of the exact distributions. If PROC LOGISTIC does not finish within that time, the procedure terminates.

Calculation of frequencies are performed in the log scale by default. This reduces the need to check for excessively large frequencies but can be slower than not scaling. You can turn off the log scaling by specifying the [NOLOGSCALE](#) option in the [EXACTOPTIONS](#) statement. If a frequency in the exact distribution is larger than the largest integer that can be held in double precision, a warning is printed to the SAS log. But

since inaccuracies due to adding small numbers to these large frequencies might have little or no effect on the statistics, the exact computations continue.

You can monitor the progress of the procedure by submitting your program with the **EXACTOPTIONS** option **STATUSTIME=**. If the procedure is too slow, you can try another method by specifying the **EXACTOPTIONS** option **METHOD=**, you can try reordering the variables in the **MODEL** statement (note that **CLASS** variables are always processed before continuous covariates), or you can try reparameterizing your classification variables as in the following statement:

```
class class-variables / param=ref ref=first order=freq;
```

---

## Displayed Output

If you use the **NOPRINT** option in the **PROC LOGISTIC** statement, the procedure does not display any output. Otherwise, the tables displayed by the LOGISTIC procedure are discussed in the following section in the order in which they appear in the output. Some of the tables appear only in conjunction with certain options or statements; see the section “**ODS Table Names**” on page 4291 for details.

**NOTE:** The **EFFECT**, **ESTIMATE**, **LSMEANS**, **LSMESTIMATE**, and **SLICE** statements also create tables, which are not listed in this section. For information about these tables, see the corresponding sections of Chapter 19, “**Shared Concepts and Topics**.”

## Table Summary

### **Model Information and the Number of Observations**

See the section “**Missing Values**” on page 4237 for information about missing-value handling, and the sections “**FREQ Statement**” on page 4203 and “**WEIGHT Statement**” on page 4236 for information about valid frequencies and weights.

### **Response Profile**

Displays the Ordered Value assigned to each response level. See the section “**Response Level Ordering**” on page 4237 for details.

### **Class Level Information**

Displays the design values for each **CLASS** explanatory variable. See the section “**Other Parameterizations**” on page 387 in Chapter 19, “**Shared Concepts and Topics**,” for details.

### **Simple Statistics Tables**

The following tables are displayed if you specify the **SIMPLE** option in the **PROC LOGISTIC** statement:

- **Descriptive Statistics for Continuous Explanatory Variables**
- **Frequency Distribution of Class Variables**
- **Weight Distribution of Class Variables**  
Displays if you also specify a **WEIGHT** statement.

### **Strata Tables for (Exact) Conditional Logistic Regression**

The following tables are displayed if you specify a **STRATA** statement:

- **Strata Summary**  
Shows the pattern of the number of events and the number of nonevents in a stratum. See the section “**STRATA Statement**” on page 4233 for more information.
- **Strata Information**  
Displays if you specify the **INFO** option in a **STRATA** statement.

### **Maximum Likelihood Iteration History**

Displays if you specify the **ITPRINT** option in the **MODEL** statement. See the sections “**Iterative Algorithms for Model Fitting**” on page 4240, “**Convergence Criteria**” on page 4242, and “**Existence of Maximum Likelihood Estimates**” on page 4242 for details.

### **Deviance and Pearson Goodness-of-Fit Statistics**

Displays if you specify the **SCALE=** option in the **MODEL** statement. Small  $p$ -values reject the null hypothesis that the fitted model is adequate. See the section “**Overdispersion**” on page 4257 for details.

### **Score Test for the Equal Slopes (Proportional Odds) Assumption**

Tests the parallel lines assumption if you fit an ordinal response model with the **LINK=CLOGLOG** or **LINK=PROBIT** options. If you specify **LINK=LOGIT**, this is called the “Proportional Odds” assumption. The table is not displayed if you specify the **UNEQUALSLOPES** option in the **MODEL** statement. Small  $p$ -values reject the null hypothesis that the slope parameters for each explanatory variable are constant across all the response functions. See the section “**Testing the Parallel Lines Assumption**” on page 4248 for details.

### **Model Fit Statistics**

Computes various fit criteria based on a model with intercepts only and a model with intercepts and explanatory variables. If you specify the **NOINT** option in the **MODEL** statement, these statistics are calculated without considering the intercept parameters. See the section “**Model Fitting Information**” on page 4245 for details.

### **Testing Global Null Hypothesis: BETA=0**

Tests the joint effect of the explanatory variables included in the model. Small  $p$ -values reject the null hypothesis that all slope parameters are equal to zero,  $H_0: \beta = \mathbf{0}$ . See the sections “**Model Fitting Information**” on page 4245, “**Residual Chi-Square**” on page 4247, and “**Testing Linear Hypotheses about the Regression Coefficients**” on page 4262 for details. If you also specify the **RSQUARE** option in the **MODEL** statement, two generalized R Square measures are included; see the section “**Generalized Coefficient of Determination**” on page 4246 for details.

### **Score Test for Global Null Hypothesis**

Displays instead of the “Testing Global Null Hypothesis: BETA=0” table if the **NOFIT** option is specified in the **MODEL** statement. The global score test evaluates the joint significance of the effects in the **MODEL** statement. Small  $p$ -values reject the null hypothesis that all slope parameters are equal to zero,  $H_0: \beta = \mathbf{0}$ . See the section “**Residual Chi-Square**” on page 4247 for details.

### Model Selection Tables

The tables in this section are produced when the **SELECTION=** option is specified in the **MODEL** statement. See the section “[Effect-Selection Methods](#)” on page 4244 for more information.

- **Residual Chi-Square Test**

Displays if you specify **SELECTION=FORWARD**, **BACKWARD**, or **STEPWISE** in the **MODEL** statement. Small  $p$ -values reject the null hypothesis that the reduced model is adequate. See the section “[Residual Chi-Square](#)” on page 4247 for details.

- **Analysis of Effects Eligible for Entry**

Displays if you specify the **DETAILS** option and the **SELECTION=FORWARD** or **STEPWISE** option in the **MODEL** statement. Small  $p$ -values reject  $H_0: \beta_i \neq 0$ . The score chi-square is used to determine entry; see the section “[Testing Individual Effects Not in the Model](#)” on page 4247 for details.

- **Analysis of Effects Eligible for Removal**

Displays if you specify the **SELECTION=BACKWARD** or **STEPWISE** option in the **MODEL** statement. Small  $p$ -values reject  $H_0: \beta_i = 0$ . The Wald chi-square is used to determine removal; see the section “[Testing Linear Hypotheses about the Regression Coefficients](#)” on page 4262 for details.

- **Analysis of Effects Removed by Fast Backward Elimination**

Displays if you specify the **FAST** option and the **SELECTION=BACKWARD** or **STEPWISE** option in the **MODEL** statement. This table gives the approximate chi-square statistic for the variable removed, the corresponding  $p$ -value with respect to a chi-square distribution with one degree of freedom, the residual chi-square statistic for testing the joint significance of the variable and the preceding ones, the degrees of freedom, and the  $p$ -value of the residual chi-square with respect to a chi-square distribution with the corresponding degrees of freedom.

- **Summary of Forward, Backward, and Stepwise Selection**

Displays if you specify **SELECTION=FORWARD**, **BACKWARD**, or **STEPWISE** in the **MODEL** statement. The score chi-square is used to determine entry; see the section “[Testing Individual Effects Not in the Model](#)” on page 4247 for details. The Wald chi-square is used to determine removal; see the section “[Testing Linear Hypotheses about the Regression Coefficients](#)” on page 4262 for details.

- **Regression Models Selected by Score Criterion**

Displays the score chi-square for all models if you specify the **SELECTION=SCORE** option in the **MODEL** statement. Small  $p$ -values reject the null hypothesis that the fitted model is adequate. See the section “[Effect-Selection Methods](#)” on page 4244 for details.

### Type 3 Analysis of Effect

Displays if the model contains a **CLASS** variable. Performs Wald chi-square tests of the joint effect of the parameters for each **CLASS** variable in the model. Small  $p$ -values reject  $H_0: \beta_i = 0$ . See the section “[Testing Linear Hypotheses about the Regression Coefficients](#)” on page 4262 for details.

### Analysis of Maximum Likelihood Estimates

**CLASS** effects are identified by their (nonreference) level. For generalized logit models, a response variable column displays the nonreference level of the logit. The table includes the following:

- the estimated standard error of the parameter estimate, computed as the square root of the corresponding diagonal element of the estimated covariance matrix
- the Wald chi-square statistic, computed by squaring the ratio of the parameter estimate divided by its standard error estimate. See the section “[Testing Linear Hypotheses about the Regression Coefficients](#)” on page 4262 for details.
- the  $p$ -value tests the null hypothesis  $H_0: \beta_i = 0$ ; small values reject the null.
- the standardized estimate for the slope parameter, if you specify the **STB** option in the **MODEL** statement. See the **STB** option on page 4220 for details.
- exponentiated values of the estimates of the slope parameters, if you specify the **EXPB** option in the **MODEL** statement. See the **EXPB** option on page 4212 for details.
- the label of the variable, if you specify the **PARMLABEL** option in the **MODEL** statement and if space permits. Due to constraints on the line size, the variable label might be suppressed in order to display the table in one panel. Use the SAS system option **LINESIZE=** to specify a larger line size to accommodate variable labels. A shorter line size can break the table into two panels allowing labels to be displayed.

### ***Odds Ratio Estimates***

Displays the odds ratio estimates and the corresponding 95% Wald confidence intervals for variables that are not involved in nestings or interactions. For continuous explanatory variables, these odds ratios correspond to a unit increase in the risk factors. See the section “[Odds Ratio Estimation](#)” on page 4250 for details.

### ***Association of Predicted Probabilities and Observed Responses***

See the section “[Rank Correlation of Observed Responses and Predicted Probabilities](#)” on page 4253 for details.

### ***Parameter Estimates and Profile-Likelihood or Wald Confidence Intervals***

Displays if you specify the **CLPARM=** option in the **MODEL** statement. See the section “[Confidence Intervals for Parameters](#)” on page 4248 for details.

### ***Odds Ratio Estimates and Profile-Likelihood or Wald Confidence Intervals***

Displays if you specify the **ODDSRATIO** statement for any effects with any class parameterizations. Also displays if you specify the **CLODDS=** option in the **MODEL** statement, except odds ratios are computed only for main effects not involved in interactions or nestings, and if the main effect is a **CLASS** variable, the parameterization must be **EFFECT**, **REFERENCE**, or **GLM**. See the section “[Odds Ratio Estimation](#)” on page 4250 for details.

### ***Estimated Covariance or Correlation Matrix***

Displays if you specify the **COVB** or **CORRB** option in the **MODEL** statement. See the section “[Iterative Algorithms for Model Fitting](#)” on page 4240 for details.



**Contrast Test Results**

Displays the Wald test for each specified **CONTRAST** statement. Small  $p$ -values reject  $H_0: \mathbf{L}\boldsymbol{\beta} = \mathbf{0}$ . The “Coefficients of Contrast” table displays the contrast matrix if you specify the **E** option, and the “Contrast Estimation and Testing Results by Row” table displays estimates and Wald tests for each row of the contrast matrix if you specify the **ESTIMATE=** option. See the sections “**CONTRAST Statement**” on page 4190, “**Testing Linear Hypotheses about the Regression Coefficients**” on page 4262, and “**Linear Predictor, Predicted Probability, and Confidence Limits**” on page 4253 for details.

**Linear Hypotheses Testing Results**

Displays the Wald test for each specified **TEST** statement. See the sections “**Testing Linear Hypotheses about the Regression Coefficients**” on page 4262 and “**TEST Statement**” on page 4234 for details.

**Hosmer and Lemeshow Goodness-of-Fit Test**

Displays if you specify the **LACKFIT** option in the **MODEL** statement. Small  $p$ -values reject the null hypothesis that the fitted model is adequate. The “Partition for the Hosmer and Lemeshow Test” table displays the grouping used in the test. See the section “**The Hosmer-Lemeshow Goodness-of-Fit Test**” on page 4259 for details.

**Classification Table**

Displays if you use the **CTABLE** option in the **MODEL** statement. If you specify a list of cutpoints with the **PPROB=** option, then the cutpoints are displayed in the Prob Level column. If you specify the prior event probabilities with the **PEVENT=** option, then the probabilities are displayed in the Prob Event column. The Correct column displays the number of correctly classified events and nonevents, the Incorrect Event column displays the number of nonevents incorrectly classified as events, and the Incorrect Nonevent column gives the number of nonevents incorrectly classified as events. See the section “**Classification Table**” on page 4255 for more details.

**Regression Diagnostics**

Displays if you specify the **INFLUENCE** option in the **MODEL** statement. See the section “**Regression Diagnostics**” on page 4263 for more information about diagnostics from an unconditional analysis, and the section “**Regression Diagnostic Details**” on page 4272 for information about diagnostics from a conditional analysis.

**Fit Statistics for SCORE Data**

Displays if you specify the **FITSTAT** option in the **SCORE** statement. See the section “**Scoring Data Sets**” on page 4266 for details.

**ROC Association Statistic and Contrast Tables**

Displayed if a **ROC** statement or a **ROCONTRAST** statement is specified. See the section “**ROC Computations**” on page 4261 for details about the Mann-Whitney statistics and the test and estimation computations, and see the section “**Rank Correlation of Observed Responses and Predicted Probabilities**” on page 4253 for details about the other statistics.

**Exact Conditional Logistic Regression Tables**

The tables in this section are produced when the **EXACT** statement is specified. If the **METHOD=NETWORKMC** option is specified, the test and estimate tables are renamed “Monte Carlo” tables and a Monte Carlo standard error column ( $\sqrt{p(1-p)/n}$ ) is displayed.



- **Sufficient Statistics**

Displays if you request an **OUTDIST=** data set in an **EXACT** statement. The table lists the parameters and their observed sufficient statistics.

- **(Monte Carlo) Conditional Exact Tests**

See the section “[Hypothesis Tests](#)” on page 4276 for details.

- **(Monte Carlo) Exact Parameter Estimates**

Displays if you specify the **ESTIMATE** option in the **EXACT** statement. This table gives individual parameter estimates for each variable (conditional on the values of all the other parameters in the model), confidence limits, and a two-sided  $p$ -value (twice the one-sided  $p$ -value) for testing that the parameter is zero. See the section “[Inference for a Single Parameter](#)” on page 4277 for details.

- **(Monte Carlo) Exact Odds Ratios**

Displays if you specify the **ESTIMATE=ODDS** or **ESTIMATE=BOTH** option in the **EXACT** statement. See the section “[Inference for a Single Parameter](#)” on page 4277 for details.

## ODS Table Names

PROC LOGISTIC assigns a name to each table it creates. You can use these names to reference the table when using the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in [Table 54.13](#). For more information about ODS, see Chapter 20, “[Using the Output Delivery System](#).”

The EFFECT, ESTIMATE, LSMEANS, LSMESTIMATE, and SLICE statements also create tables, which are not listed in [Table 54.13](#). For information about these tables, see the corresponding sections of Chapter 19, “[Shared Concepts and Topics](#).”

**Table 54.13** ODS Tables Produced by PROC LOGISTIC

| ODS Table Name | Description                                                      | Statement                 | Option                       |
|----------------|------------------------------------------------------------------|---------------------------|------------------------------|
| Association    | Association of predicted probabilities and observed responses    | MODEL<br>(without STRATA) | Default                      |
| BestSubsets    | Best subset selection                                            | MODEL                     | SELECTION=SCORE              |
| ClassFreq      | Frequency breakdown of CLASS variables                           | PROC                      | Simple<br>(with CLASS vars)  |
| ClassLevelInfo | CLASS variable levels and design variables                       | MODEL                     | Default<br>(with CLASS vars) |
| Classification | Classification table                                             | MODEL                     | CTABLE                       |
| ClassWgt       | Weight breakdown of CLASS variables                              | PROC, WEIGHT              | Simple<br>(with CLASS vars)  |
| CLOddsPL       | Odds ratio estimates and profile-likelihood confidence intervals | MODEL                     | CLODDS=PL                    |
| CLOddsWald     | Odds ratio estimates and Wald confidence intervals               | MODEL                     | CLODDS=WALD                  |

Table 54.13 continued

| ODS Table Name       | Description                                                     | Statement | Option                                       |
|----------------------|-----------------------------------------------------------------|-----------|----------------------------------------------|
| CLParmPL             | Parameter estimates and profile-likelihood confidence intervals | MODEL     | CLPARM=PL                                    |
| CLParmWald           | Parameter estimates and Wald confidence intervals               | MODEL     | CLPARM=WALD                                  |
| ContrastCoeff        | L matrix from CONTRAST                                          | CONTRAST  | E                                            |
| ContrastEstimate     | Estimates from CONTRAST                                         | CONTRAST  | ESTIMATE=                                    |
| ContrastTest         | Wald test for CONTRAST                                          | CONTRAST  | Default                                      |
| ConvergenceStatus    | Convergence status                                              | MODEL     | Default                                      |
| CorrB                | Estimated correlation matrix of parameter estimators            | MODEL     | CORRB                                        |
| CovB                 | Estimated covariance matrix of parameter estimators             | MODEL     | COVB                                         |
| CumulativeModelTest  | Test of the cumulative model assumption                         | MODEL     | (Ordinal response)                           |
| EffectNotInModel     | Test for effects not in model                                   | MODEL     | SELECTION=SIF                                |
| ExactOddsRatio       | Exact odds ratios                                               | EXACT     | ESTIMATE=ODDS,<br>ESTIMATE=BOTH              |
| ExactParmEst         | Parameter estimates                                             | EXACT     | ESTIMATE,<br>ESTIMATE=PARM,<br>ESTIMATE=BOTH |
| ExactTests           | Conditional exact tests                                         | EXACT     | Default                                      |
| FastElimination      | Fast backward elimination                                       | MODEL     | SELECTION=B,FAST                             |
| FitStatistics        | Model fit statistics                                            | MODEL     | Default                                      |
| GlobalScore          | Global score test                                               | MODEL     | NOFIT                                        |
| GlobalTests          | Test for global null hypothesis                                 | MODEL     | Default                                      |
| GoodnessOfFit        | Pearson and deviance goodness-of-fit tests                      | MODEL     | SCALE                                        |
| IndexPlots           | Batch capture of the index plots                                | MODEL     | IPLOTS                                       |
| Influence            | Regression diagnostics                                          | MODEL     | INFLUENCE                                    |
| IterHistory          | Iteration history                                               | MODEL     | ITPRINT                                      |
| LackFitChiSq         | Hosmer-Lemeshow chi-square test results                         | MODEL     | LACKFIT                                      |
| LackFitPartition     | Partition for the Hosmer-Lemeshow test                          | MODEL     | LACKFIT                                      |
| LastGradient         | Last evaluation of gradient                                     | MODEL     | ITPRINT                                      |
| Linear               | Linear combination                                              | PROC      | Default                                      |
| LogLikeChange        | Final change in the log likelihood                              | MODEL     | ITPRINT                                      |
| ModelBuildingSummary | Summary of model building                                       | MODEL     | SELECTION=BIFIS                              |
| ModelInfo            | Model information                                               | PROC      | Default                                      |
| NObs                 | Number of observations                                          | PROC      | Default                                      |

Table 54.13 continued

| ODS Table Name      | Description                                                                                 | Statement   | Option                            |
|---------------------|---------------------------------------------------------------------------------------------|-------------|-----------------------------------|
| OddsEst             | Adjusted odds ratios                                                                        | UNITS       | Default                           |
| OddsRatios          | Odds ratio estimates                                                                        | MODEL       | Default                           |
| OddsRatiosWald      | Odds ratio estimates and Wald confidence intervals                                          | ODDSRATIOS  | CL=WALD                           |
| OddsRatiosPL        | Odds ratio estimates and PL confidence intervals                                            | ODDSRATIOS  | CL=PL                             |
| ParameterEstimates  | Maximum likelihood estimates of model parameters                                            | MODEL       | Default                           |
| RSquare             | R-square                                                                                    | MODEL       | RSQUARE                           |
| ResidualChiSq       | Residual chi-square                                                                         | MODEL       | SELECTION=FIB                     |
| ResponseProfile     | Response profile                                                                            | PROC        | Default                           |
| ROCAssociation      | Association table for ROC models                                                            | ROC         | Default                           |
| ROCContrastCoeff    | L matrix from ROCCONTRAST                                                                   | ROCCONTRAST | E                                 |
| ROCContrastCov      | Covariance of ROCCONTRAST rows                                                              | ROCCONTRAST | COV                               |
| ROCContrastEstimate | Estimates from ROCCONTRAST                                                                  | ROCCONTRAST | ESTIMATE=                         |
| ROCContrastTest     | Wald test from ROCCONTRAST                                                                  | ROCCONTRAST | Default                           |
| ROCCov              | Covariance between ROC curves                                                               | ROCCONTRAST | COV                               |
| ScoreFitStat        | Fit statistics for Scored data                                                              | SCORE       | FITSTAT                           |
| SimpleStatistics    | Summary statistics for explanatory variables                                                | PROC        | SIMPLE                            |
| StrataSummary       | Number of strata with specific response frequencies                                         | STRATA      | Default                           |
| StrataInfo          | Event and nonevent frequencies for each stratum                                             | STRATA      | INFO                              |
| SuffStats           | Sufficient statistics                                                                       | EXACT       | OUTDIST=                          |
| TestPrint1          | $L[Cov(\mathbf{b})]L'$ and $L\mathbf{b}-\mathbf{c}$                                         | TEST        | PRINT                             |
| TestPrint2          | $G_{inv}(L[Cov(\mathbf{b})]L')$ and $G_{inv}(L[Cov(\mathbf{b})]L')(\mathbf{Lb}-\mathbf{c})$ | TEST        | PRINT                             |
| TestStmts           | Linear hypotheses testing results                                                           | TEST        | Default                           |
| Type3               | Type 3 tests of effects                                                                     | MODEL       | Default<br>(with CLASS variables) |

## ODS Graphics

Statistical procedures use ODS Graphics to create graphs as part of their output. ODS Graphics is described in detail in Chapter 21, “[Statistical Graphics Using ODS](#).”

Before you create graphs, ODS Graphics must be enabled (for example, by specifying the ODS GRAPHICS ON statement). For more information about enabling and disabling ODS Graphics, see the section “[Enabling and Disabling ODS Graphics](#)” on page 600 in Chapter 21, “[Statistical Graphics Using ODS](#).”

The overall appearance of graphs is controlled by ODS styles. Styles and other aspects of using ODS Graphics are discussed in the section “[A Primer on ODS Statistical Graphics](#)” on page 599 in Chapter 21, “[Statistical Graphics Using ODS](#).”

You must also specify the options in the PROC LOGISTIC statement that are indicated in [Table 54.14](#).

When ODS Graphics is enabled, then the EFFECT, EFFECTPLOT, ESTIMATE, LSMEANS, LSMESTIMATE, and SLICE statements can produce plots that are associated with their analyses. For information about these plots, see the corresponding sections of Chapter 19, “[Shared Concepts and Topics](#).”

PROC LOGISTIC assigns a name to each graph it creates using ODS. You can use these names to reference the graphs when using ODS. The names are listed in [Table 54.14](#).

**Table 54.14** Graphs Produced by PROC LOGISTIC

| ODS Graph Name                          | Plot Description                                                                                      | Statement or Option                               |
|-----------------------------------------|-------------------------------------------------------------------------------------------------------|---------------------------------------------------|
| <a href="#">DfBetasPlot</a>             | Panel of dfbetas by case number                                                                       | PLOTS=DFBETAS<br>or MODEL / INFLUENCE or IPLOTS   |
| <a href="#">DPCPlot</a>                 | Effect dfbetas by case number<br>Difchisq and/or difdev by predicted probability by CI displacement C | PLOTS=DFBETAS(UNPACK)<br>PLOTS=DPC                |
| <a href="#">EffectPlot</a>              | Predicted probability                                                                                 | PLOTS=EFFECT                                      |
| <a href="#">InfluencePlots</a>          | Panel of influence statistics by case number                                                          | PLOTS=INFLUENCE<br>or MODEL / INFLUENCE or IPLOTS |
| <a href="#">CBarPlot</a>                | CI displacement Cbar by case number                                                                   | PLOTS=INFLUENCE(UNPACK)                           |
| <a href="#">CPlot</a>                   | CI displacement C by case number                                                                      | PLOTS=INFLUENCE(UNPACK)                           |
| <a href="#">DevianceResidualPlot</a>    | Deviance residual by case number                                                                      | PLOTS=INFLUENCE(UNPACK)                           |
| <a href="#">DifChisqPlot</a>            | Difchisq by case number                                                                               | PLOTS=INFLUENCE(UNPACK)                           |
| <a href="#">DifDeviancePlot</a>         | Difdev by case number                                                                                 | PLOTS=INFLUENCE(UNPACK)                           |
| <a href="#">LeveragePlot</a>            | Hat diagonal by case number                                                                           | PLOTS=INFLUENCE(UNPACK)                           |
| <a href="#">LikelihoodResidualPlot</a>  | Likelihood residual by case number                                                                    | PLOTS=INFLUENCE(UNPACK<br>STDRES)                 |
| <a href="#">PearsonResidualPlot</a>     | Pearson chi-square residual by case number                                                            | PLOTS=INFLUENCE(UNPACK)                           |
| <a href="#">StdDevianceResidualPlot</a> | Standardized deviance residual by case number                                                         | PLOTS=INFLUENCE(UNPACK<br>STDRES)                 |
| <a href="#">StdPearsonResidualPlot</a>  | Standardized Pearson chi-square residual by case number                                               | PLOTS=INFLUENCE(UNPACK<br>STDRES)                 |

Table 54.14 continued

| ODS Graph Name       | Plot Description                            | Statement or Option                                                  |
|----------------------|---------------------------------------------|----------------------------------------------------------------------|
| <b>LeveragePlots</b> | Panel of influence statistics by leverage   | PLOTS=LEVERAGE                                                       |
| LeverageCPlot        | CI displacement C by leverage               | PLOTS=LEVERAGE(UNPACK)                                               |
| LeverageDifChisqPlot | Difchisq by leverage                        | PLOTS=LEVERAGE(UNPACK)                                               |
| LeverageDifDevPlot   | Difdev by leverage                          | PLOTS=LEVERAGE(UNPACK)                                               |
| LeveragePhatPlot     | Predicted probability by leverage           | PLOTS=LEVERAGE(UNPACK)                                               |
| <b>ORPlot</b>        | Odds ratios                                 | Default or<br>PLOTS=ODDSRATIO and<br>MODEL / CLODDS=<br>or ODDSRATIO |
| <b>PhatPlots</b>     | Panel of influence by predicted probability | PLOTS=PHAT                                                           |
| PhatCPlot            | CI displacement C by predicted probability  | PLOTS=PHAT(UNPACK)                                                   |
| PhatDifChisqPlot     | Difchisq by predicted probability           | PLOTS=PHAT(UNPACK)                                                   |
| PhatDifDevPlot       | Difdev by predicted probability             | PLOTS=PHAT(UNPACK)                                                   |
| PhatLeveragePlot     | Leverage by predicted probability           | PLOTS=PHAT(UNPACK)                                                   |
| <b>ROCCurve</b>      | Receiver operating characteristics curve    | PLOTS=ROC<br>or MODEL / OUTROC=<br>or SCORE OUTROC=<br>or ROC        |
| <b>ROCOverlay</b>    | ROC curves for comparisons                  | PLOTS=ROC and<br>MODEL / SELECTION=<br>or ROC                        |

## Examples: LOGISTIC Procedure

### Example 54.1: Stepwise Logistic Regression and Predicted Values

Consider a study on cancer remission (Lee 1974). The data consist of patient characteristics and whether or not cancer remission occurred. The following DATA step creates the data set Remission containing seven variables. The variable remiss is the cancer remission indicator variable with a value of 1 for remission and a value of 0 for nonremission. The other six variables are the risk factors thought to be related to cancer remission.

```
data Remission;
  input remiss cell smear infil li blast temp;
  label remiss='Complete Remission';
  datalines;
1   .8   .83   .66   1.9   1.1       .996
1   .9   .36   .32   1.4   .74       .992
```

```

0   .8   .88   .7   .8   .176   .982
0  1     .87   .87   .7  1.053   .986
1   .9   .75   .68  1.3   .519   .98
0  1     .65   .65   .6   .519   .982
1   .95   .97   .92  1     1.23   .992
0   .95   .87   .83  1.9  1.354  1.02
0  1     .45   .45   .8   .322   .999
0   .95   .36   .34   .5  0       1.038
0   .85   .39   .33   .7   .279   .988
0   .7   .76   .53  1.2   .146   .982
0   .8   .46   .37   .4   .38   1.006
0   .2   .39   .08   .8   .114   .99
0  1     .9   .9   1.1  1.037   .99
1  1     .84   .84  1.9  2.064  1.02
0   .65   .42   .27   .5   .114  1.014
0  1     .75   .75  1     1.322  1.004
0   .5   .44   .22   .6   .114   .99
1  1     .63   .63  1.1  1.072   .986
0  1     .33   .33   .4   .176  1.01
0   .9   .93   .84   .6  1.591  1.02
1  1     .58   .58  1     .531  1.002
0   .95   .32   .3   1.6   .886   .988
1  1     .6   .6   1.7   .964   .99
1  1     .69   .69   .9   .398   .986
0  1     .73   .73   .7   .398   .986
;

```

The following invocation of PROC LOGISTIC illustrates the use of [stepwise selection](#) to identify the prognostic factors for cancer remission. A significance level of 0.3 is required to allow a variable into the model ([SLENTRY=0.3](#)), and a significance level of 0.35 is required for a variable to stay in the model ([SLSTAY=0.35](#)). A detailed account of the variable selection process is requested by specifying the [DETAILS](#) option. The Hosmer and Lemeshow goodness-of-fit test for the final selected model is requested by specifying the [LACKFIT](#) option. The [OUTEST=](#) and [COVOUT](#) options in the PROC LOGISTIC statement create a data set that contains parameter estimates and their covariances for the final selected model. The response variable option [EVENT=](#) chooses remiss=1 (remission) as the event so that the probability of remission is modeled. The [OUTPUT](#) statement creates a data set that contains the cumulative predicted probabilities and the corresponding confidence limits, and the individual and cross validated predicted probabilities for each observation.

```

title 'Stepwise Regression on Cancer Remission Data';
proc logistic data=Remission outest=betas covout;
    model remiss(event='1')=cell smear infil li blast temp
        / selection=stepwise
          slentry=0.3
          slstay=0.35
          details
          lackfit;
    output out=pred p=phat lower=lcl upper=ucl
          predprob=(individual crossvalidate);
run;

```

```

proc print data=betas;
  title2 'Parameter Estimates and Covariance Matrix';
run;

proc print data=pred;
  title2 'Predicted Probabilities and 95% Confidence Limits';
run;

```

In stepwise selection, an attempt is made to remove any insignificant variables from the model before adding a significant variable to the model. Each addition or deletion of a variable to or from a model is listed as a separate step in the displayed output, and at each step a new model is fitted. Details of the model selection steps are shown in Outputs 54.1.1 through 54.1.5.

Prior to the first step, the intercept-only model is fit and individual score statistics for the potential variables are evaluated (Output 54.1.1).

#### Output 54.1.1 Startup Model

| Stepwise Regression on Cancer Remission Data  |    |            |                |                 |            |
|-----------------------------------------------|----|------------|----------------|-----------------|------------|
| The LOGISTIC Procedure                        |    |            |                |                 |            |
| Step 0. Intercept entered:                    |    |            |                |                 |            |
| Model Convergence Status                      |    |            |                |                 |            |
| Convergence criterion (GCONV=1E-8) satisfied. |    |            |                |                 |            |
| -2 Log L = 34.372                             |    |            |                |                 |            |
| Analysis of Maximum Likelihood Estimates      |    |            |                |                 |            |
| Parameter                                     | DF | Estimate   | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                     | 1  | -0.6931    | 0.4082         | 2.8827          | 0.0895     |
| Residual Chi-Square Test                      |    |            |                |                 |            |
|                                               |    | Chi-Square | DF             | Pr > ChiSq      |            |
|                                               |    | 9.4609     | 6              | 0.1493          |            |

**Output 54.1.1** *continued*

| Analysis of Effects Eligible for Entry |    |            |            |
|----------------------------------------|----|------------|------------|
| Effect                                 | DF | Score      |            |
|                                        |    | Chi-Square | Pr > ChiSq |
| cell                                   | 1  | 1.8893     | 0.1693     |
| smear                                  | 1  | 1.0745     | 0.2999     |
| infil                                  | 1  | 1.8817     | 0.1701     |
| li                                     | 1  | 7.9311     | 0.0049     |
| blast                                  | 1  | 3.5258     | 0.0604     |
| temp                                   | 1  | 0.6591     | 0.4169     |

In Step 1 (Output 54.1.2), the variable *li* is selected into the model since it is the most significant variable among those to be chosen ( $p = 0.0049 < 0.3$ ). The intermediate model that contains an intercept and *li* is then fitted. *li* remains significant ( $p = 0.0146 < 0.35$ ) and is not removed.

**Output 54.1.2** Step 1 of the Stepwise Analysis

```

Step 1. Effect li entered:

Model Convergence Status

Convergence criterion (GCONV=1E-8) satisfied.

Model Fit Statistics

Criterion              Intercept          Intercept
                        Only              and
                        Only              Covariates

AIC                    36.372          30.073
SC                     37.668          32.665
-2 Log L               34.372          26.073

Testing Global Null Hypothesis: BETA=0

Test                   Chi-Square        DF        Pr > ChiSq

Likelihood Ratio      8.2988           1          0.0040
Score                 7.9311           1          0.0049
Wald                  5.9594           1          0.0146

Analysis of Maximum Likelihood Estimates

Parameter    DF      Estimate      Standard      Wald
              DF      Estimate      Error        Chi-Square    Pr > ChiSq

Intercept    1      -3.7771      1.3786        7.5064        0.0061
li           1       2.8973      1.1868        5.9594        0.0146

```



**Output 54.1.2** *continued*

| Odds Ratio Estimates |                |                            |         |
|----------------------|----------------|----------------------------|---------|
| Effect               | Point Estimate | 95% Wald Confidence Limits |         |
| li                   | 18.124         | 1.770                      | 185.563 |

Association of Predicted Probabilities and Observed Responses

|                    |      |           |       |
|--------------------|------|-----------|-------|
| Percent Concordant | 84.0 | Somers' D | 0.710 |
| Percent Discordant | 13.0 | Gamma     | 0.732 |
| Percent Tied       | 3.1  | Tau-a     | 0.328 |
| Pairs              | 162  | c         | 0.855 |

Residual Chi-Square Test

|            |    |            |
|------------|----|------------|
| Chi-Square | DF | Pr > ChiSq |
| 3.1174     | 5  | 0.6819     |

Analysis of Effects Eligible for Removal

| Effect | DF | Wald Chi-Square | Pr > ChiSq |
|--------|----|-----------------|------------|
| li     | 1  | 5.9594          | 0.0146     |

NOTE: No effects for the model in Step 1 are removed.

Analysis of Effects Eligible for Entry

| Effect | DF | Score Chi-Square | Pr > ChiSq |
|--------|----|------------------|------------|
| cell   | 1  | 1.1183           | 0.2903     |
| smear  | 1  | 0.1369           | 0.7114     |
| infil  | 1  | 0.5715           | 0.4497     |
| blast  | 1  | 0.0932           | 0.7601     |
| temp   | 1  | 1.2591           | 0.2618     |

In Step 2 ([Output 54.1.3](#)), the variable temp is added to the model. The model then contains an intercept and the variables li and temp. Both li and temp remain significant at 0.35 level; therefore, neither li nor temp is removed from the model.

**Output 54.1.3** Step 2 of the Stepwise Analysis

Step 2. Effect temp entered:

Model Convergence Status

Convergence criterion (GCONV=1E-8) satisfied.

Model Fit Statistics

| Criterion | Intercept<br>Only | Intercept<br>and<br>Covariates |
|-----------|-------------------|--------------------------------|
| AIC       | 36.372            | 30.648                         |
| SC        | 37.668            | 34.535                         |
| -2 Log L  | 34.372            | 24.648                         |

Testing Global Null Hypothesis: BETA=0

| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 9.7239     | 2  | 0.0077     |
| Score            | 8.3648     | 2  | 0.0153     |
| Wald             | 5.9052     | 2  | 0.0522     |

Analysis of Maximum Likelihood Estimates

| Parameter | DF | Estimate | Standard<br>Error | Wald<br>Chi-Square | Pr > ChiSq |
|-----------|----|----------|-------------------|--------------------|------------|
| Intercept | 1  | 47.8448  | 46.4381           | 1.0615             | 0.3029     |
| li        | 1  | 3.3017   | 1.3593            | 5.9002             | 0.0151     |
| temp      | 1  | -52.4214 | 47.4897           | 1.2185             | 0.2697     |

Odds Ratio Estimates

| Effect | Point<br>Estimate | 95% Wald<br>Confidence Limits |          |
|--------|-------------------|-------------------------------|----------|
| li     | 27.158            | 1.892                         | 389.856  |
| temp   | <0.001            | <0.001                        | >999.999 |

Association of Predicted Probabilities and Observed Responses

|                    |      |           |       |
|--------------------|------|-----------|-------|
| Percent Concordant | 87.0 | Somers' D | 0.747 |
| Percent Discordant | 12.3 | Gamma     | 0.752 |
| Percent Tied       | 0.6  | Tau-a     | 0.345 |
| Pairs              | 162  | c         | 0.873 |

**Output 54.1.3** *continued*

| Residual Chi-Square Test                 |    |                    |            |
|------------------------------------------|----|--------------------|------------|
| Chi-Square                               | DF | Pr > ChiSq         |            |
| 2.1429                                   | 4  | 0.7095             |            |
| Analysis of Effects Eligible for Removal |    |                    |            |
| Effect                                   | DF | Wald<br>Chi-Square | Pr > ChiSq |
| li                                       | 1  | 5.9002             | 0.0151     |
| temp                                     | 1  | 1.2185             | 0.2697     |

NOTE: No effects for the model in Step 2 are removed.

| Analysis of Effects Eligible for Entry |    |                     |            |
|----------------------------------------|----|---------------------|------------|
| Effect                                 | DF | Score<br>Chi-Square | Pr > ChiSq |
| cell                                   | 1  | 1.4700              | 0.2254     |
| smear                                  | 1  | 0.1730              | 0.6775     |
| infil                                  | 1  | 0.8274              | 0.3630     |
| blast                                  | 1  | 1.1013              | 0.2940     |

In Step 3 ([Output 54.1.4](#)), the variable cell is added to the model. The model then contains an intercept and the variables li, temp, and cell. None of these variables are removed from the model since all are significant at the 0.35 level.

