SAS/ETS® 14.1 User's Guide

The HPCOUNTREG Procedure

This document is an individual chapter from *SAS/ETS® 14.1 User's Guide*.

The correct bibliographic citation for this manual is as follows: SAS Institute Inc. 2015. *SAS/ETS® 14.1 User's Guide*. Cary, NC: SAS Institute Inc.

SAS/ETS® 14.1 User's Guide

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July 2015

Chapter 20

The HPCOUNTREG Procedure

Contents

Overview: HPCOUNTREG Procedure

The HPCOUNTREG procedure is a high-performance version of the COUNTREG procedure in SAS/ETS software. Like the COUNTREG procedure, the HPCOUNTREG procedure fits regression models in which the dependent variable takes on nonnegative integer or count values. Unlike the COUNTREG procedure, which can be run only on an individual workstation, the HPCOUNTREG procedure takes advantage of a computing environment that enables it to distribute the optimization task among one or more nodes. In addition, each node can use one or more threads to carry out the optimization on its subset of the data. When several nodes are employed, with each node using several threads to carry out its part of the work, the result is a highly parallel computation that provides a dramatic gain in performance.

The HPCOUNTREG procedure enables you to read and write data in distributed form and perform analyses in distributed mode and single-machine mode. For information about how to affect the execution mode of SAS high-performance analytical procedures, see the section "Processing Modes" on page 62 in Chapter 3, "Shared Concepts and Topics."

The HPCOUNTREG procedure is specifically designed to operate in the high-performance distributed environment. By default, PROC HPCOUNTREG performs computations in multiple threads.

PROC HPCOUNTREG Features

The HPCOUNTREG procedure estimates the parameters of a count regression model by maximum likelihood techniques.

The HPCOUNTREG procedure supports the following models for count data:

- Poisson regression
- Conway-Maxwell-Poisson regression
- negative binomial regression with quadratic and linear variance functions (Cameron and Trivedi 1986)
- zero-inflated Poisson (ZIP) model (Lambert 1992)
- zero-inflated Conway-Maxwell-Poisson (ZICMP) model
- zero-inflated negative binomial (ZINB) model
- fixed-effects and random-effects Poisson models for panel data
- fixed-effects and random-effects negative binomial models for panel data

The following list summarizes some basic features of the HPCOUNTREG procedure:

- can perform analysis on a massively parallel high-performance appliance
- reads input data in parallel and writes output data in parallel when the data source is the appliance database
- is highly multithreaded during all phases of analytic execution
- has model-building syntax that uses CLASS and effect-based MODEL statements familiar from SAS/ETS analytic procedures
- performs maximum likelihood estimation
- supports multiple link functions
- uses the WEIGHT statement for weighted analysis
- uses the FREQ statement for grouped analysis
- uses the OUTPUT statement to produce a data set that contains predicted probabilities and other observationwise statistics

Getting Started: HPCOUNTREG Procedure

Except for its ability to operate in the high-performance distributed environment, the HPCOUNTREG procedure is similar in use to other regression model procedures in the SAS System. For example, the following statements are used to estimate a Poisson regression model:

```
proc hpcountreg data=one ;
   model y = x / dist=poisson ;
run;
```

The response variable *y* is numeric and has nonnegative integer values.

This section illustrates two simple examples that use PROC HPCOUNTREG. The data are taken from Long (1997). This study examines how factors such as gender (fem), marital status (mar), number of young children (kid5), prestige of the graduate program (phd), and number of articles published by a scientist's mentor (ment) affect the number of articles (art) published by the scientist.

The first 10 observations are shown in Figure 20.1.

Figure 20.1 Article Count Data

Obs	art	fem	mar	kid5	phd	ment
1	3	0	1	2	1.38000	8.0000
2	0	0	0	0	4.29000	7.0000
3	4	0	0	0	3.85000	47.0000
4	1	0	1	1	3.59000	19.0000
5	1	0	1	0	1.81000	0.0000
6	1	0	1	1	3.59000	6.0000
7	0	0	1	1	2.12000	10.0000
8	0	0	1	0	4.29000	2.0000
9	3	0	1	2	2.58000	2.0000
10	3	0	1	1	1.80000	4.0000

The following SAS statements estimate the Poisson regression model. The model is executed in the distributed computing environment with two threads and four nodes.

```
/*-- Poisson Regression --*/
proc hpcountreg data=long97data;
   model art = fem mar kid5 phd ment / dist=poisson method=quanew;
   performance nthreads=2 nodes=4 details;
run;
```

The "Model Fit Summary" table that is shown in Figure 20.2 lists several details about the model. By default, the HPCOUNTREG procedure uses the Newton-Raphson optimization technique. The maximum log-likelihood value is shown, in addition to two information measures—Akaike's information criterion (AIC) and Schwarz's Bayesian information criterion (SBC)—which can be used to compare competing Poisson models. Smaller values of these criteria indicate better models.

Figure 20.2 Estimation Summary Table for a Poisson Regression

The HPCOUNTREG Procedure

Model Fit Summary	
Dependent Variable	art
Number of Observations	915
Data Set	WORK.LONG97DATA
Model	Poisson
Log Likelihood	-1651
Maximum Absolute Gradient	0.0002080
Number of Iterations	13
Optimization Method	Quasi-Newton
AIC	3314
SBC	3343

Figure 20.3 shows the parameter estimates of the model and their standard errors. All covariates are significant predictors of the number of articles, except for the prestige of the program (phd), which has a *p*-value of 0.6271.

Figure 20.3 Parameter Estimates of Poisson Regression

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.3046	0.1030	2.96	0.0031
fem	1	-0.2246	0.05461	-4.11	<.0001
mar	1	0.1552	0.06137	2.53	0.0114
kid5	1	-0.1849	0.04013	-4.61	<.0001
phd	1	0.01282	0.02640	0.49	0.6271
ment	1	0.02554	0.002006	12.73	<.0001

To allow for variance greater than the mean, you can fit the negative binomial model instead of the Poisson model by specifying the DIST=NEGBIN option, as shown in the following statements. Whereas the Poisson model requires that the conditional mean and conditional variance be equal, the negative binomial model allows for overdispersion, in which the conditional variance can exceed the conditional mean.

```
/*-- Negative Binomial Regression --*/
proc hpcountreg data=long97data;
   model art = fem mar kid5 phd ment / dist=negbin(p=2) method=quanew;
   performance nthreads=2 nodes=4 details;
run;
```

Figure 20.4 shows the fit summary and Figure 20.5 shows the parameter estimates.

Figure 20.4 Estimation Summary Table for a Negative Binomial Regression

The HPCOUNTREG Procedure

Model Fit Summary

Dependent Variable	art
Number of Observations	915
Data Set	WORK.LONG97DATA
Model	NegBin
Log Likelihood	-1561
Maximum Absolute Gradient	0.0000666
Number of Iterations	16
Optimization Method	Quasi-Newton
AIC	3136
SBC	3170

Figure 20.5 Parameter Estimates of Negative Binomial Regression

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.2561	0.1386	1.85	0.0645
fem	1	-0.2164	0.07267	-2.98	0.0029
mar	1	0.1505	0.08211	1.83	0.0668
kid5	1	-0.1764	0.05306	-3.32	0.0009
phd	1	0.01527	0.03604	0.42	0.6718
ment	1	0.02908	0.003470	8.38	<.0001
_Alpha	1	0.4416	0.05297	8.34	<.0001

The parameter estimate for _Alpha of *0.4416* is an estimate of the dispersion parameter in the negative binomial distribution. A t test for the hypothesis $H_0 : \alpha = 0$ is provided. It is highly significant, indicating overdispersion ($p < 0.0001$).

The null hypothesis $H_0 : \alpha = 0$ can be also tested against the alternative $\alpha > 0$ by using the likelihood ratio test, as described by Cameron and Trivedi (1998, pp. 45, 77–78). The likelihood ratio test statistic is equal to $-2(\mathcal{L}_P - \mathcal{L}_{NB}) = -2(-1651 + 1561) = 180$, which is highly significant, providing strong evidence of overdispersion.

Syntax: HPCOUNTREG Procedure

The following statements are available in the HPCOUNTREG procedure. Items within angle brackets (< >) or square brackets ([ ]) are optional.

PROC HPCOUNTREG <*options*> ;
  BOUNDS *bound1* [, *bound2* . . .] ;
  BY *variables* ;
  CLASS *variables* ;
  DISPMODEL *dependent variable* ∼ < *dispersion-related regressors* > ;
  FREQ *freq-variable* ;
  INIT *initialization1* < , *initialization2* . . . > ;
  MODEL *dependent-variable* = *regressors* </ *options*> ;
  OUTPUT <*output-options*> ;
  PERFORMANCE *performance-options* ;
  RESTRICT *restriction1* [, *restriction2* . . .] ;
  TEST *equation1* < , *equation2*. . . > / < *test-options* > ;
  WEIGHT *variable* </ *option*> ;
  ZEROMODEL *dependent-variable* ∼ *zero-inflated-regressors* </ *options*> ;

There can be only one MODEL statement. The ZEROMODEL statement, if used, must appear after the MODEL statement. If a FREQ or WEIGHT statement is specified more than once, the variable specified in the first instance is used.

Functional Summary

Table 20.1 summarizes the statements and options used with the HPCOUNTREG procedure.

Table 20.1 PROC HPCOUNTREG Functional Summary

Description	Statement	Option
Data Set Options		
Specifies the input data set	HPCOUNTREG	DATA=
Specifies the identification variable for panel data analysis	HPCOUNTREG	GROUPID=
Writes parameter estimates to an output data set	HPCOUNTREG	OUTEST=
Writes estimates to an output data set	OUTPUT	OUT=
Specifies BY-group processing	BY	
Specifies an optional frequency variable	FREQ	
Specifies an optional weight variable	WEIGHT	
Printing Control Options		
Prints the correlation matrix of the estimates	HPCOUNTREG	CORRB
Prints the covariance matrix of the estimates	HPCOUNTREG	COVB
Suppresses the normal printed output	HPCOUNTREG	NOPRINT
Requests all printing options	HPCOUNTREG	PRINTALL
Options to Control the Optimization Process		
Specifies maximum number of iterations allowed	HPCOUNTREG	MAXITER=
Selects the iterative minimization method to use	HPCOUNTREG	METHOD=
Specifies maximum number of iterations allowed	HPCOUNTREG	MAXITER=
Specifies maximum number of function calls	HPCOUNTREG	MAXFUNC=
Specifies the upper limit of CPU time in seconds	HPCOUNTREG	MAXTIME=
Specifies absolute function convergence criterion	HPCOUNTREG	ABSCONV=
Specifies absolute function convergence criterion	HPCOUNTREG	ABSFCONV=
Specifies absolute gradient convergence criterion	HPCOUNTREG	ABSGCONV=
Specifies relative function convergence criterion	HPCOUNTREG	FCONV=
Specifies relative gradient convergence criterion	HPCOUNTREG	GCONV=
Specifies absolute parameter convergence criterion	HPCOUNTREG	ABSXCONV=
Specifies matrix singularity criterion	HPCOUNTREG	SINGULAR=
Sets boundary restrictions on parameters	BOUNDS	
Sets initial values for parameters	INIT	
Sets linear restrictions on parameters	RESTRICT	
Model Estimation Options		
Specifies the dispersion variables	DISPMODEL	
Specifies the type of model	HPCOUNTREG	DIST=
Specifies the type of covariance matrix	HPCOUNTREG	COVEST=
Specifies the type of error components model for panel data	MODEL	ERRORCOMP=

