SAS/ETS® 14.1 User's Guide
The ENTROPY Procedure

This document is an individual chapter from *SAS/ETS® 14.1 User's Guide*.

The correct bibliographic citation for this manual is as follows: SAS Institute Inc. 2015. *SAS/ETS® 14.1 User's Guide*. Cary, NC: SAS Institute Inc.

SAS/ETS® 14.1 User's Guide

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July 2015

Chapter 14
The ENTROPY Procedure (Experimental)

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Overview: ENTROPY Procedure

The ENTROPY procedure implements a parametric method of linear estimation based on generalized maximum entropy. The ENTROPY procedure is suitable when there are outliers in the data and robustness is required, when the model is ill-posed or under-determined for the observed data, or for regressions that involve small data sets.

The main features of the ENTROPY procedure are as follows:

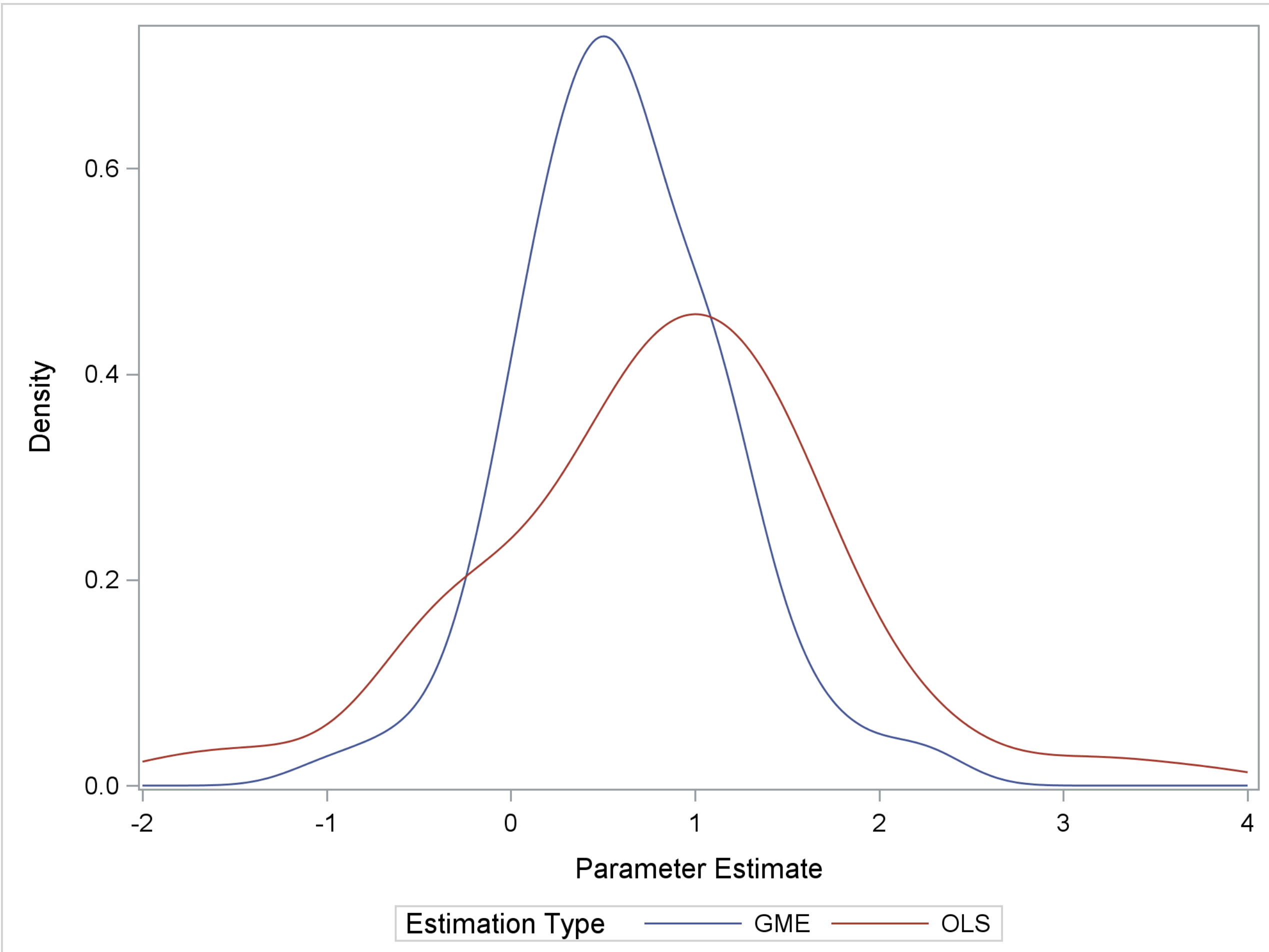
- estimation of simultaneous systems of linear regression models
- estimation of Markov models
- estimation of seemingly unrelated regression (SUR) models
- estimation of unordered multinomial discrete Choice models
- solution of pure inverse problems
- allowance of bounds and restrictions on parameters
- performance of tests on parameters
- allowance of data and moment constrained generalized cross entropy

It is often the case that the statistical/economic model of interest is ill-posed or under-determined for the observed data. For the general linear model, this can imply that high degrees of collinearity exist among explanatory variables or that there are more parameters to estimate than observations available to estimate them. These conditions lead to high variances or non-estimability for traditional generalized least squares (GLS) estimates.

Under these situations it might be in the researcher's or practitioner's best interest to consider a nontraditional technique for model fitting. The principle of maximum entropy is the foundation for an estimation methodology that is characterized by its robustness to ill-conditioned designs and its ability to fit over-parameterized models. See Mittelhammer, Judge, and Miller (2000) and Golan, Judge, and Miller (1996) for a discussion of Shannon's maximum entropy measure and the related Kullback-Leibler information.

Generalized maximum entropy (GME) is a means of selecting among probability distributions to choose the distribution that maximizes uncertainty or uniformity remaining in the distribution, subject to information already known about the distribution. Information takes the form of data or moment constraints in the estimation procedure. PROC ENTROPY creates a GME distribution for each parameter in the linear model, based upon support points supplied by the user. The mean of each distribution is used as the estimate of the parameter. Estimates tend to be biased, as they are a type of shrinkage estimate, but typically portray smaller variances than ordinary least squares (OLS) counterparts, making them more desirable from a mean squared error viewpoint (see Figure 14.1).

Figure 14.1 Distribution of Maximum Entropy Estimates versus OLS



Maximum entropy techniques are most widely used in the econometric and time series fields. Some important uses of maximum entropy include the following:

- size distribution of firms
- stationary Markov Process
- social accounting matrix (SAM)
- consumer brand preference
- exchange rate regimes
- wage-dependent firm relocation
- oil market dynamics

Getting Started: ENTROPY Procedure

This section introduces the ENTROPY procedure and shows how to use PROC ENTROPY for several kinds of statistical analyses.

Simple Regression Analysis

The ENTROPY procedure is similar in syntax to the other regression procedures in SAS. To demonstrate the similarity, suppose the endogenous/dependent variable is y, and x1 and x2 are two exogenous/independent variables of interest. To estimate the parameters in this single equation model using PROC ENTROPY, use the following SAS statements:

```
proc entropy;
   model y = x1 x2;
run;
```

Test Scores Data Set

Consider the following test score data compiled by Coleman et al. (1966):

```
title "Test Scores compiled by Coleman et al. (1966)";
data coleman;
   input test_score 6.2 teach_sal 6.2 prcnt_prof 8.2
         socio_stat 9.2 teach_score 8.2 mom_ed 7.2;
   label test_score="Average sixth grade test scores in observed district";
   label teach_sal="Average teacher salaries per student (1000s of dollars)";
   label prcnt_prof="Percent of students' fathers with professional employment";
   label socio_stat="Composite measure of socio-economic status in the district";
   label teach_score="Average verbal score for teachers";
   label mom_ed="Average level of education (years) of the students' mothers";
datalines;
37.01   3.83   28.87     7.20   26.60   6.19

   ... more lines ...
```

This data set contains outliers, and the condition number of the matrix of regressors, **X**, is large, which indicates collinearity among the regressors. Since the maximum entropy estimates are both robust with respect to the outliers and also less sensitive to a high condition number of the **X** matrix, maximum entropy estimation is a good choice for this problem.

To fit a simple linear model to this data by using PROC ENTROPY, use the following statements:

```
proc entropy data=coleman;
   model test_score = teach_sal prcnt_prof socio_stat teach_score mom_ed;
run;
```

This requests the estimation of a linear model for TEST_SCORE with the following form:

$$\begin{aligned} test\_score &= intercept + a * teach\_sal + b * prcnt\_prof + c * socio\_stat \\ &\quad + d * teach\_score + e * mom\_ed + \epsilon; \end{aligned}$$

This estimation produces the "Model Summary" table in Figure 14.2, which shows the equation variables used in the estimation.

Figure 14.2 Model Summary Table

Test Scores compiled by Coleman et al. (1966)

The ENTROPY Procedure

Variables(Supports(Weights))	teach_sal prcnt_prof socio_stat teach_score mom_ed Intercept
Equations(Supports(Weights))	test_score

Since support points and prior weights are not specified in this example, they are not shown in the "Model Summary" table. The next four pieces of information displayed in Figure 14.3 are: the "Data Set Options," the "Minimization Summary," the "Final Information Measures," and the "Observations Processed."

Figure 14.3 Estimation Summary Tables

Test Scores compiled by Coleman et al. (1966)

The ENTROPY Procedure
GME Estimation Summary

Data Set Options	
DATA=	WORK.COLEMAN

Minimization Summary	
Parameters Estimated	6
Covariance Estimator	GME
Entropy Type	Shannon
Entropy Form	Dual
Numerical Optimizer	Quasi Newton

Final Information Measures	
Objective Function Value	9.553699
Signal Entropy	9.569484
Noise Entropy	-0.01578
Normed Entropy (Signal)	0.990976
Normed Entropy (Noise)	0.999786
Parameter Information Index	0.009024
Error Information Index	0.000214

Observations Processed	
Read	20
Used	20

The item labeled “Objective Function Value” is the value of the entropy estimation criterion for this estimation problem. This measure is analogous to the log-likelihood value in a maximum likelihood estimation. The “Parameter Information Index” and the “Error Information Index” are normalized entropy values that measure the proximity of the solution to the prior or target distributions.

The next table displayed is the ANOVA table, shown in Figure 14.4. This is in the same form as the ANOVA table for the MODEL procedure, since this is also a multivariate procedure.

Figure 14.4 Summary of Residual Errors

GME Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj RSq
test_score	6	14	175.8	8.7881	2.9645	0.7266	0.6290

The last table displayed is the “Parameter Estimates” table, shown in Figure 14.5. The difference between this parameter estimates table and the parameter estimates table produced by other regression procedures is that the standard error and the probabilities are labeled as approximate.

Figure 14.5 Parameter Estimates

GME Variable Estimates

Variable	Estimate	Approx Std Err	t Value	Approx Pr > |t|
teach_sal	0.287979	0.00551	52.26	<.0001
prcnt_prof	0.02266	0.00323	7.01	<.0001
socio_stat	0.199777	0.0308	6.48	<.0001
teach_score	0.497137	0.0180	27.61	<.0001
mom_ed	1.644472	0.0921	17.85	<.0001
Intercept	10.5021	0.3958	26.53	<.0001

The parameter estimates produced by the REG procedure for this same model are shown in Figure 14.6. Note that the parameters and standard errors from PROC REG are much different than estimates produced by PROC ENTROPY.

```
symbol v=dot h=1 c=green;

proc reg data=coleman;
   model test_score = teach_sal prcnt_prof socio_stat teach_score mom_ed;
   plot rstudent.*obs.
        / vref= -1.714 1.714 cvref=blue lvref=1
          HREF=0 to 30 by 5 cHREF=red  cframe=ligr;
run;
```

Figure 14.6 REG Procedure Parameter Estimates

Test Scores compiled by Coleman et al. (1966)

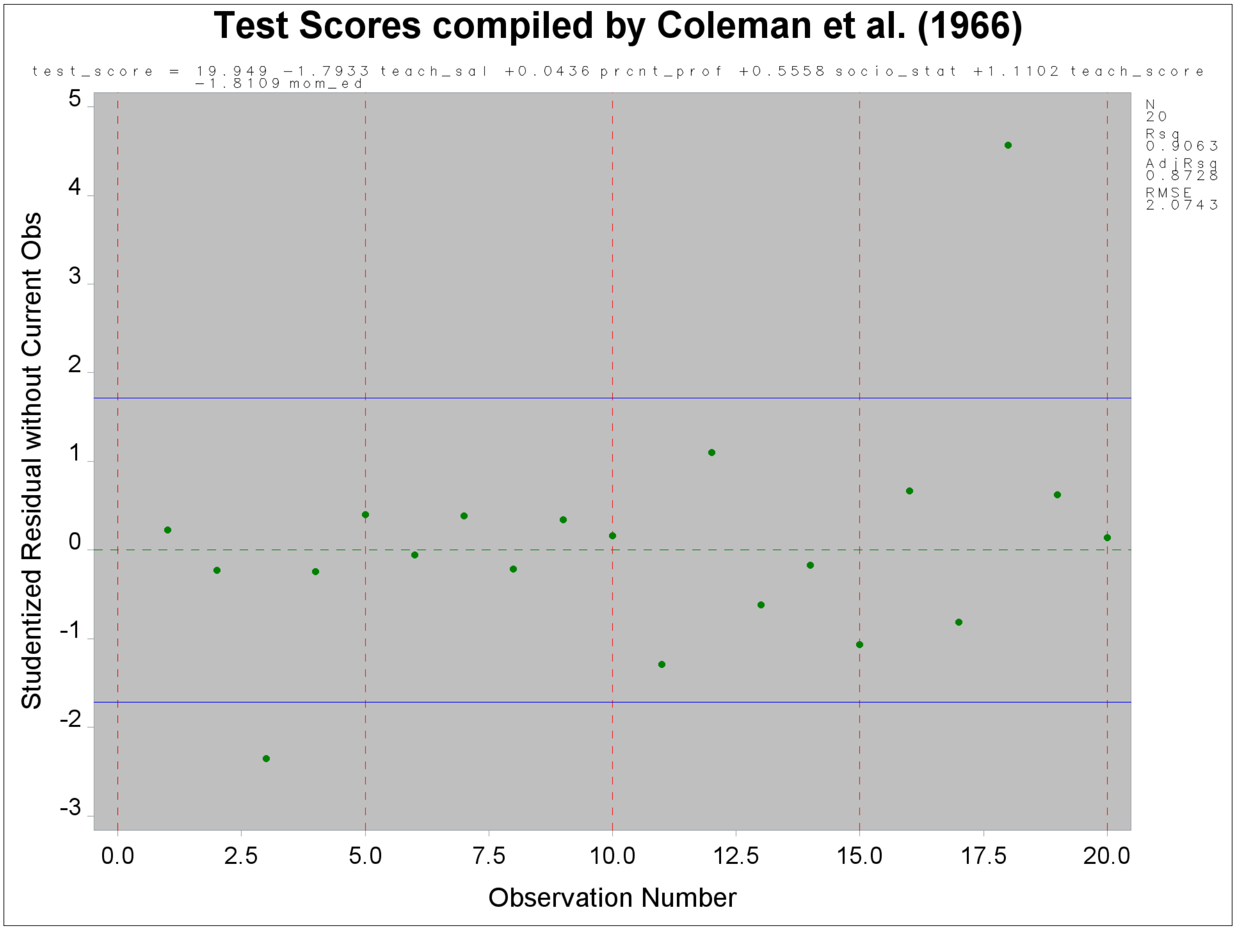
The REG Procedure
Model: MODEL1
Dependent Variable: test_score

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	19.94857	13.62755	1.46	0.1653
teach_sal	1	-1.79333	1.23340	-1.45	0.1680
prcnt_prof	1	0.04360	0.05326	0.82	0.4267
socio_stat	1	0.55576	0.09296	5.98	<.0001
teach_score	1	1.11017	0.43377	2.56	0.0227
mom_ed	1	-1.81092	2.02739	-0.89	0.3868

This data set contains two outliers, observations 3 and 18. These can be seen in a plot of the residuals shown in Figure 14.7

Figure 14.7 PROC REG Residuals with Outliers



The presence of outliers suggests that a robust estimator such as *M*-estimator in the ROBUSTREG procedure should be used. The following statements use the ROBUSTREG procedure to estimate the model.

```
proc robustreg data=coleman;
   model test_score = teach_sal prcnt_prof
                      socio_stat teach_score mom_ed;
run;
```

The results of the estimation are shown in Figure 14.8.

