SAS/QC® 13.2 User’s Guide
The CUSUM Procedure
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## Chapter 6
The CUSUM Procedure

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</table>
Introduction: CUSUM Procedure

The CUSUM procedure creates cumulative sum control charts, also known as *cusum charts*, which display cumulative sums of the deviations of measurements or subgroup means from a target value. Cusum charts are used to decide whether a process is in statistical control by detecting a shift in the process mean.

You can use the CUSUM procedure to

- apply a *one-sided cusum scheme*, also referred to as a *decision interval scheme*, which detects a shift in one direction from the target mean. You can specify the scheme with the decision interval \( h \) and the reference value \( k \).
- apply a *two-sided cusum scheme* with a V-mask, which detects a shift in either direction from the target mean. You can specify the scheme with geometric parameters (\( h \) and \( k \)) for the V-mask or with error probabilities (\( \alpha \) and \( \beta \)).
- implement cusum schemes graphically or computationally
- specify the shift to be detected as a multiple of standard error or in data units
- estimate the process standard deviation \( \sigma \) using a variety of methods
- compute average run lengths (ARLs)
- read raw data (actual measurements) or summarized data (subgroup means and standard deviations)
- analyze multiple process variables. If used with a BY statement, PROC CUSUM produces charts separately for groups of observations.
- save cusums and cusum scheme parameters in output data sets
- tabulate the information displayed on the chart
- read cusum scheme parameters from an input data set
- read numeric- or character-valued subgroup variables
- display subgroups with date and time formats
- enhance cusum charts with special legends and symbol markers that indicate the levels of stratification variables
- superimpose plotted points with stars (polygons) whose vertices indicate the values of multivariate data related to the process
• display a trend chart below the cusum chart that plots a systematic or fitted trend in the data

• produce charts as traditional graphics, ODS Graphics output, or legacy line printer charts. Line printer charts can use special formatting characters that improve the appearance of the chart. Traditional graphics can be annotated, saved, and replayed.

---

Learning about the CUSUM Procedure

If you are using the CUSUM procedure for the first time, begin by reading “PROC CUSUM Statement” on page 541 to learn about input data sets. Then turn to “Getting Started: XCHART Statement” on page 547 in “XCHART Statement: CUSUM Procedure” on page 546. This chapter also provides syntax information and advanced examples.

If you are not familiar with cusum charts, read “Formulas for Cumulative Sums” on page 577 “Defining the Decision Interval for a One-Sided Cusum Scheme” on page 579 and “Defining the V-Mask for a Two-Sided Cusum Scheme” on page 580 in the section “Details: XCHART Statement” on page 576. References lists articles and textbooks that provide more detailed information on cusum charts. The expository articles by Lucas (1976) and Goel (1982) and the textbooks by Montgomery (1996) and Ryan (1989) are recommended introductory reading.

---

PROC CUSUM Statement

Overview: PROC CUSUM Statement

The PROC CUSUM statement starts the CUSUM procedure and it identifies input data sets.

After the PROC CUSUM statement, you provide an XCHART statement that specifies the cusum chart you want to create and the variables in the input data set that you want to analyze. For example, the following statements request a one-sided (decision interval) cusum chart:

```plaintext
proc cusum data=values;
   xchart weight*lot / scheme = onesided
                         mu0    = 8.100
                         sigma0 = 0.050
                         delta  = 1
                         h      = 2.2
                         k      = 0.5;
run;
```

In this example, the DATA= option specifies an input data set (values) that contains the process measurement variable weight and the subgroup-variable lot.

You can use options in the PROC CUSUM statement to do the following:

• specify input data sets containing variables to be analyzed, parameters for cusum schemes, or annotation information
• specify a graphics catalog for saving traditional graphics output
• specify that line printer charts are to be produced
• define characters used for features on line printer charts

In addition to the XCHART statement, you can provide BY statements, ID statements, TITLE statements, and FOOTNOTE statements. If you are producing traditional graphics, you can also provide graphics enhancement statements, such as SYMBOLn statements, which are described in SAS/GRAPH: Reference.


NOTE: If you are using the CUSUM procedure for the first time, you should read both this chapter and the section “Getting Started: XCHART Statement” on page 547 in “XCHART Statement: CUSUM Procedure” on page 546.

Syntax: PROC CUSUM Statement

The syntax for the PROC CUSUM statement is as follows:

```
PROC CUSUM <options> ;
```

The PROC CUSUM statement starts the CUSUM procedure, and it optionally identifies various data sets. You can specify the following options in the PROC CUSUM statement.

**ANNOTATE=SAS-data-set**

specifies an input data set that contains appropriate annotate variables, as described in SAS/GRAPH: Reference. The ANNOTATE= option enables you to add features to a cusum chart (for example, labels that explain out-of-control points). The ANNOTATE= data set is used only when the chart is created as traditional graphics; it is ignored when the LINEPRINTER option is specified or ODS Graphics is enabled. The data set specified with the ANNOTATE= option in the PROC CUSUM statement is a “global” annotate data set in the sense that the information in this data set is displayed on every chart produced in the current run of the CUSUM procedure.

**ANNOTATE2=SAS-data-set**

specifies an input data set that contains appropriate annotate variables that add features to the trend chart (secondary chart) produced with the TRENDVAR= option in the XCHART statement. This option applies only when you produce traditional graphics.

**DATA=SAS-data-set**

names an input data set that contains raw data (measurements) as observations. If the values of the subgroup-variable are numeric, you need to sort the data set so that these values are in increasing order (within BY groups). The DATA= data set can contain more than one observation for each value of the subgroup-variable.

You cannot use a DATA= data set with a HISTORY= data set. If you do not specify a DATA= or HISTORY= data set, PROC CUSUM uses the most recently created data set as a DATA= data set. For more information, see “DATA= Data Set” on page 592.
**FORMCHAR**(index)='string'

defines characters used for features on legacy line printer charts, where *index* is a list of numbers ranging from 1 to 17 and *string* is a character or hexadecimal string. This option applies only if you also specify the LINEPRINTER option.

The *index* identifies which features are controlled with the *string* characters, as described in the following table. If you specify the FORMCHAR= option and omit the *index*, the *string* controls all 17 features.

<table>
<thead>
<tr>
<th>Value of <em>index</em></th>
<th>Description of Character</th>
<th>Chart Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>vertical bar</td>
<td>frame</td>
</tr>
<tr>
<td>2</td>
<td>horizontal bar</td>
<td>frame, central line</td>
</tr>
<tr>
<td>3</td>
<td>box character (upper left)</td>
<td>frame</td>
</tr>
<tr>
<td>4</td>
<td>box character (upper middle)</td>
<td>serifs, tick (horizontal axis)</td>
</tr>
<tr>
<td>5</td>
<td>box character (upper right)</td>
<td>frame</td>
</tr>
<tr>
<td>6</td>
<td>box character (middle left)</td>
<td>not used</td>
</tr>
<tr>
<td>7</td>
<td>box character (middle middle)</td>
<td>serifs</td>
</tr>
<tr>
<td>8</td>
<td>box character (middle right)</td>
<td>tick (vertical axis)</td>
</tr>
<tr>
<td>9</td>
<td>box character (lower left)</td>
<td>frame</td>
</tr>
<tr>
<td>10</td>
<td>box character (lower middle)</td>
<td>serifs</td>
</tr>
<tr>
<td>11</td>
<td>box character (lower right)</td>
<td>frame</td>
</tr>
<tr>
<td>12</td>
<td>vertical bar</td>
<td>control limits</td>
</tr>
<tr>
<td>13</td>
<td>horizontal bar</td>
<td>control limits</td>
</tr>
<tr>
<td>14</td>
<td>box character (upper right)</td>
<td>control limits</td>
</tr>
<tr>
<td>15</td>
<td>box character (lower left)</td>
<td>control limits</td>
</tr>
<tr>
<td>16</td>
<td>box character (lower right)</td>
<td>control limits</td>
</tr>
<tr>
<td>17</td>
<td>box character (upper left)</td>
<td>control limits</td>
</tr>
</tbody>
</table>

Not all printers can produce the characters in the preceding list. By default, the form character list specified by the SAS system option FORMCHAR= is used; otherwise, the default is FORMCHAR='|—+|—|====='. If you print to a PC screen or if your device supports the ASCII symbol set (1 or 2), the following is recommended:

```
formchar='B3,C4,DA,C2,BF,C3,B5,B4,C0,C1,D9,BA,CD,BB,C8,BCD9'X
```

Note that you can use the FORMCHAR= option to temporarily override the values of the SAS system FORMCHAR= option. The values of the SAS system FORMCHAR= option are not altered by the FORMCHAR= option in the PROC CUSUM statement.

**GOUT=**graphics-catalog

specifies the graphics catalog for traditional graphics output from PROC CUSUM. This is useful if you want to save the output. The GOUT= option is used only when the chart is created using traditional graphics; it is ignored when the LINEPRINTER option is specified or ODS Graphics is enabled.

**HISTORY=**SAS-data-set

**HIST=**SAS-data-set

names an input data set that contains subgroup summary statistics (means, standard deviations, and sample sizes). Typically, this data set is created as an OUTHISTORY= data set in a previous run of
PROC CUSUM or PROC SHEWHART, but it can also be created with a SAS summarization procedure such as PROC MEANS.

If the values of the subgroup-variable are numeric, you need to sort the data set so that these values are in increasing order (within BY groups). A HISTORY= data set can contain only one observation for each value for the subgroup-variable.

You cannot use a HISTORY= data set together with a DATA= data set. If you do not specify a HISTORY= or DATA= data set, PROC CUSUM uses the most recently created data set as a DATA= data set. For more information on HISTORY= data sets, see “HISTORY= Data Set” on page 593.

LIMITS=SAS-data-set

names an input data set that contains a set of decision interval or V-mask parameters. Each observation in a LIMITS= data set contains the parameters for a process.

If you are using SAS 6.09 or an earlier release of SAS/QC software, you must specify the options READLIMITS or READINDEX= in the XCHART statement to read the parameters from the LIMITS= data set. In SAS 6.10 and later releases, these options are not needed.

For details about the variables needed in a LIMITS= data set, see “LIMITS= Data Set” on page 592. If you do not provide a LIMITS= data set, you must specify the parameters with options in the XCHART statement.

LINEPRINTER

requests that legacy line printer charts be produced.

BY Statement

BY variables ;

You can specify a BY statement with PROC CUSUM to obtain separate analyses of observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the CUSUM procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

For more information about BY-group processing, see the discussion in SAS Language Reference: Concepts. For more information about the DATASETS procedure, see the discussion in the Base SAS Procedures Guide.
Input and Output Data Sets: CUSUM Procedure

Figure 6.1 summarizes the data sets used with the CUSUM procedure.

Figure 6.1 Input and Output Data Sets in the CUSUM Procedure
Overview: XCHART Statement

The XCHART statement creates cumulative sum control charts from subgroup means or individual measurements. You can create these charts for one-sided cusum (decision interval) schemes or for two-sided (V-mask) schemes. A one-sided scheme is designed to detect either a positive or a negative shift from the target mean, and a two-sided scheme is designed to detect positive and negative shifts from the target mean.

You can use options in the XCHART statement to

- specify parameters for a decision interval or V-mask
- specify the shift \( \delta \) to be detected
- specify the target mean \( \mu_0 \)
- specify a known (standard) value \( \sigma_0 \) for the process standard deviation or estimate the standard deviation from the data using various methods
- tabulate the information displayed on the chart
- save the information displayed on the chart in an output data set
- read parameters for the cusum scheme from a data set
- display a secondary chart that plots a time trend that has been removed from the data
- add block legends and special symbol markers to reveal stratification in process data
- superimpose stars at each point to represent related multivariate factors
- display vertical and horizontal reference lines
- modify the axis values and labels
- modify the chart layout and appearance

You have three alternatives for producing cumulative sum control charts with the XCHART statement:

- ODS Graphics output is produced if ODS Graphics is enabled, for example by specifying the ODS GRAPHICS ON statement prior to the PROC statement.
- Otherwise, traditional graphics are produced by default if SAS/GRAPH® is licensed.
- Legacy line printer charts are produced when you specify the LINEPRINTER option in the PROC statement.

See Chapter 3, “SAS/QC Graphics,” for more information about producing these different kinds of graphs.
Getting Started: XCHART Statement

This section introduces the XCHART statement with simple examples that illustrate the most commonly used options. Complete syntax for the XCHART statement is presented in the section “Syntax: XCHART Statement” on page 561, and advanced examples are given in the section “Examples: XCHART Statement” on page 594.

Creating a V-Mask Cusum Chart from Raw Data

**NOTE:** See *Two-sided Cusum Chart with V-Mask* in the SAS/QC Sample Library.

A machine fills eight-ounce cans of two-cycle engine oil additive. The filling process is believed to be in statistical control, and the process is set so that the average weight of a filled can is $\mu_0 = 8.100$ ounces. Previous analysis shows that the standard deviation of fill weights is $\sigma_0 = 0.050$ ounces. A two-sided cusum chart is used to detect shifts of at least one standard deviation in either the positive or negative direction from the target mean of 8.100 ounces.

Subgroup samples of four cans are selected every hour for twelve hours. The cans are weighed, and their weights are saved in a SAS data set named Oil.

```sas
data Oil;
  label Hour = 'Hour';
  input Hour @;
  do i=1 to 4;
    input Weight @;
    output;
  end;
  drop i;
  datalines;
  1 8.024 8.135 8.151 8.065
  2 7.971 8.165 8.077 8.157
  3 8.125 8.031 8.198 8.050
  4 8.123 8.107 8.154 8.095
  5 8.068 8.093 8.116 8.128
  6 8.177 8.011 8.102 8.030
  7 8.129 8.060 8.125 8.144
  8 8.072 8.010 8.097 8.153
  9 8.066 8.067 8.055 8.059
 10 8.089 8.064 8.170 8.086
 11 8.058 8.098 8.114 8.156
 12 8.147 8.116 8.116 8.018
;
```

The data set Oil is partially listed in Figure 6.2.
Each observation contains one value of Weight along with its associated value of Hour, and the values of Hour are in increasing order. The CUSUM procedure assumes that DATA= input data sets are sorted in this “strung-out” form.

The following statements request a two-sided cusum chart with a V-mask for the average weights:

```sas
ods graphics off;
title 'Cusum Chart for Average Weights of Cans';
proc cusum data=Oil;
xchart Weight*Hour /
   mu0 = 8.100 /* Target mean for process */
   sigma0 = 0.050 /* Known standard deviation */
   delta = 1 /* Shift to be detected */
   alpha = 0.10 /* Type I error probability */
   vaxis = -5 to 3 ;
   label Weight = 'Cumulative Sum';
run;
```

The CUSUM procedure is invoked with the PROC CUSUM statement. The DATA= option in the PROC CUSUM statement specifies that the SAS data set Oil is to be read. The variables to be analyzed are specified in the XCHART statement. The process measurement variable (Weight) is specified before the asterisk (this variable is referred to more generally as a process). The time variable (Hour) is specified after the asterisk (this variable is referred to more generally as a subgroup-variable because it determines how the measurements are classified into rational subgroups).

The option ALPHA=0.10 specifies the probability of a Type 1 error for the cusum scheme (the probability of detecting a shift when none occurs).

The cusum chart is shown in Figure 6.3.
The cusum $S_1$ plotted at Hour=1 is simply the standardized deviation of the first subgroup mean from the target mean.

$$S_1 = \frac{8.09375 - 8.100}{0.050/\sqrt{4}} = -0.250$$

The cusum $S_2$ plotted at Hour=2 is $S_1$ plus the standardized deviation of the second subgroup mean from the target mean.

$$S_2 = S_1 + \frac{8.0925 - 8.100}{0.050/\sqrt{4}} = -0.550$$

In general, the cusum plotted at Hour=$t$ is $S_{t-1}$ plus the standardized deviation of the $t$th subgroup mean from the target mean.

$$S_t = S_{t-1} + \frac{\bar{X}_t - \mu_0}{\sigma_0/\sqrt{n}}$$

For further details, see “Two-Sided Cusum Schemes” on page 578.