Description	Statement	Option
Suppresses the intercept parameter	MODEL	NOINT
Specifies the offset variable	MODEL	OFFSET=
Specifies the parameterization for the Conway-Maxwell-Poisson (CMP) model	MODEL	PARAMETER=
Specifies the zero-inflated offset variable	ZEROMODEL	OFFSET=
Specifies the zero-inflated link function	ZEROMODEL	LINK=
Output Control Options		
Includes covariances in the OUTEST= data set	HPCOUNTREG	COVOUT
Includes correlations in the OUTEST= data set	HPCOUNTREG	CORROUT
Outputs SAS variables to the output data set	OUTPUT	COPYVAR=
Outputs the estimates of dispersion for the CMP model	OUTPUT	DISPERSION
Outputs the estimates of GDelta $= \mathbf{g}_i'\boldsymbol{\delta}$ for the CMP model	OUTPUT	GDELTA=
Outputs the estimates of λ for the CMP model	OUTPUT	LAMBDA=
Outputs the estimates of ν for the CMP model	OUTPUT	NU=
Outputs the estimates of μ for the CMP model	OUTPUT	MU=
Outputs the estimates of mode for the CMP model	OUTPUT	MODE=
Outputs the probability that the response variable will take the current value	OUTPUT	PROB=
Outputs probabilities for particular response values	OUTPUT	PROBCOUNT( )
Outputs expected value of response variable	OUTPUT	PRED=
Outputs the estimates of variance for the CMP model	OUTPUT	VARIANCE=
Outputs estimates of XBeta $= \mathbf{x}_i'\boldsymbol{\beta}$	OUTPUT	XBETA=
Outputs estimates of ZGamma $= \mathbf{z}_i'\boldsymbol{\gamma}$	OUTPUT	ZGAMMA=
Outputs probability of a zero value as a result of the zero-generating process	OUTPUT	PROBZERO=
Performance Options		
Requests a table that shows a timing breakdown	PERFORMANCE	DETAILS
Specifies the number of threads to use	PERFORMANCE	NTHREADS=
Specifies the number of nodes to use on the SAS appliance	PERFORMANCE	NODES=

PROC HPCOUNTREG Statement

PROC HPCOUNTREG <*options*> **;**

The following *options* can be used in the PROC HPCOUNTREG statement.

Input Data Set Options

DATA=*SAS-data-set*
specifies the input SAS data set. If the DATA= option is not specified, PROC HPCOUNTREG uses the most recently created SAS data set.

GROUPID=*variable*
specifies an identification variable when a panel data model is estimated. The identification variable is used as a cross-sectional ID variable.

Output Data Set Options

OUTEST=*SAS-data-set*
writes the parameter estimates to the specified output data set.

CORROUT
writes the correlation matrix for the parameter estimates to the OUTEST= data set. This option is valid only if the OUTEST= option is specified.

COVOUT
writes the covariance matrix for the parameter estimates to the OUTEST= data set. This option is valid only if the OUTEST= option is specified.

Printing Options

You can specify the following options in either the PROC HPCOUNTREG statement or the MODEL statement:

CORRB
prints the correlation matrix of the parameter estimates.

COVB
prints the covariance matrix of the parameter estimates.

NOPRINT
suppresses all printed output.

PRINTALL
requests all printing options.

Estimation Control Options

You can specify the following options in either the PROC HPCOUNTREG statement or the MODEL statement:

COVEST=HESSIAN | OP | QML
specifies the type of covariance matrix for the parameter estimates.

The default is COVEST=HESSIAN. You can specify the following values:

HESSIAN	specifies the covariance from the Hessian matrix.
OP	specifies the covariance from the outer product matrix.
QML	specifies the covariance from the outer product and Hessian matrices.

Optimization Control Options

PROC HPCOUNTREG uses the nonlinear optimization (NLO) subsystem to perform nonlinear optimization tasks. You can specify the following *options* in either the PROC HPCOUNTREG statement or the MODEL statement.

ABSCONV=*r*

ABSTOL=*r*

specifies an absolute function value convergence criterion by which minimization stops when $f(\theta^{(k)}) \leq r$. The default value of *r* is the negative square root of the largest double-precision value, which serves only as a protection against overflows.

ABSFCONV=*r*

ABSFTOL=*r*

specifies an absolute function difference convergence criterion by which minimization stops when the function value has a small change in successive iterations:

$$|f(\theta^{(k-1)}) - f(\theta^{(k)})| \leq r$$

The default is 0.

ABSGCONV=*r*

ABSGTOL=*r*

specifies an absolute gradient convergence criterion. Optimization stops when the maximum absolute gradient element is small:

$$\max_j |g_j(\theta^{(k)})| \leq r$$

The default is 1E–5.

ABSXCONV=*r*

ABSXTOL=*r*

specifies an absolute parameter convergence criterion. Optimization stops when the Euclidean distance between successive parameter vectors is small:

$$\parallel \theta^{(k)} - \theta^{(k-1)} \parallel_2 \leq r$$

The default is 0.

FCONV=*r*

FTOL=*r*

specifies a relative function convergence criterion. Optimization stops when a relative change of the function value in successive iterations is small:

$$\frac{|f(\theta^{(k)}) - f(\theta^{(k-1)})|}{|f(\theta^{(k-1)})|} \leq r$$

The default value is 2ϵ, where ϵ denotes the machine precision constant, which is the smallest double-precision floating-point number such that $1 + \epsilon > 1$.

GCONV=*r*

GTOL=*r*

specifies a relative gradient convergence criterion. For all techniques except CONGRA, optimization stops when the normalized predicted function reduction is small:

$$\frac{g(\theta^{(k)})^T [H^{(k)}]^{-1} g(\theta^{(k)})}{|f(\theta^{(k)})|} \leq r$$

For the CONGRA technique (where a reliable Hessian estimate H is not available), the following criterion is used:

$$\frac{\| g(\theta^{(k)}) \|_2^2 \quad \| s(\theta^{(k)}) \|_2}{\| g(\theta^{(k)}) - g(\theta^{(k-1)}) \|_2 \, |f(\theta^{(k)})|} \leq r$$

The default is 1E–8.

MAXFUNC=*i*

MAXFU=*i*

specifies the maximum number of function calls in the optimization process. The default is 1,000.

The optimization can terminate only after completing a full iteration. Therefore, the number of function calls that are actually performed can exceed the number of calls that are specified by this option.

MAXITER=*i*

MAXIT=*i*

specifies the maximum number of iterations in the optimization process. The default is 200.

MAXTIME=*r*

specifies an upper limit of *r* seconds of CPU time for the optimization process. The default value is the largest floating-point double representation of your computer. The time that is specified by this option is checked only once at the end of each iteration. Therefore, the actual run time can be much longer than *r*. The actual run time includes the remaining time needed to finish the iteration and the time needed to generate the output of the results.

METHOD=*value*

specifies the iterative minimization method to use. The default is METHOD=NEWRAP. You can specify the following *values*:

CONGRA	specifies the conjugate-gradient method.
DBLDOG	specifies the double-dogleg method.
NEWRAP	specifies the Newton-Raphson method (this is the default).
NONE	specifies that no optimization be performed beyond using the ordinary least squares method to compute the parameter estimates.
NRRIDG	specifies the Newton-Raphson Ridge method.
QUANEW	specifies the quasi-Newton method.
TRUREG	specifies the trust region method.

SINGULAR=*r*
specifies the general singularity criterion that is applied by the HPCOUNTREG procedure in sweeps and inversions. The default is 1E–8.

BOUNDS Statement

BOUNDS *bound1* [, *bound2 . . .*] **;**

The BOUNDS statement imposes simple boundary constraints on the parameter estimates. You can specify any number of BOUNDS statements.

Each *bound* is composed of parameter names, constants, and inequality operators as follows:

item operator item [*operator item* [*operator item . . .*]]

Each *item* is a constant, a parameter name, or a list of parameter names. Each *operator* is <, >, <=, or >=. Parameter names are as shown in the Parameter column of the "Parameter Estimates" table. Please refer to the section "Parameter Naming Conventions for the RESTRICT, TEST, BOUNDS, and INIT Statements" on page 1103 for more information on how parameters are named in the BOUNDS statement.

You can use both the BOUNDS statement and the RESTRICT statement to impose boundary constraints. However, the BOUNDS statement provides a simpler syntax for specifying these kinds of constraints. For more information, see the section "RESTRICT Statement" on page 1088.

The following BOUNDS statement illustrates the use of parameter lists to specify boundary constraints. It constrains the estimates of the parameter for z to be negative, the parameters for x1 through x10 to be between 0 and 1, and the parameter for x1 in the zero-inflation model to be less than 1.

```
bounds z < 0,
       0 < x1-x10 < 1,
       Inf_x1 < 1;
```

BY Statement

BY *variables* **;**

A BY statement can be used with PROC HPCOUNTREG to obtain separate analyses on observations in groups defined by the BY variables. When a BY statement appears, the input data set should be sorted in order of the BY variables.

BY statement processing is not supported when the HPCOUNTREG procedure runs alongside the database or alongside the Hadoop Distributed File System (HDFS). These modes are used if the input data are stored in a database or HDFS and the grid host is the appliance that houses the data.

CLASS Statement

CLASS *variables* **;**

The CLASS statement names the classification variables to be used in the analysis. Classification variables can be either character or numeric.

Class levels are determined from the formatted values of the CLASS variables. Thus, you can use formats to group values into levels. For more information, see the discussion of the FORMAT procedure in *SAS Language Reference: Dictionary*.

DISPMODEL Statement

DISPMODEL *dependent-variable* ~ < *dispersion-related-regressors* > **;**

The DISPMODEL statement specifies the *dispersion-related-regressors* that are used to model dispersion. This statement is ignored unless you specify DIST=CMPOISSON in the MODEL statement. The *dependent-variable* in the DISPMODEL statement must be the same as the *dependent-variable* in the MODEL statement.

The *dependent-variable* that appears in the DISPMODEL statement is directly used to model dispersion. Each of the q variables to the right of the tilde (**~**) has a parameter to be estimated in the regression. For example, let $\mathbf{g}_i'$ be the ith observation's $1 \times (q+1)$ vector of values of the q dispersion explanatory variables (q_0 is set to 1 for the intercept term). Then the dispersion is a function of $\mathbf{g}_i'\boldsymbol{\delta}$, where $\boldsymbol{\delta}$ is the $(q+1) \times 1$ vector of parameters to be estimated, the dispersion model intercept is δ_0, and the coefficients for the q dispersion covariates are $\delta_1, \ldots, \delta_q$. If you specify DISP=CMPOISSON in the MODEL statement but do not include a DISPMODEL statement, then only the intercept term δ_0 is estimated. The "Parameter Estimates" table in the displayed output shows the estimates for the dispersion intercept and dispersion explanatory variables; they are labeled with the prefix "Disp_". For example, the dispersion intercept is labeled "Disp_Intercept". If you specify Age (a variable in your data set) as a dispersion explanatory variable, then the "Parameter Estimates" table labels the corresponding parameter estimate "Disp_Age". The following statements fit a Conway-Maxwell-Poisson model by using the regressors SEX, ILLNESS, and INCOME and by using AGE as a dispersion-related regressor:

```
proc hpcountreg data=docvisit;
   model doctorvisits=sex illness income / dist=cmpoisson;
   dispmodel doctorvisits ~ age;
run;
```

FREQ Statement

FREQ *freq-variable* **;**

The FREQ statement identifies a variable (*freq-variable*) that contains the frequency of occurrence of each observation. PROC HPCOUNTREG treats each observation as if it appears n times, where n is the value of *freq-variable* for the observation. If the value for the observation is not an integer, it is truncated to an integer. If the value is less than 1 or missing, the observation is not used in the model fitting. When the FREQ statement is not specified, each observation is assigned a frequency of 1.

INIT Statement

INIT *initialization1 < , initialization2 . . . >* **;**

The INIT statement sets initial values for parameters in the optimization.

Each *initialization* is written as a parameter or parameter list, followed by an optional equal sign (=), followed by a number:

parameter <=> number

Parameter names are as shown in the Parameter column of the "Parameter Estimates" table. Please refer to the section "Parameter Naming Conventions for the RESTRICT, TEST, BOUNDS, and INIT Statements" on page 1103 for more information on how parameters are named in the INIT statement.