Figure 14.8 *M*-Estimation Results

Test Scores compiled by Coleman et al. (1966)

The ROBUSTREG Procedure

Parameter Estimates							
Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	29.3416	6.0381	17.5072	41.1761	23.61	<.0001
teach_sal	1	-1.6329	0.5465	-2.7040	-0.5618	8.93	0.0028
prcnt_prof	1	0.0823	0.0236	0.0361	0.1286	12.17	0.0005
socio_stat	1	0.6653	0.0412	0.5846	0.7461	260.95	<.0001
teach_score	1	1.1744	0.1922	0.7977	1.5510	37.34	<.0001
mom_ed	1	-3.9706	0.8983	-5.7312	-2.2100	19.54	<.0001
Scale	1	0.6966					

Note that TEACH_SAL(VAR1) and MOM_ED(VAR5) change greatly when the robust estimation is used. Unfortunately, these two coefficients are negative, which implies that the test scores increase with decreasing teacher salaries and decreasing levels of the mother's education. Since ROBUSTREG is robust to outliers, they are not causing the counterintuitive parameter estimates.

The condition number of the regressor matrix $\mathbf{X}$ also plays a important role in parameter estimation. The condition number of the matrix can be obtained by specifying the COLLIN option in the PROC ENTROPY statement.

```
proc entropy data=coleman collin;
   model test_score = teach_sal prcnt_prof socio_stat teach_score mom_ed;
run;
```

The output produced by the COLLIN option is shown in Figure 14.9.

Figure 14.9 Collinearity Diagnostics

Test Scores compiled by Coleman et al. (1966)

The ENTROPY Procedure

Collinearity Diagnostics								
			Proportion of Variation					
Number	Eigenvalue	Condition Number	teach_sal	prcnt_prof	socio_stat	teach_score	mom_ed	Intercept
1	4.978128	1.0000	0.0007	0.0012	0.0026	0.0001	0.0001	0.0000
2	0.937758	2.3040	0.0006	0.0028	0.2131	0.0001	0.0000	0.0001
3	0.066023	8.6833	0.0202	0.3529	0.6159	0.0011	0.0000	0.0003
4	0.016036	17.6191	0.7961	0.0317	0.0534	0.0059	0.0083	0.0099
5	0.001364	60.4112	0.1619	0.3242	0.0053	0.7987	0.3309	0.0282
6	0.000691	84.8501	0.0205	0.2874	0.1096	0.1942	0.6607	0.9614

The condition number of the $\mathbf{X}$ matrix is reported to be 84.85. This means that the condition number of $\mathbf{X}'\mathbf{X}$ is $84.85^2 = 7199.5$, which is very large.

Ridge regression can be used to offset some of the problems associated with ill-conditioned $\mathbf{X}$ matrices. Using the formula for the ridge value as

$$\lambda_R = \frac{kS^2}{\hat{\beta}'\hat{\beta}} \approx 0.9$$

where $\hat{\beta}$ and S^2 are the least squares estimators of β and σ^2 and $k = 6$. A ridge regression of the test score model was performed by using the data set with the outliers removed. The following PROC REG code performs the ridge regression:

```
data coleman;
   set coleman;
   if _n_ = 3 or _n_ = 18 then delete;
run;

proc reg data=coleman ridge=0.9 outest=t noprint;
   model test_score = teach_sal prcnt_prof socio_stat teach_score mom_ed;
run;

proc print data=t;
run;
```

The results of the estimation are shown in Figure 14.10.

Figure 14.10 Ridge Regression Estimates

Test Scores compiled by Coleman et al. (1966)

Obs	_MODEL_	_TYPE_	_DEPVAR_	_RIDGE_	_PCOMIT_	_RMSE_	Intercept	teach_sal
1	MODEL1	PARMS	test_score	.	.	0.78236	29.7577	-1.69854
2	MODEL1	RIDGE	test_score	0.9	.	3.19679	9.6698	-0.08892

Obs	prcnt_prof	socio_stat	teach_score	mom_ed	test_score
1	0.085118	0.66617	1.18400	-4.06675	-1
2	0.041889	0.23223	0.60041	1.32168	-1

Note that the ridge regression estimates are much closer to the estimates produced by the ENTROPY procedure that uses the original data set. Ridge regressions are not robust to outliers as maximum entropy estimates are. This might explain why the estimates still differ for TEACH_SAL.

Using Prior Information

You can use prior information about the parameters or the residuals to improve the efficiency of the estimates. Some authors prefer the terms *pre-sample* or *pre-data* over the term *prior* when used with maximum entropy to avoid confusion with Bayesian methods. The maximum entropy method described here does not use Bayes' rule when including prior information in the estimation.

To perform regression, the ENTROPY procedure uses a generalization of maximum entropy called *generalized maximum entropy*. In maximum entropy estimation, the unknowns are probabilities. Generalized maximum entropy expands the set of problems that can be solved by introducing the concept of *support points*.

Generalized maximum entropy still estimates probabilities, but these are the probabilities of a support point. Support points are used to map the (0, 1) domain of the maximum entropy to the any finite range of values.

Prior information, such as expected ranges for the parameters or the residuals, is added by specifying support points for the parameters or the residuals. Support points are points in one dimension that specify the expected domain of the parameter or the residual. The wider the domain specified, the less efficient your parameter estimates are (the more variance they have). Specifying more support points in the same width interval also improves the efficiency of the parameter estimates at the cost of more computation. Golan, Judge, and Miller (1996) show that the gains in efficiency fall off for adding more than five support points. You can specify between 2 to 256 support points in the ENTROPY procedure.

If you have only a small amount of data, the estimates are very sensitive to your selection of support points and weights. For larger data sets, incorrect priors are discounted if they are not supported by the data.

Consider the data set generated by the following SAS statements:

```
data prior;
   do by = 1 to 100;
      do t = 1 to 10;
         y = 2*t + 5 * rannor(4);
         output;
      end;
   end;
run;
```

The PRIOR data set contains 100 samples of 10 observations each from the population

$$
\begin{aligned}
y &= 2 * t + \epsilon \\
\epsilon &\sim N(0, 5)
\end{aligned}
$$

You can estimate these samples using PROC ENTROPY as

```
proc entropy data=prior outest=parm1 noprint;
   model y = t ;
   by by;
run;
```

The 100 estimates are summarized by using the following SAS statements:

```
proc univariate data=parm1;
   var t;
run;
```

The summary statistics from PROC UNIVARIATE are shown in Output 14.11. The true value of the coefficient T is 2.0, demonstrating that maximum entropy estimates tend to be biased.

Figure 14.11 No Prior Information Monte Carlo Summary

Test Scores compiled by Coleman et al. (1966)

The UNIVARIATE Procedure
Variable: t

Basic Statistical Measures			
Location		Variability	
Mean	1.693608	**Std Deviation**	0.30199
Median	1.707653	**Variance**	0.09120
Mode	.	**Range**	1.46194
		Interquartile Range	0.32329

Now assume that you have prior information about the slope and the intercept for this model. You are reasonably confident that the slope is 2 and you are less confident that intercept is zero. To specify prior information about the parameters, use the PRIORS statement.

There are two parts to the prior information specified in the PRIORS statement. The first part is the support points for a parameter. The support points specify the domain of the parameter. For example, the following statement sets the support points -1000 and 1000 for the parameter associated with variable T:

```
priors t -1000 1000;
```

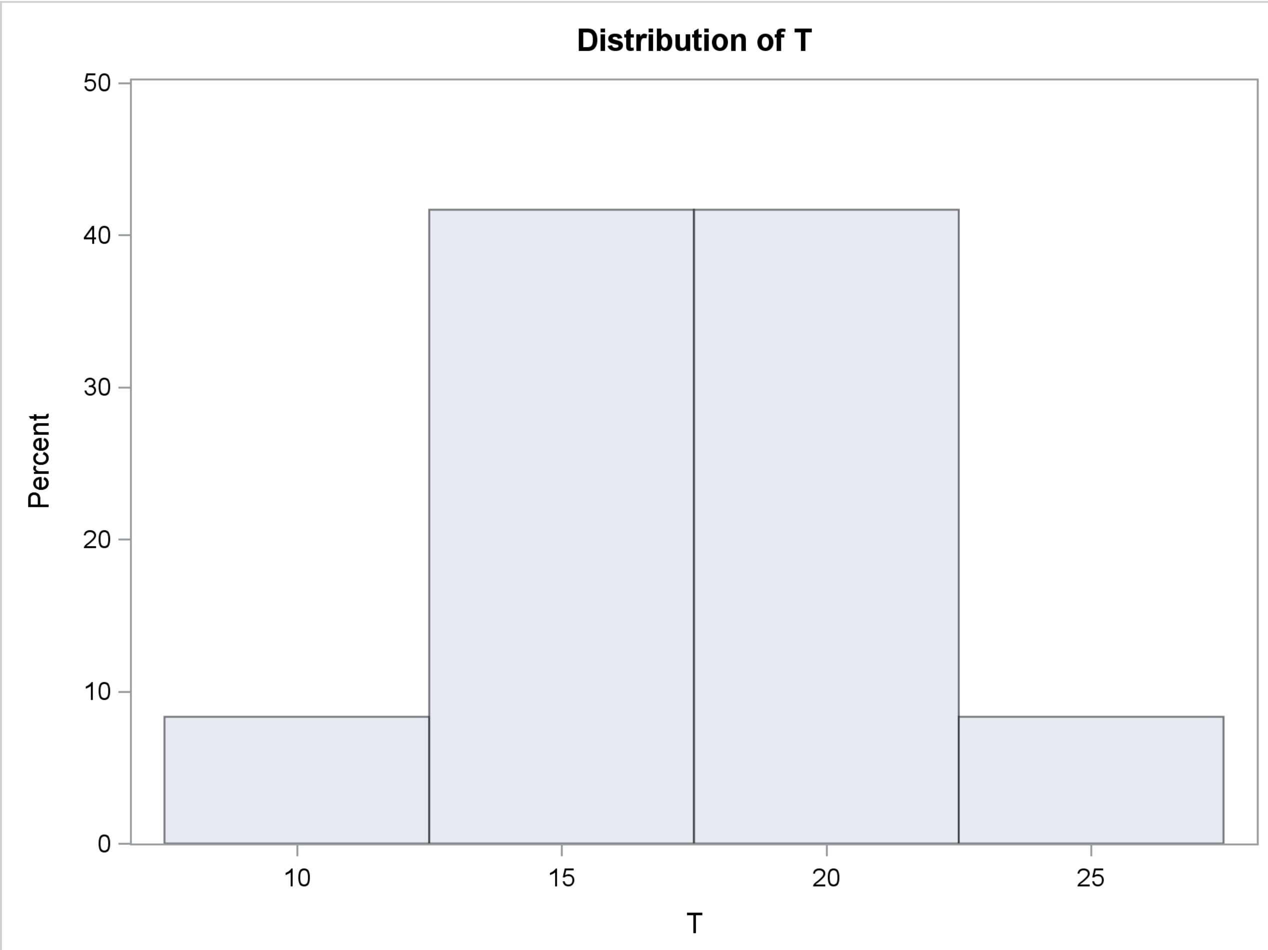
This means that the coefficient lies in the interval $[-1000, 1000]$. If the estimated value of the coefficient is actually outside of this interval, the estimation will not converge. In the previous PRIORS statement, no weights were specified for the support points, so uniform weights are assumed. This implies that the coefficient has a uniform probability of being in the interval $[-1000, 1000]$.

The second part of the prior information is the weights on the support points. For example, the following statements sets the support points 10, 15, 20, and 25 with weights 1, 5, 5, and 1 respectively for the coefficient of T:

```
priors t 10(1) 15(5) 20(5) 25(1);
```

This creates the prior distribution on the coefficient shown in Figure 14.12. The weights are automatically normalized so that they sum to one.

Figure 14.12 Prior Distribution of Parameter T



For the PRIOR data set created previously, the expected value of the coefficient of T is 2. The following SAS statements reestimate the parameters with a prior weight specified for each one.

```
proc entropy data=prior outest=parm2 noprint;
   priors t 0(1) 2(3) 4(1)
          intercept -100(.5) -10(1.5) 0(2) 10(1.5) 100(0.5);
   model y = t;
   by by;
run;
```

The priors on the coefficient of T express a confident view of the value of the coefficient. The priors on INTERCEPT express a more diffuse view on the value of the intercept. The following PROC UNIVARIATE statement computes summary statistics from the estimations:

```
proc univariate data=parm2;
   var t;
run;
```

The summary statistics for the distribution of the estimates of T are shown in Figure 14.13.

Figure 14.13 Prior Information Monte Carlo Summary

Prior Distribution of Parameter T

The UNIVARIATE Procedure
Variable: t

Basic Statistical Measures			
Location		Variability	
Mean	1.999953	Std Deviation	0.01436
Median	2.001423	Variance	0.0002061
Mode	.	Range	0.08525
		Interquartile Range	0.01855

The prior information improves the estimation of the coefficient of T dramatically. The downside of specifying priors comes when they are incorrect. For example, say the priors for this model were specified as

```
priors t -2(1) 0(3) 2(1);
```

to indicate a prior centered on zero instead of two.

The resulting summary statistics shown in Figure 14.14 indicate how the estimation is biased away from the solution.