You can interpret the chart by comparing the points with the V-mask whose right edge is centered at the most recent point (Hour=12). Since none of the points cross the arms of the V-mask, there is no evidence
that a shift has occurred, and the fluctuations in the cusums can be attributed to chance variation. In general, crossing the lower arm is evidence of an increase in the process mean, whereas crossing the upper arm is evidence of a decrease in the mean.

Creating a V-Mask Cusum Chart from Subgroup Summary Data

**NOTE:** See *Two-sided Cusum Chart with V-Mask* in the SAS/QC Sample Library.

The previous example illustrates how you can create a cusum chart using raw process measurements read from a DATA= data set. In many applications, however, the data are provided in *summarized form* as subgroup means. This example illustrates the use of the XCHART statement when the input data set is a HISTORY= data set.

The following data set provides the subgroup means, standard deviations, and sample sizes corresponding to the variable *Weight* in the data set *Oil* (see the section “Creating a V-Mask Cusum Chart from Raw Data” on page 547:

```sas
   data Oilstat;
      label Hour = 'Hour';
      input Hour WeightX WeightS WeightN;
   datalines;
   1 8.0938 0.0596 4
   2 8.0925 0.0902 4
   3 8.1010 0.0763 4
   4 8.1198 0.0256 4
   5 8.1013 0.0265 4
   6 8.0800 0.0756 4
   7 8.1145 0.0372 4
   8 8.0830 0.0593 4
   9 8.0618 0.0057 4
  10 8.1023 0.0465 4
  11 8.1065 0.0405 4
  12 8.0993 0.0561 4
   ;
```

The data set *Oilstat* is listed in **Figure 6.4**.

**Figure 6.4** Listing of the Data Set Oilstat

<table>
<thead>
<tr>
<th>Obs</th>
<th>Hour</th>
<th>WeightX</th>
<th>WeightS</th>
<th>WeightN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>8.0938</td>
<td>0.0596</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>8.0925</td>
<td>0.0902</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>8.1010</td>
<td>0.0763</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8.1198</td>
<td>0.0256</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>8.1013</td>
<td>0.0265</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>8.0800</td>
<td>0.0756</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>8.1145</td>
<td>0.0372</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8.0830</td>
<td>0.0593</td>
<td>4</td>
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<td>9</td>
<td>9</td>
<td>8.0618</td>
<td>0.0057</td>
<td>4</td>
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<tr>
<td>10</td>
<td>10</td>
<td>8.1023</td>
<td>0.0465</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>8.1065</td>
<td>0.0405</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>8.0993</td>
<td>0.0561</td>
<td>4</td>
</tr>
</tbody>
</table>
Since the data set contains a subgroup variable, a mean variable, a standard deviation variable, and a sample size variable, it can be read as a HISTORY= data set. Note that the names WeightX, WeightS, and WeightN satisfy the naming conventions for summary variables since they begin with a common prefix (Weight) and end with the suffix letters X, S, and N.

The following statements create the cusum chart:

```plaintext
title 'Cusum Chart for Average Weights of Cans';
proc cusum history=Oilstat;
   xchart Weight*Hour /
       mu0 = 8.100 /* target mean */
       sigma0 = 0.050 /* known standard deviation */
       delta = 1 /* shift to be detected */
       alpha = 0.10 /* Type 1 error probability */
       vaxis = -5 to 3 ;
       label WeightX = 'Cumulative Sum';
run;
```

Note that the process Weight specified in the XCHART statement is the prefix of the summary variable names in Oilstat. Also note that the vertical axis label is specified by associating a variable label with the subgroup mean variable (WeightX). The chart (not shown here) is identical to the one in Figure 6.2.

In general, a HISTORY= input data set used with the XRCHART statement must contain the following four variables:

- subgroup variable
- subgroup mean variable
- subgroup range variable
- subgroup sample size variable

Furthermore, the names of subgroup mean, standard deviation, and sample size variables must begin with the prefix process specified in the XRCHART statement and end with the special suffix characters X, S, and N, respectively.

Note that the interpretation of process depends on the input data set specified in the PROC CUSUM statement.

- If raw data are read using the DATA= option (as in the previous example), process is the name of the SAS variable containing the process measurements.
- If summary data are read using the HISTORY= option (as in this example), process is the common prefix for the names containing the summary statistics.

For more information, see “DATA= Data Set” on page 592 and “HISTORY= Data Set” on page 593.
Saving Summary Statistics

NOTE: See Two-sided Cusum Chart with V-Mask in the SAS/QC Sample Library.

In this example, the CUSUM procedure is used to save summary statistics and cusums in an output data set. The summary statistics can subsequently be analyzed by the CUSUM procedure (as in the preceding example). The following statements read the raw measurements from the data set Oil (see “Creating a V-Mask Cusum Chart from Raw Data” on page 547) and create a summary data set named Oilhist:

```sas
title 'Cusum Chart for Average Weights of Cans';
proc cusum data=Oil;
   xchart Weight*Hour /
      nochart
      outhistory = Oilhist
      mu0 = 8.100 /* Target mean for process */
      sigma0 = 0.050 /* Known standard deviation */
      delta = 1 /* Shift to be detected */
      alpha = 0.10 /* Type I error probability */
      vaxis = -5 to 3;
      label Weight = 'Cumulative Sum';
run;
```

The OUTHISTORY= option names the SAS data set containing the summary information, and the NOCHART option suppresses the display of the charts (since the purpose here is simply to create an output data set). Figure 6.5 lists the data set Oilhist.

Figure 6.5 Listing of the Data Set Oilhist

<table>
<thead>
<tr>
<th>Obs</th>
<th>Hour</th>
<th>WeightX</th>
<th>WeightS</th>
<th>WeightC</th>
<th>WeightN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>8.0938</td>
<td>0.0596</td>
<td>-2.500</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>8.0925</td>
<td>0.0902</td>
<td>-5.500</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>8.1010</td>
<td>0.0763</td>
<td>-5.100</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8.1198</td>
<td>0.0256</td>
<td>0.2800</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>8.1013</td>
<td>0.0265</td>
<td>0.3300</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>8.0800</td>
<td>0.0756</td>
<td>-4.700</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>8.1145</td>
<td>0.0372</td>
<td>0.1100</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>8.0830</td>
<td>0.0593</td>
<td>-5.700</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>8.0618</td>
<td>0.0057</td>
<td>-2.100</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>8.1023</td>
<td>0.0405</td>
<td>-2.010</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>8.1065</td>
<td>0.0405</td>
<td>-1.750</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>8.0993</td>
<td>0.0561</td>
<td>-1.780</td>
<td>4</td>
</tr>
</tbody>
</table>

There are five variables in the data set.

- Hour contains the subgroup index
- WeightX contains the subgroup means
- WeightS contains the subgroup standard deviations
- WeightC contains the cumulative sums
• WeightN contains the subgroup sample sizes

Note that the variables in the OUTHISTORY= data set are named by adding the suffix characters X, S, N, and C to the process Weight specified in the XCHART statement. In other words, the variable naming convention for OUTHISTORY= data sets is the same as for HISTORY= data sets.

For more information, see “OUTHISTORY= Data Set” on page 589.

Creating a One-Sided Cusum Chart with a Decision Interval

NOTE: See One-sided Cusum Chart in the SAS/QC Sample Library.

An alternative to the V-mask cusum chart is the one-sided cusum chart with a decision interval, which is sometimes referred to as the “computational form of the cusum chart.” This example illustrates how you can create a one-sided cusum chart for individual measurements.

A can of oil is selected every hour for fifteen hours. The cans are weighed, and their weights are saved in a SAS data set named Cans:

```sas
   data Cans;
   length comment $16;
   label Hour = 'Hour';
   input Hour Weight comment $16. ;
   datalines;
   1 8.024
   2 7.971
   3 8.125
   4 8.123
   5 8.068
   6 8.177 Pump Adjusted
   7 8.229 Pump Adjusted
   8 8.072
   9 8.066
  10 8.089
  11 8.058
  12 8.147
  13 8.141
  14 8.047
  15 8.125
;```

Suppose the problem is to detect a positive shift in the process mean of one standard deviation ($\delta = 1$) from the target of 8.100 ounces. Furthermore, suppose that

• a known value $\sigma_0 = 0.050$ is available for the process standard deviation
• an in-control average run length (ARL) of approximately 100 is required
• an ARL of approximately five is appropriate for detecting the shift

---

1This data set is used by later examples in this chapter.
Table 6.4 indicates that these ARLs can be achieved with the decision interval $h = 3$ and the reference value $k = 0.5$. The following statements use these parameters to create the chart and tabulate the cusum scheme:

```plaintext
options nogstyle;
goptions ftext=swiss;
symbol v=dot color=salmon h=1.8 pct;
title "One-Sided Cusum Analysis";
proc cusum data=Cans;
xchart Weight*Hour /
  mu0 = 8.100 /* target mean for process */
  sigma0 = 0.050 /* known standard deviation */
  delta = 1 /* shift to be detected */
  h = 3 /* cusum parameter h */
  k = 0.5 /* cusum parameter k */
  scheme = onesided /* one-sided decision interval */
  tableall /* table */
  cinfill = ywh
  cframe = bigb
  cout = salmon
  cconnect = salmon
  climits = black
  coutfill = bilg;
  label Weight = 'Cusum of Weight';
run;
options gstyle;
```

The NOGSTYLE system option causes ODS styles not to affect traditional graphics. Instead, the SYMBOL statement, GOPTIONS, and XCHART statement options control the appearance of the graph. The GSTYLE system option restores the use of ODS styles for traditional graphics produced subsequently. The chart is shown in Figure 6.6.
The cusum plotted at \( t \) is

\[
S_t = \max(0, S_{t-1} + (z_t - k))
\]

where \( S_0 = 0 \), and \( z_t \) is the standardized deviation of the \( t \)th measurement from the target.

\[
 z_t = \frac{x_t - \mu_0}{\sigma_0}
\]

The cusum \( S_t \) is referred to as an upper cumulative sum. A shift is signaled at the seventh hour since \( S_7 \) exceeds \( h \). For further details, see “One-Sided Cusum Schemes” on page 577.

The option TABLEALL requests the tables shown in Figure 6.7, Figure 6.8, and Figure 6.9. The table in Figure 6.7 summarizes the cusum scheme, and it confirms that an in-control ARL of 117.6 and an ARL of 6.4 at \( \delta = 1 \) are achieved with the specified \( h \) and \( k \).
Figure 6.7 Summary Table

<table>
<thead>
<tr>
<th>Cusum Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process Variable</td>
</tr>
<tr>
<td>Subgroup Variable</td>
</tr>
<tr>
<td>Scheme</td>
</tr>
<tr>
<td>Target Mean (Mu0)</td>
</tr>
<tr>
<td>Sigma0</td>
</tr>
<tr>
<td>Delta</td>
</tr>
<tr>
<td>Nominal Sample Size</td>
</tr>
<tr>
<td>h</td>
</tr>
<tr>
<td>k</td>
</tr>
<tr>
<td>Average Run Length (Delta)</td>
</tr>
<tr>
<td>Average Run Length (0)</td>
</tr>
</tbody>
</table>

The table in Figure 6.8 tabulates the information displayed in Figure 6.6.

Figure 6.8 Tabulation of One-Sided Chart

One-Sided Cusum Analysis

The CUSUM Procedure

<table>
<thead>
<tr>
<th>Cumulative Sum Chart Summary for Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subgroup Sample Size</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>Hour</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>14</td>
</tr>
<tr>
<td>15</td>
</tr>
</tbody>
</table>

The table in Figure 6.9 presents the computational form of the cusum scheme described by Lucas (1976).
Following the method of Lucas (1976), the process average at the out-of-control point (Hour=7) can be estimated as

\[
\mu_0 + \sigma_0 (N_7 k + S_7)/(N_7 \sqrt{n})
\]

\[
= 8.10 + 0.05(2(0.5) + 3.12)/2
\]

\[
= 8.203 \text{ ounces}
\]

where \( S_7 = 3.12 \) is the upper sum at Hour=7, and \( N_7 = 2 \) is the number of successive positive upper sums at Hour=7.

**Saving Cusum Scheme Parameters**

**NOTE:** See *One-sided Cusum Chart* in the SAS/QC Sample Library.

This example is a continuation of the previous example that illustrates how to save cusum scheme parameters in a data set specified with the OUTLIMITS= option. This enables you to apply the parameters to future data or to subsequently modify the parameters with a DATA step program.
ods graphics on;
title 'One-Sided Cusum Analysis';
proc cusum data=Cans;
   xchart Weight*Hour /
      mu0 = 8.100 /* target mean for process */
      sigma0 = 0.050 /* known standard deviation */
      delta = 1 /* shift to be detected */
      h = 3 /* cusum parameter h */
      k = 0.5 /* cusum parameter k */
      scheme = onesided /* one-sided decision interval */
      outlimits = cusparm
      odstitle = title
      markers;
   label Weight = 'Cusum of Weight';
run;

The chart, shown in Figure 6.10, is similar to the one in Figure 6.6 but is created by using ODS Graphics because the ODS GRAPHICS ON statement is specified before the PROC CUSUM statement.

**Figure 6.10** One-Sided Cusum Scheme with Decision Interval

The OUTLIMITS= data set is listed in Figure 6.11.

Subgroup Sizes  $n = 1$
Parameters   $\mu_0 = 8.1, \delta = 1, h = 3$
The data set contains one observation with the parameters for process Weight. The variables _TYPE_, _H_, _K_, _MU0_, _DELTA_, and _STDDEV_ save the parameters specified with the options SCHEME=, H=, K=, MU0=, DELTA=, and SIGMA0=, respectively. The variable _MEAN_ saves an estimate of the process mean, and the variable _LIMITN_ saves the nominal sample size. The variables _ARLIN_ and _ARLOUT_ save the average run lengths for $\delta = 0$ and for $\delta = 1$.

For more information, see “OUTLIMITS= Data Set” on page 588.

Reading Cusum Scheme Parameters

NOTE: See One-sided Cusum Chart in the SAS/QC Sample Library.

This example shows how the cusum parameters saved in the previous example can be applied to new measurements saved in a data set named Cans2:

```sas
   data Cans2;
      length pump $ 8;
      label Hour = 'Hour';
      input Hour Weight pump $ 8. ;
   datalines;
16  8.1765 Pump 3
17  8.0949 Pump 3
18  8.1393 Pump 3
19  8.1491 Pump 3
20  8.0473 Pump 1
21  8.1602 Pump 1
22  8.0633 Pump 1
23  8.0921 Pump 1
24  8.1573 Pump 1
25  8.1304 Pump 1
26  8.0979 Pump 1
27  8.2407 Pump 1
28  8.0730 Pump 1
29  8.0986 Pump 2
30  8.0785 Pump 2
31  8.2308 Pump 2
32  8.0986 Pump 2
33  8.0782 Pump 2
34  8.1435 Pump 2
35  8.0666 Pump 2
;```
The following statements create a one-sided cusum chart for the measurements in Cans2 using the parameters in cusparm:

```sas
title "One-Sided Cusum Analysis for New Data";
proc cusum data=Cans2 limits=cusparm;
    xchart Weight*Hour( pump ) / odstitle=title markers;
    label Weight = 'Cusum of Weight';
run;
```

ODS Graphics remains enabled until it is disabled with the ODS GRAPHICS OFF statement, so this cusum chart is also created using ODS Graphics.

The LIMITS= option in the PROC CUSUM statement specifies the data set containing preestablished cusum parameters. The chart, shown in Figure 6.12, indicates that the process is in control. Levels of the variable pump (referred to as a block-variable) do not enter into the analysis but are displayed in a block legend across the top of the chart. See Block Variable Legend Options.