MODEL Statement

MODEL *dependent-variable* **=** *regressors </ options>* **;**

The MODEL statement specifies the dependent variable and independent regressor variables for the regression model. The dependent count variable should take only nonnegative integer values from the input data set. PROC HPCOUNTREG rounds any positive noninteger count value to the nearest integer. PROC HPCOUNTREG discards any observation that has a negative count.

Only one MODEL statement can be specified. You can specify the following *options* in the MODEL statement after a slash (/).

DIST=*value*

specifies a type of model to be analyzed. You can specify the following *values*:

POISSON | P specifies the Poisson regression model.

CMPOISSON | C | CMP specifies a Conway-Maxwell-Poisson regression model.

NEGBIN(P=1) specifies the negative binomial regression model that uses a linear variance function.

NEGBIN(P=2) | NEGBIN specifies the negative binomial regression model that uses a quadratic variance function.

ZIPOISSON | ZIP specifies zero-inflated Poisson regression.

ZICMPOISSON | ZICMP specifies a zero-inflated Conway-Maxwell-Poisson regression. The ZEROMODEL statement must be specified when this model type is specified.

ZINEGBIN | ZINB specifies zero-inflated negative binomial regression.

You can also specify the DIST option in the HPCOUNTREG statement.

ERRORCOMP=FIXED | RANDOM

specifies a type of conditional panel model to be analyzed. You can specify the following model types:

FIXED specifies a fixed-effect error component regression model.

RANDOM specifies a random-effect error component regression model.

NOINT

suppresses the intercept parameter.

OFFSET=*offset-variable*

specifies a variable in the input data set to be used as an offset variable. The *offset-variable* is used to allow the observational units to vary across observations. For example, when the number of shipping accidents could be measured across different time periods or the number of students who participate in an activity could be reported across different class sizes, the observational units need to be adjusted to a common denominator by using the offset variable. The offset variable appears as a covariate in the model with its parameter restricted to 1. The offset variable cannot be the response variable, the zero-inflation offset variable (if any), or any of the explanatory variables. The "Model Fit Summary" table gives the name of the data set variable that is used as the offset variable; it is labeled "Offset."

PARAMETER=MU | LAMBDA

specifies the parameterization for the Conway-Maxwell-Poisson model. The following parameterizations are supported:

LAMBDA	estimates the original Conway-Maxwell-Poisson model (Shmueli et al. 2005).
MU	reparameterizes λ as documented by Guikema and Coffelt (2008), where $\mu = \lambda^{1/\nu}$ and the integral part of μ represents the mode, which can be considered a measure of central tendency (mean).

By default, PARAMETER=MU.

Printing Options

You can specify the following options in either the PROC HPCOUNTREG statement or the MODEL statement:

CORRB

prints the correlation matrix of the parameter estimates.

COVB

prints the covariance matrix of the parameter estimates.

NOPRINT

suppresses all printed output.

PRINTALL

requests all printing options.

OUTPUT Statement

OUTPUT < *output-options* > **;**

The OUTPUT statement creates a new SAS data set that includes variables created by the *output-options*. These variables include the estimates of $\mathbf{x}_i'\boldsymbol{\beta}$, the expected value of the response variable, and the probability of the response variable taking on the current value. Furthermore, if a zero-inflated model was fit, you can request that the output data set contain the estimates of $\mathbf{z}_i'\boldsymbol{\gamma}$ and the probability that the response is zero as a result of the zero-generating process. For the Conway-Maxwell-Poisson model, the estimates of $\mathbf{g}_i'\boldsymbol{\delta}$, λ, ν, μ, mode, variance, and dispersion are also available. Except for the probability of the current value, these statistics can be computed for all observations in which the regressors are not missing, even if the response is missing. By adding observations that have missing response values to the input data set, you can compute these statistics for new observations or for settings of the regressors that are not present in the data without affecting the model fit.

You can specify only one OUTPUT statement. You can specify the following *output-options*:

OUT=*SAS-data-set*
names the output data set

COPYVAR=*SAS-variable-names*
COPYVARS=*SAS-variable-names*
adds SAS variables to the output data set.

XBETA=*name*
names the variable to contain estimates of $\mathbf{x}_i'\boldsymbol{\beta}$.

PRED=*name*
MEAN=*name*
names the variable to contain the predicted value of the response variable.

PROB=*name*
names the variable to contain the probability that the response variable will take the actual value, $\Pr(Y = y_i)$.

PROBCOUNT(*value1 < value2* **. . .** **>)**
outputs the probability that the response variable will take particular values. Each *value* should be a nonnegative integer. If you specify a noninteger, it is rounded to the nearest integer. The *value* can also be a list of the form X TO Y BY Z. For example, PROBCOUNT(0 1 2 TO 10 BY 2 15) requests predicted probabilities for counts 0, 1, 2, 4, 5, 6, 8, 10, and 15. This option is not available for the fixed-effects and random-effects panel models.

ZGAMMA=*name*
names the variable to contain estimates of $\mathbf{z}_i'\boldsymbol{\gamma}$.

PROBZERO=*name*
names the variable to contain the value of φ_i, which is the probability that the response variable will take the value of 0 as a result of the zero-generating process. This variable is written to the output file only if the model is zero-inflated.

GDELTA=*name*
assigns a name to the variable that contains estimates of $\mathbf{g}_i'\boldsymbol{\delta}$ for the Conway-Maxwell-Poisson distribution.

LAMBDA=*name*
assigns a name to the variable that contains the estimate of λ for the Conway-Maxwell-Poisson distribution.

NU=*name*
: assigns a name to the variable that contains the estimate of ν for the Conway-Maxwell-Poisson distribution.

MU=*name*
: assigns a name to the variable that contains the estimate of μ for the Conway-Maxwell-Poisson distribution.

MODE=*name*
: assigns a name to the variable that contains the integral part of μ (mode) for the Conway-Maxwell-Poisson distribution.

VARIANCE=*name*
: assigns a name to the variable that contains the estimate of variance for the Conway-Maxwell-Poisson distribution.

DISPERSION=*name*
: assigns a name to the variable that contains the value of dispersion for the Conway-Maxwell-Poisson distribution.

PERFORMANCE Statement

PERFORMANCE < *performance-options* > **;**

The PERFORMANCE statement specifies options to control the multithreaded and distributed computing environment and requests detailed results about the performance characteristics of the HPCOUNTREG procedure. You can also use the PERFORMANCE statement to control whether the HPCOUNTREG procedure executes in single-machine or distributed mode. The most commonly used *performance-options* in the PERFORMANCE statement are as follows:

DETAILS
: requests a table that shows a timing breakdown of the procedure steps.

NODES=*n*
: specifies the number of nodes in the distributed computing environment, provided that the data are not processed alongside the database.

NTHREADS=*n*
: specifies the number of threads for analytic computations and overrides the SAS system option THREADS | NOTHREADS. If you do not specify the NTHREADS= option, PROC HPCOUNTREG creates one thread per CPU for the analytic computations.

For more information about the PERFORMANCE statement for high-performance analytical procedures, see the section “PERFORMANCE Statement” on page 87 of Chapter 3, “Shared Concepts and Topics.”

RESTRICT Statement

RESTRICT *restriction1* [*, restriction2 . . .*] **;**

The RESTRICT statement imposes linear restrictions on the parameter estimates. You can specify any number of RESTRICT statements.

Each *restriction* is written as an expression, followed by an equality operator (=) or an inequality operator (<, >, <=, >=) and then by a second expression, as follows:

expression operator expression

The *operator* can be =, <, >, <=, or >=.

Restriction expressions can be composed of parameter names, constants, and the following operators: times ($*$), plus ($+$), and minus ($-$). Parameter names are as shown in the Effect column of the "Parameter Estimates" table. The restriction expressions must be a linear function of the variables.

Parameter names are as shown in the Parameter column of the "Parameter Estimates" table. Please refer to the section "Parameter Naming Conventions for the RESTRICT, TEST, BOUNDS, and INIT Statements" on page 1103 for more information on how parameters are named in the RESTRICT statement.

Lagrange multipliers are reported in the "Parameter Estimates" table for all the active linear constraints. They are identified by the names Restrict1, Restrict2, and so on. The probabilities of these Lagrange multipliers are computed using a beta distribution (LaMotte 1994). Nonactive (nonbinding) restrictions have no effect on the estimation results and are not noted in the output.

The following RESTRICT statement constrains the negative binomial dispersion parameter α to 1, which restricts the conditional variance to be $\mu + \mu^2$:

```
restrict _Alpha = 1;
```

TEST Statement

<label:> **TEST** *<'string'> equation1 < , equation2. . . > / < test-options>* **;**

The TEST statement performs Wald, Lagrange multiplier, and likelihood ratio tests of linear hypotheses about the regression parameters that are specified in the preceding MODEL statement.

You can add a label (which is printed in the output) to a TEST statement in two ways: add an unquoted *label* followed by a colon before the TEST keyword, or add a quoted *string* after the TEST keyword. The unquoted *label* cannot contain any spaces. If you include both an unquoted *label* and a quoted *string*, PROC HPCOUNTREG uses the unquoted *label*. If you specify neither an unquoted *label* nor a quoted *string*, PROC HPCOUNTREG automatically labels the tests.

Each *equation* specifies a linear hypothesis to be tested and consists of regression parameter names and relational operators. The regression parameter names are as shown in the Parameter column of the "Parameter Estimates" table. For more information about how parameters are named in the TEST statement, the section "Parameter Naming Conventions for the RESTRICT, TEST, BOUNDS, and INIT Statements" on page 1103. Only linear equality restrictions and tests are permitted in PROC COUNTREG. Test *equations* can consist only of algebraic operations that involve the addition symbol (+), subtraction symbol (-), and multiplication symbol (*).

All hypotheses in one TEST statement are tested jointly.

You can specify the following *test-options* after a slash (/):

ALL
: requests Wald, Lagrange multiplier, and likelihood ratio tests.

LM
: requests the Lagrange multiplier test.

LR
: requests the likelihood ratio test.

WALD
: requests the Wald test.

By default, the Wald test is performed.

The following illustrates the use of the TEST statement:

```
proc hpcountreg;
   model y = x1 x2 x3;
   test x1 = 0, x2 * .5 + 2 * x3 = 0;
   test _int: test intercept = 0, x3 = 0;
run;
```

The first test investigates the joint hypothesis that

$$\beta_1 = 0$$

and

$$0.5\beta_2 + 2\beta_3 = 0$$

Only linear equality restrictions and tests are permitted in PROC HPCOUNTREG. Tests expressions can consist only of algebraic operations that involve the addition symbol (+), subtraction symbol (-), and multiplication symbol (*).

WEIGHT Statement

WEIGHT *variable* < / *option*> **;**

The WEIGHT statement specifies a variable to supply weighting values to use for each observation in estimating parameters. The log likelihood for each observation is multiplied by the corresponding weight variable value.

If the weight of an observation is nonpositive, that observation is not used in the estimation.

The following *option* can be added to the WEIGHT statement after a slash (/).

NONORMALIZE

does not normalize the weights. (By default, the weights are normalized so that they add up to the actual sample size. The weights w_i are normalized by multiplying them by $\frac{n}{\sum_{i=1}^{n} w_i}$, where n is the sample size.) If the weights are required to be used as they are, then specify the NONORMALIZE option.

ZEROMODEL Statement

ZEROMODEL *dependent-variable* ~ *zero-inflated-regressors* < / *options* > **;**

The ZEROMODEL statement is required if either ZIP or ZINB is specified in the DIST= option in the MODEL statement. If ZIP or ZINB is specified, then the ZEROMODEL statement must follow the MODEL statement. The dependent variable in the ZEROMODEL statement must be the same as the dependent variable in the MODEL statement.

