SAS/OR® 14.3 User's Guide
Mathematical Programming
The OPTMILP Procedure

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Chapter 13
The OPTMILP Procedure

Contents

Overview: OPTMILP Procedure

The OPTMILP procedure solves general mixed integer linear programs (MILPs).

A standard mixed integer linear program has the formulation

$$\begin{array}{rl} \min & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x}\ \{\geq, =, \leq\}\ \mathbf{b} \\ & \mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \\ & \mathbf{x}_i \in \mathbb{Z} \quad \forall i \in \mathcal{S} \end{array} \qquad \text{(MILP)}$$

where

$\mathbf{x}$	$\in$	$\mathbb{R}^n$	is the vector of structural variables
$\mathbf{A}$	$\in$	$\mathbb{R}^{m \times n}$	is the matrix of technological coefficients
$\mathbf{c}$	$\in$	$\mathbb{R}^n$	is the vector of objective function coefficients
$\mathbf{b}$	$\in$	$\mathbb{R}^m$	is the vector of constraints' right-hand sides (RHS)
$\mathbf{l}$	$\in$	$\mathbb{R}^n$	is the vector of lower bounds on variables
$\mathbf{u}$	$\in$	$\mathbb{R}^n$	is the vector of upper bounds on variables
$\mathcal{S}$			is a nonempty subset of the set $\{1 \ldots, n\}$ of indices

The OPTMILP procedure implements a linear-programming-based branch-and-cut algorithm. This divide-and-conquer approach attempts to solve the original problem by solving linear programming relaxations of a sequence of smaller subproblems. The OPTMILP procedure also implements advanced techniques such as presolving, generating cutting planes, and applying primal heuristics to improve the efficiency of the overall algorithm.

The OPTMILP procedure requires a mixed integer linear program to be specified using a SAS data set that adheres to the mathematical programming system (MPS) format, a widely accepted format in the optimization community. Chapter 17 discusses the MPS format in detail. It is also possible to input an incumbent solution in MPS format; see the section "Warm Start" on page 630 for details.

The OPTMILP procedure provides various control options and solution strategies. In particular, you can enable, disable, or set levels for the advanced techniques previously mentioned.

The OPTMILP procedure outputs an optimal solution or the best feasible solution found, if any, in SAS data sets. This enables you to generate solution reports and perform additional analyses by using SAS software.

Getting Started: OPTMILP Procedure

The following example illustrates the use of the OPTMILP procedure to solve mixed integer linear programs. For more examples, see the section "Examples: OPTMILP Procedure" on page 644. Suppose you want to solve the following problem:

$$
\begin{array}{lrcrcrcrl}
\min & 2x_1 & - & 3x_2 & - & 4x_3 & & & \\
\text{s.t.} & & - & 2x_2 & - & 3x_3 & \geq & -5 & \text{(R1)} \\
& x_1 & + & x_2 & + & 2x_3 & \leq & 4 & \text{(R2)} \\
& x_1 & + & 2x_2 & + & 3x_3 & \leq & 7 & \text{(R3)} \\
& & & x_1, & & x_2, \; x_3 & \geq & 0 & \\
& & & x_1, & & x_2, \; x_3 & \in & \mathbb{Z} &
\end{array}
$$

The corresponding MPS-format data set is created as follows:

```
data ex_mip;
   input field1 $ field2 $ field3 $ field4 field5 $ field6;
   datalines;
NAME          .       EX_MIP       .    .          .
ROWS          .         .          .    .          .
N             COST      .          .    .          .
G             R1        .          .    .          .
L             R2        .          .    .          .
L             R3        .          .    .          .
COLUMNS       .         .          .    .          .
.             MARK00  'MARKER'     .    'INTORG'   .
.             X1      COST         2    R2         1
.             X1      R3           1    .          .
.             X2      COST        -3    R1        -2
.             X2      R2           1    R3         2
.             X3      COST        -4    R1        -3
.             X3      R2           2    R3         3
.             MARK01  'MARKER'     .    'INTEND'   .
RHS           .       .            .    .          .
.             RHS     R1          -5    R2         4
.             RHS     R3           7    .          .
ENDATA        .       .            .    .          .
;
```

You can also create this SAS data set from an MPS-format flat file (ex_mip.mps) by using the following SAS macro:

```
%mps2sasd(mpsfile = "ex_mip.mps", outdata = ex_mip);
```

This problem can be solved by using the following statement to call the OPTMILP procedure:

```
proc optmilp data = ex_mip
   objsense   = min
   primalout  = primal_out
   dualout    = dual_out
   presolver  = automatic
   heuristics = automatic;
run;
```

The DATA= option names the MPS-format SAS data set that contains the problem data. The OBJSENSE= option specifies whether to maximize or minimize the objective function. The PRIMALOUT= option names the SAS data set to contain the optimal solution or the best feasible solution found by the solver. The DUALOUT= option names the SAS data set to contain the constraint activities. The PRESOLVER= and HEURISTICS= options specify the levels for presolving and applying heuristics, respectively. In this example, each option is set to its default value AUTOMATIC, meaning that the solver automatically determines the appropriate levels for presolve and heuristics.

The optimal integer solution and its corresponding constraint activities, stored in the data sets primal_out and dual_out, respectively, are displayed in Figure 13.1 and Figure 13.2.

Figure 13.1 Optimal Solution

The OPTMILP Procedure
Primal Integer Solution

Obs	Objective Function ID	RHS ID	Variable Name	Variable Type	Objective Coefficient	Lower Bound	Upper Bound	Variable Value
1	COST	RHS	X1	B	2	0	1	0
2	COST	RHS	X2	B	-3	0	1	1
3	COST	RHS	X3	B	-4	0	1	1

Figure 13.2 Constraint Activities

The OPTMILP Procedure
Constraint Information

Obs	Objective Function ID	RHS ID	Constraint Name	Constraint Type	Constraint RHS	Constraint Lower Bound	Constraint Upper Bound	Constraint Activity
1	COST	RHS	R1	G	-5	.	.	-5
2	COST	RHS	R2	L	4	.	.	3
3	COST	RHS	R3	L	7	.	.	5

The solution summary stored in the macro variable _OROPTMILP_ can be viewed by issuing the following statement:

```
%put &_OROPTMILP_;
```

This produces the output shown in Figure 13.3.

Figure 13.3 Macro Output

```
STATUS=OK ALGORITHM=BAC SOLUTION_STATUS=OPTIMAL OBJECTIVE=-7 RELATIVE_GAP=0
ABSOLUTE_GAP=0 PRIMAL_INFEASIBILITY=0 BOUND_INFEASIBILITY=0
INTEGER_INFEASIBILITY=0 BEST_BOUND=-7 NODES=0 ITERATIONS=0 PRESOLVE_TIME=0.01
SOLUTION_TIME=0.01
```

See the section "Data Input and Output" on page 628 for details about the type and status codes displayed for variables and constraints.

Syntax: OPTMILP Procedure

The following statements are available in the OPTMILP procedure:

PROC OPTMILP < *options* > ;
  DECOMP < *options* > ;
  DECOMPMASTER < *options* > ;
  DECOMPMASTER_IP < *options* > ;
  DECOMPSUBPROB < *options* > ;

Functional Summary

Table 13.1 summarizes the options available for the OPTMILP procedure, classified by function.

Table 13.1 Options for the OPTMILP Procedure

Description	Option
Data Set Options	
Specifies the input data set	DATA=
Specifies the constraint activities output data set	DUALOUT=
Specifies whether the MILP model is a maximization or minimization problem	OBJSENSE=
Specifies the primal solution input data set (warm start)	PRIMALIN=
Specifies the primal solution output data set	PRIMALOUT=
Presolve Option	
Specifies the type of presolve	PRESOLVER=
Control Options	
Specifies the stopping criterion based on absolute objective gap	ABSOBJGAP=
Specifies the cutoff value for node removal	CUTOFF=
Emphasizes feasibility or optimality	EMPHASIS=
Specifies the maximum violation on variables and constraints	FEASTOL=
Specifies the maximum allowed difference between an integer variable's value and an integer	INTTOL=
Specifies the frequency of printing the node log	LOGFREQ=
Specifies the detail of solution progress printed in log	LOGLEVEL=
Specifies the maximum number of nodes to be processed	MAXNODES=
Specifies the maximum number of solutions to be found	MAXSOLS=
Specifies the time limit for the optimization process	MAXTIME=
Specifies the tolerance used in determining the optimality of nodes in the branch-and-bound tree	OPTTOL=
Toggles ODS output	PRINTLEVEL=
Specifies the probing level	PROBE=
Specifies the stopping criterion based on relative objective gap	RELOBJGAP=
Enables the use of scaling on the problem matrix	SCALE=
Specifies the initial seed for the random number generator	SEED=
Specifies the stopping criterion based on target objective value	TARGET=
Specifies whether time units are CPU time or real time	TIMETYPE=
Heuristics Option	
Specifies the primal heuristics level	HEURISTICS=

Table 13.1 (continued)

Description	Option
Search Options	
Specifies the level of conflict search	CONFLICTSEARCH=
Specifies the node selection strategy	NODESEL=
Enables use of variable priorities	PRIORITY=
Specifies the restarting strategy	RESTARTS=
Specifies the number of simplex iterations performed on each variable in the strong branching variable selection strategy	STRONGITER=
Specifies the number of candidates for the strong branching variable selection strategy	STRONGLEN=
Specifies the level of symmetry detection	SYMMETRY=
Specifies the rule for selecting the branching variable	VARSEL=
Cut Options	
Specifies the cut level for all cuts	ALLCUTS=
Specifies the clique cut level	CUTCLIQUE=
Specifies the flow cover cut level	CUTFLOWCOVER=
Specifies the flow path cut level	CUTFLOWPATH=
Specifies the Gomory cut level	CUTGOMORY=
Specifies the generalized upper bound (GUB) cover cut level	CUTGUB=
Specifies the implied bounds cut level	CUTIMPLIED=
Specifies the knapsack cover cut level	CUTKNAPSACK=
Specifies the lift-and-project cut level	CUTLAP=
Specifies the mixed lifted 0-1 cut level	CUTMILIFTED=
Specifies the mixed integer rounding (MIR) cut level	CUTMIR=
Specifies the multicommodity network flow cut level	CUTMULTICOMMODITY=
Specifies the row multiplier factor for cuts	CUTSFACTOR=
Specifies the overall cut aggressiveness	CUTSTRATEGY=
Specifies the zero-half cut level	CUTZEROHALF=
Parallel Options	
Specifies the number of threads for the parallel OPTMILP procedure to use	NTHREADS=

PROC OPTMILP Statement

PROC OPTMILP < *options* > **;**

You can specify the following options in the PROC OPTMILP statement.

Data Set Options

DATA=*SAS-data-set*

specifies the input data set that corresponds to the MILP model. If this option is not specified, PROC OPTMILP uses the most recently created SAS data set. See Chapter 17, "The MPS-Format SAS Data Set," for more details about the input data set.

DUALOUT=*SAS-data-set*

DOUT=*SAS-data-set*

specifies the output data set to contain the constraint activities.

OBJSENSE=MIN | MAX

specifies whether the MILP model is a minimization or a maximization problem. You can use OBJSENSE=MIN for a minimization problem and OBJSENSE=MAX for a maximization problem. Alternatively, you can specify the objective sense in the input data set. This option supersedes the objective sense specified in the input data set. If the objective sense is not specified anywhere, then PROC OPTMILP interprets and solves the MILP as a minimization problem.

PRIMALIN=*SAS-data-set*

enables you to input a warm start solution in a SAS data set. PROC OPTMILP validates both the data set and the solution stored in the data set. If the data set is not valid, then the PRIMALIN= data are ignored. If the solution stored in a valid PRIMALIN= data set is a feasible integer solution, then it provides an incumbent solution and a bound for the branch-and-bound algorithm. If the solution stored in a valid PRIMALIN= data set is infeasible, contains missing values, or contains fractional values for integer variables, PROC OPTMILP tries to repair the solution with a number of specialized repair heuristics. See the section "Warm Start" on page 630 for details.

PRIMALOUT=*SAS-data-set*

POUT=*SAS-data-set*

specifies the output data set for the primal solution. This data set contains the primal solution information. See the section "Data Input and Output" on page 628 for details.

Presolve Option

PRESOLVER=AUTOMATIC | NONE | BASIC | MODERATE | AGGRESSIVE

specifies a presolve level. You can specify the following values:

AUTOMATIC	applies the default level of presolve processing.
NONE	disables the presolver.
BASIC	performs minimal presolve processing.
MODERATE	applies a higher level of presolve processing.
AGGRESSIVE	applies the highest level of presolve processing.

By default, PRESOLVER=AUTOMATIC.

Control Options

ABSOBJGAP=*number*

ABSOLUTEOBJECTIVEGAP=*number*

specifies a stopping criterion. When the absolute difference between the best integer objective and the best bound on the objective function value becomes smaller than the value of *number*, the procedure stops. The value of *number* can be any nonnegative number; the default value is 1E–6.

CUTOFF=*number*

cuts off any nodes that have an objective value equal to or worse than *number*. The value of *number* can be any number; the default value is the largest (smallest) number that can be represented by a double.

EMPHASIS=BALANCE | OPTIMAL | FEASIBLE

specifies the type of search emphasis. You can specify the following values:

BALANCE performs a balanced search.

OPTIMAL emphasizes optimality over feasibility.

FEASIBLE emphasizes feasibility over optimality.

By default, EMPHASIS=BALANCE.