Figure 14.14 Incorrect Prior Information Monte Carlo Summary

Prior Distribution of Parameter T

The UNIVARIATE Procedure
Variable: t

Basic Statistical Measures			
Location		Variability	
Mean	0.062550	Std Deviation	0.00920
Median	0.062527	Variance	0.0000847
Mode	.	Range	0.05442
		Interquartile Range	0.01112

The more data available for estimation, the less sensitive the parameters are to the priors. If the number of observations in each sample is 50 instead of 10, then the summary statistics shown in Figure 14.15 are produced. The prior information is not supported by the data, so it is discounted.

Figure 14.15 Incorrect Prior Information with More Data

Prior Distribution of Parameter T

The UNIVARIATE Procedure
Variable: t

Basic Statistical Measures			
Location		**Variability**	
Mean	0.652921	**Std Deviation**	0.00933
Median	0.653486	**Variance**	0.0000870
Mode	.	**Range**	0.04351
		Interquartile Range	0.01498

Pure Inverse Problems

A special case of systems of equations estimation is the pure inverse problem. A pure problem is one that contains an exact relationship between the dependent variable and the independent variables and does not have an error component. A pure inverse problem can be written as

$$\mathbf{y} = X\beta$$

where $\mathbf{y}$ is a n-dimensional vector of observations, $\mathbf{X}$ is a $n \times k$ matrix of regressors, and β is a k-dimensional vector of unknowns. Notice that there is no error term.

A classic example is a dice problem (Jaynes 1963). Given a six-sided die that can take on the values $x = 1, 2, 3, 4, 5, 6$ and the average outcome of the die $y = A$, compute the probabilities $\beta = (p_1, p_2, \cdots, p_6)'$ of rolling each number. This infers six values from two pieces of information. The data points are the expected value of y, and the sum of the probabilities is one. Given $E(y) = 4.0$, this problem is solved by using the following SAS code:

```
data one;
   array x[6] ( 1 2 3 4 5 6 );
   y=4.0;
run;

proc entropy data=one pure;
   priors x1 0 1 x2 0 1 x3 0 1 x4 0 1 x5 0 1 x6 0 1;
   model y = x1-x6/ noint;
   restrict x1 + x2 +x3 +x4 + x5 + x6 =1;
run;
```

The probabilities are given in Figure 14.16.

Figure 14.16 Jaynes' Dice Pure Inverse Problem

Prior Distribution of Parameter T

The ENTROPY Procedure

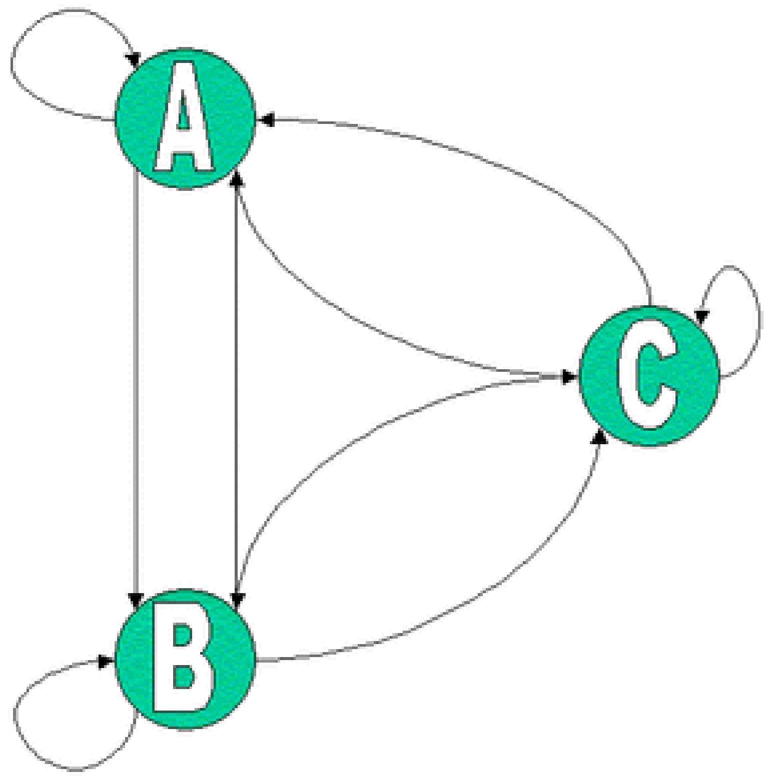
GME Variable Estimates			
Variable	**Estimate**	**Information Index**	**Label**
x1	0.101763	0.5254	
x2	0.122658	0.4630	
x3	0.147141	0.3974	
x4	0.175533	0.3298	
x5	0.208066	0.2622	
x6	0.244839	0.1970	
Restrict0	2.388082	.	x1 + x2 + x3 + x4 + x5 + x6 = 1

Note how the probabilities are skewed to the higher values because of the high average roll provided in the input data.

First-Order Markov Process Estimation

A more useful inverse problem is the first-order markov process. Companies have a share of the marketplace where they do business. Generally, customers for a specific market space can move from company to company. The movement of customers can be visualized graphically as a flow diagram, as in Figure 14.17. The arrows represent movements of customers from one company to another.

Figure 14.17 Markov Transition Diagram



You can model the probability that a customer moves from one company to another using a first-order Markov model. Mathematically the model is:

$$y_t = P y_{t-1}$$

where y_t is a vector of k market shares at time t and P is a $k \times k$ matrix of unknown transition probabilities. The value p_{ij} represents the probability that a customer who is currently using company j at time $t-1$ moves to company i at time t. The diagonal elements then represent the probability that a customer stays with the current company. The columns in P sum to one.

Given market share information over time, you can estimate the transition probabilities P. In order to estimate P using traditional methods, you need at least k observations. If you have fewer than k transitions, you can use the ENTROPY procedure to estimate the probabilities.

Suppose you are studying the market share for four companies. If you want to estimate the transition probabilities for these four companies, you need a time series with four observations of the shares. Assume the current transition probability matrix is as follows:

$$\begin{bmatrix} 0.7 & 0.4 & 0.0 & 0.1 \\ 0.1 & 0.5 & 0.4 & 0.0 \\ 0.0 & 0.1 & 0.6 & 0.0 \\ 0.2 & 0.0 & 0.0 & 0.9 \end{bmatrix}$$

The following SAS DATA step statements generate a series of market shares from this probability matrix. A transition is represented as the current period shares, `y`, and the previous period shares, `x`.

```
data m;
            /* Known Transition matrix */
   array p[4,4] (0.7 .4 .0 .1
                 0.1 .5 .4 .0
                 0.0 .1 .6 .0
                 0.2 .0 .0 .9 ) ;
            /* Initial Market shares */
   array y[4] y1-y4 ( .4 .3 .2 .1 );
   array x[4] x1-x4;
   drop p1-p16 i;
   do i = 1 to 3;
       x[1] = y[1]; x[2] = y[2];
       x[3] = y[3]; x[4] = y[4];
       y[1] = p[1,1] * x1 + p[1,2] * x2 + p[1,3] * x3 + p[1,4] * x4;
       y[2] = p[2,1] * x1 + p[2,2] * x2 + p[2,3] * x3 + p[2,4] * x4;
       y[3] = p[3,1] * x1 + p[3,2] * x2 + p[3,3] * x3 + p[3,4] * x4;
       y[4] = p[4,1] * x1 + p[4,2] * x2 + p[4,3] * x3 + p[4,4] * x4;
       output;
   end;
run;
```

The following SAS statements estimate the transition matrix by using only the first transition.

```
proc entropy markov pure data=m(obs=1);
   model y1-y4 = x1-x4;
run;
```

The MARKOV option implies NOINT for each model, that the sum of the parameters in each column is one, and chooses support points of 0 and 1. This model can be expressed equivalently as

```
proc entropy pure data=m(obs=1) ;
   priors y1.x1 0 1 y1.x2 0 1 y1.x3 0 1 y1.x4 0 1;
   priors y2.x1 0 1 y2.x2 0 1 y2.x3 0 1 y2.x4 0 1;
   priors y3.x1 0 1 y3.x2 0 1 y3.x3 0 1 y3.x4 0 1;
   priors y4.x1 0 1 y4.x2 0 1 y4.x3 0 1 y4.x4 0 1;

   model y1 = x1-x4 / noint;
   model y2 = x1-x4 / noint;
   model y3 = x1-x4 / noint;
   model y4 = x1-x4 / noint;

   restrict y1.x1 + y2.x1 + y3.x1 + y4.x1 = 1;
   restrict y1.x2 + y2.x2 + y3.x2 + y4.x2 = 1;
   restrict y1.x3 + y2.x3 + y3.x3 + y4.x3 = 1;
   restrict y1.x4 + y2.x4 + y3.x4 + y4.x4 = 1;
run;
```

The transition matrix is given in Figure 14.18.

Figure 14.18 Estimate of *P* by Using One Transition

Prior Distribution of Parameter T

The ENTROPY Procedure

GME Variable Estimates

Variable	Estimate	Information Index
y1.x1	0.463407	0.0039
y1.x2	0.41055	0.0232
y1.x3	0.356272	0.0605
y1.x4	0.302163	0.1161
y2.x1	0.272755	0.1546
y2.x2	0.271459	0.1564
y2.x3	0.267252	0.1625
y2.x4	0.260084	0.1731
y3.x1	0.119926	0.4709
y3.x2	0.148481	0.3940
y3.x3	0.180224	0.3194
y3.x4	0.214394	0.2502
y4.x1	0.143903	0.4056
y4.x2	0.169504	0.3434
y4.x3	0.196252	0.2856
y4.x4	0.223364	0.2337

Note that *P* varies greatly from the true solution.

If two transitions are used instead (OBS=2), the resulting transition matrix is shown in Figure 14.19.

```
proc entropy markov pure data=m(obs=2);
   model y1-y4 = x1-x4;
run;
```

Figure 14.19 Estimate of *P* by Using Two Transitions

Prior Distribution of Parameter T

The ENTROPY Procedure

GME Variable Estimates		
Variable	Estimate	Information Index
y1.x1	0.721012	0.1459
y1.x2	0.355703	0.0609
y1.x3	0.026095	0.8256
y1.x4	0.096654	0.5417
y2.x1	0.083987	0.5839
y2.x2	0.53886	0.0044
y2.x3	0.373668	0.0466
y2.x4	0.000133	0.9981
y3.x1	0.000062	0.9990
y3.x2	0.099848	0.5315
y3.x3	0.600104	0.0291
y3.x4	7.871E-8	1.0000
y4.x1	0.194938	0.2883
y4.x2	0.00559	0.9501
y4.x3	0.000133	0.9981
y4.x4	0.903214	0.5413

This transition matrix is much closer to the actual transition matrix.

If, in addition to the transitions, you had other information about the transition matrix, such as your own company's transition values, that information can be added as restrictions to the parameter estimates. For noisy data, the PURE option should be dropped. Note that this example has six zero probabilities in the transition matrix; the accurate estimation of transition matrices with fewer zero probabilities generally requires more transition observations.

Analyzing Multinomial Response Data

Multinomial discrete choice models suffer the same problems with collinearity of the regressors and small sample sizes as linear models. Unordered multinomial discrete choice models can be estimated using a variant of GME for discrete models called GME-D.

Consider the model shown in Golan, Judge, and Perloff (1996). In this model, there are five occupational categories, and the categories are considered a function of four individual characteristics. The sample contains 337 individuals.

```
data kpdata;
   input job x1 x2 x3 x4;
datalines;
   0 1 3 11 1

   ... more lines ...
```

The dependent variable in this data, job, takes on values 0 through 4. Support points are used only for the error terms; so error supports are specified on the MODEL statement.

```
proc entropy data=kpdata gmed tech=nra;
   model job = x1 x2 x3 x4 / noint
         esupports=( -.1 -0.0666 -0.0333 0 0.0333 0.0666 .1 );
run;
```

Figure 14.20 Estimate of Jobs Model by Using GME-D

Prior Distribution of Parameter T

The ENTROPY Procedure

GME-D Variable Estimates

Variable	Estimate	Approx Std Err	t Value	Approx Pr > |t|
x1_1	1.802572	1.3610	1.32	0.1863
x2_1	-0.00251	0.0154	-0.16	0.8705
x3_1	-0.17282	0.0885	-1.95	0.0517
x4_1	1.054659	0.6986	1.51	0.1321
x1_2	0.089156	1.2764	0.07	0.9444
x2_2	0.019947	0.0146	1.37	0.1718
x3_2	0.010716	0.0830	0.13	0.8974
x4_2	0.288629	0.5775	0.50	0.6176
x1_3	-4.62047	1.6476	-2.80	0.0053
x2_3	0.026175	0.0166	1.58	0.1157
x3_3	0.245198	0.0986	2.49	0.0134
x4_3	1.285466	0.8367	1.54	0.1254
x1_4	-9.72734	1.5813	-6.15	<.0001
x2_4	0.027382	0.0156	1.75	0.0805
x3_4	0.660836	0.0947	6.98	<.0001
x4_4	1.47479	0.6970	2.12	0.0351

Note there are five estimates of the parameters produced for each regressor, one for each choice. The first choice is restricted to zero for normalization purposes. PROC ENTROPY drops the zeroed regressors. PROC ENTROPY also generates tables of marginal effects for each regressor. The following statements generate the marginal effects table for the previous analysis at the means of the variables.

```
proc entropy data=kpdata gmed tech=nra;
   model job = x1 x2 x3 x4 / noint
         esupports=( -.1 -0.0666 -0.0333 0 0.0333 0.0666 .1 )
         marginals;
run;
```

Figure 14.21 Estimate of Jobs Model by Using GME-D (Marginals)

Prior Distribution of Parameter T

The ENTROPY Procedure

GME-D Variable Marginal Effects Table

Variable	Marginal Effect	Mean
x1_0	0.338758	1
x2_0	-0.0019	20.50148
x3_0	-0.02129	13.09496
x4_0	-0.09917	0.916914
x1_1	0.859883	1
x2_1	-0.00345	20.50148
x3_1	-0.0648	13.09496
x4_1	0.034396	0.916914
x1_2	0.86101	1
x2_2	0.000963	20.50148
x3_2	-0.04948	13.09496
x4_2	-0.16297	0.916914
x1_3	-0.25969	1
x2_3	0.0015	20.50148
x3_3	0.009289	13.09496
x4_3	0.065569	0.916914
x1_4	-1.79996	1
x2_4	0.00288	20.50148
x3_4	0.126283	13.09496
x4_4	0.162172	0.916914

The marginals are derivatives of the probabilities with respect to each variable and so summarize how a small change in each variable affects the overall probability.