The zero-inflated (ZI) regressors appear in the equation that determines the probability (φ_i) of a zero count. Each of these q variables has a parameter to be estimated in the regression. For example, let $\mathbf{z}_i'$ be the ith observation's $1 \times (q + 1)$ vector of values of the q ZI explanatory variables (w_0 is set to 1 for the intercept term). Then φ_i is a function of $\mathbf{z}_i'\boldsymbol{\gamma}$, where $\boldsymbol{\gamma}$ is the $(q + 1) \times 1$ vector of parameters to be estimated. (The zero-inflated intercept is γ_0; the coefficients for the q zero-inflated covariates are $\gamma_1, \ldots, \gamma_q$.) If q is equal to 0 (no ZI explanatory variables are provided), then only the intercept term γ_0 is estimated. The "Parameter Estimates" table in the displayed output shows the estimates for the ZI intercept and ZI explanatory variables; they are labeled with the prefix "Inf_". For example, the ZI intercept is labeled "Inf_intercept". If you specify Age (a variable in your data set) as a ZI explanatory variable, then the "Parameter Estimates" table labels the corresponding parameter estimate "Inf_Age".

You can specify the following *options* in the ZEROMODEL statement after a slash (/):

LINK=LOGISTIC | NORMAL

specifies the distribution function used to compute probability of zeros. The supported distribution functions are as follows:

LOGISTIC	specifies logistic distribution.
NORMAL	specifies standard normal distribution.

If this option is omitted, then the default ZI link function is logistic.

OFFSET=*zero-inflated-offset-variable*

specifies a variable in the input data set to be used as a zero-inflated (ZI) offset variable. The ZI offset variable *zero-inflated-offset-variable* is included as a term, with coefficient restricted to 1, in the equation that determines the probability (φ_i) of a zero count and represents an adjustment to a common observational unit. The ZI offset variable cannot be the response variable, the offset variable (if any), or any of the explanatory variables. The name of the data set variable that is used as the ZI offset variable is displayed in the "Model Fit Summary" table, where it is labeled as "Inf_offset".

Details: HPCOUNTREG Procedure

Missing Values

Any observations in the input data set that have a missing value for one or more of the regressors are ignored by PROC HPCOUNTREG and not used in the model fit. PROC HPCOUNTREG rounds any positive noninteger count values to the nearest integer and ignores any observations that have a negative count.

If the input data set contains any observations that have missing response values but nonmissing regressors, PROC HPCOUNTREG can compute several statistics and store them in an output data set by using the OUTPUT statement. For example, you can request that the output data set contain the estimates of $\mathbf{x}_i'\boldsymbol{\beta}$, the expected value of the response variable, and the probability that the response variable will take the current value. Furthermore, if a zero-inflated model was fit, you can request that the output data set contain the estimates of $\mathbf{z}_i'\boldsymbol{\gamma}$, and the probability that the response is 0 as a result of the zero-generating process. Note that the presence of such observations (that have missing response values) does not affect the model fit.

Poisson Regression

The most widely used model for count data analysis is Poisson regression. Poisson regression assumes that y_i, given the vector of covariates $\mathbf{x}_i$, is independently Poisson distributed with

$$P(Y_i = y_i|\mathbf{x}_i) = \frac{e^{-\mu_i}\mu_i^{y_i}}{y_i!}, \quad y_i = 0, 1, 2, \ldots$$

and the mean parameter—that is, the mean number of events per period—is given by

$$\mu_i = \exp(\mathbf{x}_i'\boldsymbol{\beta})$$

where $\boldsymbol{\beta}$ is a $(k+1) \times 1$ parameter vector. (The intercept is β_0; the coefficients for the k regressors are $\beta_1, \ldots, \beta_k$.) Taking the exponential of $\mathbf{x}_i'\boldsymbol{\beta}$ ensures that the mean parameter μ_i is nonnegative. It can be shown that the conditional mean is given by

$$E(y_i|\mathbf{x}_i) = \mu_i = \exp(\mathbf{x}_i'\boldsymbol{\beta})$$

Note that the conditional variance of the count random variable is equal to the conditional mean in the Poisson regression model:

$$V(y_i|\mathbf{x}_i) = E(y_i|\mathbf{x}_i) = \mu_i$$

The equality of the conditional mean and variance of y_i is known as *equidispersion*.

The standard estimator for the Poisson model is the maximum likelihood estimator (MLE). Because the observations are independent, the log-likelihood function is written as

$$\mathcal{L} = \sum_{i=1}^{N}(-\mu_i + y_i \ln \mu_i - \ln y_i!) = \sum_{i=1}^{N}(-e^{\mathbf{x}_i'\boldsymbol{\beta}} + y_i\mathbf{x}_i'\boldsymbol{\beta} - \ln y_i!)$$

For more information about the Poisson regression model, see the section "Poisson Regression" on page 622.

The Poisson model has been criticized for its restrictive property that the conditional variance equals the conditional mean. Real-life data are often characterized by *overdispersion*—that is, the variance exceeds the mean. Allowing for overdispersion can improve model predictions because the Poisson restriction of equal mean and variance results in the underprediction of zeros when overdispersion exists. The most commonly used model that accounts for overdispersion is the negative binomial model. Conway-Maxwell-Poisson regression enables you to model both overdispersion and underdispersion.

Conway-Maxwell-Poisson Regression

The Conway-Maxwell-Poisson (CMP) distribution is a generalization of the Poisson distribution that enables you to model both underdispersed and overdispersed data. It was originally proposed by Conway and Maxwell (1962), but its implementation to model under- and overdispersed count data is attributed to Shmueli et al. (2005).

Recall that y_i, given the vector of covariates $\mathbf{x}_i$, is independently Poisson-distributed as

$$P(Y_i = y_i|\mathbf{x}_i) = \frac{e^{-\lambda_i}\lambda_i^{y_i}}{y_i!}, \quad y_i = 0, 1, 2, \ldots$$

The Conway-Maxwell-Poisson distribution is defined as

$$P(Y_i = y_i|\mathbf{x}_i, \mathbf{z}_i) = \frac{1}{Z(\lambda_i, \nu_i)} \frac{\lambda_i^{y_i}}{(y_i!)^{\nu_i}}, \quad y_i = 0, 1, 2, \ldots$$

where the normalization factor is

$$Z(\lambda_i, \nu_i) = \sum_{n=0}^{\infty} \frac{\lambda_i^n}{(n!)^{\nu_i}}$$

and

$$\lambda_i = \exp(\mathbf{x}_i'\boldsymbol{\beta})$$

$$\nu_i = -\exp(\mathbf{g}_i'\delta)$$

The $\boldsymbol{\beta}$ vector is a $(k + 1) \times 1$ parameter vector. (The intercept is β_0, and the coefficients for the k regressors are $\beta_1, \ldots, \beta_k$.) The δ vector is an $(m + 1) \times 1$ parameter vector. (The intercept is represented by δ_0, and the coefficients for the m regressors are $\delta_1, \ldots, \delta_k$.) The covariates are represented by x_i and g_i vectors.

One of the restrictive properties of the Poisson model is that the conditional mean and variance must be equal:

$$E(y_i|\mathbf{x}_i) = V(y_i|\mathbf{x}_i) = \lambda_i = \exp(\mathbf{x}_i'\boldsymbol{\beta})$$

The CMP distribution overcomes this restriction by defining an additional parameter, ν, which governs the rate of decay of successive ratios of probabilities such that

$$P(Y_i = y_i - 1)/P(Y_i = y_i) = \frac{(y_i)^{\nu_i}}{\lambda_i}$$

The introduction of the additional parameter, ν, allows for flexibility in modeling the tail behavior of the distribution. If $\nu = 1$, the ratio is equal to the rate of decay of the Poisson distribution. If $\nu < 1$, the rate of decay decreases, enabling you to model processes that have longer tails than the Poisson distribution (overdispersed data). If $\nu > 1$, the rate of decay increases in a nonlinear fashion, thus shortening the tail of the distribution (underdispersed data).

There are several special cases of the Conway-Maxwell-Poisson distribution. If $\lambda < 1$ and $\nu \to \infty$, the Conway-Maxwell-Poisson results in the Bernoulli distribution. In this case, the data can take only the values 0 and 1, which represents an extreme underdispersion. If $\nu = 1$, the Poisson distribution is recovered with its equidispersion property. When $\nu = 0$ and $\lambda < 1$, the normalization factor is convergent and forms a geometric series,

$$Z(\lambda_i, 0) = \frac{1}{1 - \lambda_i}$$

and the probability density function becomes

$$P(Y = y_i; \lambda_i, \nu_i = 0) = (1 - \lambda_i)\lambda_i^{y_i}$$

The geometric distribution represents a case of severe overdispersion.

Mean, Variance, and Dispersion for the Conway-Maxwell-Poisson Model

The mean and the variance of the Conway-Maxwell-Poisson distribution are defined as

$$E[Y] = \frac{\partial \ln Z}{\partial \ln \lambda}$$

$$V[Y] = \frac{\partial^2 \ln Z}{\partial^2 \ln \lambda}$$

The Conway-Maxwell-Poisson distribution does not have closed-form expressions for its moments in terms of its parameters λ and ν. However, the moments can be approximated. Shmueli et al. (2005) use asymptotic expressions for Z to derive $E(Y)$ and $V(Y)$ as

$$E[Y] \approx \lambda^{1/\nu} + \frac{1}{2\nu} - \frac{1}{2}$$

$$V[Y] \approx \frac{1}{\nu}\lambda^{1/\nu}$$

In the Conway-Maxwell-Poisson model, the summation of infinite series is evaluated using a logarithmic expansion. The mean and variance are calculated as follows for the Shmueli et al. (2005) model:

$$E(Y) = \frac{1}{Z(\lambda, \nu)} \sum_{j=0}^{\infty} \frac{j\lambda^j}{(j!)^\nu}$$

$$V(Y) = \frac{1}{Z(\lambda, \nu)} \sum_{j=0}^{\infty} \frac{j^2\lambda^j}{(j!)^\nu} - E(Y)^2$$

The dispersion is defined as

$$D(Y) = \frac{V(Y)}{E(Y)}$$

Likelihood Function for the Conway-Maxwell-Poisson Model

The likelihood for a set of n independently and identically distributed variables $y_1, y_2, \ldots, y_n$ is written as

$$\begin{aligned} L(y_1, y_2, \ldots, y_n|\lambda, \nu) &= \frac{\prod_{i=1}^{n} \lambda^{y_i}}{(\prod_{i=1}^{n} y_i!)^{\nu}} Z(\lambda, \nu)^{-n} \\ &= \lambda^{\sum_{i=1}^{n} y_i} \exp\left(-\nu \sum_{i=1}^{n} \ln(y_i!)\right) Z(\lambda, \nu)^{-n} \\ &= \lambda^{S_1} \exp\left(-\nu S_2\right) Z(\lambda, \nu)^{-n} \end{aligned}$$

where S_1 and S_2 are sufficient statistics for $y_1, y_2, \ldots, y_n$. You can see from the preceding equation that the Conway-Maxwell-Poisson distribution is a member of the exponential family. The log-likelihood function can be written as

$$\mathcal{L} = -n \ln(Z(\lambda, \nu)) + \sum_{i=1}^{n} (y_i \ln(\lambda) - \nu \ln(y_i!))$$

The gradients can be written as

$$\mathcal{L}_{\beta} = \left(\sum_{k=1}^{N} y_k - n \frac{\lambda Z(\lambda, \nu)_{\lambda}}{Z(\lambda, \nu)} \right) \mathbf{x}$$

$$\mathcal{L}_{\delta} = \left(\sum_{k=1}^{N} \ln(y_k!) - n \frac{Z(\lambda, \nu)_{\nu}}{Z(\lambda, \nu)} \right) \nu \mathbf{z}$$