FEASTOL=*number*

specifies the tolerance that PROC OPTMILP uses to check the feasibility of a solution. This tolerance applies both to the maximum violation of bounds on variables and to the difference between the right-hand sides and left-hand sides of constraints. The value of *number* can be any value between 1E–4 and 1E–9, inclusive. However, the value of *number* cannot be larger than the integer feasibility tolerance. If the value of *number* is larger than the value of the INTTOL= option, then PROC OPTMILP sets FEASTOL= to the value of INTTOL=. The default value is 1E–6.

If PROC OPTMILP fails to find a feasible solution within this tolerance but does find a solution that has some violation, then the procedure stops with a solution status of OPTIMAL_COND (see the section "Macro Variable _OROPTMILP_" on page 641).

INTTOL=*number*

INTEGERTOLERANCE=*number*

specifies the amount by which an integer variable value can differ from an integer and still be considered integer feasible. The value of *number* can be any number between 1E–9 and 0.5, inclusive. PROC OPTMILP attempts to find an optimal solution whose integer infeasibility is less than *number*. The default value is 1E–5.

If the best solution that PROC OPTMILP finds has an integer infeasibility larger than the value of *number*, then PROC OPTMILP stops with a solution status of OPTIMAL_COND (see the section "Macro Variable _OROPTMILP_" on page 641).

LOGFREQ=*k*

PRINTFREQ=*k*

prints information in the node log every *k* seconds, where *k* is any nonnegative integer up to the largest four-byte signed integer, which is $2^{31} - 1$. If *k*=0, then the node log is disabled. If *k* is positive, then the root node processing information is printed and, if possible, an entry is made every *k* seconds. An entry is also made each time a better integer solution is found.

By default, LOGFREQ=5.

LOGLEVEL=NONE | BASIC | MODERATE | AGGRESSIVE

controls the amount of information displayed in the SAS log by the solver. You can specify the following values:

NONE turns off all solver-related messages in the SAS log.

BASIC displays a solver summary after stopping.

MODERATE prints a solver summary and a node log at the interval specified in the LOGFREQ= option.

AGGRESSIVE prints a detailed solver summary and a node log at the interval specified in the LOGFREQ= option.

By default, LOGLEVEL=MODERATE.

MAXNODES=*number*
specifies the maximum number of branch-and-bound nodes to be processed, where *number* can be any nonnegative integer up to the largest four-byte signed integer, which is $2^{31} - 1$. The default value is $2^{31} - 1$.

MAXSOLS=*number*
specifies a stopping criterion, where *number* can be any positive integer up to the largest four-byte signed integer, which is $2^{31} - 1$. If *number* of solutions have been found, then the procedure stops. The default value of *number* is $2^{31} - 1$.

MAXTIME=*t*
specifies an upper limit of *t* seconds of time for reading in the data and performing the optimization process. The value of the TIMETYPE= option determines the type of units used. If you do not specify MAXTIME= option, the procedure does not stop based on the amount of time elapsed. The value of *t* can be any positive number; the default value is the largest number that can be represented by a double.

OPTTOL=*number*
specifies the tolerance that is used to determine the optimality of nodes in the branch-and-bound tree. The value of *number* can be any value between (and including) 1E–4 and 1E–9. The default value is 1E–6.

PRINTLEVEL=0 | 1 | 2
specifies whether to print a summary of the problem and solution. You can specify the following values:

0 does not produce or print any Output Delivery System (ODS) tables.

1 produces and prints the following ODS tables: ProblemSummary and SolutionSummary.

2 produces and prints the following ODS tables: ProblemSummary, SolutionSummary, ProblemStatistics, and Timing table.

By default, PRINTLEVEL=1.

For more information about the ODS tables created by PROC OPTMILP, see the section "ODS Tables" on page 637.

PROBE=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the probing strategy. You can specify the following values:

AUTOMATIC uses the probing strategy that is determined by PROC OPTMILP.

NONE disables probing.

MODERATE uses probing moderately.

AGGRESSIVE uses probing aggressively.

By default, PROBE=AUTOMATIC. For more information, see the section "Presolve and Probing" on page 633.

RELOBJGAP=*number*

specifies a stopping criterion based on the best integer objective (BestInteger) and the best bound on the objective function value (BestBound). The relative objective gap is equal to

$$|\text{BestInteger} - \text{BestBound}| / (1\text{E}{-}10 + |\text{BestBound}|)$$

When this value becomes smaller than the specified gap size *number*, the procedure stops. The value of *number* can be any nonnegative number; the default value is 1E–4.

SCALE=AUTOMATIC | NONE

indicates whether to scale the problem matrix. You can specify the following values:

AUTOMATIC scales the matrix as determined by PROC OPTMILP.

NONE disables scaling.

By default, SCALE=AUTOMATIC.

SEED=*number*

specifies the initial seed of the random number generator. This option affects the perturbation in the simplex solvers; thus it might result in a different optimal solution and a different solver path. This option usually has a significant, but unpredictable, effect on the solution time. The value of *number* can be any positive integer up to the largest four-byte signed integer, which is $2^{31} - 1$. The default value of the seed is 100.

TARGET=*number*

specifies a stopping criterion for a minimization or maximization problem. If the best integer objective is better than or equal to *number*, the procedure stops. The value of *number* can be any number; the default value is the largest (in magnitude) negative number (for a minimization problem) or the largest (in magnitude) positive number (for a maximization problem) that can be represented by a double.

TIMETYPE=CPU | REAL

specifies whether CPU time or real time is used for the MAXTIME= option and the _OROPTMILP_ macro variable in a PROC OPTMILP call. You can specify the following values:

CPU specifies that units are in CPU time.

REAL specifies that units are in real time.

The default value of the TIMETYPE= option depends on the algorithm used and on various options. When the solver is used with distributed or multithreaded processing, then by default TIMETYPE= REAL. Otherwise, by default TIMETYPE= CPU. Table 13.2 describes the detailed logic for determining the default; the first context in the table that applies determines the default value.

Table 13.2 Default Value for TIMETYPE= Option

Context	Default
NTHREADS= value is greater than 1	REAL
NTHREADS= 1	CPU

Heuristics Option

HEURISTICS=AUTOMATIC | NONE | BASIC | MODERATE | AGGRESSIVE
controls the level of primal heuristics applied by PROC OPTMILP. This level determines how frequently primal heuristics are applied during the branch-and-bound tree search. It also affects the maximum number of iterations allowed in iterative heuristics. Some computationally expensive heuristics might be disabled by the solver at less aggressive levels. You can specify the following values:

AUTOMATIC applies the default level of heuristics, similar to MODERATE.

NONE disables all primal heuristics. This value does not disable the heuristics that repair an infeasible input solution that is specified in a PRIMALIN= data table.

BASIC applies basic primal heuristics at low frequency.

MODERATE applies most primal heuristics at moderate frequency.

AGGRESSIVE applies all primal heuristics at high frequency.

By default, HEURISTICS=AUTOMATIC. For more information about primal heuristics, see the section "Primal Heuristics" on page 635.

Search Options

CONFLICTSEARCH=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of conflict search performed by PROC OPTMILP. Conflict search is used to find clauses resulting from infeasible subproblems that arise in the search tree. You can specify the following values:

AUTOMATIC performs conflict search based on a strategy that is determined by PROC OPTMILP.

NONE disables conflict search.

MODERATE performs a moderate conflict search.

AGGRESSIVE performs an aggressive conflict search.

By default, CONFLICTSEARCH=AUTOMATIC.

NODESEL=AUTOMATIC | BESTBOUND | BESTESTIMATE | DEPTH
specifies the node selection strategy. You can specify the following values:

AUTOMATIC uses automatic node selection.

BESTBOUND chooses the node with the best relaxed objective (best-bound-first strategy).

BESTESTIMATE chooses the node with the best estimate of the integer objective value (best-estimate-first strategy).

DEPTH chooses the most recently created node (depth-first strategy).

By default, NODESEL=AUTOMATIC. For more information about node selection, see the section "Node Selection" on page 632.

PRIORITY=TRUE | FALSE
indicates whether to use specified branching priorities for integer variables. You can specify the following values:

TRUE uses priorities when they exist.

FALSE ignores variable priorities.

By default, PRIORITY=TRUE. For more information, see the section "Branching Priorities" on page 633.

RESTARTS=AUTOMATIC | NONE | BASIC | MODERATE | AGGRESSIVE
specifies the strategy for restarting the processing of the root node. You can specify the following values:

AUTOMATIC uses a restarting strategy determined by PROC OPTMILP.

NONE disables restarting.

BASIC uses a basic restarting strategy.

MODERATE uses a moderate restarting strategy.

AGGRESSIVE uses an aggressive restarting strategy.

By default, RESTARTS=AUTOMATIC.

STRONGITER=*number* | AUTOMATIC
specifies the number of simplex iterations performed for each variable in the candidate list when using the strong branching variable selection strategy. The value of *number* can be any positive integer up to the largest four-byte signed integer, which is $2^{31} - 1$. If you specify the keyword AUTOMATIC, PROC OPTMILP uses the default value; this value is calculated automatically.

STRONGLEN=*number* | AUTOMATIC
specifies the number of candidates used when performing the strong branching variable selection strategy. The value of *number* can be any positive integer up to the largest four-byte signed integer, which is $2^{31} - 1$. If you specify the keyword AUTOMATIC, PROC OPTMILP uses the default value; this value is calculated automatically.

SYMMETRY=AUTOMATIC | NONE | BASIC | MODERATE | AGGRESSIVE
specifies the level of symmetry detection. Symmetry detection identifies groups of equivalent decision variables and uses this information to solve the problem more efficiently. You can specify the following values:

AUTOMATIC performs symmetry detection based on a strategy that is determined by PROC OPTMILP.

NONE disables symmetry detection.

BASIC performs a basic symmetry detection.

MODERATE performs a moderate symmetry detection.

AGGRESSIVE performs an aggressive symmetry detection.

By default, SYMMETRY=AUTOMATIC. For more information about symmetry detection, see (Ostrowski 2008).

VARSEL=AUTOMATIC | MAXINFEAS | MININFEAS | PSEUDO | STRONG
specifies the rule for selecting the branching variable. You can specify the following values:

AUTOMATIC uses automatic branching variable selection.

MAXINFEAS chooses the variable with maximum infeasibility.

MININFEAS chooses the variable with minimum infeasibility.

PSEUDO chooses a branching variable based on pseudocost.

STRONG uses a strong branching variable selection strategy.

By default, VARSEL=AUTOMATIC. For details about variable selection, see the section "Variable Selection" on page 632.

Cut Options

Table 13.3 describes the *string* values for the cut options in PROC OPTMILP.

Table 13.3 Values for Individual Cut Options

string	**Description**
AUTOMATIC	Generates cutting planes based on a strategy determined by PROC OPTMILP
NONE	Disables generation of cutting planes
MODERATE	Uses a moderate cut strategy
AGGRESSIVE	Uses an aggressive cut strategy

You can specify the CUTSTRATEGY= option to set the overall aggressiveness of the cut generation in PROC OPTMILP. Alternatively, you can use the ALLCUTS= option to set all cut types to the same level. You can override the ALLCUTS= value by using the options that correspond to particular cut types. For example, if you want PROC OPTMILP to generate only Gomory cuts, specify ALLCUTS=NONE and CUTGOMORY=AUTOMATIC. If you want to generate all cuts aggressively but generate no lift-and-project cuts, set ALLCUTS=AGGRESSIVE and CUTLAP=NONE.

ALLCUTS=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
provides a shorthand way of setting all the cuts-related options in one setting. In other words, ALLCUTS=*string* is equivalent to setting each of the individual cuts parameters to the same value *string*. Thus, ALLCUTS=AUTOMATIC has the effect of setting CUTCLIQUE=AUTOMATIC, CUTFLOWCOVER=AUTOMATIC, ..., and CUTZEROHALF=AUTOMATIC. Table 13.3 lists the values that can be assigned to *string*. In addition, you can override levels for individual cuts with the CUTCLIQUE=, CUTFLOWCOVER=, CUTFLOWPATH=, CUTGOMORY=, CUTGUB=,

CUTIMPLIED=, CUTKNAPSACK=, CUTLAP=, CUTMILIFTED=, CUTMIR=, CUTMULTICOMMODITY=, and CUTZEROHALF= options. If the ALLCUTS= option is not specified, all the cuts-related options are either set to their individually specified values (if the corresponding option is specified) or to their default values (if that option is not specified).

CUTCLIQUE=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of clique cuts generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTCLIQE=AUTOMATIC.

CUTFLOWCOVER=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of flow cover cuts generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTFLOWCOVER=AUTOMATIC.

CUTFLOWPATH=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of flow path cuts generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTFLOWPATH=AUTOMATIC.

CUTGOMORY=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of Gomory cuts generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTGOMORY=AUTOMATIC.

CUTGUB=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of generalized upper bound (GUB) cover cuts generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTGUB=AUTOMATIC.

CUTIMPLIED=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of implied bound cuts generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTIMPLIED=AUTOMATIC.

CUTKNAPSACK=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of knapsack cover cuts generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTKNAPSACK=AUTOMATIC.

CUTLAP=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of lift-and-project (LAP) cuts generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTLAP=NONE.

CUTMILIFTED=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of mixed lifted 0-1 cuts that are generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTMILIFTED=AUTOMATIC.

CUTMIR=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of mixed integer rounding (MIR) cuts generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTMIR=AUTOMATIC.

CUTMULTICOMMODITY=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of multicommodity network flow cuts generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTMULTICOMMODITY=AUTOMATIC.

CUTSFACTOR=*number*
specifies a row multiplier factor for cuts. The number of cuts that are added is limited to *number* times the original number of rows. The value of *number* can be any nonnegative number less than or equal to 100; the default value is automatically calculated by PROC OPTMILP.