PROC ENTROPY also enables the user to specify where the derivative is evaluated, as shown below:

```
proc entropy data=kpdata gmed tech=nra;
   model job = x1 x2 x3 x4 / noint
         esupports=( -.1 -0.0666 -0.0333 0 0.0333 0.0666 .1 )
         marginals=( x2=.4 x3=10 x4=0);
run;
```

Figure 14.22 Estimate of Jobs Model by Using GME-D (Marginals)

Prior Distribution of Parameter T

The ENTROPY Procedure

GME-D Variable Marginal Effects Table				
Variable	**Marginal Effect**	**Mean**	**Marginal Effect at User Supplied Values**	**User Supplied Values**
x1_0	0.338758	1	-0.0901	1
x2_0	-0.0019	20.50148	-0.00217	0.4
x3_0	-0.02129	13.09496	0.009586	10
x4_0	-0.09917	0.916914	-0.14204	0
x1_1	0.859883	1	0.463181	1
x2_1	-0.00345	20.50148	-0.00311	0.4
x3_1	-0.0648	13.09496	-0.04339	10
x4_1	0.034396	0.916914	0.174876	0
x1_2	0.86101	1	-0.07894	1
x2_2	0.000963	20.50148	0.004405	0.4
x3_2	-0.04948	13.09496	0.015555	10
x4_2	-0.16297	0.916914	-0.072	0
x1_3	-0.25969	1	-0.16459	1
x2_3	0.0015	20.50148	0.000623	0.4
x3_3	0.009289	13.09496	0.00929	10
x4_3	0.065569	0.916914	0.02648	0
x1_4	-1.79996	1	-0.12955	1
x2_4	0.00288	20.50148	0.000256	0.4
x3_4	0.126283	13.09496	0.008956	10
x4_4	0.162172	0.916914	0.012684	0

In this example, you evaluate the derivative when x1=1, x2=0.4, x3=10, and x4=0. If the user neglects a variable, PROC ENTROPY uses its mean value.

Syntax: ENTROPY Procedure

The following statements can be used with the ENTROPY procedure:

PROC ENTROPY *options* ;
  BOUNDS *bound1* < *, bound2, . . .* > ;
  BY *variable* < *variable . . .* > ;
  ID *variable* < *variable . . .* > ;
  MODEL *variable = variable* < *variable* > *. . .* < */ options* > ;
  PRIORS *variable* < *support points* > *variable* < *value* > *. . .* ;
  RESTRICT *restriction1* < *, restriction2 . . .* > ;
  TEST < *"name"* > *test1* < *, test2 . . .* > < */ options* > ;
  WEIGHT *variable* ;

Functional Summary

The statements and options in the ENTROPY procedure are summarized in the following table.

Description	Statement	Option
Data Set Options		
specify the input data set for the variables	ENTROPY	DATA=
specify the input data set for support points and priors	ENTROPY	PDATA=
specify the output data set for residual, predicted, and actual values	ENTROPY	OUT=
specify the output data set for the support points and priors	ENTROPY	OUTP=
write the covariance matrix of the estimates to OUTEST= data set	ENTROPY	OUTCOV
write the parameter estimates to a data set	ENTROPY	OUTEST=
write the Lagrange multiplier estimates to a data set	ENTROPY	OUTL=
write the covariance matrix of the equation errors to a data set	ENTROPY	OUTS=
write the **S** matrix used in the objective function definition to a data set	ENTROPY	OUTSUSED=
read the covariance matrix of the equation errors	ENTROPY	SDATA=
Printing Options		
request that the procedure produce graphics via the Output Delivery System	ENTROPY	PLOTS=
print collinearity diagnostics	ENTROPY	COLLIN
suppress the normal printed output	ENTROPY	NOPRINT

Description	Statement	Option
Options to Control Iteration Output		
print a summary iteration listing	ENTROPY	ITPRINT
Options to Control the Minimization Process		
specify the convergence criteria	ENTROPY	CONVERGE=
specify the maximum number of iterations allowed	ENTROPY	MAXITER=
specify the maximum number of subiterations allowed	ENTROPY	MAXSUBITER=
select the iterative minimization method to use	ENTROPY	METHOD=
Statements That Declare Variables		
specify BY-group processing	BY	
specify a weight variable	WEIGHT	
specify identifying variables	ID	
General PROC ENTROPY Statement Options		
specify seemingly unrelated regression	ENTROPY	SUR
specify iterated seemingly unrelated regression	ENTROPY	ITSUR
specify data-constrained generalized maximum entropy	ENTROPY	GME
specify moment generalized maximum entropy	ENTROPY	GMEM
specify the denominator for computing variances and covariances	ENTROPY	VARDEF=
General TEST Statement Options		
specify that a Wald test be computed	TEST	WALD
specify that a Lagrange multiplier test be computed	TEST	LM
specify that a likelihood ratio test be computed	TEST	LR
request all three types of tests	TEST	ALL

PROC ENTROPY Statement

PROC ENTROPY *options* **;**

The following options can be specified in the PROC ENTROPY statement.

General Options

COLLIN
requests that the collinearity diagnostics of the $X'X$ matrix be printed.

COVBEST=CROSS | GME | GMEM
specifies the method for producing the covariance matrix of parameters for output and for standard error calculations. GMEM and GME are aliases and are the default.

GME | GCE
requests generalized maximum entropy or generalized cross entropy. This is the default estimation method.

GMEM | GCEM
requests moment maximum entropy or the moment cross entropy.

GMED
requests a variant of GME suitable for multinomial discrete choice models.

MARKOV
specifies that the model is a first-order Markov model.

PURE
specifies a regression without an error term.

SUR | ITSUR
specifies seemingly unrelated regression or iterated seemingly unrelated regression.

VARDEF=N | WGT | DF | WDF
specifies the denominator to be used in computing variances and covariances. VARDEF=N specifies that the number of nonmissing observations be used. VARDEF=WGT specifies that the sum of the weights be used. VARDEF=DF specifies that the number of nonmissing observations minus the model degrees of freedom (number of parameters) be used. VARDEF=WDF specifies that the sum of the weights minus the model degrees of freedom be used. The default is VARDEF=DF.

Data Set Options

DATA=*SAS-data-set*
specifies the input data set. Values for the variables in the model are read from this data set.

PDATA=*SAS-data-set*
names the SAS data set that contains the data about priors and supports.

OUT=*SAS-data-set*
names the SAS data set to contain the residuals from each estimation.

OUTCOV
COVOUT
writes the covariance matrix of the estimates to the OUTEST= data set in addition to the parameter estimates. The OUTCOV option is applicable only if the OUTEST= option is also specified.

OUTEST=*SAS-data-set*
names the SAS data set to contain the parameter estimates and optionally the covariance of the estimates.

OUTL=*SAS-data-set*
names the SAS data set to contain the estimated Lagrange multipliers for the models.

OUTP=*SAS-data-set*
names the SAS data set to contain the support points and estimated probabilities.

OUTS=*SAS-data-set*
names the SAS data set to contain the estimated covariance matrix of the equation errors. This is the covariance of the residuals computed from the parameter estimates.

OUTSUSED=*SAS-data-set*
names the SAS data set to contain the **S** matrix used in the objective function definition. The OUTSUSED= data set is the same as the OUTS= data set for the methods that iterate the **S** matrix.

SDATA=*SAS-data-set*
specifies a data set that provides the covariance matrix of the equation errors. The matrix read from the SDATA= data set is used for the equation error covariance matrix (**S** matrix) in the estimation. The SDATA= matrix is used to provide only the initial estimate of **S** for the methods that iterate the S matrix.

Printing Options

ITPRINT
prints the parameter estimates, objective function value, and convergence criteria at each iteration.

NOPRINT
suppresses the normal printed output but does not suppress error listings. Using any other print option turns the NOPRINT option off.

PLOTS=*global-plot-options* **|** *plot-request*
controls the plots that the ENTROPY procedure produces. (For general information about ODS Graphics, see Chapter 21, "Statistical Graphics Using ODS" (*SAS/STAT User's Guide*).) The *global-plot-options* apply to all relevant plots generated by the ENTROPY procedure.

The *global-plot-options* supported by the ENTROPY procedure are as follows:

ONLY suppresses the default plots. Only the plots specifically requested are produced.

UNPACKPANEL displays each graph separately. (By default, some graphs can appear together in a single panel.)

The specific *plot-request* values supported by the ENTROPY procedure are as follows:

ALL	requests that all plots appropriate for the particular analysis be produced. ALL is equivalent to specifying FITPLOT, COOKSD, QQ, RESIDUALHISTOGRAM, and STUDENTRESIDUAL.
FITPLOT	plots the predicted and actual values.
COOKSD	produces the Cook's *D* plot.
QQ	produces a Q-Q plot of residuals.
RESIDUALHISTOGRAM	plots the histogram of residuals.
STUDENTRESIDUAL	plots the studentized residuals.
NONE	suppresses all plots.

The default behavior is to plot all plots appropriate for the particular analysis (ALL) in a panel.

Options to Control the Minimization Process

The following options can be helpful if a convergence problem occurs for a given model and set of data. The ENTROPY procedure uses the nonlinear optimization subsystem (NLO) to perform the model optimizations. In addition to the options listed below, all options supported in the NLO subsystem can be specified on the ENTROPY procedure statement. See Chapter 7, "Nonlinear Optimization Methods," for more details.

CONVERGE=*value*

GCONV=*value*

specifies the convergence criteria for *S*-iterated methods. The convergence measure computed during model estimation must be less than *value* before convergence is assumed. The default value is CONVERGE=0.001.

DUAL | PRIMAL

specifies whether the optimization problem is solved using the dual or primal form. The dual form is the default.

MAXITER=*n*

specifies the maximum number of iterations allowed. The default is MAXITER=100.

MAXSUBITER=*n*

specifies the maximum number of subiterations allowed for an iteration. The MAXSUBITER= option limits the number of step halvings. The default is MAXSUBITER=30.

METHOD=TR | NEWRAP | NRR | QN | CONGR | NSIMP | DBLDOG | LEVMAR

TECHNIQUE=TR | NEWRAP | NRR | QN | CONGR | NSIMP | DBLDOG | LEVMAR

TECH=TR | NEWRAP | NRR | QN | CONGR | NSIMP | DBLDOG | LEVMAR

specifies the iterative minimization method to use. METHOD=TR specifies the trust region method, METHOD=NEWRAP specifies the Newton-Raphson method, METHOD=NRR specifies the Newton-Raphson ridge method, and METHOD=QN specifies the quasi-Newton method. See Chapter 7, "Nonlinear Optimization Methods," for more details about optimization methods. The default is METHOD=QN for the dual form and METHOD=NEWRAP for the primal form.

BOUNDS Statement

BOUNDS *bound1* < , *bound2* . . . > **;**

The BOUNDS statement imposes simple boundary constraints on the parameter estimates. BOUNDS statement constraints refer to the parameters estimated by the ENTROPY procedure. You can specify any number of BOUNDS statements.

Each *boundary constraint* is composed of variables, constants, and inequality operators in the following form:

```
item operator item <,operator item <,operator item ...> >
```

Each *item* is a constant, the name of a regressor variable, or a list of regressor names. Each *operator* is <, >, <=, or >=.

You can use either the BOUNDS statement or the RESTRICT statement to impose boundary constraints; the BOUNDS statement provides a simpler syntax for specifying inequality constraints. See section "RESTRICT Statement" on page 817 for more information about the computational details of estimation with inequality restrictions.

Lagrange multipliers are reported for all the active boundary constraints. In the printed output and in the OUTEST= data set, the Lagrange multiplier estimates are identified with the names BOUND1, BOUND2, and so forth. The probability of the Lagrange multipliers are computed using a beta distribution (LaMotte 1994). Nonactive or nonbinding bounds have no effect on the estimation results and are not noted in the output. To give the constraints more descriptive names, use the RESTRICT statement instead of the BOUNDS statement.

The following BOUNDS statement constrains the estimates of the coefficients of WAGE and TARGET and the 10 coefficients of x1 through x10 to be between zero and one. This example illustrates the use of parameter lists to specify boundary constraints.

```
bounds 0 < wage target x1-x10 < 1;
```

The following is an example of the use of the BOUNDS statement to impose boundary constraints on the variables X1, X2, and X3:

```
proc entropy data=zero;
   bounds .1 <= x1 <= 100,
           0 <= x2 <=  25.6,
           0 <= x3 <=   5;

   model y = x1 x2 x3;
run;
```

The parameter estimates from this run are shown in Figure 14.23.

Figure 14.23 Output from Bounded Estimation

Prior Distribution of Parameter T

The ENTROPY Procedure

Variables(Supports(Weights))	x1 x2 x3 Intercept
Equations(Supports(Weights))	y

Prior Distribution of Parameter T

The ENTROPY Procedure
GME Estimation Summary

Data Set Options	
DATA=	WORK.ZERO

Minimization Summary	
Parameters Estimated	4
Covariance Estimator	GME
Entropy Type	Shannon
Entropy Form	Dual
Numerical Optimizer	Newton-Raphson

Final Information Measures	
Objective Function Value	6.292861
Signal Entropy	6.375715
Noise Entropy	-0.08285
Normed Entropy (Signal)	0.990364
Normed Entropy (Noise)	1.004172
Parameter Information Index	0.009636
Error Information Index	-0.00417

Observations Processed	
Read	20
Used	20

NOTE: At GME Iteration 20 convergence criteria met.

GME Summary of Residual Errors

Equation	DF Model	DF Error	SSE	MSE	Root MSE	R-Square	Adj RSq
y	4	16	1665620	83281.0	288.6	-0.0013	-0.1891

Figure 14.23 *continued*

GME Variable Estimates

Variable	Estimate	Approx Std Err	t Value	Approx Pr > |t|	Label
x1	0.1	0	.	.	
x2	0	0	.	.	
x3	3.33E-16	0	.	.	
Intercept	-0.00432	3.406E-6	-1269.3	<.0001	
	1.25731	9130.3	0.00	0.9999	0.1 <= x1
	0.009384	0	.	.	0 <= x2
	0.000025	0	.	.	0 <= x3

BY Statement

BY *variables* **;**

A BY statement is used to obtain separate estimates for observations in groups defined by the BY variables. To save parameter estimates for each BY group, use the OUTEST= option.

ID Statement

ID *variables* **;**

The ID statement specifies variables to identify observations in error messages or other listings and in the OUT= data set. The ID variables are normally SAS date or datetime variables. If more than one ID variable is used, the first variable is used to identify the observations and the remaining variables are added to the OUT= data set.

MODEL Statement

MODEL *dependent = regressors* < */ options* > **;**

The MODEL statement specifies the dependent variable and independent regressor variables for the regression model. If no independent variables are specified in the MODEL statement, only the mean (intercept) is estimated. To model a system of equations, specify more than one MODEL statement.

The following options can be used in the MODEL statement after a slash (/).

ESUPPORTS=(*support (prior) . . .* **)**

specifies the support points and prior weights on the residuals for the specified equation. The default is the following five support values:

$$-10 * value, -value, 0, value, 10 * value$$

where *value* is computed as

$$value = (max(y) - \bar{y}) * multiplier$$

for GME, where y is the dependent variable, and

$$value = (max(y) - \bar{y}) * multiplier * nobs * max(X) * 0.1$$

for generalized maximum entropy—moments (GME-M), where $\mathbf{X}$ is the information matrix, and *nobs* is the number of observations. The *multiplier* depends on the MULTIPLIER= option. The MULTIPLIER= option defaults to 2 for unrestricted models and to 4 for restricted models. The prior probabilities default to the following:

$$0.0005, 0.333, 0.333, 0.333, 0.0005$$

The support points and prior weights are selected so that hypothesis tests can be performed without adding significant bias to the estimation. These prior probability values are ad hoc.