Conway-Maxwell-Poisson Regression: Guikema and Coffelt (2008) Reparameterization

Guikema and Coffelt (2008) propose a reparameterization of the Shmueli et al. (2005) Conway-Maxwell-Poisson model to provide a measure of central tendency that can be interpreted in the context of the generalized linear model. By substituting $\lambda = \mu^{\nu}$, the Guikema and Coffelt (2008) formulation is written as

$$P(Y = y_i; \mu, \nu) = \frac{1}{S(\mu, \nu)} \left(\frac{\mu^{y_i}}{y_i!} \right)^{\nu}$$

where the new normalization factor is defined as

$$S(\mu, \nu) = \sum_{j=0}^{\infty} \left(\frac{\mu^j}{j!} \right)^{\nu}$$

In terms of their new formulations, the mean and variance of Y are given as

$$E[Y] = \frac{1}{\nu} \frac{\partial \ln S}{\partial \ln \mu}$$

$$V[Y] = \frac{1}{\nu^2} \frac{\partial^2 \ln S}{\partial^2 \ln \mu}$$

They can be approximated as

$$E[Y] \approx \mu + \frac{1}{2}\nu - \frac{1}{2}$$

$$V[Y] \approx \frac{\mu}{\nu}$$

In the HPCOUNTREG procedure, the mean and variance are calculated according to the following formulas, respectively, for the Guikema and Coffelt (2008) model:

$$E(Y) = \frac{1}{Z(\lambda, \mu)} \sum_{j=0}^{\infty} \frac{j\mu^{\nu j}}{(j!)^{\nu}}$$

$$V(Y) = \frac{1}{Z(\lambda, \mu)} \sum_{j=0}^{\infty} \frac{j^2\mu^{\nu j}}{(j!)^{\nu}} - E(Y)^2$$

In terms of the new parameter μ, the log-likelihood function is specified as

$$\mathcal{L} = \ln(S(\mu, \nu)) + \nu \sum_{i=1}^{N} (y_i \ln(\mu) - \ln(y_i!))$$

and the gradients are calculated as

$$\mathcal{L}_{\beta} = \left(\nu \sum_{i=1}^{N} y_i - \frac{\mu S(\mu, \nu)_{\mu}}{S(\mu, \nu)} \right) \mathbf{x}$$

$$\mathcal{L}_{\delta} = \left(\sum_{i=1}^{N} (y_i \ln(\mu) - \ln(y_i!)) - \frac{S(\mu, \nu)_{\nu}}{S(\mu, \nu)} \right) \nu \mathbf{g}$$

By default, the HPCOUNTREG procedure uses the Guikema and Coffelt (2008) specification. The Shmueli et al. (2005) model can be estimated by specifying the PARAMETER=LAMBDA option. If you specify DISP=CMPOISSON in the MODEL statement and you omit the DISPMODEL statement, the model is estimated according to the Lord, Guikema, and Geedipally (2008) specification, where ν represents a single parameter that does not depend on any covariates. The Lord, Guikema, and Geedipally (2008) specification makes the model comparable to the negative binomial model because it has only one parameter.

The dispersion is defined as

$$D(Y) = \frac{V(Y)}{E(Y)}$$

Using the Guikema and Coffelt (2008) specification results in the integral part of μ representing the mode, which is a reasonable approximation for the mean. The dispersion can be written as

$$D(Y) = \frac{V(Y)}{E(Y)} \approx \frac{\frac{\mu}{\nu}}{\mu + \frac{1}{2}\nu - \frac{1}{2}} \approx \frac{1}{\nu}$$

When $\nu < 1$, the variance can be shown to be greater than the mean and the dispersion greater than 1. This is a result of overdispersed data. When $\nu = 1$ and the mean and variance are equal, the dispersion is equal to 1 (Poisson model). When $\nu > 1$, the variance is smaller than the mean and the dispersion is less than 1. This is a result of underdispersed data.

All Conway-Maxwell-Poisson models in the HPCOUNTREG procedure are parameterized in terms of dispersion, where

$$-\ln(\nu) = \delta_0 + \sum_{n=1}^{q} \delta_n g_n$$

Negative values of $\ln(\nu)$ indicate that the data are approximately overdispersed, and positive values of $\ln(\nu)$ indicate that the data are approximately underdispersed.

Negative Binomial Regression

The Poisson regression model can be generalized by introducing an unobserved heterogeneity term for observation i. Thus, the individuals are assumed to differ randomly in a manner that is not fully accounted for by the observed covariates. This is formulated as

$$E(y_i|\mathbf{x}_i, \tau_i) = \mu_i \tau_i \ = e^{\mathbf{x}_i' \boldsymbol{\beta} + \epsilon_i}$$

where the unobserved heterogeneity term $\tau_i \ = e^{\epsilon_i}$ is independent of the vector of regressors $\mathbf{x}_i$. Then the distribution of y_i conditional on $\mathbf{x}_i$ and τ_i is Poisson with conditional mean and conditional variance $\mu_i \tau_i$:

$$f(y_i|\mathbf{x}_i, \tau_i) = \frac{\exp(-\mu_i \tau_i)(\mu_i \tau_i)^{y_i}}{y_i!}$$

Let $g(\tau_i)$ be the probability density function of τ_i. Then, the distribution $f(y_i|\mathbf{x}_i)$ (no longer conditional on τ_i) is obtained by integrating $f(y_i|\mathbf{x}_i, \tau_i)$ with respect to τ_i:

$$f(y_i|\mathbf{x}_i) = \int_0^\infty f(y_i|\mathbf{x}_i, \tau_i) g(\tau_i) d\tau_i$$

An analytical solution to this integral exists when τ_i is assumed to follow a gamma distribution. This solution is the negative binomial distribution. If the model contains a constant term, then in order to identify the mean of the distribution, it is necessary to assume that $E(e^{\epsilon_i}) = E(\tau_i) = 1$. Thus, it is assumed that τ_i follows a gamma(θ, θ) distribution with $E(\tau_i) = 1$ and $V(\tau_i) = 1/\theta$,

$$g(\tau_i) = \frac{\theta^\theta}{\Gamma(\theta)} \tau_i^{\theta - 1} \exp(-\theta \tau_i)$$

where $\Gamma(x) = \int_0^\infty z^{x-1}\exp(-z)dz$ is the gamma function and θ is a positive parameter. Then, the density of y_i given $\mathbf{x}_i$ is derived as

$$\begin{aligned} f(y_i|\mathbf{x}_i) &= \int_0^\infty f(y_i|\mathbf{x}_i,\tau_i)g(\tau_i)d\tau_i \\ &= \frac{\theta^\theta \mu_i^{y_i}}{y_i!\Gamma(\theta)}\int_0^\infty e^{-(\mu_i+\theta)\tau_i}\tau_i^{\theta+y_i-1}d\tau_i \\ &= \frac{\theta^\theta \mu_i^{y_i}\Gamma(y_i+\theta)}{y_i!\Gamma(\theta)(\theta+\mu_i)^{\theta+y_i}} \\ &= \frac{\Gamma(y_i+\theta)}{y_i!\Gamma(\theta)}\left(\frac{\theta}{\theta+\mu_i}\right)^\theta\left(\frac{\mu_i}{\theta+\mu_i}\right)^{y_i} \end{aligned}$$

If you make the substitution $\alpha = \frac{1}{\theta}$ ($\alpha > 0$), the negative binomial distribution can then be rewritten as

$$f(y_i|\mathbf{x}_i) = \frac{\Gamma(y_i+\alpha^{-1})}{y_i!\Gamma(\alpha^{-1})}\left(\frac{\alpha^{-1}}{\alpha^{-1}+\mu_i}\right)^{\alpha^{-1}}\left(\frac{\mu_i}{\alpha^{-1}+\mu_i}\right)^{y_i}, \quad y_i = 0,1,2,\ldots$$

Thus, the negative binomial distribution is derived as a gamma mixture of Poisson random variables. It has the conditional mean

$$E(y_i|\mathbf{x}_i) = \mu_i = e^{\mathbf{x}_i'\boldsymbol{\beta}}$$

and the conditional variance

$$V(y_i|\mathbf{x}_i) = \mu_i[1+\frac{1}{\theta}\mu_i] = \mu_i[1+\alpha\mu_i] > E(y_i|\mathbf{x}_i)$$

The conditional variance of the negative binomial distribution exceeds the conditional mean. Overdispersion results from neglected unobserved heterogeneity. The negative binomial model with variance function $V(y_i|\mathbf{x}_i) = \mu_i + \alpha\mu_i^2$, which is quadratic in the mean, is referred to as the NEGBIN2 model Cameron and Trivedi (1986). To estimate this model, specify DIST=NEGBIN(P=2) in the MODEL statement. The Poisson distribution is a special case of the negative binomial distribution where $\alpha = 0$. A test of the Poisson distribution can be carried out by testing the hypothesis that $\alpha = \frac{1}{\theta_i} = 0$. A Wald test of this hypothesis is provided (it is the reported t statistic for the estimated α in the negative binomial model).

The log-likelihood function of the negative binomial regression model (NEGBIN2) is given by

$$\begin{aligned} \mathcal{L} &= \sum_{i=1}^N \Bigg\{ \sum_{j=0}^{y_i-1}\ln(j+\alpha^{-1}) - \ln(y_i!) \\ &\quad -(y_i+\alpha^{-1})\ln(1+\alpha\exp(\mathbf{x}_i'\boldsymbol{\beta})) + y_i\ln(\alpha) + y_i\mathbf{x}_i'\boldsymbol{\beta}\Bigg\} \end{aligned}$$

where use of the following fact is made if y is an integer:

$$\Gamma(y+a)/\Gamma(a) = \prod_{j=0}^{y-1}(j+a)$$

Cameron and Trivedi (1986) consider a general class of negative binomial models that have mean μ_i and variance function $\mu_i + \alpha\mu_i^p$. The NEGBIN2 model, with $p = 2$, is the standard formulation of the negative binomial model. Models that have other values of p, $-\infty < p < \infty$, have the same density $f(y_i|\mathbf{x}_i)$, except that α^{-1} is replaced everywhere by $\alpha^{-1}\mu^{2-p}$. The negative binomial model NEGBIN1, which sets $p = 1$, has the variance function $V(y_i|\mathbf{x}_i) = \mu_i + \alpha\mu_i$, which is linear in the mean. To estimate this model, specify DIST=NEGBIN(P=1) in the MODEL statement.

The log-likelihood function of the NEGBIN1 regression model is given by

$$\begin{aligned}\mathcal{L} &= \sum_{i=1}^{N} \Bigg\{ \sum_{j=0}^{y_i-1} \ln\left(j + \alpha^{-1}\exp(\mathbf{x}_i'\boldsymbol{\beta})\right) \\ &\quad - \ln(y_i!) - \left(y_i + \alpha^{-1}\exp(\mathbf{x}_i'\boldsymbol{\beta})\right)\ln(1+\alpha) + y_i\ln(\alpha) \Bigg\}\end{aligned}$$

For more information about the negative binomial regression model, see the section "Negative Binomial Regression" on page 627.

Zero-Inflated Count Regression Overview

The main motivation for using zero-inflated count models is that real-life data frequently display overdispersion and excess zeros. Zero-inflated count models provide a way to both model the excess zeros and allow for overdispersion. In particular, there are two possible data generation processes for each observation. The result of a Bernoulli trial is used to determine which of the two processes to use. For observation i, Process 1 is chosen with probability φ_i and Process 2 with probability $1 - \varphi_i$. Process 1 generates only zero counts. Process 2 generates counts from either a Poisson or a negative binomial model. In general,

$$y_i \sim \begin{cases} 0 & \text{with probability} \quad \varphi_i \\ g(y_i) & \text{with probability} \quad 1 - \varphi_i \end{cases}$$

Therefore, the probability of $\{Y_i = y_i\}$ can be described as

$$\begin{aligned} P(y_i = 0|\mathbf{x}_i) &= \varphi_i + (1-\varphi_i)g(0) \\ P(y_i|\mathbf{x}_i) &= (1-\varphi_i)g(y_i), \quad y_i > 0 \end{aligned}$$

where $g(y_i)$ follows either the Poisson or the negative binomial distribution.