CUTSTRATEGY=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
CUTS=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the overall aggressiveness of the cut generation in the procedure. By default, CUTSTRATEGY=AUTOMATIC. Setting a nondefault value adjusts a number of cut parameters such that the cut generation is none, moderate, or aggressive compared to the default value.

CUTZEROHALF=AUTOMATIC | NONE | MODERATE | AGGRESSIVE
specifies the level of zero-half cuts that are generated by PROC OPTMILP. Table 13.3 describes the possible values. This option overrides the ALLCUTS= option. By default, CUTZEROHALF=AUTOMATIC.

Parallel Options

NTHREADS=*number*
specifies the maximum number of threads to use for multithreaded processing. The branch-and-cut algorithm can take advantage of multicore machines and can potentially run faster when *number* is greater than 1. The value of *number* can be any integer between 1 and 256, inclusive. The default is the number of cores on the machine that executes the process or the number of cores permissible based on your installation (whichever is less). The number of simultaneously active CPUs is limited by your installation and license configuration.

Decomposition Algorithm Statements

The following statements are available for the decomposition algorithm in the OPTMILP procedure:

DECOMP < *options* > **;**

DECOMPMASTER < *options* > **;**

DECOMPMASTER_IP < *options* > **;**

DECOMPSUBPROB < *options* > **;**

For more information about these statements, see Chapter 15, "The Decomposition Algorithm."

TUNER Statement

TUNER < *performance-options* > **;**

The TUNER statement invokes the OPTMILP option tuner. The option tuner is a tool that enables you to explore alternative (and potentially better) option configurations for your optimization problems. For more information about this feature, see Chapter 16, "The OPTMILP Option Tuner."

Details: OPTMILP Procedure

Data Input and Output

This subsection describes the PRIMALIN= data set required to warm start PROC OPTMILP, in addition to the PRIMALOUT= and DUALOUT= data sets.

Definitions of Variables in the PRIMALIN= Data Set

The PRIMALIN= data set has two required variables defined as follows:

VAR
specifies the variable (column) names of the problem. The values should match the column names in the DATA= data set for the current problem.

VALUE
specifies the solution value for each variable in the problem.

NOTE: If PROC OPTMILP produces a feasible solution, the primal output data set from that run can be used as the PRIMALIN= data set for a subsequent run, provided that the variable names are the same. If this input solution is not feasible for the subsequent run, the solver automatically tries to repair it. See the section "Warm Start" on page 630 for more details.

Definitions of Variables in the PRIMALOUT= Data Set

PROC OPTMILP stores the current best integer feasible solution of the problem in the data set specified by the PRIMALOUT= option. The variables in this data set are defined as follows:

_OBJ_ID_
specifies the identifier of the objective function.

_RHS_ID_
specifies the identifier of the right-hand side.

VAR
specifies the variable (column) names.

TYPE
specifies the variable type. _TYPE_ can take one of the following values:

C continuous variable

I general integer variable

B binary variable (0 or 1)

OBJCOEF
specifies the coefficient of the variable in the objective function.

LBOUND
specifies the lower bound on the variable.

UBOUND
specifies the upper bound on the variable.

VALUE
specifies the value of the variable in the current solution.

Definitions of the DUALOUT= Data Set Variables

The DUALOUT= data set contains the constraint activities that correspond to the primal solution in the PRIMALOUT= data set. Information about additional objective rows of the MILP problem is not included. The variables in this data set are defined as follows:

_OBJ_ID_
specifies the identifier of the objective function from the input data set.

_RHS_ID_
specifies the identifier of the right-hand side from the input data set.

ROW
specifies the constraint (row) name.

TYPE
specifies the constraint type. _TYPE_ can take one of the following values:

L "less than or equal" constraint

E equality constraint

G "greater than or equal" constraint

R ranged constraint (both "less than or equal" and "greater than or equal")

RHS
specifies the value of the right-hand side of the constraint. It takes a missing value for a ranged constraint.

_L_RHS_
specifies the lower bound of a ranged constraint. It takes a missing value for a non-ranged constraint.

_U_RHS_
specifies the upper bound of a ranged constraint. It takes a missing value for a non-ranged constraint.

ACTIVITY
specifies the activity of a constraint for a given primal solution. In other words, the value of _ACTIVITY_ for the ith constraint is equal to $\mathbf{a}_i^T \mathbf{x}$, where $\mathbf{a}_i$ refers to the ith row of the constraint matrix and $\mathbf{x}$ denotes the vector of the current primal solution.

Warm Start

PROC OPTMILP enables you to input a warm start solution by using the PRIMALIN= option. PROC OPTMILP checks that the decision variables named in _VAR_ are the same as those in the MPS-format SAS data set. If they are not the same, PROC OPTMILP issues a warning and ignores the input solution. PROC OPTMILP also checks whether the solution is infeasible, contains missing values, or contains fractional values for integer variables. If this is the case, PROC OPTMILP attempts to repair the solution with a number of specialized repair heuristics. The success of the attempt largely depends both on the specific model and on the proximity between the input solution and an integer feasible solution. An infeasible input solution can be considered a hint for PROC OPTMILP that might or might not help to solve the problem.

An integer feasible or repaired input solution provides an incumbent solution in addition to an upper (min) or lower (max) bound for the branch-and-bound algorithm. PROC OPTMILP uses the input solution to reduce the search space and to guide the search process. When it is difficult to find a good integer feasible solution for a problem, warm start can reduce solution time significantly.

Branch-and-Bound Algorithm

The branch-and-bound algorithm, first proposed by Land and Doig (1960), is an effective approach to solving mixed integer linear programs. The following discussion outlines the approach and explains how PROC OPTMILP enhances the basic algorithm by using several advanced techniques.

The branch-and-bound algorithm solves a mixed integer linear program by dividing the search space and generating a sequence of subproblems. The search space of a mixed integer linear program can be represented by a tree. Each node in the tree is identified with a subproblem derived from previous subproblems on the path that leads to the root of the tree. The subproblem (MILP^0) associated with the root is identical to the original problem, which is called (MILP), given in the section "Overview: OPTMILP Procedure" on page 613.

The linear programming relaxation (LP^0) of (MILP^0) can be written as

$$\begin{array}{rl} \min & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & \mathbf{A}\mathbf{x}\ \{\geq, =, \leq\}\ \mathbf{b} \\ & \mathbf{l} \leq \mathbf{x} \leq \mathbf{u} \end{array}$$

The branch-and-bound algorithm generates subproblems along the nodes of the tree by using the following scheme. Consider $\bar{x}^0$, the optimal solution to (LP^0), which is usually obtained by using the dual simplex algorithm. If $\bar{x}_i^0$ is an integer for all $i \in \mathcal{S}$, then $\bar{x}^0$ is an optimal solution to (MILP). Suppose that for some $i \in \mathcal{S}$, $\bar{x}_i^0$ is nonintegral. In that case the algorithm defines two new subproblems (MILP^1) and (MILP^2), descendants of the parent subproblem (MILP^0). The subproblem (MILP^1) is identical to (MILP^0) except for the additional constraint

$$x_i \leq \lfloor \bar{x}_i^0 \rfloor$$

and the subproblem (MILP^2) is identical to (MILP^0) except for the additional constraint

$$x_i \geq \lceil \bar{x}_i^0 \rceil$$

The notation $\lfloor y \rfloor$ represents the largest integer that is less than or equal to y, and the notation $\lceil y \rceil$ represents the smallest integer that is greater than or equal to y. The two preceding constraints can be handled by modifying the bounds of the variable x_i rather than by explicitly adding the constraints to the constraint matrix. The two new subproblems do not have $\bar{x}^0$ as a feasible solution, but the integer solution to (MILP) must satisfy one of the preceding constraints. The two subproblems thus defined are called *active nodes* in the branch-and-bound tree, and the variable x_i is called the *branching variable*.

In the next step the branch-and-bound algorithm chooses one of the active nodes and attempts to solve the linear programming relaxation of that subproblem. The relaxation might be infeasible, in which case the subproblem is dropped (fathomed). If the subproblem can be solved and the solution is *integer feasible* (that is, x_i is an integer for all $i \in \mathcal{S}$), then its objective value provides an *upper bound* for the objective value in the minimization problem (MILP); if the solution is not integer feasible, then it defines two new subproblems. Branching continues in this manner until there are no active nodes. At this point the best integer solution found is an optimal solution for (MILP). If no integer solution has been found, then (MILP) is integer infeasible. You can specify other criteria to stop the branch-and-bound algorithm before it processes all the active nodes; see the section "Controlling the Branch-and-Bound Algorithm" on page 631 for details.

Upper bounds from integer feasible solutions can be used to *fathom* or *cut off* active nodes. Since the objective value of an optimal solution cannot be greater than an upper bound, active nodes with lower bounds higher than an existing upper bound can be safely deleted. In particular, if z is the objective value of the current best integer solution, then any active subproblems whose relaxed objective value is greater than or equal to z can be discarded.

It is important to realize that mixed integer linear programs are nondeterministic polynomial-time hard (NP-hard). Roughly speaking, this means that the effort required to solve a mixed integer linear program grows exponentially with the size of the problem. For example, a problem with 10 binary variables can generate in the worst case $2^{10} = 1,024$ nodes in the branch-and-bound tree. A problem with 20 binary variables can generate in the worst case $2^{20} = 1,048,576$ nodes in the branch-and-bound tree. Although it is unlikely that the branch-and-bound algorithm has to generate every single possible node, the need to explore even a small fraction of the potential number of nodes for a large problem can be resource-intensive.

A number of techniques can speed up the search progress of the branch-and-bound algorithm. Heuristics are used to find feasible solutions, which can improve the upper bounds on solutions of mixed integer linear programs. Cutting planes can reduce the search space and thus improve the lower bounds on solutions of mixed integer linear programs. When using cutting planes, the branch-and-bound algorithm is also called the *branch-and-cut algorithm*. Preprocessing can reduce problem size and improve problem solvability. PROC OPTMILP employs various heuristics, cutting planes, preprocessing, and other techniques, which you can control through corresponding options.

Controlling the Branch-and-Bound Algorithm

There are numerous strategies that can be used to control the branch-and-bound search (see Linderoth and Savelsbergh 1998, Achterberg, Koch, and Martin 2005). PROC OPTMILP implements the most widely used strategies and provides several options that enable you to direct the choice of the next active node and of the branching variable. In the discussion that follows, let (LP^k) be the linear programming relaxation of subproblem (MILP^k). Also, let

$$f_i(k) = \bar{x}_i^k - \lfloor \bar{x}_i^k \rfloor$$

where $\bar{x}^k$ is the optimal solution to the relaxation problem (LP^k) solved at node k.

Node Selection

The NODESEL= option specifies the strategy used to select the next active node. The valid keywords for this option are AUTOMATIC, BESTBOUND, BESTESTIMATE, and DEPTH. The following list describes the strategy associated with each keyword:

AUTOMATIC — allows PROC OPTMILP to choose the best node selection strategy based on problem characteristics and search progress. This is the default setting.

BESTBOUND — chooses the node with the smallest (or largest, in the case of a maximization problem) relaxed objective value. The best-bound strategy tends to reduce the number of nodes to be processed and can improve lower bounds quickly. However, if there is no good upper bound, the number of active nodes can be large. This can result in the solver running out of memory.

BESTESTIMATE — chooses the node with the smallest (or largest, in the case of a maximization problem) objective value of the estimated integer solution. Besides improving lower bounds, the best-estimate strategy also attempts to process nodes that can yield good feasible solutions.

DEPTH — chooses the node that is deepest in the search tree. Depth-first search is effective in locating feasible solutions, since such solutions are usually deep in the search tree. Compared to the costs of the best-bound and best-estimate strategies, the cost of solving LP relaxations is less in the depth-first strategy. The number of active nodes is generally small, but it is possible that the depth-first search will remain in a portion of the search tree with no good integer solutions. This occurrence is computationally expensive.

Variable Selection

The VARSEL= option specifies the strategy used to select the next branching variable. The valid keywords for this option are AUTOMATIC, MAXINFEAS, MININFEAS, PSEUDO, and STRONG. The following list describes the action taken in each case when $\bar{x}^k$, a relaxed optimal solution of (MILP^k), is used to define two active subproblems. In the following list, "INTTOL" refers to the value assigned using the INTTOL= option. For details about the INTTOL= option, see the section "Control Options" on page 619.

AUTOMATIC — enables PROC OPTMILP to choose the best variable selection strategy based on problem characteristics and search progress. This is the default setting.

MAXINFEAS — chooses as the branching variable the variable x_i such that i maximizes

$$\{\min\{f_i(k), 1 - f_i(k)\} \mid i \in \mathcal{S} \text{ and}$$

$$\text{INTTOL} \leq f_i(k) \leq 1 - \text{INTTOL}\}$$

MININFEAS — chooses as the branching variable the variable x_i such that i minimizes

$$\{\min\{f_i(k), 1 - f_i(k)\} \mid i \in \mathcal{S} \text{ and}$$

$$\text{INTTOL} \leq f_i(k) \leq 1 - \text{INTTOL}\}$$

PSEUDO chooses as the branching variable the variable x_i such that i maximizes the weighted up and down pseudocosts. Pseudocost branching attempts to branch on significant variables first, quickly improving lower bounds. Pseudocost branching estimates significance based on historical information; however, this approach might not be accurate for future search.

STRONG chooses as the branching variable the variable x_i such that i maximizes the estimated improvement in the objective value. Strong branching first generates a list of candidates, then branches on each candidate and records the improvement in the objective value. The candidate with the largest improvement is chosen as the branching variable. Strong branching can be effective for combinatorial problems, but it is usually computationally expensive.