**Output 54.1.4** Step 3 of the Stepwise Analysis

| Step 3. Effect cell entered:                  |                   |                                |
|-----------------------------------------------|-------------------|--------------------------------|
| Model Convergence Status                      |                   |                                |
| Convergence criterion (GCONV=1E-8) satisfied. |                   |                                |
| Model Fit Statistics                          |                   |                                |
| Criterion                                     | Intercept<br>Only | Intercept<br>and<br>Covariates |
| AIC                                           | 36.372            | 29.953                         |
| SC                                            | 37.668            | 35.137                         |
| -2 Log L                                      | 34.372            | 21.953                         |

## Output 54.1.4 continued

## Testing Global Null Hypothesis: BETA=0

| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 12.4184    | 3  | 0.0061     |
| Score            | 9.2502     | 3  | 0.0261     |
| Wald             | 4.8281     | 3  | 0.1848     |

## Analysis of Maximum Likelihood Estimates

| Parameter | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
|-----------|----|----------|----------------|-----------------|------------|
| Intercept | 1  | 67.6339  | 56.8875        | 1.4135          | 0.2345     |
| cell      | 1  | 9.6521   | 7.7511         | 1.5507          | 0.2130     |
| li        | 1  | 3.8671   | 1.7783         | 4.7290          | 0.0297     |
| temp      | 1  | -82.0737 | 61.7124        | 1.7687          | 0.1835     |

## Odds Ratio Estimates

| Effect | Point Estimate | 95% Wald Confidence Limits |
|--------|----------------|----------------------------|
| cell   | >999.999       | 0.004 >999.999             |
| li     | 47.804         | 1.465 >999.999             |
| temp   | <0.001         | <0.001 >999.999            |

## Association of Predicted Probabilities and Observed Responses

|                    |      |           |       |
|--------------------|------|-----------|-------|
| Percent Concordant | 88.9 | Somers' D | 0.778 |
| Percent Discordant | 11.1 | Gamma     | 0.778 |
| Percent Tied       | 0.0  | Tau-a     | 0.359 |
| Pairs              | 162  | c         | 0.889 |

## Residual Chi-Square Test

| Chi-Square | DF | Pr > ChiSq |
|------------|----|------------|
| 0.1831     | 3  | 0.9803     |

## Analysis of Effects Eligible for Removal

| Effect | DF | Wald Chi-Square | Pr > ChiSq |
|--------|----|-----------------|------------|
| cell   | 1  | 1.5507          | 0.2130     |
| li     | 1  | 4.7290          | 0.0297     |
| temp   | 1  | 1.7687          | 0.1835     |

**Output 54.1.4** *continued*

NOTE: No effects for the model in Step 3 are removed.

Analysis of Effects Eligible for Entry

| Effect | DF | Score      |            |
|--------|----|------------|------------|
|        |    | Chi-Square | Pr > ChiSq |
| smear  | 1  | 0.0956     | 0.7572     |
| infil  | 1  | 0.0844     | 0.7714     |
| blast  | 1  | 0.0208     | 0.8852     |

Finally, none of the remaining variables outside the model meet the entry criterion, and the stepwise selection is terminated. A summary of the stepwise selection is displayed in [Output 54.1.5](#).

**Output 54.1.5** Summary of the Stepwise Selection

Summary of Stepwise Selection

| Step | Effect  |         | DF | Number<br>In | Score      |            | Wald |        |
|------|---------|---------|----|--------------|------------|------------|------|--------|
|      | Entered | Removed |    |              | Chi-Square | Chi-Square | Pr > | ChiSq  |
| 1    | li      |         | 1  | 1            | 7.9311     |            |      | 0.0049 |
| 2    | temp    |         | 1  | 2            | 1.2591     |            |      | 0.2618 |
| 3    | cell    |         | 1  | 3            | 1.4700     |            |      | 0.2254 |

Results of the Hosmer and Lemeshow test are shown in [Output 54.1.6](#). There is no evidence of a lack of fit in the selected model ( $p = 0.5054$ ).

**Output 54.1.6** Display of the LACKFIT Option

Partition for the Hosmer and Lemeshow Test

| Group | Total | remiss = 1 |          | remiss = 0 |          |
|-------|-------|------------|----------|------------|----------|
|       |       | Observed   | Expected | Observed   | Expected |
| 1     | 3     | 0          | 0.00     | 3          | 3.00     |
| 2     | 3     | 0          | 0.01     | 3          | 2.99     |
| 3     | 3     | 0          | 0.19     | 3          | 2.81     |
| 4     | 3     | 0          | 0.56     | 3          | 2.44     |
| 5     | 4     | 1          | 1.09     | 3          | 2.91     |
| 6     | 3     | 2          | 1.35     | 1          | 1.65     |
| 7     | 3     | 2          | 1.84     | 1          | 1.16     |
| 8     | 3     | 3          | 2.15     | 0          | 0.85     |
| 9     | 2     | 1          | 1.80     | 1          | 0.20     |

**Output 54.1.6** *continued*

| Hosmer and Lemeshow Goodness-of-Fit Test |    |            |
|------------------------------------------|----|------------|
| Chi-Square                               | DF | Pr > ChiSq |
| 6.2983                                   | 7  | 0.5054     |

The data set betas created by the **OUTEST=** and **COVOUT** options is displayed in [Output 54.1.7](#). The data set contains parameter estimates and the covariance matrix for the final selected model. Note that all explanatory variables listed in the **MODEL** statement are included in this data set; however, variables that are not included in the final model have all missing values.

**Output 54.1.7** Data Set of Estimates and Covariances

| Stepwise Regression on Cancer Remission Data<br>Parameter Estimates and Covariance Matrix |        |        |             |           |           |          |           |
|-------------------------------------------------------------------------------------------|--------|--------|-------------|-----------|-----------|----------|-----------|
| Obs                                                                                       | _LINK_ | _TYPE_ | _STATUS_    | _NAME_    | Intercept | cell     |           |
| 1                                                                                         | LOGIT  | PARMS  | 0 Converged | remiss    | 67.63     | 9.652    |           |
| 2                                                                                         | LOGIT  | COV    | 0 Converged | Intercept | 3236.19   | 157.097  |           |
| 3                                                                                         | LOGIT  | COV    | 0 Converged | cell      | 157.10    | 60.079   |           |
| 4                                                                                         | LOGIT  | COV    | 0 Converged | smear     | .         | .        |           |
| 5                                                                                         | LOGIT  | COV    | 0 Converged | infil     | .         | .        |           |
| 6                                                                                         | LOGIT  | COV    | 0 Converged | li        | 64.57     | 6.945    |           |
| 7                                                                                         | LOGIT  | COV    | 0 Converged | blast     | .         | .        |           |
| 8                                                                                         | LOGIT  | COV    | 0 Converged | temp      | -3483.23  | -223.669 |           |
| Obs                                                                                       | smear  | infil  | li          | blast     | temp      | _LNLIKE_ | _ESTTYPE_ |
| 1                                                                                         | .      | .      | 3.8671      | .         | -82.07    | -10.9767 | MLE       |
| 2                                                                                         | .      | .      | 64.5726     | .         | -3483.23  | -10.9767 | MLE       |
| 3                                                                                         | .      | .      | 6.9454      | .         | -223.67   | -10.9767 | MLE       |
| 4                                                                                         | .      | .      | .           | .         | .         | -10.9767 | MLE       |
| 5                                                                                         | .      | .      | .           | .         | .         | -10.9767 | MLE       |
| 6                                                                                         | .      | .      | 3.1623      | .         | -75.35    | -10.9767 | MLE       |
| 7                                                                                         | .      | .      | .           | .         | .         | -10.9767 | MLE       |
| 8                                                                                         | .      | .      | -75.3513    | .         | 3808.42   | -10.9767 | MLE       |

The data set `pred` created by the `OUTPUT` statement is displayed in [Output 54.1.8](#). It contains all the variables in the input data set, the variable `phat` for the (cumulative) predicted probability, the variables `lcl` and `ucl` for the lower and upper confidence limits for the probability, and four other variables (`IP_1`, `IP_0`, `XP_1`, and `XP_0`) for the `PREDPROBS=` option. The data set also contains the variable `_LEVEL_`, indicating the response value to which `phat`, `lcl`, and `ucl` refer. For instance, for the first row of the `OUTPUT` data set, the values of `_LEVEL_` and `phat`, `lcl`, and `ucl` are 1, 0.72265, 0.16892, and 0.97093, respectively; this means that the estimated probability that `remiss=1` is 0.723 for the given explanatory variable values, and the corresponding 95% confidence interval is (0.16892, 0.97093). The variables `IP_1` and `IP_0` contain the predicted probabilities that `remiss=1` and `remiss=0`, respectively. Note that values of `phat` and `IP_1` are identical since they both contain the probabilities that `remiss=1`. The variables `XP_1` and `XP_0` contain the cross validated predicted probabilities that `remiss=1` and `remiss=0`, respectively.

**Output 54.1.8** Predicted Probabilities and Confidence Intervals

| Stepwise Regression on Cancer Remission Data<br>Predicted Probabilities and 95% Confidence Limits |         |         |         |         |         |         |         |        |        |         |
|---------------------------------------------------------------------------------------------------|---------|---------|---------|---------|---------|---------|---------|--------|--------|---------|
| Obs                                                                                               | remiss  | cell    | smear   | infil   | li      | blast   | temp    | _FROM_ | _INTO_ | IP_0    |
| 1                                                                                                 | 1       | 0.80    | 0.83    | 0.66    | 1.9     | 1.100   | 0.996   | 1      | 1      | 0.27735 |
| 2                                                                                                 | 1       | 0.90    | 0.36    | 0.32    | 1.4     | 0.740   | 0.992   | 1      | 1      | 0.42126 |
| 3                                                                                                 | 0       | 0.80    | 0.88    | 0.70    | 0.8     | 0.176   | 0.982   | 0      | 0      | 0.89540 |
| 4                                                                                                 | 0       | 1.00    | 0.87    | 0.87    | 0.7     | 1.053   | 0.986   | 0      | 0      | 0.71742 |
| 5                                                                                                 | 1       | 0.90    | 0.75    | 0.68    | 1.3     | 0.519   | 0.980   | 1      | 1      | 0.28582 |
| 6                                                                                                 | 0       | 1.00    | 0.65    | 0.65    | 0.6     | 0.519   | 0.982   | 0      | 0      | 0.72911 |
| 7                                                                                                 | 1       | 0.95    | 0.97    | 0.92    | 1.0     | 1.230   | 0.992   | 1      | 0      | 0.67844 |
| 8                                                                                                 | 0       | 0.95    | 0.87    | 0.83    | 1.9     | 1.354   | 1.020   | 0      | 1      | 0.39277 |
| 9                                                                                                 | 0       | 1.00    | 0.45    | 0.45    | 0.8     | 0.322   | 0.999   | 0      | 0      | 0.83368 |
| 10                                                                                                | 0       | 0.95    | 0.36    | 0.34    | 0.5     | 0.000   | 1.038   | 0      | 0      | 0.99843 |
| 11                                                                                                | 0       | 0.85    | 0.39    | 0.33    | 0.7     | 0.279   | 0.988   | 0      | 0      | 0.92715 |
| 12                                                                                                | 0       | 0.70    | 0.76    | 0.53    | 1.2     | 0.146   | 0.982   | 0      | 0      | 0.82714 |
| Obs                                                                                               | IP_1    | XP_0    | XP_1    | _LEVEL_ | phat    | lcl     | ucl     |        |        |         |
| 1                                                                                                 | 0.72265 | 0.43873 | 0.56127 | 1       | 0.72265 | 0.16892 | 0.97093 |        |        |         |
| 2                                                                                                 | 0.57874 | 0.47461 | 0.52539 | 1       | 0.57874 | 0.26788 | 0.83762 |        |        |         |
| 3                                                                                                 | 0.10460 | 0.87060 | 0.12940 | 1       | 0.10460 | 0.00781 | 0.63419 |        |        |         |
| 4                                                                                                 | 0.28258 | 0.67259 | 0.32741 | 1       | 0.28258 | 0.07498 | 0.65683 |        |        |         |
| 5                                                                                                 | 0.71418 | 0.36901 | 0.63099 | 1       | 0.71418 | 0.25218 | 0.94876 |        |        |         |
| 6                                                                                                 | 0.27089 | 0.67269 | 0.32731 | 1       | 0.27089 | 0.05852 | 0.68951 |        |        |         |
| 7                                                                                                 | 0.32156 | 0.72923 | 0.27077 | 1       | 0.32156 | 0.13255 | 0.59516 |        |        |         |
| 8                                                                                                 | 0.60723 | 0.09906 | 0.90094 | 1       | 0.60723 | 0.10572 | 0.95287 |        |        |         |
| 9                                                                                                 | 0.16632 | 0.80864 | 0.19136 | 1       | 0.16632 | 0.03018 | 0.56123 |        |        |         |
| 10                                                                                                | 0.00157 | 0.99840 | 0.00160 | 1       | 0.00157 | 0.00000 | 0.68962 |        |        |         |
| 11                                                                                                | 0.07285 | 0.91723 | 0.08277 | 1       | 0.07285 | 0.00614 | 0.49982 |        |        |         |
| 12                                                                                                | 0.17286 | 0.63838 | 0.36162 | 1       | 0.17286 | 0.00637 | 0.87206 |        |        |         |

Output 54.1.8 continued

| Stepwise Regression on Cancer Remission Data<br>Predicted Probabilities and 95% Confidence Limits |         |         |         |         |         |         |         |        |        |         |
|---------------------------------------------------------------------------------------------------|---------|---------|---------|---------|---------|---------|---------|--------|--------|---------|
| Obs                                                                                               | remiss  | cell    | smear   | infil   | li      | blast   | temp    | _FROM_ | _INTO_ | IP_0    |
| 13                                                                                                | 0       | 0.80    | 0.46    | 0.37    | 0.4     | 0.380   | 1.006   | 0      | 0      | 0.99654 |
| 14                                                                                                | 0       | 0.20    | 0.39    | 0.08    | 0.8     | 0.114   | 0.990   | 0      | 0      | 0.99982 |
| 15                                                                                                | 0       | 1.00    | 0.90    | 0.90    | 1.1     | 1.037   | 0.990   | 0      | 1      | 0.42878 |
| 16                                                                                                | 1       | 1.00    | 0.84    | 0.84    | 1.9     | 2.064   | 1.020   | 1      | 1      | 0.28530 |
| 17                                                                                                | 0       | 0.65    | 0.42    | 0.27    | 0.5     | 0.114   | 1.014   | 0      | 0      | 0.99938 |
| 18                                                                                                | 0       | 1.00    | 0.75    | 0.75    | 1.0     | 1.322   | 1.004   | 0      | 0      | 0.77711 |
| 19                                                                                                | 0       | 0.50    | 0.44    | 0.22    | 0.6     | 0.114   | 0.990   | 0      | 0      | 0.99846 |
| 20                                                                                                | 1       | 1.00    | 0.63    | 0.63    | 1.1     | 1.072   | 0.986   | 1      | 1      | 0.35089 |
| 21                                                                                                | 0       | 1.00    | 0.33    | 0.33    | 0.4     | 0.176   | 1.010   | 0      | 0      | 0.98307 |
| 22                                                                                                | 0       | 0.90    | 0.93    | 0.84    | 0.6     | 1.591   | 1.020   | 0      | 0      | 0.99378 |
| 23                                                                                                | 1       | 1.00    | 0.58    | 0.58    | 1.0     | 0.531   | 1.002   | 1      | 0      | 0.74739 |
| 24                                                                                                | 0       | 0.95    | 0.32    | 0.30    | 1.6     | 0.886   | 0.988   | 0      | 1      | 0.12989 |
| Obs                                                                                               | IP_1    | XP_0    | XP_1    | _LEVEL_ | phat    | lcl     | ucl     |        |        |         |
| 13                                                                                                | 0.00346 | 0.99644 | 0.00356 | 1       | 0.00346 | 0.00001 | 0.46530 |        |        |         |
| 14                                                                                                | 0.00018 | 0.99981 | 0.00019 | 1       | 0.00018 | 0.00000 | 0.96482 |        |        |         |
| 15                                                                                                | 0.57122 | 0.35354 | 0.64646 | 1       | 0.57122 | 0.25303 | 0.83973 |        |        |         |
| 16                                                                                                | 0.71470 | 0.47213 | 0.52787 | 1       | 0.71470 | 0.15362 | 0.97189 |        |        |         |
| 17                                                                                                | 0.00062 | 0.99937 | 0.00063 | 1       | 0.00062 | 0.00000 | 0.62665 |        |        |         |
| 18                                                                                                | 0.22289 | 0.73612 | 0.26388 | 1       | 0.22289 | 0.04483 | 0.63670 |        |        |         |
| 19                                                                                                | 0.00154 | 0.99842 | 0.00158 | 1       | 0.00154 | 0.00000 | 0.79644 |        |        |         |
| 20                                                                                                | 0.64911 | 0.42053 | 0.57947 | 1       | 0.64911 | 0.26305 | 0.90555 |        |        |         |
| 21                                                                                                | 0.01693 | 0.98170 | 0.01830 | 1       | 0.01693 | 0.00029 | 0.50475 |        |        |         |
| 22                                                                                                | 0.00622 | 0.99348 | 0.00652 | 1       | 0.00622 | 0.00003 | 0.56062 |        |        |         |
| 23                                                                                                | 0.25261 | 0.84423 | 0.15577 | 1       | 0.25261 | 0.06137 | 0.63597 |        |        |         |
| 24                                                                                                | 0.87011 | 0.03637 | 0.96363 | 1       | 0.87011 | 0.40910 | 0.98481 |        |        |         |
| Stepwise Regression on Cancer Remission Data<br>Predicted Probabilities and 95% Confidence Limits |         |         |         |         |         |         |         |        |        |         |
| Obs                                                                                               | remiss  | cell    | smear   | infil   | li      | blast   | temp    | _FROM_ | _INTO_ | IP_0    |
| 25                                                                                                | 1       | 1.00    | 0.60    | 0.60    | 1.7     | 0.964   | 0.990   | 1      | 1      | 0.06868 |
| 26                                                                                                | 1       | 1.00    | 0.69    | 0.69    | 0.9     | 0.398   | 0.986   | 1      | 0      | 0.53949 |
| 27                                                                                                | 0       | 1.00    | 0.73    | 0.73    | 0.7     | 0.398   | 0.986   | 0      | 0      | 0.71742 |
| Obs                                                                                               | IP_1    | XP_0    | XP_1    | _LEVEL_ | phat    | lcl     | ucl     |        |        |         |
| 25                                                                                                | 0.93132 | 0.08017 | 0.91983 | 1       | 0.93132 | 0.44114 | 0.99573 |        |        |         |
| 26                                                                                                | 0.46051 | 0.62312 | 0.37688 | 1       | 0.46051 | 0.16612 | 0.78529 |        |        |         |
| 27                                                                                                | 0.28258 | 0.67259 | 0.32741 | 1       | 0.28258 | 0.07498 | 0.65683 |        |        |         |

Next, a different variable selection method is used to select prognostic factors for cancer remission, and an efficient algorithm is employed to eliminate insignificant variables from a model. The following statements invoke PROC LOGISTIC to perform the backward elimination analysis:



```

title 'Backward Elimination on Cancer Remission Data';
proc logistic data=Remission;
    model remiss(event='1')=temp cell li smear blast
        / selection=backward fast slstay=0.2 ctable;
run;

```

The backward elimination analysis (**SELECTION=BACKWARD**) starts with a model that contains all explanatory variables given in the **MODEL** statement. By specifying the **FAST** option, **PROC LOGISTIC** eliminates insignificant variables without refitting the model repeatedly. This analysis uses a significance level of 0.2 to retain variables in the model (**SLSTAY=0.2**), which is different from the previous stepwise analysis where **SLSTAY=.35**. The **CTABLE** option is specified to produce classifications of input observations based on the final selected model.

Results of the fast elimination analysis are shown in [Output 54.1.9](#) and [Output 54.1.10](#). Initially, a full model containing all six risk factors is fit to the data ([Output 54.1.9](#)). In the next step ([Output 54.1.10](#)), **PROC LOGISTIC** removes **blast**, **smear**, **cell**, and **temp** from the model all at once. This leaves **li** and the intercept as the only variables in the final model. Note that in this analysis, only parameter estimates for the final model are displayed because the **DETAILS** option has not been specified.

**Output 54.1.9** Initial Step in Backward Elimination

| Backward Elimination on Cancer Remission Data |                  |                    |
|-----------------------------------------------|------------------|--------------------|
| The LOGISTIC Procedure                        |                  |                    |
| Model Information                             |                  |                    |
| Data Set                                      | WORK.REMISSION   |                    |
| Response Variable                             | remiss           | Complete Remission |
| Number of Response Levels                     | 2                |                    |
| Model                                         | binary logit     |                    |
| Optimization Technique                        | Fisher's scoring |                    |
| Number of Observations Read                   |                  | 27                 |
| Number of Observations Used                   |                  | 27                 |
| Response Profile                              |                  |                    |
| Ordered Value                                 | remiss           | Total Frequency    |
| 1                                             | 0                | 18                 |
| 2                                             | 1                | 9                  |
| Probability modeled is remiss=1.              |                  |                    |

**Output 54.1.9** *continued***Backward Elimination Procedure**

Step 0. The following effects were entered:

Intercept temp cell li smear blast

**Model Convergence Status**

Convergence criterion (GCONV=1E-8) satisfied.

**Model Fit Statistics**

| Criterion | Intercept<br>Only | Intercept<br>and<br>Covariates |
|-----------|-------------------|--------------------------------|
| AIC       | 36.372            | 33.857                         |
| SC        | 37.668            | 41.632                         |
| -2 Log L  | 34.372            | 21.857                         |

**Testing Global Null Hypothesis: BETA=0**

| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 12.5146    | 5  | 0.0284     |
| Score            | 9.3295     | 5  | 0.0966     |
| Wald             | 4.7284     | 5  | 0.4499     |

**Output 54.1.10** Fast Elimination Step

Step 1. Fast Backward Elimination:

**Analysis of Effects Removed by Fast Backward Elimination**

| Effect<br>Removed | Chi-Square | DF | Pr > ChiSq | Residual<br>Chi-Square | DF | Pr ><br>Residual<br>ChiSq |
|-------------------|------------|----|------------|------------------------|----|---------------------------|
| blast             | 0.0008     | 1  | 0.9768     | 0.0008                 | 1  | 0.9768                    |
| smear             | 0.0951     | 1  | 0.7578     | 0.0959                 | 2  | 0.9532                    |
| cell              | 1.5134     | 1  | 0.2186     | 1.6094                 | 3  | 0.6573                    |
| temp              | 0.6535     | 1  | 0.4189     | 2.2628                 | 4  | 0.6875                    |

**Model Convergence Status**

Convergence criterion (GCONV=1E-8) satisfied.

## Output 54.1.10 continued

## Model Fit Statistics

| Criterion | Intercept<br>Only | Intercept<br>and<br>Covariates |
|-----------|-------------------|--------------------------------|
| AIC       | 36.372            | 30.073                         |
| SC        | 37.668            | 32.665                         |
| -2 Log L  | 34.372            | 26.073                         |

## Testing Global Null Hypothesis: BETA=0

| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 8.2988     | 1  | 0.0040     |
| Score            | 7.9311     | 1  | 0.0049     |
| Wald             | 5.9594     | 1  | 0.0146     |

## Residual Chi-Square Test

| Chi-Square | DF | Pr > ChiSq |
|------------|----|------------|
| 2.8530     | 4  | 0.5827     |

## Summary of Backward Elimination

| Step | Effect<br>Removed | DF | Number<br>In | Wald<br>Chi-Square | Pr > ChiSq |
|------|-------------------|----|--------------|--------------------|------------|
| 1    | blast             | 1  | 4            | 0.0008             | 0.9768     |
| 1    | smear             | 1  | 3            | 0.0951             | 0.7578     |
| 1    | cell              | 1  | 2            | 1.5134             | 0.2186     |
| 1    | temp              | 1  | 1            | 0.6535             | 0.4189     |

## Analysis of Maximum Likelihood Estimates

| Parameter | DF | Estimate | Standard<br>Error | Wald<br>Chi-Square | Pr > ChiSq |
|-----------|----|----------|-------------------|--------------------|------------|
| Intercept | 1  | -3.7771  | 1.3786            | 7.5064             | 0.0061     |
| li        | 1  | 2.8973   | 1.1868            | 5.9594             | 0.0146     |

## Odds Ratio Estimates

| Effect | Point<br>Estimate | 95% Wald<br>Confidence Limits |
|--------|-------------------|-------------------------------|
| li     | 18.124            | 1.770 185.563                 |

**Output 54.1.10** *continued*

| Association of Predicted Probabilities and Observed Responses |      |           |       |
|---------------------------------------------------------------|------|-----------|-------|
| Percent Concordant                                            | 84.0 | Somers' D | 0.710 |
| Percent Discordant                                            | 13.0 | Gamma     | 0.732 |
| Percent Tied                                                  | 3.1  | Tau-a     | 0.328 |
| Pairs                                                         | 162  | c         | 0.855 |

Note that you can also use the FAST option when [SELECTION=STEPWISE](#). However, the FAST option operates only on backward elimination steps. In this example, the stepwise process only adds variables, so the FAST option would not be useful.

Results of the [CTABLE](#) option are shown in [Output 54.1.11](#).

**Output 54.1.11** Classifying Input Observations

| Classification Table |         |               |           |               |         |                  |                  |              |              |
|----------------------|---------|---------------|-----------|---------------|---------|------------------|------------------|--------------|--------------|
| Prob<br>Level        | Correct |               | Incorrect |               | Correct | Percentages      |                  | False<br>POS | False<br>NEG |
|                      | Event   | Non-<br>Event | Event     | Non-<br>Event |         | Sensi-<br>tivity | Speci-<br>ficity |              |              |
| 0.060                | 9       | 0             | 18        | 0             | 33.3    | 100.0            | 0.0              | 66.7         | .            |
| 0.080                | 9       | 2             | 16        | 0             | 40.7    | 100.0            | 11.1             | 64.0         | 0.0          |
| 0.100                | 9       | 4             | 14        | 0             | 48.1    | 100.0            | 22.2             | 60.9         | 0.0          |
| 0.120                | 9       | 4             | 14        | 0             | 48.1    | 100.0            | 22.2             | 60.9         | 0.0          |
| 0.140                | 9       | 7             | 11        | 0             | 59.3    | 100.0            | 38.9             | 55.0         | 0.0          |
| 0.160                | 9       | 10            | 8         | 0             | 70.4    | 100.0            | 55.6             | 47.1         | 0.0          |
| 0.180                | 9       | 10            | 8         | 0             | 70.4    | 100.0            | 55.6             | 47.1         | 0.0          |
| 0.200                | 8       | 13            | 5         | 1             | 77.8    | 88.9             | 72.2             | 38.5         | 7.1          |
| 0.220                | 8       | 13            | 5         | 1             | 77.8    | 88.9             | 72.2             | 38.5         | 7.1          |
| 0.240                | 8       | 13            | 5         | 1             | 77.8    | 88.9             | 72.2             | 38.5         | 7.1          |
| 0.260                | 6       | 13            | 5         | 3             | 70.4    | 66.7             | 72.2             | 45.5         | 18.8         |
| 0.280                | 6       | 13            | 5         | 3             | 70.4    | 66.7             | 72.2             | 45.5         | 18.8         |
| 0.300                | 6       | 13            | 5         | 3             | 70.4    | 66.7             | 72.2             | 45.5         | 18.8         |
| 0.320                | 6       | 14            | 4         | 3             | 74.1    | 66.7             | 77.8             | 40.0         | 17.6         |
| 0.340                | 5       | 14            | 4         | 4             | 70.4    | 55.6             | 77.8             | 44.4         | 22.2         |
| 0.360                | 5       | 14            | 4         | 4             | 70.4    | 55.6             | 77.8             | 44.4         | 22.2         |
| 0.380                | 5       | 15            | 3         | 4             | 74.1    | 55.6             | 83.3             | 37.5         | 21.1         |
| 0.400                | 5       | 15            | 3         | 4             | 74.1    | 55.6             | 83.3             | 37.5         | 21.1         |
| 0.420                | 5       | 15            | 3         | 4             | 74.1    | 55.6             | 83.3             | 37.5         | 21.1         |
| 0.440                | 5       | 15            | 3         | 4             | 74.1    | 55.6             | 83.3             | 37.5         | 21.1         |
| 0.460                | 4       | 16            | 2         | 5             | 74.1    | 44.4             | 88.9             | 33.3         | 23.8         |
| 0.480                | 4       | 16            | 2         | 5             | 74.1    | 44.4             | 88.9             | 33.3         | 23.8         |
| 0.500                | 4       | 16            | 2         | 5             | 74.1    | 44.4             | 88.9             | 33.3         | 23.8         |
| 0.520                | 4       | 16            | 2         | 5             | 74.1    | 44.4             | 88.9             | 33.3         | 23.8         |
| 0.540                | 3       | 16            | 2         | 6             | 70.4    | 33.3             | 88.9             | 40.0         | 27.3         |
| 0.560                | 3       | 16            | 2         | 6             | 70.4    | 33.3             | 88.9             | 40.0         | 27.3         |
| 0.580                | 3       | 16            | 2         | 6             | 70.4    | 33.3             | 88.9             | 40.0         | 27.3         |
| 0.600                | 3       | 16            | 2         | 6             | 70.4    | 33.3             | 88.9             | 40.0         | 27.3         |
| 0.620                | 3       | 16            | 2         | 6             | 70.4    | 33.3             | 88.9             | 40.0         | 27.3         |
| 0.640                | 3       | 16            | 2         | 6             | 70.4    | 33.3             | 88.9             | 40.0         | 27.3         |
| 0.660                | 3       | 16            | 2         | 6             | 70.4    | 33.3             | 88.9             | 40.0         | 27.3         |
| 0.680                | 3       | 16            | 2         | 6             | 70.4    | 33.3             | 88.9             | 40.0         | 27.3         |
| 0.700                | 3       | 16            | 2         | 6             | 70.4    | 33.3             | 88.9             | 40.0         | 27.3         |
| 0.720                | 2       | 16            | 2         | 7             | 66.7    | 22.2             | 88.9             | 50.0         | 30.4         |
| 0.740                | 2       | 16            | 2         | 7             | 66.7    | 22.2             | 88.9             | 50.0         | 30.4         |
| 0.760                | 2       | 16            | 2         | 7             | 66.7    | 22.2             | 88.9             | 50.0         | 30.4         |
| 0.780                | 2       | 16            | 2         | 7             | 66.7    | 22.2             | 88.9             | 50.0         | 30.4         |
| 0.800                | 2       | 17            | 1         | 7             | 70.4    | 22.2             | 94.4             | 33.3         | 29.2         |
| 0.820                | 2       | 17            | 1         | 7             | 70.4    | 22.2             | 94.4             | 33.3         | 29.2         |
| 0.840                | 0       | 17            | 1         | 9             | 63.0    | 0.0              | 94.4             | 100.0        | 34.6         |
| 0.860                | 0       | 17            | 1         | 9             | 63.0    | 0.0              | 94.4             | 100.0        | 34.6         |
| 0.880                | 0       | 17            | 1         | 9             | 63.0    | 0.0              | 94.4             | 100.0        | 34.6         |
| 0.900                | 0       | 17            | 1         | 9             | 63.0    | 0.0              | 94.4             | 100.0        | 34.6         |
| 0.920                | 0       | 17            | 1         | 9             | 63.0    | 0.0              | 94.4             | 100.0        | 34.6         |
| 0.940                | 0       | 17            | 1         | 9             | 63.0    | 0.0              | 94.4             | 100.0        | 34.6         |
| 0.960                | 0       | 18            | 0         | 9             | 66.7    | 0.0              | 100.0            | .            | 33.3         |

Each row of the “Classification Table” corresponds to a cutpoint applied to the predicted probabilities, which is given in the Prob Level column. The  $2 \times 2$  frequency tables of observed and predicted responses are given by the next four columns. For example, with a cutpoint of 0.5, 4 events and 16 nonevents were classified correctly. On the other hand, 2 nonevents were incorrectly classified as events and 5 events were incorrectly classified as nonevents. For this cutpoint, the correct classification rate is 20/27 (=74.1%), which is given in the sixth column. Accuracy of the classification is summarized by the sensitivity, specificity, and false positive and negative rates, which are displayed in the last four columns. You can control the number of cutpoints used, and their values, by using the `PPROB=` option.

## Example 54.2: Logistic Modeling with Categorical Predictors

Consider a study of the analgesic effects of treatments on elderly patients with neuralgia. Two test treatments and a placebo are compared. The response variable is whether the patient reported pain or not. Researchers recorded the age and gender of 60 patients and the duration of complaint before the treatment began. The following DATA step creates the data set Neuralgia:

```
data Neuralgia;
  input Treatment $ Sex $ Age Duration Pain $ @@;
  datalines;
P F 68 1 No B M 74 16 No P F 67 30 No
P M 66 26 Yes B F 67 28 No B F 77 16 No
A F 71 12 No B F 72 50 No B F 76 9 Yes
A M 71 17 Yes A F 63 27 No A F 69 18 Yes
B F 66 12 No A M 62 42 No P F 64 1 Yes
A F 64 17 No P M 74 4 No A F 72 25 No
P M 70 1 Yes B M 66 19 No B M 59 29 No
A F 64 30 No A M 70 28 No A M 69 1 No
B F 78 1 No P M 83 1 Yes B F 69 42 No
B M 75 30 Yes P M 77 29 Yes P F 79 20 Yes
A M 70 12 No A F 69 12 No B F 65 14 No
B M 70 1 No B M 67 23 No A M 76 25 Yes
P M 78 12 Yes B M 77 1 Yes B F 69 24 No
P M 66 4 Yes P F 65 29 No P M 60 26 Yes
A M 78 15 Yes B M 75 21 Yes A F 67 11 No
P F 72 27 No P F 70 13 Yes A M 75 6 Yes
B F 65 7 No P F 68 27 Yes P M 68 11 Yes
P M 67 17 Yes B M 70 22 No A M 65 15 No
P F 67 1 Yes A M 67 10 No P F 72 11 Yes
A F 74 1 No B M 80 21 Yes A F 69 3 No
;
```

The data set Neuralgia contains five variables: Treatment, Sex, Age, Duration, and Pain. The last variable, Pain, is the response variable. A specification of Pain=Yes indicates there was pain, and Pain=No indicates no pain. The variable Treatment is a categorical variable with three levels: A and B represent the two test treatments, and P represents the placebo treatment. The gender of the patients is given by the categorical variable Sex. The variable Age is the age of the patients, in years, when treatment began. The duration of complaint, in months, before the treatment began is given by the variable Duration.

The following statements use the LOGISTIC procedure to fit a two-way logit with interaction model for the effect of Treatment and Sex, with Age and Duration as covariates. The categorical variables Treatment and

Sex are declared in the **CLASS** statement.

```
proc logistic data=Neuralgia;
  class Treatment Sex;
  model Pain= Treatment Sex Treatment*Sex Age Duration / expb;
run;
```

In this analysis, PROC LOGISTIC models the probability of no pain (Pain=No). By default, effect coding is used to represent the CLASS variables. Two design variables are created for Treatment and one for Sex, as shown in [Output 54.2.1](#).

**Output 54.2.1** Effect Coding of CLASS Variables

| The LOGISTIC Procedure  |       |                  |    |
|-------------------------|-------|------------------|----|
| Class Level Information |       |                  |    |
| Class                   | Value | Design Variables |    |
| Treatment               | A     | 1                | 0  |
|                         | B     | 0                | 1  |
|                         | P     | -1               | -1 |
| Sex                     | F     | 1                |    |
|                         | M     | -1               |    |

PROC LOGISTIC displays a table of the Type 3 analysis of effects based on the Wald test ([Output 54.2.2](#)). Note that the Treatment\*Sex interaction and the duration of complaint are not statistically significant ( $p = 0.9318$  and  $p = 0.8752$ , respectively). This indicates that there is no evidence that the treatments affect pain differently in men and women, and no evidence that the pain outcome is related to the duration of pain.

**Output 54.2.2** Wald Tests of Individual Effects

| Type 3 Analysis of Effects |    |            |            |
|----------------------------|----|------------|------------|
| Effect                     | DF | Wald       |            |
|                            |    | Chi-Square | Pr > ChiSq |
| Treatment                  | 2  | 11.9886    | 0.0025     |
| Sex                        | 1  | 5.3104     | 0.0212     |
| Treatment*Sex              | 2  | 0.1412     | 0.9318     |
| Age                        | 1  | 7.2744     | 0.0070     |
| Duration                   | 1  | 0.0247     | 0.8752     |

Parameter estimates are displayed in [Output 54.2.3](#). The Exp(Est) column contains the exponentiated parameter estimates requested with the **EXPB** option. These values can, but do not necessarily, represent odds ratios for the corresponding variables. For continuous explanatory variables, the Exp(Est) value corresponds to the odds ratio for a unit increase of the corresponding variable. For CLASS variables that use effect coding, the Exp(Est) values have no direct interpretation as a comparison of levels. However, when

the reference coding is used, the Exp(Est) values represent the odds ratio between the corresponding level and the reference level. Following the parameter estimates table, PROC LOGISTIC displays the odds ratio estimates for those variables that are not involved in any interaction terms. If the variable is a CLASS variable, the odds ratio estimate comparing each level with the reference level is computed regardless of the coding scheme. In this analysis, since the model contains the Treatment\*Sex interaction term, the odds ratios for Treatment and Sex were not computed. The odds ratio estimates for Age and Duration are precisely the values given in the Exp(Est) column in the parameter estimates table.

**Output 54.2.3** Parameter Estimates with Effect Coding

| Analysis of Maximum Likelihood Estimates |     |    |          |                |                 |            |           |
|------------------------------------------|-----|----|----------|----------------|-----------------|------------|-----------|
| Parameter                                |     | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq | Exp (Est) |
| Intercept                                |     | 1  | 19.2236  | 7.1315         | 7.2661          | 0.0070     | 2.232E8   |
| Treatment                                | A   | 1  | 0.8483   | 0.5502         | 2.3773          | 0.1231     | 2.336     |
| Treatment                                | B   | 1  | 1.4949   | 0.6622         | 5.0956          | 0.0240     | 4.459     |
| Sex                                      | F   | 1  | 0.9173   | 0.3981         | 5.3104          | 0.0212     | 2.503     |
| Treatment*Sex                            | A F | 1  | -0.2010  | 0.5568         | 0.1304          | 0.7180     | 0.818     |
| Treatment*Sex                            | B F | 1  | 0.0487   | 0.5563         | 0.0077          | 0.9302     | 1.050     |
| Age                                      |     | 1  | -0.2688  | 0.0996         | 7.2744          | 0.0070     | 0.764     |
| Duration                                 |     | 1  | 0.00523  | 0.0333         | 0.0247          | 0.8752     | 1.005     |

| Odds Ratio Estimates |                |                            |       |
|----------------------|----------------|----------------------------|-------|
| Effect               | Point Estimate | 95% Wald Confidence Limits |       |
| Age                  | 0.764          | 0.629                      | 0.929 |
| Duration             | 1.005          | 0.942                      | 1.073 |

The following PROC LOGISTIC statements illustrate the use of forward selection on the data set Neuralgia to identify the effects that differentiate the two Pain responses. The option **SELECTION=FORWARD** is specified to carry out the forward selection. The term Treatment|Sex@2 illustrates another way to specify main effects and two-way interactions. (Note that, in this case, the “@2” is unnecessary because no interactions besides the two-way interaction are possible).

```
proc logistic data=Neuralgia;
  class Treatment Sex;
  model Pain=Treatment|Sex@2 Age Duration
    /selection=forward expb;
run;
```

Results of the forward selection process are summarized in [Output 54.2.4](#). The variable Treatment is selected first, followed by Age and then Sex. The results are consistent with the previous analysis ([Output 54.2.2](#)) in which the Treatment\*Sex interaction and Duration are not statistically significant.



**Output 54.2.4** Effects Selected into the Model

| The LOGISTIC Procedure       |                |    |           |                  |            |
|------------------------------|----------------|----|-----------|------------------|------------|
| Summary of Forward Selection |                |    |           |                  |            |
| Step                         | Effect Entered | DF | Number In | Score Chi-Square | Pr > ChiSq |
| 1                            | Treatment      | 2  | 1         | 13.7143          | 0.0011     |
| 2                            | Age            | 1  | 2         | 10.6038          | 0.0011     |
| 3                            | Sex            | 1  | 3         | 5.9959           | 0.0143     |

Output 54.2.5 shows the Type 3 analysis of effects, the parameter estimates, and the odds ratio estimates for the selected model. All three variables, Treatment, Age, and Sex, are statistically significant at the 0.05 level ( $p=0.0018$ ,  $p=0.0213$ , and  $p=0.0057$ , respectively). Since the selected model does not contain the Treatment\*Sex interaction, odds ratios for Treatment and Sex are computed. The estimated odds ratio is 24.022 for treatment A versus placebo, 41.528 for Treatment B versus placebo, and 6.194 for female patients versus male patients. Note that these odds ratio estimates are not the same as the corresponding values in the Exp(Est) column in the parameter estimates table because effect coding was used. From Output 54.2.5, it is evident that both Treatment A and Treatment B are better than the placebo in reducing pain; females tend to have better improvement than males; and younger patients are faring better than older patients.