If the probability φ_i depends on the characteristics of observation i, then φ_i is written as a function of $\mathbf{z}_i'\boldsymbol{\gamma}$, where $\mathbf{z}_i'$ is the $1 \times (q+1)$ vector of zero-inflated covariates and $\boldsymbol{\gamma}$ is the $(q+1) \times 1$ vector of zero-inflated coefficients to be estimated. (The zero-inflated intercept is γ_0; the coefficients for the q zero-inflated covariates are $\gamma_1, \ldots, \gamma_q$.) The function F that relates the product $\mathbf{z}_i'\boldsymbol{\gamma}$ (which is a scalar) to the probability φ_i is called the zero-inflated link function,

$$\varphi_i = F_i = F(\mathbf{z}_i'\boldsymbol{\gamma})$$

In the HPCOUNTREG procedure, the zero-inflated covariates are indicated in the ZEROMODEL statement. Furthermore, the zero-inflated link function F can be specified as either the logistic function,

$$F(\mathbf{z}_i'\boldsymbol{\gamma}) = \Lambda(\mathbf{z}_i'\boldsymbol{\gamma}) = \frac{\exp(\mathbf{z}_i'\boldsymbol{\gamma})}{1+\exp(\mathbf{z}_i'\boldsymbol{\gamma})}$$

or the standard normal cumulative distribution function (also called the probit function),

$$F(\mathbf{z}_i'\boldsymbol{\gamma}) = \Phi(\mathbf{z}_i'\boldsymbol{\gamma}) = \int_0^{\mathbf{z}_i'\boldsymbol{\gamma}} \frac{1}{\sqrt{2\pi}} \exp(-u^2/2)du$$

The zero-inflated link function is indicated by using the LINK= option in the ZEROMODEL statement. The default ZI link function is the logistic function.

Zero-Inflated Poisson Regression

In the zero-inflated Poisson (ZIP) regression model, the data generation process that is referred to earlier as Process 2 is

$$g(y_i) = \frac{\exp(-\mu_i)\mu_i^{y_i}}{y_i!}$$

where $\mu_i = e^{\mathbf{x}_i'\boldsymbol{\beta}}$. Thus the ZIP model is defined as

$$\begin{aligned} P(y_i = 0|\mathbf{x}_i, \mathbf{z}_i) &= F_i + (1-F_i)\exp(-\mu_i) \\ P(y_i|\mathbf{x}_i, \mathbf{z}_i) &= (1-F_i)\frac{\exp(-\mu_i)\mu_i^{y_i}}{y_i!}, \quad y_i > 0 \end{aligned}$$

The conditional expectation and conditional variance of y_i are given by

$$E(y_i|\mathbf{x}_i, \mathbf{z}_i) = \mu_i(1-F_i)$$

$$V(y_i|\mathbf{x}_i, \mathbf{z}_i) = E(y_i|\mathbf{x}_i, \mathbf{z}_i)(1+\mu_i F_i)$$

Note that the ZIP model (in addition to the ZINB model) exhibits overdispersion because $V(y_i|\mathbf{x}_i, \mathbf{z}_i) > E(y_i|\mathbf{x}_i, \mathbf{z}_i)$.

In general, the log-likelihood function of the ZIP model is

$$\mathcal{L} = \sum_{i=1}^{N} \ln\left[P(y_i|\mathbf{x}_i, \mathbf{z}_i)\right]$$

After a specific link function (either logistic or standard normal) for the probability φ_i is chosen, it is possible to write the exact expressions for the log-likelihood function and the gradient.

ZIP Model with Logistic Link Function

First, consider the ZIP model in which the probability φ_i is expressed by a logistic link function, namely

$$\varphi_i = \frac{\exp(\mathbf{z}_i'\boldsymbol{\gamma})}{1+\exp(\mathbf{z}_i'\boldsymbol{\gamma})}$$

The log-likelihood function is

$$\begin{aligned}\mathcal{L} &= \sum_{\{i:y_i=0\}} \ln\left[\exp(\mathbf{z}_i'\boldsymbol{\gamma}) + \exp(-\exp(\mathbf{x}_i'\boldsymbol{\beta}))\right] \\ &\quad + \sum_{\{i:y_i>0\}} \left[y_i\mathbf{x}_i'\boldsymbol{\beta} - \exp(\mathbf{x}_i'\boldsymbol{\beta}) - \sum_{k=2}^{y_i} \ln(k) \right] \\ &\quad - \sum_{i=1}^{N} \ln\left[1+\exp(\mathbf{z}_i'\boldsymbol{\gamma})\right]\end{aligned}$$

ZIP Model with Standard Normal Link Function

Next, consider the ZIP model in which the probability φ_i is expressed by a standard normal link function: $\varphi_i = \Phi(\mathbf{z}_i'\boldsymbol{\gamma})$. The log-likelihood function is

$$\begin{aligned}\mathcal{L} &= \sum_{\{i:y_i=0\}} \ln\left\{\Phi(\mathbf{z}_i'\boldsymbol{\gamma}) + \left[1-\Phi(\mathbf{z}_i'\boldsymbol{\gamma})\right]\exp(-\exp(\mathbf{x}_i'\boldsymbol{\beta}))\right\} \\ &+ \sum_{\{i:y_i>0\}} \left\{ \ln\left[(1-\Phi(\mathbf{z}_i'\boldsymbol{\gamma}))\right] - \exp(\mathbf{x}_i'\boldsymbol{\beta}) + y_i\mathbf{x}_i'\boldsymbol{\beta} - \sum_{k=2}^{y_i} \ln(k) \right\}\end{aligned}$$

For more information about the zero-inflated Poisson regression model, see the section "Zero-Inflated Poisson Regression" on page 630.

Zero-Inflated Conway-Maxwell-Poisson Regression

In the Conway-Maxwell-Poisson regression model, the data generation process is defined as

$$P(Y_i = y_i|\mathbf{x}_i, \mathbf{z}_i) = \frac{1}{Z(\lambda_i,\nu_i)}\frac{\lambda_i^{y_i}}{(y_i!)^{\nu_i}}, \quad y_i = 0, 1, 2, \ldots$$

where the normalization factor is

$$Z(\lambda_i,\nu_i) = \sum_{n=0}^{\infty} \frac{\lambda_i^n}{(n!)^{\nu_i}}$$

and

$$\lambda_i = \exp(\mathbf{x}_i'\boldsymbol{\beta})$$

$$\nu_i = -\exp(\mathbf{g}_i'\delta)$$

The zero-inflated Conway-Maxwell-Poisson model can be written as

$$\begin{aligned} P(y_i|\mathbf{x}_i, \mathbf{z}_i) &= F_i + (1 - F_i)\frac{1}{Z(\lambda_i, \nu_i)}, \quad y_i = 0 \\ P(y_i|\mathbf{x}_i, \mathbf{z}_i) &= (1 - F_i)\frac{1}{Z(\lambda_i, \nu_i)}\frac{\lambda_i^{y_i}}{(y_i!)^{\nu_i}}, \quad y_i > 0 \end{aligned}$$

The conditional expectation and conditional variance of y_i are given respectively by

$$E(y_i|\mathbf{x}_i, \mathbf{z}_i) = (1 - F_i)\frac{1}{Z(\lambda, \nu)}\sum_{j=0}^{\infty}\frac{j\lambda^j}{(j!)^\nu}$$

$$V(y_i|\mathbf{x}_i, \mathbf{z}_i) = (1 - F_i)\frac{1}{Z(\lambda, \nu)}\sum_{j=0}^{\infty}\frac{j^2\lambda^j}{(j!)^\nu} - E(y_i|\mathbf{x}_i, \mathbf{z}_i)^2$$

The general form of the log-likelihood function for the Conway-Maxwell-Poisson zero-inflated model is

$$\mathcal{L} = \sum_{i=1}^{N} w_i \ln[P(y_i|\mathbf{x}_i, \mathbf{z}_i)]$$

Zero-Inflated Conway-Maxwell-Poisson Model with Logistic Link Function

For this model, the probability φ_i is expressed by using a logistic link function as

$$\varphi_i = \Lambda(\mathbf{z}_i'\boldsymbol{\gamma}) = \frac{\exp(\mathbf{z}_i'\boldsymbol{\gamma})}{1 + \exp(\mathbf{z}_i'\boldsymbol{\gamma})}$$

The log-likelihood function is

$$\begin{aligned} \mathcal{L} &= \sum_{\{i:y_i=0\}} w_i \ln\left\{\Lambda(\mathbf{z}_i'\boldsymbol{\gamma}) + \left[1 - \Lambda(\mathbf{z}_i'\boldsymbol{\gamma})\right]\frac{1}{Z(\lambda_i, \nu_i)}\right\} \\ &+ \sum_{\{i:y_i>0\}} w_i \left\{\ln\left[\left(1 - \Lambda(\mathbf{z}_i'\boldsymbol{\gamma})\right)\right] - ln(Z(\lambda, \nu)) + (y_i \ln(\lambda) - \nu \ln(y_i!)\right\} \end{aligned}$$

Zero-Inflated Conway-Maxwell-Poisson Model with Normal Link Function

For this model, the probability φ_i is specified by using the standard normal distribution function (probit function): $\varphi_i = \Phi(\mathbf{z}_i'\boldsymbol{\gamma})$.

The log-likelihood function is written as

$$\begin{aligned} \mathcal{L} &= \sum_{\{i:y_i=0\}} w_i \ln\left\{\Phi(\mathbf{z}_i'\boldsymbol{\gamma}) + \left[1 - \Phi(\mathbf{z}_i'\boldsymbol{\gamma})\right]\frac{1}{Z(\lambda_i, \nu_i)}\right\} \\ &+ \sum_{\{i:y_i>0\}} w_i \left\{\ln\left[\left(1 - \Phi(\mathbf{z}_i'\boldsymbol{\gamma})\right)\right] - ln(Z(\lambda, \nu)) + (y_i \ln(\lambda) - \nu \ln(y_i!)\right\} \end{aligned}$$

Zero-Inflated Negative Binomial Regression

The zero-inflated negative binomial (ZINB) model in PROC HPCOUNTREG is based on the negative binomial model that has a quadratic variance function (when DIST=NEGBIN in the MODEL or PROC HPCOUNTREG statement). The ZINB model is obtained by specifying a negative binomial distribution for the data generation process referred to earlier as Process 2:

$$g(y_i) = \frac{\Gamma(y_i + \alpha^{-1})}{y_i!\Gamma(\alpha^{-1})} \left(\frac{\alpha^{-1}}{\alpha^{-1} + \mu_i}\right)^{\alpha^{-1}} \left(\frac{\mu_i}{\alpha^{-1} + \mu_i}\right)^{y_i}$$

Thus the ZINB model is defined to be

$$\begin{aligned} P(y_i = 0|\mathbf{x}_i, \mathbf{z}_i) &= F_i + (1 - F_i)(1 + \alpha\mu_i)^{-\alpha^{-1}} \\ P(y_i|\mathbf{x}_i, \mathbf{z}_i) &= (1 - F_i)\frac{\Gamma(y_i + \alpha^{-1})}{y_i!\Gamma(\alpha^{-1})}\left(\frac{\alpha^{-1}}{\alpha^{-1} + \mu_i}\right)^{\alpha^{-1}} \\ &\times \left(\frac{\mu_i}{\alpha^{-1} + \mu_i}\right)^{y_i}, \quad y_i > 0 \end{aligned}$$

In this case, the conditional expectation (E) and conditional variance (V) of y_i are

$$E(y_i|\mathbf{x}_i, \mathbf{z}_i) = \mu_i(1 - F_i)$$

$$V(y_i|\mathbf{x}_i, \mathbf{z}_i) = E(y_i|\mathbf{x}_i, \mathbf{z}_i)\,[1 + \mu_i(F_i + \alpha)]$$

Like the ZIP model, the ZINB model exhibits overdispersion because the conditional variance exceeds the conditional mean.