Branching Priorities

In some cases, it is possible to speed up the branch-and-bound algorithm by branching on variables in a specific order. You can accomplish this in PROC OPTMILP by attaching branching priorities to the integer variables in your model.

You can set branching priorities for use by PROC OPTMILP in two ways. You can specify the branching priorities directly in the input MPS-format data set; see the section "BRANCH Section (Optional)" on page 823 for details. If you are constructing a model in PROC OPTMODEL, you can set branching priorities for integer variables by using the .priority suffix. More information about this suffix is available in the section "Integer Variable Suffixes" on page 136 in Chapter 5. For an example in which branching priorities are used, see Example 8.3.

Presolve and Probing

PROC OPTMILP includes a variety of presolve techniques to reduce problem size, improve numerical stability, and detect infeasibility or unboundedness (Andersen and Andersen 1995; Gondzio 1997). During presolve, redundant constraints and variables are identified and removed. Presolve can further reduce the problem size by substituting variables. Variable substitution is a very effective technique, but it might occasionally increase the number of nonzero entries in the constraint matrix. Presolve might also modify the constraint coefficients to tighten the formulation of the problem.

In most cases, using presolve is very helpful in reducing solution times. You can enable presolve at different levels by specifying the PRESOLVER= option.

Probing is a technique that tentatively sets each binary variable to 0 or 1, then explores the logical consequences (Savelsbergh 1994). Probing can expedite the solution of a difficult problem by fixing variables and improving the model. However, probing is often computationally expensive and can significantly increase the solution time in some cases. You can enable probing at different levels by specifying the PROBE= option.

Cutting Planes

The feasible region of every linear program forms a *polyhedron*. Every polyhedron in n-space can be written as a finite number of half-spaces (equivalently, inequalities). In the notation used in this chapter, this polyhedron is defined by the set $\mathcal{Q} = \{x \in \mathbb{R}^n \mid Ax \leq b, l \leq x \leq u\}$. After you add the restriction that

some variables must be integral, the set of feasible solutions, $\mathcal{F} = \{x \in \mathcal{Q} \mid x_i \in \mathbb{Z} \ \forall i \in \mathcal{S}\}$, no longer forms a polyhedron.

The *convex hull* of a set X is the minimal convex set that contains X. In solving a mixed integer linear program, in order to take advantage of LP-based algorithms you want to find the convex hull, $\text{conv}(\mathcal{F})$, of $\mathcal{F}$. If you can find $\text{conv}(\mathcal{F})$ and describe it compactly, then you can solve a mixed integer linear program with a linear programming solver. This is generally very difficult, so you must be satisfied with finding an approximation. Typically, the better the approximation, the more efficiently the LP-based branch-and-bound algorithm can perform.

As described in the section "Branch-and-Bound Algorithm" on page 630, the branch-and-bound algorithm begins by solving the linear programming relaxation over the polyhedron $\mathcal{Q}$. Clearly, $\mathcal{Q}$ contains the convex hull of the feasible region of the original integer program; that is, $\text{conv}(\mathcal{F}) \subseteq \mathcal{Q}$.

Cutting plane techniques are used to tighten the linear relaxation to better approximate $\text{conv}(\mathcal{F})$. Assume you are given a solution $\bar{x}$ to some intermediate linear relaxation during the branch-and-bound algorithm. A cut, or valid inequality ($\pi x \leq \pi^0$), is some half-space with the following characteristics:

- The half-space contains $\text{conv}(\mathcal{F})$; that is, every integer feasible solution is feasible for the cut ($\pi x \leq \pi^0, \forall x \in \mathcal{F}$).
- The half-space does not contain the current solution $\bar{x}$; that is, $\bar{x}$ is not feasible for the cut ($\pi \bar{x} > \pi^0$).

Cutting planes were first made popular by Dantzig, Fulkerson, and Johnson (1954) in their work on the traveling salesman problem. The two major classifications of cutting planes are *generic cuts* and *structured cuts*. Generic cuts are based solely on algebraic arguments and can be applied to any relaxation of any integer program. Structured cuts are specific to certain structures that can be found in some relaxations of the mixed integer linear program. These structures are automatically discovered during the cut initialization phase of PROC OPTMILP. Table 13.4 lists the various types of cutting planes that are built into PROC OPTMILP. Included in each type are algorithms for numerous variations based on different relaxations and lifting techniques. For a survey of cutting plane techniques for mixed integer programming, see Marchand et al. (1999). For a survey of lifting techniques, see Atamturk (2004).

Table 13.4 Cutting Planes in PROC OPTMILP

Generic Cutting Planes	Structured Cutting Planes
Gomory mixed integer	Cliques
Lift-and-project	Flow cover
Mixed integer rounding	Flow path
Mixed lifted 0-1	Generalized upper bound cover
Zero-half	Implied bound
	Knapsack cover
	Multicommodity network flow

You can set levels for individual cuts by using the CUTCLIQUE=, CUTFLOWCOVER=, CUTFLOWPATH=, CUTGOMORY=, CUTGUB=, CUTIMPLIED=, CUTKNAPSACK=, CUTLAP=, CUTMIR=, CUTMULTICOMMODITY=, and CUTZEROHALF= options. The valid levels for these options are given in Table 13.3.

The cut level determines the internal strategy used by PROC OPTMILP for generating the cutting planes. The strategy consists of several factors, including how frequently the cut search is called, the number of cuts allowed, and the aggressiveness of the search algorithms.

Sophisticated cutting planes, such as those included in PROC OPTMILP, can take a great deal of CPU time. Usually, the additional tightening of the relaxation helps speed up the overall process because it provides better bounds for the branch-and-bound tree and helps guide the LP solver toward integer solutions. In rare cases, shutting off cutting planes completely might lead to faster overall run times.

The default settings of PROC OPTMILP have been tuned to work well for most instances. However, problem-specific expertise might suggest adjusting one or more of the strategies. These options give you that flexibility.

Primal Heuristics

Primal heuristics, an important component of PROC OPTMILP, are applied during the branch-and-bound algorithm. They are used to find integer feasible solutions early in the search tree, thereby improving the upper bound for a minimization problem. Primal heuristics play a role that is complementary to cutting planes in reducing the gap between the upper and lower bounds, thus reducing the size of the branch-and-bound tree.

Applying primal heuristics in the branch-and-bound algorithm assists in the following areas:

- finding a good upper bound early in the tree search (this can lead to earlier fathoming, resulting in fewer subproblems to be processed)
- locating a reasonably good feasible solution when that is sufficient (sometimes a good feasible solution is the best the solver can produce within certain time or resource limits)
- providing upper bounds for some bound-tightening techniques

The OPTMILP procedure implements several heuristic methodologies. Some algorithms, such as rounding and iterative rounding (diving) heuristics, attempt to construct an integer feasible solution by using fractional solutions to the continuous relaxation at each node of the branch-and-cut tree. Other algorithms start with an incumbent solution and attempt to find a better solution within a neighborhood of the current best solution.

The HEURISTICS= option enables you to control the level of primal heuristics that are applied by PROC OPTMILP. This level determines how frequently primal heuristics are applied during the tree search. Some expensive heuristics might be disabled by the solver at less aggressive levels. Setting the HEURISTICS= option to a lower level also reduces the maximum number of iterations that are allowed in iterative heuristics.

Parallel Processing

You can run the decomposition algorithm, the branch-and-cut algorithm, and the option tuner in either single-machine or distributed mode. In distributed mode, the computation is executed on multiple computing nodes in a distributed computing environment.

NOTE: Distributed mode requires the SAS Optimization product on the SAS Viya platform.

Node Log

The following information about the status of the branch-and-bound algorithm is printed in the node log:

Node	indicates the sequence number of the current node in the search tree.
Active	indicates the current number of active nodes in the branch-and-bound tree.
Sols	indicates the number of feasible solutions found so far.
BestInteger	indicates the best upper bound (assuming minimization) found so far.
BestBound	indicates the best lower bound (assuming minimization) found so far.
Gap	indicates the relative gap between BestInteger and BestBound, displayed as a percentage. If the relative gap is larger than 1,000, then the absolute gap is displayed. If no active nodes remain, the value of Gap is 0.
Time	indicates the elapsed real or CPU time.

The LOGFREQ= and LOGLEVEL= options can be used to control the amount of information printed in the node log. By default, the root node processing information is printed and, if possible, an entry is made every five seconds. A new entry is also included each time a better integer solution is found. The LOGFREQ= option enables you to change the interval between entries in the node log. Figure 13.4 shows a sample node log.

Figure 13.4 Sample Node Log

```
NOTE: The problem ex1data has 10 variables (0 binary, 10 integer, 0 free, 0
      fixed).
NOTE: The problem has 2 constraints (2 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 20 constraint coefficients.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 2 variables and 0 constraints.
NOTE: The MILP presolver removed 4 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 8 variables, 2 constraints, and 16 constraint
      coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 2 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      3     85.0000000    158.0000000   46.20%       0
             0        1      3     85.0000000     88.0955497    3.51%       0
             0        1      3     85.0000000     87.4545455    2.81%       0
             0        1      3     85.0000000     87.4545455    2.81%       0
NOTE: The MILP presolver is applied again.
             0        1      4     87.0000000     87.4545455    0.52%       0
             0        0      4     87.0000000     87.0000000    0.00%       0
NOTE: Optimal.
NOTE: Objective = 87.
NOTE: There were 43 observations read from the data set WORK.EX1DATA.
NOTE: The data set WORK.EX1SOLN has 10 observations and 8 variables.
```

ODS Tables

PROC OPTMILP creates two Output Delivery System (ODS) tables by default. The first table, ProblemSummary, is a summary of the input MILP problem. The second table, SolutionSummary, is a brief summary of the solution status. You can use ODS table names to select tables and create output data sets. For more information about ODS, see *SAS Output Delivery System: User's Guide*.

If you specify a value of 2 for the PRINTLEVEL= option, then the ProblemStatistics table and a Timing table are produced. The ProblemStatistics table contains information about the problem data. See the section "Problem Statistics" on page 641 for more information. The Timing table contains detailed information about the solution time.

Table 13.5 lists all the ODS tables that can be produced by the OPTMILP procedure, along with the statement and option specifications required to produce each table.

Table 13.5 ODS Tables Produced by PROC OPTMILP

ODS Table Name	Description	Statement	Option
ProblemSummary	Summary of the input MILP problem	PROC OPTMILP	PRINTLEVEL=1 (default)
SolutionSummary	Summary of the solution status	PROC OPTMILP	PRINTLEVEL=1 (default)
ProblemStatistics	Description of input problem data	PROC OPTMILP	PRINTLEVEL=2
Timing	Detailed solution timing	PROC OPTMILP	PRINTLEVEL=2

A typical ProblemSummary table is shown in Figure 13.5.

Figure 13.5 Example PROC OPTMILP Output: Problem Summary

The OPTMILP Procedure

Problem Summary	
Problem Name	EX_MIP
Objective Sense	Minimization
Objective Function	COST
RHS	RHS
Number of Variables	3
Bounded Above	0
Bounded Below	0
Bounded Above and Below	3
Free	0
Fixed	0
Binary	3
Integer	0
Number of Constraints	3
LE (<=)	2
EQ (=)	0
GE (>=)	1
Range	0
Constraint Coefficients	8

A typical SolutionSummary table is shown in Figure 13.6.

Figure 13.6 Example PROC OPTMILP Output: Solution Summary

The OPTMILP Procedure

Solution Summary	
Solver	MILP
Algorithm	Branch and Cut
Objective Function	COST
Solution Status	Optimal
Objective Value	-7
Relative Gap	0
Absolute Gap	0
Primal Infeasibility	0
Bound Infeasibility	0
Integer Infeasibility	0
Best Bound	-7
Nodes	0
Iterations	0
Presolve Time	0.00
Solution Time	0.00

You can create output data sets from these tables by using the ODS OUTPUT statement. The output data sets from the preceding example are displayed in Figure 13.7 and Figure 13.8, where you can also find variable names for the tables used in the ODS template of the OPTMILP procedure.

Figure 13.7 ODS Output Data Set: Problem Summary

Problem Summary

Obs	Label1	cValue1	nValue1
1	Problem Name	EX_MIP	.
2	Objective Sense	Minimization	.
3	Objective Function	COST	.
4	RHS	RHS	.
5			.
6	Number of Variables	3	3.000000
7	Bounded Above	0	0
8	Bounded Below	0	0
9	Bounded Above and Below	3	3.000000
10	Free	0	0
11	Fixed	0	0
12	Binary	3	3.000000
13	Integer	0	0
14			.
15	Number of Constraints	3	3.000000
16	LE (<=)	2	2.000000
17	EQ (=)	0	0
18	GE (>=)	1	1.000000
19	Range	0	0
20			.
21	Constraint Coefficients	8	8.000000

Figure 13.8 ODS Output Data Set: Solution Summary

Solution Summary

Obs	Label1	cValue1	nValue1
1	Solver	MILP	.
2	Algorithm	Branch and Cut	.
3	Objective Function	COST	.
4	Solution Status	Optimal	.
5	Objective Value	-7	-7.000000
6			.
7	Relative Gap	0	0
8	Absolute Gap	0	0
9	Primal Infeasibility	0	0
10	Bound Infeasibility	0	0
11	Integer Infeasibility	0	0
12			.
13	Best Bound	-7	-7.000000
14	Nodes	0	0
15	Iterations	0	0
16	Presolve Time	0.00	0
17	Solution Time	0.00	0

Problem Statistics

Optimizers can encounter difficulty when solving poorly formulated models. Information about data magnitude provides a simple gauge to determine how well a model is formulated. For example, a model whose constraint matrix contains one very large entry (on the order of 10^9) can cause difficulty when the remaining entries are single-digit numbers. The PRINTLEVEL=2 option in the OPTMILP procedure causes the ODS table ProblemStatistics to be generated. This table provides basic data magnitude information that enables you to improve the formulation of your models.