**Output 54.2.5** Type 3 Effects and Parameter Estimates with Effect Coding

| Type 3 Analysis of Effects               |    |          |                 |                 |            |           |
|------------------------------------------|----|----------|-----------------|-----------------|------------|-----------|
| Effect                                   | DF |          | Wald Chi-Square | Pr > ChiSq      |            |           |
| Treatment                                | 2  |          | 12.6928         | 0.0018          |            |           |
| Sex                                      | 1  |          | 5.3013          | 0.0213          |            |           |
| Age                                      | 1  |          | 7.6314          | 0.0057          |            |           |
| Analysis of Maximum Likelihood Estimates |    |          |                 |                 |            |           |
| Parameter                                | DF | Estimate | Standard Error  | Wald Chi-Square | Pr > ChiSq | Exp (Est) |
| Intercept                                | 1  | 19.0804  | 6.7882          | 7.9007          | 0.0049     | 1.9343E8  |
| Treatment A                              | 1  | 0.8772   | 0.5274          | 2.7662          | 0.0963     | 2.404     |
| Treatment B                              | 1  | 1.4246   | 0.6036          | 5.5711          | 0.0183     | 4.156     |
| Sex F                                    | 1  | 0.9118   | 0.3960          | 5.3013          | 0.0213     | 2.489     |
| Age                                      | 1  | -0.2650  | 0.0959          | 7.6314          | 0.0057     | 0.767     |

Output 54.2.5 continued

| Odds Ratio Estimates |                |                            |         |
|----------------------|----------------|----------------------------|---------|
| Effect               | Point Estimate | 95% Wald Confidence Limits |         |
| Treatment A vs P     | 24.022         | 3.295                      | 175.121 |
| Treatment B vs P     | 41.528         | 4.500                      | 383.262 |
| Sex F vs M           | 6.194          | 1.312                      | 29.248  |
| Age                  | 0.767          | 0.636                      | 0.926   |

Finally, the following statements refit the previously selected model, except that reference coding is used for the CLASS variables instead of effect coding:

```
ods graphics on;
proc logistic data=Neuralgia plots(only)=(oddsratio(range=clip));
  class Treatment Sex /param=ref;
  model Pain= Treatment Sex Age;
  oddsratio Treatment;
  oddsratio Sex;
  oddsratio Age;
  contrast 'Pairwise A vs P' Treatment 1 0 / estimate=exp;
  contrast 'Pairwise B vs P' Treatment 0 1 / estimate=exp;
  contrast 'Pairwise A vs B' Treatment 1 -1 / estimate=exp;
  contrast 'Female vs Male' Sex 1 / estimate=exp;
  effectplot / at(Sex=all) noobs;
  effectplot slicefit(sliceby=Sex plotby=Treatment) / noobs;
run;
ods graphics off;
```

The **ODDSRATIO** statements compute the odds ratios for the covariates. Four **CONTRAST** statements are specified; they provide another method of producing the odds ratios. The three contrasts labeled 'Pairwise' specify a contrast vector,  $L$ , for each of the pairwise comparisons between the three levels of Treatment. The contrast labeled 'Female vs Male' compares female to male patients. The option **ESTIMATE=EXP** is specified in all **CONTRAST** statements to exponentiate the estimates of  $L'\beta$ . With the given specification of contrast coefficients, the first of the 'Pairwise' **CONTRAST** statements corresponds to the odds ratio of A versus P, the second corresponds to B versus P, and the third corresponds to A versus B. You can also specify the 'Pairwise' contrasts in a single contrast statement with three rows. The 'Female vs Male' **CONTRAST** statement corresponds to the odds ratio that compares female to male patients.

The **PLOTS(ONLY)=** option displays only the requested odds ratio plot when ODS Graphics is enabled. The **EFFECTPLOT** statements do not honor the **ONLY** option, and display the fitted model. The first **EFFECTPLOT** statement by default produces a plot of the predicted values against the continuous Age variable, grouped by the Treatment levels. The **AT** option produces one plot for males and another for females; the **NOOBS** option suppresses the display of the observations. In the second **EFFECTPLOT** statement, a **SLICEFIT** plot is specified to display the Age variable on the X axis, the fits are grouped by the Sex levels, and the **PLOTBY=** option produces a panel of plots that displays each level of the Treatment variable.

The reference coding is shown in [Output 54.2.6](#). The Type 3 analysis of effects, the parameter estimates for the reference coding, and the odds ratio estimates are displayed in [Output 54.2.7](#). Although the parameter

estimates are different because of the different parameterizations, the “Type 3 Analysis of Effects” table and the “Odds Ratio” table remain the same as in [Output 54.2.5](#). With effect coding, the treatment A parameter estimate (0.8772) estimates the effect of treatment A compared to the average effect of treatments A, B, and placebo. The treatment A estimate (3.1790) under the reference coding estimates the difference in effect of treatment A and the placebo treatment.

#### Output 54.2.6 Reference Coding of CLASS Variables

| The LOGISTIC Procedure  |       |                  |   |
|-------------------------|-------|------------------|---|
| Class Level Information |       |                  |   |
| Class                   | Value | Design Variables |   |
| Treatment               | A     | 1                | 0 |
|                         | B     | 0                | 1 |
|                         | P     | 0                | 0 |
| Sex                     | F     | 1                |   |
|                         | M     | 0                |   |

#### Output 54.2.7 Type 3 Effects and Parameter Estimates with Reference Coding

| Type 3 Analysis of Effects |    |                 |            |
|----------------------------|----|-----------------|------------|
| Effect                     | DF | Wald Chi-Square | Pr > ChiSq |
| Treatment                  | 2  | 12.6928         | 0.0018     |
| Sex                        | 1  | 5.3013          | 0.0213     |
| Age                        | 1  | 7.6314          | 0.0057     |

| Analysis of Maximum Likelihood Estimates |    |          |                |                 |            |
|------------------------------------------|----|----------|----------------|-----------------|------------|
| Parameter                                | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                | 1  | 15.8669  | 6.4056         | 6.1357          | 0.0132     |
| Treatment A                              | 1  | 3.1790   | 1.0135         | 9.8375          | 0.0017     |
| Treatment B                              | 1  | 3.7264   | 1.1339         | 10.8006         | 0.0010     |
| Sex F                                    | 1  | 1.8235   | 0.7920         | 5.3013          | 0.0213     |
| Age                                      | 1  | -0.2650  | 0.0959         | 7.6314          | 0.0057     |

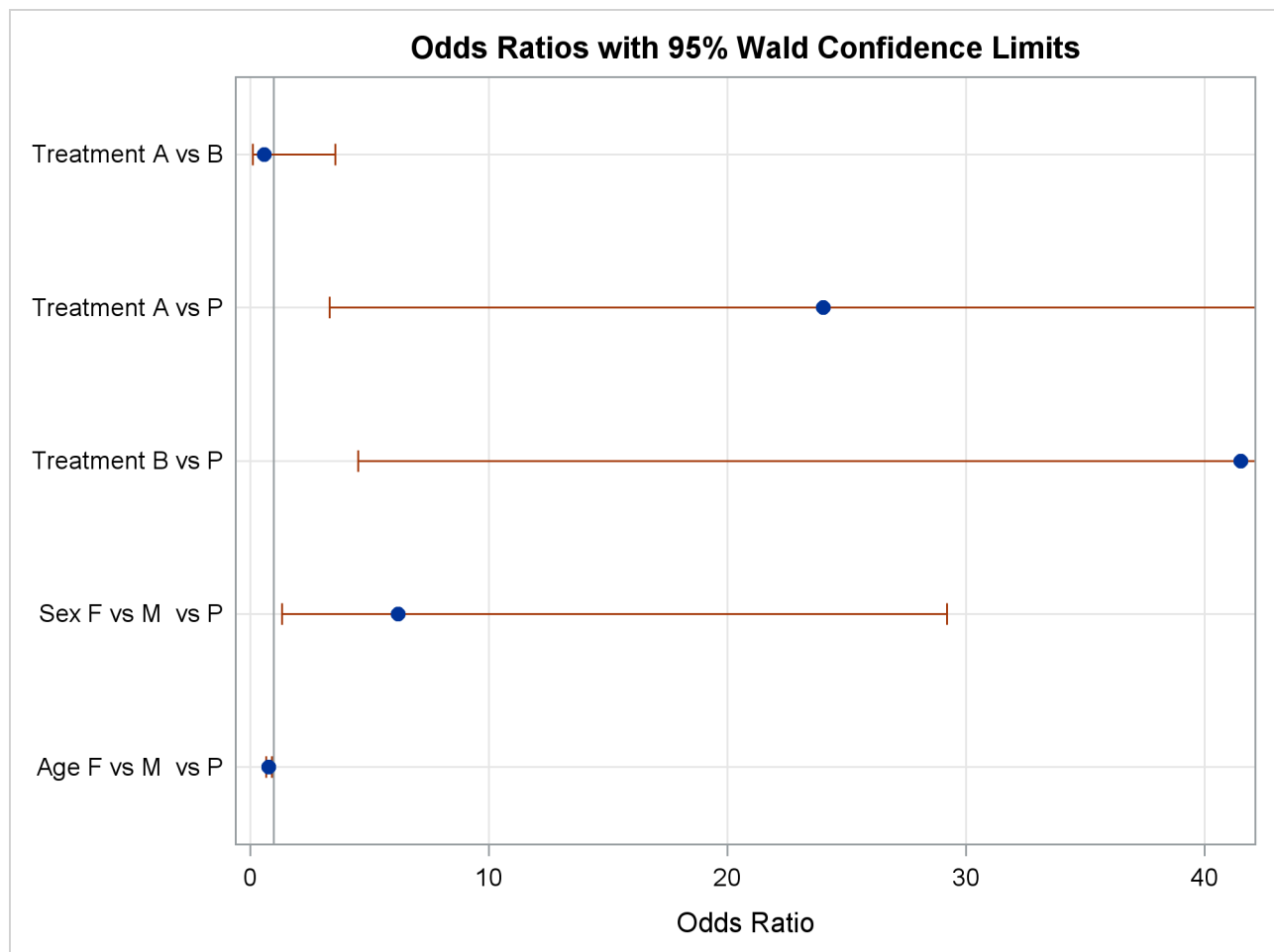
The **ODDSRATIO** statement results are shown in [Output 54.2.8](#), and the resulting plot is displayed in [Output 54.2.9](#). Note in [Output 54.2.9](#) that the odds ratio confidence limits are truncated due to specifying the **RANGE=CLIP** option; this enables you to see which intervals contain “1” more clearly. The odds ratios are identical to those shown in the “Odds Ratio Estimates” table in [Output 54.2.7](#) with the addition of the odds ratio for “Treatment A vs B”. Both treatments A and B are highly effective over placebo in reducing pain, as can be seen from the odds ratios comparing treatment A against P and treatment B against P (the second

and third rows in the table). However, the 95% confidence interval for the odds ratio comparing treatment A to B is (0.0932, 3.5889), indicating that the pain reduction effects of these two test treatments are not very different. Again, the 'Sex F vs M' odds ratio shows that female patients fared better in obtaining relief from pain than male patients. The odds ratio for Age shows that a patient one year older is 0.77 times as likely to show no pain; that is, younger patients have more improvement than older patients.

**Output 54.2.8** Results from the ODDSRATIO Statements

| Odds Ratio Estimates and Wald Confidence Intervals |          |                       |         |
|----------------------------------------------------|----------|-----------------------|---------|
| Label                                              | Estimate | 95% Confidence Limits |         |
| Treatment A vs B                                   | 0.578    | 0.093                 | 3.589   |
| Treatment A vs P                                   | 24.022   | 3.295                 | 175.121 |
| Treatment B vs P                                   | 41.528   | 4.500                 | 383.262 |
| Sex F vs M                                         | 6.194    | 1.312                 | 29.248  |
| Age                                                | 0.767    | 0.636                 | 0.926   |

**Output 54.2.9** Plot of the ODDSRATIO Statement Results



Output 54.2.10 contains two tables: the “Contrast Test Results” table and the “Contrast Estimation and Testing Results by Row” table. The former contains the overall Wald test for each CONTRAST statement. The latter table contains estimates and tests of individual contrast rows. The estimates for the first two rows of the ‘Pairwise’ CONTRAST statements are the same as those given in the two preceding odds ratio tables (Output 54.2.7 and Output 54.2.8). The third row estimates the odds ratio comparing A to B, agreeing with Output 54.2.8, and the last row computes the odds ratio comparing pain relief for females to that for males.

**Output 54.2.10** Results of CONTRAST Statements

| Contrast Test Results |    |                    |            |  |  |  |
|-----------------------|----|--------------------|------------|--|--|--|
| Contrast              | DF | Wald<br>Chi-Square | Pr > ChiSq |  |  |  |
| Pairwise A vs P       | 1  | 9.8375             | 0.0017     |  |  |  |
| Pairwise B vs P       | 1  | 10.8006            | 0.0010     |  |  |  |
| Pairwise A vs B       | 1  | 0.3455             | 0.5567     |  |  |  |
| Female vs Male        | 1  | 5.3013             | 0.0213     |  |  |  |

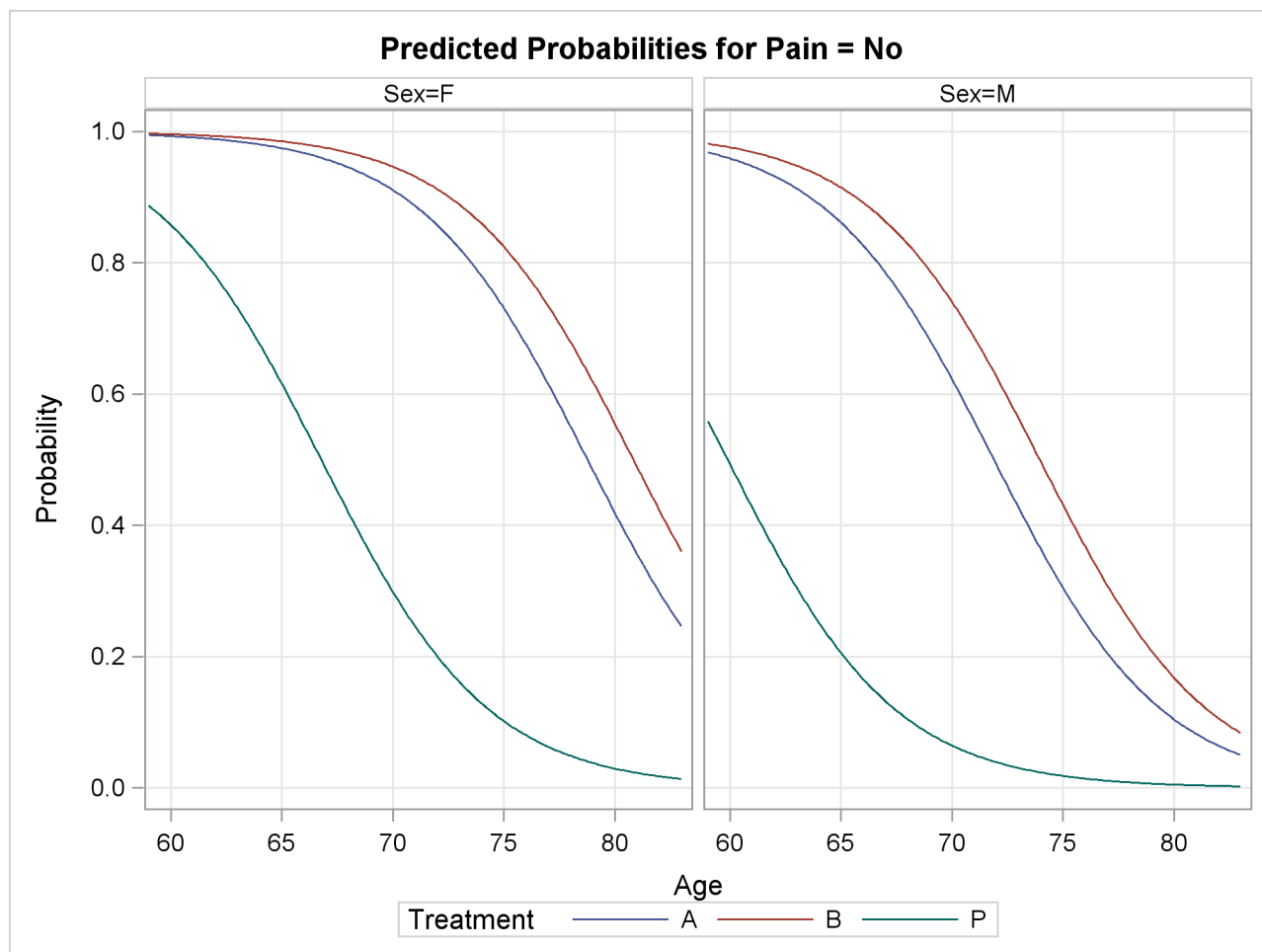
  

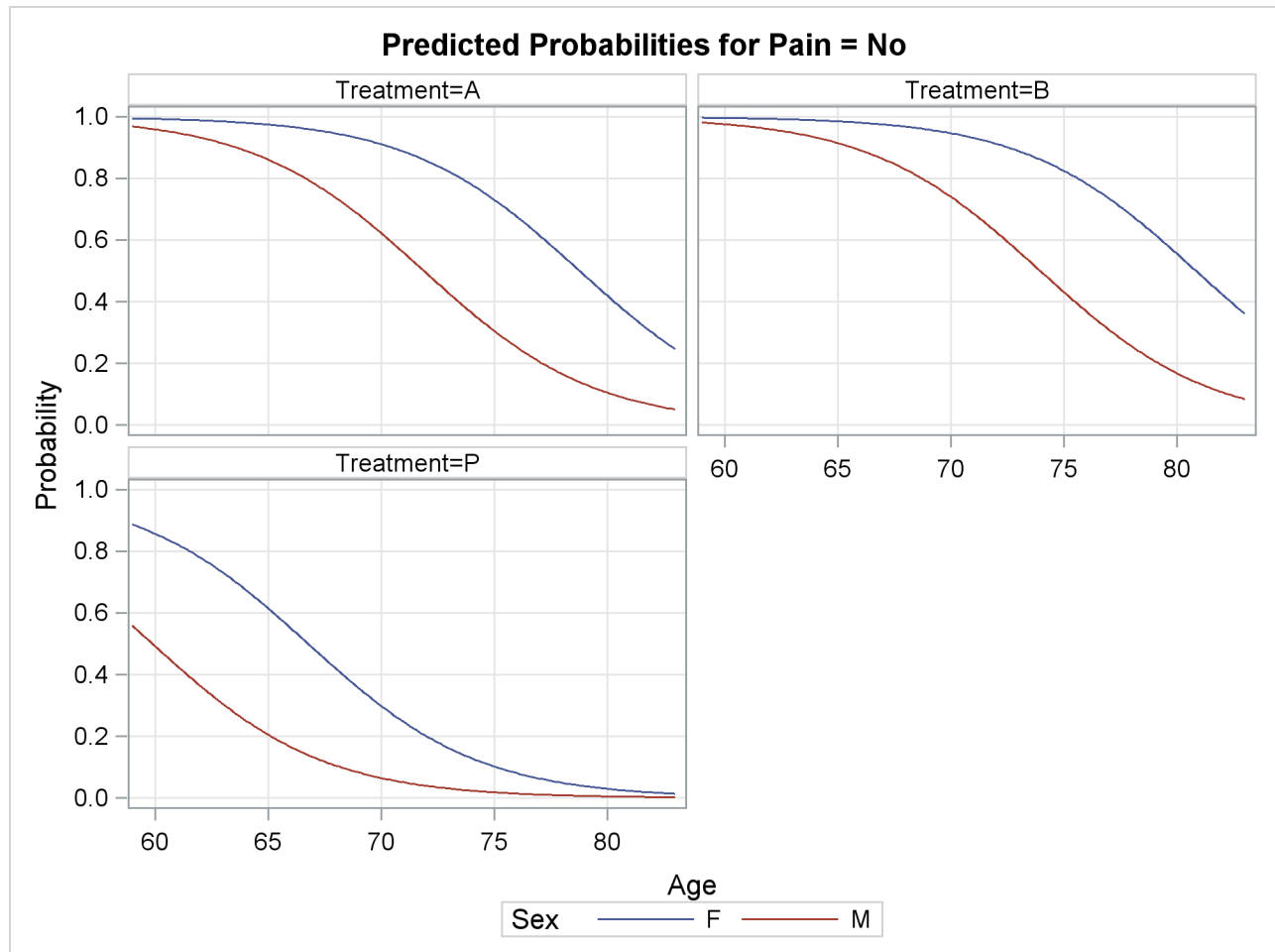
| Contrast Estimation and Testing Results by Row |      |     |          |                   |       |                   |         |
|------------------------------------------------|------|-----|----------|-------------------|-------|-------------------|---------|
| Contrast                                       | Type | Row | Estimate | Standard<br>Error | Alpha | Confidence Limits |         |
| Pairwise A vs P                                | EXP  | 1   | 24.0218  | 24.3473           | 0.05  | 3.2951            | 175.1   |
| Pairwise B vs P                                | EXP  | 1   | 41.5284  | 47.0877           | 0.05  | 4.4998            | 383.3   |
| Pairwise A vs B                                | EXP  | 1   | 0.5784   | 0.5387            | 0.05  | 0.0932            | 3.5889  |
| Female vs Male                                 | EXP  | 1   | 6.1937   | 4.9053            | 0.05  | 1.3116            | 29.2476 |

| Contrast Estimation and Testing Results by Row |      |     |                    |            |  |
|------------------------------------------------|------|-----|--------------------|------------|--|
| Contrast                                       | Type | Row | Wald<br>Chi-Square | Pr > ChiSq |  |
| Pairwise A vs P                                | EXP  | 1   | 9.8375             | 0.0017     |  |
| Pairwise B vs P                                | EXP  | 1   | 10.8006            | 0.0010     |  |
| Pairwise A vs B                                | EXP  | 1   | 0.3455             | 0.5567     |  |
| Female vs Male                                 | EXP  | 1   | 5.3013             | 0.0213     |  |

ANCOVA-style plots of the model-predicted probabilities against the Age variable for each combination of Treatment and Sex are displayed in Output 54.2.11 and Output 54.2.12. These plots confirm that females always have a higher probability of pain reduction in each treatment group, the placebo treatment has a lower probability of success than the other treatments, and younger patients respond to treatment better than older patients.

**Output 54.2.11** Model-Predicted Probabilities by Sex

**Output 54.2.12** Model-Predicted Probabilities by Treatment

### Example 54.3: Ordinal Logistic Regression

Consider a study of the effects on taste of various cheese additives. Researchers tested four cheese additives and obtained 52 response ratings for each additive. Each response was measured on a scale of nine categories ranging from strong dislike (1) to excellent taste (9). The data, given in McCullagh and Nelder (1989, p. 175) in the form of a two-way frequency table of additive by rating, are saved in the data set `Cheese` by using the following program. The variable `y` contains the response rating. The variable `Additive` specifies the cheese additive (1, 2, 3, or 4). The variable `freq` gives the frequency with which each additive received each rating.

```
data Cheese;
  do Additive = 1 to 4;
    do y = 1 to 9;
      input freq @@;
      output;
    end;
  end;
  label y='Taste Rating';
  datalines;
```

```

0 0 1 7 8 8 19 8 1
6 9 12 11 7 6 1 0 0
1 1 6 8 23 7 5 1 0
0 0 0 1 3 7 14 16 11
;

```

The response variable  $y$  is ordinally scaled. A cumulative logit model is used to investigate the effects of the cheese additives on taste. The following statements invoke PROC LOGISTIC to fit this model with  $y$  as the response variable and three indicator variables as explanatory variables, with the fourth additive as the reference level. With this parameterization, each Additive parameter compares an additive to the fourth additive. The COVB option displays the estimated covariance matrix. The ODDSRATIO statement computes odds ratios for all combinations of the Additive levels. The PLOTS option produces a graphical display of the odds ratios, and the EFFECTPLOT statement displays the predicted probabilities.

```

ods graphics on;
proc logistic data=Cheese plots(only)=oddsratio(range=clip);
  freq freq;
  class Additive (param=ref ref='4');
  model y=Additive / covb;
  oddsratio Additive;
  effectplot / polybar;
  title 'Multiple Response Cheese Tasting Experiment';
run;
ods graphics off;

```

The “Response Profile” table in [Output 54.3.1](#) shows that the strong dislike ( $y=1$ ) end of the rating scale is associated with lower Ordered Values in the “Response Profile” table; hence the probability of disliking the additives is modeled.

The score chi-square for testing the proportional odds assumption is 17.287, which is not significant with respect to a chi-square distribution with 21 degrees of freedom ( $p = 0.694$ ). This indicates that the proportional odds assumption is reasonable. The positive value (1.6128) for the parameter estimate for Additive1 indicates a tendency toward the lower-numbered categories of the first cheese additive relative to the fourth. In other words, the fourth additive tastes better than the first additive. The second and third additives are both less favorable than the fourth additive. The relative magnitudes of these slope estimates imply the preference ordering: fourth, first, third, second.

**Output 54.3.1** Proportional Odds Model Regression Analysis

| Multiple Response Cheese Tasting Experiment |                  |              |
|---------------------------------------------|------------------|--------------|
| The LOGISTIC Procedure                      |                  |              |
| Model Information                           |                  |              |
| Data Set                                    | WORK.CHEESE      |              |
| Response Variable                           | y                | Taste Rating |
| Number of Response Levels                   | 9                |              |
| Frequency Variable                          | freq             |              |
| Model                                       | cumulative logit |              |
| Optimization Technique                      | Fisher's scoring |              |



**Output 54.3.1** *continued*

|                             |     |
|-----------------------------|-----|
| Number of Observations Read | 36  |
| Number of Observations Used | 28  |
| Sum of Frequencies Read     | 208 |
| Sum of Frequencies Used     | 208 |

**Response Profile**

| Ordered<br>Value | y | Total<br>Frequency |
|------------------|---|--------------------|
| 1                | 1 | 7                  |
| 2                | 2 | 10                 |
| 3                | 3 | 19                 |
| 4                | 4 | 27                 |
| 5                | 5 | 41                 |
| 6                | 6 | 28                 |
| 7                | 7 | 39                 |
| 8                | 8 | 25                 |
| 9                | 9 | 12                 |

Probabilities modeled are cumulated over the lower Ordered Values.

NOTE: 8 observations having nonpositive frequencies or weights were excluded since they do not contribute to the analysis.

**Class Level Information**

| Class    | Value | Design Variables |
|----------|-------|------------------|
| Additive | 1     | 1    0    0      |
|          | 2     | 0    1    0      |
|          | 3     | 0    0    1      |
|          | 4     | 0    0    0      |

**Model Convergence Status**

Convergence criterion (GCONV=1E-8) satisfied.

**Score Test for the Proportional Odds Assumption**

| Chi-Square | DF | Pr > ChiSq |
|------------|----|------------|
| 17.2866    | 21 | 0.6936     |

## Output 54.3.1 continued

| Model Fit Statistics                                          |                   |                                |                   |                    |            |
|---------------------------------------------------------------|-------------------|--------------------------------|-------------------|--------------------|------------|
| Criterion                                                     | Intercept<br>Only | Intercept<br>and<br>Covariates |                   |                    |            |
| AIC                                                           | 875.802           | 733.348                        |                   |                    |            |
| SC                                                            | 902.502           | 770.061                        |                   |                    |            |
| -2 Log L                                                      | 859.802           | 711.348                        |                   |                    |            |
| Testing Global Null Hypothesis: BETA=0                        |                   |                                |                   |                    |            |
| Test                                                          | Chi-Square        | DF                             | Pr > ChiSq        |                    |            |
| Likelihood Ratio                                              | 148.4539          | 3                              | <.0001            |                    |            |
| Score                                                         | 111.2670          | 3                              | <.0001            |                    |            |
| Wald                                                          | 115.1504          | 3                              | <.0001            |                    |            |
| Type 3 Analysis of Effects                                    |                   |                                |                   |                    |            |
| Effect                                                        | DF                | Wald<br>Chi-Square             | Pr > ChiSq        |                    |            |
| Additive                                                      | 3                 | 115.1504                       | <.0001            |                    |            |
| Analysis of Maximum Likelihood Estimates                      |                   |                                |                   |                    |            |
| Parameter                                                     | DF                | Estimate                       | Standard<br>Error | Wald<br>Chi-Square | Pr > ChiSq |
| Intercept 1                                                   | 1                 | -7.0801                        | 0.5624            | 158.4851           | <.0001     |
| Intercept 2                                                   | 1                 | -6.0249                        | 0.4755            | 160.5500           | <.0001     |
| Intercept 3                                                   | 1                 | -4.9254                        | 0.4272            | 132.9484           | <.0001     |
| Intercept 4                                                   | 1                 | -3.8568                        | 0.3902            | 97.7087            | <.0001     |
| Intercept 5                                                   | 1                 | -2.5205                        | 0.3431            | 53.9704            | <.0001     |
| Intercept 6                                                   | 1                 | -1.5685                        | 0.3086            | 25.8374            | <.0001     |
| Intercept 7                                                   | 1                 | -0.0669                        | 0.2658            | 0.0633             | 0.8013     |
| Intercept 8                                                   | 1                 | 1.4930                         | 0.3310            | 20.3439            | <.0001     |
| Additive 1                                                    | 1                 | 1.6128                         | 0.3778            | 18.2265            | <.0001     |
| Additive 2                                                    | 1                 | 4.9645                         | 0.4741            | 109.6427           | <.0001     |
| Additive 3                                                    | 1                 | 3.3227                         | 0.4251            | 61.0931            | <.0001     |
| Association of Predicted Probabilities and Observed Responses |                   |                                |                   |                    |            |
| Percent Concordant                                            | 67.6              | Somers' D                      | 0.578             |                    |            |
| Percent Discordant                                            | 9.8               | Gamma                          | 0.746             |                    |            |
| Percent Tied                                                  | 22.6              | Tau-a                          | 0.500             |                    |            |
| Pairs                                                         | 18635             | c                              | 0.789             |                    |            |

The odds ratio results in [Output 54.3.2](#) show the preferences more clearly. For example, the “Additive 1 vs 4” odds ratio says that the first additive has 5.017 times the odds of receiving a lower score than the fourth

additive; that is, the first additive is 5.017 times more likely than the fourth additive to receive a lower score. [Output 54.3.3](#) displays the odds ratios graphically; the range of the confidence limits is truncated by the `RANGE=CLIP` option, so you can see that “1” is not contained in any of the intervals.

**Output 54.3.2** Odds Ratios of All Pairs of Additive Levels

| Odds Ratio Estimates and Wald Confidence Intervals |          |                       |         |
|----------------------------------------------------|----------|-----------------------|---------|
| Label                                              | Estimate | 95% Confidence Limits |         |
| Additive 1 vs 2                                    | 0.035    | 0.015                 | 0.080   |
| Additive 1 vs 3                                    | 0.181    | 0.087                 | 0.376   |
| Additive 1 vs 4                                    | 5.017    | 2.393                 | 10.520  |
| Additive 2 vs 3                                    | 5.165    | 2.482                 | 10.746  |
| Additive 2 vs 4                                    | 143.241  | 56.558                | 362.777 |
| Additive 3 vs 4                                    | 27.734   | 12.055                | 63.805  |

**Output 54.3.3** Plot of Odds Ratios for Additive



The estimated covariance matrix of the parameters is displayed in [Output 54.3.4](#).

**Output 54.3.4** Estimated Covariance Matrix

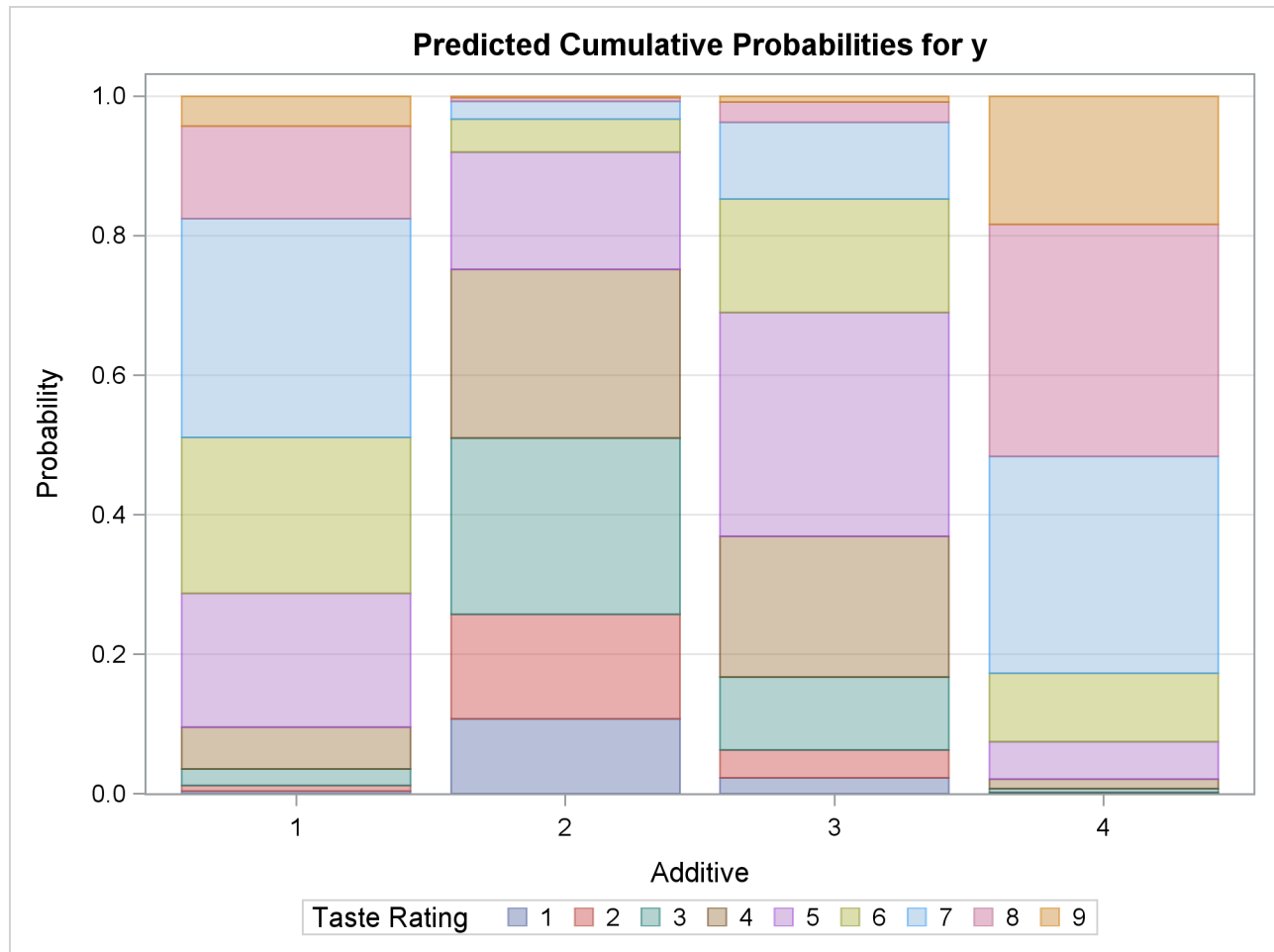
| Estimated Covariance Matrix |                 |                 |                 |                 |                 |
|-----------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Parameter                   | Intercept_<br>1 | Intercept_<br>2 | Intercept_<br>3 | Intercept_<br>4 | Intercept_<br>5 |
| Intercept_1                 | 0.316291        | 0.219581        | 0.176278        | 0.147694        | 0.114024        |
| Intercept_2                 | 0.219581        | 0.226095        | 0.177806        | 0.147933        | 0.11403         |
| Intercept_3                 | 0.176278        | 0.177806        | 0.182473        | 0.148844        | 0.114092        |
| Intercept_4                 | 0.147694        | 0.147933        | 0.148844        | 0.152235        | 0.114512        |
| Intercept_5                 | 0.114024        | 0.11403         | 0.114092        | 0.114512        | 0.117713        |
| Intercept_6                 | 0.091085        | 0.091081        | 0.091074        | 0.091109        | 0.091821        |
| Intercept_7                 | 0.057814        | 0.057813        | 0.057807        | 0.05778         | 0.057721        |
| Intercept_8                 | 0.041304        | 0.041304        | 0.0413          | 0.041277        | 0.041162        |
| Additive1                   | -0.09419        | -0.09421        | -0.09427        | -0.09428        | -0.09246        |
| Additive2                   | -0.18686        | -0.18161        | -0.1687         | -0.14717        | -0.11415        |
| Additive3                   | -0.13565        | -0.13569        | -0.1352         | -0.13118        | -0.11207        |

| Estimated Covariance Matrix |                 |                 |                 |           |           |           |
|-----------------------------|-----------------|-----------------|-----------------|-----------|-----------|-----------|
| Parameter                   | Intercept_<br>6 | Intercept_<br>7 | Intercept_<br>8 | Additive1 | Additive2 | Additive3 |
| Intercept_1                 | 0.091085        | 0.057814        | 0.041304        | -0.09419  | -0.18686  | -0.13565  |
| Intercept_2                 | 0.091081        | 0.057813        | 0.041304        | -0.09421  | -0.18161  | -0.13569  |
| Intercept_3                 | 0.091074        | 0.057807        | 0.0413          | -0.09427  | -0.1687   | -0.1352   |
| Intercept_4                 | 0.091109        | 0.05778         | 0.041277        | -0.09428  | -0.14717  | -0.13118  |
| Intercept_5                 | 0.091821        | 0.057721        | 0.041162        | -0.09246  | -0.11415  | -0.11207  |
| Intercept_6                 | 0.09522         | 0.058312        | 0.041324        | -0.08521  | -0.09113  | -0.09122  |
| Intercept_7                 | 0.058312        | 0.07064         | 0.04878         | -0.06041  | -0.05781  | -0.05802  |
| Intercept_8                 | 0.041324        | 0.04878         | 0.109562        | -0.04436  | -0.0413   | -0.04143  |
| Additive1                   | -0.08521        | -0.06041        | -0.04436        | 0.142715  | 0.094072  | 0.092128  |
| Additive2                   | -0.09113        | -0.05781        | -0.0413         | 0.094072  | 0.22479   | 0.132877  |
| Additive3                   | -0.09122        | -0.05802        | -0.04143        | 0.092128  | 0.132877  | 0.180709  |

Output 54.3.5 displays the probability of each taste rating  $y$  within each additive. You can see that Additive=1 mostly receives ratings of 5 to 7, Additive=2 mostly receives ratings of 2 to 5, Additive=3 mostly receives ratings of 4 to 6, and Additive=4 mostly receives ratings of 7 to 9, which also confirms the previously discussed preference orderings.

**Output 54.3.5** Model-Predicted Probabilities



## Example 54.4: Nominal Response Data: Generalized Logits Model

Over the course of one school year, third graders from three different schools are exposed to three different styles of mathematics instruction: a self-paced computer-learning style, a team approach, and a traditional class approach. The students are asked which style they prefer and their responses, classified by the type of program they are in (a regular school day versus a regular day supplemented with an afternoon school program), are displayed in [Table 54.15](#). The data set is from Stokes, Davis, and Koch (2012), and is also analyzed in the section “Generalized Logits Model” on page 1784 of Chapter 30, “The CATMOD Procedure.”