**Figure 6.12** Cusum Chart with Decision Interval for New Data
In general, the parameters for a specified process and subgroup-variable are read from the first observation in the LIMITS= data set for which

- the value of _VAR_ matches the process (in this case, Weight)
- the value of _SUBGRP_ matches the subgroup-variable name (in this case, Hour)

If you are maintaining more than one set of cusum parameters for a particular process, you will find it convenient to include a special identifier variable named _INDEX_ in the LIMITS= data set. This must be a character variable of length 16. Then, if you specify READINDEX='value' in the XCHART statement, the parameters for a specified process and subgroup-variable are read from the first observation in the LIMITS= data set for which

- the value of _VAR_ matches process
- the value of _SUBGRP_ matches the subgroup-variable name
- the value of _INDEX_ matches value

In this example, the LIMITS= data set was created in a previous run of the CUSUM procedure. You can also create a LIMITS= data set with the DATA step. See “LIMITS= Data Set” on page 592 for details concerning the variables that you must provide.

---

**Syntax: XCHART Statement**

The basic syntax for a one-sided (decision interval) scheme using the XCHART statement is as follows:

```
XCHART process * subgroup-variable / SCHEME=ONESIDED MU0=target DELTA=shift H=h < options > ;
```

The general form of this syntax is as follows:

```
XCHART processes * subgroup-variable < (block-variables) >
<=symbol-variable = 'character' > / SCHEME=ONESIDED MU0=target DELTA=shift H=h < options > ;
```

Note that the options SCHEME=ONESIDED, MU0=, DELTA=, and H= are required unless their values are read from a LIMITS= data set.

The basic syntax for a two-sided (V-mask) scheme is as follows:

```
XCHART process * subgroup-variable / MU0=target DELTA=shift ALPHA=alpha H=h < options > ;
```

The general form of this syntax is as follows:

```
XCHART processes * subgroup-variable < (block-variables) >
<=symbol-variable | = 'character' > / MU0=target DELTA=shift ALPHA=alpha H=h < options > ;
```

Note that the options MU0=, DELTA=, and either ALPHA= or H= are required unless their values are read from a LIMITS= data set.

You can use any number of XCHART statements in the CUSUM procedure. The components of the XCHART statement are described as follows.
process

processes identify one or more processes to be analyzed. The specification of process depends on the input data set specified in the PROC CUSUM statement.

- If raw data are read from a DATA= data set, process must be the name of the variable containing the raw measurements. For an example, see “Creating a V-Mask Cusum Chart from Raw Data” on page 547.
- If summary data are read from a HISTORY= data set, process must be the common prefix of the summary variables in the HISTORY= data set. For an example, see “Creating a V-Mask Cusum Chart from Subgroup Summary Data” on page 550.

A process is required. If more than one process is specified, enclose the list in parentheses. The parameters specified in the XCHART statement are applied to all of the processes.2

subgroup-variable is the variable that classifies the data into subgroups. The subgroup-variable is required. In the examples “Creating a V-Mask Cusum Chart from Raw Data” on page 547 and “Creating a V-Mask Cusum Chart from Subgroup Summary Data” on page 550, Hour is the subgroup variable.

block-variables are optionally specified variables that group the data into blocks of consecutive subgroups. The blocks are labeled in a legend, and each block-variable provides one level of labels in the legend. See Figure 6.12 for an example.

symbol-variable is an optionally specified variable whose levels (unique values) determine the plotting character or symbol marker used to plot the cusums.

- If you produce a line printer chart, an ‘A’ marks points corresponding to the first level of the symbol-variable, a ‘B’ marks points corresponding to the second level, and so on.
- If you produce traditional graphics, distinct symbol markers are displayed for points corresponding to the various levels of the symbol-variable. You can specify the symbol markers with SYMBOLn statements.

character specifies a plotting character for line printer charts. See Figure 6.10 for an example.

options specify optional cusum parameters, enhance the appearance of the chart, request additional analyses, save results in data sets, and so on. The section “Summary of Options”, which follows, lists all options by function.

---

2 For this reason, it may be preferable to read distinct cusum parameters for each process from a LIMITS= data set.
Summary of Options

The following tables list the XCHART statement options by function. Options unique to the CUSUM procedure are listed in Table 6.1, and are described in detail in the section “Dictionary of Special Options” on page 570. Options that are common to both the CUSUM and SHEWHART procedures are listed in Table 6.2. They are described in detail in “Dictionary of Options: SHEWHART Procedure” on page 1946.

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Options for Specifying a One-Sided (Decision Interval) Cusum Scheme</strong></td>
<td></td>
</tr>
<tr>
<td>DELTA=</td>
<td>specifies shift to be detected as a multiple of standard error</td>
</tr>
<tr>
<td>H=</td>
<td>specifies decision interval $h$ ($h &gt; 0$) as a multiple of standard error</td>
</tr>
<tr>
<td>HEADSTART=</td>
<td>specifies headstart value $S_0$ as a multiple of standard error</td>
</tr>
<tr>
<td>K=</td>
<td>specifies reference value $k$ ($k &gt; 0$)</td>
</tr>
<tr>
<td>LIMITN=</td>
<td>specifies fixed nominal sample size for cusum scheme</td>
</tr>
<tr>
<td>LIMITN=</td>
<td>specifies that cusums are to be computed for all subgroups regardless of sample size</td>
</tr>
<tr>
<td>MU0=</td>
<td>specifies target $\mu_0$ for mean</td>
</tr>
<tr>
<td>NOREADLIMITS</td>
<td>specifies that cusum parameters are not to be read from LIMITS= data set (SAS 6.10 and later releases)</td>
</tr>
<tr>
<td>READINDEX=</td>
<td>reads cusum scheme parameters from a LIMITS= data set</td>
</tr>
<tr>
<td>READLIMITS</td>
<td>specifies that cusum parameters are to be read from LIMITS= data set (SAS 6.09 and earlier releases)</td>
</tr>
<tr>
<td>SCHEME=ONESIDED</td>
<td>specifies a one-sided scheme</td>
</tr>
<tr>
<td>SHIFT=</td>
<td>specifies shift to be detected in data units</td>
</tr>
<tr>
<td>SIGMA0=</td>
<td>specifies standard (known) value $\sigma_0$ for process standard deviation</td>
</tr>
<tr>
<td><strong>Options for Specifying a Two-Sided (V-Mask) Cusum Scheme</strong></td>
<td></td>
</tr>
<tr>
<td>ALPHA=</td>
<td>specifies probability of Type 1 error</td>
</tr>
<tr>
<td>BETA=</td>
<td>specifies probability of Type 2 error</td>
</tr>
<tr>
<td>H=</td>
<td>specifies vertical distance between V-mask origin and upper (or lower) arm</td>
</tr>
<tr>
<td>K=</td>
<td>specifies slope of lower arm of V-mask</td>
</tr>
<tr>
<td>LIMITN=</td>
<td>specifies fixed nominal sample size for cusum scheme</td>
</tr>
<tr>
<td>LIMITN=</td>
<td>specifies that cusums are to be computed for all subgroups regardless of sample size</td>
</tr>
<tr>
<td>NOREADLIMITS</td>
<td>specifies that cusum parameters are not to be read from LIMITS= data set (SAS 6.10 and later releases)</td>
</tr>
<tr>
<td>READINDEX=</td>
<td>reads cusum scheme parameters from a LIMITS= data set</td>
</tr>
<tr>
<td>READLIMITS</td>
<td>specifies that cusum parameters are to be read from LIMITS= data set (SAS 6.09 and earlier releases)</td>
</tr>
<tr>
<td>READSIGMAS</td>
<td>reads <em>SIGMAS</em> instead of <em>ALPHA</em> from LIMITS= data set when both variables are available</td>
</tr>
<tr>
<td>SIGMAS=</td>
<td>specifies probability of Type 1 error as probability that standard normally distributed variable exceeds value in absolute value</td>
</tr>
</tbody>
</table>
### Table 6.1  
**continued**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Options for Estimating Process Standard Deviation</strong></td>
<td></td>
</tr>
<tr>
<td>SMETHOD=</td>
<td>specifies method for estimating process standard deviation ( \sigma )</td>
</tr>
<tr>
<td>TYPE=</td>
<td>identifies whether _STDDEV_ in OUTLIMITS= data set is an estimate or standard, and specifies value of _TYPE_ in OUTLIMITS= data set</td>
</tr>
<tr>
<td><strong>Options for Displaying Decision Interval or V-Mask</strong></td>
<td></td>
</tr>
<tr>
<td>CINFILL=</td>
<td>specifies color for area under decision interval line or inside V-mask</td>
</tr>
<tr>
<td>CLIMITS=</td>
<td>specifies color of decision interval line</td>
</tr>
<tr>
<td>CMASK=</td>
<td>specifies color of V-mask outline</td>
</tr>
<tr>
<td>LLIMITS=</td>
<td>specifies line type for decision interval</td>
</tr>
<tr>
<td>LMASK=</td>
<td>specifies line type for V-mask arms</td>
</tr>
<tr>
<td>NOMASK=</td>
<td>suppresses display of V-mask</td>
</tr>
<tr>
<td>ORIGIN=</td>
<td>specifies value of subgroup-variable locating origin of V-mask</td>
</tr>
<tr>
<td>WLIMITS=</td>
<td>specifies line width for decision interval</td>
</tr>
<tr>
<td>WMASK=</td>
<td>specifies line width for V-mask</td>
</tr>
<tr>
<td><strong>Tabulation Options</strong></td>
<td></td>
</tr>
<tr>
<td>TABLEALL</td>
<td>specifies the options TABLECHART, TABLECOMP, TABLEID, TABLEOUT, and TABLESUMMARY</td>
</tr>
<tr>
<td>TABLECHART</td>
<td>tabulates the information displayed in the cusum chart</td>
</tr>
<tr>
<td>TABLECOMP</td>
<td>tabulates the computational form of the cusum scheme as described by Lucas (1976) and Lucas and Crosier (1982)</td>
</tr>
<tr>
<td>TABLEID</td>
<td>augments TABLECHART and TABLECOMP tables with columns for ID variables</td>
</tr>
<tr>
<td>TABLEOUT</td>
<td>augments TABLECHART table with a column indicating if the decision interval or V-mask was exceeded</td>
</tr>
<tr>
<td>TABLESUMMARY</td>
<td>tabulates the parameters for the cusum scheme and the average run lengths corresponding to shifts of zero and ( \delta )</td>
</tr>
</tbody>
</table>

### Table 6.2  
**XCHART Statement General Options**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Options for Plotting and Labeling Points</strong></td>
<td></td>
</tr>
<tr>
<td>ALLLABEL=</td>
<td>labels every point on cusum chart</td>
</tr>
<tr>
<td>ALLLABEL2=</td>
<td>labels every point on trend chart</td>
</tr>
<tr>
<td>CLABEL=</td>
<td>specifies color for labels</td>
</tr>
<tr>
<td>CCONNECT=</td>
<td>specifies color for line segments that connect points on chart</td>
</tr>
<tr>
<td>CFRAMELAB=</td>
<td>specifies fill color for frame around labeled points</td>
</tr>
<tr>
<td>COUT=</td>
<td>specifies color for portions of line segments that connect points outside control limits</td>
</tr>
<tr>
<td>COUTFILL=</td>
<td>specifies color for shading areas between the connected points and control limits outside the limits</td>
</tr>
<tr>
<td>LABELANGLE=</td>
<td>specifies angle at which labels are drawn</td>
</tr>
</tbody>
</table>
### Syntax: XCHART Statement

#### Table 6.2 continued

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LABELFONT=</td>
<td>specifies software font for labels</td>
</tr>
<tr>
<td>LABELHEIGHT=</td>
<td>specifies height of labels</td>
</tr>
<tr>
<td>NOCONNECT</td>
<td>suppresses line segments that connect points on chart</td>
</tr>
<tr>
<td>NOTRENDCONNECT</td>
<td>suppresses line segments that connect points on trend chart</td>
</tr>
<tr>
<td>OUTLABEL=</td>
<td>labels points outside control limits</td>
</tr>
<tr>
<td>SYMBOLLEGEND=</td>
<td>specifies LEGEND statement for levels of symbol-variable</td>
</tr>
<tr>
<td>SYMBOLORDER=</td>
<td>specifies order in which symbols are assigned for levels of symbol-variable</td>
</tr>
<tr>
<td>TURNALL</td>
<td>TURNOUT</td>
</tr>
</tbody>
</table>

#### Axis and Axis Label Options

| CAXIS=                  | specifies color for axis lines and tick marks                           |
| CFRAME=                 | specifies fill colors for frame for plot area                            |
| CTEXT=                  | specifies color for tick mark values and axis labels                     |
| DISCRETE                | produces horizontal axis for discrete numeric group values              |
| HAXIS=                  | specifies major tick mark values for horizontal axis                    |
| HEIGHT=                 | specifies height of axis label and axis legend text                      |
| HMINOR=                 | specifies number of minor tick marks between major tick marks on horizontal axis |
| HOFFSET=                | specifies length of offset at both ends of horizontal axis              |
| INTSTART=               | specifies first major tick mark value on horizontal axis when a date, time, or datetime format is associated with numeric subgroup variable |
| NOHLABEL                | suppresses label for horizontal axis                                     |
| NOTICKREP               | specifies that only the first occurrence of repeated, adjacent subgroup values is to be labeled on horizontal axis |
| NOVANGLE                | requests vertical axis labels that are strung out vertically             |
| SKIPHLABELS=            | specifies thinning factor for tick mark labels on horizontal axis        |
| SPLIT=                  | specifies splitting character for axis labels                            |
| TURNHLABELS             | requests horizontal axis labels that are strung out vertically           |
| VAXIS=                  | specifies major tick mark values for vertical axis of cusum chart       |
| VAXIS2=                 | specifies major tick mark values for vertical axis of trend chart        |
| VFORMAT=                | specifies format for primary vertical axis tick mark labels             |
| VFORMAT2=               | specifies format for secondary vertical axis tick mark labels           |
| VMINOR=                 | specifies number of minor tick marks between major tick marks on vertical axis |
| VOFFSET=                | specifies length of offset at both ends of vertical axis                |
Table 6.2  continued

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WAXIS</strong>=</td>
<td>specifies width of axis lines</td>
</tr>
<tr>
<td><strong>Plot Layout Options</strong></td>
<td>plots means for all subgroups</td>
</tr>
<tr>
<td>ALLN</td>
<td>plots means for all subgroups</td>
</tr>
<tr>
<td>BILEVEL</td>
<td>creates control charts using half-screens and half-pages</td>
</tr>
<tr>
<td>EXCHART</td>
<td>creates control charts for a process only when exceptions occur</td>
</tr>
<tr>
<td>INTERVAL=</td>
<td>natural time interval between consecutive subgroup positions when time, date, or datetime format is associated with a numeric subgroup variable</td>
</tr>
<tr>
<td>MAXPANELS=</td>
<td>maximum number of pages or screens for chart</td>
</tr>
<tr>
<td>NMARKERS</td>
<td>requests special markers for points corresponding to sample sizes not equal to nominal sample size for fixed control limits</td>
</tr>
<tr>
<td>NOCHART</td>
<td>suppresses creation of chart</td>
</tr>
<tr>
<td>NOFRAME</td>
<td>suppresses frame for plot area</td>
</tr>
<tr>
<td>NOLEGEND</td>
<td>suppresses legend for subgroup sample sizes</td>
</tr>
<tr>
<td>NPANELPOS=</td>
<td>specifies number of subgroup positions per panel on each chart</td>
</tr>
<tr>
<td>REPEAT</td>
<td>repeats last subgroup position on panel as first subgroup position of next panel</td>
</tr>
<tr>
<td>TOTPANELS=</td>
<td>specifies number of pages or screens to be used to display chart</td>
</tr>
<tr>
<td>TRENDVAR=</td>
<td>specifies list of trend variables</td>
</tr>
<tr>
<td>YPCT1=</td>
<td>specifies length of vertical axis on cusum chart as a percentage of sum of lengths of vertical axes for cusum and trend charts</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Reference Line Options</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CHREF=</td>
<td>specifies color for lines requested by HREF= and HREF2= options</td>
</tr>
<tr>
<td>CVREF=</td>
<td>specifies color for lines requested by VREF= and VREF2= options</td>
</tr>
<tr>
<td>HREF=</td>
<td>specifies position of reference lines perpendicular to horizontal axis on cusum chart</td>
</tr>
<tr>
<td>HREF2=</td>
<td>specifies position of reference lines perpendicular to horizontal axis on trend chart</td>
</tr>
<tr>
<td>HREFDATA=</td>
<td>specifies position of reference lines perpendicular to horizontal axis on cusum chart</td>
</tr>
<tr>
<td>HREF2DATA=</td>
<td>specifies position of reference lines perpendicular to horizontal axis on trend chart</td>
</tr>
<tr>
<td>HREFLABELS=</td>
<td>specifies labels for HREF= lines</td>
</tr>
<tr>
<td>HREF2LABELS=</td>
<td>specifies labels for HREF2= lines</td>
</tr>
<tr>
<td>HREFLABPOS=</td>
<td>specifies position of HREFLABELS= and HREF2LABELS= labels</td>
</tr>
<tr>
<td>LHREF=</td>
<td>specifies line type for HREF= and HREF2= lines</td>
</tr>
</tbody>
</table>
Table 6.2  continued