The example output in Figure 13.9 demonstrates the contents of the ODS table ProblemStatistics.

Figure 13.9 ODS Table ProblemStatistics

ProblemStatistics

Obs	Label1	cValue1	nValue1
1	Number of Constraint Matrix Nonzeros	8	8.000000
2	Maximum Constraint Matrix Coefficient	3	3.000000
3	Minimum Constraint Matrix Coefficient	1	1.000000
4	Average Constraint Matrix Coefficient	1.875	1.875000
5			.
6	Number of Objective Nonzeros	3	3.000000
7	Maximum Objective Coefficient	4	4.000000
8	Minimum Objective Coefficient	2	2.000000
9	Average Objective Coefficient	3	3.000000
10			.
11	Number of RHS Nonzeros	3	3.000000
12	Maximum RHS	7	7.000000
13	Minimum RHS	4	4.000000
14	Average RHS	5.3333333333	5.333333
15			.
16	Maximum Number of Nonzeros per Column	3	3.000000
17	Minimum Number of Nonzeros per Column	2	2.000000
18	Average Number of Nonzeros per Column	2.67	2.666667
19			.
20	Maximum Number of Nonzeros per Row	3	3.000000
21	Minimum Number of Nonzeros per Row	2	2.000000
22	Average Number of Nonzeros per Row	2.67	2.666667

The variable names in the ODS table ProblemStatistics are Label1, cValue1, and nValue1.

Macro Variable _OROPTMILP_

The OPTMILP procedure defines a macro variable named _OROPTMILP_. This variable contains a character string that indicates the status of the OPTMILP procedure upon termination. The various terms of the variable are interpreted as follows.

STATUS
indicates the solver status at termination. It can take one of the following values:

OK	The procedure terminated normally.
SYNTAX_ERROR	Incorrect syntax was used.
DATA_ERROR	The input data was inconsistent.
OUT_OF_MEMORY	Insufficient memory was allocated to the procedure.
IO_ERROR	A problem occurred in reading or writing data.
ERROR	The status cannot be classified into any of the preceding categories.

ALGORITHM
indicates the algorithm that produced the solution data in the macro variable. This term only appears when STATUS=OK. It can take one of the following values:

BAC	The branch-and-cut algorithm produced the solution data.
DECOMP	The decomposition algorithm produced the solution data.

SOLUTION_STATUS
indicates the solution status at termination. It can take one of the following values:

OPTIMAL	The solution is optimal.
OPTIMAL_AGAP	The solution is optimal within the absolute gap specified by the ABSOBJGAP= option.
OPTIMAL_RGAP	The solution is optimal within the relative gap specified by the RELOBJGAP= option.
OPTIMAL_COND	The solution is optimal, but some infeasibilities (primal, bound, or integer) exceed tolerances due to scaling or choice of a small INTTOL= value.
TARGET	The solution is not worse than the target specified by the TARGET= option.
INFEASIBLE	The problem is infeasible.
UNBOUNDED	The problem is unbounded.
INFEASIBLE_OR_UNBOUNDED	The problem is infeasible or unbounded.
SOLUTION_LIM	The solver reached the maximum number of solutions specified by the MAXSOLS= option.
NODE_LIM_SOL	The solver reached the maximum number of nodes specified by the MAXNODES= option and found a solution.
NODE_LIM_NOSOL	The solver reached the maximum number of nodes specified by the MAXNODES= option and did not find a solution.
TIME_LIM_SOL	The solver reached the execution time limit specified by the MAXTIME= option and found a solution.

TIME_LIM_NOSOL	The solver reached the execution time limit specified by the MAXTIME= option and did not find a solution.
ABORT_SOL	The solver was stopped by the user but still found a solution.
ABORT_NOSOL	The solver was stopped by the user and did not find a solution.
OUTMEM_SOL	The solver ran out of memory but still found a solution.
OUTMEM_NOSOL	The solver ran out of memory and either did not find a solution or failed to output the solution due to insufficient memory.
FAIL_SOL	The solver stopped due to errors but still found a solution.
FAIL_NOSOL	The solver stopped due to errors and did not find a solution.

OBJECTIVE
indicates the objective value obtained by the solver at termination.

RELATIVE_GAP
indicates the relative gap between the best integer objective (BestInteger) and the best bound on the objective function value (BestBound) upon termination of the OPTMILP procedure. The relative gap is equal to

$$|\text{BestInteger} - \text{BestBound}| / (1\text{E}{-}10 + |\text{BestBound}|)$$

ABSOLUTE_GAP
indicates the absolute gap between the best integer objective (BestInteger) and the best bound on the objective function value (BestBound) upon termination of the OPTMILP procedure. The absolute gap is equal to $|\text{BestInteger} - \text{BestBound}|$.

PRIMAL_INFEASIBILITY
indicates the maximum (absolute) violation of the primal constraints by the solution.

BOUND_INFEASIBILITY
indicates the maximum (absolute) violation by the solution of the lower or upper bounds (or both).

INTEGER_INFEASIBILITY
indicates the maximum (absolute) violation of the integrality of integer variables returned by the OPTMILP procedure.

BEST_BOUND
indicates the best bound on the objective function value at termination. A missing value indicates that the OPTMILP procedure was not able to obtain such a bound.

NODES
indicates the number of nodes enumerated by the OPTMILP procedure when using the branch-and-bound algorithm.

ITERATIONS
indicates the number of simplex iterations taken to solve the problem.

PRESOLVE_TIME
indicates the time (in seconds) used in preprocessing.

SOLUTION_TIME
indicates the time (in seconds) taken to solve the problem, including preprocessing time.

NOTE: The time reported in PRESOLVE_TIME and SOLUTION_TIME is either CPU time or real time. The type is determined by the TIMETYPE= option.

Examples: OPTMILP Procedure

This section contains examples that illustrate the options and syntax of PROC OPTMILP. Example 13.1 demonstrates a model contained in an MPS-format SAS data set and finds an optimal solution by using PROC OPTMILP. Example 13.2 illustrates the use of standard MPS files in PROC OPTMILP. Example 13.3 demonstrates how to warm start PROC OPTMILP. More detailed examples of mixed integer linear programs, along with example SAS code, are given in Chapter 8.

Example 13.1: Simple Integer Linear Program

This example illustrates a model in an MPS-format SAS data set. This data set is passed to PROC OPTMILP, and a solution is found.

Consider a scenario where you have a container with a set of limiting attributes (volume V and weight W) and a set I of items that you want to pack. Each item type i has a certain value p_i, a volume v_i, and a weight w_i. You must choose at most four items of each type so that the total value is maximized and all the chosen items fit into the container. Let x_i be the number of items of type i to be included in the container. This model can be formulated as the following integer linear program:

$$
\begin{array}{lllll}
\max & \displaystyle\sum_{i \in I} p_i x_i & & & \\
\text{s.t.} & \displaystyle\sum_{i \in I} v_i x_i & \leq & V & \text{(volume\_con)} \\
& \displaystyle\sum_{i \in I} w_i x_i & \leq & W & \text{(weight\_con)} \\
& x_i & \leq & 4 \quad \forall i \in I & \\
& x_i \in \mathbb{Z}^+ & & \forall i \in I &
\end{array}
$$

Constraint (volume_con) enforces the volume capacity limit, while constraint (weight_con) enforces the weight capacity limit. An instance of this problem can be saved in an MPS-format SAS data set by using the following code:

```
data ex1data;
    input field1 $ field2 $ field3 $ field4 field5 $ field6;
    datalines;
NAME        .             ex1data          .     .                .
ROWS        .             .                .     .                .
MAX         z             .                .     .                .
L           volume_con    .                .     .                .
L           weight_con    .                .     .                .
COLUMNS     .             .                .     .                .
.           .MRK0         'MARKER'         .     'INTORG'         .
.           x[1]          z                1     volume_con       10
.           x[1]          weight_con       12    .                .
.           x[2]          z                2     volume_con       300
.           x[2]          weight_con       15    .                .
.           x[3]          z                3     volume_con       250
.           x[3]          weight_con       72    .                .
.           x[4]          z                4     volume_con       610
.           x[4]          weight_con       100   .                .
.           x[5]          z                5     volume_con       500
.           x[5]          weight_con       223   .                .
.           x[6]          z                6     volume_con       120
.           x[6]          weight_con       16    .                .
.           x[7]          z                7     volume_con       45
.           x[7]          weight_con       73    .                .
.           x[8]          z                8     volume_con       100
.           x[8]          weight_con       12    .                .
.           x[9]          z                9     volume_con       200
.           x[9]          weight_con       200   .                .
.           x[10]         z                10    volume_con       61
.           x[10]         weight_con       110   .                .
.           .MRK1         'MARKER'         .     'INTEND'         .
RHS         .             .                .     .                .
.           .RHS.         volume_con       1000  .                .
.           .RHS.         weight_con       500   .                .
BOUNDS      .             .                .     .                .
UP          .BOUNDS.      x[1]             4     .                .
UP          .BOUNDS.      x[2]             4     .                .
UP          .BOUNDS.      x[3]             4     .                .
UP          .BOUNDS.      x[4]             4     .                .
UP          .BOUNDS.      x[5]             4     .                .
UP          .BOUNDS.      x[6]             4     .                .
UP          .BOUNDS.      x[7]             4     .                .
UP          .BOUNDS.      x[8]             4     .                .
UP          .BOUNDS.      x[9]             4     .                .
UP          .BOUNDS.      x[10]            4     .                .
ENDATA      .             .                .     .                .
;
```

In the COLUMNS section of this data set, the name of the objective is z, and the objective coefficients p_i appear in field4. The coefficients v_i of (volume_con) appear in field6. The coefficients w_i of (weight_con) appear in field4. In the RHS section, the bounds V and W appear in field4.

This problem can be solved by using the following statements to call the OPTMILP procedure:

```
proc optmilp data=ex1data primalout=ex1soln;
run;
```

The progress of the solver is shown in Output 13.1.1.

Output 13.1.1 Simple Integer Linear Program PROC OPTMILP Log

```
NOTE: The problem ex1data has 10 variables (0 binary, 10 integer, 0 free, 0
      fixed).
NOTE: The problem has 2 constraints (2 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 20 constraint coefficients.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 2 variables and 0 constraints.
NOTE: The MILP presolver removed 4 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 8 variables, 2 constraints, and 16 constraint
      coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 2 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      3     85.0000000    158.0000000   46.20%       0
             0        1      3     85.0000000     88.0955497    3.51%       0
             0        1      3     85.0000000     87.4545455    2.81%       0
             0        1      3     85.0000000     87.4545455    2.81%       0
NOTE: The MILP presolver is applied again.
             0        1      4     87.0000000     87.4545455    0.52%       0
             0        0      4     87.0000000     87.0000000    0.00%       0
NOTE: Optimal.
NOTE: Objective = 87.
NOTE: There were 43 observations read from the data set WORK.EX1DATA.
NOTE: The data set WORK.EX1SOLN has 10 observations and 8 variables.
```

The data set ex1soln is shown in Output 13.1.2.

Output 13.1.2 Simple Integer Linear Program Solution

Example 1 Solution Data

Objective Function ID	RHS ID	Variable Name	Variable Type	Objective Coefficient	Lower Bound	Upper Bound	Variable Value
z	.RHS.	x[1]	I	1	0	4	0
z	.RHS.	x[2]	I	2	0	4	0
z	.RHS.	x[3]	I	3	0	4	0
z	.RHS.	x[4]	I	4	0	4	0
z	.RHS.	x[5]	I	5	0	4	0
z	.RHS.	x[6]	I	6	0	4	3
z	.RHS.	x[7]	I	7	0	4	1
z	.RHS.	x[8]	I	8	0	4	4
z	.RHS.	x[9]	I	9	0	4	0
z	.RHS.	x[10]	I	10	0	4	3

The optimal solution is $x_6 = 3, x_7 = 1, x_8 = 4$, and $x_{10} = 3$, with a total value of 87. From this solution, you can compute the total volume used, which is 988 ($\leq V = 1000$); the total weight used is 499 ($\leq W = 500$). The problem summary and solution summary are shown in Output 13.1.3.

Output 13.1.3 Simple Integer Linear Program Summary

The OPTMILP Procedure

Problem Summary	
Problem Name	ex1data
Objective Sense	Maximization
Objective Function	z
RHS	.RHS.
Number of Variables	10
Bounded Above	0
Bounded Below	0
Bounded Above and Below	10
Free	0
Fixed	0
Binary	0
Integer	10
Number of Constraints	2
LE (<=)	2
EQ (=)	0
GE (>=)	0
Range	0
Constraint Coefficients	20

Output 13.1.3 *continued*

Solution Summary	
Solver	MILP
Algorithm	Branch and Cut
Objective Function	z
Solution Status	Optimal
Objective Value	87
Relative Gap	0
Absolute Gap	0
Primal Infeasibility	0
Bound Infeasibility	0
Integer Infeasibility	0
Best Bound	87
Nodes	1
Iterations	14
Presolve Time	0.01
Solution Time	0.09

Example 13.2: MIPLIB Benchmark Instance

The following example illustrates the conversion of a standard MPS-format file into an MPS-format SAS data set. The problem is re-solved several times, each time by using a different control option. For such a small example, it is necessary to disable cuts and heuristics in order to see the computational savings gained by using other options. For larger or more complex examples, the benefits of using the various control options are more pronounced.