**Table 54.15** School Program Data

| School | Program   | Learning Style Preference |      |       |
|--------|-----------|---------------------------|------|-------|
|        |           | Self                      | Team | Class |
| 1      | Regular   | 10                        | 17   | 26    |
| 1      | Afternoon | 5                         | 12   | 50    |
| 2      | Regular   | 21                        | 17   | 26    |
| 2      | Afternoon | 16                        | 12   | 36    |
| 3      | Regular   | 15                        | 15   | 16    |
| 3      | Afternoon | 12                        | 12   | 20    |

The levels of the response variable (self, team, and class) have no essential ordering, so a logistic regression is performed on the generalized logits. The model to be fit is

$$\log\left(\frac{\pi_{hij}}{\pi_{hir}}\right) = \alpha_j + \mathbf{x}_{hi}'\boldsymbol{\beta}_j$$

where  $\pi_{hij}$  is the probability that a student in school  $h$  and program  $i$  prefers teaching style  $j$ ,  $j \neq r$ , and style  $r$  is the baseline style (in this case, class). There are separate sets of intercept parameters  $\alpha_j$  and regression parameters  $\boldsymbol{\beta}_j$  for each logit, and the vector  $\mathbf{x}_{hi}$  is the set of explanatory variables for the  $h$ th population. Thus, two logits are modeled for each school and program combination: the logit comparing self to class and the logit comparing team to class.

The following statements create the data set `school` and request the analysis. The `LINK=GLOGIT` option forms the generalized logits. The response variable option `ORDER=DATA` means that the response variable levels are ordered as they exist in the data set: self, team, and class; thus, the logits are formed by comparing self to class and by comparing team to class. The `ODDSRATIO` statement produces odds ratios in the presence of interactions, and a graphical display of the requested odds ratios is produced when ODS Graphics is enabled.

```
data school;
  length Program $ 9;
  input School Program $ Style $ Count @@;
  datalines;
1 regular    self 10  1 regular    team 17  1 regular    class 26
1 afternoon  self  5  1 afternoon  team 12  1 afternoon  class 50
2 regular    self 21  2 regular    team 17  2 regular    class 26
2 afternoon  self 16  2 afternoon  team 12  2 afternoon  class 36
3 regular    self 15  3 regular    team 15  3 regular    class 16
3 afternoon  self 12  3 afternoon  team 12  3 afternoon  class 20
;

ods graphics on;
proc logistic data=school;
  freq Count;
  class School Program(ref=first);
  model Style(order=data)=School Program School*Program / link=glogit;
  oddsratio program;
run;
ods graphics off;
```

Summary information about the model, the response variable, and the classification variables are displayed in [Output 54.4.1](#).

#### Output 54.4.1 Analysis of Saturated Model

| The LOGISTIC Procedure                                      |                   |           |     |
|-------------------------------------------------------------|-------------------|-----------|-----|
| Model Information                                           |                   |           |     |
| Data Set                                                    | WORK.SCHOOL       |           |     |
| Response Variable                                           | Style             |           |     |
| Number of Response Levels                                   | 3                 |           |     |
| Frequency Variable                                          | Count             |           |     |
| Model                                                       | generalized logit |           |     |
| Optimization Technique                                      | Newton-Raphson    |           |     |
|                                                             |                   |           |     |
| Number of Observations Read                                 |                   |           | 18  |
| Number of Observations Used                                 |                   |           | 18  |
| Sum of Frequencies Read                                     |                   |           | 338 |
| Sum of Frequencies Used                                     |                   |           | 338 |
|                                                             |                   |           |     |
| Response Profile                                            |                   |           |     |
| Ordered                                                     |                   | Total     |     |
| Value                                                       | Style             | Frequency |     |
| 1                                                           | self              | 79        |     |
| 2                                                           | team              | 85        |     |
| 3                                                           | class             | 174       |     |
| Logits modeled use Style='class' as the reference category. |                   |           |     |
|                                                             |                   |           |     |
| Class Level Information                                     |                   |           |     |
| Class                                                       | Value             | Design    |     |
|                                                             |                   | Variables |     |
| School                                                      | 1                 | 1         | 0   |
|                                                             | 2                 | 0         | 1   |
|                                                             | 3                 | -1        | -1  |
| Program                                                     | afternoon         | -1        |     |
|                                                             | regular           | 1         |     |
|                                                             |                   |           |     |
| Model Convergence Status                                    |                   |           |     |
| Convergence criterion (GCONV=1E-8) satisfied.               |                   |           |     |

The “Testing Global Null Hypothesis: BETA=0” table in [Output 54.4.2](#) shows that the parameters are significantly different from zero.

**Output 54.4.2** Analysis of Saturated Model

| Model Fit Statistics                   |                   |                                |            |
|----------------------------------------|-------------------|--------------------------------|------------|
| Criterion                              | Intercept<br>Only | Intercept<br>and<br>Covariates |            |
| AIC                                    | 699.404           | 689.156                        |            |
| SC                                     | 707.050           | 735.033                        |            |
| -2 Log L                               | 695.404           | 665.156                        |            |
| Testing Global Null Hypothesis: BETA=0 |                   |                                |            |
| Test                                   | Chi-Square        | DF                             | Pr > ChiSq |
| Likelihood Ratio                       | 30.2480           | 10                             | 0.0008     |
| Score                                  | 28.3738           | 10                             | 0.0016     |
| Wald                                   | 25.6828           | 10                             | 0.0042     |

However, the “Type 3 Analysis of Effects” table in [Output 54.4.3](#) shows that the interaction effect is clearly nonsignificant.

**Output 54.4.3** Analysis of Saturated Model

| Type 3 Analysis of Effects               |         |         |            |            |                |                 |            |
|------------------------------------------|---------|---------|------------|------------|----------------|-----------------|------------|
| Effect                                   |         | DF      | Wald       |            |                |                 |            |
|                                          |         |         | Chi-Square | Pr > ChiSq |                |                 |            |
| School                                   |         | 4       | 14.5522    | 0.0057     |                |                 |            |
| Program                                  |         | 2       | 10.4815    | 0.0053     |                |                 |            |
| School*Program                           |         | 4       | 1.7439     | 0.7827     |                |                 |            |
| Analysis of Maximum Likelihood Estimates |         |         |            |            |                |                 |            |
| Parameter                                |         | Style   | DF         | Estimate   | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                |         | self    | 1          | -0.8097    | 0.1488         | 29.5989         | <.0001     |
| Intercept                                |         | team    | 1          | -0.6585    | 0.1366         | 23.2449         | <.0001     |
| School                                   | 1       | self    | 1          | -0.8194    | 0.2281         | 12.9066         | 0.0003     |
| School                                   | 1       | team    | 1          | -0.2675    | 0.1881         | 2.0233          | 0.1549     |
| School                                   | 2       | self    | 1          | 0.2974     | 0.1919         | 2.4007          | 0.1213     |
| School                                   | 2       | team    | 1          | -0.1033    | 0.1898         | 0.2961          | 0.5863     |
| Program                                  | regular | self    | 1          | 0.3985     | 0.1488         | 7.1684          | 0.0074     |
| Program                                  | regular | team    | 1          | 0.3537     | 0.1366         | 6.7071          | 0.0096     |
| School*Program                           | 1       | regular | self       | 0.2751     | 0.2281         | 1.4547          | 0.2278     |
| School*Program                           | 1       | regular | team       | 0.1474     | 0.1881         | 0.6143          | 0.4332     |
| School*Program                           | 2       | regular | self       | -0.0998    | 0.1919         | 0.2702          | 0.6032     |
| School*Program                           | 2       | regular | team       | -0.0168    | 0.1898         | 0.0079          | 0.9293     |

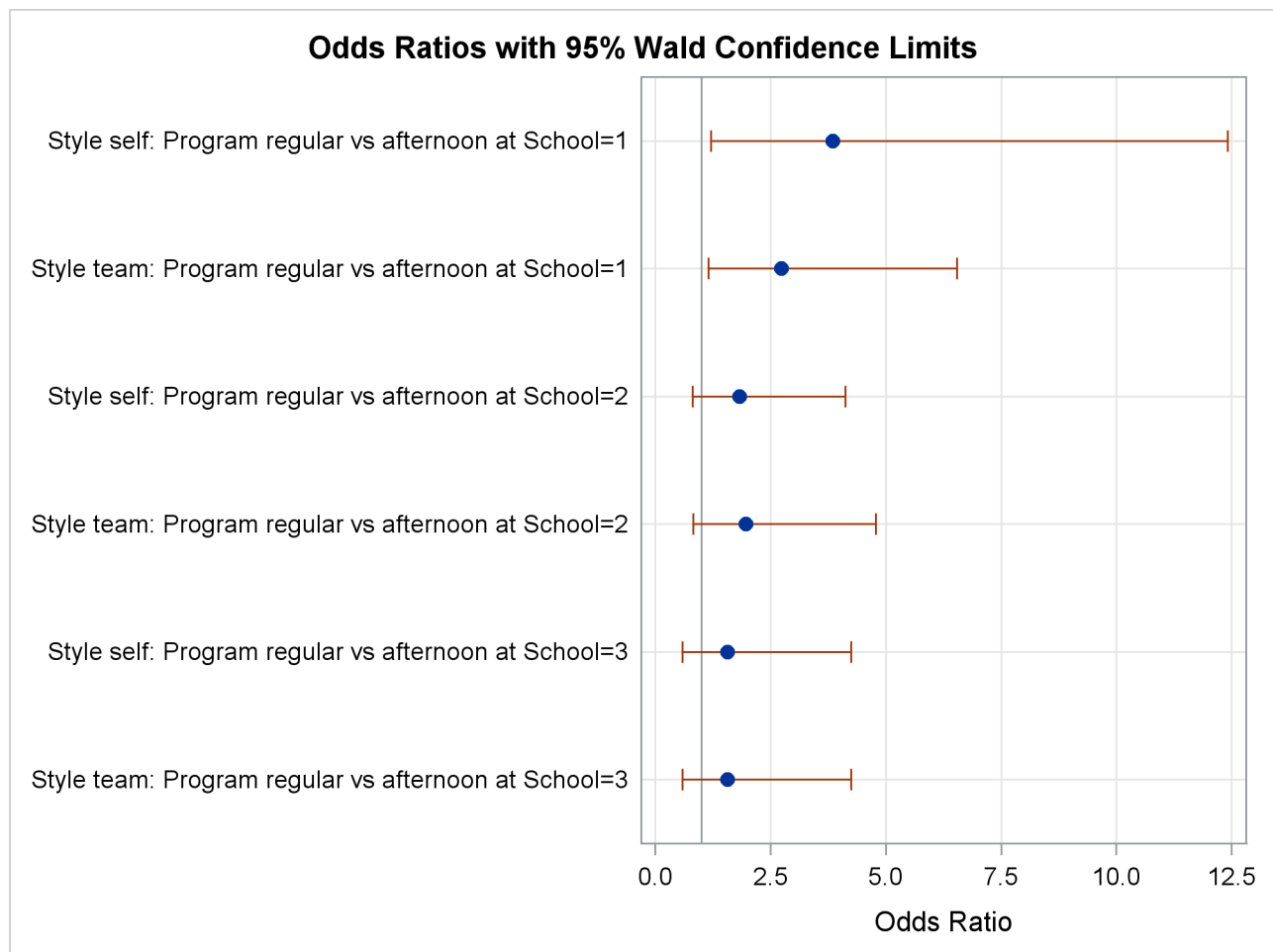


The table produced by the **ODDSRATIO** statement is displayed in **Output 54.4.4**. The differences between the program preferences are small across all the styles (logits) compared to their variability as displayed by the confidence limits in **Output 54.4.5**, confirming that the interaction effect is nonsignificant.

**Output 54.4.4** Odds Ratios for Style

| Odds Ratio Estimates and Wald Confidence Intervals   |          |                       |        |
|------------------------------------------------------|----------|-----------------------|--------|
| Label                                                | Estimate | 95% Confidence Limits |        |
| Style self: Program regular vs afternoon at School=1 | 3.846    | 1.190                 | 12.435 |
| Style team: Program regular vs afternoon at School=1 | 2.724    | 1.132                 | 6.554  |
| Style self: Program regular vs afternoon at School=2 | 1.817    | 0.798                 | 4.139  |
| Style team: Program regular vs afternoon at School=2 | 1.962    | 0.802                 | 4.799  |
| Style self: Program regular vs afternoon at School=3 | 1.562    | 0.572                 | 4.265  |
| Style team: Program regular vs afternoon at School=3 | 1.562    | 0.572                 | 4.265  |

**Output 54.4.5** Plot of Odds Ratios for Style



Since the interaction effect is clearly nonsignificant, a main-effects model is fit with the following statements. The **EFFECTPLOT** statement creates a plot of the predicted values versus the levels of the School

variable at each level of the Program variables. The **CLM** option adds confidence bars, and the **NOOBS** option suppresses the display of the observations.

```
ods graphics on;
proc logistic data=school;
  freq Count;
  class School Program(ref=first);
  model Style(order=data)=School Program / link=glogit;
  effectplot interaction(plotby=Program) / clm noobs;
run;
ods graphics off;
```

All of the global fit tests in [Output 54.4.6](#) suggest the model is significant, and the Type 3 tests show that the school and program effects are also significant.

**Output 54.4.6** Analysis of Main-Effects Model

| The LOGISTIC Procedure                        |                   |                                |            |
|-----------------------------------------------|-------------------|--------------------------------|------------|
| Model Convergence Status                      |                   |                                |            |
| Convergence criterion (GCONV=1E-8) satisfied. |                   |                                |            |
| Model Fit Statistics                          |                   |                                |            |
| Criterion                                     | Intercept<br>Only | Intercept<br>and<br>Covariates |            |
| AIC                                           | 699.404           | 682.934                        |            |
| SC                                            | 707.050           | 713.518                        |            |
| -2 Log L                                      | 695.404           | 666.934                        |            |
| Testing Global Null Hypothesis: BETA=0        |                   |                                |            |
| Test                                          | Chi-Square        | DF                             | Pr > ChiSq |
| Likelihood Ratio                              | 28.4704           | 6                              | <.0001     |
| Score                                         | 27.1190           | 6                              | 0.0001     |
| Wald                                          | 25.5881           | 6                              | 0.0003     |
| Type 3 Analysis of Effects                    |                   |                                |            |
| Effect                                        | DF                | Wald<br>Chi-Square             | Pr > ChiSq |
| School                                        | 4                 | 14.8424                        | 0.0050     |
| Program                                       | 2                 | 10.9160                        | 0.0043     |

The parameter estimates, tests for individual parameters, and odds ratios are displayed in [Output 54.4.7](#). The Program variable has nearly the same effect on both logits, while School=1 has the largest effect of the schools.

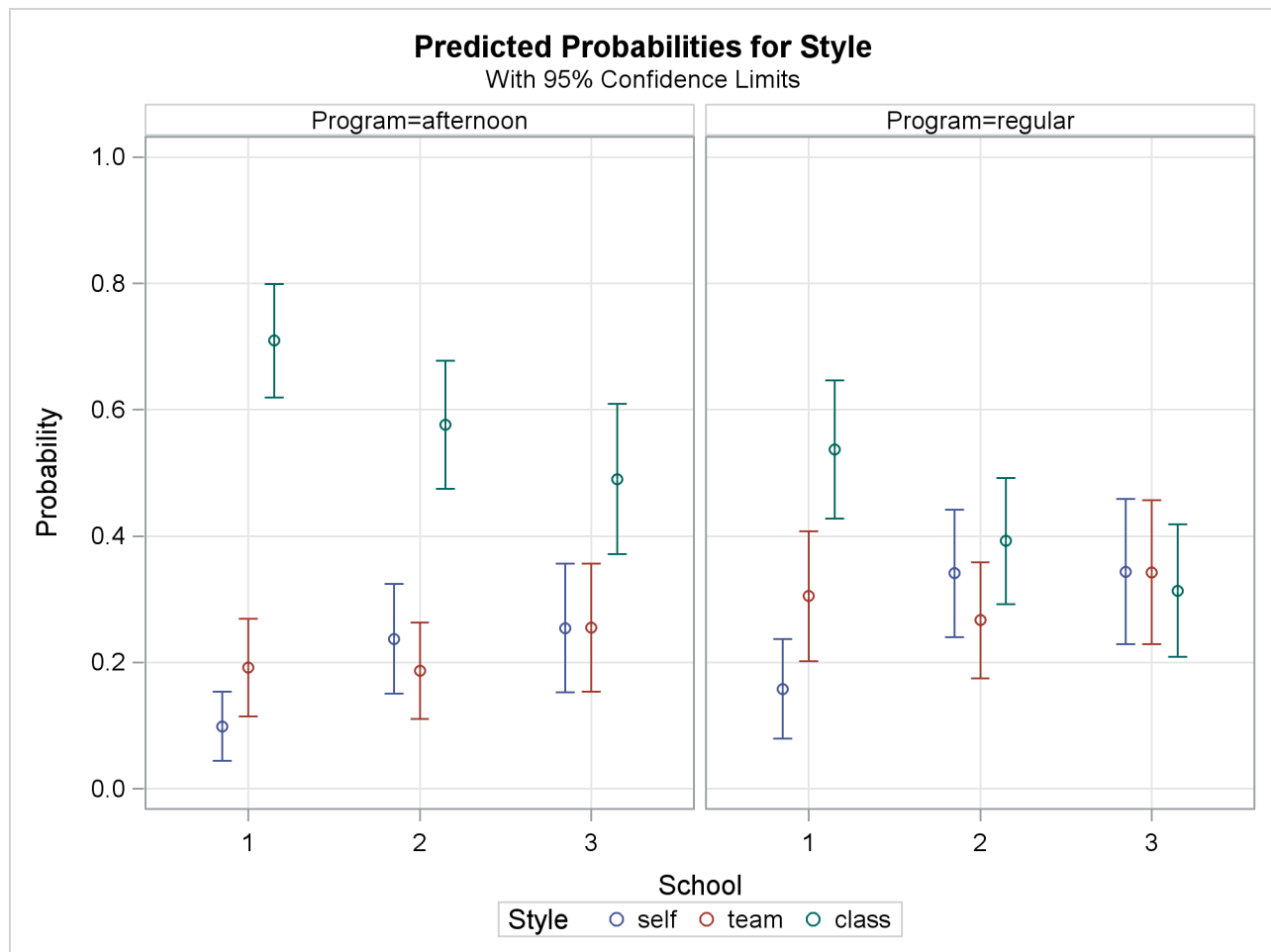
**Output 54.4.7** Estimates

| Analysis of Maximum Likelihood Estimates |       |    |          |                |                 |            |
|------------------------------------------|-------|----|----------|----------------|-----------------|------------|
| Parameter                                | Style | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                | self  | 1  | -0.7978  | 0.1465         | 29.6502         | <.0001     |
| Intercept                                | team  | 1  | -0.6589  | 0.1367         | 23.2300         | <.0001     |
| School 1                                 | self  | 1  | -0.7992  | 0.2198         | 13.2241         | 0.0003     |
| School 1                                 | team  | 1  | -0.2786  | 0.1867         | 2.2269          | 0.1356     |
| School 2                                 | self  | 1  | 0.2836   | 0.1899         | 2.2316          | 0.1352     |
| School 2                                 | team  | 1  | -0.0985  | 0.1892         | 0.2708          | 0.6028     |
| Program regular                          | self  | 1  | 0.3737   | 0.1410         | 7.0272          | 0.0080     |
| Program regular                          | team  | 1  | 0.3713   | 0.1353         | 7.5332          | 0.0061     |

| Odds Ratio Estimates         |       |                |                            |       |
|------------------------------|-------|----------------|----------------------------|-------|
| Effect                       | Style | Point Estimate | 95% Wald Confidence Limits |       |
| School 1 vs 3                | self  | 0.269          | 0.127                      | 0.570 |
| School 1 vs 3                | team  | 0.519          | 0.267                      | 1.010 |
| School 2 vs 3                | self  | 0.793          | 0.413                      | 1.522 |
| School 2 vs 3                | team  | 0.622          | 0.317                      | 1.219 |
| Program regular vs afternoon | self  | 2.112          | 1.215                      | 3.670 |
| Program regular vs afternoon | team  | 2.101          | 1.237                      | 3.571 |

The interaction plots in [Output 54.4.8](#) show that School=1 and Program=afternoon have a preference for the traditional classroom style. Of course, since these are not simultaneous confidence intervals, the nonoverlapping 95% confidence limits do not take the place of an actual test.

**Output 54.4.8** Model-Predicted Probabilities

### Example 54.5: Stratified Sampling

Consider the hypothetical example in Fleiss (1981, pp. 6–7), in which a test is applied to a sample of 1,000 people known to have a disease and to another sample of 1,000 people known not to have the same disease. In the diseased sample, 950 test positive; in the nondiseased sample, only 10 test positive. If the true disease rate in the population is 1 in 100, specifying **PEVENT=0.01** results in the correct false positive and negative rates for the stratified sampling scheme. Omitting the **PEVENT=** option is equivalent to using the overall sample disease rate ( $1000/2000 = 0.5$ ) as the value of the **PEVENT=** option, which would ignore the stratified sampling.

The statements to create the data set and perform the analysis are as follows:

```
data Screen;
  do Disease='Present', 'Absent';
    do Test=1,0;
      input Count @@;
      output;
    end;
  end;
```

```

    datalines;
950  50
10  990
;

proc logistic data=Screen;
    freq Count;
    model Disease(event='Present')=Test
        / pevent=.5 .01 ctable pprob=.5;
run;

```

The response variable option **EVENT=** indicates that `Disease='Present'` is the event. The **CTABLE** option is specified to produce a classification table. Specifying **PPROB=0.5** indicates a cutoff probability of 0.5. A list of two probabilities, 0.5 and 0.01, is specified for the **PEVENT=** option; 0.5 corresponds to the overall sample disease rate, and 0.01 corresponds to a true disease rate of 1 in 100.

The classification table is shown in [Output 54.5.1](#).

**Output 54.5.1** False Positive and False Negative Rates

| The LOGISTIC Procedure |               |         |               |           |               |         |                  |                  |              |              |
|------------------------|---------------|---------|---------------|-----------|---------------|---------|------------------|------------------|--------------|--------------|
| Classification Table   |               |         |               |           |               |         |                  |                  |              |              |
| Prob<br>Event          | Prob<br>Level | Correct |               | Incorrect |               | Correct | Percentages      |                  |              |              |
|                        |               | Event   | Non-<br>Event | Event     | Non-<br>Event |         | Sensi-<br>tivity | Speci-<br>ficity | False<br>POS | False<br>NEG |
| 0.500                  | 0.500         | 950     | 990           | 10        | 50            | 97.0    | 95.0             | 99.0             | 1.0          | 4.8          |
| 0.010                  | 0.500         | 950     | 990           | 10        | 50            | 99.0    | 95.0             | 99.0             | 51.0         | 0.1          |

In the classification table, the column “Prob Level” represents the cutoff values (the settings of the **PPROB=** option) for predicting whether an observation is an event. The “Correct” columns list the numbers of subjects that are correctly predicted as events and nonevents, respectively, and the “Incorrect” columns list the number of nonevents incorrectly predicted as events and the number of events incorrectly predicted as nonevents, respectively. For **PEVENT=0.5**, the false positive rate is 1% and the false negative rate is 4.8%. These results ignore the fact that the samples were stratified and incorrectly assume that the overall sample proportion of disease (which is 0.5) estimates the true disease rate. For a true disease rate of 0.01, the false positive rate and the false negative rate are 51% and 0.1%, respectively, as shown in the second line of the classification table.

## Example 54.6: Logistic Regression Diagnostics

In a controlled experiment to study the effect of the rate and volume of air intake on a transient reflex vasoconstriction in the skin of the digits, 39 tests under various combinations of rate and volume of air intake were obtained (Finney 1947). The endpoint of each test is whether or not vasoconstriction occurred. Pregibon (1981) uses this set of data to illustrate the diagnostic measures he proposes for detecting influential observations and to quantify their effects on various aspects of the maximum likelihood fit.

The vasoconstriction data are saved in the data set vaso:

```
data vaso;
  length Response $12;
  input Volume Rate Response @@;
  LogVolume=log(Volume);
  LogRate=log(Rate);
  datalines;
3.70 0.825 constrict      3.50 1.09 constrict
1.25 2.50 constrict      0.75 1.50 constrict
0.80 3.20 constrict      0.70 3.50 constrict
0.60 0.75 no_constrict   1.10 1.70 no_constrict
0.90 0.75 no_constrict   0.90 0.45 no_constrict
0.80 0.57 no_constrict   0.55 2.75 no_constrict
0.60 3.00 no_constrict   1.40 2.33 constrict
0.75 3.75 constrict      2.30 1.64 constrict
3.20 1.60 constrict      0.85 1.415 constrict
1.70 1.06 no_constrict   1.80 1.80 constrict
0.40 2.00 no_constrict   0.95 1.36 no_constrict
1.35 1.35 no_constrict   1.50 1.36 no_constrict
1.60 1.78 constrict      0.60 1.50 no_constrict
1.80 1.50 constrict      0.95 1.90 no_constrict
1.90 0.95 constrict      1.60 0.40 no_constrict
2.70 0.75 constrict      2.35 0.03 no_constrict
1.10 1.83 no_constrict   1.10 2.20 constrict
1.20 2.00 constrict      0.80 3.33 constrict
0.95 1.90 no_constrict   0.75 1.90 no_constrict
1.30 1.625 constrict
;
```

In the data set vaso, the variable Response represents the outcome of a test. The variable LogVolume represents the log of the volume of air intake, and the variable LogRate represents the log of the rate of air intake.

The following statements invoke PROC LOGISTIC to fit a logistic regression model to the vasoconstriction data, where Response is the response variable, and LogRate and LogVolume are the explanatory variables. Regression diagnostics are displayed when ODS Graphics is enabled, and the [INFLUENCE](#) option is specified to display a table of the regression diagnostics.

```
ods graphics on;
title 'Occurrence of Vasoconstriction';
proc logistic data=vaso;
  model Response=LogRate LogVolume/influence iplots;
run;
ods graphics off;
```

Results of the model fit are shown in [Output 54.6.1](#). Both LogRate and LogVolume are statistically significant to the occurrence of vasoconstriction ( $p = 0.0131$  and  $p = 0.0055$ , respectively). Their positive parameter estimates indicate that a higher inspiration rate or a larger volume of air intake is likely to increase the probability of vasoconstriction.

**Output 54.6.1** Logistic Regression Analysis for Vasoconstriction Data

| Occurrence of Vasoconstriction                |                  |                          |            |
|-----------------------------------------------|------------------|--------------------------|------------|
| The LOGISTIC Procedure                        |                  |                          |            |
| Model Information                             |                  |                          |            |
| Data Set                                      | WORK.VASO        |                          |            |
| Response Variable                             | Response         |                          |            |
| Number of Response Levels                     | 2                |                          |            |
| Model                                         | binary logit     |                          |            |
| Optimization Technique                        | Fisher's scoring |                          |            |
|                                               |                  |                          |            |
| Number of Observations Read                   | 39               |                          |            |
| Number of Observations Used                   | 39               |                          |            |
|                                               |                  |                          |            |
| Response Profile                              |                  |                          |            |
| Ordered Value                                 | Response         | Total Frequency          |            |
| 1                                             | constrict        | 20                       |            |
| 2                                             | no_constrict     | 19                       |            |
|                                               |                  |                          |            |
| Probability modeled is Response='constrict'.  |                  |                          |            |
|                                               |                  |                          |            |
| Model Convergence Status                      |                  |                          |            |
| Convergence criterion (GCONV=1E-8) satisfied. |                  |                          |            |
|                                               |                  |                          |            |
| Model Fit Statistics                          |                  |                          |            |
| Criterion                                     | Intercept Only   | Intercept and Covariates |            |
| AIC                                           | 56.040           | 35.227                   |            |
| SC                                            | 57.703           | 40.218                   |            |
| -2 Log L                                      | 54.040           | 29.227                   |            |
|                                               |                  |                          |            |
| Testing Global Null Hypothesis: BETA=0        |                  |                          |            |
| Test                                          | Chi-Square       | DF                       | Pr > ChiSq |
| Likelihood Ratio                              | 24.8125          | 2                        | <.0001     |
| Score                                         | 16.6324          | 2                        | 0.0002     |
| Wald                                          | 7.8876           | 2                        | 0.0194     |

Output 54.6.1 *continued*

| Analysis of Maximum Likelihood Estimates |    |          |                |                 |            |
|------------------------------------------|----|----------|----------------|-----------------|------------|
| Parameter                                | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                | 1  | -2.8754  | 1.3208         | 4.7395          | 0.0295     |
| LogRate                                  | 1  | 4.5617   | 1.8380         | 6.1597          | 0.0131     |
| LogVolume                                | 1  | 5.1793   | 1.8648         | 7.7136          | 0.0055     |

| Odds Ratio Estimates |                |                            |          |
|----------------------|----------------|----------------------------|----------|
| Effect               | Point Estimate | 95% Wald Confidence Limits |          |
| LogRate              | 95.744         | 2.610                      | >999.999 |
| LogVolume            | 177.562        | 4.592                      | >999.999 |

| Association of Predicted Probabilities and Observed Responses |      |           |       |  |
|---------------------------------------------------------------|------|-----------|-------|--|
| Percent Concordant                                            | 93.7 | Somers' D | 0.874 |  |
| Percent Discordant                                            | 6.3  | Gamma     | 0.874 |  |
| Percent Tied                                                  | 0.0  | Tau-a     | 0.448 |  |
| Pairs                                                         | 380  | c         | 0.937 |  |

The INFLUENCE option displays the values of the explanatory variables (LogRate and LogVolume) for each observation, a column for each diagnostic produced, and the *case number* that represents the sequence number of the observation (Output 54.6.2).



**Output 54.6.2** Regression Diagnostics from the INFLUENCE Option

| Regression Diagnostics |            |               |                     |                      |                           |                     |                   |
|------------------------|------------|---------------|---------------------|----------------------|---------------------------|---------------------|-------------------|
| Case<br>Number         | Covariates |               | Pearson<br>Residual | Deviance<br>Residual | Hat<br>Matrix<br>Diagonal | Intercept<br>DfBeta | LogRate<br>DfBeta |
|                        | LogRate    | Log<br>Volume |                     |                      |                           |                     |                   |
| 1                      | -0.1924    | 1.3083        | 0.2205              | 0.3082               | 0.0927                    | -0.0165             | 0.0193            |
| 2                      | 0.0862     | 1.2528        | 0.1349              | 0.1899               | 0.0429                    | -0.0134             | 0.0151            |
| 3                      | 0.9163     | 0.2231        | 0.2923              | 0.4049               | 0.0612                    | -0.0492             | 0.0660            |
| 4                      | 0.4055     | -0.2877       | 3.5181              | 2.2775               | 0.0867                    | 1.0734              | -0.9302           |
| 5                      | 1.1632     | -0.2231       | 0.5287              | 0.7021               | 0.1158                    | -0.0832             | 0.1411            |
| 6                      | 1.2528     | -0.3567       | 0.6090              | 0.7943               | 0.1524                    | -0.0922             | 0.1710            |
| 7                      | -0.2877    | -0.5108       | -0.0328             | -0.0464              | 0.00761                   | -0.00280            | 0.00274           |
| 8                      | 0.5306     | 0.0953        | -1.0196             | -1.1939              | 0.0559                    | -0.1444             | 0.0613            |
| 9                      | -0.2877    | -0.1054       | -0.0938             | -0.1323              | 0.0342                    | -0.0178             | 0.0173            |
| 10                     | -0.7985    | -0.1054       | -0.0293             | -0.0414              | 0.00721                   | -0.00245            | 0.00246           |
| 11                     | -0.5621    | -0.2231       | -0.0370             | -0.0523              | 0.00969                   | -0.00361            | 0.00358           |
| 12                     | 1.0116     | -0.5978       | -0.5073             | -0.6768              | 0.1481                    | -0.1173             | 0.0647            |
| 13                     | 1.0986     | -0.5108       | -0.7751             | -0.9700              | 0.1628                    | -0.0931             | -0.00946          |
| 14                     | 0.8459     | 0.3365        | 0.2559              | 0.3562               | 0.0551                    | -0.0414             | 0.0538            |
| 15                     | 1.3218     | -0.2877       | 0.4352              | 0.5890               | 0.1336                    | -0.0940             | 0.1408            |
| 16                     | 0.4947     | 0.8329        | 0.1576              | 0.2215               | 0.0402                    | -0.0198             | 0.0234            |
| 17                     | 0.4700     | 1.1632        | 0.0709              | 0.1001               | 0.0172                    | -0.00630            | 0.00701           |
| 18                     | 0.3471     | -0.1625       | 2.9062              | 2.1192               | 0.0954                    | 0.9595              | -0.8279           |
| 19                     | 0.0583     | 0.5306        | -1.0718             | -1.2368              | 0.1315                    | -0.2591             | 0.2024            |
| 20                     | 0.5878     | 0.5878        | 0.2405              | 0.3353               | 0.0525                    | -0.0331             | 0.0421            |
| 21                     | 0.6931     | -0.9163       | -0.1076             | -0.1517              | 0.0373                    | -0.0180             | 0.0158            |
| 22                     | 0.3075     | -0.0513       | -0.4193             | -0.5691              | 0.1015                    | -0.1449             | 0.1237            |
| 23                     | 0.3001     | 0.3001        | -1.0242             | -1.1978              | 0.0761                    | -0.1961             | 0.1275            |
| 24                     | 0.3075     | 0.4055        | -1.3684             | -1.4527              | 0.0717                    | -0.1281             | 0.0410            |
| 25                     | 0.5766     | 0.4700        | 0.3347              | 0.4608               | 0.0587                    | -0.0403             | 0.0570            |
| 26                     | 0.4055     | -0.5108       | -0.1595             | -0.2241              | 0.0548                    | -0.0366             | 0.0329            |
| 27                     | 0.4055     | 0.5878        | 0.3645              | 0.4995               | 0.0661                    | -0.0327             | 0.0496            |
| 28                     | 0.6419     | -0.0513       | -0.8989             | -1.0883              | 0.0647                    | -0.1423             | 0.0617            |
| 29                     | -0.0513    | 0.6419        | 0.8981              | 1.0876               | 0.1682                    | 0.2367              | -0.1950           |
| 30                     | -0.9163    | 0.4700        | -0.0992             | -0.1400              | 0.0507                    | -0.0224             | 0.0227            |
| 31                     | -0.2877    | 0.9933        | 0.6198              | 0.8064               | 0.2459                    | 0.1165              | -0.0996           |
| 32                     | -3.5066    | 0.8544        | -0.00073            | -0.00103             | 0.000022                  | -3.22E-6            | 3.405E-6          |
| 33                     | 0.6043     | 0.0953        | -1.2062             | -1.3402              | 0.0510                    | -0.0882             | -0.0137           |
| 34                     | 0.7885     | 0.0953        | 0.5447              | 0.7209               | 0.0601                    | -0.0425             | 0.0877            |
| 35                     | 0.6931     | 0.1823        | 0.5404              | 0.7159               | 0.0552                    | -0.0340             | 0.0755            |
| 36                     | 1.2030     | -0.2231       | 0.4828              | 0.6473               | 0.1177                    | -0.0867             | 0.1381            |
| 37                     | 0.6419     | -0.0513       | -0.8989             | -1.0883              | 0.0647                    | -0.1423             | 0.0617            |
| 38                     | 0.6419     | -0.2877       | -0.4874             | -0.6529              | 0.1000                    | -0.1395             | 0.1032            |
| 39                     | 0.4855     | 0.2624        | 0.7053              | 0.8987               | 0.0531                    | 0.0326              | 0.0190            |