ZINB Model with Logistic Link Function

In this model, the probability φ_i is given by the logistic function, namely

$$\varphi_i = \frac{\exp(\mathbf{z}_i'\boldsymbol{\gamma})}{1 + \exp(\mathbf{z}_i'\boldsymbol{\gamma})}$$

The log-likelihood function is

$$\begin{aligned} \mathcal{L} &= \sum_{\{i:y_i=0\}} \ln\left[\exp(\mathbf{z}_i'\boldsymbol{\gamma}) + (1 + \alpha\exp(\mathbf{x}_i'\boldsymbol{\beta}))^{-\alpha^{-1}}\right] \\ &+ \sum_{\{i:y_i>0\}} \sum_{j=0}^{y_i-1} \ln(j + \alpha^{-1}) \\ &+ \sum_{\{i:y_i>0\}} \left\{-\ln(y_i!) - (y_i + \alpha^{-1})\ln(1 + \alpha\exp(\mathbf{x}_i'\boldsymbol{\beta})) + y_i\ln(\alpha) + y_i\mathbf{x}_i'\boldsymbol{\beta}\right\} \\ &- \sum_{i=1}^{N} \ln\left[1 + \exp(\mathbf{z}_i'\boldsymbol{\gamma})\right] \end{aligned}$$

ZINB Model with Standard Normal Link Function

For this model, the probability φ_i is expressed by the standard normal distribution function (probit function): $\varphi_i = \Phi(\mathbf{z}_i'\boldsymbol{\gamma})$. The log-likelihood function is

$$
\begin{aligned}
\mathcal{L} = & \sum_{\{i:y_i=0\}} \ln\left\{\Phi(\mathbf{z}_i'\boldsymbol{\gamma}) + \left[1-\Phi(\mathbf{z}_i'\boldsymbol{\gamma})\right](1+\alpha\exp(\mathbf{x}_i'\boldsymbol{\beta}))^{-\alpha^{-1}}\right\} \\
& + \sum_{\{i:y_i>0\}} \ln\left[1-\Phi(\mathbf{z}_i'\boldsymbol{\gamma})\right] \\
& + \sum_{\{i:y_i>0\}} \sum_{j=0}^{y_i-1} \{\ln(j+\alpha^{-1})\} \\
& - \sum_{\{i:y_i>0\}} \ln(y_i!) \\
& - \sum_{\{i:y_i>0\}} (y_i+\alpha^{-1})\ln(1+\alpha\exp(\mathbf{x}_i'\boldsymbol{\beta})) \\
& + \sum_{\{i:y_i>0\}} y_i\ln(\alpha) \\
& + \sum_{\{i:y_i>0\}} y_i\mathbf{x}_i'\boldsymbol{\beta}
\end{aligned}
$$

For more information about the zero-inflated negative binomial regression model, see the section "Zero-Inflated Negative Binomial Regression" on page 634.

Parameter Naming Conventions for the RESTRICT, TEST, BOUNDS, and INIT Statements

This section describes how you can refer to the parameters in the MODEL, ZEROMODEL, and DISPMODEL statements when you use the RESTRICT, TEST, BOUNDS, or INIT statement. The following examples use the RESTRICT statement, but the same remarks apply to naming parameters when you use the TEST, BOUNDS, or INIT statement. The names of the parameters can be seen in the OUTEST= data set.

To impose a restriction on a parameter that is related to a regressor in the MODEL statement, you simply use the name of the regressor itself to refer to its associated parameter. Suppose your model is:

```
model y = x1 x2 x5;
```

where x1 through x5 are continuous variables. If you want to restrict the parameter associated with the regressor x5 to be greater than 1.7, then you should the following statement:

```
RESTRICT x5 > 1.7;
```

To impose a restriction on a parameter associated with a regressor in the ZEROMODEL statement, you can form the name of the parameter by prefixing Inf_ to the name of the regressor. Suppose your MODEL and ZEROMODEL statements are as follows:

```
model y = x1 x2 x5;
zeromodel y ~ x3 x5;
```

If you want to restrict the parameter related to the x5 regressor in the ZEROMODEL statement to be less than 1.0, then you refer to the parameter as Inf_x5 and provide the following statement:

```
RESTRICT Inf_x5 < 1.0;
```

Even though the regressor x5 appears in both the MODEL and ZEROMODEL statements, the parameter associated with x5 in the MODEL statement is, of course, different from the parameter associated with x5 in the ZEROMODEL statement. Thus, when the name of a regressor is used in a RESTRICT statement without any prefix, it refers to the parameter associated with that regressor in the MODEL statement. Meanwhile, when the name of a regressor is used in a RESTRICT statement with the prefix Inf_, it refers to the parameter associated with that regressor in the ZEROMODEL statement. The parameter associated with the intercept in the ZEROMODEL is named Inf_Intercept.

In a similar way, you can form the name of a parameter associated with a regressor in the DISPMODEL statement by prefixing Dsp_ to the name of the regressor. The parameter associated with the intercept in the DISPMODEL is named Dsp_Intercept.

Referring to Class-Level Parameters

When your MODEL includes a classification variable, you can impose restrictions on the parameters associated with each of the levels that are related to the classification variable as follows.

Suppose your classification variable is named C and that it has three levels: 0, 1, 2. Suppose your model is the following:

```
class C;
model y = x1 x2 C;
```

Adding a classification variable as a regressor to your model introduces additional parameters into your model, each of which is associated with one of the levels of the classification variable. You can form the name of the parameter associated with a particular level of your class variable by inserting the underscore character between the name of the classification variable and the value of the level. Thus, to restrict the parameter associated with level 0 of the classification variable C to always be greater than 0.7, you refer to the parameter as C_0 and provide the following statement:

```
RESTRICT C_0 > 0.7;
```

Referring to Parameters Associated with Interactions between Regressors

When a regressor in your model involves an interaction between other regressors, you can impose restrictions on the parameters associated with the interaction.

Suppose you have the following model:

```
model y = x1 x2 x3*x4;
```

You can form the name of the parameter associated with the interaction regressor x3*x4 by replacing the multiplication sign with an underscore. Thus, x3_x4 refers to the parameter that is associated with the interaction regressor x3*x4.

Referring to interactions between regressors and classification variables is handled in the same way. Suppose you have a classification variable that is named C and has three levels: 0, 1, 2. Suppose that your model is the following:

```
class C;
model y = x1 x2 C*x3;
```

The interaction between the continuous variable x3 and the classification variable C introduces three additional parameters, which are named: x3_C_0, x3_C_1, and x3_C_2. Note how, although the order of the terms in the interaction is C followed by x3, the name of the parameter associated with the interaction is formed by placing the name of the continuous variable x3 first, followed by an underscore, followed by the name of the classification variable C, followed by an underscore, and then followed by the level value. Once again, depending on the parameterization you specify in your CLASS statement, for each interaction in your model that involves a classification variable, one of the parameters associated with that interaction might be dropped from your model prior to optimization.

The name of a parameter associated with a nested interaction is formed in a slightly different way. Suppose you have a classification variable that is named C and has three levels: 0, 1, 2. Suppose that your model is the following:

```
class C;
model y = x1 x2 x3(C);
```

The nested interaction between the continuous variable x3 and the classification variable C introduces three additional parameters, which are named: x3_C__0, x3_C__1, and x3_C__2. Note how the name in each case is formed from the name of the regressor by replacing the left and right parentheses with underscores and then appending another underscore followed by the level value.

Referring to Class Level Parameters with Negative Values

When the value of a level is a negative number, you must replace the minus sign with an underscore when you form the name of the parameter that is associated with that particular level of the classification variable. For example, suppose your classification variable is named D and has four levels: –1, 0, 1, 2. Suppose your model is the following:

```
class D;
model y = x1 x2 D;
```

To restrict the parameter that is associated with level –1 of the classification variable D to always be less than 0.4, you refer to the parameter as D__1 (note that there are two underscores in this parameter name: one to connect the name of the classification variable to its value and the other to replace the minus sign in the value itself) and provide following statement:

```
RESTRICT D__1 < 0.4;
```

Dropping a Class Level Parameter to Avoid Collinearity

Depending on the parameterization you impose on your classification variable, one of the parameters associated with its levels might be dropped from your model prior to optimization in order to avoid collinearity. For example, when the default parameterization GLM is imposed, the parameter that is associated with the last level of your classification variable is dropped prior to optimization. If you attempt to impose a restriction on a dropped parameter by using the RESTRICT statement, PROC COUNTREG issues an error message in the log.

For example, suppose again that your classification variable is named C and that it has three levels: 0, 1, 2. Suppose your model is the following:

```
class C;
model y = x1 x2 C;
```

Because no additional options are specified in the CLASS statement, GLM parameterization is assumed. This means that the parameter named C_2 (which is the parameter associated with the last level of your classification variable) will be dropped from your model before the optimizer is invoked. Therefore, an error will be issued if you attempt to restrict the C_2 parameter in any way by referring to it in a RESTRICT statement. For example, the following RESTRICT statement will generate an error:

```
RESTRICT C_2 < 0.3;
```

Referring to Implicit Parameters

For certain model types, one or more implicit parameters will be added to your model prior to optimization. You can impose restrictions on these implicit parameters.

For the Poisson model for which ERRORCOMP=RANDOM is specified, PROC COUNTREG automatically adds the _Alpha parameter to your model.

If no ERRORCOMP= option is specified, for zero-inflated binomial and negative binomial models, PROC COUNTREG adds the _Alpha parameter to the model. If ERRORCOMP=RANDOM is specified for the zero-inflated binomial and negative binomial models, then PROC COUNTREG adds two implicit parameters to the model: _Alpha and _Beta.

For Conway-Maxwell Poisson models that do not include a DISPMODEL statement, the _lnNu parameter is added to the model.

Whenever your model type dictates the addition of one or more of these implicit parameters, you can impose restrictions on the implicit parameters by referring to them by name in a RESTRICT statement. For example, if your model type implies the existence of the _Alpha parameter, you can restrict _Alpha to be greater than 0.2 as follows:

```
RESTRICT _Alpha > 0.2;
```

Computational Resources

The time and memory that PROC HPCOUNTREG requires are proportional to the number of parameters in the model and the number of observations in the data set being analyzed. Less time and memory are required for smaller models and fewer observations. When PROC HPCOUNTREG is run in the high-performance distributed environment, the amount of time required is also affected by the number of nodes and the number of threads per node as specified in the PERFORMANCE statement.

The method that is chosen to calculate the variance-covariance matrix and the optimization method also affect the time and memory resources. All optimization methods available through the METHOD= option have similar memory use requirements. The processing time might differ for each method, depending on the number of iterations and functional calls needed. The data set is read into memory to save processing time. If not enough memory is available to hold the data, the HPCOUNTREG procedure stores the data in a utility file on disk and rereads the data as needed from this file, substantially increasing the execution time of the procedure. The gradient and the variance-covariance matrix must be held in memory. If the model has p parameters including the intercept, then at least $8 * (p + p * (p + 1)/2)$ bytes of memory are needed. The processing time is also a function of the number of iterations needed to converge to a solution for the model parameters. The number of iterations that are needed cannot be known in advance. You can use the MAXITER= option to limit the number of iterations that PROC HPCOUNTREG executes. You can alter the convergence criteria by using the nonlinear optimization options available in the PROC HPCOUNTREG statement. For a list of all the nonlinear optimization options, see "Optimization Control Options" on page 1080.

Covariance Matrix Types

The COVEST= option in the PROC HPCOUNTREG statement enables you to specify the estimation method for the covariance matrix. COVEST=HESSIAN estimates the covariance matrix that is based on the inverse of the Hessian matrix; COVEST=OP uses the outer product of gradients; and COVEST=QML produces the covariance matrix that is based on both the Hessian and outer product matrices. Although all three methods produce asymptotically equivalent results, they differ in computational intensity and produce results that might differ in finite samples. The COVEST=OP option provides the covariance matrix that is typically the easiest to compute. In some cases, the OP approximation is considered more efficient than the Hessian or QML approximation because it contains fewer random elements. The QML approximation is computationally the most complex because it requires both the outer product of gradients and the Hessian matrix. In most cases, the OP or Hessian approximation is preferred to QML. The need for QML approximation arises in cases where the model is misspecified and the information matrix equality does not hold. The default is COVEST=HESSIAN.