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LVREF=</td>
<td>specifies line type for VREF= and VREF2= lines</td>
</tr>
<tr>
<td>NOBYREF</td>
<td>specifies that reference line information in a data set applies uniformly to charts created for all BY groups</td>
</tr>
<tr>
<td>VREF=</td>
<td>specifies position of reference lines perpendicular to vertical axis on cusum chart</td>
</tr>
<tr>
<td>VREF2=</td>
<td>specifies position of reference lines perpendicular to vertical axis on trend chart</td>
</tr>
<tr>
<td>VREFLABELS=</td>
<td>specifies labels for VREF= lines</td>
</tr>
<tr>
<td>VREF2LABELS=</td>
<td>specifies labels for VREF2= lines</td>
</tr>
<tr>
<td>VREFLABPOS=</td>
<td>position of VREFLABELS= and VREF2LABELS= labels</td>
</tr>
</tbody>
</table>

**Grid Options**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CGRID=</td>
<td>specifies color for grid requested with GRID or ENDGRID option</td>
</tr>
<tr>
<td>ENDGRID</td>
<td>adds grid after last plotted point</td>
</tr>
<tr>
<td>GRID</td>
<td>adds grid to control chart</td>
</tr>
<tr>
<td>LENDGRID=</td>
<td>specifies line type for grid requested with the ENDGRID option</td>
</tr>
<tr>
<td>LGRID=</td>
<td>specifies line type for grid requested with the GRID option</td>
</tr>
<tr>
<td>WGRID=</td>
<td>specifies width of grid lines</td>
</tr>
</tbody>
</table>

**Graphical Enhancement Options**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANNOTATE=</td>
<td>specifies annotate data set that adds features to cusum chart</td>
</tr>
<tr>
<td>ANNOTATE2=</td>
<td>specifies annotate data set that adds features to trend chart</td>
</tr>
<tr>
<td>DESCRIPTION=</td>
<td>specifies description of cusum chart’s GRSEG catalog entry</td>
</tr>
<tr>
<td>FONT=</td>
<td>specifies software font for labels and legends on charts</td>
</tr>
<tr>
<td>NAME=</td>
<td>specifies name of cusum chart’s GRSEG catalog entry</td>
</tr>
<tr>
<td>PAGENUM=</td>
<td>specifies the form of the label used in pagination</td>
</tr>
<tr>
<td>PAGENUMPOS=</td>
<td>specifies the position of the page number requested with the PAGENUM= option</td>
</tr>
<tr>
<td>WTREND=</td>
<td>specifies width of line segments connecting points on trend chart</td>
</tr>
</tbody>
</table>

**Options for Producing Graphs Using ODS Styles**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLOCKVAR=</td>
<td>specifies one or more variables whose values define colors for filling background of block-variable legend</td>
</tr>
<tr>
<td>CFRAMELAB</td>
<td>draws a frame around labeled points</td>
</tr>
<tr>
<td>COUT</td>
<td>draw portions of line segments that connect points outside control limits in a contrasting color</td>
</tr>
<tr>
<td>CSTAROUT</td>
<td>specifies that portions of stars exceeding inner or outer circles are drawn using a different color</td>
</tr>
</tbody>
</table>
Table 6.2  continued

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUTFILL</td>
<td>shades areas between control limits and connected points lying outside the limits</td>
</tr>
<tr>
<td>STARFILL=</td>
<td>specifies a variable identifying groups of stars filled with different colors</td>
</tr>
<tr>
<td>STARS=</td>
<td>specifies a variable identifying groups of stars whose outlines are drawn with different colors</td>
</tr>
</tbody>
</table>

**Options for ODS Graphics**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLOCKREFTRANSPARENCY=</td>
<td>specifies the wall fill transparency for blocks and phases</td>
</tr>
<tr>
<td>INFILLTRANSPARENCY=</td>
<td>specifies the control limit infill transparency</td>
</tr>
<tr>
<td>MARKERS</td>
<td>plots subgroup points with markers</td>
</tr>
<tr>
<td>NOBLOCKREF</td>
<td>suppresses block and phase reference lines</td>
</tr>
<tr>
<td>NOBLOCKREFFILL</td>
<td>suppresses block and phase wall fills</td>
</tr>
<tr>
<td>NOFILLLEGEND</td>
<td>suppresses legend for levels of a STARFILL= variable</td>
</tr>
<tr>
<td>NOPHASEREF</td>
<td>suppresses block and phase reference lines</td>
</tr>
<tr>
<td>NOPHASEREFFILL</td>
<td>suppresses block and phase wall fills</td>
</tr>
<tr>
<td>NOREF</td>
<td>suppresses block and phase reference lines</td>
</tr>
<tr>
<td>NOREFFILL</td>
<td>suppresses block and phase wall fills</td>
</tr>
<tr>
<td>NOSTARFILLLEGEND</td>
<td>suppresses legend for levels of a STARFILL= variable</td>
</tr>
<tr>
<td>NOTRANSparency</td>
<td>disables transparency in ODS Graphics output</td>
</tr>
<tr>
<td>ODSFOOTNOTE=</td>
<td>specifies a graph footnote</td>
</tr>
<tr>
<td>ODSFOOTNOTE2=</td>
<td>specifies a secondary graph footnote</td>
</tr>
<tr>
<td>ODSLEGENDEXPAND</td>
<td>specifies that legend entries contain all levels observed in the data</td>
</tr>
<tr>
<td>ODSITLE=</td>
<td>specifies a graph title</td>
</tr>
<tr>
<td>ODSITLE2=</td>
<td>specifies a secondary graph title</td>
</tr>
<tr>
<td>OUTFILLTRANSPARENCY=</td>
<td>specifies control limit outfill transparency</td>
</tr>
<tr>
<td>OVERLAYURL=</td>
<td>specifies URLs to associate with overlay points</td>
</tr>
<tr>
<td>OVERLAY2URL=</td>
<td>specifies URLs to associate with overlay points on secondary chart</td>
</tr>
<tr>
<td>PHASEPOS=</td>
<td>specifies vertical position of phase legend</td>
</tr>
<tr>
<td>PHASEREFLEVEL=</td>
<td>associates phase and block reference lines with either innermost or the outermost level</td>
</tr>
<tr>
<td>PHASEREFTRANSPARENCY=</td>
<td>specifies the wall fill transparency for blocks and phases</td>
</tr>
<tr>
<td>REFFILLTRANSPARENCY=</td>
<td>specifies the wall fill transparency for blocks and phases</td>
</tr>
<tr>
<td>SIMULATEQCFONT</td>
<td>draws central line labels using a simulated software font</td>
</tr>
<tr>
<td>STARTRANSPARENCY=</td>
<td>specifies star fill transparency</td>
</tr>
<tr>
<td>URL=</td>
<td>specifies a variable whose values are URLs to be associated with subgroups</td>
</tr>
<tr>
<td>URL2=</td>
<td>specifies a variable whose values are URLs to be associated with subgroups on secondary chart</td>
</tr>
</tbody>
</table>

**Input Data Set Options**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MISSBREAK</td>
<td>specifies that observations with missing values are not to be processed</td>
</tr>
</tbody>
</table>
### Table 6.2 continued

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output Data Set Options</strong></td>
<td></td>
</tr>
<tr>
<td>OUTHISTORY=</td>
<td>creates output data set containing subgroup summary statistics</td>
</tr>
<tr>
<td>OUTINDEX=</td>
<td>specifies value of <em>INDEX</em> in the OUTLIMITS= data set</td>
</tr>
<tr>
<td>OUTLIMITS=</td>
<td>creates output data set containing control limits</td>
</tr>
<tr>
<td>OUTTABLE=</td>
<td>creates output data set containing subgroup summary statistics and control limits</td>
</tr>
<tr>
<td><strong>Specification Limit Options</strong></td>
<td></td>
</tr>
<tr>
<td>CIINDICES</td>
<td>specifies $\alpha$ value and type for computing capability index</td>
</tr>
<tr>
<td>LSL=</td>
<td>specifies list of lower specification limits</td>
</tr>
<tr>
<td>TARGET=</td>
<td>specifies list of target values</td>
</tr>
<tr>
<td>USL=</td>
<td>specifies list of upper specification limits</td>
</tr>
<tr>
<td><strong>Block Variable Legend Options</strong></td>
<td></td>
</tr>
<tr>
<td>BLOCKLABELPOS=</td>
<td>specifies position of label for block-variable legend</td>
</tr>
<tr>
<td>BLOCKLABTYPE=</td>
<td>specifies text size of block-variable legend</td>
</tr>
<tr>
<td>BLOCKPOS=</td>
<td>specifies vertical position of block-variable legend</td>
</tr>
<tr>
<td>BLOCKREP</td>
<td>repeats identical consecutive labels in block-variable legend</td>
</tr>
<tr>
<td>CBLOCKLAB=</td>
<td>specifies fill colors for frames enclosing variable labels</td>
</tr>
<tr>
<td>CBLOCKVAR=</td>
<td>specifies one or more variables whose values are colors for filling background of block-variable legend</td>
</tr>
<tr>
<td><strong>Phase Options</strong></td>
<td></td>
</tr>
<tr>
<td>CPHASELEG=</td>
<td>specifies text color for phase legend</td>
</tr>
<tr>
<td>OUTPHASE=</td>
<td>specifies value of <em>PHASE</em> in the OUTHISTORY= data set</td>
</tr>
<tr>
<td>PHASEBREAK</td>
<td>disconnects last point in a phase from first point in next phase</td>
</tr>
<tr>
<td>PHASELABTYPE=</td>
<td>specifies text size of phase legend</td>
</tr>
<tr>
<td>PHASELEGEND</td>
<td>displays phase labels in a legend across top of chart</td>
</tr>
<tr>
<td>PHASELIMITS</td>
<td>labels control limits for each phase, provided they are constant within that phase</td>
</tr>
<tr>
<td>PHASEREF</td>
<td>delineates phases with vertical reference lines</td>
</tr>
<tr>
<td>READPHASES=</td>
<td>specifies phases to be read from an input data set</td>
</tr>
<tr>
<td><strong>Star Options</strong></td>
<td></td>
</tr>
<tr>
<td>CSTARCIRCLES=</td>
<td>specifies color for STARCIRCLES= circles</td>
</tr>
<tr>
<td>CSTARFILL=</td>
<td>specifies color for filling stars</td>
</tr>
<tr>
<td>CSTAROUT=</td>
<td>specifies outline color for stars exceeding inner or outer circles</td>
</tr>
<tr>
<td>CSTARS=</td>
<td>specifies color for outlines of stars</td>
</tr>
<tr>
<td>LSTARCIRCLES=</td>
<td>specifies line types for STARCIRCLES= circles</td>
</tr>
</tbody>
</table>
Table 6.2  continued

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSTARS=</td>
<td>specifies line types for outlines of STARVERTICES= stars</td>
</tr>
<tr>
<td>STARBDRADIUS=</td>
<td>specifies radius of outer bound circle for vertices of stars</td>
</tr>
<tr>
<td>STARCIRCLES=</td>
<td>specifies reference circles for stars</td>
</tr>
<tr>
<td>STARINRADIUS=</td>
<td>specifies inner radius of stars</td>
</tr>
<tr>
<td>STARLABEL=</td>
<td>specifies vertices to be labeled</td>
</tr>
<tr>
<td>STARLEGEND=</td>
<td>specifies style of legend for star vertices</td>
</tr>
<tr>
<td>STARLEGENDLAB=</td>
<td>specifies label for STARLEGEND= legend</td>
</tr>
<tr>
<td>STAROUTRADIUS=</td>
<td>specifies outer radius of stars</td>
</tr>
<tr>
<td>STARSPECS=</td>
<td>specifies method used to standardize vertex variables</td>
</tr>
<tr>
<td>STARSTART=</td>
<td>specifies angle for first vertex</td>
</tr>
<tr>
<td>STARTYPE=</td>
<td>specifies graphical style of star</td>
</tr>
<tr>
<td>STARVERTICES=</td>
<td>superimposes star at each point on cusum chart</td>
</tr>
<tr>
<td>WSTARCIRCLES=</td>
<td>specifies width of STARCIRCLES= circles</td>
</tr>
<tr>
<td>WSTARS=</td>
<td>specifies width of STARVERTICES= stars</td>
</tr>
</tbody>
</table>

Options for Interactive Control Charts

HTML= specifies a variable whose values create links to be associated with subgroups

HTML2= specifies variable whose values create links to be associated with subgroups on secondary chart

HTML_LEGEND= specifies a variable whose values create links to be associated with symbols in the symbol legend

WEBOUT= creates an OUTTABLE= data set with additional graphics coordinate data

Options for Line Printer Charts

CONNECTCHAR= specifies character used to form line segments that connect points on chart

HREFCHAR= specifies line character for HREF= and HREF2= lines

SYMBOLCHARS= specifies characters indicating symbol-variable

VREFCHAR= specifies line character for VREF= and VREF2= lines

Dictionary of Special Options

**General Options**

You can specify the following options when you use either ODS Graphics or traditional graphics:

**ALPHA=value** specifies the probability \( \alpha \) of incorrectly deciding that a shift has occurred when the process mean is equal to the target mean. This is known as the probability of a Type 1 error. The value must be between zero and one, and it is typically set at 0.05 or 0.10. If you specify the ALPHA= option, the error probability approach is used to determine the V-mask. For details, see “Defining the V-Mask for a Two-Sided Cusum Scheme” on page 580.

The ALPHA= option is applicable only with two-sided cusum schemes. As an alternative to the ALPHA= value, you can specify the percentile \( z_{1-\alpha/2} \) from a standard normal distribution with the
SIGMAS= option. As a second alternative, you can specify the geometric parameter $h$ for the V-mask (in standard error units) with the H= option.

In addition to the ALPHA= option, you can optionally specify the probability of a Type 2 error with the BETA= option.

**BETA=value**

specifies the probability $\beta$ of failing to discover that the specified shift has occurred. This is known as the probability of a Type 2 error. The *value* must be between zero and one. The BETA= option is used in conjunction with either the ALPHA= option or the SIGMAS= option.

The interpretation of $\beta$ is based on the analogy between cusum charts and sequential probability ratio tests, and it is inexact since the cusum chart does not provide an acceptance region. Refer to Johnson (1961) and van Dobben de Bruyn (1968) for further details.

**DATAUNITS**

computes cumulative sums without standardizing the subgroup means or individual measurements. As a result, the vertical axis of the cusum chart is scaled in the same units as the data.