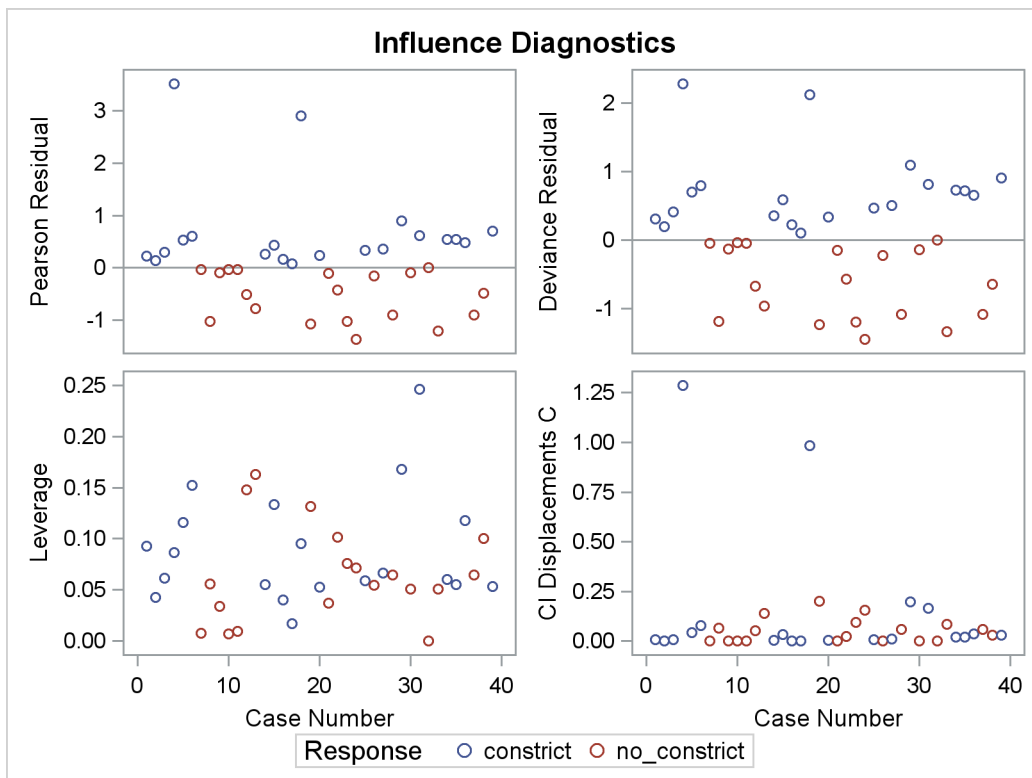
Output 54.6.2 continued

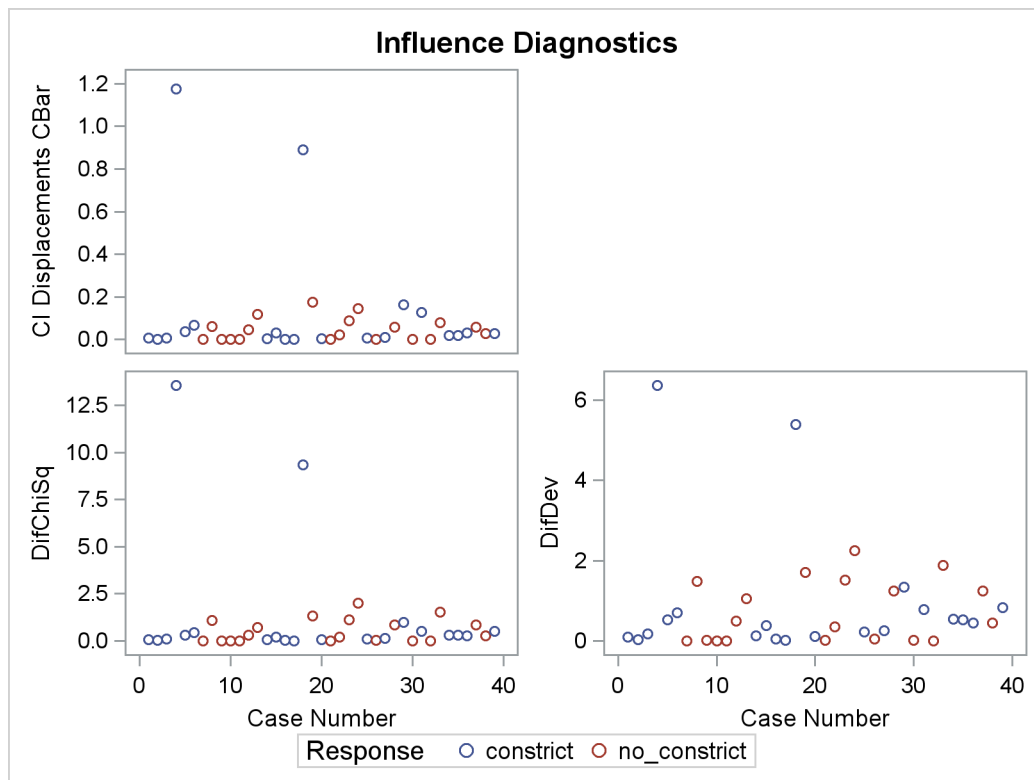
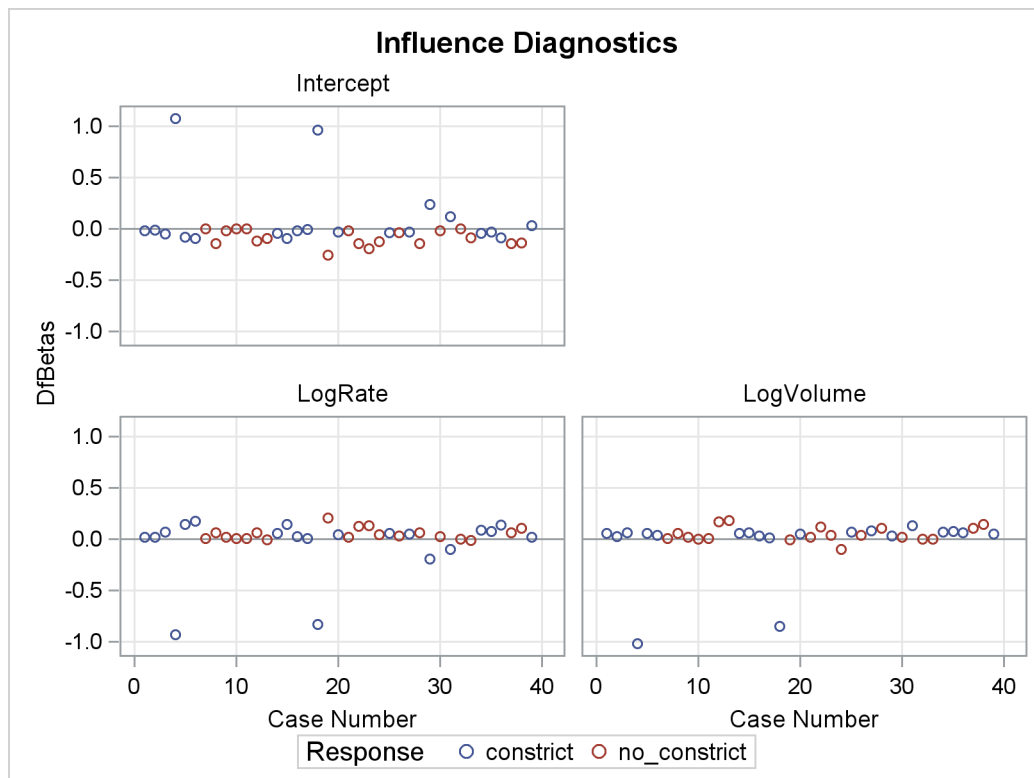
| Regression Diagnostics |                   |                                    |                                       |                |                  |
|------------------------|-------------------|------------------------------------|---------------------------------------|----------------|------------------|
| Case Number            | Log Volume DfBeta | Confidence Interval Displacement C | Confidence Interval Displacement CBar | Delta Deviance | Delta Chi-Square |
| 1                      | 0.0556            | 0.00548                            | 0.00497                               | 0.1000         | 0.0536           |
| 2                      | 0.0261            | 0.000853                           | 0.000816                              | 0.0369         | 0.0190           |
| 3                      | 0.0589            | 0.00593                            | 0.00557                               | 0.1695         | 0.0910           |
| 4                      | -1.0180           | 1.2873                             | 1.1756                                | 6.3626         | 13.5523          |
| 5                      | 0.0583            | 0.0414                             | 0.0366                                | 0.5296         | 0.3161           |
| 6                      | 0.0381            | 0.0787                             | 0.0667                                | 0.6976         | 0.4376           |
| 7                      | 0.00265           | 8.321E-6                           | 8.258E-6                              | 0.00216        | 0.00109          |
| 8                      | 0.0570            | 0.0652                             | 0.0616                                | 1.4870         | 1.1011           |
| 9                      | 0.0153            | 0.000322                           | 0.000311                              | 0.0178         | 0.00911          |
| 10                     | 0.00211           | 6.256E-6                           | 6.211E-6                              | 0.00172        | 0.000862         |
| 11                     | 0.00319           | 0.000014                           | 0.000013                              | 0.00274        | 0.00138          |
| 12                     | 0.1651            | 0.0525                             | 0.0447                                | 0.5028         | 0.3021           |
| 13                     | 0.1775            | 0.1395                             | 0.1168                                | 1.0577         | 0.7175           |
| 14                     | 0.0527            | 0.00404                            | 0.00382                               | 0.1307         | 0.0693           |
| 15                     | 0.0643            | 0.0337                             | 0.0292                                | 0.3761         | 0.2186           |
| 16                     | 0.0307            | 0.00108                            | 0.00104                               | 0.0501         | 0.0259           |
| 17                     | 0.00914           | 0.000089                           | 0.000088                              | 0.0101         | 0.00511          |
| 18                     | -0.8477           | 0.9845                             | 0.8906                                | 5.3817         | 9.3363           |
| 19                     | -0.00488          | 0.2003                             | 0.1740                                | 1.7037         | 1.3227           |
| 20                     | 0.0518            | 0.00338                            | 0.00320                               | 0.1156         | 0.0610           |
| 21                     | 0.0208            | 0.000465                           | 0.000448                              | 0.0235         | 0.0120           |
| 22                     | 0.1179            | 0.0221                             | 0.0199                                | 0.3437         | 0.1956           |
| 23                     | 0.0357            | 0.0935                             | 0.0864                                | 1.5212         | 1.1355           |
| 24                     | -0.1004           | 0.1558                             | 0.1447                                | 2.2550         | 2.0171           |
| 25                     | 0.0708            | 0.00741                            | 0.00698                               | 0.2193         | 0.1190           |
| 26                     | 0.0373            | 0.00156                            | 0.00147                               | 0.0517         | 0.0269           |
| 27                     | 0.0788            | 0.0101                             | 0.00941                               | 0.2589         | 0.1423           |
| 28                     | 0.1025            | 0.0597                             | 0.0559                                | 1.2404         | 0.8639           |
| 29                     | 0.0286            | 0.1961                             | 0.1631                                | 1.3460         | 0.9697           |
| 30                     | 0.0159            | 0.000554                           | 0.000526                              | 0.0201         | 0.0104           |
| 31                     | 0.1322            | 0.1661                             | 0.1253                                | 0.7755         | 0.5095           |
| 32                     | 2.48E-6           | 1.18E-11                           | 1.18E-11                              | 1.065E-6       | 5.324E-7         |
| 33                     | -0.00216          | 0.0824                             | 0.0782                                | 1.8744         | 1.5331           |
| 34                     | 0.0671            | 0.0202                             | 0.0190                                | 0.5387         | 0.3157           |
| 35                     | 0.0711            | 0.0180                             | 0.0170                                | 0.5295         | 0.3091           |
| 36                     | 0.0631            | 0.0352                             | 0.0311                                | 0.4501         | 0.2641           |
| 37                     | 0.1025            | 0.0597                             | 0.0559                                | 1.2404         | 0.8639           |
| 38                     | 0.1397            | 0.0293                             | 0.0264                                | 0.4526         | 0.2639           |
| 39                     | 0.0489            | 0.0295                             | 0.0279                                | 0.8355         | 0.5254           |

The index plots produced by the IPLOTS option are essentially the same line-printer plots as those produced by the INFLUENCE option, but with a 90-degree rotation and perhaps on a more refined scale. Since ODS Graphics is enabled, the line-printer plots from the INFLUENCE and IPLOTS options are suppressed and ODS Graphics versions of the plots are displayed in Outputs 54.6.3 through 54.6.5. For general information about ODS Graphics, see Chapter 21, “[Statistical Graphics Using ODS](#).” For specific information about the graphics available in the LOGISTIC procedure, see the section “[ODS Graphics](#)” on page 4294. The vertical axis of an index plot represents the value of the diagnostic, and the horizontal axis represents the sequence (case number) of the observation. The index plots are useful for identification of extreme values.

The index plots of the Pearson residuals and the deviance residuals ([Output 54.6.3](#)) indicate that case 4 and case 18 are poorly accounted for by the model. The index plot of the diagonal elements of the hat matrix ([Output 54.6.3](#)) suggests that case 31 is an extreme point in the design space. The index plots of DFBETAS ([Output 54.6.5](#)) indicate that case 4 and case 18 are causing instability in all three parameter estimates. The other four index plots in Outputs 54.6.3 and 54.6.4 also point to these two cases as having a large impact on the coefficients and goodness of fit.

**Output 54.6.3** Residuals, Hat Matrix, and CI Displacement C

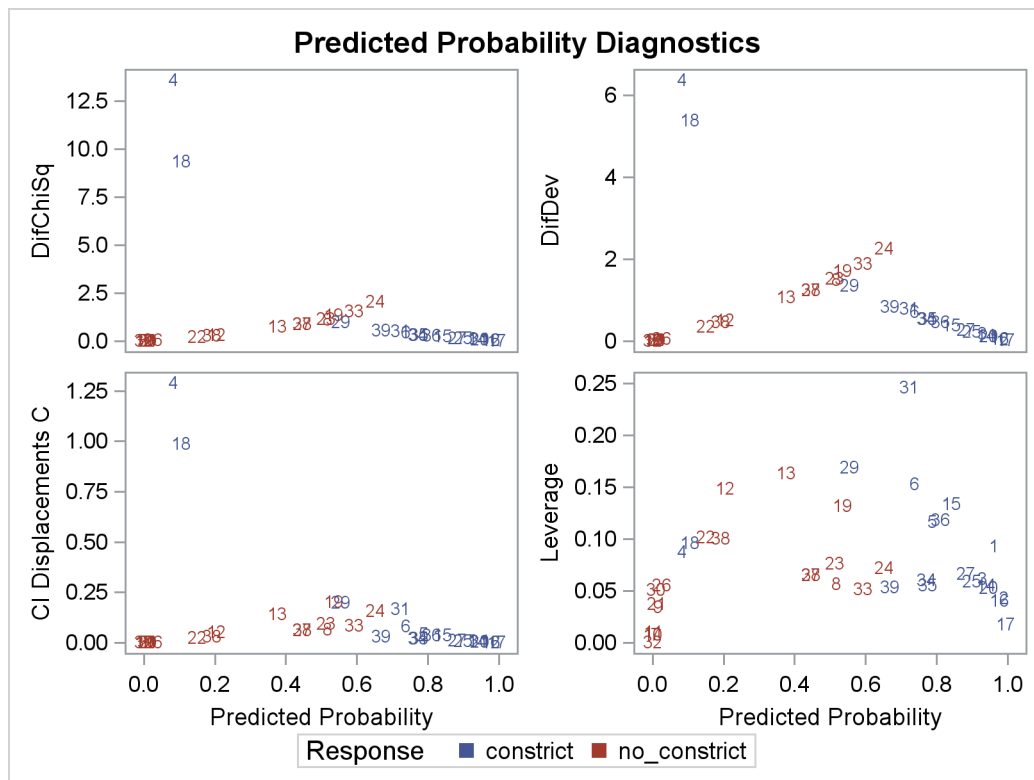


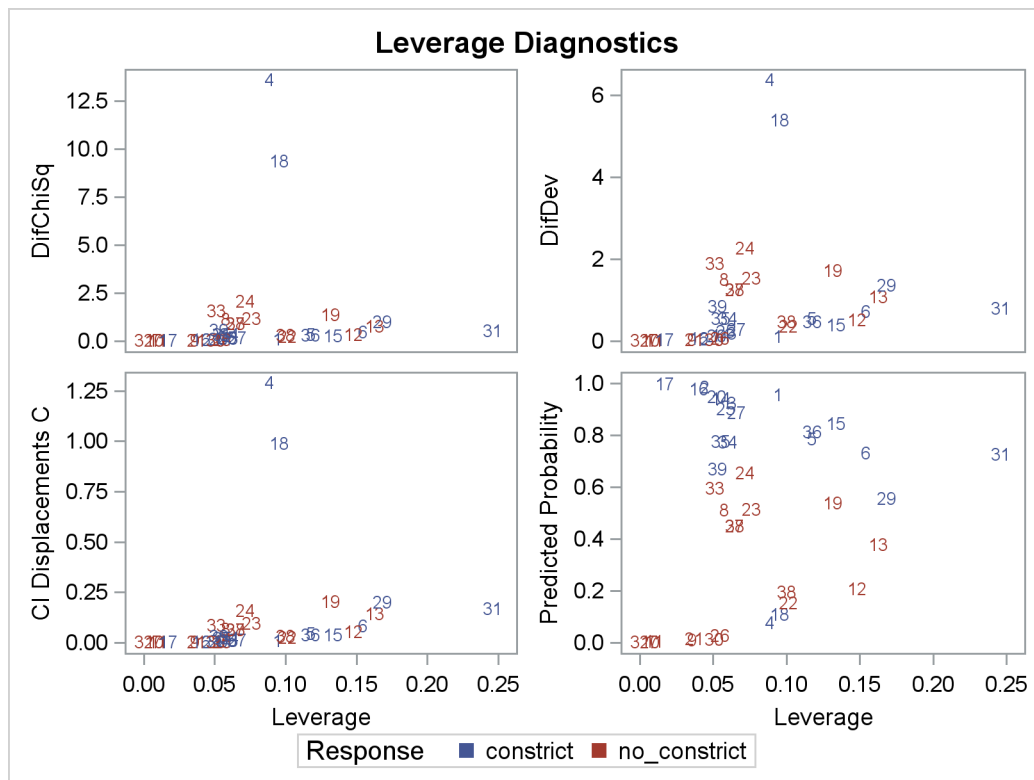
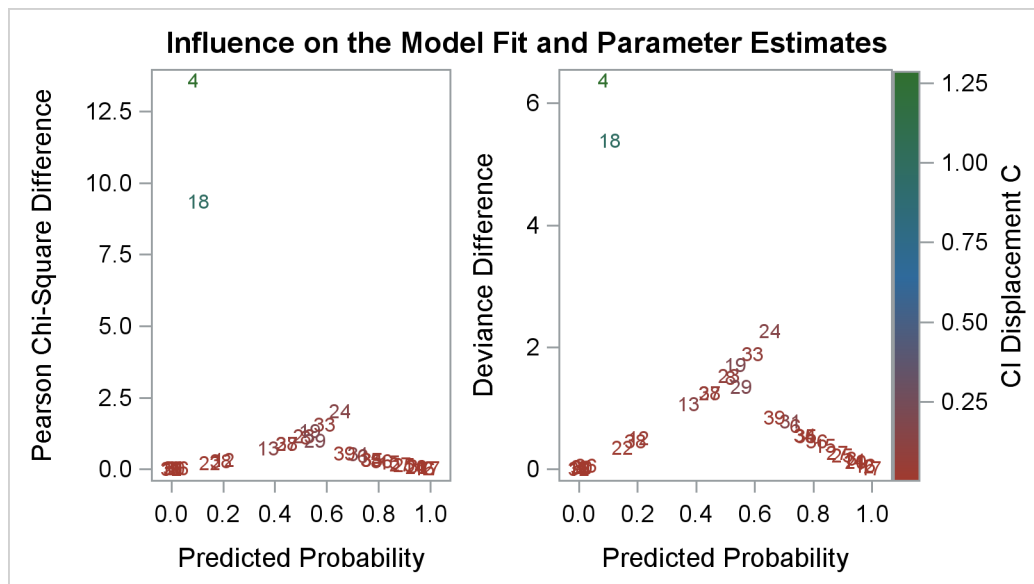
**Output 54.6.4** CI Displacement CBar, Change in Deviance and Pearson Chi-Square**Output 54.6.5** DFBETAS Plots

Other versions of diagnostic plots can be requested by specifying the appropriate options in the **PLOTS=** option. For example, the following statements produce three other sets of influence diagnostic plots: the **PHAT** option plots several diagnostics against the predicted probabilities ([Output 54.6.6](#)), the **LEVERAGE** option plots several diagnostics against the leverage ([Output 54.6.7](#)), and the **DPC** option plots the deletion diagnostics against the predicted probabilities and colors the observations according to the confidence interval displacement diagnostic ([Output 54.6.8](#)). The **LABEL** option displays the observation numbers on the plots. In all plots, you are looking for the outlying observations, and again cases 4 and 18 are noted.

```
ods graphics on;
proc logistic data=vaso plots(only label)=(phat leverage dpc);
  model Response=LogRate LogVolume;
run;
ods graphics off;
```

**Output 54.6.6** Diagnostics versus Predicted Probability



**Output 54.6.7** Diagnostics versus Leverage**Output 54.6.8** Three Diagnostics

## Example 54.7: ROC Curve, Customized Odds Ratios, Goodness-of-Fit Statistics, R-Square, and Confidence Limits

This example plots an ROC curve, estimates a customized odds ratio, produces the traditional goodness-of-fit analysis, displays the generalized R Square measures for the fitted model, calculates the normal confidence intervals for the regression parameters, and produces a display of the probability function and prediction curves for the fitted model. The data consist of three variables: *n* (number of subjects in the sample), *disease* (number of diseased subjects in the sample), and *age* (age for the sample). A linear logistic regression model is used to study the effect of age on the probability of contracting the disease. The statements to produce the data set and perform the analysis are as follows:

```
data Data1;
    input disease n age;
    datalines;
    0 14 25
    0 20 35
    0 19 45
    7 18 55
    6 12 65
    17 17 75
;

ods graphics on;
proc logistic data=Data1 plots(only)=roc(id=obs);
    model disease/n=age / scale=none
                        clparm=wald
                        clodds=pl
                        rsquare;

    units age=10;
    effectplot;
run;
ods graphics off;
```

The option **SCALE=NONE** is specified to produce the deviance and Pearson goodness-of-fit analysis without adjusting for overdispersion. The **RSQUARE** option is specified to produce generalized R Square measures of the fitted model. The **CLPARM=WALD** option is specified to produce the Wald confidence intervals for the regression parameters. The **UNITS** statement is specified to produce customized odds ratio estimates for a change of 10 years in the age variable, and the **CLODDS=PL** option is specified to produce profile-likelihood confidence limits for the odds ratio. The **PLOTS=** option with ODS Graphics enabled produces a graphical display of the ROC curve, and the **EFFECTPLOT** statement displays the model fit.

The results in [Output 54.7.1](#) show that the deviance and Pearson statistics indicate no lack of fit in the model.

**Output 54.7.1** Deviance and Pearson Goodness-of-Fit Analysis

| The LOGISTIC Procedure                          |        |    |          |            |
|-------------------------------------------------|--------|----|----------|------------|
| Deviance and Pearson Goodness-of-Fit Statistics |        |    |          |            |
| Criterion                                       | Value  | DF | Value/DF | Pr > ChiSq |
| Deviance                                        | 7.7756 | 4  | 1.9439   | 0.1002     |
| Pearson                                         | 6.6020 | 4  | 1.6505   | 0.1585     |
| Number of events/trials observations: 6         |        |    |          |            |

Output 54.7.2 shows that the R-square for the model is 0.74. The odds of an event increases by a factor of 7.9 for each 10-year increase in age.

**Output 54.7.2** R-Square, Confidence Intervals, and Customized Odds Ratio

| Model Fit Statistics                     |            |                          |                     |                 |            |
|------------------------------------------|------------|--------------------------|---------------------|-----------------|------------|
| Criterion                                | Intercept  | Intercept and Covariates |                     |                 |            |
|                                          | Only       | Log Likelihood           | Full Log Likelihood |                 |            |
| AIC                                      | 124.173    | 52.468                   | 18.075              |                 |            |
| SC                                       | 126.778    | 57.678                   | 23.285              |                 |            |
| -2 Log L                                 | 122.173    | 48.468                   | 14.075              |                 |            |
| R-Square                                 | 0.5215     | Max-rescaled R-Square    | 0.8925              |                 |            |
| Testing Global Null Hypothesis: BETA=0   |            |                          |                     |                 |            |
| Test                                     | Chi-Square | DF                       | Pr > ChiSq          |                 |            |
| Likelihood Ratio                         | 73.7048    | 1                        | <.0001              |                 |            |
| Score                                    | 55.3274    | 1                        | <.0001              |                 |            |
| Wald                                     | 23.3475    | 1                        | <.0001              |                 |            |
| Analysis of Maximum Likelihood Estimates |            |                          |                     |                 |            |
| Parameter                                | DF         | Estimate                 | Standard Error      | Wald Chi-Square | Pr > ChiSq |
| Intercept                                | 1          | -12.5016                 | 2.5555              | 23.9317         | <.0001     |
| age                                      | 1          | 0.2066                   | 0.0428              | 23.3475         | <.0001     |



**Output 54.7.2** *continued***Association of Predicted Probabilities and Observed Responses**

|                    |      |           |       |
|--------------------|------|-----------|-------|
| Percent Concordant | 92.6 | Somers' D | 0.906 |
| Percent Discordant | 2.0  | Gamma     | 0.958 |
| Percent Tied       | 5.4  | Tau-a     | 0.384 |
| Pairs              | 2100 | c         | 0.953 |

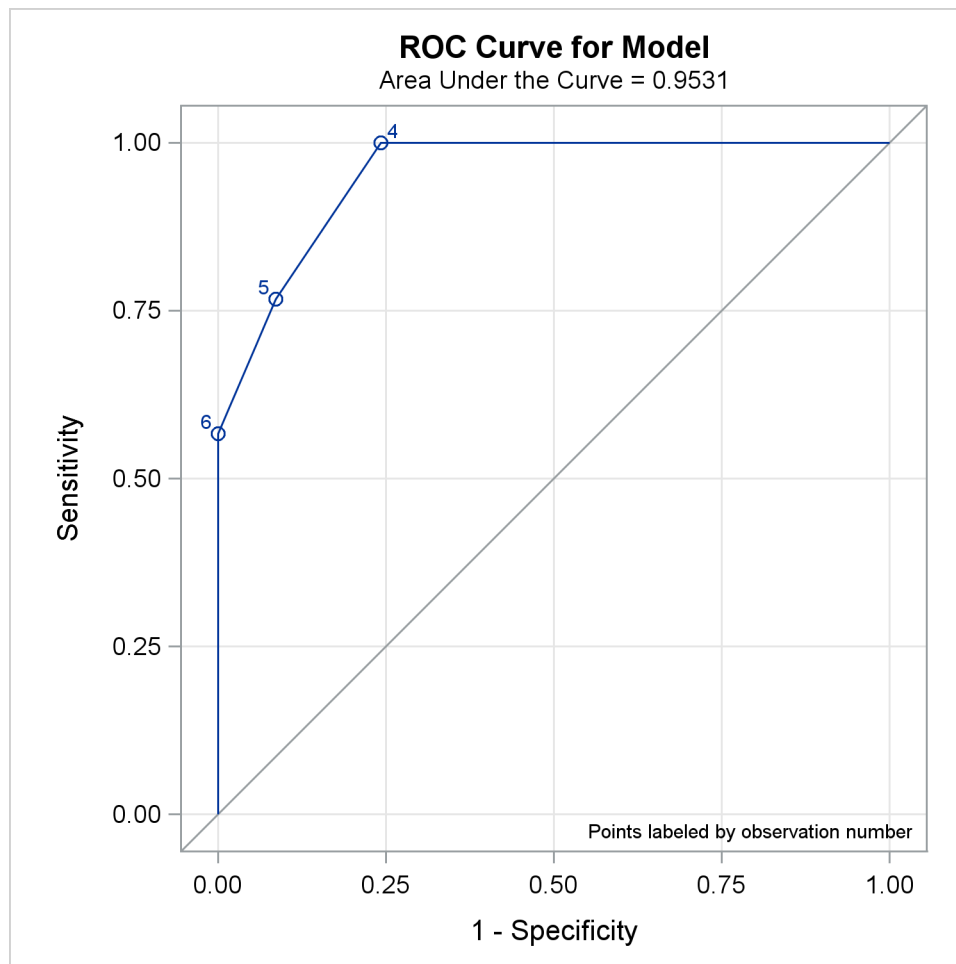
**Parameter Estimates and Wald  
Confidence Intervals**

| Parameter | Estimate | 95% Confidence Limits |         |
|-----------|----------|-----------------------|---------|
| Intercept | -12.5016 | -17.5104              | -7.4929 |
| age       | 0.2066   | 0.1228                | 0.2904  |

**Odds Ratio Estimates and Profile-Likelihood Confidence Intervals**

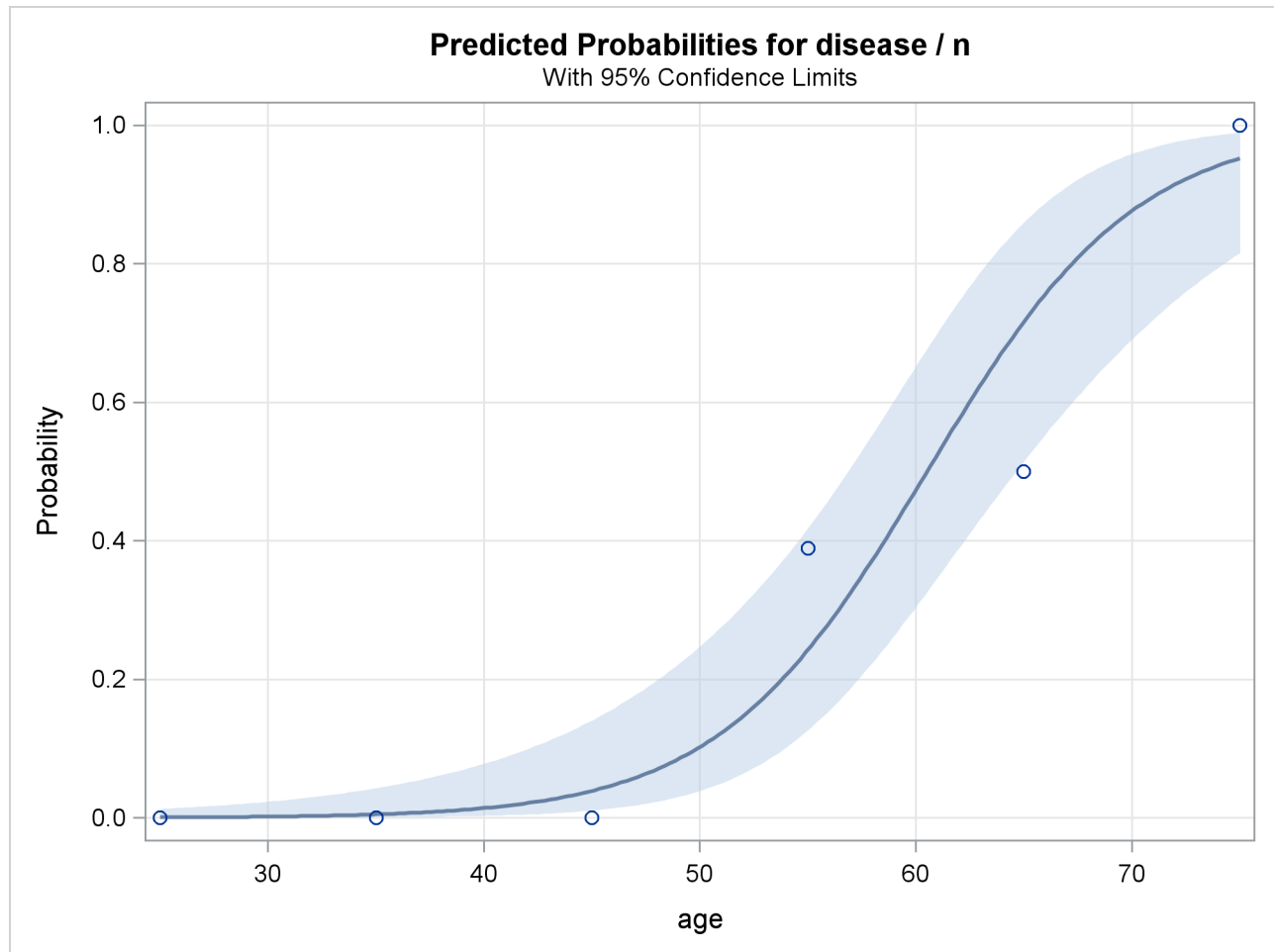
| Effect | Unit    | Estimate | 95% Confidence Limits |        |
|--------|---------|----------|-----------------------|--------|
| age    | 10.0000 | 7.892    | 3.881                 | 21.406 |

Since ODS Graphics is enabled, a graphical display of the ROC curve is produced as shown in [Output 54.7.3](#).

**Output 54.7.3** Receiver Operating Characteristic Curve

Note that the area under the ROC curve is estimated by the statistic  $c$  in the “Association of Predicted Probabilities and Observed Responses” table. In this example, the area under the ROC curve is 0.953.

Since there is only one continuous covariate and since ODS Graphics is enabled, the [EFFECTPLOT](#) statement produces a graphical display of the predicted probability curve with bounding 95% confidence limits as shown in [Output 54.7.4](#).

**Output 54.7.4** Predicted Probability and 95% Prediction Limits

### Example 54.8: Comparing Receiver Operating Characteristic Curves

DeLong, DeLong, and Clarke-Pearson (1988) report on 49 patients with ovarian cancer who also suffer from an intestinal obstruction. Three (correlated) screening tests are measured to determine whether a patient will benefit from surgery. The three tests are the K-G score and two measures of nutritional status: total protein and albumin. The data are as follows:

```
data roc;
  input alb tp totscore popind @@;
  totscore = 10 - totscore;
  datalines;
3.0 5.8 10 0   3.2 6.3 5 1   3.9 6.8 3 1   2.8 4.8 6 0
3.2 5.8 3 1   0.9 4.0 5 0   2.5 5.7 8 0   1.6 5.6 5 1
3.8 5.7 5 1   3.7 6.7 6 1   3.2 5.4 4 1   3.8 6.6 6 1
4.1 6.6 5 1   3.6 5.7 5 1   4.3 7.0 4 1   3.6 6.7 4 0
2.3 4.4 6 1   4.2 7.6 4 0   4.0 6.6 6 0   3.5 5.8 6 1
3.8 6.8 7 1   3.0 4.7 8 0   4.5 7.4 5 1   3.7 7.4 5 1
3.1 6.6 6 1   4.1 8.2 6 1   4.3 7.0 5 1   4.3 6.5 4 1
3.2 5.1 5 1   2.6 4.7 6 1   3.3 6.8 6 0   1.7 4.0 7 0
```

```

3.7 6.1 5 1    3.3 6.3 7 1    4.2 7.7 6 1    3.5 6.2 5 1
2.9 5.7 9 0    2.1 4.8 7 1    2.8 6.2 8 0    4.0 7.0 7 1
3.3 5.7 6 1    3.7 6.9 5 1    3.6 6.6 5 1
;

```

In the following statements, the **NOFIT** option is specified in the **MODEL** statement to prevent PROC LOGISTIC from fitting the model with three covariates. Each **ROC** statement lists one of the covariates, and PROC LOGISTIC then fits the model with that single covariate. Note that the original data set contains six more records with missing values for one of the tests, but PROC LOGISTIC ignores all records with missing values; hence there is a common sample size for each of the three models. The **ROCCONTRAST** statement implements the nonparametric approach of DeLong, DeLong, and Clarke-Pearson (1988) to compare the three ROC curves, the **REFERENCE** option specifies that the K-G Score curve is used as the reference curve in the contrast, the **E** option displays the contrast coefficients, and the **ESTIMATE** option computes and tests each comparison. With ODS Graphics enabled, the **plots=roc(id=prob)** specification in the PROC LOGISTIC statement displays several plots, and the plots of individual ROC curves have certain points labeled with their predicted probabilities.

```

ods graphics on;
proc logistic data=roc plots=roc(id=prob);
  model popind(event='0') = alb tp totscore / nofit;
  roc 'Albumin' alb;
  roc 'K-G Score' totscore;
  roc 'Total Protein' tp;
  roccontrast reference('K-G Score') / estimate e;
run;
ods graphics off;

```

The initial model information is displayed in [Output 54.8.1](#).

**Output 54.8.1** Initial LOGISTIC Output

| The LOGISTIC Procedure           |                  |                    |
|----------------------------------|------------------|--------------------|
| Model Information                |                  |                    |
| Data Set                         | WORK.ROC         |                    |
| Response Variable                | popind           |                    |
| Number of Response Levels        | 2                |                    |
| Model                            | binary logit     |                    |
| Optimization Technique           | Fisher's scoring |                    |
|                                  |                  |                    |
| Number of Observations Read      | 43               |                    |
| Number of Observations Used      | 43               |                    |
|                                  |                  |                    |
| Response Profile                 |                  |                    |
| Ordered<br>Value                 | popind           | Total<br>Frequency |
| 1                                | 0                | 12                 |
| 2                                | 1                | 31                 |
|                                  |                  |                    |
| Probability modeled is popind=0. |                  |                    |

**Output 54.8.1** *continued***Score Test for Global Null Hypothesis**

| Chi-Square | DF | Pr > ChiSq |
|------------|----|------------|
| 10.7939    | 3  | 0.0129     |

For each ROC model, the model fitting details in Outputs 54.8.2, 54.8.4, and 54.8.6 can be suppressed with the **ROCOPTIONS(NODETAILS)** option; however, the convergence status is always displayed.

The ROC curves for the three models are displayed in Outputs 54.8.3, 54.8.5, and 54.8.7. Note that the labels on the ROC curve are produced by specifying the **ID=PROB** option, and are the predicted probabilities for the cutpoints.

**Output 54.8.2** Fit Tables for Popind=Alb**Model Convergence Status**

Convergence criterion (GCONV=1E-8) satisfied.

**Model Fit Statistics**

| Criterion | Intercept<br>Only | Intercept<br>and<br>Covariates |
|-----------|-------------------|--------------------------------|
| AIC       | 52.918            | 49.384                         |
| SC        | 54.679            | 52.907                         |
| -2 Log L  | 50.918            | 45.384                         |

**Testing Global Null Hypothesis: BETA=0**

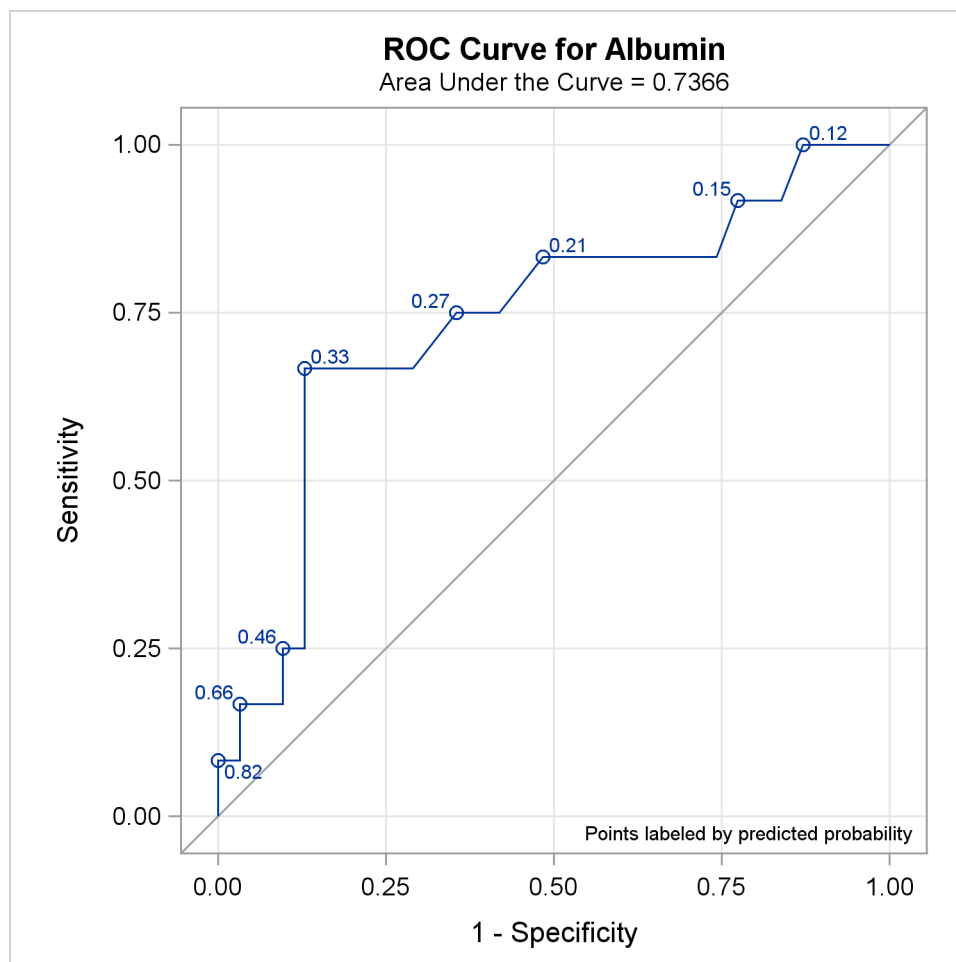
| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 5.5339     | 1  | 0.0187     |
| Score            | 5.6893     | 1  | 0.0171     |
| Wald             | 4.6869     | 1  | 0.0304     |

**Analysis of Maximum Likelihood Estimates**

| Parameter | DF | Estimate | Standard<br>Error | Wald<br>Chi-Square | Pr > ChiSq |
|-----------|----|----------|-------------------|--------------------|------------|
| Intercept | 1  | 2.4646   | 1.5913            | 2.3988             | 0.1214     |
| alb       | 1  | -1.0520  | 0.4859            | 4.6869             | 0.0304     |

**Output 54.8.2** *continued*

| Odds Ratio Estimates |                |                            |       |
|----------------------|----------------|----------------------------|-------|
| Effect               | Point Estimate | 95% Wald Confidence Limits |       |
| alb                  | 0.349          | 0.135                      | 0.905 |

**Output 54.8.3** ROC Curve for Popind=Alb

**Output 54.8.4** Fit Tables for Popind=Totscore**Model Convergence Status**

Convergence criterion (GCONV=1E-8) satisfied.