The standard set of MILP benchmark cases is called MIPLIB (Bixby et al. 1998, Achterberg, Koch, and Martin 2003) and can be found at http://miplib.zib.de/. The following statement uses the %MPS2SASD macro to convert an example from MIPLIB to a SAS data set:

```
%mps2sasd(mpsfile="bell3a.mps", outdata=mpsdata);
```

The problem can then be solved using PROC OPTMILP on the data set created by the conversion:

```
proc optmilp data=mpsdata allcuts=none heuristics=none logfreq=10000;
run;
```

The resulting log is shown in Output 13.2.1.

Output 13.2.1 MIPLIB PROC OPTMILP Log

```
NOTE: The problem BELL3A has 133 variables (39 binary, 32 integer, 0 free, 0
      fixed).
NOTE: The problem has 123 constraints (123 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 347 constraint coefficients.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 45 variables and 59 constraints.
NOTE: The MILP presolver removed 144 constraint coefficients.
NOTE: The MILP presolver modified 25 constraint coefficients.
NOTE: The presolved problem has 88 variables, 64 constraints, and 203
      constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 2 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      0              .         869515        .       0
            88       86      1         925387         871182    6.22%       0
          1055       95      2         881651         874836    0.78%       0
          1095      106      3         878651         874836    0.44%       0
          1709      298      4         878430         874836    0.41%       0
          1853      339      5         878430         874836    0.41%       0
          5061        1      5         878430         878400    0.00%       0
NOTE: Optimal within relative gap.
NOTE: Objective = 878430.316.
NOTE: There were 475 observations read from the data set WORK.MPSDATA.
```

Suppose you do not have a bound for the solution. If there is an objective value that, even if it is not optimal, satisfies your requirements, then you can save time by using the TARGET= option. The following PROC OPTMILP call solves the problem with a target value of 880,000:

```
proc optmilp data=mpsdata allcuts=none heuristics=none logfreq=5000
             target=880000;
run;
```

The relevant results from this run are displayed in Output 13.2.2. In this case, there is a decrease in CPU time, but the objective value has increased.

Output 13.2.2 MIPLIB PROC OPTMILP Log with TARGET= Option

```
NOTE: The problem BELL3A has 133 variables (39 binary, 32 integer, 0 free, 0
      fixed).
NOTE: The problem has 123 constraints (123 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 347 constraint coefficients.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 45 variables and 59 constraints.
NOTE: The MILP presolver removed 144 constraint coefficients.
NOTE: The MILP presolver modified 25 constraint coefficients.
NOTE: The presolved problem has 88 variables, 64 constraints, and 203
      constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 2 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      0              .         869515        .       0
            88       86      1         925387         871182    6.22%       0
          1055       95      2         881651         874836    0.78%       0
          1095      106      3         878651         874836    0.44%       0
NOTE: Target reached.
NOTE: Objective of the best integer solution found = 878651.068.
NOTE: There were 475 observations read from the data set WORK.MPSDATA.
```

When the objective value of a solution is within a certain relative gap of the optimal objective value, the procedure stops. The acceptable relative gap can be changed using the RELOBJGAP= option, as demonstrated in the following example:

```
proc optmilp data=mpsdata allcuts=none heuristics=none relobjgap=0.01;
run;
```

The relevant results from this run are displayed in Output 13.2.3. In this case, since the specified RELOBJGAP= value is larger than the default value, the number of nodes and the CPU time have decreased from their values in the original run. Note that these savings are exchanged for an increase in the objective value of the solution.

Output 13.2.3 MIPLIB PROC OPTMILP Log with RELOBJGAP= Option

```
NOTE: The problem BELL3A has 133 variables (39 binary, 32 integer, 0 free, 0
      fixed).
NOTE: The problem has 123 constraints (123 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 347 constraint coefficients.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 45 variables and 59 constraints.
NOTE: The MILP presolver removed 144 constraint coefficients.
NOTE: The MILP presolver modified 25 constraint coefficients.
NOTE: The presolved problem has 88 variables, 64 constraints, and 203
      constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 2 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      0              .         869515        .       0
            88       86      1         925387         871182    6.22%       0
          1055       95      2         881651         874836    0.78%       0
NOTE: Optimal within relative gap.
NOTE: Objective = 881650.93.
NOTE: There were 475 observations read from the data set WORK.MPSDATA.
```

The MAXTIME= option enables you to accept the best solution produced by PROC OPTMILP in a specified amount of time. The following example illustrates the use of the MAXTIME= option:

```
proc optmilp data=mpsdata allcuts=none heuristics=none maxtime=0.1;
run;
```

The relevant results from this run are displayed in Output 13.2.4. Once again, a reduction in solution time is traded for an increase in objective value.

Output 13.2.4 MIPLIB PROC OPTMILP Log with MAXTIME= Option

```
NOTE: The problem BELL3A has 133 variables (39 binary, 32 integer, 0 free, 0
      fixed).
NOTE: The problem has 123 constraints (123 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 347 constraint coefficients.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 45 variables and 59 constraints.
NOTE: The MILP presolver removed 144 constraint coefficients.
NOTE: The MILP presolver modified 25 constraint coefficients.
NOTE: The presolved problem has 88 variables, 64 constraints, and 203
      constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 2 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      0              .         869515        .       0
            88       86      1         925387         871182    6.22%       0
            90       87      1         925387         871698    6.16%       0
NOTE: Real time limit reached.
NOTE: Objective of the best integer solution found = 925387.42.
NOTE: There were 475 observations read from the data set WORK.MPSDATA.
```

The MAXNODES= option enables you to limit the number of nodes generated by PROC OPTMILP. The following example illustrates the use of the MAXNODES= option:

```
proc optmilp data=mpsdata allcuts=none heuristics=none maxnodes=1000;
run;
```

The relevant results from this run are displayed in Output 13.2.5. PROC OPTMILP displays the best objective value of all the solutions produced.

Output 13.2.5 MIPLIB PROC OPTMILP Log with MAXNODES= Option

```
NOTE: The problem BELL3A has 133 variables (39 binary, 32 integer, 0 free, 0
      fixed).
NOTE: The problem has 123 constraints (123 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 347 constraint coefficients.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 45 variables and 59 constraints.
NOTE: The MILP presolver removed 144 constraint coefficients.
NOTE: The MILP presolver modified 25 constraint coefficients.
NOTE: The presolved problem has 88 variables, 64 constraints, and 203
      constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 2 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      0              .         869515        .       0
            88       86      1         925387         871182    6.22%       0
           999       72      1         925387         874836    5.78%       0
NOTE: Node limit reached.
NOTE: Objective of the best integer solution found = 925387.42.
NOTE: There were 475 observations read from the data set WORK.MPSDATA.
```

Example 13.3: Facility Location

This advanced example demonstrates how to warm start PROC OPTMILP by using the PRIMALIN= option. The model is constructed in PROC OPTMODEL and saved in an MPS-format SAS data set for use in PROC OPTMILP. This problem can also be solved from within PROC OPTMODEL; see Chapter 8 for details.

Consider the classical facility location problem. Given a set L of customer locations and a set F of candidate facility sites, you must decide on which sites to build facilities and assign coverage of customer demand to these sites so as to minimize cost. All customer demand d_i must be satisfied, and each facility has a demand capacity limit C. The total cost is the sum of the distances c_{ij} between facility j and its assigned customer i, plus a fixed charge f_j for building a facility at site j. Let $y_j = 1$ represent choosing site j to build a facility, and 0 otherwise. Also, let $x_{ij} = 1$ represent the assignment of customer i to facility j, and 0 otherwise. This model can be formulated as the following integer linear program:

$$
\begin{array}{lllll}
\min & \displaystyle\sum_{i\in L}\sum_{j\in F} c_{ij}x_{ij} + \sum_{j\in F} f_j y_j & & & \\
\text{s.t.} & \displaystyle\sum_{j\in F} x_{ij} = 1 & \forall i \in L & \text{(assign\_def)} \\
& x_{ij} \le y_j & \forall i \in L, j \in F & \text{(link)} \\
& \displaystyle\sum_{i\in L} d_i x_{ij} \le C y_j & \forall j \in F & \text{(capacity)} \\
& x_{ij} \in \{0,1\} & \forall i \in L, j \in F & \\
& y_j \in \{0,1\} & \forall j \in F &
\end{array}
$$

Constraint (assign_def) ensures that each customer is assigned to exactly one site. Constraint (link) forces a facility to be built if any customer has been assigned to that facility. Finally, constraint (capacity) enforces the capacity limit at each site.

Consider also a variation of this same problem where there is no cost for building a facility. This problem is typically easier to solve than the original problem. For this variant, let the objective be

$$\min \sum_{i \in L} \sum_{j \in F} c_{ij} x_{ij}$$

First, construct a random instance of this problem by using the following DATA steps:

```
%let NumCustomers  = 50;
%let NumSites      = 10;
%let SiteCapacity  = 35;
%let MaxDemand     = 10;
%let xmax          = 200;
%let ymax          = 100;
%let seed          = 423;

/* generate random customer locations */
data cdata(drop=i);
   call streaminit(&seed);
   length name $8;
   do i = 1 to &NumCustomers;
      name = compress('C'||put(i,best.));
      x = rand('UNIFORM') * &xmax;
      y = rand('UNIFORM') * &ymax;
      demand = rand('UNIFORM') * &MaxDemand;
      output;
   end;
run;

/* generate random site locations and fixed charge */
data sdata(drop=i);
   call streaminit(&seed);
   length name $8;
   do i = 1 to &NumSites;
      name = compress('SITE'||put(i,best.));
      x = rand('UNIFORM') * &xmax;
      y = rand('UNIFORM') * &ymax;
      fixed_charge = 30 * (abs(&xmax/2-x) + abs(&ymax/2-y));
      output;
   end;
run;
```

The following PROC OPTMODEL statements generate the model and define both variants of the cost function:

```
proc optmodel;
   set <str> CUSTOMERS;
   set <str> SITES init {};
   /* x and y coordinates of CUSTOMERS and SITES */
   num x {CUSTOMERS union SITES};
   num y {CUSTOMERS union SITES};
   num demand {CUSTOMERS};
   num fixed_charge {SITES};
   /* distance from customer i to site j */
   num dist {i in CUSTOMERS, j in SITES}
       = sqrt((x[i] - x[j])^2 + (y[i] - y[j])^2);
   read data cdata into CUSTOMERS=[name] x y demand;
   read data sdata into SITES=[name] x y fixed_charge;
   var Assign {CUSTOMERS, SITES} binary;
   var Build {SITES} binary;
   /* each customer assigned to exactly one site */
   con assign_def {i in CUSTOMERS}:
       sum {j in SITES} Assign[i,j] = 1;
   /* if customer i assigned to site j, then facility must be */
   /* built at j                                              */
   con link {i in CUSTOMERS, j in SITES}:
       Assign[i,j] <= Build[j];
   /* each site can handle at most &SiteCapacity demand */
   con capacity {j in SITES}:
       sum {i in CUSTOMERS} demand[i] * Assign[i,j]
       <= &SiteCapacity * Build[j];
   min CostNoFixedCharge
       = sum {i in CUSTOMERS, j in SITES} dist[i,j] * Assign[i,j];
   save mps nofcdata;
   min CostFixedCharge
       = CostNoFixedCharge
         + sum {j in SITES} fixed_charge[j] * Build[j];
   save mps fcdata;
quit;
```

First solve the problem for the model with no fixed charge by using the following statements. The first PROC SQL call populates the macro variables varcostNo. This macro variable displays the objective value when the results are plotted. The second PROC SQL call generates a data set that is used to plot the results. The information printed in the log by PROC OPTMILP is displayed in Output 13.3.1.

```
proc optmilp data=nofcdata primalout=nofcout;
run;
proc sql noprint;
   select put(sum(_objcoef_ * _value_),6.1) into :varcostNo
   from nofcout;
quit;

proc sql;
   create table CostNoFixedCharge_Data as
   select
      scan(p._var_,2,'[],') as customer,
      scan(p._var_,3,'[],') as site,
      c.x as x1, c.y as y1, s.x as x2, s.y as y2
   from
```

```
      cdata as c,
      sdata as s,
      nofcout(where=(substr(_var_,1,6)='Assign' and
         round(_value_) = 1)) as p
   where calculated customer = c.name and calculated site = s.name;
quit;
```

Output 13.3.1 PROC OPTMILP Log for Facility Location with No Fixed Charges

```
NOTE: The problem nofcdata has 510 variables (510 binary, 0 integer, 0 free, 0
      fixed).
NOTE: The problem has 560 constraints (510 LE, 50 EQ, 0 GE, 0 range).
NOTE: The problem has 2010 constraint coefficients.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 10 variables and 500 constraints.
NOTE: The MILP presolver removed 1010 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 500 variables, 60 constraints, and 1000
      constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 2 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      2   1331.1324031              0   1331.1       0
             0        1      2   1331.1324031   1177.1539196   13.08%       0
             0        1      2   1331.1324031   1189.9519987   11.86%       0
             0        1      3   1192.6273240   1192.6252526    0.00%       0
NOTE: The MILP solver added 6 cuts with 179 cut coefficients at the root.
NOTE: Optimal within relative gap.
NOTE: Objective = 1192.627324.
NOTE: There were 2389 observations read from the data set WORK.NOFCDATA.
NOTE: The data set WORK.NOFCOUT has 510 observations and 8 variables.
```