The DATAUNITS option requires constant subgroup sample sizes. If your data do not have constant subgroup sample sizes, you need to specify a constant nominal sample size $n$ for the V-mask or decision interval with the LIMITN= option or with the variable _LIMITN_ in the LIMITS= data set.

**DELTA=value**

specifies the absolute value of the smallest shift to be detected as a multiple $\delta$ of the process standard deviation $\sigma$ or the standard error $\sigma \bar{X}$, depending on whether $\delta$ is viewed as a shift in the population mean or a shift in the sampling distribution of the subgroup mean $\bar{X}$, respectively.

If you specify SCHEME=ONESIDED (see the SCHEME= option later in this list) and the *value* is positive, a shift above the process mean is to be detected, whereas if the *value* is negative, a shift below the process mean is to be detected.

As an alternative to specifying the DELTA= option, you can specify the shift in the same units as the data with the SHIFT= option.

**H=value**

specifies the decision interval $h$ for a one-sided cusum scheme. This type of scheme is completely specified by the parameters $h$ and $k$ (see the K= option later in this list). You can also specify the H= option as an alternative to the ALPHA= or SIGMAS= options for a two-sided cusum scheme with a V-mask. In this case, the H= option specifies the vertical distance $h$ between the origin for the V-mask and the upper or lower arm of the V-mask. In either case, the H=value must be positive and must be expressed as a multiple of standard error.

You can use a table of average run lengths to choose $h$ (this is typically between zero and 10). See Table 6.4 and Table 6.5

**HEADSTART=value**

**HSTART=value**

specifies a headstart value $S_0$ for a one-sided cusum scheme. The value must be expressed as a multiple of standard error. See the section “Headstart Values” on page 578, and refer to Lucas and Crosier (1982), Ryan (1989), and Montgomery (1996).
**K=value**

specifies the reference value $k$ for a one-sided (decision interval) cusum scheme. This type of scheme is completely specified by the parameters $k$ and $h$ (see the H= option earlier in this list). You can also specify the K= and H= options as geometric parameters for a two-sided cusum scheme with a V-mask. In this case, the K= option specifies the slope of the lower arm of the V-mask, and the K= and H= options together are alternatives to the error probability options ALPHA=, SIGMAS=, and BETA=. In either case, the K= value must be positive and must be expressed as a multiple of standard error.

You can use a table of average run lengths to choose $k$ and $h$ ($k$ is typically between zero and two). See Table 6.4 and Table 6.5.

For a one-sided scheme, the default K= value is $\delta/2$, which is referred to as the central reference value. For a two-sided scheme where the V-mask is specified geometrically with the H= option, the default K= value is $\delta/2$. If, however, the V-mask is specified by an error probability with the ALPHA= option, then the K= option should not be specified.

**CAUTION:** The interpretation of the K= value depends on the subgroup-variable and the interval between subgroups that is specified with the INTERVAL= option. For a two-sided scheme, the value is the increase in the lower V-mask arm per unit change on the subgroup axis, so the value depends on how the subgroup-variable is scaled.

- If integer values are assigned to the subgroup-variable, then a unit change is defined as one.
- If the subgroup-variable has character values, then a unit change is defined as the increment between adjacent values of the subgroup-variable.
- If the subgroup-variable is numeric and is formatted with a SAS date or time format, then a unit change is defined as the default value for the INTERVAL= option. For example, if a DATE7. format is associated with the subgroup-variable, then a unit change is defined as one day.

You can use the INTERVAL= option to modify the definition of a unit change. For example, if a DATE7. format is associated with the subgroup-variable but subgroups are collected hourly, then INTERVAL=HOUR defines a unit change as one hour rather than one day.

**LIMITN=n**

**LIMITN=VARYING**

specifies either a fixed or varying nominal sample size for the control limits. If you specify LIMITN=n, cusums are calculated and displayed only for those subgroups with a sample size equal to $n$, although you can specify the ALLN option to force all cusums to be plotted. If you specify LIMITN=VARYING, cusums are calculated and displayed for all subgroups, regardless of sample size.

**MU0=value**

specifies the target mean $\mu_0$ for the process. The target mean must be scaled in the same units as the data.

**NOARL**

suppresses calculation of average run lengths. By default, this calculation is performed if you specify the TABLESUMMARY option or an OUTLIMITS= data set.
NOMASK
suppresses the display of the V-mask on charts for two-sided schemes. This option does not affect computations of cusums or V-mask parameters.

NOREADLIMITS
specifies that the cusum scheme parameters for each process listed in the chart statement are not to be read from the LIMITS= data set specified in the PROC CUSUM statement. The NOREADLIMITS option is available only in SAS 6.10 and later releases. See the READLIMITS option later in this list.

ORIGIN=value
specifies the origin of the V-mask, which is defined as the horizontal coordinate of the right edge of the V-mask. If a date, time, or datetime format is associated with the subgroup-variable, you must specify the value as a date, time, or datetime constant, respectively. If the subgroup variable is character, you must specify the value as a quoted string. The default value is the last (most recent) value of the subgroup-variable.

Note that estimates for the process mean and standard deviation are calculated only from subgroups up to and including the origin subgroup.

READINDEX=’value’
reads cusum scheme parameters from a LIMITS= data set (specified in the PROC CUSUM statement) for each process listed in the chart statement. The ith set of control limits for a particular process is read from the first observation in the LIMITS= data set for which

• the value of _VAR_ matches process
• the value of _SUBGRP_ matches the subgroup-variable
• the value of _INDEX_ matches value

The value can be up to 16 characters and must be enclosed in quotes.

READLIMITS
specifies that cusum scheme parameters are to be read from a LIMITS= data set specified in the PROC CUSUM statement. The parameters for a particular process are read from the first observation in the LIMITS= data set for which

• the value of _VAR_ matches process
• the value of _SUBGRP_ matches the subgroup variable

The use of the READLIMITS option depends on which release of SAS/QC software you are using.

• In SAS 6.10 and later releases, the READLIMITS option is not necessary. To read cusum scheme parameters as described previously, you simply specify a LIMITS= data set. However, even though the READLIMITS option is redundant, it continues to function as in earlier releases.
• In SAS 6.09 and earlier releases, you must specify the READLIMITS option to read cusum scheme parameters as described previously. If you specify a LIMITS= data set without specifying the READLIMITS option (or the READINDEX= option), the cusum scheme parameters are computed from the data.
READSIGMAS
specifies that the variable _SIGMAS_ (instead of _ALPHA_) is to be read from a LIMITS= data set that contains both variables. The variables _SIGMAS_ and _ALPHA_ provide the same parameters as the SIGMAS= and ALPHA= options. By default, _ALPHA_ is read from the LIMITS= data set.

SCHEME=ONESIDED
SCHEME=TWOSIDED
indicates whether the cusum scheme is a one-sided (decision interval) scheme or a two-sided scheme with a V-mask. By default, SCHEME=TWOSIDED.

SHIFT=value
specifies the shift to be detected in the same units as the data. The value is interpreted as the shift in the mean of the sampling distribution of the subgroup mean. The SHIFT= option is an alternative to the DELTA= option. To specify the SHIFT= option, one of the following must be true:

- The subgroup sample sizes are constant.
- A constant nominal sample size $n$ is provided for the cusum scheme with the LIMITN= option or the _LIMITN_ variable in a LIMITS= data set.

The relationship between the SHIFT= value (denoted by $\Delta$) and the DELTA= value (denoted by $\delta$) is $\delta = \Delta / (\sigma / \sqrt{n})$, where $\sigma$ is the process standard deviation.

SIGMA0=value
specifies a known standard deviation $\sigma_0$ for the process standard deviation $\sigma$. The value must be positive. By default, PROC CUSUM estimates $\sigma$ from the data using the formulas given in “Methods for Estimating the Standard Deviation” on page 586. You can use the variable _STDDEV_ in a LIMITS= data set as an alternative to the SIGMA0= option.

SIGMAS=value
specifies the probability $\alpha$ of false detection for a two-sided cusum scheme with a V-mask as the probability that the absolute value of a standard normally distributed variable is greater than the value. For example, SIGMAS=3 corresponds to the probability $\alpha = 0.0027$. The value must be positive. The SIGMAS= option is an alternative to the ALPHA= and H= options, and only one of these three options can be specified.

The SIGMAS= option is useful for defining cusum charts that correspond to Shewhart charts whose control limits are defined with the same value as the multiple of $\sigma$. Refer to Johnson and Leone (1962, 1974).

SMETHOD=NOWEIGHT | MVLUE | RMSDF
specifies a method for estimating the process standard deviation from subgroup observations, $\sigma$, as summarized by the following table.

<table>
<thead>
<tr>
<th>Keyword</th>
<th>Method for Estimating Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOWEIGHT</td>
<td>estimates $\sigma$ as an unweighted average of unbiased subgroup estimates of $\sigma$</td>
</tr>
<tr>
<td>MVLUE</td>
<td>calculates a minimum variance linear unbiased estimate for $\sigma$</td>
</tr>
<tr>
<td>RMSDF</td>
<td>calculates a root-mean square estimate for $\sigma$</td>
</tr>
</tbody>
</table>

For formulas, see “Methods for Estimating the Standard Deviation” on page 586.
Syntax: XCHART Statement

TABLEALL
requests all the tables specified by the options TABLECHART, TABLECOMP, TABLEID, TABLEOUT, and TABLESUMMARY.

TABLECHART < (EXCEPTIONS) > creates a table of the subgroup variable, the subgroup sample sizes, the subgroup means, the cumulative sums, and the decision interval or V-mask limits. A table is produced for each process specified in the XCHART statement. The keyword EXCEPTIONS (enclosed in parentheses) is optional and restricts the tabulation to those subgroups for which the decision interval or V-mask values are exceeded.

TABLECOMP tabulates the computational form of the cusum scheme as described by Lucas (1976) and Lucas and Crosier (1982). Upper or lower cumulative sums (or both) are tabulated for each process given in the XCHART statement. See “Formulas for Cumulative Sums” on page 577 for more information.

TABLEID augments the tables specified by the TABLECHART and TABLECOMP options with a column for each of the ID variables.

TABLEOUT augments the table specified by the TABLECHART option with a column indicating whether the decision interval or V-mask values are exceeded.

TABLESUMMARY produces a table that summarizes the cusum scheme. The table lists the parameters of the scheme and the average run lengths corresponding to shifts of zero and δ. The average run lengths are computed using the method of Goel and Wu (1971). A table is produced for each process. You can save the summary in a data set by specifying the OUTLIMITS= option. See “OUTLIMITS= Data Set” on page 588 for details.

TYPE=ESTIMATE
TYPE=STANDARD specifies the value of _TYPE_ in an OUTLIMITS= data set. The variable _TYPE_ indicates whether the variable _STDDEV_ in the OUTLIMITS= data set represents an estimate or a standard (known) value. The default is ‘STANDARD’ if the SIGMA0= option is specified; otherwise, the default is ‘ESTIMATE’.

Options for Traditional Graphics
You can specify the following options when you produce traditional graphics:

CINFILL=color specifies the color for the area under the decision interval or inside the V-mask arms. See also the COUTFILL= option.

CLIMITS=color specifies the color for the decision interval line.
**CMASK=**<color>
specifies the color for the V-mask arms.

**LLIMITS=**<linetype>
specifies the line type for the decision interval.

**LMASK=**<linetype>
specifies the line type for the V-mask arms.

**WLIMITS=**<linetype>
specifies the width (in pixels) of the decision interval line.

**WMASK=**<linetype>
specifies the width (in pixels) of the V-mask arms.

---

**Details: XCHART Statement**

**Basic Notation for Cusum Charts**

The following notation is used in this chapter:

\[ \mu \] denotes the mean of the population, also referred to as the *process mean* or the *process level*.

\[ \mu_0 \] denotes the target mean (goal) for the population. Goel and Wu (1971) refer to \( \mu_0 \) as the “acceptable quality level” and use the symbol \( \mu_a \) instead. The symbol \( \bar{X}_0 \) is used for \( \mu_0 \) in *Glossary and Tables for Statistical Quality Control*. You can provide \( \mu_0 \) with the MU0= option or with the variable _MU0_ in a LIMITS= data set.

\[ \sigma \] denotes the population standard deviation. You can provide \( \sigma \) with the variable _STDDEV_ in a LIMITS= data set (where _TYPE_=’STANDARD’).

\[ \sigma_0 \] denotes a known standard deviation. You can provide \( \sigma_0 \) with the SIGMA0= option or the variable _STDDEV_ in a LIMITS= data set.

\[ \hat{\sigma} \] denotes an estimate of \( \sigma \). You can provide \( \hat{\sigma} \) with the SIGMA0= option or the variable _STDDEV_ in a LIMITS= data set. To identify this value as an estimate, specify TYPE=ESTIMATE or assign the value ‘ESTIMATE’ to the variable _TYPE_ in a LIMITS= data set.

\[ n \] denotes the nominal sample size for the cusum scheme. You can provide \( n \) with the LIMITN= option or the variable _LIMITN_ in a LIMITS= data set.

\[ \delta \] denotes the shift in \( \mu \) to be detected, expressed as a multiple of the standard deviation. You can provide \( \delta \) with the DELTA= option or the variable _DELTA_ in a LIMITS= data set.
\[ \Delta \] denotes the shift in \( \mu \) to be detected, expressed in data units. If the sample size \( n \) is constant across subgroups, then \( \Delta = \delta \bar{x} = (\delta \sigma)/\sqrt{n} \).

Some authors use the symbol \( D \) instead of \( \Delta \); for example, refer to Johnson and Leone (1962, 1974) and Wadsworth, Stephens, and Godfrey (1986). You can provide \( \Delta \) with the \text{SHIFT=} option. Although it may be more natural to specify the shift in data units, it is preferable to specify the shift as \( \delta \), since this generalizes to data with unequal subgroup sample sizes.

### Formulas for Cumulative Sums

#### One-Sided Cusum Schemes

**Positive Shifts** If the shift \( \delta \) to be detected is positive, the cusum computed for the \( t \)th subgroup is

\[ S_t = \max(0, S_{t-1} + (z_t - k)) \]

for \( t=1, 2, \ldots, n \), where \( S_0=0 \), \( z_t \) is defined as for two-sided schemes, and the parameter \( k \), termed the \textit{reference value}, is positive. The cusum \( S_t \) is referred to as an \textit{upper cumulative sum}. Since \( S_t \) can be written as

\[ \max \left( 0, S_{t-1} + \frac{\bar{X}_t - (\mu_0 + k \sigma \bar{X}_t)}{\sigma \bar{X}_t} \right) \]

the sequence \( S_t \) cumulates deviations in the subgroup means greater than \( k \) standard errors from \( \mu_0 \). If \( S_t \) exceeds a positive value \( h \) (referred to as the \textit{decision interval}), a shift or out-of-control condition is signaled. This formulation follows that of Lucas (1976), Lucas and Crosier (1982), and Montgomery (1996).

**Negative Shifts** If the shift \( \delta \) to be detected is negative, the cusum computed for the \( r \)th subgroup is

\[ S_r = \max(0, S_{r-1} - (z_r + k)) \]

for \( r=1, 2, \ldots, n \), where \( S_0=0 \), \( z_r \) is defined as for two-sided cusum schemes, and the parameter \( k \), termed the \textit{reference value}, is positive. The cusum \( S_r \) is referred to as a \textit{lower cumulative sum}. Since \( S_r \) can be written as

\[ \max \left( 0, S_{r-1} - \frac{\bar{X}_r - (\mu_0 - k \sigma \bar{X}_r)}{\sigma \bar{X}_r} \right) \]

the sequence \( S_r \) cumulates the absolute value of deviations in the subgroup means less than \( k \) standard errors from \( \mu_0 \). If \( S_r \) exceeds a positive value \( h \) (referred to as the \textit{decision interval}), a shift or out-of-control condition is signaled.