NOINT
suppresses the intercept parameter.

MARGINALS = *(variable = value, . . . , variable = value)*
requests that the marginal effects of each variable be calculated for GME-D. Specifying the MARGINALS option with an optional list of values calculates the marginals at that vector of values. For example, if x1–x4 are explanatory variables, then including

MARGINALS = (x1 = 2, x2 = 4, x3 = –1, x4 = 5)

calculates the marginal effects at that vector. A skipped variable implies that its mean value is to be used.

CENSORED *((UB | LB) = (variable | value)*, **ESUPPORTS** *=(support (prior) . . .))*
specifies that the dependent variable be observed with censoring and specifies the censoring thresholds and the supports of the censored observations.

CATEGORY= *variable*
specifies the variable that keeps track of the categories the dependent variable is in when there is range censoring. When the actual value is observed, this variable should be set to MISSING.

RANGE (ID = (QS | INT) L = (NUMBER) R = (NUMBER) , ESUPPORTS=(*support < (prior)> . . .* **))**
specifies that the dependent variable be range bound. The RANGE option defines the range and the key (RANGE) that is used to identify the observation as being range bound. The RANGE = value should be some value in the CATEGORY= variable. The L and R define, respectively, the left endpoint of the range and the right endpoint of the range. ESUPPORTS sets the error supports on the variable.

PRIORS Statement

PRIORS *variable < support points < (priors) > > variable < support points < (priors) > > . . .* **;**

The PRIORS statement specifies the support points and prior weights for the coefficients on the variables.

Support points for coefficients default to five points, determined as follows:

$$-2 * value, -value, 0, value, 2 * value$$

where *value* is computed as

$$value = (\|mean\| + 3 * stderr) * multiplier$$

where the *mean* and the *stderr* are obtained from OLS and the *multiplier* depends on the MULTIPLIER= option. The MULTIPLIER= option defaults to 2 for unrestricted models and to 4 for restricted models. The prior probabilities for each support point default to the uniform distribution.

The number of support points must be at least two. If priors are specified, they must be positive and there must be the same number of priors as there are support points. Priors and support points can also be specified through the PDATA= data set.

RESTRICT Statement

RESTRICT *restriction1* < , *restriction2* . . . > **;**

The RESTRICT statement is used to impose linear restrictions on the parameter estimates. You can specify any number of RESTRICT statements.

Each *restriction* is written as an optional name, followed by an expression, followed by an equality operator (=) or an inequality operator (<, >, <=, >=), followed by a second expression:

<"name" > expression operator expression

The optional *"name"* is a string used to identify the restriction in the printed output and in the OUTEST= data set. The *operator* can be =, <, >, <= , or >=. The operator and second expression are optional, as in the TEST statement, where they default to = 0.

Restriction expressions can be composed of variable names, multiplication ($*$), and addition ($+$) operators, and constants. Variable names in restriction expressions must be among the variables whose coefficients are estimated by the model. The restriction expressions must be a linear function of the variables.

The following is an example of the use of the RESTRICT statement:

```
proc entropy data=one;
   restrict y1.x1*2 <= x2 + y2.x1;
   model y1 = x1 x2;
   model y2 = x1 x3;
run;
```

This example illustrates the use of compound names, y1.x1, to specify coefficients of specific equations.

TEST Statement

TEST < *"name"*> *test1* < , *test2* . . . > < ,/ *options* > **;**

The TEST statement performs tests of linear hypotheses on the model parameters.

The TEST statement applies only to parameters estimated in the model. You can specify any number of TEST statements.

Each *test* is written as an expression optionally followed by an equal sign (=) and a second expression:

expression <= expression>

Test expressions can be composed of variable names, multiplication (∗), addition (+), and subtraction (−) operators, and constants. Variables named in test expressions must be among the variables estimated by the model.

If you specify only one expression in a TEST statement, that expression is tested against zero. For example, the following two TEST statements are equivalent:

```
test a + b;

test a + b = 0;
```

When you specify multiple tests on the same TEST statement, a joint test is performed. For example, the following TEST statement tests the joint hypothesis that both of the coefficients on a and b are equal to zero:

```
test a, b;
```

To perform separate tests rather than a joint test, use separate TEST statements. For example, the following TEST statements test the two separate hypotheses that a is equal to zero and that b is equal to zero:

```
test a;
test b;
```

You can use the following options in the TEST statement:

WALD

specifies that a Wald test be computed. WALD is the default.

LM

RAO

LAGRANGE

specifies that a Lagrange multiplier test be computed.

LR

LIKE

specifies that a pseudo-likelihood ratio test be computed.

ALL

requests all three types of tests.

OUT=

specifies the name of an output SAS data set that contains the test results. The format of the OUT= data set produced by the TEST statement is similar to that of the OUTEST= data set.

WEIGHT Statement

WEIGHT *variable* ;

The WEIGHT statement specifies a variable to supply weighting values to use for each observation in estimating parameters.

If the weight of an observation is nonpositive, that observation is not used for the estimation. *Variable* must be a numeric variable in the input data set. The regressors and the dependent variables are multiplied by the square root of the weight variable to form the weighted **X** matrix and the weighted dependent variable. The same weight is used for all MODEL statements.

Details: ENTROPY Procedure

Shannon's measure of entropy for a distribution is given by

$$\begin{aligned} \text{maximize} \quad & -\sum_{i=1}^{n} p_i \ \ln(p_i) \\ \text{subject to} \quad & \sum_{i=1}^{n} p_i \ = \ 1 \end{aligned}$$

where p_i is the probability associated with the ith support point. Properties that characterize the entropy measure are set forth by Kapur and Kesavan (1992).

The objective is to maximize the entropy of the distribution with respect to the probabilities p_i and subject to constraints that reflect any other known information about the distribution (Jaynes 1957). This measure, in the absence of additional information, reaches a maximum when the probabilities are uniform. A distribution other than the uniform distribution arises from information already known.

Generalized Maximum Entropy

Reparameterization of the errors in a regression equation is the process of specifying a support for the errors, observation by observation. If a two-point support is used, the error for the tth observation is reparameterized by setting $e_t = w_{t1}\, v_{t1} + w_{t2}\, v_{t2}$, where v_{t1} and v_{t2} are the upper and lower bounds for the tth error e_t, and w_{t1} and w_{t2} represent the weight associated with the point v_{t1} and v_{t2}. The error distribution is usually chosen to be symmetric, centered around zero, and the same across observations so that $v_{t1} = -v_{t2} = R$, where R is the support value chosen for the problem (Golan, Judge, and Miller 1996).

The generalized maximum entropy (GME) formulation was proposed for the ill-posed or underdetermined case where there is insufficient data to estimate the model with traditional methods. β is reparameterized by defining a support for β (and a set of weights in the cross entropy case), which defines a prior distribution for β.

In the simplest case, each β_k is reparameterized as $\beta_k = p_{k1}\, z_{k1} + p_{k2}\, z_{k2}$, where p_{k1} and p_{k2} represent the probabilities ranging from [0,1] for each β, and z_{k1} and z_{k2} represent the lower and upper bounds placed on β_k. The support points, z_{k1} and z_{k2}, are usually distributed symmetrically around the most likely value for β_k based on some prior knowledge.

With these reparameterizations, the GME estimation problem is

$$\begin{aligned} \text{maximize} \quad H(p,w) &= -p' \ln(p) - w' \ln(w) \\ \text{subject to} \quad y &= X\,Z\,p + V\,w \\ 1_K &= (I_K \otimes 1'_L)\,p \\ 1_T &= (I_T \otimes 1'_L)\,w \end{aligned}$$

where y denotes the column vector of length T of the dependent variable; $\mathbf{X}$ denotes the $(T \times K)$ matrix of observations of the independent variables; p denotes the LK column vector of weights associated with the points in Z; w denotes the LT column vector of weights associated with the points in V; 1_K, 1_L, and 1_T are K-, L-, and T-dimensional column vectors, respectively, of ones; and I_K and I_T are $(K \times K)$ and $(T \times T)$ dimensional identity matrices.

These equations can be rewritten using set notation as follows:

$$\text{maximize} \quad H(p,w) = -\sum_{l=1}^{L}\sum_{k=1}^{K} p_{kl} \ln(p_{kl}) - \sum_{l=1}^{L}\sum_{t=1}^{T} w_{tl} \ln(w_{tl})$$

$$\text{subject to} \quad y_t = \sum_{l=1}^{L}\left[\sum_{k=1}^{K} (X_{kt}\,Z_{kl}\,p_{kl}) + V_{tl}\,w_{tl}\right]$$

$$\sum_{l=1}^{L} p_{kl} = 1 \quad \text{and} \quad \sum_{l=1}^{L} w_{tl} = 1$$

The subscript l denotes the support point (l=1, 2, ..., L), k denotes the parameter (k=1, 2, ..., K), and t denotes the observation (t=1, 2, ..., T).

The GME objective is strictly concave; therefore, a unique solution exists. The optimal estimated probabilities, p and w, and the prior supports, Z and V, can be used to form the point estimates of the unknown parameters, β, and the unknown errors, e.

Generalized Cross Entropy

Kullback and Leibler (1951) cross entropy measures the "discrepancy" between one distribution and another. Cross entropy is called a measure of discrepancy rather than distance because it does not satisfy some of the properties one would expect of a distance measure. (See Kapur and Kesavan (1992) for a discussion of cross entropy as a measure of discrepancy.) Mathematically, cross entropy is written as

$$\text{minimize} \quad \sum_{i=1}^{n} p_i \ln(p_i / q_i)$$

$$\text{subject to} \quad \sum_{i=1}^{n} p_i = 1,$$

where q_i is the probability associated with the ith point in the distribution from which the discrepancy is measured. The q_i (in conjunction with the support) are often referred to as the prior distribution. The measure is nonnegative and is equal to zero when p_i equals q_i. The properties of the cross entropy measure are examined by Kapur and Kesavan (1992).

The principle of minimum cross entropy (Kullback 1959; Good 1963) states that one should choose probabilities that are as close as possible to the prior probabilities. That is, out of all probability distributions that satisfy a given set of constraints which reflect known information about the distribution, choose the distribution that is closest (as measured by $p(\ln(p) - \ln(q))$) to the prior distribution. When the prior distribution is uniform, maximum entropy and minimum cross entropy produce the same results (Kapur and Kesavan 1992), where the higher values for entropy correspond exactly with the lower values for cross entropy.

If the prior distributions are nonuniform, the problem can be stated as a generalized cross entropy (GCE) formulation. The cross entropy terminology specifies weights, q_i and u_i, for the points Z and V, respectively. Given informative prior distributions on Z and V, the GCE problem is

$$
\begin{aligned}
\text{minimize} \quad I(p,q,w,u) &= p' \ln(p/q) + w' \ln(w/u) \\
\text{subject to} \quad y &= X\,Z\,p \;+\; V\,w \\
1_K &= (I_K \otimes 1'_L)\,p \\
1_T &= (I_T \otimes 1'_L)\,w
\end{aligned}
$$

where y denotes the T column vector of observations of the dependent variables; **X** denotes the $(T \times K)$ matrix of observations of the independent variables; q and p denote LK column vectors of prior and posterior weights, respectively, associated with the points in Z; u and w denote the LT column vectors of prior and posterior weights, respectively, associated with the points in V; 1_K, 1_L, and 1_T are K-, L-, and T-dimensional column vectors, respectively, of ones; and $\mathbf{I_K}$ and $\mathbf{I_T}$ are $(K \times K)$ and $(T \times T)$ dimensional identity matrices.

The optimization problem can be rewritten using set notation as follows

$$
\text{minimize} \quad I(p,q,w,u) = \sum_{l=1}^{L} \sum_{k=1}^{K} p_{kl} \ln(p_{kl}/q_{kl}) + \sum_{l=1}^{L} \sum_{t=1}^{T} w_{tl} \ln(w_{tl}/u_{tl})
$$

$$
\text{subject to} \quad y_t = \sum_{l=1}^{L} \left[\sum_{k=1}^{K} (X_{kt}\, Z_{kl}\, p_{kl}) + V_{tl}\, w_{tl} \right]
$$

$$
\sum_{l=1}^{L} p_{kl} = 1 \;\text{ and }\; \sum_{l=1}^{L} w_{tl} = 1
$$

The subscript l denotes the support point (l=1, 2, ..., L), k denotes the parameter (k=1, 2, ..., K), and t denotes the observation (t=1, 2, ..., T).

The objective function is strictly convex; therefore, there is a unique global minimum for the problem (Golan, Judge, and Miller 1996). The optimal estimated weights, p and w, and the prior supports, Z and V, can be used to form the point estimates of the unknown parameters, β, and the unknown errors, e, by using

$$\beta \;=\; Z\,p \;=\; \begin{bmatrix} z_{11} & \cdots & z_{L1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z_{12} & \cdots & z_{L2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & z_{1K} & \cdots & z_{LK} \end{bmatrix} \begin{bmatrix} p_{11} \\ \vdots \\ p_{L1} \\ p_{12} \\ \vdots \\ p_{L2} \\ \vdots \\ p_{1K} \\ \vdots \\ p_{LK} \end{bmatrix}$$

$$e \;=\; V\,w \;=\; \begin{bmatrix} v_{11} & \cdots & v_{L1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_{12} & \cdots & v_{L2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & v_{1T} & \cdots & v_{LT} \end{bmatrix} \begin{bmatrix} w_{11} \\ \vdots \\ w_{L1} \\ w_{12} \\ \vdots \\ w_{L2} \\ \vdots \\ w_{1T} \\ \vdots \\ w_{LT} \end{bmatrix}$$

Computational Details

This constrained estimation problem can be solved either directly (primal) or by using the dual form. Either way, it is prudent to factor out one probability for each parameter and each observation as the sum of the other probabilities. This factoring reduces the computational complexity significantly. If the primal formalization is used and two support points are used for the parameters and the errors, the resulting GME problem is $O((nparms + nobs)^3)$. For the dual form, the problem is $O((nobs)^3)$. Therefore for large data sets, GME-M should be used instead of GME.

Moment Generalized Maximum Entropy

The default estimation technique is moment generalized maximum entropy (GME-M). This is simply GME with the data constraints modified by multiplying both sides by X'. GME-M then becomes

$$\begin{aligned} \text{maximize} \quad & H(p,w) \;=\; -p'\ln(p) \;-\; w'\ln(w) \\ \text{subject to} \quad & X'y \;=\; X'X\,Z\,p \;+\; X'V\,w \\ & 1_K \;=\; (I_K \otimes 1'_L)\,p \\ & 1_T \;=\; (I_T \otimes 1'_L)\,w \end{aligned}$$

There is also the cross entropy version of GME-M, which has the same form as GCE but with the moment constraints.