Next, solve the fixed-charge model by using the following statements. Note that the solution to the model with no fixed charge is feasible for the fixed-charge model and should provide a good starting point for PROC OPTMILP. The PRIMALIN= option provides an incumbent solution ("warm start"). The two PROC SQL calls perform the same functions as in the case with no fixed charges. The results from this approach are shown in Output 13.3.2.

```
proc optmilp data=fcdata primalin=nofcout primalout=fcout;
run;
proc sql noprint;
   select put(sum(_objcoef_ * _value_), 6.1) into :varcost
   from fcout(where=(substr(_var_,1,6)='Assign'));
   select put(sum(_objcoef_ * _value_), 5.1) into :fixcost
   from fcout(where=(substr(_var_,1,5)='Build'));
   select put(sum(_objcoef_ * _value_), 6.1) into :totalcost
   from fcout;
quit;
proc sql;
   create table CostFixedCharge_Data as
   select
      scan(p._var_,2,'[],') as customer,
      scan(p._var_,3,'[],') as site,
      c.x as x1, c.y as y1, s.x as x2, s.y as y2
   from
      cdata as c,
      sdata as s,
      fcout(where=(substr(_var_,1,6)='Assign' and
         round(_value_) = 1)) as p
   where calculated customer = c.name and calculated site = s.name;
quit;
```

Output 13.3.2 PROC OPTMILP Log for Facility Location with Fixed Charges, Using Warm Start

```
NOTE: The problem fcdata has 510 variables (510 binary, 0 integer, 0 free, 0
      fixed).
NOTE: The problem has 560 constraints (510 LE, 50 EQ, 0 GE, 0 range).
NOTE: The problem has 2010 constraint coefficients.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 0 constraints.
NOTE: The MILP presolver removed 0 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 510 variables, 560 constraints, and 2010
      constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 2 threads.
          Node   Active   Sols    BestInteger      BestBound      Gap    Time
             0        1      3  24086.8916716              0    24087       0
             0        1      3  24086.8916716  19197.7909681   25.47%       0
             0        1      3  24086.8916716  19204.4310169   25.42%       0
             0        1      3  24086.8916716  19209.7654194   25.39%       0
             0        1      3  24086.8916716  19216.6357753   25.34%       0
             0        1      3  24086.8916716  19222.1729500   25.31%       0
             0        1      5  21638.2071053  19224.9103955   12.55%       0
             0        1      5  21638.2071053  19225.8681982   12.55%       0
             0        1      5  21638.2071053  19227.0274850   12.54%       0
             0        1      7  21552.3564314  19229.2654855   12.08%       0
NOTE: The MILP solver added 24 cuts with 898 cut coefficients at the root.
           121        6      8  21550.6969404  21535.4344018    0.07%       2
           201       41      9  21550.2574461  21538.1436015    0.06%       2
           214       45     10  21550.0038126  21538.9052973    0.05%       2
           310       59     11  21549.5715653  21540.7667184    0.04%       2
           314       59     12  21548.1764622  21540.7667184    0.03%       2
           422       10     12  21548.1764622  21546.3220713    0.01%       2
NOTE: Optimal within relative gap.
NOTE: Objective = 21548.176462.
NOTE: There were 2389 observations read from the data set WORK.FCDATA.
NOTE: There were 510 observations read from the data set WORK.NOFCOUT.
NOTE: The data set WORK.FCOUT has 510 observations and 8 variables.
```

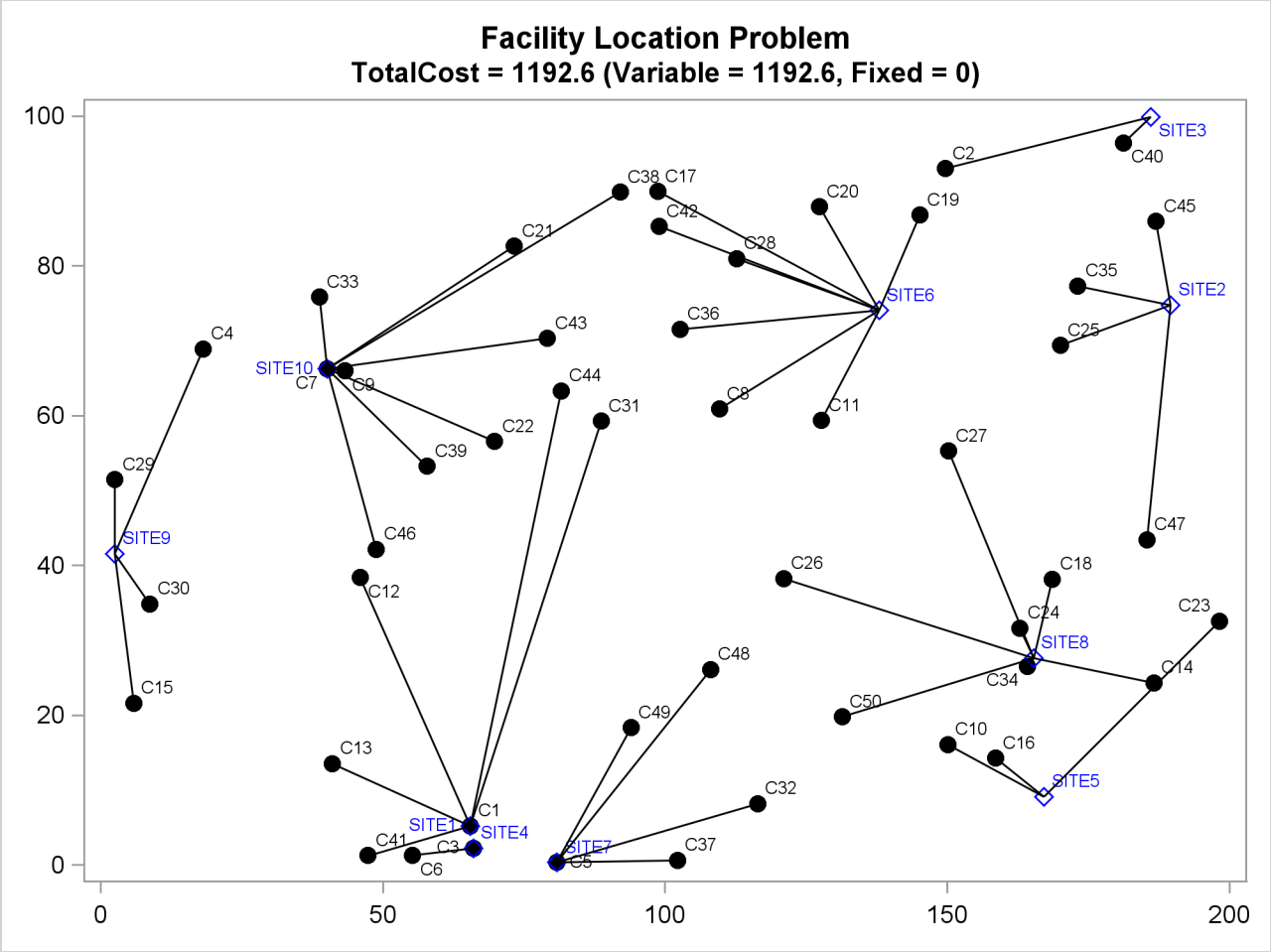
The following two SAS programs produce a plot of the solutions for both variants of the model, using data sets produced by PROC SQL from the PRIMALOUT= data sets produced by PROC OPTMILP.

NOTE: Execution of this code requires SAS/GRAPH software.

```
title1 "Facility Location Problem";
title2 "TotalCost = &varcostNo (Variable = &varcostNo, Fixed = 0)";
data csdata;
   set cdata(rename=(y=cy)) sdata(rename=(y=sy));
run;
/* create Annotate data set to draw line between customer and */
/* assigned site                                              */
data anno;
   retain function "line" drawspace "datavalue"
      linethickness 1 linecolor "black";
   set CostNoFixedCharge_Data(keep=x1 y1 x2 y2);
run;
proc sgplot data=csdata sganno=anno noautolegend;
   scatter x=x y=cy / datalabel=name datalabelattrs=(size=6pt)
      markerattrs=(symbol=circlefilled color=black size=6pt);
   scatter x=x y=sy / datalabel=name datalabelattrs=(size=6pt)
      markerattrs=(symbol=diamond color=blue size=6pt);
   xaxis display=(nolabel);
   yaxis display=(nolabel);
run;
quit;
```

The output from the first program appears in Output 13.3.3.

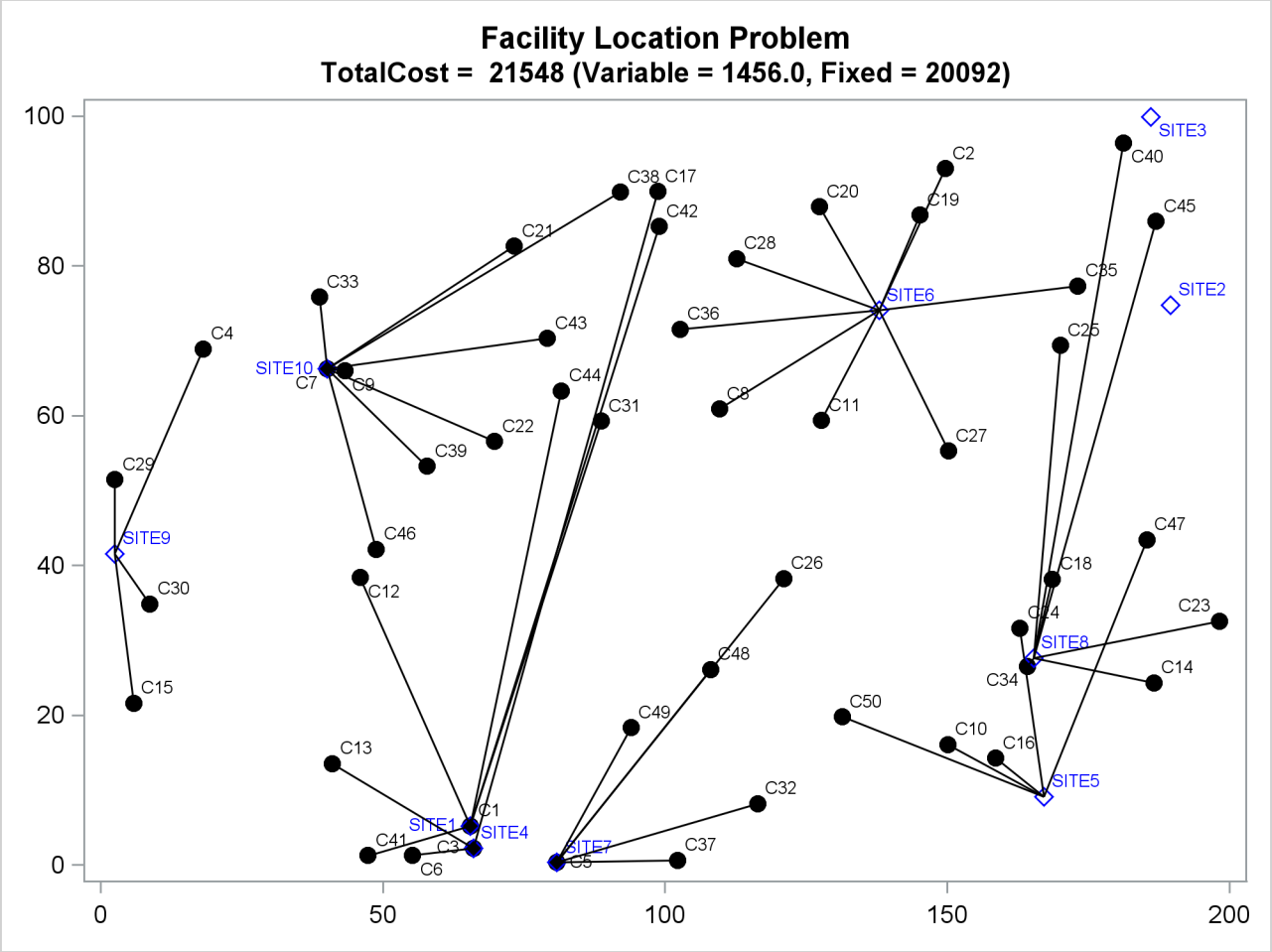
Output 13.3.3 Solution Plot for Facility Location with No Fixed Charges




```
title1 "Facility Location Problem";
title2 "TotalCost = &totalcost (Variable = &varcost, Fixed = &fixcost)";
/* create Annotate data set to draw line between customer and */
/* assigned site                                              */
data anno;
   retain function "line" drawspace "datavalue"
      linethickness 1 linecolor "black";
   set CostFixedCharge_Data(keep=x1 y1 x2 y2);
run;
proc sgplot data=csdata sganno=anno noautolegend;
   scatter x=x y=cy / datalabel=name datalabelattrs=(size=6pt)
      markerattrs=(symbol=circlefilled color=black size=6pt);
   scatter x=x y=sy / datalabel=name datalabelattrs=(size=6pt)
      markerattrs=(symbol=diamond color=blue size=6pt);
   xaxis display=(nolabel);
   yaxis display=(nolabel);
run;
quit;
```

The output from the second program appears in Output 13.3.4.

Output 13.3.4 Solution Plot for Facility Location with Fixed Charges



The economic tradeoff for the fixed-charge model forces you to build fewer sites and push more demand to each site.

Example 13.4: Scheduling

This example is intended for users who prefer to use the SAS DATA step, PROC SQL, and similar programming methods to prepare data for input to SAS/OR optimization procedures. SAS/OR users who prefer to use the algebraic modeling capabilities of PROC OPTMODEL to specify optimization models should consult Example 8.1 in Chapter 8, "The Mixed Integer Linear Programming Solver," for a discussion of the same business problem in a PROC OPTMODEL context.

Scheduling is an application area where techniques in model generation can be valuable. Problems that involve scheduling are often solved with integer programming and are similar to assignment problems. In this example, you have eight one-hour time slots in each of five days. You have to assign four people to these time slots so that each slot is covered every day. You allow the people to specify preference data for each slot on each day. In addition, there are constraints that must be satisfied:

- Each person has some slots for which they are unavailable.