**Model Fit Statistics**

| Criterion | Intercept<br>Only | Intercept<br>and<br>Covariates |
|-----------|-------------------|--------------------------------|
| AIC       | 52.918            | 46.262                         |
| SC        | 54.679            | 49.784                         |
| -2 Log L  | 50.918            | 42.262                         |

**Testing Global Null Hypothesis: BETA=0**

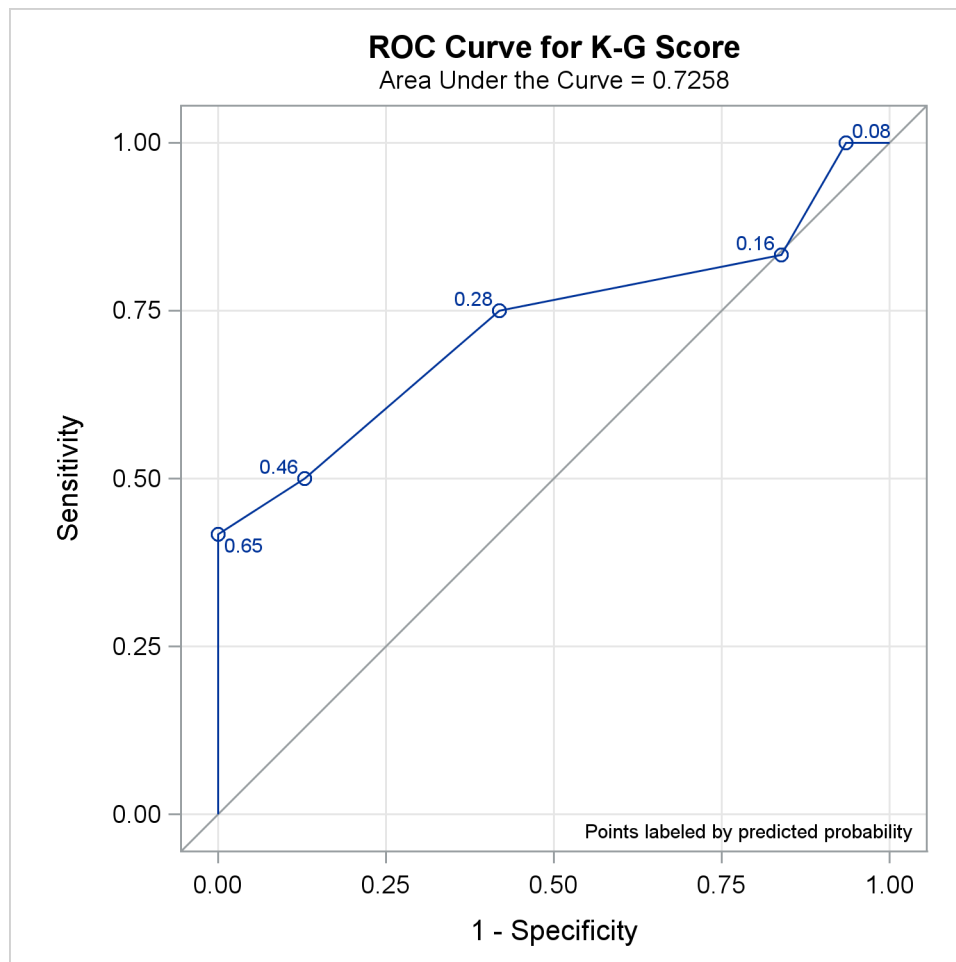
| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 8.6567     | 1  | 0.0033     |
| Score            | 8.3613     | 1  | 0.0038     |
| Wald             | 6.3845     | 1  | 0.0115     |

**Analysis of Maximum Likelihood Estimates**

| Parameter | DF | Estimate | Standard<br>Error | Wald<br>Chi-Square | Pr > ChiSq |
|-----------|----|----------|-------------------|--------------------|------------|
| Intercept | 1  | 2.1542   | 1.2477            | 2.9808             | 0.0843     |
| totscore  | 1  | -0.7696  | 0.3046            | 6.3845             | 0.0115     |

**Odds Ratio Estimates**

| Effect   | Point<br>Estimate | 95% Wald<br>Confidence Limits |       |
|----------|-------------------|-------------------------------|-------|
| totscore | 0.463             | 0.255                         | 0.841 |

**Output 54.8.5** ROC Curve for Popind=Totscore**Output 54.8.6** Fit Tables for Popind=Tp

| Model Convergence Status                      |                |                          |
|-----------------------------------------------|----------------|--------------------------|
| Convergence criterion (GCONV=1E-8) satisfied. |                |                          |
| Model Fit Statistics                          |                |                          |
| Criterion                                     | Intercept Only | Intercept and Covariates |
| AIC                                           | 52.918         | 51.794                   |
| SC                                            | 54.679         | 55.316                   |
| -2 Log L                                      | 50.918         | 47.794                   |



**Output 54.8.6** *continued***Testing Global Null Hypothesis: BETA=0**

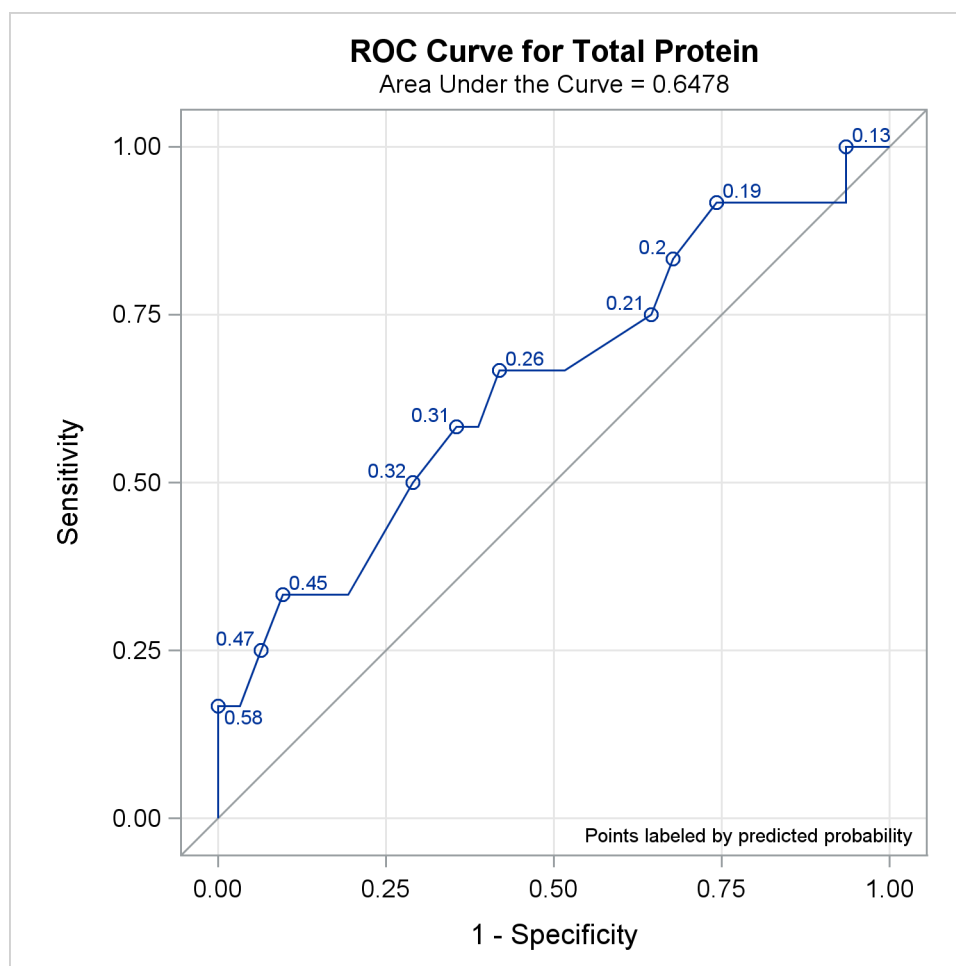
| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 3.1244     | 1  | 0.0771     |
| Score            | 3.1123     | 1  | 0.0777     |
| Wald             | 2.9059     | 1  | 0.0883     |

**Analysis of Maximum Likelihood Estimates**

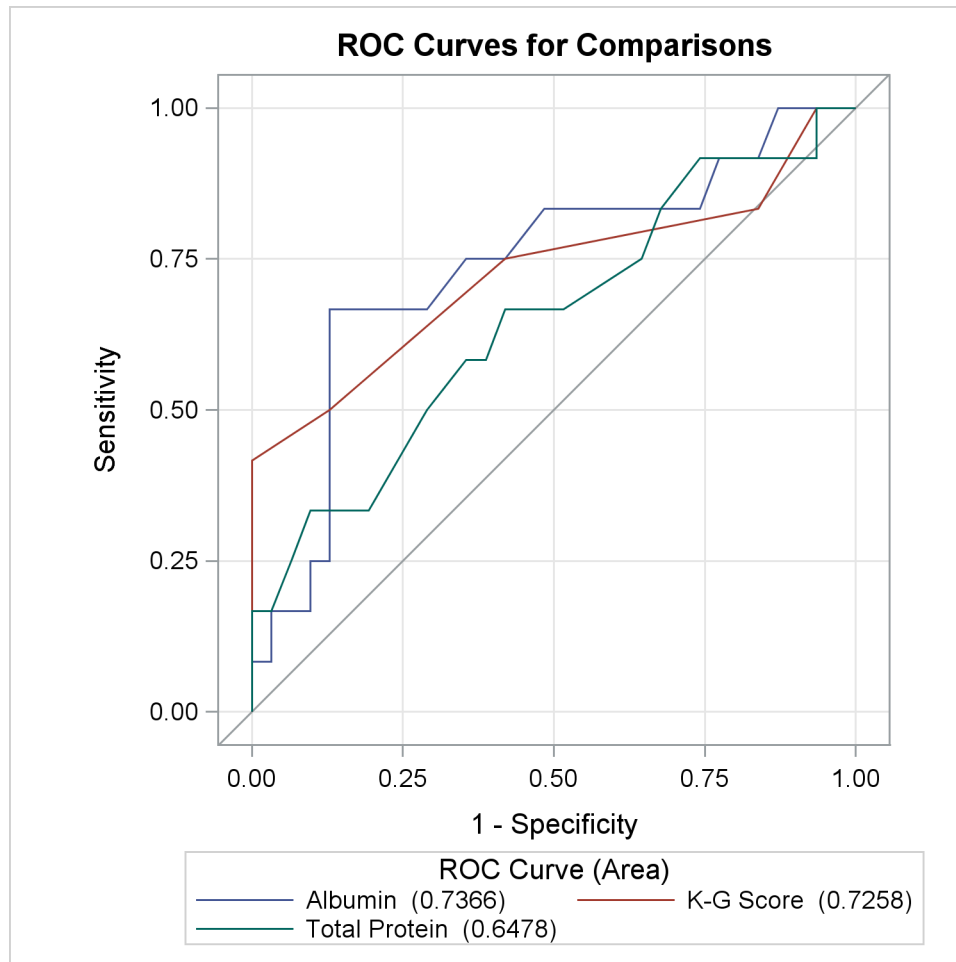
| Parameter | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
|-----------|----|----------|----------------|-----------------|------------|
| Intercept | 1  | 2.8295   | 2.2065         | 1.6445          | 0.1997     |
| tp        | 1  | -0.6279  | 0.3683         | 2.9059          | 0.0883     |

**Odds Ratio Estimates**

| Effect | Point Estimate | 95% Wald Confidence Limits |
|--------|----------------|----------------------------|
| tp     | 0.534          | 0.259 1.099                |

**Output 54.8.7** ROC Curve for Popind=Tp

All ROC curves being compared are also overlaid on the same plot, as shown in [Output 54.8.8](#).

**Output 54.8.8** Overlay of All Models Being Compared

Output 54.8.9 displays the association statistics, and displays the area under the ROC curve along with its standard error and a confidence interval for each model in the comparison. The confidence interval for Total Protein contains 0.50; hence it is not significantly different from random guessing, which is represented by the diagonal line in the preceding ROC plots.

**Output 54.8.9** ROC Association Table

| ROC Association Statistics |        |                          |                               |        |                     |        |        |
|----------------------------|--------|--------------------------|-------------------------------|--------|---------------------|--------|--------|
| ROC Model                  | Area   | ----- Mann-Whitney ----- |                               |        | Somers' D<br>(Gini) | Gamma  | Tau-a  |
|                            |        | Standard<br>Error        | 95% Wald<br>Confidence Limits |        |                     |        |        |
| Albumin                    | 0.7366 | 0.0927                   | 0.5549                        | 0.9182 | 0.4731              | 0.4809 | 0.1949 |
| K-G Score                  | 0.7258 | 0.1028                   | 0.5243                        | 0.9273 | 0.4516              | 0.5217 | 0.1860 |
| Total Protein              | 0.6478 | 0.1000                   | 0.4518                        | 0.8439 | 0.2957              | 0.3107 | 0.1218 |

Output 54.8.10 shows that the contrast used 'K-G Score' as the reference level. This table is produced by specifying the **E** option in the **ROCCONTRAST** statement.

**Output 54.8.10** ROC Contrast Coefficients

| ROC Contrast Coefficients |      |      |
|---------------------------|------|------|
| ROC Model                 | Row1 | Row2 |
| Albumin                   | 1    | 0    |
| K-G Score                 | -1   | -1   |
| Total Protein             | 0    | 1    |

Output 54.8.11 shows that the 2-degrees-of-freedom test that the 'K-G Score' is different from at least one other test is not significant at the 0.05 level.

**Output 54.8.11** ROC Test Results (2 Degrees of Freedom)

| ROC Contrast Test Results |    |            |            |
|---------------------------|----|------------|------------|
| Contrast                  | DF | Chi-Square | Pr > ChiSq |
| Reference = K-G Score     | 2  | 2.5340     | 0.2817     |

Output 54.8.12 is produced by specifying the **ESTIMATE** option in the **ROCCONTRAST** statement. Each row shows that the curves are not significantly different.

**Output 54.8.12** ROC Contrast Row Estimates (1-Degree-of-Freedom Tests)

| ROC Contrast Estimation and Testing Results by Row |          |                |                            |            |            |  |
|----------------------------------------------------|----------|----------------|----------------------------|------------|------------|--|
| Contrast                                           | Estimate | Standard Error | 95% Wald Confidence Limits | Chi-Square | Pr > ChiSq |  |
| Albumin - K-G Score                                | 0.0108   | 0.0953         | -0.1761 0.1976             | 0.0127     | 0.9102     |  |
| Total Protein - K-G Score                          | -0.0780  | 0.1046         | -0.2830 0.1271             | 0.5554     | 0.4561     |  |

## Example 54.9: Goodness-of-Fit Tests and Subpopulations

A study is done to investigate the effects of two binary factors, A and B, on a binary response, Y. Subjects are randomly selected from subpopulations defined by the four possible combinations of levels of A and B. The number of subjects responding with each level of Y is recorded, and the following DATA step creates the data set One:

```

data One;
  do A=0,1;
    do B=0,1;
      do Y=1,2;
        input F @@;
        output;
      end;
    end;
  end;
datalines;
23 63 31 70 67 100 70 104
;

```

The following statements fit a full model to examine the main effects of A and B as well as the interaction effect of A and B:

```

proc logistic data=One;
  freq F;
  model Y=A B A*B;
run;

```

Results of the model fit are shown in [Output 54.9.1](#). Notice that neither the A\*B interaction nor the B main effect is significant.

**Output 54.9.1** Full Model Fit

| The LOGISTIC Procedure      |                  |                 |
|-----------------------------|------------------|-----------------|
| Model Information           |                  |                 |
| Data Set                    | WORK.ONE         |                 |
| Response Variable           | Y                |                 |
| Number of Response Levels   | 2                |                 |
| Frequency Variable          | F                |                 |
| Model                       | binary logit     |                 |
| Optimization Technique      | Fisher's scoring |                 |
|                             |                  |                 |
| Number of Observations Read | 8                |                 |
| Number of Observations Used | 8                |                 |
| Sum of Frequencies Read     | 528              |                 |
| Sum of Frequencies Used     | 528              |                 |
|                             |                  |                 |
| Response Profile            |                  |                 |
| Ordered Value               | Y                | Total Frequency |
| 1                           | 1                | 191             |
| 2                           | 2                | 337             |
|                             |                  |                 |
| Probability modeled is Y=1. |                  |                 |

## Output 54.9.1 continued

| Model Convergence Status                      |                   |                                |                   |                    |            |
|-----------------------------------------------|-------------------|--------------------------------|-------------------|--------------------|------------|
| Convergence criterion (GCONV=1E-8) satisfied. |                   |                                |                   |                    |            |
| Model Fit Statistics                          |                   |                                |                   |                    |            |
| Criterion                                     | Intercept<br>Only | Intercept<br>and<br>Covariates |                   |                    |            |
| AIC                                           | 693.061           | 691.914                        |                   |                    |            |
| SC                                            | 697.330           | 708.990                        |                   |                    |            |
| -2 Log L                                      | 691.061           | 683.914                        |                   |                    |            |
| Testing Global Null Hypothesis: BETA=0        |                   |                                |                   |                    |            |
| Test                                          | Chi-Square        | DF                             | Pr > ChiSq        |                    |            |
| Likelihood Ratio                              | 7.1478            | 3                              | 0.0673            |                    |            |
| Score                                         | 6.9921            | 3                              | 0.0721            |                    |            |
| Wald                                          | 6.9118            | 3                              | 0.0748            |                    |            |
| Analysis of Maximum Likelihood Estimates      |                   |                                |                   |                    |            |
| Parameter                                     | DF                | Estimate                       | Standard<br>Error | Wald<br>Chi-Square | Pr > ChiSq |
| Intercept                                     | 1                 | -1.0074                        | 0.2436            | 17.1015            | <.0001     |
| A                                             | 1                 | 0.6069                         | 0.2903            | 4.3714             | 0.0365     |
| B                                             | 1                 | 0.1929                         | 0.3254            | 0.3515             | 0.5533     |
| A*B                                           | 1                 | -0.1883                        | 0.3933            | 0.2293             | 0.6321     |

Pearson and deviance goodness-of-fit tests cannot be obtained for this model since a full model containing four parameters is fit, leaving no residual degrees of freedom. For a binary response model, the goodness-of-fit tests have  $m - q$  degrees of freedom, where  $m$  is the number of subpopulations and  $q$  is the number of model parameters. In the preceding model,  $m = q = 4$ , resulting in zero degrees of freedom for the tests.

The following statements fit a reduced model containing only the A effect, so two degrees of freedom become available for testing goodness of fit. Specifying the **SCALE=NONE** option requests the Pearson and deviance statistics. With single-trial syntax, the **AGGREGATE=** option is needed to define the subpopulations in the study. Specifying **AGGREGATE=(A B)** creates subpopulations of the four combinations of levels of A and B. Although the B effect is being dropped from the model, it is still needed to define the original subpopulations in the study. If **AGGREGATE=(A)** were specified, only two subpopulations would be created from the levels of A, resulting in  $m = q = 2$  and zero degrees of freedom for the tests.

```
proc logistic data=One;
  freq F;
  model Y=A / scale=none aggregate=(A B);
run;
```

The goodness-of-fit tests in [Output 54.9.2](#) show that dropping the B main effect and the A\*B interaction simultaneously does not result in significant lack of fit of the model. The tests' large  $p$ -values indicate insufficient evidence for rejecting the null hypothesis that the model fits.

**Output 54.9.2** Reduced Model Fit

| The LOGISTIC Procedure                          |        |    |          |            |
|-------------------------------------------------|--------|----|----------|------------|
| Deviance and Pearson Goodness-of-Fit Statistics |        |    |          |            |
| Criterion                                       | Value  | DF | Value/DF | Pr > ChiSq |
| Deviance                                        | 0.3541 | 2  | 0.1770   | 0.8377     |
| Pearson                                         | 0.3531 | 2  | 0.1765   | 0.8382     |
| Number of unique profiles: 4                    |        |    |          |            |

## Example 54.10: Overdispersion

In a seed germination test, seeds of two cultivars were planted in pots of two soil conditions. The following statements create the data set `seeds`, which contains the observed proportion of seeds that germinated for various combinations of cultivar and soil condition. The variable `n` represents the number of seeds planted in a pot, and the variable `r` represents the number germinated. The indicator variables `cult` and `soil` represent the cultivar and soil condition, respectively.

```
data seeds;
  input pot n r cult soil;
  datalines;
1 16      8      0      0
2 51     26      0      0
3 45     23      0      0
4 39     10      0      0
5 36      9      0      0
6 81     23      1      0
7 30     10      1      0
8 39     17      1      0
9 28      8      1      0
10 62     23      1      0
11 51     32      0      1
12 72     55      0      1
13 41     22      0      1
14 12      3      0      1
15 13     10      0      1
16 79     46      1      1
17 30     15      1      1
18 51     32      1      1
19 74     53      1      1
20 56     12      1      1
;
```

PROC LOGISTIC is used as follows to fit a logit model to the data, with cult, soil, and cult  $\times$  soil interaction as explanatory variables. The option **SCALE=NONE** is specified to display goodness-of-fit statistics.

```
proc logistic data=seeds;
  model r/n=cult soil cult*soil/scale=none;
  title 'Full Model With SCALE=NONE';
run;
```

Results of fitting the full factorial model are shown in [Output 54.10.1](#). Both Pearson  $\chi^2$  and deviance are highly significant ( $p < 0.0001$ ), suggesting that the model does not fit well.

**Output 54.10.1** Results of the Model Fit for the Two-Way Layout

| Full Model With SCALE=NONE                      |            |                          |                     |                 |            |
|-------------------------------------------------|------------|--------------------------|---------------------|-----------------|------------|
| The LOGISTIC Procedure                          |            |                          |                     |                 |            |
| Deviance and Pearson Goodness-of-Fit Statistics |            |                          |                     |                 |            |
| Criterion                                       | Value      | DF                       | Value/DF            | Pr > ChiSq      |            |
| Deviance                                        | 68.3465    | 16                       | 4.2717              | <.0001          |            |
| Pearson                                         | 66.7617    | 16                       | 4.1726              | <.0001          |            |
| Number of events/trials observations: 20        |            |                          |                     |                 |            |
| Model Fit Statistics                            |            |                          |                     |                 |            |
| Criterion                                       | Intercept  | Intercept and Covariates |                     |                 |            |
|                                                 | Only       | Log Likelihood           | Full Log Likelihood |                 |            |
| AIC                                             | 1256.852   | 1213.003                 | 156.533             |                 |            |
| SC                                              | 1261.661   | 1232.240                 | 175.769             |                 |            |
| -2 Log L                                        | 1254.852   | 1205.003                 | 148.533             |                 |            |
| Testing Global Null Hypothesis: BETA=0          |            |                          |                     |                 |            |
| Test                                            | Chi-Square | DF                       | Pr > ChiSq          |                 |            |
| Likelihood Ratio                                | 49.8488    | 3                        | <.0001              |                 |            |
| Score                                           | 49.1682    | 3                        | <.0001              |                 |            |
| Wald                                            | 47.7623    | 3                        | <.0001              |                 |            |
| Analysis of Maximum Likelihood Estimates        |            |                          |                     |                 |            |
| Parameter                                       | DF         | Estimate                 | Standard Error      | Wald Chi-Square | Pr > ChiSq |
| Intercept                                       | 1          | -0.3788                  | 0.1489              | 6.4730          | 0.0110     |
| cult                                            | 1          | -0.2956                  | 0.2020              | 2.1412          | 0.1434     |
| soil                                            | 1          | 0.9781                   | 0.2128              | 21.1234         | <.0001     |
| cult*soil                                       | 1          | -0.1239                  | 0.2790              | 0.1973          | 0.6569     |



If the link function and the model specification are correct and if there are no outliers, then the lack of fit might be due to overdispersion. Without adjusting for the overdispersion, the standard errors are likely to be underestimated, causing the Wald tests to be too sensitive. In PROC LOGISTIC, there are three `SCALE=` options to accommodate overdispersion. With unequal sample sizes for the observations, `SCALE=WILLIAMS` is preferred. The Williams model estimates a scale parameter  $\phi$  by equating the value of Pearson  $\chi^2$  for the full model to its approximate expected value. The full model considered in the following statements is the model with cultivar, soil condition, and their interaction. Using a full model reduces the risk of contaminating  $\phi$  with lack of fit due to incorrect model specification.

```
proc logistic data=seeds;
  model r/n=cult soil cult*soil / scale=williams;
  title 'Full Model With SCALE=WILLIAMS';
run;
```

Results of using Williams' method are shown in [Output 54.10.2](#). The estimate of  $\phi$  is 0.075941 and is given in the formula for the Weight Variable at the beginning of the displayed output.

**Output 54.10.2** Williams' Model for Overdispersion

| Full Model With SCALE=WILLIAMS                |                                  |                 |              |
|-----------------------------------------------|----------------------------------|-----------------|--------------|
| The LOGISTIC Procedure                        |                                  |                 |              |
| Model Information                             |                                  |                 |              |
| Data Set                                      | WORK.SEEDS                       |                 |              |
| Response Variable (Events)                    | r                                |                 |              |
| Response Variable (Trials)                    | n                                |                 |              |
| Weight Variable                               | $1 / ( 1 + 0.075941 * (n - 1) )$ |                 |              |
| Model                                         | binary logit                     |                 |              |
| Optimization Technique                        | Fisher's scoring                 |                 |              |
|                                               |                                  |                 |              |
| Number of Observations Read                   |                                  |                 | 20           |
| Number of Observations Used                   |                                  |                 | 20           |
| Sum of Frequencies Read                       |                                  |                 | 906          |
| Sum of Frequencies Used                       |                                  |                 | 906          |
| Sum of Weights Read                           |                                  |                 | 198.3216     |
| Sum of Weights Used                           |                                  |                 | 198.3216     |
|                                               |                                  |                 |              |
| Response Profile                              |                                  |                 |              |
| Ordered Value                                 | Binary Outcome                   | Total Frequency | Total Weight |
| 1                                             | Event                            | 437             | 92.95346     |
| 2                                             | Nonevent                         | 469             | 105.36819    |
|                                               |                                  |                 |              |
| Model Convergence Status                      |                                  |                 |              |
| Convergence criterion (GCONV=1E-8) satisfied. |                                  |                 |              |

## Output 54.10.2 continued

| Deviance and Pearson Goodness-of-Fit Statistics                                                                                                                                              |            |                          |                |                 |            |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|--------------------------|----------------|-----------------|------------|
| Criterion                                                                                                                                                                                    | Value      | DF                       | Value/DF       | Pr > ChiSq      |            |
| Deviance                                                                                                                                                                                     | 16.4402    | 16                       | 1.0275         | 0.4227          |            |
| Pearson                                                                                                                                                                                      | 16.0000    | 16                       | 1.0000         | 0.4530          |            |
| Number of events/trials observations: 20                                                                                                                                                     |            |                          |                |                 |            |
| NOTE: Since the Williams method was used to accommodate overdispersion, the Pearson chi-squared statistic and the deviance can no longer be used to assess the goodness of fit of the model. |            |                          |                |                 |            |
| Model Fit Statistics                                                                                                                                                                         |            |                          |                |                 |            |
|                                                                                                                                                                                              |            | Intercept and Covariates |                |                 |            |
|                                                                                                                                                                                              | Intercept  | Log                      | Full Log       |                 |            |
| Criterion                                                                                                                                                                                    | Only       | Likelihood               | Likelihood     |                 |            |
| AIC                                                                                                                                                                                          | 276.155    | 273.586                  | 44.579         |                 |            |
| SC                                                                                                                                                                                           | 280.964    | 292.822                  | 63.815         |                 |            |
| -2 Log L                                                                                                                                                                                     | 274.155    | 265.586                  | 36.579         |                 |            |
| Testing Global Null Hypothesis: BETA=0                                                                                                                                                       |            |                          |                |                 |            |
| Test                                                                                                                                                                                         | Chi-Square | DF                       | Pr > ChiSq     |                 |            |
| Likelihood Ratio                                                                                                                                                                             | 8.5687     | 3                        | 0.0356         |                 |            |
| Score                                                                                                                                                                                        | 8.4856     | 3                        | 0.0370         |                 |            |
| Wald                                                                                                                                                                                         | 8.3069     | 3                        | 0.0401         |                 |            |
| Analysis of Maximum Likelihood Estimates                                                                                                                                                     |            |                          |                |                 |            |
| Parameter                                                                                                                                                                                    | DF         | Estimate                 | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                                                                                                                                                                    | 1          | -0.3926                  | 0.2932         | 1.7932          | 0.1805     |
| cult                                                                                                                                                                                         | 1          | -0.2618                  | 0.4160         | 0.3963          | 0.5290     |
| soil                                                                                                                                                                                         | 1          | 0.8309                   | 0.4223         | 3.8704          | 0.0491     |
| cult*soil                                                                                                                                                                                    | 1          | -0.0532                  | 0.5835         | 0.0083          | 0.9274     |

Since neither cult nor cult  $\times$  soil is statistically significant ( $p = 0.5290$  and  $p = 0.9274$ , respectively), a reduced model that contains only the soil condition factor is fitted, with the observations weighted by  $1/(1 + 0.075941(N - 1))$ . This can be done conveniently in PROC LOGISTIC by including the scale estimate in the SCALE=WILLIAMS option as follows:

```
proc logistic data=seeds;
  model r/n=soil / scale=williams(0.075941);
  title 'Reduced Model With SCALE=WILLIAMS(0.075941)';
run;
```

Results of the reduced model fit are shown in [Output 54.10.3](#). Soil condition remains a significant factor ( $p = 0.0064$ ) for the seed germination.

**Output 54.10.3** Reduced Model with Overdispersion Controlled

| Reduced Model With SCALE=WILLIAMS(0.075941) |    |          |                |                 |            |
|---------------------------------------------|----|----------|----------------|-----------------|------------|
| The LOGISTIC Procedure                      |    |          |                |                 |            |
| Analysis of Maximum Likelihood Estimates    |    |          |                |                 |            |
| Parameter                                   | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                   | 1  | -0.5249  | 0.2076         | 6.3949          | 0.0114     |
| soil                                        | 1  | 0.7910   | 0.2902         | 7.4284          | 0.0064     |

## Example 54.11: Conditional Logistic Regression for Matched Pairs Data

In matched pairs, or *case-control*, studies, conditional logistic regression is used to investigate the relationship between an outcome of being an event (case) or a nonevent (control) and a set of prognostic factors.

The following data are a subset of the data from the Los Angeles Study of the Endometrial Cancer Data in Breslow and Day (1980). There are 63 matched pairs, each consisting of a case of endometrial cancer (Outcome=1) and a control (Outcome=0). The case and corresponding control have the same ID. Two prognostic factors are included: Gall (an indicator variable for gall bladder disease) and Hyper (an indicator variable for hypertension). The goal of the case-control analysis is to determine the relative risk for gall bladder disease, controlling for the effect of hypertension.

```
data Data1;
  do ID=1 to 63;
    do Outcome = 1 to 0 by -1;
      input Gall Hyper @@;
      output;
    end;
  end;
  datalines;
0 0 0 0    0 0 0 0    0 1 0 1    0 0 1 0    1 0 0 1
0 1 0 0    1 0 0 0    1 1 0 1    0 0 0 0    0 0 0 0
1 0 0 0    0 0 0 1    1 0 0 1    1 0 1 0    1 0 0 1
0 1 0 0    0 0 1 1    0 0 1 1    0 0 0 1    0 1 0 0
0 0 1 1    0 1 0 1    0 1 0 0    0 0 0 0    0 0 0 0
0 0 0 1    1 0 0 1    0 0 0 1    1 0 0 0    0 1 0 0
0 1 0 0    0 1 0 0    0 1 0 0    0 0 0 0    1 1 1 1
0 0 0 1    0 1 0 0    0 1 0 1    0 1 0 1    0 1 0 0
0 0 0 0    0 1 1 0    0 0 0 1    0 0 0 0    1 0 0 0
0 0 0 0    1 1 0 0    0 1 0 0    0 0 0 0    0 1 0 1
0 0 0 0    0 1 0 1    0 1 0 0    0 1 0 0    1 0 0 0
0 0 0 0    1 1 1 0    0 0 0 0    0 0 0 0    1 1 0 0
1 0 1 0    0 1 0 0    1 0 0 0
;
```

There are several ways to approach this problem with PROC LOGISTIC:

- Specify the **STRATA** statement to perform a conditional logistic regression.
- Specify **EXACT** and **STRATA** statements to perform an exact logistic regression on the original data set, if you believe the data set is too small or too sparse for the usual asymptotics to hold.
- Transform each matched pair into a single observation, and then specify a PROC LOGISTIC statement on this transformed data without a STRATA statement; this also performs a conditional logistic regression and produces essentially the same results.
- Specify an **EXACT** statement on the transformed data.

SAS statements and selected results for these four approaches are given in the remainder of this example.

### Conditional Analysis Using the STRATA Statement

In the following statements, PROC LOGISTIC is invoked with the ID variable declared in the **STRATA** statement to obtain the conditional logistic model estimates for a model containing Gall as the only predictor variable:

```
proc logistic data=Data1;
  strata ID;
  model outcome(event='1')=Gall;
run;
```

Results from the conditional logistic analysis are shown in [Output 54.11.1](#). Note that there is no intercept term in the “Analysis of Maximum Likelihood Estimates” tables.

The odds ratio estimate for Gall is 2.60, which is marginally significant ( $p = 0.0694$ ) and which is an estimate of the relative risk for gall bladder disease. A 95% confidence interval for this relative risk is (0.927, 7.293).

**Output 54.11.1** Conditional Logistic Regression (Gall as Risk Factor)

| The LOGISTIC Procedure      |                      |
|-----------------------------|----------------------|
| Conditional Analysis        |                      |
| Model Information           |                      |
| Data Set                    | WORK.DATA1           |
| Response Variable           | Outcome              |
| Number of Response Levels   | 2                    |
| Number of Strata            | 63                   |
| Model                       | binary logit         |
| Optimization Technique      | Newton-Raphson ridge |
|                             |                      |
| Number of Observations Read | 126                  |
| Number of Observations Used | 126                  |

**Output 54.11.1** *continued***Response Profile**

| Ordered Value | Outcome | Total Frequency |
|---------------|---------|-----------------|
| 1             | 0       | 63              |
| 2             | 1       | 63              |

Probability modeled is Outcome=1.

**Strata Summary**

| Response Pattern | Outcome |   | Number of Strata | Frequency |
|------------------|---------|---|------------------|-----------|
|                  | 0       | 1 |                  |           |
| 1                | 1       | 1 | 63               | 126       |

**Newton-Raphson Ridge Optimization**

Without Parameter Scaling

Convergence criterion (GCONV=1E-8) satisfied.

**Model Fit Statistics**

| Criterion | Without Covariates | With Covariates |
|-----------|--------------------|-----------------|
| AIC       | 87.337             | 85.654          |
| SC        | 87.337             | 88.490          |
| -2 Log L  | 87.337             | 83.654          |

**Testing Global Null Hypothesis: BETA=0**

| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 3.6830     | 1  | 0.0550     |
| Score            | 3.5556     | 1  | 0.0593     |
| Wald             | 3.2970     | 1  | 0.0694     |

**Analysis of Conditional Maximum Likelihood Estimates**

| Parameter | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
|-----------|----|----------|----------------|-----------------|------------|
| Gall      | 1  | 0.9555   | 0.5262         | 3.2970          | 0.0694     |

**Output 54.11.1** *continued*

| Odds Ratio Estimates |                |                            |       |
|----------------------|----------------|----------------------------|-------|
| Effect               | Point Estimate | 95% Wald Confidence Limits |       |
| Gall                 | 2.600          | 0.927                      | 7.293 |

### Exact Analysis Using the STRATA Statement

When you believe there are not enough data or that the data are too sparse, you can perform a stratified exact logistic regression. The following statements perform stratified exact logistic regressions on the original data set by specifying both the **STRATA** and **EXACT** statements:

```
proc logistic data=Data1 exactonly;
  strata ID;
  model outcome(event='1')=Gall;
  exact Gall / estimate=both;
run;
```

**Output 54.11.2** Exact Logistic Regression (Gall as Risk Factor)

| The LOGISTIC Procedure     |             |                       |                       |                   |                   |
|----------------------------|-------------|-----------------------|-----------------------|-------------------|-------------------|
| Exact Conditional Analysis |             |                       |                       |                   |                   |
| Exact Conditional Tests    |             |                       |                       |                   |                   |
| Effect                     | Test        | Statistic             | --- p-Value ---       |                   |                   |
|                            |             |                       | Exact                 | Mid               |                   |
| Gall                       | Score       | 3.5556                | 0.0963                | 0.0799            |                   |
|                            | Probability | 0.0327                | 0.0963                | 0.0799            |                   |
| Exact Parameter Estimates  |             |                       |                       |                   |                   |
| Parameter                  | Estimate    | Standard Error        | 95% Confidence Limits |                   | Two-sided p-Value |
| Gall                       | 0.9555      | 0.5262                | -0.1394               | 2.2316            | 0.0963            |
| Exact Odds Ratios          |             |                       |                       |                   |                   |
| Parameter                  | Estimate    | 95% Confidence Limits |                       | Two-sided p-Value |                   |
| Gall                       | 2.600       | 0.870                 | 9.315                 | 0.0963            |                   |

Note that the score statistic in the “Conditional Exact Tests” table in [Output 54.11.2](#) is identical to the score statistic in [Output 54.11.1](#) from the conditional analysis. The exact odds ratio confidence interval is much wider than its conditional analysis counterpart, but the parameter estimates are similar. The exact analysis confirms the marginal significance of Gall as a predictor variable.

### Conditional Analysis Using Transformed Data

When each matched set consists of one event and one nonevent, the conditional likelihood is given by

$$\prod_i (1 + \exp(-\beta'(\mathbf{x}_{i1} - \mathbf{x}_{i0})))^{-1}$$

where  $\mathbf{x}_{i1}$  and  $\mathbf{x}_{i0}$  are vectors representing the prognostic factors for the event and nonevent, respectively, of the  $i$ th matched set. This likelihood is identical to the likelihood of fitting a logistic regression model to a set of data with constant response, where the model contains no intercept term and has explanatory variables given by  $\mathbf{d}_i = \mathbf{x}_{i1} - \mathbf{x}_{i0}$  (Breslow 1982).

To apply this method, the following DATA step transforms each matched pair into a single observation, where the variables Gall and Hyper contain the differences between the corresponding values for the case and the control (case-control). The variable Outcome, which will be used as the response variable in the logistic regression model, is given a constant value of 0 (which is the Outcome value for the control, although any constant, numeric or character, will suffice).

```
data Data2;
    set Data1;
    drop id1 gall1 hyper1;
    retain id1 gall1 hyper1 0;
    if (ID = id1) then do;
        Gall=gall1-Gall; Hyper=hyper1-Hyper;
        output;
    end;
    else do;
        id1=ID; gall1=Gall; hyper1=Hyper;
    end;
run;
```

Note that there are 63 observations in the data set, one for each matched pair. Since the number of observations  $n$  is halved, statistics that depend on  $n$  such as R Square (see the “[Generalized Coefficient of Determination](#)” on page 4246 section) will be incorrect. The variable Outcome has a constant value of 0.

In the following statements, PROC LOGISTIC is invoked with the **NOINT** option to obtain the conditional logistic model estimates. Because the option **CLODDS=PL** is specified, PROC LOGISTIC computes a 95% profile-likelihood confidence interval for the odds ratio for each predictor variable; note that profile-likelihood confidence intervals are not currently available when a **STRATA** statement is specified.

```
proc logistic data=Data2;
    model outcome=Gall / noint clodds=PL;
run;
```

The results are not displayed here.

## Exact Analysis Using Transformed Data

Sometimes the original data set in a matched-pairs study is too large for the exact methods to handle. In such cases it might be possible to use the transformed data set. The following statements perform exact logistic regressions on the transformed data set. The results are not displayed here.

```
proc logistic data=Data2 exactonly;
  model outcome=Gall / noint;
  exact Gall / estimate=both;
run;
```

---

## Example 54.12: Firth's Penalized Likelihood Compared with Other Approaches

Firth's penalized likelihood approach is a method of addressing issues of separability, small sample sizes, and bias of the parameter estimates. This example performs some comparisons between results from using the **FIRTH** option to results from the usual unconditional, conditional, and exact logistic regression analyses. When the sample size is large enough, the unconditional estimates and the Firth penalized-likelihood estimates should be nearly the same. These examples show that Firth's penalized likelihood approach compares favorably with unconditional, conditional, and exact logistic regression; however, this is not an exhaustive analysis of Firth's method. For more detailed analyses with separable data sets, see Heinze (2006, 1999) and Heinze and Schemper (2002).

## Comparison on 2x2 Tables with One Zero Cell

A 2×2 table with one cell having zero frequency, where the rows of the table are the levels of a covariate while the columns are the levels of the response variable, is an example of a quasi-completely separated data set. The parameter estimate for the covariate under unconditional logistic regression will move off to infinity, although PROC LOGISTIC will stop the iterations at an earlier point in the process. An exact logistic regression is sometimes performed to determine the importance of the covariate in describing the variation in the data, but the median-unbiased parameter estimate, while finite, might not be near the true value, and one confidence limit (for this example, the upper) is always infinite.