This formulation follows that of Lucas (1976), Lucas and Crosier (1982), and Montgomery (1996). Note that \( S_t \) is always positive and \( h \) is always positive, regardless of whether \( \delta \) is positive or negative. For schemes designed to detect a negative shift, some authors, including van Dobben de Bruyn (1968) and Wadsworth, Stephens, and Godfrey (1986), define a reflected version of \( S_t \) for which a shift is signaled when \( S_t \) is less than a negative limit.
**Headstart Values** Lucas and Crosier (1982) describe the properties of a fast initial response (FIR) feature for cusum schemes in which the initial cusum $S_0$ is set to a “headstart” value. Average run length calculations given by Lucas and Crosier (1982) show that the FIR feature has little effect when the process is in control and that it leads to a faster response to an initial out-of-control condition than a standard cusum scheme. You can provide headstart value $S_0$ with the HEADSTART= option or the variable _HSTART_ in a LIMITS= data set.

**Constant Sample Sizes** When the subgroup sample sizes are constant ($= n$), it may be preferable to compute cusums that are scaled in the same units as the data. Refer to Montgomery (1996) and Wadsworth, Stephens, and Godfrey (1986). To request this, specify the DATAUNITS option. Cusums are then computed as

$$S_t = \max(0, S_{t-1} + (\bar{X}_i - (\mu_0 + k\sigma/\sqrt{n})))$$

for $\delta > 0$ and the equation

$$S_t = \max(0, S_{t-1} - (\bar{X}_i - (\mu_0 - k\sigma/\sqrt{n})))$$

for $\delta < 0$. In either case, a shift is signaled if $S_t$ exceeds $h = h'\sigma/\sqrt{n}$. Wadsworth, Stephens, and Godfrey (1986) use the symbol $H$ for $h'$.

If the subgroup sample sizes are not constant, you can specify a constant nominal sample size $n$ with the LIMITN= option or the variable _LIMITN_ in a LIMITS= data set. In this case, only those subgroups with sample size $n$ are analyzed unless you also specify the option ALLN. You can further specify the option NMARKERS to request special symbol markers for points corresponding to sample sizes not equal to $n$.

**Two-Sided Cusum Schemes**

If the cusum scheme is two-sided, the cumulative sum $S_t$ plotted for the $t$th subgroup is

$$S_t = S_{t-1} + z_t$$

for $t=1, 2, \ldots, n$. Here $S_0=0$, and the term $z_t$ is calculated as

$$z_t = (\bar{X}_t - \mu_0)/(\sigma/\sqrt{n_t})$$

where $\bar{X}_t$ is the $t$th subgroup average, and $n_t$ is the $t$th subgroup sample size. If the subgroup samples consist of individual measurements $x_t$, the term $z_t$ simplifies to

$$z_t = (x_t - \mu_0)/\sigma$$

Since the first equation can be rewritten as

$$S_t = \sum_{i=1}^{t} z_i = \sum_{i=1}^{t} (\bar{X}_i - \mu_0)/\sigma \bar{X}_i$$

the sequence $S_t$ cumulates standardized deviations of the subgroup averages from the target mean $\mu_0$. 
In many applications, the subgroup sample sizes $n_i$ are constant ($n_i = n$), and the equation for $S_t$ can be simplified.

$$S_t = \frac{1}{\sigma \bar{X}} \sum_{i=1}^{t} (\bar{X}_i - \mu_0) = \left(\frac{\sqrt{n}}{\sigma}\right) \sum_{i=1}^{t} (\bar{X}_i - \mu_0)$$

In some applications, it may be preferable to compute $S_t$ as

$$S_t = \sum_{i=1}^{t} (\bar{X}_i - \mu_0)$$

which is scaled in the same units as the data. Refer to Montgomery (1996), Wadsworth, Stephens, and Godfrey (1986), and American Society for Quality Control (1983). If the subgroup sample sizes are constant ($= n$) and if you specify the DATAUNITS option in the XCHART statement, the CUSUM procedure computes cusums using the final equation above. In this case, the procedure rescales the V-mask parameters $h$ and $k$ to $h' = h \sigma / \sqrt{n}$ and $k' = k \sigma / \sqrt{n}$, respectively. Wadsworth, Stephens, and Godfrey (1986) use the symbols $F$ for $k'$ and $H$ for $h'$.

If the subgroup sample sizes are not constant, you can specify a constant nominal sample size $n$ with the LIMITN= option or with the variable _LIMITN_ in a LIMITS= data set. In this case, only those subgroups with sample size $n$ are analyzed unless you also specify the option ALLN. You can further specify the option NMARKERS to request special symbol markers for points corresponding to sample sizes not equal to $n$.

If the process is in control and the mean $\mu$ is at or near the target $\mu_0$, the points will not exhibit a trend since positive and negative displacements from $\mu_0$ tend to cancel each other. If $\mu$ shifts in the positive direction, the points exhibit an upward trend, and if $\mu$ shifts in the negative direction, the points exhibit a downward trend.

### Defining the Decision Interval for a One-Sided Cusum Scheme

The height of the decision interval is $h$, expressed as a multiple of the standard error of the subgroup mean. You can specify $h$ with the H= option in the XCHART statement or with the variable _H_ in a LIMITS= data set. The decision interval is displayed as a horizontal line on the cusum chart, as illustrated in Figure 6.13.
Interpreting One-Sided Cusum Charts

A shift or out-of-control condition is signaled at time $t$ if the cusum $S_t$ plotted at time $t$ exceeds the decision interval line.

Defining the V-Mask for a Two-Sided Cusum Scheme

The dimensions of the V-mask can be specified using two distinct sets of two parameters.

- $\theta$, defined as half of the angle formed by the V-mask arms, and $d$, the distance between the origin and the vertex, as shown in Figure 6.14. This parameterization is used by many authors, including Johnson and Leone (1962, 1974) and Montgomery (1996).

- $h$, the vertical distance between the origin and the upper (or lower) V-mask arm, and $k$, the rise (drop) in the lower (upper) arm corresponding to an interval of one subgroup unit on the horizontal axis. You can specify the definition of an interval with the INTERVAL= option. This parameterization is used by Lucas (1976) and Wadsworth, Stephens, and Godfrey (1986). Lucas (1976) uses the symbols $h^*$ for $h$ and $k^*$ for $k$, and Wadsworth, Stephens, and Godfrey (1986) use the symbol $f$ in place of $k$.

The two parameterizations are related by the equations

\[
\theta = \arctan(k/a)
\]
\[
d = h/k
\]
where the aspect ratio $a$ is the number of units on the vertical axis corresponding to one unit on the horizontal axis. The CUSUM procedure uses the $h$ and $k$ parameterization because it eliminates the need for working with aspect ratios. Furthermore, $h$ and $k$ are also useful for average run length computations and for parameterizing one-sided cusum schemes.

**Figure 6.14** V-Mask Parameters

You can specify the V-mask in two ways:

- geometrically, by providing $h$ and $k$ (or simply $h$) with the H= and K= options or with the variables _H_ and _K_ in a LIMITS= data set
- in terms of error probabilities, by providing $\alpha$ and $\beta$ (or simply $\alpha$) with the ALPHA= and BETA= options or with the variables _ALPHA_ and _BETA_ in a LIMITS= data set. The SIGMAS= option is an alternative to the ALPHA= option, and the variable _SIGMAS_ is an alternative to the variable _ALPHA_ (if the READSIGMAS option is specified).

If you provide $\alpha$ and $\beta$, $h$ and $k$ are computed using the formulas

$$h = |\delta|^{-1} \log((1 - \beta)/(\alpha/2))$$
\[ k = |\delta|/2 \]

If you provide \( \alpha \) but not \( \beta \), \( h \) and \( k \) are computed using the formulas

\[ h = -|\delta|^{-1} \log(\alpha/2) \]

\[ k = |\delta|/2 \]

In the preceding equations, the error probability \( \alpha \) is divided by two because two-sided deviations from the target mean are detected. Refer to Johnson and Leone (1962, 1974).

**Interpreting Two-Sided Cusum Charts**

The origin of the V-mask is located at the most recently plotted point, as illustrated in Figure 6.14. As additional data are collected and the cumulative sum sequence is updated, the origin is relocated at the newest point. A shift or out-of-control condition is signaled at time \( t \) if one or more of the points plotted up to time \( t \) cross an arm of the V-mask. An upward shift is signaled by points crossing the lower arm, and a downward shift is signaled by points crossing the upper arm. The time at which the shift occurred corresponds to the time at which a distinct change is observed in the slope of the plotted points.

**Designing a Cusum Scheme**

There are three main methods for designing a cusum scheme: the average run length (ARL) approach, the error probability approach, and the economic design approach.

**Average Run Length (ARL) Approach**

With the ARL approach, the parameters \( h \) and \( k \) are chosen to yield desired average run lengths when the process is operating at the target mean and when a shift of magnitude \( \delta \) has occurred. The average run length is the expected number of samples taken before an out-of-control condition is signaled. Ideally, the ARL should be long when \( \mu = \mu_0 \) and short when \( \mu \) shifts away from \( \mu_0 \).


For one-sided charts, average run lengths are tabulated as a function of \( h \), \( k \), and \( \delta \) in Table 6.4. No headstart is assumed in this table. For two-sided charts, average run lengths are tabulated as a function of \( h \), \( k \), and \( \delta \) in Table 6.5, which is formatted similarly to Table 2 given by Lucas (1976).

The ARLs in Table 6.4 and Table 6.5 were calculated with the DATA step function CUSUMARL (see the section “CUSUMARL Function” on page 2176). This function uses the method of Goel and Wu (1971). You can use this function to generate more detailed, interpolated versions of the tables or to compute ARLs with headstart values.
It can be shown that the two-sided (V-mask) cusum scheme parameterized by \( h \) and \( k \) is equivalent to two simultaneously operating one-sided cusum schemes, one that computes an upper cusum and one that computes a lower cusum. Both one-sided schemes use the same parameters \( h \) and \( k \).

You can specify \( h, k, \) and \( \delta \) with the options H=, K=, and DELTA= or with the variables \_H_, \_K_, and \_DELTA_ in a LIMITS= data set. The reference value \( k \) is optional, and its default value is \( k = |\delta|/2 \), referred to as the central reference value.

**Error Probability Approach**

This approach is available only for two-sided cusum schemes. Values of \( \alpha \) (the probability of incorrectly signaling the occurrence of a shift) and \( \beta \) (the probability of failing to detect a shift) are specified, and \( h \) and \( k \) are computed from \( \alpha \) and \( \beta \) as described in “Defining the V-Mask for a Two-Sided Cusum Scheme” on page 580. The error probability approach interprets the cusum as a sequence of reversed sequential probability ratio tests. Refer to Johnson (1961), Johnson and Leone (1962, 1974), van Dobben de Bruyn (1968), Montgomery (1996), and Wadsworth, Stephens, and Godfrey (1986).

Although the error probability method is intuitively appealing, the actual error probabilities achieved may not be close to those specified since the V-mask does not provide for an acceptance region. This has been pointed out by various authors, including Johnson (1961) and van Dobben de Bruyn (1968). If you follow this approach, it is recommended that you examine the average run lengths for the cusum scheme (these are tabulated by the TABLESUMMARY option and are saved in OUTLIMITS= data sets).

You can specify \( \alpha \) and \( \beta \) with the ALPHA= and BETA= options or with the variables \_ALPHA_ and \_BETA_ in a LIMITS= data set. It is not necessary to specify \( \beta \), and the interpretation of \( \beta \) is somewhat questionable. The SIGMAS= option is an alternative to the ALPHA= option, and the variable \_SIGMAS_ is an alternative to the variable \_ALPHA_ (if you specify the READSIGMAS option).

**Economic Design**

The parameters \( n, h, \) and \( k \) are chosen so that the long-run average cost of the cusum scheme is minimized. Refer to Chiu (1974), Montgomery (1980), Svoboda (1991), and Ho and Case (1994) for reviews of the literature on economic design. This approach typically requires numerical optimization techniques, which are available in SAS/IML software and in the NLP procedure in SAS/OR software.

You can pass the optimal parameters to the CUSUM procedure as values of the variables \_LIMITN_, \_H_, and \_K_ in a LIMITS= data set.
Table 6.4  Average Run Lengths for One-Sided V-Mask Cusum Charts as a Function of $h$, $k$, and $\delta$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\delta$ (shift in mean)</th>
<th>$h$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.00</td>
<td>0.25</td>
</tr>
<tr>
<td>2.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.00</td>
<td></td>
<td></td>
<td></td>
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Table 6.5  Average Run Lengths for Two-Sided V-Mask Cusum Charts as a Function of \( h \), \( k \), and \( \delta \).

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Chapter 6: The CUSUM Procedure

Cusum Charts Compared with Shewhart Charts

Although cusum charts and Shewhart charts are both used to detect shifts in the process mean, there are important differences in the two methods.

- Each point on a Shewhart chart is based on information for a single subgroup sample or measurement. Each point on a cusum chart is based on information from all samples (measurements) up to and including the current sample (measurement).
- On a Shewhart chart, upper and lower control limits are used to decide whether a point signals an out-of-control condition. On a cusum chart, the limits take the form of a decision interval or a V-mask.
- On a Shewhart chart, the control limits are commonly computed as $3\sigma$ limits. On a cusum chart, the limits are determined from average run length specifications, specified error probabilities, or an economic design.

A cusum chart offers several advantages over a Shewhart chart.

- A cusum chart is more efficient for detecting small shifts in the process mean, in particular, shifts of 0.5 to 2 standard deviations from the target mean (refer to Montgomery 1996). Lucas (1976) noted that “a V-mask designed to detect a $1\sigma$ shift will detect it about four times as fast as a competing Shewhart chart.”
- Shifts in the process mean are visually easy to detect on a cusum chart since they produce a change in the slope of the plotted points. The point at which the slope changes is the point at which the shift has occurred.

These advantages are not as pronounced if the Shewhart chart is augmented by the tests for special causes described by Nelson (1984, 1985). Also see “Tests for Special Causes: SHEWHART Procedure” on page 2073. Moreover,

- cusum schemes are more complicated to design.
- a cusum chart can be slower to detect large shifts in the process mean.
- it can be difficult to interpret point patterns on a cusum chart since the cusums are correlated.

Methods for Estimating the Standard Deviation

It is recommended practice to provide a stable estimate or standard value for $\sigma$ with either the SIGMA0= option or the variable _STDDEV_ in a LIMITS= data set. However, if such a value is not available, you can compute an estimate $\hat{\sigma}$ from the data, as described in this section.

This section provides formulas for various methods used to estimate the standard deviation $\sigma$. One method is applicable with individual measurements, and three are applicable with subgrouped data. The methods can be requested with the SMETHOD= option.
Method for Individual Measurements

When the cumulative sums are calculated from individual observations

\[ x_1, x_2, \ldots, x_N \]

rather than subgroup samples of two or more observations, the CUSUM procedure estimates \( \sigma \) as \( \sqrt{\hat{\sigma}^2} \), where

\[ \hat{\sigma}^2 = \frac{1}{2(N - 1)} \sum_{i=1}^{N-1} (x_{i+1} - x_i)^2 \]

where \( N \) is the number of observations. Wetherill (1977) states that the estimate of the variance is biased if the measurements are autocorrelated.

Note that you can compute alternative estimates (for instance, robust estimates or estimates based on variance components models) by analyzing the data with SAS modeling procedures or your own DATA step program. Such estimates can be passed to the CUSUM procedure as values of the variable _STDDEV_ in a LIMITS= data set.