Displayed Output

PROC HPCOUNTREG produces the following displayed output.

Model Fit Summary

The "Model Fit Summary" table contains the following information:

- dependent (count) variable name
- number of observations used
- number of missing values in data set, if any
- data set name
- type of model that was fit
- parameterization for the Conway-Maxwell-Poisson model
- offset variable name, if any
- zero-inflated link function, if any
- zero-inflated offset variable name, if any
- log-likelihood value at solution
- maximum absolute gradient at solution
- number of iterations
- AIC value at solution (smaller value indicates better fit)
- SBC value at solution (smaller value indicates better fit)

A line in the "Model Fit Summary" table indicates whether the algorithm successfully converged.

Parameter Estimates

The "Parameter Estimates" table gives the estimates of the model parameters. In zero-inflated (ZI) models, estimates are also given for the ZI intercept and ZI regressor parameters, which are labeled with the prefix "Inf_". For example, the ZI intercept is labeled "Inf_intercept". If you specify "Age" as a ZI regressor, then the "Parameter Estimates" table labels the corresponding parameter estimate "Inf_Age". If you do not list any ZI regressors, then only the ZI intercept term is estimated.

If the DISPMODEL statement is specified for the Conway-Maxwell-Poisson model, the estimates are given for the dispersion intercept, and parameters are labeled with the prefix "Dsp_". For example, the dispersion model intercept is labeled "Dsp_Intercept". If you specify "Education" as a dispersion model regressor, then the "Parameter Estimates" table labels the corresponding parameter estimate "Dsp_Education". If you do not list any dispersion regressors, then only the dispersion intercept is estimated.

"_Alpha" is the negative binomial dispersion parameter. The *t* statistic that is given for "_Alpha" is a test of overdispersion.

Covariance of Parameter Estimates

If you specify the COVB option in the PROC HPCOUNTREG or MODEL statement, the HPCOUNTREG procedure displays the estimated covariance matrix, which is defined as the inverse of the information matrix at the final iteration.

Correlation of Parameter Estimates

If you specify the CORRB option in the PROC HPCOUNTREG or MODEL statement, the HPCOUNTREG procedure displays the estimated correlation matrix, which is based on the Hessian matrix used at the final iteration.

OUTPUT OUT= Data Set

The OUTPUT statement creates a new SAS data set that contains various estimates that you specify. You can request that the output data set contain the estimates of $\mathbf{x}_i'\boldsymbol{\beta}$, the expected value of the response variable, and the probability that the response variable will take the current value. In a zero-inflated model, you can also request that the output data set contain the estimates of $\mathbf{z}_i'\boldsymbol{\gamma}$, and the probability that the response is zero as a result of the zero-generating process. In a Conway-Maxwell-Poisson model, you can also request that the output data set contains estimates of $\mathbf{g}_i'\boldsymbol{\delta}$, λ, ν, μ, mode, variance and dispersion.

Except for the probability of the current value, these statistics can be computed for all observations in which the regressors are not missing, even if the response is missing. By adding observations with missing response values to the input data set, you can compute these statistics for new observations or for settings of the regressors that are not present in the data without affecting the model fit. Because of potential space limitations on the client workstation, the data set that is created by the OUTPUT statement does not contain the variables in the input data set.

OUTEST= Data Set

The OUTEST= data set is made up of at least two rows: the first row (with _TYPE_=‘PARM’) contains each of the parameter estimates in the model, and the second row (with _TYPE_=‘STD’) contains the standard errors for the parameter estimates in the model.

If you use the COVOUT option in the PROC HPCOUNTREG statement, the OUTEST= data set also contains the covariance matrix for the parameter estimates. The covariance matrix appears in the observations with _TYPE_=‘COV’, and the _NAME_ variable labels the rows with the parameter names.

ODS Table Names

PROC HPCOUNTREG assigns a name to each table that it creates. You can use these names to denote the table when you use the Output Delivery System (ODS) to select tables and create output data sets. These table names are listed in Table 20.2.

Table 20.2 ODS Tables Produced in PROC HPCOUNTREG

ODS Table Name	Description	Option
ODS Tables Created by the MODEL Statement		
FitSummary	Summary of nonlinear estimation	Default
ConvergenceStatus	Convergence status	Default
ParameterEstimates	Parameter estimates	Default
CovB	Covariance of parameter estimates	COVB
CorrB	Correlation of parameter estimates	CORRB

Examples: The HPCOUNTREG Procedure

Example 20.1: High-Performance Zero-Inflated Poisson Model

This example shows the use of the HPCOUNTREG procedure with an emphasis on large data set processing and the performance improvements that are achieved by executing in the high-performance distributed environment.

The following DATA step generates one million replicates from the zero-inflated Poisson (ZIP) model. The model contains seven variables and three variables that correspond to the zero-inflated process.

```
data simulate;
   call streaminit(12345);
   array vars x1-x7;
   array zero_vars z1-z3;

   array parms{7}  (.3 .4 .2 .4 -.3 -.5 -.3);
   array zero_parms{3} (-.6 .3 .2);

   intercept=2;
   z_intercept=-1;
   theta=0.5;

   do i=1 to 1000000;
      sum_xb=0;
      sum_gz=0;
      do j=1 to 7;
         vars[j]=rand('NORMAL',0,1);
         sum_xb=sum_xb+parms[j]*vars[j];
      end;
      mu=exp(intercept+sum_xb);
      y_p=rand('POISSON', mu);

      do j=1 to 3;
```

```
               zero_vars[j]=rand('NORMAL',0,1);
               sum_gz = sum_gz+zero_parms[j]*zero_vars[j];
            end;
            z_gamma = z_intercept+sum_gz;
            pzero = cdf('LOGISTIC',z_gamma);
            cut=rand('UNIFORM');
            if cut<pzero then y_p=0;
            output;
         end;
      keep y_p x1-x7 z1-z3;
      run;
```

The following statements estimate a zero-inflated Poisson model.

```
   option set=GRIDHOST="&GRIDHOST";
   option set=GRIDINSTALLLOC="&GRIDINSTALLLOC";

   proc hpcountreg data=simulate dist=zip;
      performance nthreads=2 nodes=1 details
              host="&GRIDHOST" install="&GRIDINSTALLLOC";
      model y_p=x1-x7;
      zeromodel y_p ~ z1-z3;
   run;
```

The model is executed in the distributed computing environment on two threads and only one node. These settings are used to obtain a hypothetical environment that might resemble running the HPCOUNTREG procedure on a desktop workstation with a dual-core CPU. To run these statements successfully, you need to set the macro variables GRIDHOST and GRIDINSTALLLOC to resolve to appropriate values, or you can replace the references to the macro variables in the example with the appropriate values. Output 20.1.1 shows the “Performance Information” table for this hypothetical scenario.

Output 20.1.1 Performance Information with One Node and One Thread

Performance Information	
Host Node	<< your grid host >>
Install Location	<< your grid install location >>
Execution Mode	Distributed
Number of Compute Nodes	1
Number of Threads per Node	2

Output 20.1.2 shows the results for the zero-inflated Poisson model. The “Model Fit Summary” table shows detailed information about the model and indicates that all one million observations were used to fit the model. All parameter estimates in the “Parameter Estimates” table are highly significant and correspond to their theoretical values set during the data generating process. The optimization of the model that contains one million observations took 40.77 seconds.

Output 20.1.2 Zero-Inflated Poisson Model Execution on One Node and Two Threads

Model Fit Summary	
Dependent Variable	y_p
Number of Observations	1000000
Data Set	WORK.SIMULATE
Model	ZIP
ZI Link Function	Logistic
Log Likelihood	-2215238
Maximum Absolute Gradient	2.044E-8
Number of Iterations	7
Optimization Method	Newton-Raphson
AIC	4430500
SBC	4430642

Convergence criterion (FCONV=2.220446E-16) satisfied.

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	2.0005	0.000492	4069.80	<.0001
x1	1	0.2995	0.000352	850.17	<.0001
x2	1	0.3998	0.000353	1132.23	<.0001
x3	1	0.2008	0.000352	570.27	<.0001
x4	1	0.3994	0.000353	1132.85	<.0001
x5	1	-0.2995	0.000353	-848.95	<.0001
x6	1	-0.5000	0.000353	-1414.9	<.0001
x7	1	-0.3002	0.000352	-852.14	<.0001
Inf_Intercept	1	-0.9993	0.002521	-396.45	<.0001
Inf_z1	1	-0.6024	0.002585	-233.02	<.0001
Inf_z2	1	0.2976	0.002454	121.25	<.0001
Inf_z3	1	0.1974	0.002430	81.20	<.0001

Procedure Task Timing

Task	Seconds	Percent
Reading and Levelizing Data	0.32	0.78%
Communication to Client	0.03	0.06%
Optimization	40.77	98.52%
Post-Optimization	0.26	0.63%

In the following statements, the PERFORMANCE statement is modified to use a grid with 10 nodes, with each node capable of spawning eight threads:

```
proc hpcountreg data=simulate dist=zip;
   performance nthreads=8 nodes=10 details
               host="&GRIDHOST" install="&GRIDINSTALLLOC";
   model y_p=x1-x7;
   zeromodel y_p ~ z1-z3;
run;
```

Because the two models being estimated are identical, it is reasonable to expect that Output 20.1.2 and Output 20.1.3 would show the same results. However, you can see a significant difference in performance between the two models. The second model, which was run on a grid that used 10 nodes with eight threads each, took only 3.54 seconds instead of 40.77 seconds to optimize.

In certain circumstances, you might observe slight numerical differences in the results, depending on the number of nodes and threads involved. This happens because the order in which partial results are accumulated can make a difference in the final result, owing to the limits of numerical precision and the propagation of error in numerical computations.

Output 20.1.3 Zero-Inflated Poisson Model Execution on 10 Nodes with Eight Threads Each

The HPCOUNTREG Procedure

Model Fit Summary

Dependent Variable	y_p
Number of Observations	1000000
Data Set	WORK.SIMULATE
Model	ZIP
ZI Link Function	Logistic
Log Likelihood	-2215238
Maximum Absolute Gradient	2.0608E-8
Number of Iterations	7
Optimization Method	Newton-Raphson
AIC	4430500
SBC	4430642

Convergence criterion (FCONV=2.220446E-16) satisfied.

Parameter Estimates

Parameter	DF	Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	2.0005	0.000492	4069.80	<.0001
x1	1	0.2995	0.000352	850.17	<.0001
x2	1	0.3998	0.000353	1132.23	<.0001
x3	1	0.2008	0.000352	570.27	<.0001
x4	1	0.3994	0.000353	1132.85	<.0001
x5	1	-0.2995	0.000353	-848.95	<.0001
x6	1	-0.5000	0.000353	-1414.9	<.0001
x7	1	-0.3002	0.000352	-852.14	<.0001
Inf_Intercept	1	-0.9993	0.002521	-396.45	<.0001
Inf_z1	1	-0.6024	0.002585	-233.02	<.0001
Inf_z2	1	0.2976	0.002454	121.25	<.0001
Inf_z3	1	0.1974	0.002430	81.20	<.0001

Procedure Task Timing

Task	Seconds	Percent
Reading and Levelizing Data	0.02	0.61%
Communication to Client	0.06	1.44%
Optimization	3.54	90.99%
Post-Optimization	0.27	6.96%

As this example suggests, increasing the number of nodes and the number of threads per node improves performance significantly. When you use the parallelism afforded by a high-performance distributed environment, you can see an even more dramatic reduction in the time required for the optimization as the number of observations in the data set increases. When the data set is extremely large, the computations might not even be possible in some cases, given the typical memory resources and computational constraints of a desktop computer. Under such circumstances the high-performance distributed environment becomes a necessity.

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