GME versus GME-M

GME-M is more computationally attractive than GME for large data sets because the computational complexity of the estimation problem depends primarily on the number of parameters and not on the number of observations. GME-M is based on the first moment of the data, whereas GME is based on the data itself. If the distribution of the residuals is well defined by its first moment, then GME-M is a good choice. So if the residuals are normally distributed or exponentially distributed, then GME-M should be used. On the other hand if the distribution is Cauchy, lognormal, or some other distribution where the first moment does not describe the distribution, then use GME. See Example 14.1 for an illustration of this point.

Maximum Entropy-Based Seemingly Unrelated Regression

In a multivariate regression model, the errors in different equations might be correlated. In this case, the efficiency of the estimation can be improved by taking these cross-equation correlations into account. Seemingly unrelated regression (SUR), also called joint generalized least squares (JGLS) or Zellner estimation, is a generalization of OLS for multi-equation systems.

Like SUR in the least squares setting, the generalized maximum entropy SUR (GME-SUR) method assumes that all the regressors are independent variables and uses the correlations among the errors in different equations to improve the regression estimates. The GME-SUR method requires an initial entropy regression to compute residuals. The entropy residuals are used to estimate the cross-equation covariance matrix.

In the iterative GME-SUR (ITGME-SUR) case, the preceding process is repeated by using the residuals from the GME-SUR estimation to estimate a new cross-equation covariance matrix. ITGME-SUR method alternates between estimating the system coefficients and estimating the cross-equation covariance matrix until the estimated coefficients and covariance matrix converge.

The estimation problem becomes the generalized maximum entropy system adapted for multi-equations as follows:

$$
\begin{array}{lrcl}
\text{maximize} & H(p,w) & = & -p' \ln(p) \; - \; w' \ln(w) \\
\text{subject to} & y & = & X\,Z\,p \; + \; V\,w \\
 & 1_{KM} & = & (I_{KM} \otimes 1'_L)\,p \\
 & 1_{MT} & = & (I_{MT} \otimes 1'_L)\,w
\end{array}
$$

where

$$\beta \; = \; Z\,p$$

$$Z = \begin{bmatrix} z_{11}^1 & \cdots & z_{L1}^1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_{11}^K & \cdots & z_{L1}^K & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z_{1M}^1 & \cdots & z_{LM}^1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z_{1M}^K & \cdots & z_{LM}^K \end{bmatrix}$$

$$p = [\ p_{11}^1 \ \cdot \ p_{L1}^1 \ \cdot \ p_{11}^K \ \cdot \ p_{L1}^K \ \cdot \ p_{1M}^1 \ \cdot \ p_{LM}^1 \ \cdot \ p_{1M}^K \ \cdot \ p_{LM}^K \ ]'$$

$$e = V\, w$$

$$V = \begin{bmatrix} v_{11}^1 & \cdots & v_{11}^L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{1T}^1 & \cdots & v_{1T}^L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v_{M1}^1 & \cdots & v_{M1}^L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v_{MT}^1 & \cdots & v_{MT}^L \end{bmatrix}$$

$$w = [\ w_{11}^1 \ \cdot \ w_{11}^L \ \cdot \ w_{1T}^1 \ \cdot \ w_{1T}^L \ \cdot \ w_{M1}^1 \ \cdot \ w_{M1}^L \ \cdot \ w_{MT}^1 \ \cdot \ w_{MT}^L \ ]'$$

y denotes the *MT* column vector of observations of the dependent variables; **X** denotes the (*MT* x *KM*) matrix of observations for the independent variables; *p* denotes the *LKM* column vector of weights associated with the points in *Z*; *w* denotes the *LMT* column vector of weights associated with the points in *V*; 1_L, 1_{KM}, and 1_{MT} are *L*-, *KM*-, and *MT*-dimensional column vectors, respectively, of ones; and $\mathbf{I_{KM}}$ and $\mathbf{I_{MT}}$ are (*KM* x *KM*) and (*MT* x *MT*) dimensional identity matrices. The subscript *l* denotes the support point $(l = 1, 2, \ldots, L)$, *k* denotes the parameter $(k = 1, 2, \ldots, K)$, *m* denotes the equation $(m = 1, 2, \ldots, M)$, and *t* denotes the observation $(t = 1, 2, \ldots, T)$.

Using this notation, the maximum entropy problem that is analogous to the OLS problem used as the initial step of the traditional SUR approach is

$$\begin{aligned} \text{maximize} \quad & H(p, w) \; = \; -p' \ln(p) \; - \; w' \ln(w) \\ \text{subject to} \quad & (y \; - \; X\, Z\, p) \; = \; \sqrt{\Sigma}\, V\, w \\ & 1_{KM} \; = \; (I_{KM} \otimes 1_L')\, p \\ & 1_{MT} \; = \; (I_{MT} \otimes 1_L')\, w \end{aligned}$$

The results are GME-SUR estimates with independent errors, the analog of OLS. The covariance matrix $\hat{\Sigma}$ is computed based on the residual of the equations, $Vw = e$. An $L'L$ factorization of the $\hat{\Sigma}$ is used to compute the square root of the matrix.

After solving this problem, these entropy-based estimates are analogous to the Aitken two-step estimator. For iterative GME-SUR, the covariance matrix of the errors is recomputed, and a new $\hat{\Sigma}$ is computed and factored. As in traditional ITSUR, this process repeats until the covariance matrix and the parameter estimates converge.

The estimation of the parameters for the normed-moment version of SUR (GME-SUR-NM) uses an identical process. The constraints for GME-SUR-NM is defined as:

$$X'y \;=\; X'(\mathbf{S}^{-1}\otimes\mathbf{I})X\,Z\,p \;+\; X'(\mathbf{S}^{-1}\otimes\mathbf{I})V\,w$$

The estimation of the parameters for GME-SUR-NM uses an identical process as outlined previously for GME-SUR.

Generalized Maximum Entropy for Multinomial Discrete Choice Models

Multinomial discrete choice models take the form of an experiment that consists of n trials. On each trial, one of k alternatives is observed. If y_{ij} is the random variable that takes on the value 1 when alternative j is selected for the ith trial and 0 otherwise, then the probability that y_{ij} is 1, conditional on a vector of regressors X_i and unknown parameter vector β_j, is

$$\Pr(y_{ij} = 1|X_i, \beta_j) = G(X_i'\beta_j)$$

where $G()$ is a link function. For noisy data the model becomes:

$$y_{ij} = G(X_i'\beta_j) + \epsilon_{ij} = p_{ij} + \epsilon_{ij}$$

The standard maximum likelihood approach for multinomial logit is equivalent to the maximum entropy solution for discrete choice models. The generalized maximum entropy approach avoids an assumption of the form of the link function $G()$.

The generalized maximum entropy for discrete choice models (GME-D) is written in primal form as

$$\begin{array}{lll} \text{maximize} & H(p,w) & = \; -p'\,\ln(p) \;-\; w'\,\ln(w) \\ \text{subject to} & (I_j \otimes X'y) & = \; (I_j \otimes X')p \;+\; (I_j \otimes X')V\,w \\ & \sum_j^k p_{ij} & = 1 \quad \text{for } i \;=1 \text{ to } N \\ & \sum_m^L w_{ijm} & = 1 \quad \text{for } i \;=1 \text{ to } N \;\text{ and }\; j \;=1 \text{ to } k \end{array}$$

Golan, Judge, and Miller (1996) have shown that the dual unconstrained formulation of the GME-D can be viewed as a general class of logit models. Additionally, as the sample size increases, the solution of the dual problem approaches the maximum likelihood solution. Because of these characteristics, only the dual approach is available for the GME-D estimation method.

The parameters β_j are the Lagrange multipliers of the constraints. The covariance matrix of the parameter estimates is computed as the inverse of the Hessian of the dual form of the objective function.

Censored or Truncated Dependent Variables

In practice, you might find that variables are not always measured throughout their natural ranges. A given variable might be recorded continuously in a range, but, outside of that range, only the endpoint is denoted. In other words, say that the data generating process is:

$$y_i = \mathbf{x}_{i_\epsilon} + \epsilon.$$

However, you observe the following:

$$y_i^\star = \begin{cases} ub & : \quad y_i \geq ub \\ \mathbf{x}_{i_\epsilon} + \epsilon & : \quad lb < y_i < ub \\ lb & : \quad y_i \leq lb \end{cases}$$

The primal problem is simply a slight modification of the primal formulation for GME-GCE. You specify different supports for the errors in the truncated or censored region, perhaps reflecting some nonsample information. Then the data constraints are modified. The constraints that arise in the censored areas are changed to inequality constraints (Golan, Judge, and Perloff 1997). Let the variable $\mathbf{X}^u$ denote the observations of the explanatory variable where censoring occurs from the top, $\mathbf{X}^l$ from the bottom, and $\mathbf{X}^a$ in the middle region (no censoring). Let, $\mathbf{V}^u$ be the supports for the observations at the upper bound, $\mathbf{V}^l$ lower bound, and $\mathbf{V}^a$ in the middle.

You have:

$$\begin{bmatrix} \mathbf{y}^u \geq ub \\ \mathbf{y}^a \\ \mathbf{y}^l \leq lb \end{bmatrix} = \begin{bmatrix} \mathbf{X}^u \\ \mathbf{X}^a \\ \mathbf{X}^l \end{bmatrix} \mathbf{Zp} + \begin{bmatrix} \mathbf{V}^u \mathbf{w}^u \\ \mathbf{V}^a \mathbf{w}^a \\ \mathbf{V}^l \mathbf{w}^l \end{bmatrix}$$

The primal problem then becomes

$$\begin{aligned} \text{maximize} \quad & H(p,w) \;=\; -p' \ln(p) \;-\; w' \ln(w) \\ \text{subject to} \quad & \mathbf{y}^a \;=\; \mathbf{X}^a\, \mathbf{V}^a\; p \;+\; \mathbf{V}^a\, \mathbf{w}^a \\ & \mathbf{y}^u \;\geq\; \mathbf{X}^u\, \mathbf{V}^u\; p \;+\; \mathbf{V}^u\, \mathbf{w}^u \\ & \mathbf{y}^l \;\leq\; \mathbf{X}^l\, \mathbf{V}^l\; p \;+\; \mathbf{V}^l\, \mathbf{w}^l \\ & 1_K \;=\; (I_K \,\otimes\, 1'_L)\, p \\ & 1_T \;=\; (I_T \,\otimes\, 1'_L)\, w \end{aligned}$$

PROC ENTROPY requires that the number of supports be identical for all three regions.

Alternatively, you can think of cases where the dependent variable is observed continuously for most of its range. However, the variable's range is reported for some observations. Such data is often found in highly disaggregated state level employment measures.

$$y_i^\star = \begin{cases} \textit{missing} & : \quad l_1 \leq y \leq r_1 \\ \vdots & : \quad \vdots \\ \textit{missing} & : \quad l_k \leq y \leq r_k \\ \mathbf{x}_{i_\epsilon} + \epsilon & : \quad \textit{otherwise} \end{cases}$$

Just as in the censored case, each range yields two inequality constraints for each observation in that range.

Information Measures

PROC ENTROPY returns several measures of fit. First, the value of the objective function is returned. Next, the signal entropy is provided followed by the noise entropy. The sum of the noise and signal entropies should equal the value of the objective function. The next two metrics that follow are the normed entropies of both the signal and the noise.

Normalized entropy (NE) measures the relative informational content of both the signal and noise components through p and w, respectively (Golan, Judge, and Miller 1996). Let S denote the normalized entropy of the signal, $X\beta$, defined as:

$$S(\tilde{p}) = \frac{-\tilde{p}' \ln(\tilde{p})}{-q' \ln(q)}$$

where $S(\tilde{p}) \epsilon [0, 1]$. In the case of GME, where uniform priors are assumed, S can be written as:

$$S(\tilde{p}) = \frac{-\tilde{p}' \ln(\tilde{p})}{\sum_i \ln(M_i)}$$

where M_i is the number of support points for parameter i. A value of 0 for S implies that there is no uncertainty regarding the parameters; hence, it is a degenerate situation. However, a value of 1 implies that the posterior distributions equal the priors, which indicates total uncertainty if the priors are uniform.

Because NE is relative, it can be used for comparing various situations. Consider adding a data point to the model. If $S_{T+1} = S_T$, then there is no additional information contained within that data constraint. However, if $S_{T+1} < S_T$, then the data point gives a more informed set of parameter estimates.

NE can be used for determining the importance of particular variables with regard to the reduction of the uncertainty they bring to the model. Each of the k parameters that is estimated has an associated NE defined as

$$S(\tilde{p_k}) = \frac{-\tilde{p}_k' \ln(\tilde{p}_k)}{-\ln(q_k)}$$

or, in the GME case,

$$S(\tilde{p_k}) = \frac{-\tilde{p}_k' \ln(\tilde{p}_k)}{\ln(M)}$$

where $\tilde{p_k}$ is the vector of supports for parameter β_k and M is the corresponding number of support points. Since a value of 1 implies no relative information for that particular sample, Golan, Judge, and Miller (1996) suggest an exclusion criteria of $S(\tilde{p_k}) > 0.99$ as an acceptable means of selecting noninformative variables. See Golan, Judge, and Miller (1996) for some simulation results.

The final set of measures of fit are the parameter information index and error information index. These measures can be best summarized as 1 – the appropriate normed entropy.

Parameter Covariance For GCE

For the cross-entropy problem, the estimate of the asymptotic variance of the signal parameter is given by:

$$\hat{Var}(\hat{\beta}) = \frac{\hat{\sigma_{\gamma}^2}(\hat{\beta})}{\hat{\psi}^2(\hat{\beta})}(X'X)^{-1}$$

where

$$\hat{\sigma_{\gamma}^2}(\hat{\beta}) = \frac{1}{N}\sum_{i=1}^{N}\gamma_i^2$$

and γ_i is the Lagrange multiplier associated with the i th row of the Vw constraint matrix. Also,

$$\hat{\psi}^2(\hat{\beta}) = \left[\frac{1}{N}\sum_{i=1}^{N}\left(\sum_{j=1}^{J}v_{ij}^2 w_{ij} - (\sum_{j=1}^{J}v_{ij}w_{ij})^2\right)^{-1}\right]^2$$

Parameter Covariance For GCE-M

Golan, Judge, and Miller (1996) give the finite approximation to the asymptotic variance matrix of the moment formulation as:

$$\hat{Var}(\hat{\beta}) = \Sigma_z X'XC^{-1}DC^{-1}X'X\Sigma_z$$

where

$$C = X'X\Sigma_z X'X + \Sigma_v$$

and

$$D = X'\Sigma_e X$$

Recall that in the moment formulation, V is the support of $\frac{X'e}{T}$, which implies that Σ_v is a k-dimensional variance matrix. Σ_z and Σ_v are both diagonal matrices with the form

$$\Sigma_z = \begin{bmatrix} \sum_{l=1}^{L} z_{1l}^2 p_{1l} - (\sum_{l=1}^{L} z_{1l}p_{1l})^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sum_{l=1}^{L} z_{Kl}^2 p_{Kl} - (\sum_{l=1}^{L} z_{Kl}p_{Kl})^2 \end{bmatrix}$$

and

$$\Sigma_v = \begin{bmatrix} \sum_{j=1}^{J} v_{1j}^2 w_{jl} - (\sum_{j=1}^{J} v_{1j}w_{1j})^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sum_{j=1}^{J} v_{Kl}^2 w_{Kl} - (\sum_{j=1}^{J} v_{Kl}w_{Kl})^2 \end{bmatrix}$$

Statistical Tests

Since the GME estimates have been shown to be asymptotically normally distributed, the classical Wald, Lagrange multiplier, and likelihood ratio statistics can be used for testing linear restrictions on the parameters.