NOWEIGHT Method for Subgroup Samples

This method is the default for cusum charts for subgrouped data. The estimate is

\[ \hat{\sigma} = \frac{(s_1/c_4(n_1)) + \cdots + (s_N/c_4(n_N))}{N} \]

where \( n_i \) is the sample size of the \( i \)th subgroup, \( N \) is the number of subgroups for which \( n_i \geq 2 \), \( s_i \) is the sample standard deviation of the observations \( x_{i1}, \ldots, x_{ni} \) in the \( i \)th subgroup.

\[ s_i = \sqrt{(1/(n_i - 1)) \sum_{j=1}^{n_i} (x_{ij} - \bar{X}_i)^2} \]

and

\[ c_4(n_i) = \frac{\Gamma((n_i)/2)\sqrt{2/(n_i - 1)}}{\Gamma((n_i - 1)/2)} \]

where \( \Gamma(\cdot) \) denotes the gamma function, and \( \bar{X}_i \) denotes the \( i \)th subgroup mean. A subgroup standard deviation \( s_i \) is included in the calculation only if \( n_i \geq 2 \). If the observations are normally distributed, then the expected value of \( s_i \) is

\[ E(s_i) = c_4(n_i)\sigma \]

Thus, \( \hat{\sigma} \) is the unweighted average of \( N \) unbiased estimates of \( \sigma \). This method is described in the ASTM Manual on Presentation of Data and Control Chart Analysis.

MVLUE Method for Subgroup Samples

If you specify SMETHOD=MVLUE, a minimum variance linear unbiased estimate (MVLUE) is computed, as introduced by Burr (1969, 1976). This estimate is a weighted average of unbiased estimates of \( \sigma \) of the form

\[ s_i/c_4(n_i) \]
where

\[ s_i \] is the standard deviation of the \( i \)th subgroup.

\[ c_4(n_i) \] is the unbiasing factor defined previously.

\[ n_i \] is the \( i \)th subgroup sample size, \( i = 1, 2, \ldots, N \).

\( N \) is the number of subgroups for which \( n_i \geq 2 \).

The estimate is

\[ \hat{\sigma} = \frac{h_1 s_1 / c_4(n_1) + \cdots + h_N s_N / c_4(n_N)}{h_1 + \cdots + h_N} \]

where \( h_i = c_4^2(n_i) / (1 - c_4^2(n_i)) \). A subgroup standard deviation \( s_i \) is included in the calculation only if \( n_i \geq 2 \).

The MVLUE assigns greater weight to estimates of \( \sigma \) from subgroups with larger sample sizes and is intended for situations where the subgroup sample sizes vary. If the subgroup sample sizes are constant, the MVLUE reduces to the default estimate (NOWEIGHT).

**RMSDF Method for Subgroup Samples**

If you specify_SMETHOD=RMSDF, a weighted root-mean-square estimate is computed:

\[ \hat{\sigma} = \sqrt{\frac{(n_1 - 1)s_1^2 + \cdots + (n_N - 1)s_N^2}{c_4(n) \sqrt{n_1 + \cdots + n_N - N}}} \]

where

\( n_i \) is the sample size of the \( i \)th subgroup.

\( N \) is the number of subgroups for which \( n_i \geq 2 \).

\( s_i \) is the sample standard deviation of the \( i \)th subgroup.

\[ c_4(n_i) \] is the unbiasing factor defined previously.

\( n \) is equal to \( (n_1 + \cdots + n_N) - (N - 1) \).

The weights in the root-mean-square expression are the degrees of freedom \( n_i - 1 \). A subgroup standard deviation \( s_i \) is included in the calculation only if \( n_i \geq 2 \).

If the unknown standard deviation \( \sigma \) is constant across subgroups, the root-mean-square estimate is more efficient than the minimum variance linear unbiased estimate. However, as noted by Burr (1969), “the constancy of \( \sigma \) is the very thing under test,” and if \( \sigma \) varies across subgroups, the root-mean-square estimate tends to be more inflated than the MVLUE.

**Output Data Sets**

**OUTLIMITS= Data Set**

When you save the parameters for the cusum scheme in an OUTLIMITS= data set, the following variables are included:
### Variable Description

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>ALPHA</em></td>
<td>probability ($\alpha$) of Type 1 error</td>
</tr>
<tr>
<td><em>ARLIN</em></td>
<td>average run length for zero shift</td>
</tr>
<tr>
<td><em>ARLOUT</em></td>
<td>average run length for shift of $\delta$</td>
</tr>
<tr>
<td><em>BETA</em></td>
<td>probability ($\beta$) of Type 2 error</td>
</tr>
<tr>
<td><em>DELTA</em></td>
<td>shift ($\delta$) to be detected</td>
</tr>
<tr>
<td><em>H</em></td>
<td>decision interval $h$ for one-sided scheme; distance $h$ between origin and upper arm V-mask for two-sided scheme</td>
</tr>
<tr>
<td><em>HSTART</em></td>
<td>headstart value</td>
</tr>
<tr>
<td><em>INDEX</em></td>
<td>optional identifier for cusum parameters (if the OUTINDEX= option is specified)</td>
</tr>
<tr>
<td><em>K</em></td>
<td>reference value $k$ for one-sided scheme; slope of lower V-mask arm for two-sided scheme</td>
</tr>
<tr>
<td><em>LIMITN</em></td>
<td>nominal sample size for cusum scheme</td>
</tr>
<tr>
<td><em>MEAN</em></td>
<td>estimated process mean ($\bar{X}$)</td>
</tr>
<tr>
<td><em>MU0</em></td>
<td>target mean $\mu_0$</td>
</tr>
<tr>
<td><em>ORIGIN</em></td>
<td>origin of V-mask</td>
</tr>
<tr>
<td><em>SCHEME</em></td>
<td>type of scheme (‘ONESIDED’ or ‘TWOSIDED’)</td>
</tr>
<tr>
<td><em>SIGMAS</em></td>
<td>$\frac{z_1-\alpha/2}{\hat{\sigma}}$</td>
</tr>
<tr>
<td><em>STDDEV</em></td>
<td>estimated or known standard deviation ($\hat{\sigma}$ or $\sigma_0$)</td>
</tr>
<tr>
<td><em>SUBGRP</em></td>
<td>subgroup-variable specified in XCHART statement</td>
</tr>
<tr>
<td><em>TYPE</em></td>
<td>type (‘ESTIMATE’ or ‘STANDARD’) of <em>STDDEV</em></td>
</tr>
<tr>
<td><em>VAR</em></td>
<td>process specified in XCHART statement</td>
</tr>
</tbody>
</table>

### Notes:

1. If the subgroup sample sizes vary, the special missing value $V$ is assigned to the variable _LIMITN_.

2. If a V-mask is specified with SIGMAS=$k$, _ALPHA_ is computed as $\alpha = 2(1 - \Phi(k))$, where $\Phi(\cdot)$ is the standard normal distribution function.

3. If a V-mask is specified with ALPHA=$\alpha$, _SIGMAS_ is computed as $k = \Phi^{-1}(1 - \alpha/2)$, where $\Phi^{-1}$ is the inverse standard normal distribution function.

4. BY variables are saved in the OUTLIMITS= data set.

The OUTLIMITS= data set contains one observation for each process specified in the XCHART statement. For an example, see “Saving Cusum Scheme Parameters” on page 557.

### OUT HISTORY= Data Set

When you save subgroup summary statistics in an OUT HISTORY= data set, the following variables are included:

- the subgroup-variable
- a subgroup mean variable named by process suffixed with $X$
• a subgroup sample size variable named by `process` suffixed with `N`
• a subgroup standard deviation variable named by `process` suffixed with `S`
• a cusum variable named by `process` suffixed with `C`

Given a `process` name that contains 32 characters, the procedure first shortens the name to its first 16 characters and its last 15 characters, and then it adds the suffix.

Variables containing subgroup summary statistics are created for each `process` specified in the XCHART statement. For example, consider the following statements:

```
proc cusum data=Steel limits=Stparm;
  xchart (Width Diameter)*Lot / outhistory=Summary;
run;
```


Additionally, if specified, the following variables are included:

• BY variables
• `block-variables`
• `symbol-variable`
• ID variables
• `_PHASE_` (if the OUTPHASE= option is specified)

For an example creating an OUTHISTORY= data set, see “Saving Summary Statistics” on page 552.

**OUTTABLE= Data Set**

The OUTTABLE= data set saves subgroup means, subgroup sample sizes, cusums, and cusum limits. The following variables are included:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>CUSUM</em></td>
<td>cumulative sum</td>
</tr>
<tr>
<td><em>EXLIM</em></td>
<td>decision interval or V-mask arm exceeded</td>
</tr>
<tr>
<td><em>H</em></td>
<td>decision interval</td>
</tr>
<tr>
<td><em>MASKL</em></td>
<td>lower arm of V-mask</td>
</tr>
<tr>
<td><em>MASKU</em></td>
<td>upper arm of V-mask</td>
</tr>
<tr>
<td><code>subgroup</code></td>
<td>values of the subgroup variable</td>
</tr>
<tr>
<td><em>SUBN</em></td>
<td>subgroup sample size</td>
</tr>
<tr>
<td><em>SUBX</em></td>
<td>subgroup mean</td>
</tr>
<tr>
<td><em>SUBS</em></td>
<td>subgroup standard deviation</td>
</tr>
<tr>
<td><em>VAR</em></td>
<td><code>process</code> specified in XCHART statement</td>
</tr>
</tbody>
</table>
In addition, the following variables are saved if specified:

- BY variables
- block-variables
- ID variables
- _PHASE_ (if the READPHASES= option is specified)
- _TREND_ (if the TRENDVAR= option is specified)
- symbol-variable

Note that the variables _VAR_ and _EXLIM_ are character variables of length eight. The variable _PHASE_ is a character variable of length 16.

**ODS Tables**

The following table summarizes the ODS tables that you can request with the XCHART statement.

<table>
<thead>
<tr>
<th>Table Name</th>
<th>Description</th>
<th>Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>CompCusum</td>
<td>computational form of the cusum scheme</td>
<td>TABLEALL, TABLECOMP</td>
</tr>
<tr>
<td>Parameters</td>
<td>cusum parameters and computed average run lengths</td>
<td>TABLEALL, TABLESUMMARY</td>
</tr>
<tr>
<td>XCHART</td>
<td>cusum chart summary statistics</td>
<td>TABLEALL, TABLECHART, TABLEOUT</td>
</tr>
</tbody>
</table>

**ODS Graphics**

Before you create ODS Graphics output, ODS Graphics must be enabled (for example, by using the ODS GRAPHICS ON statement). For more information about enabling and disabling ODS Graphics, see the section “Enabling and Disabling ODS Graphics” (Chapter 21, *SAS/STAT User’s Guide*). **Note:** In SAS/QC 13.1 the CUSUM procedure does not support the creation of graphs that are editable with the ODS Graphics Editor.

The appearance of a graph produced with ODS Graphics is determined by the style associated with the ODS destination where the graph is produced. XCHART options used to control the appearance of traditional graphics are ignored for ODS Graphics output. **Options for Producing Graphs Using ODS Styles** lists options that can be used to control the appearance of graphs produced with ODS Graphics or with traditional graphics using ODS styles. **Options for ODS Graphics** lists options to be used exclusively with ODS Graphics. Detailed descriptions of these options are provided in “Dictionary of Options: SHEWHART Procedure” on page 1946.

When ODS Graphics is in effect, the XCHART statement assigns a name to the graph it creates. You can use this name to reference the graph when using ODS. The name is listed in Table 6.8.
Table 6.8 ODS Graphics Produced by the XCHART Statement

<table>
<thead>
<tr>
<th>ODS Graph Name</th>
<th>Plot Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>XChart</td>
<td>cusum chart</td>
</tr>
</tbody>
</table>


**Input Data Sets**

**DATA= Data Set**

You can read raw data (measurements) from a DATA= data set specified in the PROC CUSUM statement. Each process specified in the XCHART statement must be a SAS variable in the DATA= data set. The values of this variable are typically measurements of a quality characteristic taken on items in subgroup samples indexed by the values of the subgroup variable. The subgroup-variable specified in the XCHART statement must also be a SAS variable in the DATA= data set. Other variables that can be read from a DATA= data set include

- \_PHASE\_ (if the READPHASES= option is specified)
- block-variables
- symbol-variable
- BY variables
- ID variables

Each observation in a DATA= data set should contain a raw measurement for each process and a value for the subgroup variable. If the ith subgroup contains \( n_i \) items, there should be \( n_i \) consecutive observations for which the value of the subgroup variable is the index of the ith subgroup. For example, if each of 30 subgroup samples contains five items, the DATA= data set should contain 150 observations.

By default, the CUSUM procedure reads all of the observations in a DATA= data set. However, if the DATA= data set includes the variable \_PHASE\_, you can read selected groups of observations (referred to as phases) by specifying the READPHASES= option in the XCHART statement.

For an example of a DATA= data set, see “Creating a V-Mask Cusum Chart from Raw Data” on page 547.

**LIMITS= Data Set**

You can read cusum scheme parameters from a LIMITS= data set specified in the PROC CUSUM statement. As an alternative to specifying the parameters with options, a LIMITS= data set provides the following advantages: it facilitates reusing a permanently saved set of parameters, reading a distinct set of parameters for each process specified in the XCHART statement, and keeping track of multiple sets of parameters for the same process over time.

The LIMITS= data set can be an OUTLIMITS= data set that was created in a previous run of the CUSUM procedure. Such data sets always contain the variables required for a LIMITS= data set; consequently, this is the easiest way to construct a LIMITS= data set.
A LIMITS= data set can also be created directly using a DATA step. The variables required for the data set depend on the type of cusum scheme and how the scheme is specified. The following restrictions apply:

- The variables _VAR_, _SUBGRP_, _DELTA_, and _MU0_ are required.
- For a one-sided cusum scheme, _H_ is required.
- For a two-sided cusum scheme, one of the following three variables is required: _ALPHA_, _H_, or _SIGMAS_.
- If you plan to use the READINDEX= option, the variable _INDEX_ is required; otherwise, it is optional.
- For a one-sided scheme, the variable _SCHEME_ is required; otherwise, it is optional.
- If you want to provide a value for the process standard deviation $\sigma$, the variable _STDDEV_ is required; otherwise, it is optional.

Variable names in a LIMITS= data set are predefined; the procedure reads only variables with these predefined names. With the exception of BY variables, all names start and end with an underscore. In addition, note the following:

- The variables _VAR_, _SUBGRP_, _TYPE_, and _SCHEME_ must be character variables of length eight. The variable _INDEX_ must be a character variable of length 16.
- The variable _TYPE_ is a bookkeeping variable that uses the values ‘ESTIMATE’ and ‘STANDARD’ to record whether the value of _STDDEV_ represents an estimate or standard (known) value.
- BY variables are required if specified with a BY statement.

For an example of reading control limit information from a LIMITS= data set, see “Reading Cusum Scheme Parameters” on page 559.

**HISTORY= Data Set**

Instead of reading raw data from a DATA= data set, you can read subgroup summary statistics from a HISTORY= data set specified in the PROC CUSUM statement. This enables you to reuse OUTHISTORY= data sets that have been created in previous runs of the CUSUM, MACONTROL, or SHEWHART procedures or to read output data sets created with SAS summarization procedures such as PROC MEANS. A HISTORY= data set must contain the following variables:

- subgroup-variable
- subgroup mean variable for each process
- subgroup standard deviation variable for each process
- subgroup sample size variable for each process

The names of the subgroup mean, subgroup standard deviation, and subgroup sample size variables must be the process concatenated with the special suffix characters X, S, and N respectively.

For example, consider the following statements:
Chapter 6: The CUSUM Procedure

proc cusum history=Steel limits=Steelparm;
   xchart (Weight Yieldstrength)*Batch;
run;

The data set Steel must contain the variables Batch, WeightX, WeightS, WeightN, YieldstrengthX, YieldstrengthS, and YieldstrengthN.

Note that if you specify a process name that contains 32 characters, the names of the summary variables must be formed from the first 16 characters and the last 15 characters of the process name, suffixed with the appropriate character.