The following DATA step produces 1000 different 2×2 tables, all following an underlying probability structure, with one cell having a near zero probability of being observed:

```
%let beta0=-15;
%let beta1=16;
data one;
  keep sample X y pry;
  do sample=1 to 1000;
    do i=1 to 100;
      X=rantrb1(987987, .4, .6)-1;
      xb= &beta0 + X*&beta1;
      exb=exp(xb);
      pry= exb/(1+exb);
      cut= ranuni(393993);
      if (pry < cut) then y=1; else y=0;
      output;
    end;
  end;
```



```
end;
run;
```

The following statements perform the bias-corrected and exact logistic regression on each of the 1000 different data sets, output the odds ratio tables by using the ODS OUTPUT statement, and compute various statistics across the data sets by using the MEANS procedure:

```
ods exclude all;
proc logistic data=one;
  by sample;
  class X(param=ref);
  model y(event='1')=X / firth clodds=pl;
  ods output cloddspl=firth;
run;
proc logistic data=one exactonly;
  by sample;
  class X(param=ref);
  model y(event='1')=X;
  exact X / estimate=odds;
  ods output exactoddsratio=exact;
run;
ods select all;
proc means data=firth;
  var LowerCL OddsRatioEst UpperCL;
run;
proc means data=exact;
  var LowerCL Estimate UpperCL;
run;
```

The results of the PROC MEANS statements are summarized in [Table 54.16](#). You can see that the odds ratios are all quite large; the confidence limits on every table suggest that the covariate X is a significant factor in explaining the variability in the data.

**Table 54.16** Odds Ratio Results

| Method | Mean Estimate | Standard Error | Minimum Lower CL | Maximum Upper CL |
|--------|---------------|----------------|------------------|------------------|
| Firth  | 231.59        | 83.57          | 10.40            | 111317           |
| Exact  | 152.02        | 52.30          | 11.16            | $\infty$         |

## Comparison on Case-Control Data

Case-control models contain an intercept term for every case-control pair in the data set. This means that there are a large number of parameters compared to the number of observations. Breslow and Day (1980) note that the estimates from unconditional logistic regression are biased with the corresponding odds ratios off by a power of 2 from the true value; conditional logistic regression was developed to remedy this.

The following DATA step produces 1000 case-control data sets, with pair indicating the strata:

```

%let beta0=1;
%let beta1=2;
data one;
  do sample=1 to 1000;
    do pair=1 to 20;
      ran=ranuni(939393);
      a=3*ranuni(9384984)-1;
      pdf0= pdf('NORMAL',a,.4,1);
      pdf1= pdf('NORMAL',a,1,1);
      pry0= pdf0/(pdf0+pdf1);
      pry1= 1-pry0;
      xb= log(pry0/pry1);
      x= (xb-&beta0*pair/100) / &beta1;
      y=0;
      output;
      x= (-xb-&beta0*pair/100) / &beta1;
      y=1;
      output;
    end;
  end;
run;

```

Unconditional, conditional, exact, and Firth-adjusted analyses are performed on the data sets, and the mean, minimum, and maximum odds ratios and the mean upper and lower limits for the odds ratios are displayed in [Table 54.17](#). **CAUTION:** Due to the exact analyses, this program takes a long time and a lot of resources to run. You might want to reduce the number of samples generated.

```

ods exclude all;
proc logistic data=one;
  by sample;
  class pair / param=ref;
  model y=x pair / clodds=pl;
  ods output cloddspl=oru;
run;
data oru;
  set oru;
  if Effect='x';
  rename lowercl=lclu uppercl=uclu oddsratioest=orestu;
run;
proc logistic data=one;
  by sample;
  strata pair;
  model y=x / clodds=wald;
  ods output cloddswald=orc;
run;
data orc;
  set orc;
  if Effect='x';
  rename lowercl=lclc uppercl=uclc oddsratioest=orestc;
run;
proc logistic data=one exactonly;
  by sample;
  strata pair;

```

```

model y=x;
exact x / estimate=both;
ods output ExactOddsRatio=ore;
run;
proc logistic data=one;
  by sample;
  class pair / param=ref;
  model y=x pair / firth clodds=pl;
  ods output cloddspl=orf;
run;
data orf;
  set orf;
  if Effect='x';
  rename lowercl=lclf uppercl=uclf oddsratioest=orestf;
run;
data all;
  merge oru orc ore orf;
run;
ods select all;
proc means data=all;
run;

```

You can see from Table 54.17 that the conditional, exact, and Firth-adjusted results are all comparable, while the unconditional results are several orders of magnitude different.

**Table 54.17** Odds Ratio Estimates

| Method        | N    | Minimum | Mean   | Maximum |
|---------------|------|---------|--------|---------|
| Unconditional | 1000 | 0.00045 | 112.09 | 38038   |
| Conditional   | 1000 | 0.021   | 4.20   | 195     |
| Exact         | 1000 | 0.021   | 4.20   | 195     |
| Firth         | 1000 | 0.018   | 4.89   | 71      |

Further examination of the data set all shows that the differences between the square root of the unconditional odds ratio estimates and the conditional estimates have mean  $-0.00019$  and standard deviation  $0.0008$ , verifying that the unconditional odds ratio is about the square of the conditional odds ratio. The conditional and exact conditional odds ratios are also nearly equal, with their differences having mean  $3E-7$  and standard deviation  $6E-6$ . The differences between the Firth and the conditional odds ratios can be large (mean  $0.69$ , standard deviation  $5.40$ ), but their relative differences,  $\frac{Firth - Conditional}{Conditional}$ , have mean  $0.20$  with standard deviation  $0.19$ , so the largest differences occur with the larger estimates.

### Example 54.13: Complementary Log-Log Model for Infection Rates

Antibodies produced in response to an infectious disease like malaria remain in the body after the individual has recovered from the disease. A serological test detects the presence or absence of such antibodies. An individual with such antibodies is called seropositive. In geographic areas where the disease is endemic, the inhabitants are at fairly constant risk of infection. The probability of an individual never having been

infected in  $Y$  years is  $\exp(-\mu Y)$ , where  $\mu$  is the mean number of infections per year (see the appendix of Draper, Voller, and Carpenter 1972). Rather than estimating the unknown  $\mu$ , epidemiologists want to estimate the probability of a person living in the area being infected in one year. This infection rate  $\gamma$  is given by

$$\gamma = 1 - e^{-\mu}$$

The following statements create the data set `sero`, which contains the results of a serological survey of malarial infection. Individuals of nine age groups (Group) were tested. The variable `A` represents the midpoint of the age range for each age group. The variable `N` represents the number of individuals tested in each age group, and the variable `R` represents the number of individuals that are seropositive.

```
data sero;
  input Group A N R;
  X=log(A);
  label X='Log of Midpoint of Age Range';
  datalines;
1  1.5  123  8
2  4.0  132  6
3  7.5  182 18
4 12.5  140 14
5 17.5  138 20
6 25.0  161 39
7 35.0  133 19
8 47.0   92 25
9 60.0   74 44
;
```

For the  $i$ th group with the age midpoint  $A_i$ , the probability of being seropositive is  $p_i = 1 - \exp(-\mu A_i)$ . It follows that

$$\log(-\log(1 - p_i)) = \log(\mu) + \log(A_i)$$

By fitting a binomial model with a complementary log-log link function and by using `X=log(A)` as an offset term, you can estimate  $\alpha = \log(\mu)$  as an intercept parameter. The following statements invoke PROC LOGISTIC to compute the maximum likelihood estimate of  $\alpha$ . The `LINK=CLOGLOG` option is specified to request the complementary log-log link function. Also specified is the `CLPARM=PL` option, which requests the profile-likelihood confidence limits for  $\alpha$ .

```
proc logistic data=sero;
  model R/N= / offset=X
          link=cloglog
          clparm=pl
          scale=none;
  title 'Constant Risk of Infection';
run;
```

Results of fitting this constant risk model are shown in [Output 54.13.1](#).

### Output 54.13.1 Modeling Constant Risk of Infection

```

Constant Risk of Infection

The LOGISTIC Procedure

Model Information

Data Set                WORK.SERO
Response Variable (Events)  R
Response Variable (Trials)  N
Offset Variable           X
Model                     binary cloglog
Optimization Technique     Fisher's scoring

Number of Observations Read          9
Number of Observations Used          9
Sum of Frequencies Read              1175
Sum of Frequencies Used              1175

Response Profile

Ordered   Binary   Total
Value     Outcome   Frequency

1         Event     193
2         Nonevent  982

Intercept-Only Model Convergence Status

Convergence criterion (GCONV=1E-8) satisfied.

-2 Log L = 967.1158

Deviance and Pearson Goodness-of-Fit Statistics

Criterion      Value      DF      Value/DF      Pr > ChiSq
Deviance      41.5032      8       5.1879      <.0001
Pearson       50.6883      8       6.3360      <.0001

Number of events/trials observations: 9

```

Output 54.13.1 continued

| Analysis of Maximum Likelihood Estimates |    |          |                |                 |            |
|------------------------------------------|----|----------|----------------|-----------------|------------|
| Parameter                                | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                | 1  | -4.6605  | 0.0725         | 4133.5626       | <.0001     |
| X                                        | 0  | 1.0000   | 0              | .               | .          |

| Parameter Estimates and Profile-Likelihood Confidence Intervals |          |                       |         |
|-----------------------------------------------------------------|----------|-----------------------|---------|
| Parameter                                                       | Estimate | 95% Confidence Limits |         |
| Intercept                                                       | -4.6605  | -4.8057               | -4.5219 |

Output 54.13.1 shows that the maximum likelihood estimate of  $\alpha = \log(\mu)$  and its estimated standard error are  $\hat{\alpha} = -4.6605$  and  $\hat{\sigma}_{\hat{\alpha}} = 0.0725$ , respectively. The infection rate is estimated as

$$\hat{\gamma} = 1 - e^{-\hat{\mu}} = 1 - e^{-e^{\hat{\alpha}}} = 1 - e^{-e^{-4.6605}} = 0.00942$$

The 95% confidence interval for  $\gamma$ , obtained by back-transforming the 95% confidence interval for  $\alpha$ , is (0.0082, 0.0108); that is, there is a 95% chance that, in repeated sampling, the interval of 8 to 11 infections per thousand individuals contains the true infection rate.

The goodness-of-fit statistics for the constant risk model are statistically significant ( $p < 0.0001$ ), indicating that the assumption of constant risk of infection is not correct. You can fit a more extensive model by allowing a separate risk of infection for each age group. Suppose  $\mu_i$  is the mean number of infections per year for the  $i$ th age group. The probability of seropositive for the  $i$ th group with the age midpoint  $A_i$  is  $p_i = 1 - \exp(-\mu_i A_i)$ , so that

$$\log(-\log(1 - p_i)) = \log(\mu_i) + \log(A_i)$$

In the following statements, a complementary log-log model is fit containing Group as an explanatory classification variable with the GLM coding (so that a dummy variable is created for each age group), no intercept term, and  $X = \log(A)$  as an offset term. The ODS OUTPUT statement saves the estimates and their 95% profile-likelihood confidence limits to the ClparmPL data set. Note that  $\log(\mu_i)$  is the regression parameter associated with  $\text{Group} = i$ .

```
proc logistic data=sero;
  ods output ClparmPL=ClparmPL;
  class Group / param=glm;
  model R/N=Group / noint
        offset=X
        link=cloglog
        clparm=pl;
  title 'Infectious Rates and 95% Confidence Intervals';
run;
```

Results of fitting the model with a separate risk of infection are shown in Output 54.13.2.

**Output 54.13.2** Modeling Separate Risk of Infection

| Infectious Rates and 95% Confidence Intervals |    |          |                |                 |            |  |
|-----------------------------------------------|----|----------|----------------|-----------------|------------|--|
| The LOGISTIC Procedure                        |    |          |                |                 |            |  |
| Analysis of Maximum Likelihood Estimates      |    |          |                |                 |            |  |
| Parameter                                     | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |  |
| Group 1                                       | 1  | -3.1048  | 0.3536         | 77.0877         | <.0001     |  |
| Group 2                                       | 1  | -4.4542  | 0.4083         | 119.0164        | <.0001     |  |
| Group 3                                       | 1  | -4.2769  | 0.2358         | 328.9593        | <.0001     |  |
| Group 4                                       | 1  | -4.7761  | 0.2674         | 319.0600        | <.0001     |  |
| Group 5                                       | 1  | -4.7165  | 0.2238         | 443.9920        | <.0001     |  |
| Group 6                                       | 1  | -4.5012  | 0.1606         | 785.1350        | <.0001     |  |
| Group 7                                       | 1  | -5.4252  | 0.2296         | 558.1114        | <.0001     |  |
| Group 8                                       | 1  | -4.9987  | 0.2008         | 619.4666        | <.0001     |  |
| Group 9                                       | 1  | -4.1965  | 0.1559         | 724.3157        | <.0001     |  |
| X                                             | 0  | 1.0000   | 0              | .               | .          |  |

| Parameter Estimates and Profile-Likelihood Confidence Intervals |   |          |                       |         |
|-----------------------------------------------------------------|---|----------|-----------------------|---------|
| Parameter                                                       |   | Estimate | 95% Confidence Limits |         |
| Group 1                                                         | 1 | -3.1048  | -3.8880               | -2.4833 |
| Group 2                                                         | 1 | -4.4542  | -5.3769               | -3.7478 |
| Group 3                                                         | 1 | -4.2769  | -4.7775               | -3.8477 |
| Group 4                                                         | 1 | -4.7761  | -5.3501               | -4.2940 |
| Group 5                                                         | 1 | -4.7165  | -5.1896               | -4.3075 |
| Group 6                                                         | 1 | -4.5012  | -4.8333               | -4.2019 |
| Group 7                                                         | 1 | -5.4252  | -5.9116               | -5.0063 |
| Group 8                                                         | 1 | -4.9987  | -5.4195               | -4.6289 |
| Group 9                                                         | 1 | -4.1965  | -4.5164               | -3.9037 |

For the first age group (Group=1), the point estimate of  $\log(\mu_1)$  is -3.1048, which transforms into an infection rate of  $1 - \exp(-\exp(-3.1048)) = 0.0438$ . A 95% confidence interval for this infection rate is obtained by transforming the 95% confidence interval for  $\log(\mu_1)$ . For the first age group, the lower and upper confidence limits are  $1 - \exp(-\exp(-3.8880)) = 0.0203$  and  $1 - \exp(-\exp(-2.4833)) = 0.0801$ , respectively; that is, there is a 95% chance that, in repeated sampling, the interval of 20 to 80 infections per thousand individuals contains the true infection rate. The following statements perform this transformation on the estimates and confidence limits saved in the ClparmPL data set; the resulting estimated infection rates in one year's time for each age group are displayed in [Table 54.18](#). Note that the infection rate for the first age group is high compared to that of the other age groups.

```
data ClparmPL;
  set ClparmPL;
  Estimate=round( 1000*( 1-exp(-exp(Estimate)) ) );
  LowerCL =round( 1000*( 1-exp(-exp(LowerCL)) ) );
  UpperCL =round( 1000*( 1-exp(-exp(UpperCL)) ) );
run;
```

**Table 54.18** Infection Rate in One Year

| Age Group | Number Infected per 1,000 People |           |           |
|-----------|----------------------------------|-----------|-----------|
|           | Point Estimate                   | 95% Lower | 95% Upper |
| 1         | 44                               | 20        | 80        |
| 2         | 12                               | 5         | 23        |
| 3         | 14                               | 8         | 21        |
| 4         | 8                                | 5         | 14        |
| 5         | 9                                | 6         | 13        |
| 6         | 11                               | 8         | 15        |
| 7         | 4                                | 3         | 7         |
| 8         | 7                                | 4         | 10        |
| 9         | 15                               | 11        | 20        |

### Example 54.14: Complementary Log-Log Model for Interval-Censored Survival Times

Often survival times are not observed more precisely than the interval (for instance, a day) within which the event occurred. Survival data of this form are known as grouped or interval-censored data. A discrete analog of the continuous proportional hazards model (Prentice and Gloeckler 1978; Allison 1982) is used to investigate the relationship between these survival times and a set of explanatory variables.

Suppose  $T_i$  is the discrete survival time variable of the  $i$ th subject with covariates  $\mathbf{x}_i$ . The discrete-time hazard rate  $\lambda_{it}$  is defined as

$$\lambda_{it} = \Pr(T_i = t \mid T_i \geq t, \mathbf{x}_i), \quad t = 1, 2, \dots$$

Using elementary properties of conditional probabilities, it can be shown that

$$\Pr(T_i = t) = \lambda_{it} \prod_{j=1}^{t-1} (1 - \lambda_{ij}) \quad \text{and} \quad \Pr(T_i > t) = \prod_{j=1}^t (1 - \lambda_{ij})$$

Suppose  $t_i$  is the observed survival time of the  $i$ th subject. Suppose  $\delta_i = 1$  if  $T_i = t_i$  is an event time and 0 otherwise. The likelihood for the grouped survival data is given by

$$\begin{aligned}
 L &= \prod_i [\Pr(T_i = t_i)]^{\delta_i} [\Pr(T_i > t_i)]^{1-\delta_i} \\
 &= \prod_i \left( \frac{\lambda_{it_i}}{1 - \lambda_{it_i}} \right)^{\delta_i} \prod_{j=1}^{t_i} (1 - \lambda_{ij}) \\
 &= \prod_i \prod_{j=1}^{t_i} \left( \frac{\lambda_{ij}}{1 - \lambda_{ij}} \right)^{y_{ij}} (1 - \lambda_{ij})
 \end{aligned}$$

where  $y_{ij} = 1$  if the  $i$ th subject experienced an event at time  $T_i = j$  and 0 otherwise.



Note that the likelihood  $L$  for the grouped survival data is the same as the likelihood of a binary response model with event probabilities  $\lambda_{ij}$ . If the data are generated by a continuous-time proportional hazards model, Prentice and Gloeckler (1978) have shown that

$$\lambda_{ij} = 1 - \exp(-\exp(\alpha_j + \beta' \mathbf{x}_i))$$

which can be rewritten as

$$\log(-\log(1 - \lambda_{ij})) = \alpha_j + \beta' \mathbf{x}_i$$

where the coefficient vector  $\beta$  is identical to that of the continuous-time proportional hazards model, and  $\alpha_j$  is a constant related to the conditional survival probability in the interval defined by  $T_i = j$  at  $\mathbf{x}_i = \mathbf{0}$ . The grouped data survival model is therefore equivalent to the binary response model with complementary log-log link function. To fit the grouped survival model by using PROC LOGISTIC, you must treat each discrete time unit for each subject as a separate observation. For each of these observations, the response is dichotomous, corresponding to whether or not the subject died in the time unit.

Consider a study of the effect of insecticide on flour beetles. Four different concentrations of an insecticide were sprayed on separate groups of flour beetles. The following DATA step saves the number of male and female flour beetles dying in successive intervals in the data set Beetles:

```
data Beetles(keep=time sex conc freq);
  input time m20 f20 m32 f32 m50 f50 m80 f80;
  conc=.20; freq= m20; sex=1; output;
           freq= f20; sex=2; output;
  conc=.32; freq= m32; sex=1; output;
           freq= f32; sex=2; output;
  conc=.50; freq= m50; sex=1; output;
           freq= f50; sex=2; output;
  conc=.80; freq= m80; sex=1; output;
           freq= f80; sex=2; output;
  datalines;
1   3   0   7   1   5   0   4   2
2  11   2  10   5   8   4  10   7
3  10   4  11  11  11   6   8  15
4   7   8  16  10  15   6  14   9
5   4   9   3   5   4   3   8   3
6   3   3   2   1   2   1   2   4
7   2   0   1   0   1   1   1   1
8   1   0   0   1   1   4   0   1
9   0   0   1   1   0   0   0   0
10  0   0   0   0   0   0   1   1
11  0   0   0   0   1   1   0   0
12  1   0   0   0   0   1   0   0
13  1   0   0   0   0   1   0   0
14 101 126 19 47   7  17   2   4
;
```

The data set Beetles contains four variables: time, sex, conc, and freq. The variable time represents the interval death time; for example, time=2 is the interval between day 1 and day 2. Insects surviving the duration (13 days) of the experiment are given a time value of 14. The variable sex represents the sex of the insects (1=male, 2=female), conc represents the concentration of the insecticide (mg/cm<sup>2</sup>), and freq represents the frequency of the observations.

To use PROC LOGISTIC with the grouped survival data, you must expand the data so that each beetle has a separate record for each day of survival. A beetle that died in the third day (time=3) would contribute three observations to the analysis, one for each day it was alive at the beginning of the day. A beetle that survives the 13-day duration of the experiment (time=14) would contribute 13 observations.

The following DATA step creates a new data set named Days containing the beetle-day observations from the data set Beetles. In addition to the variables sex, conc, and freq, the data set contains an outcome variable y and a classification variable day. The variable y has a value of 1 if the observation corresponds to the day that the beetle died, and it has a value of 0 otherwise. An observation for the first day will have a value of 1 for day; an observation for the second day will have a value of 2 for day, and so on. For instance, [Output 54.14.1](#) shows an observation in the Beetles data set with time=3, and [Output 54.14.2](#) shows the corresponding beetle-day observations in the data set Days.

```
data Days;
  set Beetles;
  do day=1 to time;
    if (day < 14) then do;
      y= (day=time);
      output;
    end;
  end;
run;
```

**Output 54.14.1** An Observation with Time=3 in Beetles Data Set

| Obs | time | conc | freq | sex |
|-----|------|------|------|-----|
| 17  | 3    | 0.2  | 10   | 1   |

**Output 54.14.2** Corresponding Beetle-Day Observations in Days

| Obs | time | conc | freq | sex | day | y |
|-----|------|------|------|-----|-----|---|
| 25  | 3    | 0.2  | 10   | 1   | 1   | 0 |
| 26  | 3    | 0.2  | 10   | 1   | 2   | 0 |
| 27  | 3    | 0.2  | 10   | 1   | 3   | 1 |

The following statements invoke PROC LOGISTIC to fit a complementary log-log model for binary data with the response variable Y and the explanatory variables day, sex, and Variableconc. Specifying the **EVENT=** option ensures that the event (y=1) probability is modeled. The GLM coding in the **CLASS** statement creates an indicator column in the design matrix for each level of day. The coefficients of the indicator effects for day can be used to estimate the baseline survival function. The **NOINT** option is specified to prevent any redundancy in estimating the coefficients of day. The Newton-Raphson algorithm is used for the maximum likelihood estimation of the parameters.

```
proc logistic data=Days outest=est1;
  class day / param=glm;
  model y(event='1')= day sex conc
    / noint link=cloglog technique=newton;
  freq freq;
run;
```

Results of the model fit are given in [Output 54.14.3](#). Both sex and conc are statistically significant for the survival of beetles sprayed by the insecticide. Female beetles are more resilient to the chemical than male beetles, and increased concentration of the insecticide increases its effectiveness.

**Output 54.14.3** Parameter Estimates for the Grouped Proportional Hazards Model

| Analysis of Maximum Likelihood Estimates |    |          |                |                 |            |  |
|------------------------------------------|----|----------|----------------|-----------------|------------|--|
| Parameter                                | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |  |
| day                                      | 1  | -3.9314  | 0.2934         | 179.5602        | <.0001     |  |
| day                                      | 2  | -2.8751  | 0.2412         | 142.0596        | <.0001     |  |
| day                                      | 3  | -2.3985  | 0.2299         | 108.8833        | <.0001     |  |
| day                                      | 4  | -1.9953  | 0.2239         | 79.3960         | <.0001     |  |
| day                                      | 5  | -2.4920  | 0.2515         | 98.1470         | <.0001     |  |
| day                                      | 6  | -3.1060  | 0.3037         | 104.5799        | <.0001     |  |
| day                                      | 7  | -3.9704  | 0.4230         | 88.1107         | <.0001     |  |
| day                                      | 8  | -3.7917  | 0.4007         | 89.5233         | <.0001     |  |
| day                                      | 9  | -5.1540  | 0.7316         | 49.6329         | <.0001     |  |
| day                                      | 10 | -5.1350  | 0.7315         | 49.2805         | <.0001     |  |
| day                                      | 11 | -5.1131  | 0.7313         | 48.8834         | <.0001     |  |
| day                                      | 12 | -5.1029  | 0.7313         | 48.6920         | <.0001     |  |
| day                                      | 13 | -5.0951  | 0.7313         | 48.5467         | <.0001     |  |
| sex                                      | 1  | -0.5651  | 0.1141         | 24.5477         | <.0001     |  |
| conc                                     | 1  | 3.0918   | 0.2288         | 182.5665        | <.0001     |  |

The coefficients of parameters for the day variable are the maximum likelihood estimates of  $\alpha_1, \dots, \alpha_{13}$ , respectively. The baseline survivor function  $S_0(t)$  is estimated by

$$\hat{S}_0(t) = \hat{\Pr}(T > t) = \prod_{j \leq t} \exp(-\exp(\hat{\alpha}_j))$$

and the survivor function for a given covariate pattern (sex= $x_1$  and conc= $x_2$ ) is estimated by

$$\hat{S}(t) = [\hat{S}_0(t)]^{\exp(-0.5651x_1 + 3.0918x_2)}$$

The following statements compute the survival curves for male and female flour beetles exposed to the insecticide in concentrations of 0.20 mg/cm<sup>2</sup> and 0.80 mg/cm<sup>2</sup>:

```

data one (keep=day survival element s_m20 s_f20 s_m80 s_f80);
  array dd day1-day13;
  array sc[4] m20 f20 m80 f80;
  array s_sc[4] s_m20 s_f20 s_m80 s_f80 (1 1 1 1);
  set est1;
  m20= exp(sex + .20 * conc);
  f20= exp(2 * sex + .20 * conc);
  m80= exp(sex + .80 * conc);
  f80= exp(2 * sex + .80 * conc);
  survival=1;
  day=0;
  output;
  do over dd;
    element= exp(-exp(dd));
    survival= survival * element;
    do i=1 to 4;
      s_sc[i] = survival ** sc[i];
    end;
    day + 1;
    output;
  end;
run;

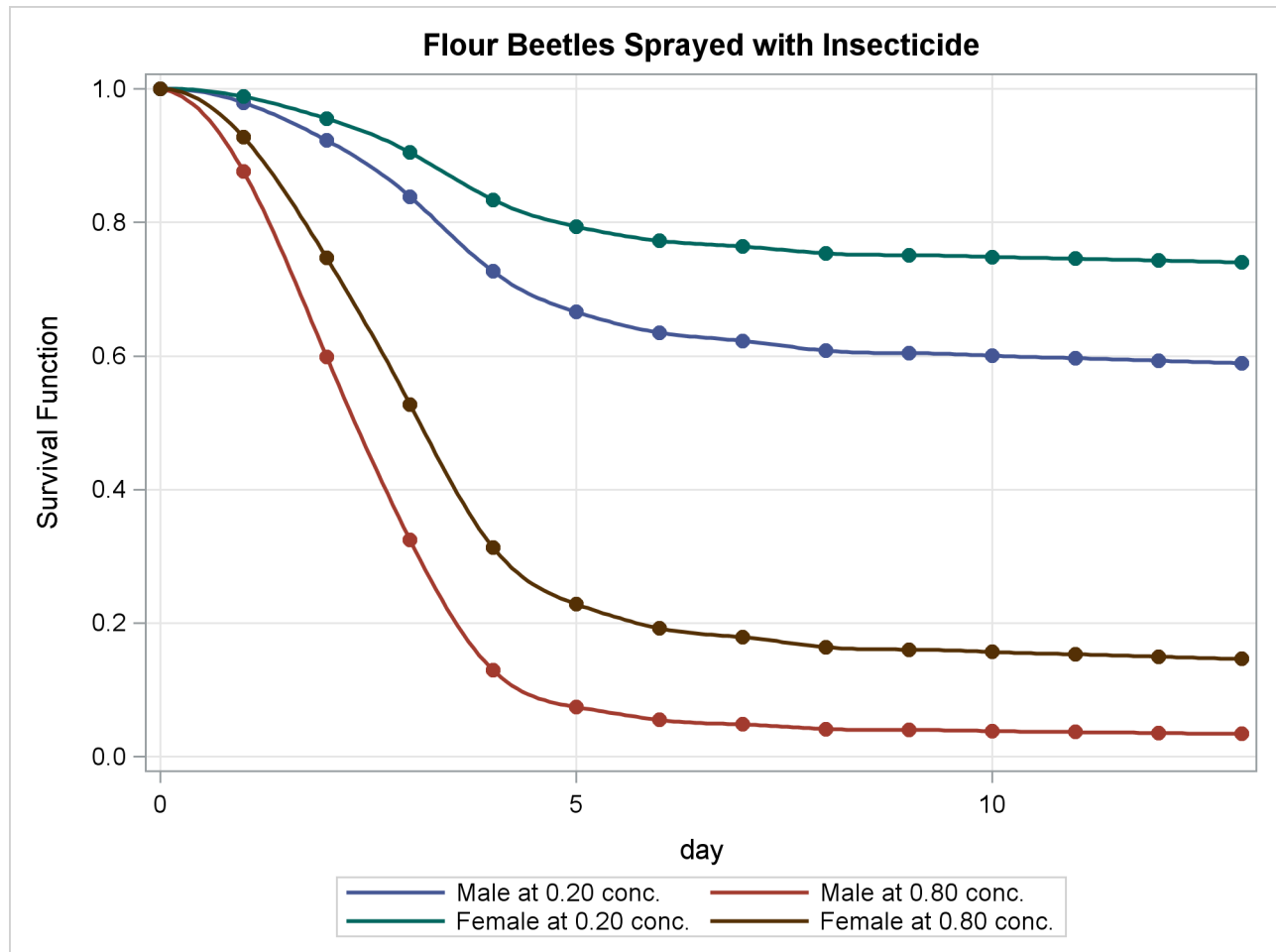
```

Instead of plotting the curves as step functions, the following statements use the PBSPLINE statement in the SGPLOT procedure to smooth the curves with a penalized B-spline. See Chapter 97, “[The TRANSREG Procedure](#),” for details about the implementation of the penalized B-spline method. The SAS autocall macro %MODSTYLE is specified to change the marker symbols for the plot. For more information about the %MODSTYLE macro, see the section “[Style Template Modification Macro](#)” on page 667 in Chapter 21, “[Statistical Graphics Using ODS](#).” The smoothed survival curves are displayed in [Output 54.14.4](#).

```

%modstyle(name=LogiStyle,parent=htmlblue,markers=circlefilled);
ods listing style=LogiStyle;
proc sgplot data=one;
  title 'Flour Beetles Sprayed with Insecticide';
  xaxis grid integer;
  yaxis grid label='Survival Function';
  pbspline y=s_m20 x=day /
    legendlabel = "Male at 0.20 conc." name="pred1";
  pbspline y=s_m80 x=day /
    legendlabel = "Male at 0.80 conc." name="pred2";
  pbspline y=s_f20 x=day /
    legendlabel = "Female at 0.20 conc." name="pred3";
  pbspline y=s_f80 x=day /
    legendlabel = "Female at 0.80 conc." name="pred4";
  discretelegend "pred1" "pred2" "pred3" "pred4" / across=2;
run;

```

**Output 54.14.4** Predicted Survival at Insecticide Concentrations of 0.20 and 0.80 mg/cm<sup>2</sup>

The probability of survival is displayed on the vertical axis. Notice that most of the insecticide effect occurs by day 6 for both the high and low concentrations.

## Example 54.15: Scoring Data Sets

This example first illustrates the syntax used for scoring data sets, then uses a previously scored data set to score a new data set. A generalized logit model is fit to the remote-sensing data set used in the section “[Example 33.4: Linear Discriminant Analysis of Remote-Sensing Data on Crops](#)” on page 2146 of Chapter 33, “[The DISCRIM Procedure](#),” to illustrate discrimination and classification methods. In the following DATA step, the response variable is Crop and the prognostic factors are x1 through x4:

```
data Crops;
  length Crop $ 10;
  infile datalines truncover;
  input Crop $ @@;
  do i=1 to 3;
    input x1-x4 @@;
    if (x1 ^= .) then output;
```

```

end;
input;
datalines;
Corn      16 27 31 33 15 23 30 30 16 27 27 26
Corn      18 20 25 23 15 15 31 32 15 32 32 15
Corn      12 15 16 73
Soybeans  20 23 23 25 24 24 25 32 21 25 23 24
Soybeans  27 45 24 12 12 13 15 42 22 32 31 43
Cotton    31 32 33 34 29 24 26 28 34 32 28 45
Cotton    26 25 23 24 53 48 75 26 34 35 25 78
Sugarbeets 22 23 25 42 25 25 24 26 34 25 16 52
Sugarbeets 54 23 21 54 25 43 32 15 26 54 2 54
Clover    12 45 32 54 24 58 25 34 87 54 61 21
Clover    51 31 31 16 96 48 54 62 31 31 11 11
Clover    56 13 13 71 32 13 27 32 36 26 54 32
Clover    53 08 06 54 32 32 62 16
;

```

In the following statements, you specify a **SCORE** statement to use the fitted model to score the Crops data. The data together with the predicted values are saved in the data set Score1. The output from the **EFFECTPLOT** statement is discussed at the end of this section.

```

ods graphics on;
proc logistic data=Crops;
  model Crop=x1-x4 / link=glogit;
  score out=Score1;
  effectplot slicefit(x=x3);
run;
ods graphics off;

```

In the following statements, the model is fit again, and the data and the predicted values are saved into the data set Score2. The **OUTMODEL=** option saves the fitted model information in the permanent SAS data set sasuser.CropModel, and the **STORE** statement saves the fitted model information into the SAS data set CropModel2. Both the **OUTMODEL=** option and the **STORE** statement are specified to illustrate their use; you would usually specify only one of these model-storing methods.

```

proc logistic data=Crops outmodel=sasuser.CropModel;
  model Crop=x1-x4 / link=glogit;
  score data=Crops out=Score2;
  store CropModel2;
run;

```

To score data without refitting the model, specify the **INMODEL=** option to identify a previously saved SAS data set of model information. In the following statements, the model is read from the sasuser.CropModel data set, and the data and the predicted values are saved in the data set Score3. Note that the data set being scored does not have to include the response variable.

```

proc logistic inmodel=sasuser.CropModel;
  score data=Crops out=Score3;
run;

```

Another method available to score the data without refitting the model is to invoke the PLM procedure. In the following statements, the stored model is named in the **SOURCE=** option. The **PREDICTED=** option computes the linear predictors, and the **ILINK** option transforms the linear predictors to the probability

scale. The SCORE statement scores the Crops data set, and the predicted probabilities are saved in the data set ScorePLM. See Chapter 69, “[The PLM Procedure](#),” for more information.

```
proc plm source=CropModel2;
    score data=Crops out=ScorePLM predicted=p / ilink;
run;
```

For each observation in the Crops data set, the ScorePLM data set contains 5 observations—one for each level of the response variable. The following statements transform this data set into a form that is similar to the other scored data sets in this example:

```
proc transpose data=ScorePLM out=Score4 prefix=P_ let;
    id _LEVEL_;
    var p;
    by x1-x4 notsorted;
run;
data Score4(drop=_NAME_ _LABEL_);
    merge Score4 Crops(keep=Crop x1-x4);
    F_Crop=Crop;
run;
proc summary data=ScorePLM nway;
    by x1-x4 notsorted;
    var p;
    output out=into maxid(p(_LEVEL_))=I_Crop;
run;
data Score4;
    merge Score4 into(keep=I_Crop);
run;
```

To set prior probabilities on the responses, specify the **PRIOR=** option to identify a SAS data set containing the response levels and their priors. In the following statements, the Prior data set contains the values of the response variable (because this example uses single-trial MODEL statement syntax) and a **\_PRIOR\_** variable containing values proportional to the default priors. The data and the predicted values are saved in the data set Score5.

```
data Prior;
    length Crop $10.;
    input Crop _PRIOR_;
    datalines;
Clover      11
Corn        7
Cotton      6
Soybeans    6
Sugarbeets  6
;

proc logistic inmodel=sasuser.CropModel;
    score data=Crops prior=prior out=Score5 fitstat;
run;
```

The “Fit Statistics for SCORE Data” table displayed in [Output 54.15.1](#) shows that 47.22% of the observations are misclassified.