Wald Tests

Let $H_0 : L\beta = m$, where L is a set of linearly independent combinations of the elements of β. Then under the null hypothesis, the Wald test statistic,

$$T_W = (L\beta - m)' \left(L(\hat{Var}(\hat{\beta}))L' \right)^{-1} (L\beta - m)$$

has a central χ^2 limiting distribution with degrees of freedom equal to the rank of L.

Pseudo-Likelihood Ratio Tests

Using the conditionally maximized entropy function as a pseudo-likelihood, F, Mittelhammer and Cardell (2000) state that:

$$\frac{2\hat{\psi}(\hat{\beta})}{\hat{\sigma_\gamma^2}(\hat{\beta})} \left(F(\hat{\beta}) - F(\tilde{\beta}) \right)$$

has the limiting distribution of the Wald statistic when testing the same hypothesis. Note that $F(\hat{\beta})$ and $F(\tilde{\beta})$ are the maximum values of the entropy objective function over the full and restricted parameter spaces, respectively.

Lagrange Multiplier Tests

Again using the GME function as a pseudo-likelihood, Mittelhammer and Cardell (2000) define the Lagrange multiplier statistic as:

$$\frac{1}{\hat{\sigma_\gamma^2}(\tilde{\beta})} G(\tilde{\beta})'(X'X)^{-1} G(\tilde{\beta})$$

where G is the gradient of F, which is being evaluated at the optimum point for the restricted parameters. This test statistic shares the same limiting distribution as the Wald and pseudo-likelihood ratio tests.

Missing Values

If an observation in the input data set contains a missing value for any of the regressors or dependent values, that observation is dropped from the analysis.

Input Data Sets

DATA= Data Set

The DATA= data set specified in the PROC ENTROPY statement is the data set that contains the data to be analyzed.

PDATA= Data Set

The PDATA= data set specified in the PROC ENTROPY statement specifies the support points and prior probabilities to be used in the estimation. The PDATA= can be used in lieu of a PRIORS statement, but is intended for use in conjunction with the OUTP= option. Once priors are entered through a PRIORS statement, they can be reused in subsequent estimations by specifying the PDATA= option.

The variables in the data set are as follows:

- BY variables (if any)
- _TYPE_, a character variable of length 8 that identifies the estimation method: GME or GMEM. This is an optional column.
- variable, a character variable of length 32 that indicates the name of the regressor. The regressor name and the equation name identify a unique coefficient. This is required.
- _OBS_, a numeric variable that is either missing when the probabilities are for coefficients or the observation number when the probabilities are for the residual terms. The _OBS_ and the equation name identify which residual the probability is associated with. This an optional column.
- equation, a character variable of length 32 indicating the name of the dependent variable. This is a required column.
- NSupport, a numeric variable that indicates the number of support points for each basis. This variable is required.
- support, a numeric variable that is the support value the probability is associated with. This is a required column.
- prior, a numeric variable that is the prior probability associated with the probability. This is a required column.
- Prb, a numeric variable that is the estimated probability. This is optional.

SDATA= Data Set

The SDATA= data set specifies a data set that provides the covariance matrix of the equation errors. The matrix read from the SDATA= data set is used for the equation covariance matrix (**S** matrix) in the estimation. (The SDATA= **S** matrix is used to provide only the initial estimate of **S** for the methods that iterate the **S** matrix.)

Output Data Sets

OUT= Data Set

The OUT= data set specified in the PROC ENTROPY statement contains residuals of the dependent variables computed from the parameter estimates. The ID and BY variables are also added to this data set.

OUTEST= Data Set

The OUTEST= data set contains parameter estimates and, if requested via the COVOUT option, estimates of the covariance of the parameter estimates.

The variables in the data set are as follows:

- BY variables
- _NAME_, a character variable of length 32, blank for observations that contain parameter estimates or a parameter name for observations that contain covariances
- _TYPE_, a character variable of length 8 that identifies the estimation method: GME or GMEM
- the parameters estimated

If the COVOUT option is specified, an additional observation is written for each row of the estimate of the covariance matrix of parameter estimates, with the _NAME_ values containing the parameter names for the rows.

OUTP= Data Set

The OUTP= data set specified in the PROC ENTROPY statement contains the probabilities estimated for each support point, as well as the support points and prior probabilities used in the estimation.

The variables in the data set are as follows:

- BY variables (if any)
- _TYPE_, a character variable of length 8 that identifies the estimation method: GME or GMEM.
- variable, a character variable of length 32 that indicates the name of the regressor. The regressor name and the equation name identify a unique coefficient.
- _OBS_, a numeric variable that is either missing when the probabilities are for coefficients or the observation number when the probabilities are for the residual terms. The _OBS_ and the equation name identify which residual the probability is associated with.
- equation, a character variable of length 32 that indicates the name of the dependent variable
- NSupport, a numeric variable that indicates the number of support points for each basis
- support, a numeric variable that is the support value the probability is associated with
- prior, a numeric variable that is the prior probability associated with the probability
- Prb, a numeric variable that is the estimated probability

OUTL= Data Set

The OUTL= data set specified in the PROC ENTROPY statement contains the Lagrange multiplier values for the underlying maximum entropy problem.

The variables in the data set are as follows:

- BY variables
- equation, a character variable of length 32 that indicates the name of the dependent variable
- variable, a character variable of length 32 that indicates the name of the regressor. The regressor name and the equation name identify a unique coefficient.
- _OBS_, a numeric variable that is either missing when the probabilities are for coefficients or the observation number when the probabilities are for the residual terms. The _OBS_ and the equation name identify which residual the Lagrange multiplier is associated with
- LagrangeMult, a numeric variable that contains the Lagrange multipliers

ODS Table Names

PROC ENTROPY assigns a name to each table it creates. You can use these names to reference the table when using the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in the following table.

Table 14.2 ODS Tables Produced in PROC ENTROPY

ODS Table Name	Description	Option
ConvCrit	Convergence criteria for estimation	default
ConvergenceStatus	Convergence status	default
DatasetOptions	Data sets used	default
MinSummary	Number of parameters, estimation kind	default
ObsUsed	Observations read, used, and missing	default
ParameterEstimates	Parameter estimates	default
ResidSummary	Summary of the SSE, MSE for the equations	default
TestResults	Test statement table	TEST statement

ODS Graphics

Statistical procedures use ODS Graphics to create graphs as part of their output. ODS Graphics is described in detail in Chapter 21, “Statistical Graphics Using ODS” (*SAS/STAT User’s Guide*).

Before you create graphs, ODS Graphics must be enabled (for example, with the ODS GRAPHICS ON statement). For more information about enabling and disabling ODS Graphics, see the section "Enabling and Disabling ODS Graphics" in that chapter.

The overall appearance of graphs is controlled by ODS styles. Styles and other aspects of using ODS Graphics are discussed in the section "A Primer on ODS Statistical Graphics" in that chapter.

This section describes the use of ODS for creating graphics with the ENTROPY procedure.

ODS Graph Names

PROC ENTROPY assigns a name to each graph it creates using ODS. You can use these names to reference the graphs when using ODS. The names are listed in Table 14.3.

To request these graphs, you must specify the ODS GRAPHICS statement.

Table 14.3 ODS Graphics Produced by PROC ENTROPY

ODS Graph Name	Plot Description
DiagnosticsPanel	Includes all the plots listed below
FitPlot	Predicted versus actual plot
CooksD	Cook's *D* plot
QQPlot	Q-Q plot of residuals
StudentResidualPlot	Studentized residual plot
ResidualHistogram	Histogram of the residuals

Examples: ENTROPY Procedure

Example 14.1: Nonnormal Error Estimation

This example illustrates the difference between GME-M and GME. One of the basic assumptions of OLS estimation is that the errors in the estimation are normally distributed. If this assumption is violated, the estimated parameters are biased. For GME-M, the story is similar. If the first moment of the distribution of the errors and a scale factor cannot be used to describe the distribution, then the parameter estimates from GME-MN are more biased. GME is much less sensitive to the underlying distribution of the errors than GME-M.

To illustrate this, data for the following model is simulated with three different error distributions:

$$y = a * x_1 + b * x_2 + \epsilon.$$

For the first simulation, ϵ is distributed normally, then a chi-squared distribution with six degrees of freedom is assumed for the second simulation, and finally ϵ is assumed to have a Cauchy distribution in the third simulation.

In each of the three simulations, 100 samples of 10 observations each were simulated. The data for the model with the Cauchy error distribution is generated using the following DATA step code:

```
data one;
   call streaminit(156789);
   do by = 1 to 100;
      do x2 = 1 to 10;
         x1 = 10 * ranuni( 512);
         y = x1 + 2*x2 + rand('cauchy');
         output;
      end;
   end;
run;
```

The statements for the other distributions are identical except for the argument to the RAND() function.

The parameters to the model were estimated by using maximum entropy with the following programming statements:

```
proc entropy data=one gme outest=parm1;
   model y = x1 x2;
   by by;
run;
```

The estimation by using moment-constrained maximum entropy was performed by changing the GME option to GMEM. For comparison, the same model was estimated by using OLS with the following PROC REG statements:

```
proc reg data=one outest=parm3;
   model y = x1 x2;
   by by;
run;
```

The 100 estimations of the coefficient on variable x1 are then summarized for each of the three error distributions by using PROC UNIVARIATE, as follows:

```
proc univariate data=parm1;
   var x1;
run;
```

The following table summarizes the results from the estimations. The true value for the coefficient on x1 is 1.0.

Estimation	**Normal**		**Chi-Squared**		**Cauchy**	
Method	**Mean**	**Std Deviation**	**Mean**	**Std Deviation**	**Mean**	**Std Deviation**
GME	0.418	0.117	0.626	.330	0.818	3.36
GME-M	0.878	0.116	0.948	0.427	3.03	13.62
OLS	0.973	0.142	1.023	0.467	5.54	26.83

For normally distributed or nearly normally distributed data, moment-constrained maximum entropy is a good choice. For distributions not well described by a normal distribution, data-constrained maximum entropy is a good choice.

Example 14.2: Unreplicated Factorial Experiments

Factorial experiments are useful for studying the effects of various factors on a response. For the practitioner constrained to the use of OLS regression, there must be replication to estimate all of the possible main and interaction effects in a factorial experiment. Using OLS regression to analyze unreplicated experimental data results in zero degrees of freedom for error in the ANOVA table, since there are as many parameters as observations. This situation leaves the experimenter unable to compute confidence intervals or perform hypothesis testing on the parameter estimates.

Several options are available when replication is impossible. The higher-order interactions can be assumed to have negligible effects, and their degrees of freedom can be pooled to create the error degrees of freedom used to perform inference on the lower-order estimates. Or, if a preliminary experiment is being run, a normal probability plot of all effects can provide insight as to which effects are significant, and therefore focused, in a later, more complete experiment.

The following example illustrates the probability plot methodology and the alternative by using PROC ENTROPY. Consider a 2^4 factorial model with no replication. The data are taken from Myers and Montgomery (1995).

```
data rate;
   do a=-1,1; do b=-1,1; do c=-1,1; do d=-1,1;
      input y @@;
      ab=a*b; ac=a*c; ad=a*d; bc=b*c; bd=b*d; cd=c*d;
      abc=a*b*c; abd=a*b*d; acd=a*c*d; bcd=b*c*d;
      abcd=a*b*c*d;
      output;
   end; end; end; end;
   datalines;
   45 71 48 65 68 60 80 65 43 100 45 104 75 86 70 96
   ;
run;
```

Analyze the data by using PROC REG, then output the resulting estimates.

```
proc reg data=rate outest=regout;
   model y=a b c d ab ac ad bc bd cd abc abd acd bcd abcd;
run;

proc transpose data=regout out=ploteff name=effect prefix=est;
   var a b c d ab ac ad bc bd cd abc abd acd bcd abcd;
run;
```

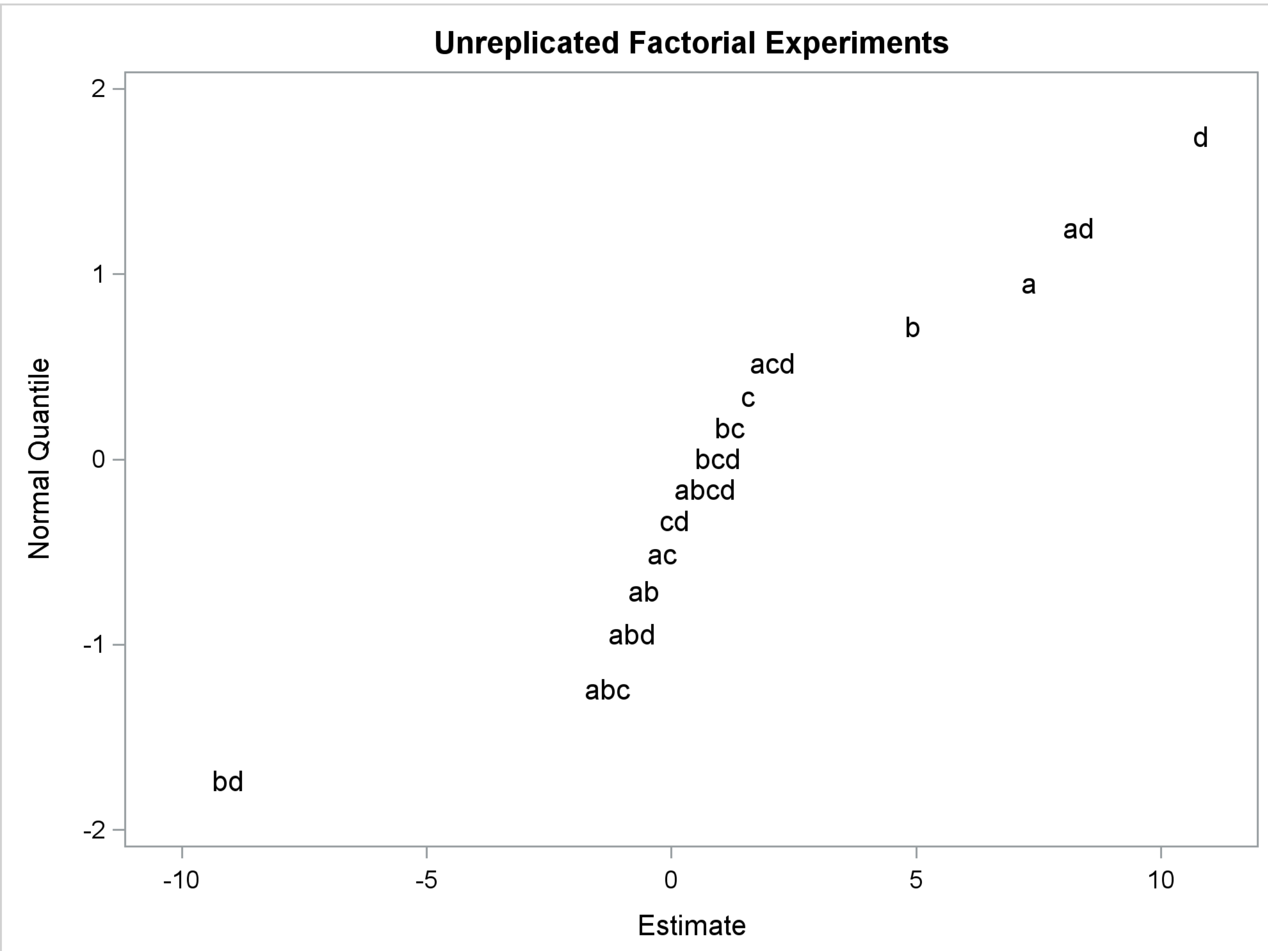
Now the normal scores for the estimates can be computed with the rank procedure as follows:

```
proc rank data=ploteff normal=blom out=qqplot;
   var est1;
   ranks normalq;
run;
```

To create the probability plot, simply plot the estimates versus their normal scores by using PROC SGPLOT as follows:

```
title "Unreplicated Factorial Experiments";
proc sgplot data=qqplot;
   scatter x=est1 y=normalq / markerchar=effect
                              markercharattrs=(size=10pt);
   xaxis label="Estimate";
   yaxis label="Normal Quantile";
run;
```

Output 14.2.1 Normal Probability Plot of Effects



The plot shown in Output 14.2.1 displays evidence that the a, b, d, ad, and bd estimates do not fit into the purely random normal model, which suggests that they may have some significant effect on the response variable. To verify this, fit a reduced model that contains only these effects.

```
proc reg data=rate;
   model y=a b d ad bd;
run;
```

The estimates for the reduced model are shown in Output 14.2.2.