Other variables that can be read from a HISTORY= data set include

- _PHASE_ (if the READPHASES= option is specified)
- block-variables
- symbol-variable
- BY variables
- ID variables

By default, the CUSUM procedure reads all of the observations in a HISTORY= data set. However, if the HISTORY= data set includes the variable _PHASE_, you can read selected groups of observations (referred to as phases) by specifying the READPHASES= option.

For an example of reading summary information from a HISTORY= data set, see “Creating a V-Mask Cusum Chart from Subgroup Summary Data” on page 550.

Missing Values

An observation read from a DATA= or HISTORY= data set is not analyzed if the value of the subgroup variable is missing. For a particular process variable, an observation read from a DATA= data set is not analyzed if the value of the process variable is missing. Missing values of process variables generally lead to unequal subgroup sample sizes. For a particular process variable, an observation read from a HISTORY= data set is not analyzed if the values of any of the corresponding summary variables are missing.

Examples: XCHART Statement

This section provides advanced examples of the XCHART statement.

Example 6.1: Cusum and Standard Deviation Charts

NOTE: See Cusum and Standard Deviation Charts in the SAS/QC Sample Library.

When you are working with subgrouped data, it can be helpful to accompany a cusum chart for means with a Shewhart s chart for monitoring the variability of the process. This example creates this combination for the variable Weight in the data set Oil (see “Creating a V-Mask Cusum Chart from Raw Data” on page 547).
The first step is to create a one-sided cusum chart for means that detects a shift of one standard error ($\delta = -1$) below the target mean.

```sas
proc cusum data=Oil;
  xchart Weight*Hour /
    nochart
    mu0=8.100 /* target mean for process */
    sigma0=0.050 /* known standard deviation */
    delta=-1 /* shift to be detected */
    h=3 /* cusum parameter h */
    k=0.5 /* cusum parameter k */
    scheme=onesided
    outtable = Tabcusum
    ( drop = _var_ _subn_ _subx_ _exlim_
      rename = ( _cusum_ = _subx_ _h_ = _uclx_ ) )
; run;
```

The results are saved in an OUTTABLE= data set named Tabcusum. The cusum variable (_CUSUM_) and the decision interval variable (_H_) are renamed to _SUBX_ and _LCLX_ so that they can later be read by the SHEWHART procedure.

The next step is to construct a Shewhart $\bar{X}$ and $s$ chart for Weight and save the results in a data set named Tabxscht.

```sas
proc shewhart data=Oil;
  xschart Weight*Hour /
    nochart
    outtable = Tabxscht
    ( drop = _subx_ _uclx_ );
run;
```

Note that the variables _SUBX_ and _UCLX_ are dropped from Tabxscht.

The third step is to merge the data sets Tabcusum and Tabxscht.

```sas
data taball;
  merge Tabxscht Tabcusum by Hour;
  _mean_ = _uclx_ * 0.5;
  _lclx_ = 0.0;
run;
```

The variable _LCLX_ is assigned the role of the lower limit for the cusums, and the variable _MEAN_ is assigned a dummy value. Now, TABALL, which is listed in Output 6.1.1, has the structure required for a TABLE= data set used with the XSCHART statement in the SHEWHART procedure (see “TABLE= Data Set” on page 1917).
The final step is to use the SHEWHART procedure to read TABALL as a TABLE= data set and to display the cusum and s charts.

ods graphics on;
  title 'Cusum Chart for Mean and s chart';
  proc shewhart table=taball;
    xschart Weight * Hour /
      nolimitslegend
      uclabel = 'h=3.0'
      odstitle = title
      markers
      noctl
      split = '/'
      nolegend;
    label _subx_ = 'Lower Cusum/Std Dev';
  run;

The central line for the primary (cusum) chart is suppressed with the NOCTL option, and the default $3\sigma$ Limits legend is suppressed with the NOLIMITLEGEND option. The charts are shown in Output 6.1.2.
The process variability is stable, and there is no signal of a downward shift in the process mean.

**Example 6.2: Upper and Lower One-Sided Cusum Charts**

**NOTE:** See *Upper and Lower One-Sided Cusum Charts* in the SAS/QC Sample Library.

This example illustrates how to combine upper and lower one-sided cusum charts for means in the same display. As in the preceding example, OUTTABLE= data sets are created with the CUSUM procedure, and the display is created with the SHEWHART procedure.

The following statements analyze the variable *Weight* in the data set *Oil* (see “Creating a V-Mask Cusum Chart from Raw Data” on page 547). The first step is to compute and save upper and lower one-sided cusums for shifts of one standard error in the positive and negative directions.

```sas
proc cusum data=Oil;
   xchart Weight*Hour / nochart
   mu0=8.100      /* target mean for process */
   sigma0=0.050   /* known standard deviation */
```
Chapter 6: The CUSUM Procedure

```sas
delta=1 /* shift to be detected */
h=3 /* cusum parameter h */
k=0.5 /* cusum parameter k */
scheme=onesided
outtable = tabupper
   ( drop = _subx_ _subs_ _exlim_
      rename = ( _cusum_ = _subx_ _h_ = _uclx_ ) )
;
xchart Weight*Hour /
   nochart
   mu0=8.100 /* target mean for process */
   sigma0=0.050 /* known standard deviation */
   delta=-1 /* shift to be detected */
   h=3 /* cusum parameter h */
   k=0.5 /* cusum parameter k */
   scheme=onesided
outtable = tablower
   ( drop = _var_ _subn_ _subx_ _subs_ _exlim_
      rename = ( _cusum_ = _subs_ _h_ = _ucls_ ) )
;
run;
```

Next, the OUTTABLE= data sets are merged.

```sas
data Tabboth;
   merge tabupper tablower; by Hour;
   _mean_ = _uclx_ * 0.5;
   _s_ = _ucls_ * 0.5;
   _lclx_ = 0.0;
   _lcls_ = 0.0;
run;
```

The variables _LCLX_ and _UCLX_ are assigned lower limits of zero for the cusums, and the variables _MEAN_ and _S_ are assigned dummy values. Now, Tabboth has the structure required for a TABLE= data set used with the XSCHART statement in the SHEWHART procedure (see “TABLE= Data Set” on page 1917).

The final step is to read Tabboth as a TABLE= data set with the SHEWHART procedure.

```sas
ods graphics on;
title 'Upper and Lower Cusums';
proc shewhart table=Tabboth;
   xschart Weight * Hour /
      nolimitslegend
      markers
      odstitle = title
      ucllabel = 'h=3.0'
      ucllabel2 = 'h=3.0'
      ypct1 = 50
      vref = 1 2
      vref2 = 1 2
      noctl
      noctl2
      split = '/'
      nolegend;
      label _subx_ = 'Upper Sum/Lower Sum';
run;
```
Example 6.3: Combined Shewhart–Cusum Scheme

NOTE: See Combined Shewhart-Cusum Scheme in the SAS/QC Sample Library.

Lucas and Crosier (1982) introduced a combined Shewhart-cusum scheme that is illustrated in this example. Also refer to Ryan (1989). The data set used here is Cans, which is created in “Creating a One-Sided Cusum Chart with a Decision Interval” on page 553.

The first step is to compute and save one-sided cusums to detect a positive shift from the mean.

```
proc cusum data=Cans;
   xchart Weight*Hour / nochart
      mu0  = 8.100  /* target mean for process */
      sigma0 = 0.050 /* known standard deviation */
      delta = 1    /* shift to be detected */
      h      = 3    /* cusum parameter h */
```
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k = 0.5 /* cusum parameter k */
scheme = onesided
outtable = Tabcus
  ( drop = _var_ _subn_ _exlim_
    rename = ( _cusum_ = _subr_ _h_ = _uclr_ ) )
;
run;

Note that a headstart value is not used here but can be specified with the HSTART= option. Several variables in the OUTTABLE= data set are dropped or renamed so that they can later be read by the SHEWHART procedure.

The next step is to construct a Shewhart chart (not shown) for individual measurements.

```
proc shewhart data=Cans;
   irchart Weight*Hour /
      nochart
      mu0 = 8.100
      sigma0 = 0.050
      outtable = Tabx
        ( drop = _subr_ _lclr_ _r_ _uclr_ );
   id comment;
run;
```

By default, 3\(\sigma\) limits are computed, but the multiple of \(\sigma\) can be modified with the SIGMAS= option. As before, the results are saved in an OUTTABLE= data set.

Next, the two OUTTABLE= data sets are merged.

```
data Combine;
   merge Tabx Tabcus; by Hour;
   _lclr_ = 0.0;
   _r_ = 0.5 * _uclr_;
run;
```

The data set Combine has the structure required for a TABLE= data set used with the IRCHART statement in the SHEWHART procedure (see the section “TABLE= Data Set” on page 1503).

Finally, the combined scheme is displayed with the SHEWHART procedure.

```
ods graphics on;
title "Combined Shewhart-Cusum Analysis for Weight";
proc shewhart table=Combine;
   irchart Weight*Hour /
      odsitle = title
      ypc1 = 50
      noc12
      markers
      uc1l = 'h=0.3'
      outlabel = ( comment )
      outlabel2 = ( comment )
      split = '/';
      label _subi_ = 'Shewhart/Cusum';
run;
```

The chart is shown in Output 6.3.1.
Note that a shift is detected by the cusum scheme but not by the Shewhart chart. The point exceeding the decision interval is labeled with the variable `comment` created in the data set `Cans`.

Lucas and Crosier (1982) tabulates average run lengths for combined Shewhart-cusum schemes. The scheme used here has an ARL of 111.1 for $\delta = 0$ and an ARL of 6.322 for $\delta = 1$.

### Overview: INSET Statement

The INSET statement enables you to enhance a cusum chart by adding a box or table (referred to as an *inset*) of summary statistics directly to the graph. A possible application of an inset is to present cusum parameters on the chart rather than displaying them in a legend. An inset can also display arbitrary values provided in a SAS data set.
Note that the INSET statement by itself does not produce a display but must be used in conjunction with an XCHART statement. Insets are not available with line printer charts, so the INSET statement is not applicable when the LINEPRINTER option is specified in the PROC CUSUM statement.

You can use options in the INSET statement to do the following:

- specify the position of the inset
- specify a header for the inset table
- specify graphical enhancements, such as background colors, text colors, text height, text font, and drop shadows

---

**Getting Started: INSET Statement**

This section introduces the INSET statement with a basic example showing how it is used. See the section “INSET and INSET2 Statements: SHEWHART Procedure” on page 1928 in Chapter 17, “The SHEWHART Procedure,” for a complete description of the INSET statement.

This example is based on the same scenario as the first example in the “Getting Started” subsection of “XCHART Statement: CUSUM Procedure” on page 546. A machine fills cans with oil additive and a two-sided cusum chart is used to detect shifts from the target mean of 8.100 ounces. The following statements create the data set Oil and request a two-sided cusum chart with an inset:

```plaintext
data Oil;
    label Hour = 'Hour';
    input Hour @;
    do i=1 to 4;
      input Weight @;
      output;
    end;
    drop i;
  datalines;
1 8.024 8.135 8.151 8.065
  2 7.971 8.165 8.077 8.157
  3 8.125 8.031 8.198 8.050
  4 8.123 8.107 8.154 8.095
  5 8.068 8.093 8.116 8.128
  6 8.177 8.011 8.102 8.030
  7 8.129 8.060 8.125 8.144
  8 8.072 8.010 8.097 8.153
  9 8.066 8.067 8.055 8.059
 10 8.089 8.064 8.170 8.086
 11 8.058 8.098 8.114 8.156
 12 8.147 8.116 8.116 8.018
;```
title 'Cusum Chart for Average Weights of Cans';
ods graphics on;
proc cusum data=Oil;
    xchart Weight*Hour /
        mu0 = 8.100 /* Target mean for process */
        sigma0 = 0.050 /* Known standard deviation */
        delta = 1 /* Shift to be detected */
        alpha = 0.10 /* Type I error probability */
        vaxis = -5 to 3
        odstitle = title
        markers
        nolegend;
    label Weight = 'Cumulative Sum';
    inset arl0 ualpha udelta h k shift sigmas / pos = sw;
run;
The ODS GRAPHICS ON statement specified before the PROC CUSUM statement enables ODS Graphics, so the cusum chart is created using ODS Graphics instead of traditional graphics.

The resulting cusum chart is shown in Figure 6.3.2.

Output 6.3.2 Two-Sided Cusum Chart with an Inset
Syntax: INSET Statement

The syntax for the INSET statement is as follows:

```
INSET keyword-list < / options > ;
```

You can use any number of INSET statements in the CUSUM procedure. However, when ODS Graphics is enabled, at most two insets are displayed inside the plot area and at most two are displayed in the chart margins. Each INSET statement produces a separate inset and must follow an XCHART statement. The inset appears on every panel (page) produced by the last XCHART statement preceding it.

Keywords specify the statistics to be displayed in an inset; options control the inset’s location and appearance. A complete description of the INSET statement syntax is given in the section “Syntax: INSET and INSET2 Statements” on page 1934 of Chapter 17, “The SHEWHART Procedure.” The INSET statement options are identical in the CUSUM and SHEWHART procedures, but the available keywords are different. The options are listed in Table 17.87. The keywords available with the CUSUM procedure are listed in Table 6.9 to Table 6.12.

<table>
<thead>
<tr>
<th>Keyword</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARL0</td>
<td>average run length for zero shift</td>
</tr>
<tr>
<td>ARLDELTA</td>
<td>average run length for shift of $\delta$</td>
</tr>
<tr>
<td>DATA=</td>
<td>arbitrary values from SAS-data-set</td>
</tr>
<tr>
<td>N</td>
<td>nominal subgroup size</td>
</tr>
<tr>
<td>NMIN</td>
<td>minimum subgroup size</td>
</tr>
<tr>
<td>NMAX</td>
<td>maximum subgroup size</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Keyword</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DELTA</td>
<td>shift to be detected as multiple of standard error</td>
</tr>
<tr>
<td>H</td>
<td>decision interval $h$ as a multiple of standard error</td>
</tr>
<tr>
<td>HEADSTART</td>
<td>headstart value $S_0$ as a multiple of standard error</td>
</tr>
<tr>
<td>K</td>
<td>reference value $k$</td>
</tr>
<tr>
<td>MU0</td>
<td>target mean $\mu_0$</td>
</tr>
<tr>
<td>SHIFT</td>
<td>shift to be detected in data units</td>
</tr>
<tr>
<td>STDDEV</td>
<td>estimated or specified process standard deviation</td>
</tr>
</tbody>
</table>
Table 6.11 Parameters for Two-Sided (V-Mask) Cusum Scheme

<table>
<thead>
<tr>
<th>Keyword</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALPHA</td>
<td>probability of Type 1 error</td>
</tr>
<tr>
<td>BETA</td>
<td>probability of Type 2 error</td>
</tr>
<tr>
<td>H</td>
<td>vertical distance between V-mask origin and upper (or lower) arm</td>
</tr>
<tr>
<td>K</td>
<td>slope of lower arm of V-mask</td>
</tr>
<tr>
<td>SIGMAS</td>
<td>probability of Type 1 error as probability that standard normally distributed variable exceeds a specified value in absolute value</td>
</tr>
</tbody>
</table>

You can use the keywords in Table 6.12 only when producing ODS Graphics output. The labels for the statistics use Greek letters.

Table 6.12 Keywords Specific to ODS Graphics Output

<table>
<thead>
<tr>
<th>Keyword</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>UALPHA</td>
<td>probability of Type 1 error</td>
</tr>
<tr>
<td>UARLDELTA</td>
<td>average run length for shift of $\delta$</td>
</tr>
<tr>
<td>UBETA</td>
<td>probability of Type 2 error</td>
</tr>
<tr>
<td>UDELTA</td>
<td>shift to be detected as multiple of standard error</td>
</tr>
<tr>
<td>UMU0</td>
<td>target mean $\mu_0$</td>
</tr>
<tr>
<td>USIGMA</td>
<td>estimated or specified process standard deviation</td>
</tr>
</tbody>
</table>

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