- Each person must have either slot 4 or 5 off for lunch.
- Each person can work only two time slots in a row.
- Each person can work only a specified number of hours in the week.

To formulate this problem, let i denote person, j denote time slot, and k denote day. Then, let $x_{ijk} = 1$ if person i is assigned to time slot j on day k, and 0 otherwise; let p_{ijk} denote the preference of person i for slot j on day k; and let h_i denote the number of hours in a week that person i will work. Then, you get

$$
\begin{array}{lll}
\max & \sum_{ijk} p_{ijk} x_{ijk} & \\
\text{subject to} & \sum_{i} x_{ijk} = 1 & \text{for all } j \text{ and } k \\
& x_{i4k} + x_{i5k} \leq 1 & \text{for all } i \text{ and } k \\
& x_{i,\ell,k} + x_{i,\ell+1,k} + x_{i,\ell+2,k} \leq 2 & \text{for all } i \text{ and } k, \text{ and } \ell = 1, \ldots, 6 \\
& \sum_{jk} x_{ijk} \leq h_i & \text{for all } i \\
& x_{ijk} = 0 \text{ or } 1 & \text{for all } i \text{ and } k \text{ such that } p_{ijk} > 0, \\
& & \quad \text{otherwise } x_{ijk} = 0
\end{array}
$$

To solve this problem, create a data set that has the hours and preference data for each individual, time slot, and day. A 10 represents the most desirable time slot, and a 1 represents the least desirable time slot. In addition, a 0 indicates that the time slot is not available.

```
data raw;
   input name $ hour slot mon tue wed thu fri;
   datalines;
marc  20   1    10 10 10 10 10
marc  20   2     9  9  9  9  9
marc  20   3     8  8  8  8  8
marc  20   4     1  1  1  1  1
marc  20   5     1  1  1  1  1
marc  20   6     1  1  1  1  1
marc  20   7     1  1  1  1  1
marc  20   8     1  1  1  1  1
mike  20   1    10  9  8  7  6
mike  20   2    10  9  8  7  6
mike  20   3    10  9  8  7  6
mike  20   4    10  3  3  3  3
mike  20   5     1  1  1  1  1
mike  20   6     1  2  3  4  5
mike  20   7     1  2  3  4  5
mike  20   8     1  2  3  4  5
bill  20   1    10 10 10 10 10
bill  20   2     9  9  9  9  9
bill  20   3     8  8  8  8  8
bill  20   4     0  0  0  0  0
bill  20   5     1  1  1  1  1
bill  20   6     1  1  1  1  1
bill  20   7     1  1  1  1  1
bill  20   8     1  1  1  1  1
bob   20   1    10  9  8  7  6
bob   20   2    10  9  8  7  6
bob   20   3    10  9  8  7  6
```

```
bob   20   4    10  3  3  3  3
bob   20   5     1  1  1  1  1
bob   20   6     1  2  3  4  5
bob   20   7     1  2  3  4  5
bob   20   8     1  2  3  4  5
;
```

These data are read by the following DATA step, and an integer program is built to solve the problem. The model is saved in the data set named MODEL, which is constructed in the following steps:

1. The objective function is built using the data saved in the RAW data set.
2. The constraints that ensure that no one works during a time slot during which they are unavailable are built.
3. The constraints that require a person to be working in each time slot are built.
4. The constraints that allow each person time for lunch are added.
5. The constraints that restrict people to only two consecutive hours are added.
6. The constraints that limit the time that any one person works in a week are added.
7. The constraints that allow a person to be assigned only to a time slot for which he is available are added.

The statements to build each of these constraints follow the formulation closely.

```
data model;
   array workweek{5} mon tue wed thu fri;
   array hours{4} hours1 hours2 hours3 hours4;
   retain hours1-hours4;

   set raw end=eof;

   length _row_ $ 8 _col_ $ 8 _type_ $ 8;
   keep _type_ _col_ _row_ _coef_;

   if      name='marc' then i=1;
   else if name='mike' then i=2;
   else if name='bill' then i=3;
   else if name='bob'  then i=4;

   hours{i}=hour;

   /* build the objective function */

   do k=1 to 5;
      _col_='x'||put(i,1.)||put(slot,1.)||put(k,1.);

      _row_='object';
      _coef_=workweek{k} * 1000;
      output;
   end;

   /* build the rest of the model */
```

```
/* cannot work during unavailable slots */
do k=1 to 5;
   if workweek{k}=0 then do;
      _row_='off'||put(i,1.)||put(slot,1.)||put(k,1.);
      _type_='eq';
      _col_='_RHS_';
      _coef_=0;
     output;
      _col_='x'||put(i,1.)||put(slot,1.)||put(k,1.);
     _coef_=1;
     _type_=' ';
     output;
   end;
end;

if eof then do;
   _coef_=.;
   _col_=' ';

   /* every hour 1 person working */
   do j=1 to 8;
      do k=1 to 5;
         _row_='work'||put(j,1.)||put(k,1.);
         _type_='eq';
         _col_='_RHS_';
         _coef_=1;
         output;
         _coef_=1;
         _type_=' ';
         do i=1 to 4;
            _col_='x'||put(i,1.)||put(j,1.)||put(k,1.);
            output;
         end;
      end;
   end;

   /* each person has a lunch */
   do i=1 to 4;
      do k=1 to 5;
         _row_='lunch'||put(i,1.)||put(k,1.);
         _type_='le';
         _col_='_RHS_';
         _coef_=1;
         output;
         _coef_=1;
         _type_=' ';
         _col_='x'||put(i,1.)||'4'||put(k,1.);
         output;
         _col_='x'||put(i,1.)||'5'||put(k,1.);
         output;
      end;
   end;
```

```
      /* work at most 2 slots in a row */
      do i=1 to 4;
         do k=1 to 5;
            do l=1 to 6;
            _row_='seq'||put(i,1.)||put(k,1.)||put(l,1.);
            _type_='le';
            _col_='_RHS_';
            _coef_=2;
             output;
            _coef_=1;
            _type_=' ';
               do j=0 to 2;
                  _col_='x'||put(i,1.)||put(l+j,1.)||put(k,1.);
                  output;
               end;
            end;
         end;
      end;

      /* work at most n hours in a week */
      do i=1 to 4;
         _row_='capacit'||put(i,1.);
         _type_='le';
         _col_='_RHS_';
         _coef_=hours{i};
         output;
         _coef_=1;
         _type_=' ';
         do j=1 to 8;
            do k=1 to 5;
               _col_='x'||put(i,1.)||put(j,1.)||put(k,1.);
               output;
            end;
         end;
      end;
   end;
run;
```

Next, this SAS data set is converted to an MPS-format SAS data set by establishing the structure of the MPS format and through very minor conversions of the data.

```
/* the following code transforms the above sparse data set */
/* into an MPS-format data set                              */

/* generate the header of the MPS-format data set */
data mps0;
   format field1 field2 field3 $10.;
   format field4 10.;
   format field5 $10.;
   format field6 10.;
   field1 = 'NAME';
   field2 = '       ';
   field3 = 'PROBLEM';
   field4 = .;
```

```
   field5 = '       ';
   field6 = .;
   output;
   field1 = 'ROWS';
   field3 = '';
   output;
   field1 = 'MAX';
   field2 = 'object';
   field3 = '';
   output;
run;

/* generate rows */
proc sql;
   create table mps1 as
      select _type_ as field1, _row_ as field2 from model
         where _row_ eq 'object' and _type_ ne '' union
      select 'E' as field1, _row_ as field2 from model
         where _type_ eq 'eq' union
      select 'L' as field1, _row_ as field2 from model
         where _type_ eq 'le' union
      select 'G' as field1, _row_ as field2 from model
         where _type_ eq 'ge';
quit;

/* indicate start of columns section and declare type of all */
/* variables as integer                                      */
data mps2;
   format field1 field2 field3  $10.;
   format field4 10.;
   format field5 $10.;
   format field6 10.;
   field1 = 'COLUMNS';
   field2 = '         ';
   field3 = '         ';
   field4 = .;
   field5 = '         ';
   field6 = .;
   output;
   field1 = '         ';
   field2 = '.MARK0';
   field3 = "'MARKER'";
   field4 = .;
   field5 = "'INTORG'";
   field6 = .;
   output;
run;

/* generate columns */
data mps3;
   set model;
   format field1 field2 field3 $10.;
   format field4 10.;
```

```
   format field5 $10.;
   format field6 10.;
   keep field1-field6;
   field1 = '      ';
   field2 = _col_;
   field3 = _row_;
   field4 = _coef_;
   field5 = '      ';
   field6 = .;
   if field2 ne '_RHS_'  then do;
      output;
   end;
run;

/* sort columns by variable names */
proc sort data=mps3;
   by field2;
run;

/* indicate the end of the columns section */
data mps4;
   format field1 field2 field3  $10.;
   format field4 10.;
   format field5 $10.;
   format field6 10.;
   field1 = '         ';
   field2 = '.MARK1';
   field3 = "'MARKER'";
   field4 = .;
   field5 = "'INTEND'";
   field6 = .;
   output;
run;

/* indicate the start of the RHS section */
data mps5;
   format field1 field2 field3  $10.;
   format field4 10.;
   format field5 $10.;
   format field6 10.;
   field1 = 'RHS';
run;

/* generate RHS entries */
data mps6;
   set model;
   format field1 field2 field3 $10.;
   format field4 10.;
   format field5 $10.;
   format field6 10.;
   keep field1-field6;
   field1 = '      ';
   field2 = _col_;
   field3 = _row_;
```

```
   field4 = _coef_;
   field5 = '       ';
   field6 = .;
   if field2 eq '_RHS_'  then do;
      output;
   end;
run;

/* denote the end of the MPS-format data set */
data mps7;
   format field1 field2 field3  $10.;
   format field4 10.;
   format field5 $10.;
   format field6 10.;
   field1 = 'ENDATA';
run;

/* merge all sections of the MPS-format data set */
data mps;
   format field1 field2 field3 $10.;
   format field4 10.;
   format field5 $10.;
   format field6 10.;
   set mps0 mps1 mps2 mps3 mps4 mps5 mps6 mps7;
run;
```

The model is solved using the OPTMILP procedure. The option PRIMALOUT=SOLUTION causes PROC OPTMILP to save the primal solution in the data set named SOLUTION.

```
/* solve the binary program */
proc optmilp data=mps
   printlevel=0 loglevel=0
   primalout=solution maxtime=1000;
run;
```

The following DATA step takes the solution data set SOLUTION and generates a report data set named ASSIGNMENTS. It restores the original interpretation (person, shift, day) of the variable names x_{ijk} so that a more meaningful report can be written. Then a DATA step and PROC PRINT are used to display a schedule that shows how the eight time slots are covered for the week.

```
/* report the solution */
title 'Reported Solution';

data assignments;
   set solution;
   keep slot mon tue wed thu fri;
   if substr(_var_,1,1)='x' then do;
      if _value_>0 then do;
         n=input(substr(_var_,2,1), 8.);
         slot=input(substr(_var_,3,1), 8.);
         d=substr(_var_,4,1);
         if      d='1' then mon=n;
         else if d='2' then tue=n;
         else if d='3' then wed=n;
         else if d='4' then thu=n;
         else               fri=n;
         output;
      end;
   end;
run;

proc format;
   value namefmt 1='Marc' 2='Mike' 3='Bill' 4='Bob' .=' ';
run;

proc sort data=assignments;
   by slot;
run;

data report;
   do until (last.slot);
      set assignments;
      by slot;
      if mon ne . then Monday = mon;
      if tue ne . then Tuesday = tue;
      if wed ne . then Wednesday = wed;
      if thu ne . then Thursday = thu;
      if fri ne . then Friday = fri;
   end;
   drop mon tue wed thu fri;
   format Monday Tuesday Wednesday Thursday Friday namefmt.;
run;

proc print data=report;
   id slot;
run;
```

Output 13.4.1 from PROC PRINT summarizes the schedule. Notice that the constraint that requires a person to be assigned to each possible time slot on each day is satisfied.

Output 13.4.1 A Scheduling Problem

Reported Solution

slot	Monday	Tuesday	Wednesday	Thursday	Friday
1	Mike	Bill	Marc	Bill	Bill
2	Mike	Mike	Bill	Bill	Bill
3	Bob	Bob	Marc	Marc	Marc
4	Mike	Mike	Mike	Mike	Mike
5	Marc	Marc	Marc	Marc	Marc
6	Mike	Mike	Mike	Mike	Mike
7	Marc	Bob	Bob	Mike	Bob
8	Marc	Mike	Bob	Bob	Bob

Recall that PROC OPTMILP puts a character string in the macro variable _OROPTMILP_ that describes the characteristics of the solution on termination. This string can be parsed using macro functions, and the information obtained can be used in report writing. The variable can be written to the log with the following command:

```
%put &_OROPTMILP_;
```

This command produces the output shown in Output 13.4.2.

Output 13.4.2 _OROPTMILP_ Macro Variable

```
STATUS=OK ALGORITHM=BAC SOLUTION_STATUS=OPTIMAL OBJECTIVE=211000 RELATIVE_GAP=0
ABSOLUTE_GAP=0 PRIMAL_INFEASIBILITY=0 BOUND_INFEASIBILITY=0
INTEGER_INFEASIBILITY=0 BEST_BOUND=211000 NODES=1 ITERATIONS=63
PRESOLVE_TIME=0.02 SOLUTION_TIME=0.06
```

From this output you learn, for example, that at termination the solution is integer-optimal and has an objective value of 211,000.

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Subject Index

Syntax Index