Output 14.2.2 Reduced Model OLS Estimates

Unreplicated Factorial Experiments

The REG Procedure
Model: MODEL1
Dependent Variable: y

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	70.06250	1.10432	63.44	<.0001
a	1	7.31250	1.10432	6.62	<.0001
b	1	4.93750	1.10432	4.47	0.0012
d	1	10.81250	1.10432	9.79	<.0001
ad	1	8.31250	1.10432	7.53	<.0001
bd	1	-9.06250	1.10432	-8.21	<.0001

These results support the probability plot methodology.

PROC ENTROPY can directly estimate the full model without having to rely upon the probability plot for insight into which effects can be significant. To illustrate this, PROC ENTROPY is run by using default parameter and error supports in the following statements:

```
proc entropy data=rate;
   model y=a b c d ab ac ad bc bd cd abc abd acd bcd abcd;
run;
```

The resulting GME estimates are shown in Output 14.2.3. Note that the parameter estimates associated with the a, b, d, ad, and bd effects are all significant.

Output 14.2.3 Full Model Entropy Results

Unreplicated Factorial Experiments

The ENTROPY Procedure

GME Variable Estimates				
Variable	Estimate	Approx Std Err	t Value	Approx Pr > |t|
a	5.688414	0.7911	7.19	<.0001
b	2.988032	0.5464	5.47	<.0001
c	0.234331	0.1379	1.70	0.1086
d	9.627308	0.9765	9.86	<.0001
ab	-0.01386	0.0270	-0.51	0.6149
ac	-0.00054	0.00325	-0.16	0.8712
ad	6.833076	0.8627	7.92	<.0001
bc	0.113908	0.0941	1.21	0.2435
bd	-7.68105	0.9053	-8.48	<.0001
cd	0.00002	0.000364	0.05	0.9569
abc	-0.14876	0.1087	-1.37	0.1900
abd	-0.0399	0.0516	-0.77	0.4509
acd	0.466938	0.1961	2.38	0.0300
bcd	0.059581	0.0654	0.91	0.3756
abcd	0.024785	0.0387	0.64	0.5312
Intercept	69.87294	1.1403	61.28	<.0001

Example 14.3: Censored Data Models in PROC ENTROPY

Data available to an analyst might sometimes be censored, where only part of the actual series is observed. Consider the case in which only observations greater than some lower bound are recorded, as defined by the following process:

$$\mathbf{y} = max\left(\mathbf{X}\boldsymbol{\beta} + \epsilon, lb\right).$$

Running ordinary least squares estimation on data generated by the preceding process is not optimal because the estimates are likely to be biased and inefficient. One alternative to estimating models with censored data is the tobit estimator. This model is supported in the QLIM procedure in SAS/ETS and in the LIFEREG procedure in SAS/STAT. PROC ENTROPY provides another alternative which can make it very easy to estimate such a model correctly.

The following DATA step generates censored data in which any negative values of the dependent variable, y, are set to a lower bound of 0.

```
data cens;
  do t = 1 to 100;
     x1 = 5 * ranuni(456);
     x2 = 10 * ranuni(456);
     y = 4.5*x1 + 2*x2 + 15 * rannor(456);
     if( y<0 ) then y = 0;
     output;
   end;
run;
```

To illustrate the effect of the censored option in PROC ENTROPY, the model is initially estimated without accounting for censoring in the following statements:

```
title "Censored Data Estimation";
proc entropy data = cens gme primal;
   priors intercept -32 32
          x1        -15 15
          x2        -15 15;
    model y = x1 x2 /
          esupports = (-25 1 25);
run;
```

Output 14.3.1 GME Estimates

Censored Data Estimation

The ENTROPY Procedure

GME Variable Estimates				
Variable	**Estimate**	**Approx Std Err**	**t Value**	**Approx Pr > |t|**
x1	2.377609	0.000503	4725.98	<.0001
x2	2.353014	0.000255	9244.87	<.0001
intercept	5.478121	0.00188	2906.41	<.0001

The previous model is reestimated by using the CENSORED option in the following statements:

```
proc entropy data = cens gme primal;
   priors intercept -32 32
          x1        -15 15
          x2        -15 15;
   model y = x1 x2 /
         esupports = (-25 1 25)
         censored(lb = 0, esupports=(-15 1 15) );
run;
```

Output 14.3.2 Entropy Estimates

Censored Data Estimation

The ENTROPY Procedure

GME Variable Estimates				
Variable	**Estimate**	**Approx Std Err**	**t Value**	**Approx Pr > |t|**
x1	4.429697	0.00690	641.85	<.0001
x2	1.46858	0.00349	420.61	<.0001
intercept	8.261412	0.0259	319.51	<.0001

The second set of entropy estimates are much closer to the true parameter estimates of 4.5 and 2. Since another alternative available for fitting a model of censored data is a tobit model, PROC QLIM is used in the following statements to fit a tobit model to the data:

```
proc qlim data=cens;
   model y = x1 x2;
   endogenous y ~ censored(lb=0);
run;
```

Output 14.3.3 QLIM Estimates

Censored Data Estimation

The QLIM Procedure

Parameter Estimates					
Parameter	**DF**	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**
Intercept	1	2.979455	3.824252	0.78	0.4359
x1	1	4.882284	1.019913	4.79	<.0001
x2	1	1.374006	0.513000	2.68	0.0074
_Sigma	1	13.723213	1.032911	13.29	<.0001

For this data and code, PROC ENTROPY produces estimates that are closer to the true parameter values than those computed by PROC QLIM.

Example 14.4: Use of the PDATA= Option

It is sometimes useful to specify priors and supports by using the PDATA= option. This example illustrates how to create a PDATA= data set which contains the priors and support points for use in a subsequent PROC ENTROPY step. In order to have a model to estimate in PROC ENTROPY, you must first have data to analyze. The following DATA step generates the data used in this analysis:

```
title "Using a PDATA= data set";
data a;
   array x[4];
   do t = 1 to 100;
      ys = -5;
      do k = 1 to 4;
         x[k] = rannor( 55372 )  ;
         ys = ys + x[k] * k;
      end;
      ys = ys + rannor( 55372 );
      output;
    end;
run;
```

Next you fit this data with some arbitrary parameter support points and priors by using the following PROC ENTROPY statements:

```
proc entropy data = a gme primal;
   priors           x1  -10(2) 30(1)
                    x2  -20(3) 30(2)
                    x3  -15(4) 30(4)
                    x4  -25(3) 30(2)
             intercept  -13(4) 30(2) ;
   model ys = x1 x2 x3 x4 / esupports=(-25 0 25);
run;
```

These statements produce the output shown in Output 14.4.1.

Output 14.4.1 Output From PROC ENTROPY

Using a PDATA= data set

The ENTROPY Procedure

GME Variable Estimates

Variable	Estimate	Approx Std Err	t Value	Approx Pr > |t|
x1	1.195688	0.1078	11.09	<.0001
x2	1.844903	0.1018	18.12	<.0001
x3	3.268396	0.1136	28.77	<.0001
x4	3.908194	0.0934	41.83	<.0001
intercept	-4.94319	0.1005	-49.21	<.0001

You can estimate the same model by first creating a PDATA= data set, which includes the same information as the PRIORS statement in the preceding PROC ENTROPY step.

A data set that defines the supports and priors for the model parameters is shown in the following statements:

```
data test;
   length Variable $ 12 Equation $ 12;
   input Variable $ Equation $ Nsupport Support Prior ;
datalines;
     Intercept   .  2 -13 0.66667
     Intercept   .  2  30 0.33333
            x1   .  2 -10 0.66667
            x1   .  2  30 0.33333
            x2   .  2 -20 0.60000
            x2   .  2  30 0.40000
            x3   .  2 -15 0.50000
            x3   .  2  30 0.50000
            x4   .  2 -25 0.60000
            x4   .  2  30 0.40000
;
```

The following statements reestimate the model by using these support points.

```
proc entropy data=a gme primal pdata=test;
   model ys = x1 x2 x3 x4 / esupports=(-25 0 25);
run;
```

These statements produce the output shown in Output 14.4.2.

Output 14.4.2 Output From PROC ENTROPY with PDATA= option

Using a PDATA= data set

The ENTROPY Procedure

GME Variable Estimates

Variable	Estimate	Approx Std Err	t Value	Approx Pr > |t|
x1	1.195686	0.1078	11.09	<.0001
x2	1.844902	0.1018	18.12	<.0001
x3	3.268395	0.1136	28.77	<.0001
x4	3.908194	0.0934	41.83	<.0001
Intercept	-4.94319	0.1005	-49.21	<.0001

These results are identical to the ones produced by the previous PROC ENTROPY step.

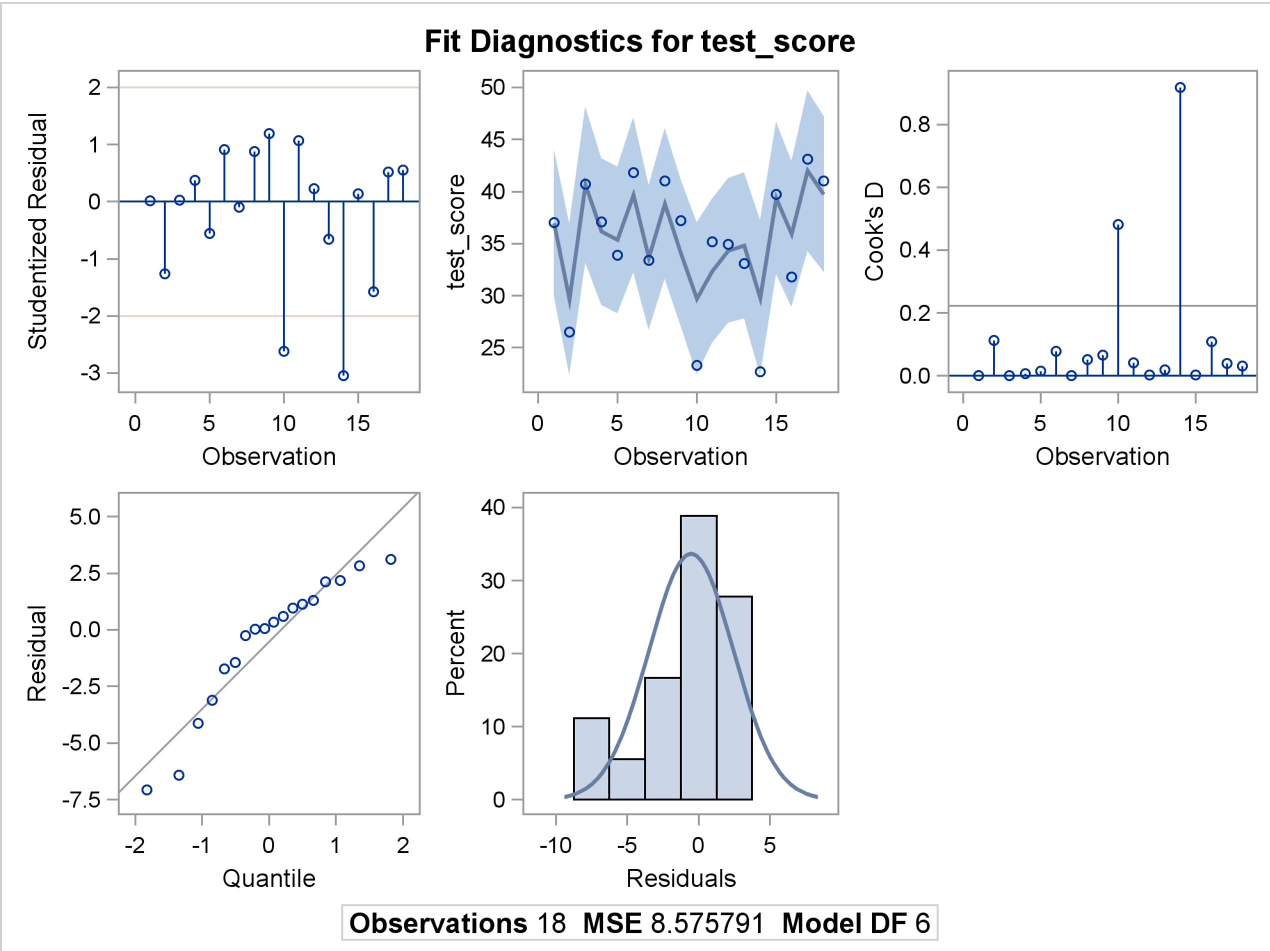
Example 14.5: Illustration of ODS Graphics

This example illustrates how to use ODS graphics in the ENTROPY procedure. This example is a continuation of the example in the section "Simple Regression Analysis" on page 788. Graphical displays are requested by specifying the ODS GRAPHICS statement. For information about the graphics available in the ENTROPY procedure, see the section "ODS Graphics" on page 832.

The following statements show how to generate ODS graphics plots with the ENTROPY procedure. The plots are displayed in Output 14.5.1.

```
proc entropy data=coleman;
   model test_score = teach_sal prcnt_prof socio_stat
                      teach_score mom_ed;
run;
```

Output 14.5.1 Model Diagnostics Plots



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