**Output 54.15.1** Fit Statistics for Data Set Prior

| Fit Statistics for SCORE Data |                    |                   |                          |          |                |          |
|-------------------------------|--------------------|-------------------|--------------------------|----------|----------------|----------|
| Data Set                      | Total<br>Frequency | Log<br>Likelihood | Error<br>Rate            | AIC      | AICC           | BIC      |
| WORK.CROPS                    | 36                 | -32.2247          | 0.4722                   | 104.4493 | 160.4493       | 136.1197 |
| Data Set                      | SC                 | R-Square          | Max-Rescaled<br>R-Square | AUC      | Brier<br>Score |          |
| WORK.CROPS                    | 136.1197           | 0.744081          | 0.777285                 | .        | 0.492712       |          |

The data sets Score1, Score2, Score3, Score4, and Score5 are identical. The following statements display the scoring results in [Output 54.15.2](#):

```
proc freq data=Score1;
  table F_Crop*I_Crop / nocol nocum nopercent;
run;
```

**Output 54.15.2** Classification of Data Used for Scoring

| Table of F_Crop by I_Crop |        |                    |        |          |            |       |
|---------------------------|--------|--------------------|--------|----------|------------|-------|
| F_Crop(From: Crop)        |        | I_Crop(Into: Crop) |        |          |            |       |
| Frequency                 |        |                    |        |          |            |       |
| Row Pct                   | Clover | Corn               | Cotton | Soybeans | Sugarbeets | Total |
| Clover                    | 6      | 0                  | 2      | 2        | 1          | 11    |
|                           | 54.55  | 0.00               | 18.18  | 18.18    | 9.09       |       |
| Corn                      | 0      | 7                  | 0      | 0        | 0          | 7     |
|                           | 0.00   | 100.00             | 0.00   | 0.00     | 0.00       |       |
| Cotton                    | 4      | 0                  | 1      | 1        | 0          | 6     |
|                           | 66.67  | 0.00               | 16.67  | 16.67    | 0.00       |       |
| Soybeans                  | 1      | 1                  | 1      | 3        | 0          | 6     |
|                           | 16.67  | 16.67              | 16.67  | 50.00    | 0.00       |       |
| Sugarbeets                | 2      | 0                  | 0      | 2        | 2          | 6     |
|                           | 33.33  | 0.00               | 0.00   | 33.33    | 33.33      |       |
| Total                     | 13     | 8                  | 4      | 8        | 3          | 36    |

The following statements use the previously fitted and saved model in the sasuser.CropModel data set to score the observations in a new data set, Test. The results of scoring the test data are saved in the ScoredTest data set and displayed in [Output 54.15.3](#).



```

data Test;
  input Crop $ 1-10 x1-x4;
  datalines;
Corn      16 27 31 33
Soybeans  21 25 23 24
Cotton    29 24 26 28
Sugarbeets 54 23 21 54
Clover    32 32 62 16
;

proc logistic noprint inmodel=sasuser.CropModel;
  score data=Test out=ScoredTest;
run;

proc print data=ScoredTest label noobs;
  var F_Crop I_Crop P_Clover P_Corn P_Cotton P_Soybeans P_Sugarbeets;
run;

```

Output 54.15.3 Classification of Test Data

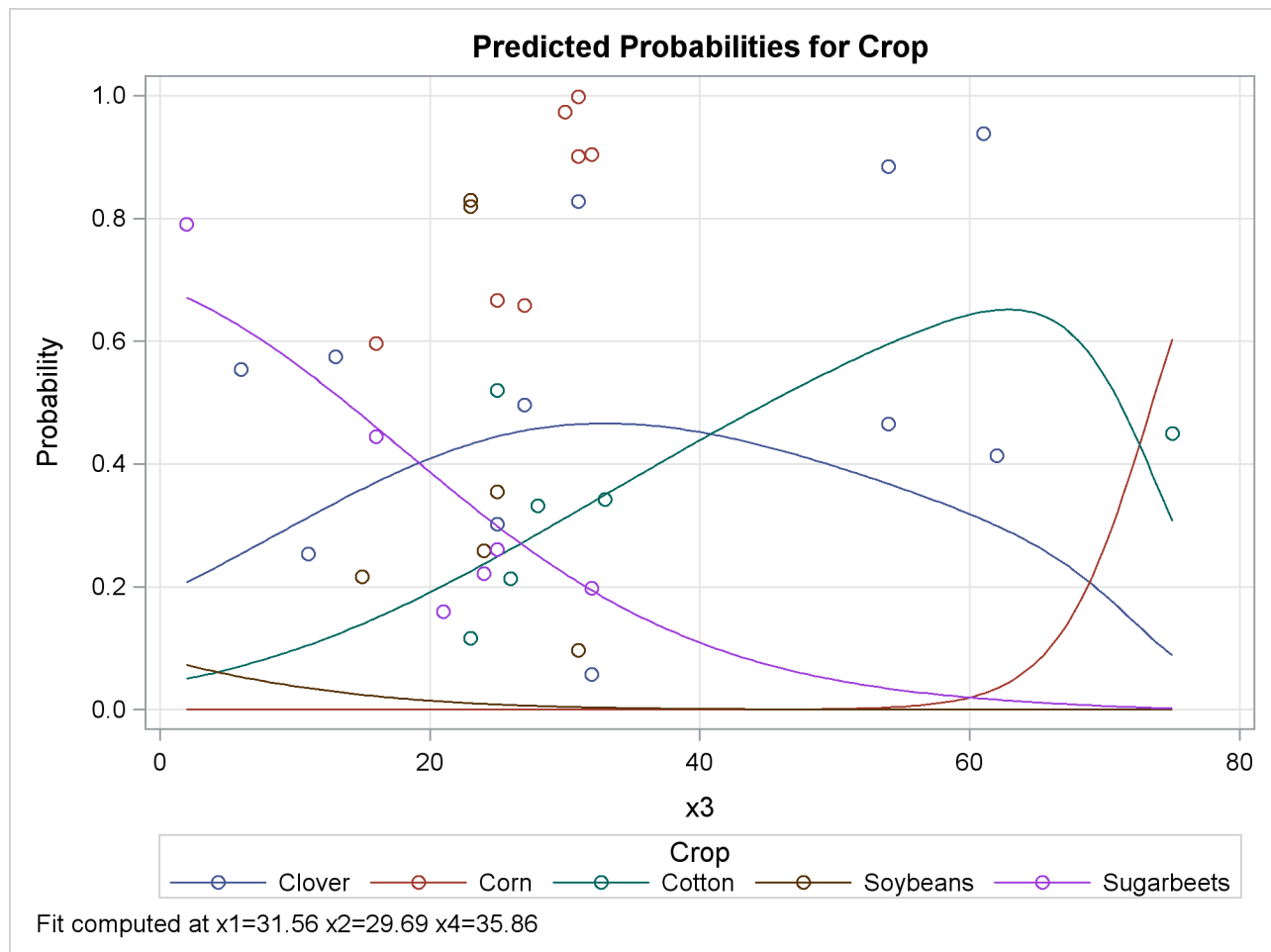
| From: Crop | Into:<br>Crop | Predicted<br>Probability:<br>Crop=Clover | Predicted<br>Probability:<br>Crop=Corn |
|------------|---------------|------------------------------------------|----------------------------------------|
| Corn       | Corn          | 0.00342                                  | 0.90067                                |
| Soybeans   | Soybeans      | 0.04801                                  | 0.03157                                |
| Cotton     | Clover        | 0.43180                                  | 0.00015                                |
| Sugarbeets | Clover        | 0.66681                                  | 0.00000                                |
| Clover     | Cotton        | 0.41301                                  | 0.13386                                |

| Predicted<br>Probability:<br>Crop=Cotton | Predicted<br>Probability:<br>Crop=Soybeans | Predicted<br>Probability:<br>Crop=Sugarbeets |
|------------------------------------------|--------------------------------------------|----------------------------------------------|
| 0.00500                                  | 0.08675                                    | 0.00416                                      |
| 0.02865                                  | 0.82933                                    | 0.06243                                      |
| 0.21267                                  | 0.07623                                    | 0.27914                                      |
| 0.17364                                  | 0.00000                                    | 0.15955                                      |
| 0.43649                                  | 0.00033                                    | 0.01631                                      |

The **EFFECTPLOT** statement that is specified in the first PROC LOGISTIC invocation produces a plot of the model-predicted probabilities versus X3 while holding the other three covariates at their means (Output 54.15.4). This plot shows how the value of X3 affects the probabilities of the various crops when the other prognostic factors are fixed at their means. If you are interested in the effect of X3 when the other covariates are fixed at a certain level—say, 10—specify the following EFFECTPLOT statement.

```
effectplot slicefit(x=x3) / at (x1=10 x2=10 x4=10)
```

**Output 54.15.4** Model-Predicted Probabilities

### Example 54.16: Using the LSMEANS Statement

Recall the main-effects model fit to the Neuralgia data set in [Example 54.2](#). The Treatment\*Sex interaction, which was previously shown to be nonsignificant, is added back into the model for this discussion.

In the following statements, the **ODDSRATIO** statement is specified to produce odds ratios of pairwise differences of the Treatment parameters in the presence of the Sex interaction. The **LSMEANS** statement is specified with several options: the **E** option displays the coefficients that are used to compute the LS-means for each Treatment level, the **DIFF** option takes all pairwise differences of the LS-means for the levels of the Treatment variable, the **ODDSRATIO** option computes odds ratios of these differences, the **CL** option produces confidence intervals for the differences and odds ratios, and the **ADJUST=BON** option performs a very conservative adjustment of the  $p$ -values and confidence intervals.

```

proc logistic data=Neuralgia;
  class Treatment Sex / param=glm;
  model Pain= Treatment|Sex Age;
  oddsratio Treatment;
  lsmeans Treatment / e diff oddsratio cl adjust=bon;
run;

```

The results from the **ODDSRATIO** statement are displayed in [Output 54.16.1](#). All pairwise differences of levels of the Treatment effect are compared. However, because of the interaction between the Treatment and Sex variables, each difference is computed at each of the two levels of the Sex variable. These results show that the difference between Treatment levels A and B is insignificant for both genders.

To compute these odds ratios, you must first construct a linear combination of the parameters,  $l'\beta$ , for each level that is compared with all other levels fixed at some value. For example, to compare Treatment=A with B for Sex=F, you fix the Age variable at its mean, 70.05, and construct the following  $l$  vectors:

|               | Intercept | Treatment |    |   | Sex |   | Treatment*Sex |    |    |    |    |    | Age   |
|---------------|-----------|-----------|----|---|-----|---|---------------|----|----|----|----|----|-------|
|               |           | A         | B  | P | F   | M | AF            | AM | BF | BM | PF | PM |       |
| $l'_A$        | 1         | 1         | 0  | 0 | 1   | 0 | 1             | 0  | 0  | 0  | 0  | 0  | 70.05 |
| $l'_B$        | 1         | 0         | 1  | 0 | 1   | 0 | 0             | 0  | 1  | 0  | 0  | 0  | 70.05 |
| $l'_A - l'_B$ | 0         | 1         | -1 | 0 | 0   | 0 | 1             | 0  | -1 | 0  | 0  | 0  | 0     |

Then the odds ratio for Treatment A versus B at Sex=F is computed as  $\exp((l'_A - l'_B)\beta)$ . Different  $l$  vectors must be similarly constructed when Sex=M because the resulting odds ratio will be different due to the interaction.

**Output 54.16.1** Odds Ratios from the ODDSRATIO Statement

| Odds Ratio Estimates and Wald Confidence Intervals |          |                       |          |
|----------------------------------------------------|----------|-----------------------|----------|
| Label                                              | Estimate | 95% Confidence Limits |          |
| Treatment A vs B at Sex=F                          | 0.398    | 0.016                 | 9.722    |
| Treatment A vs P at Sex=F                          | 16.892   | 1.269                 | 224.838  |
| Treatment B vs P at Sex=F                          | 42.492   | 2.276                 | 793.254  |
| Treatment A vs B at Sex=M                          | 0.663    | 0.078                 | 5.623    |
| Treatment A vs P at Sex=M                          | 34.766   | 1.807                 | 668.724  |
| Treatment B vs P at Sex=M                          | 52.458   | 2.258                 | >999.999 |

The results from the **LSMEANS** statement are displayed in [Output 54.16.2](#) through [Output 54.16.4](#).

The LS-means are computed by constructing each of the  $l$  coefficient vectors shown in [Output 54.16.2](#), and then computing  $l'\beta$ . The LS-means are not estimates of the event probabilities; they are estimates of the linear predictors on the logit scale. In order to obtain event probabilities, you need to apply the inverse-link transformation by specifying the **ILINK** option in the **LSMEANS** statement. Notice in [Output 54.16.2](#) that the Sex rows do not indicate either Sex=F or Sex=M. Instead, the LS-means are computed at an average of these two levels, so only one result needs to be reported. For more information about the construction of LS-means, see the section “[Construction of Least Squares Means](#)” on page 3363 of Chapter 42, “[The GLM Procedure](#).”

**Output 54.16.2** Treatment LS-Means Coefficients

| Coefficients for Treatment Least Squares Means |           |     |       |       |       |
|------------------------------------------------|-----------|-----|-------|-------|-------|
| Parameter                                      | Treatment | Sex | Row1  | Row2  | Row3  |
| Intercept: Pain=No                             |           |     | 1     | 1     | 1     |
| Treatment A                                    | A         |     | 1     |       |       |
| Treatment B                                    | B         |     |       | 1     |       |
| Treatment P                                    | P         |     |       |       | 1     |
| Sex F                                          |           | F   | 0.5   | 0.5   | 0.5   |
| Sex M                                          |           | M   | 0.5   | 0.5   | 0.5   |
| Treatment A * Sex F                            | A         | F   | 0.5   |       |       |
| Treatment A * Sex M                            | A         | M   | 0.5   |       |       |
| Treatment B * Sex F                            | B         | F   |       | 0.5   |       |
| Treatment B * Sex M                            | B         | M   |       | 0.5   |       |
| Treatment P * Sex F                            | P         | F   |       |       | 0.5   |
| Treatment P * Sex M                            | P         | M   |       |       | 0.5   |
| Age                                            |           |     | 70.05 | 70.05 | 70.05 |

The Treatment LS-means shown in [Output 54.16.3](#) are all significantly nonzero at the 0.05 level. These LS-means are *predicted population margins* of the logits; that is, they estimate the marginal means over a balanced population, and they are effectively the within-Treatment means appropriately adjusted for the other effects in the model. The LS-means are not event probabilities; in order to obtain event probabilities, you need to apply the inverse-link transformation by specifying the ILINK option in the **LSMEANS** statement. For more information about LS-means, see the section “**LSMEANS Statement**” on page 453 of Chapter 19, “**Shared Concepts and Topics**.”

**Output 54.16.3** Treatment LS-Means

| Treatment Least Squares Means |          |                |         |         |       |         |         |
|-------------------------------|----------|----------------|---------|---------|-------|---------|---------|
| Treatment                     | Estimate | Standard Error | z Value | Pr >  z | Alpha | Lower   | Upper   |
| A                             | 1.3195   | 0.6664         | 1.98    | 0.0477  | 0.05  | 0.01331 | 2.6257  |
| B                             | 1.9864   | 0.7874         | 2.52    | 0.0116  | 0.05  | 0.4431  | 3.5297  |
| P                             | -1.8682  | 0.7620         | -2.45   | 0.0142  | 0.05  | -3.3618 | -0.3747 |

Pairwise differences between the Treatment LS-means, requested with the **DIFF** option, are displayed in [Output 54.16.4](#). The LS-mean for the level that is displayed in the `_Treatment` column is subtracted from the LS-mean for the level in the `Treatment` column, so the first row displays the LS-mean for Treatment level A minus the LS-mean for Treatment level B. The `Pr > |z|` column indicates that the A and B levels are not significantly different; however, both of these levels are different from level P. If the inverse-link transformation is specified with the **ILINK** option, then these differences do not transform back to differences in probabilities.

There are two odds ratios for Treatment level A versus B in [Output 54.16.1](#); these are constructed at each level of the interacting covariate Sex. In contrast, there is only one LS-means odds ratio for Treatment level A versus B in [Output 54.16.4](#). This odds ratio is computed at an average of the interacting effects by creating the  $\mathbf{l}$  vectors shown in [Output 54.16.2](#) (the Row1 column corresponds to  $\mathbf{l}_A$  and the Row2 column corresponds to  $\mathbf{l}_B$ ) and computing  $\exp(\mathbf{l}'_A \boldsymbol{\beta} - \mathbf{l}'_B \boldsymbol{\beta})$ .

Since multiple tests are performed, you can protect yourself from falsely significant results by adjusting your  $p$ -values for multiplicity. The **ADJUST=BON** option performs the very conservative Bonferroni adjustment, and adds the columns labeled with 'Adj' to [Output 54.16.4](#). Comparing the  $\text{Pr} > |z|$  column to the Adj P column, you can see that the  $p$ -values are adjusted upwards; in this case, there is no change in your conclusions. The confidence intervals are also adjusted for multiplicity—all adjusted intervals are wider than the unadjusted intervals, but again your conclusions in this example are unchanged.

**Output 54.16.4** Differences and Odds Ratios for the Treatment LS-Means

| Differences of Treatment Least Squares Means<br>Adjustment for Multiple Comparisons: Bonferroni |            |                                       |                                       |                      |                      |            |       |
|-------------------------------------------------------------------------------------------------|------------|---------------------------------------|---------------------------------------|----------------------|----------------------|------------|-------|
| Treatment                                                                                       | _Treatment | Estimate                              | Standard Error                        | z Value              | Pr >  z              | Adj P      | Alpha |
| A                                                                                               | B          | -0.6669                               | 1.0026                                | -0.67                | 0.5059               | 1.0000     | 0.05  |
| A                                                                                               | P          | 3.1877                                | 1.0376                                | 3.07                 | 0.0021               | 0.0064     | 0.05  |
| B                                                                                               | P          | 3.8547                                | 1.2126                                | 3.18                 | 0.0015               | 0.0044     | 0.05  |
| Differences of Treatment Least Squares Means<br>Adjustment for Multiple Comparisons: Bonferroni |            |                                       |                                       |                      |                      |            |       |
| Treatment                                                                                       | _Treatment | Lower                                 | Upper                                 | Adj Lower            | Adj Upper            | Odds Ratio |       |
| A                                                                                               | B          | -2.6321                               | 1.2982                                | -3.0672              | 1.7334               | 0.513      |       |
| A                                                                                               | P          | 1.1541                                | 5.2214                                | 0.7037               | 5.6717               | 24.234     |       |
| B                                                                                               | P          | 1.4780                                | 6.2313                                | 0.9517               | 6.7576               | 47.213     |       |
| Differences of Treatment Least Squares Means<br>Adjustment for Multiple Comparisons: Bonferroni |            |                                       |                                       |                      |                      |            |       |
| Treatment                                                                                       | _Treatment | Lower Confidence Limit for Odds Ratio | Upper Confidence Limit for Odds Ratio | Adj Lower Odds Ratio | Adj Upper Odds Ratio |            |       |
| A                                                                                               | B          | 0.072                                 | 3.663                                 | 0.047                | 5.660                |            |       |
| A                                                                                               | P          | 3.171                                 | 185.195                               | 2.021                | 290.542              |            |       |
| B                                                                                               | P          | 4.384                                 | 508.441                               | 2.590                | 860.612              |            |       |

If you want to jointly test whether the active treatments are different from the placebo, you can specify a custom hypothesis test with the **LSMESTIMATE** statement. In the following statements, the LS-means for the two treatments are contrasted against the LS-mean of the placebo, and the **JOINT** option performs a joint test that the two treatments are not different from placebo.

```
proc logistic data=Neuralgia;
  class Treatment Sex / param=glm;
  model Pain= Treatment|Sex Age;
  lsmestimate treatment 1 0 -1, 0 1 -1 / joint;
run;
```

[Output 54.16.5](#) displays the results from the **LSMESTIMATE** statement. The “Least Squares Means Es-

imate” table displays the differences of the two active treatments against the placebo, and the results are identical to the second and third rows of [Output 54.16.3](#). The “Chi-Square Test for Least Squares Means Estimates” table displays the joint test. In all of these tests, you reject the null hypothesis that the treatment has the same effect as the placebo.

**Output 54.16.5** Custom LS-Mean Tests

| Least Squares Means Estimates                     |        |            |                |         |         |
|---------------------------------------------------|--------|------------|----------------|---------|---------|
| Effect                                            | Label  | Estimate   | Standard Error | z Value | Pr >  z |
| Treatment                                         | Row 1  | 3.1877     | 1.0376         | 3.07    | 0.0021  |
| Treatment                                         | Row 2  | 3.8547     | 1.2126         | 3.18    | 0.0015  |
| Chi-Square Test for Least Squares Means Estimates |        |            |                |         |         |
| Effect                                            | Num DF | Chi-Square | Pr > ChiSq     |         |         |
| Treatment                                         | 2      | 12.13      | 0.0023         |         |         |

If you want to work with LS-means but you prefer to compute the Treatment odds ratios within the Sex levels in the same fashion as the [ODDSRATIO](#) statement does, you can specify the [SLICE](#) statement. In the following statements, you specify the same options in the [SLICE](#) statement as you do in the [LSMEANS](#) statement, except that you also specify the [SLICEBY=](#) option to perform an LS-means analysis partitioned into sets that are defined by the Sex variable:

```
proc logistic data=Neuralgia;
  class Treatment Sex / param=glm;
  model Pain= Treatment|Sex Age;
  slice Treatment*Sex / sliceby=Sex diff oddsratio cl adjust=bon;
run;
```

The results for Sex=F are displayed in [Output 54.16.6](#) and [Output 54.16.7](#). The joint test in [Output 54.16.6](#) tests the equality of the LS-means of the levels of Treatment for Sex=F, and rejects equality at level 0.05. In [Output 54.16.7](#), the odds ratios and confidence intervals match those reported for Sex=F in [Output 54.16.1](#), and multiplicity adjustments are performed.

**Output 54.16.6** Joint Test of Treatment Equality for Females

| Chi-Square Test for Treatment*Sex<br>Least Squares Means Slice |        |            |            |
|----------------------------------------------------------------|--------|------------|------------|
| Slice                                                          | Num DF | Chi-Square | Pr > ChiSq |
| Sex F                                                          | 2      | 8.22       | 0.0164     |

**Output 54.16.7** Differences of the Treatment LS-Means for Females

| Simple Differences of Treatment*Sex Least Squares Means |           |            |                |                                       |                                       |           |           |
|---------------------------------------------------------|-----------|------------|----------------|---------------------------------------|---------------------------------------|-----------|-----------|
| Adjustment for Multiple Comparisons: Bonferroni         |           |            |                |                                       |                                       |           |           |
| Slice                                                   | Treatment | _Treatment | Estimate       | Standard Error                        | z Value                               | Pr >  z   | Adj P     |
| Sex F                                                   | A         | B          | -0.9224        | 1.6311                                | -0.57                                 | 0.5717    | 1.0000    |
| Sex F                                                   | A         | P          | 2.8269         | 1.3207                                | 2.14                                  | 0.0323    | 0.0970    |
| Sex F                                                   | B         | P          | 3.7493         | 1.4933                                | 2.51                                  | 0.0120    | 0.0361    |
| Simple Differences of Treatment*Sex Least Squares Means |           |            |                |                                       |                                       |           |           |
| Adjustment for Multiple Comparisons: Bonferroni         |           |            |                |                                       |                                       |           |           |
| Slice                                                   | Treatment | _Treatment | Alpha          | Lower                                 | Upper                                 | Adj Lower | Adj Upper |
| Sex F                                                   | A         | B          | 0.05           | -4.1193                               | 2.2744                                | -4.8272   | 2.9824    |
| Sex F                                                   | A         | P          | 0.05           | 0.2384                                | 5.4154                                | -0.3348   | 5.9886    |
| Sex F                                                   | B         | P          | 0.05           | 0.8225                                | 6.6761                                | 0.1744    | 7.3243    |
| Simple Differences of Treatment*Sex Least Squares Means |           |            |                |                                       |                                       |           |           |
| Adjustment for Multiple Comparisons: Bonferroni         |           |            |                |                                       |                                       |           |           |
| Slice                                                   | Treatment | _Treatment | Odds Ratio     | Lower Confidence Limit for Odds Ratio | Upper Confidence Limit for Odds Ratio |           |           |
| Sex F                                                   | A         | B          | 0.398          | 0.016                                 | 9.722                                 |           |           |
| Sex F                                                   | A         | P          | 16.892         | 1.269                                 | 224.838                               |           |           |
| Sex F                                                   | B         | P          | 42.492         | 2.276                                 | 793.254                               |           |           |
| Simple Differences of Treatment*Sex                     |           |            |                |                                       |                                       |           |           |
| Least Squares Means                                     |           |            |                |                                       |                                       |           |           |
| Adjustment for Multiple Comparisons: Bonferroni         |           |            |                |                                       |                                       |           |           |
| Slice                                                   | Treatment | _Treatment | Adj Odds Ratio | Lower                                 | Upper                                 |           |           |
| Sex F                                                   | A         | B          | 0.008          |                                       | 19.734                                |           |           |
| Sex F                                                   | A         | P          | 0.715          |                                       | 398.848                               |           |           |
| Sex F                                                   | B         | P          | 1.190          |                                       | >999.999                              |           |           |

Similarly, the results for Sex=M are shown in [Output 54.16.8](#) and [Output 54.16.9](#).

**Output 54.16.8** Joint Test of Treatment Equality for Males

| Chi-Square Test for Treatment*Sex<br>Least Squares Means Slice |           |            |            |
|----------------------------------------------------------------|-----------|------------|------------|
| Slice                                                          | Num<br>DF | Chi-Square | Pr > ChiSq |
| Sex M                                                          | 2         | 6.64       | 0.0361     |

**Output 54.16.9** Differences of the Treatment LS-Means for Males

| Simple Differences of Treatment*Sex Least Squares Means |           |            |            |                                       |                                       |             |           |
|---------------------------------------------------------|-----------|------------|------------|---------------------------------------|---------------------------------------|-------------|-----------|
| Adjustment for Multiple Comparisons: Bonferroni         |           |            |            |                                       |                                       |             |           |
| Slice                                                   | Treatment | _Treatment | Estimate   | Standard Error                        | z Value                               | Pr >  z     | Adj P     |
| Sex M                                                   | A         | B          | -0.4114    | 1.0910                                | -0.38                                 | 0.7061      | 1.0000    |
| Sex M                                                   | A         | P          | 3.5486     | 1.5086                                | 2.35                                  | 0.0187      | 0.0560    |
| Sex M                                                   | B         | P          | 3.9600     | 1.6049                                | 2.47                                  | 0.0136      | 0.0408    |
| Simple Differences of Treatment*Sex Least Squares Means |           |            |            |                                       |                                       |             |           |
| Adjustment for Multiple Comparisons: Bonferroni         |           |            |            |                                       |                                       |             |           |
| Slice                                                   | Treatment | _Treatment | Alpha      | Lower                                 | Upper                                 | Adj Lower   | Adj Upper |
| Sex M                                                   | A         | B          | 0.05       | -2.5496                               | 1.7268                                | -3.0231     | 2.2003    |
| Sex M                                                   | A         | P          | 0.05       | 0.5919                                | 6.5054                                | -0.06286    | 7.1601    |
| Sex M                                                   | B         | P          | 0.05       | 0.8145                                | 7.1055                                | 0.1180      | 7.8021    |
| Simple Differences of Treatment*Sex Least Squares Means |           |            |            |                                       |                                       |             |           |
| Adjustment for Multiple Comparisons: Bonferroni         |           |            |            |                                       |                                       |             |           |
| Slice                                                   | Treatment | _Treatment | Odds Ratio | Lower Confidence Limit for Odds Ratio | Upper Confidence Limit for Odds Ratio |             |           |
| Sex M                                                   | A         | B          | 0.663      | 0.078                                 | 5.623                                 |             |           |
| Sex M                                                   | A         | P          | 34.766     | 1.807                                 | 668.724                               |             |           |
| Sex M                                                   | B         | P          | 52.458     | 2.258                                 | >999.999                              |             |           |
| Simple Differences of Treatment*Sex                     |           |            |            |                                       |                                       |             |           |
| Least Squares Means                                     |           |            |            |                                       |                                       |             |           |
| Adjustment for Multiple Comparisons: Bonferroni         |           |            |            |                                       |                                       |             |           |
| Slice                                                   | Treatment | _Treatment | Adj Odds   | Lower Ratio                           | Adj Odds                              | Upper Ratio |           |
| Sex M                                                   | A         | B          |            | 0.049                                 |                                       | 9.028       |           |
| Sex M                                                   | A         | P          |            | 0.939                                 |                                       | >999.999    |           |
| Sex M                                                   | B         | P          |            | 1.125                                 |                                       | >999.999    |           |



## Example 54.17: Partial Proportional Odds Model

Cameron and Trivedi (1998, p. 68) studied the number of doctor visits from the Australian Health Survey 1977–78. The data set contains a dependent variable, `dvisits`, which contains the number of doctor visits in the past two weeks (0, 1, or 2, where 2 represents two or more visits) and the following explanatory variables: `sex`, which indicates whether the patient is female; `age`, which contains the patient's age in years divided by 100; `income`, which contains the patient's annual income (in units of \$10,000); `levyplus`, which indicates whether the patient has private health insurance; `freepoor`, which indicates that the patient has free government health insurance due to low income; `freerepa`, which indicates that the patient has free government health insurance for other reasons; `illness`, which contains the number of illnesses in the past two weeks; `actdays`, which contains the number of days the illness caused reduced activity; `hscore`, which is a questionnaire score; `chcond1`, which indicates a chronic condition that does not limit activity; and `chcond2`, which indicates a chronic condition that limits activity.

```
data docvisit;
  input sex age agesq income levyplus freepoor freerepa
        illness actdays hscore chcond1 chcond2 dvisits;
  if ( dvisits > 2) then dvisits = 2;
datalines;
1 0.19 0.0361 0.55 1 0 0 1 4 1 0 0 1
1 0.19 0.0361 0.45 1 0 0 1 2 1 0 0 1
0 0.19 0.0361 0.90 0 0 0 3 0 0 0 0 1

... more lines ...

1 0.37 0.1369 0.25 0 0 1 1 0 1 0 0 0
1 0.52 0.2704 0.65 0 0 0 0 0 0 0 0 0
0 0.72 0.5184 0.25 0 0 1 0 0 0 0 0 0
;
```

Because the response variable `dvisits` has three levels, the proportional odds model constructs two response functions. There is an intercept parameter for each of the two response functions,  $\alpha_1 < \alpha_2$ , and common slope parameters  $\beta = (\beta_1, \dots, \beta_{12})$  across the functions. The model can be written as

$$\text{logit}(\Pr(Y \leq i \mid \mathbf{x})) = \alpha_i + \beta' \mathbf{x}, \quad i = 1, 2$$

The following statements fit a proportional odds model to this data:

```
proc logistic data=docvisit;
  model dvisits = sex age agesq income levyplus
                freepoor freerepa illness actdays hscore
                chcond1 chcond2;
run;
```

Selected results are displayed in [Output 54.17.1](#).

**Output 54.17.1** Test of Proportional Odds Assumption

| Score Test for the Proportional Odds Assumption |            |            |            |
|-------------------------------------------------|------------|------------|------------|
| Chi-Square                                      | DF         | Pr > ChiSq |            |
| 27.4256                                         | 12         | 0.0067     |            |
| Testing Global Null Hypothesis: BETA=0          |            |            |            |
| Test                                            | Chi-Square | DF         | Pr > ChiSq |
| Likelihood Ratio                                | 734.2971   | 12         | <.0001     |
| Score                                           | 811.8964   | 12         | <.0001     |
| Wald                                            | 690.7156   | 12         | <.0001     |

The test of the proportional odds assumption in [Output 54.17.1](#) rejects the null hypothesis that all the slopes are equal across the two response functions. This test is very anticonservative; that is, it tends to reject the null hypothesis even when the proportional odds assumption is reasonable.

The proportional odds assumption for ordinal response models can be relaxed by specifying the [UNEQUALSLOPES](#) option in the MODEL statement. A fully nonproportional odds model has different slope parameters  $\beta_i = (\beta_{1,i}, \dots, \beta_{12,i})$  for every logit  $i$ :

$$\text{logit}(\Pr(Y \leq i | \mathbf{x})) = \alpha_i + \beta'_i \mathbf{x}, \quad i = 1, 2$$

The nonproportional odds model is fit with the following statements. The [TEST](#) statements test the proportional odds assumption for each of the covariates in the model.

```
proc logistic data=docvisit;
  model dvisits = sex age agesq income levyplus
               freepoor freerepa illness actdays hscore
               chcond1 chcond2 / unequalslopes;
  sex:      test sex_0      =sex_1;
  age:      test age_0      =age_1;
  agesq:    test agesq_0    =agesq_1;
  income:   test income_0   =income_1;
  levyplus: test levyplus_0 =levyplus_1;
  freepoor: test freepoor_0 =freepoor_1;
  freerepa: test freerepa_0 =freerepa_1;
  illness:  test illness_0  =illness_1;
  actdays: test actdays_0 =actdays_1;
  hscore:   test hscore_0   =hscore_1;
  chcond1:  test chcond1_0  =chcond1_1;
  chcond2:  test chcond2_0  =chcond2_1;
run;
```

Selected results from fitting the nonproportional odds model to the data are displayed in [Output 54.17.2](#).

**Output 54.17.2** Results for Nonproportional Odds Model

| Testing Global Null Hypothesis: BETA=0   |         |    |            |                |                 |            |
|------------------------------------------|---------|----|------------|----------------|-----------------|------------|
| Test                                     |         |    | Chi-Square | DF             | Pr > ChiSq      |            |
| Likelihood Ratio                         |         |    | 761.4797   | 24             | <.0001          |            |
| Score                                    |         |    | 957.6793   | 24             | <.0001          |            |
| Wald                                     |         |    | 688.2306   | 24             | <.0001          |            |
| Analysis of Maximum Likelihood Estimates |         |    |            |                |                 |            |
| Parameter                                | dvisits | DF | Estimate   | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                | 0       | 1  | 2.3238     | 0.2754         | 71.2018         | <.0001     |
| Intercept                                | 1       | 1  | 4.2862     | 0.4890         | 76.8368         | <.0001     |
| sex                                      | 0       | 1  | -0.2637    | 0.0818         | 10.3909         | 0.0013     |
| sex                                      | 1       | 1  | -0.1232    | 0.1451         | 0.7210          | 0.3958     |
| age                                      | 0       | 1  | 1.7489     | 1.5115         | 1.3389          | 0.2472     |
| age                                      | 1       | 1  | -2.0974    | 2.6003         | 0.6506          | 0.4199     |
| agesq                                    | 0       | 1  | -2.4718    | 1.6636         | 2.2076          | 0.1373     |
| agesq                                    | 1       | 1  | 2.6883     | 2.8398         | 0.8961          | 0.3438     |
| income                                   | 0       | 1  | -0.00857   | 0.1266         | 0.0046          | 0.9460     |
| income                                   | 1       | 1  | 0.6464     | 0.2375         | 7.4075          | 0.0065     |
| levyplus                                 | 0       | 1  | -0.2658    | 0.0997         | 7.0999          | 0.0077     |
| levyplus                                 | 1       | 1  | -0.2869    | 0.1820         | 2.4848          | 0.1150     |
| freepoor                                 | 0       | 1  | 0.6773     | 0.2601         | 6.7811          | 0.0092     |
| freepoor                                 | 1       | 1  | 0.9020     | 0.4911         | 3.3730          | 0.0663     |
| freerepa                                 | 0       | 1  | -0.4044    | 0.1382         | 8.5637          | 0.0034     |
| freerepa                                 | 1       | 1  | -0.0958    | 0.2361         | 0.1648          | 0.6848     |
| illness                                  | 0       | 1  | -0.2645    | 0.0287         | 84.6792         | <.0001     |
| illness                                  | 1       | 1  | -0.3083    | 0.0499         | 38.1652         | <.0001     |
| actdays                                  | 0       | 1  | -0.1521    | 0.0116         | 172.2764        | <.0001     |
| actdays                                  | 1       | 1  | -0.1863    | 0.0134         | 193.7700        | <.0001     |
| hscore                                   | 0       | 1  | -0.0620    | 0.0172         | 12.9996         | 0.0003     |
| hscore                                   | 1       | 1  | -0.0568    | 0.0252         | 5.0940          | 0.0240     |
| chcond1                                  | 0       | 1  | -0.1140    | 0.0909         | 1.5721          | 0.2099     |
| chcond1                                  | 1       | 1  | -0.2478    | 0.1743         | 2.0201          | 0.1552     |
| chcond2                                  | 0       | 1  | -0.2660    | 0.1255         | 4.4918          | 0.0341     |
| chcond2                                  | 1       | 1  | -0.3146    | 0.2116         | 2.2106          | 0.1371     |

**Output 54.17.2** *continued*

| Linear Hypotheses Testing Results |                    |    |            |
|-----------------------------------|--------------------|----|------------|
| Label                             | Wald<br>Chi-Square | DF | Pr > ChiSq |
| sex                               | 1.0981             | 1  | 0.2947     |
| age                               | 2.5658             | 1  | 0.1092     |
| agesq                             | 3.8309             | 1  | 0.0503     |
| income                            | 8.8006             | 1  | 0.0030     |
| levyplus                          | 0.0162             | 1  | 0.8989     |
| freepoor                          | 0.2569             | 1  | 0.6122     |
| freerepa                          | 2.0099             | 1  | 0.1563     |
| illness                           | 0.8630             | 1  | 0.3529     |
| actdays                           | 6.9407             | 1  | 0.0084     |
| hscore                            | 0.0476             | 1  | 0.8273     |
| chcond1                           | 0.6906             | 1  | 0.4060     |
| chcond2                           | 0.0615             | 1  | 0.8042     |

The preceding nonproportional odds model fits  $12 \times 2 = 24$  slope parameters, and the model seems to overfit the data. You can obtain a more parsimonious model by specifying a subset of the parameters to have nonproportional odds. The following statements allow the parameters for the variables in the “Linear Hypotheses Testing Results” table that have  $p$ -values less than 0.1 (actdays, agesq, and income) to vary across the response functions:

```
proc logistic data=docvisit;
  model dvisits= sex age agesq income levyplus freepoor
    freerepa illness actdays hscore chcond1 chcond2
    / unequalslopes=(actdays agesq income);
run;
```

Selected results from fitting this partial proportional odds model are displayed in [Output 54.17.3](#).

**Output 54.17.3** Results for Partial Proportional Odds Model

| Testing Global Null Hypothesis: BETA=0 |            |    |            |
|----------------------------------------|------------|----|------------|
| Test                                   | Chi-Square | DF | Pr > ChiSq |
| Likelihood Ratio                       | 752.5512   | 15 | <.0001     |
| Score                                  | 947.3269   | 15 | <.0001     |
| Wald                                   | 683.4719   | 15 | <.0001     |

## Output 54.17.3 continued

| Analysis of Maximum Likelihood Estimates |         |    |          |                |                 |            |
|------------------------------------------|---------|----|----------|----------------|-----------------|------------|
| Parameter                                | dvisits | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept                                | 0       | 1  | 2.3882   | 0.2716         | 77.2988         | <.0001     |
| Intercept                                | 1       | 1  | 3.7597   | 0.3138         | 143.5386        | <.0001     |
| sex                                      |         | 1  | -0.2485  | 0.0807         | 9.4789          | 0.0021     |
| age                                      |         | 1  | 1.3000   | 1.4864         | 0.7649          | 0.3818     |
| agesq                                    | 0       | 1  | -2.0110  | 1.6345         | 1.5139          | 0.2186     |
| agesq                                    | 1       | 1  | -0.8789  | 1.6512         | 0.2833          | 0.5945     |
| income                                   | 0       | 1  | 0.0209   | 0.1261         | 0.0275          | 0.8683     |
| income                                   | 1       | 1  | 0.4283   | 0.2221         | 3.7190          | 0.0538     |
| levyplus                                 |         | 1  | -0.2703  | 0.0989         | 7.4735          | 0.0063     |
| freepoor                                 |         | 1  | 0.6936   | 0.2589         | 7.1785          | 0.0074     |
| freerepa                                 |         | 1  | -0.3648  | 0.1358         | 7.2155          | 0.0072     |
| illness                                  |         | 1  | -0.2707  | 0.0281         | 92.7123         | <.0001     |
| actdays                                  | 0       | 1  | -0.1522  | 0.0115         | 173.5696        | <.0001     |
| actdays                                  | 1       | 1  | -0.1868  | 0.0129         | 209.7134        | <.0001     |
| hscore                                   |         | 1  | -0.0609  | 0.0166         | 13.5137         | 0.0002     |
| chcond1                                  |         | 1  | -0.1200  | 0.0901         | 1.7756          | 0.1827     |
| chcond2                                  |         | 1  | -0.2628  | 0.1227         | 4.5849          | 0.0323     |

The partial proportional odds model can be written in the same form as the nonproportional odds model by letting  $\mathbf{x} = (x_1, \dots, x_q, x_{q+1}, \dots, x_{12})$  and  $\boldsymbol{\beta}_i = (\beta_1, \dots, \beta_q, \beta_{q+1,i}, \dots, \beta_{12,i})$ , so the first  $q$  parameters have proportional odds and the remaining parameters do not. The last  $12-q$  parameters can be rewritten to have a common slope:  $\beta_{q+j} + \gamma_{q+j,i}$ ,  $j = 1, \dots, 12-q$ , where the new parameters  $\gamma_i$  contain the increments from the common slopes. The model in this form makes it obvious that the proportional odds model is a submodel of the partial proportional odds models, and both of these are submodels of the nonproportional odds model. This means that you can use likelihood ratio tests to compare models.

You can use the following statements to compute the likelihood ratio tests from the Likelihood Ratio row of the “Testing Global Null hypothesis: BETA=0” tables in the preceding outputs:

```
data a;
  label p='Pr>ChiSq';
  format p 8.6;
  input Test $10. ChiSq1 DF1 ChiSq2 DF2;
  ChiSq= ChiSq1-ChiSq2;
  DF= DF1-DF2;
  p=1-probchi(ChiSq,DF);
  keep Test ChiSq DF p;
  datalines;
Non vs PO      761.4797 24      734.2971 12
PPO vs PO      752.5512 15      734.2971 12
Non vs PPO      761.4797 24      752.5512 15
;

proc print data=a label noobs;
  var Test ChiSq DF p;
run;
```

**Output 54.17.4** Likelihood Ratio Tests

| Test       | ChiSq   | DF | Pr>ChiSq |
|------------|---------|----|----------|
| Non vs PO  | 27.1826 | 12 | 0.007273 |
| PPO vs PO  | 18.2541 | 3  | 0.000390 |
| Non vs PPO | 8.9285  | 9  | 0.443900 |

Therefore, you reject the proportional odds model in favor of both the nonproportional odds model and the partial proportional odds model, and the partial proportional odds model fits as well as the nonproportional odds model. The likelihood ratio test of the nonproportional odds model versus the proportional odds model is very similar to the score test of the proportional odds assumption in [Output 54.17.1](#) because of the large sample size (Stokes, Davis, and Koch 2000, p. 249).

**NOTE:** The proportional odds model has increasing intercepts, which ensures the increasing nature of the cumulative response functions. However, none of the parameters in the partial or nonproportional odds models are constrained. Because of this, sometimes during the optimization process a predicted individual probability can be negative; the optimization continues because it might recover from this situation. Sometimes your final model will predict negative individual probabilities for some of the observations; in this case a message is displayed, and you should check your data for outliers and possibly redefine your model. Other times the model fits your data well, but if you try to score new data you can get negative individual probabilities. This means the model is not appropriate for the data you are trying to score, a message is displayed, and the estimates are set to missing.

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