THE POWER TO KNOW®

SAS/STAT® 9.22 User's Guide

The STDIZE Procedure

(Book Excerpt)

SAS® Documentation

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Chapter 82

The STDIZE Procedure

Contents

Overview: STDIZE Procedure

The STDIZE procedure standardizes one or more numeric variables in a SAS data set by subtracting a location measure and dividing by a scale measure. A variety of location and scale measures are provided, including estimates that are resistant to outliers and clustering. Some of the well-known standardization methods such as mean, median, standard deviation, range, Huber's estimate, Tukey's biweight estimate, and Andrew's wave estimate are available in the STDIZE procedure.

In addition, you can multiply each standardized value by a constant and add a constant. Thus, the final output value is

$$result = add + multiply \times \frac{original - location}{scale}$$

where

result	= final output value
add	= constant to add (ADD= option)
multiply	= constant to multiply by (MULT= option)
original	= original input value
location	= location measure
scale	= scale measure

PROC STDIZE can also find quantiles in one pass of the data, a capability that is especially useful for very large data sets. With such data sets, the UNIVARIATE procedure might have high or excessive memory or time requirements.

Getting Started: STDIZE Procedure

The following example demonstrates how you can use the STDIZE procedure to obtain location and scale measures of your data.

In the following hypothetical data set, a random sample of grade twelve students is selected from a number of coeducational schools. Each school is classified as one of two types: Urban or Rural. There are 40 observations.

The variables are id (student identification), Type (type of school attended: 'urban'=urban area and 'rural'=rural area), and total (total assessment scores in History, Geometry, and Chemistry).

The following DATA step creates the SAS data set TotalScores.

```
data TotalScores;
   title 'High School Scores Data';
   input id Type $ total @@;
   datalines;
 1 rural 135   2 rural 125   3 rural 223   4 rural 224   5 rural 133
 6 rural 253   7 rural 144   8 rural 193   9 rural 152  10 rural 178
11 rural 120  12 rural 180  13 rural 154  14 rural 184  15 rural 187
16 rural 111  17 rural 190  18 rural 128  19 rural 110  20 rural 217
21 urban 192  22 urban 186  23 urban  64  24 urban 159  25 urban 133
26 urban 163  27 urban 130  28 urban 163  29 urban 189  30 urban 144
31 urban 154  32 urban 198  33 urban 150  34 urban 151  35 urban 152
36 urban 151  37 urban 127  38 urban 167  39 urban 170  40 urban 123
;
```

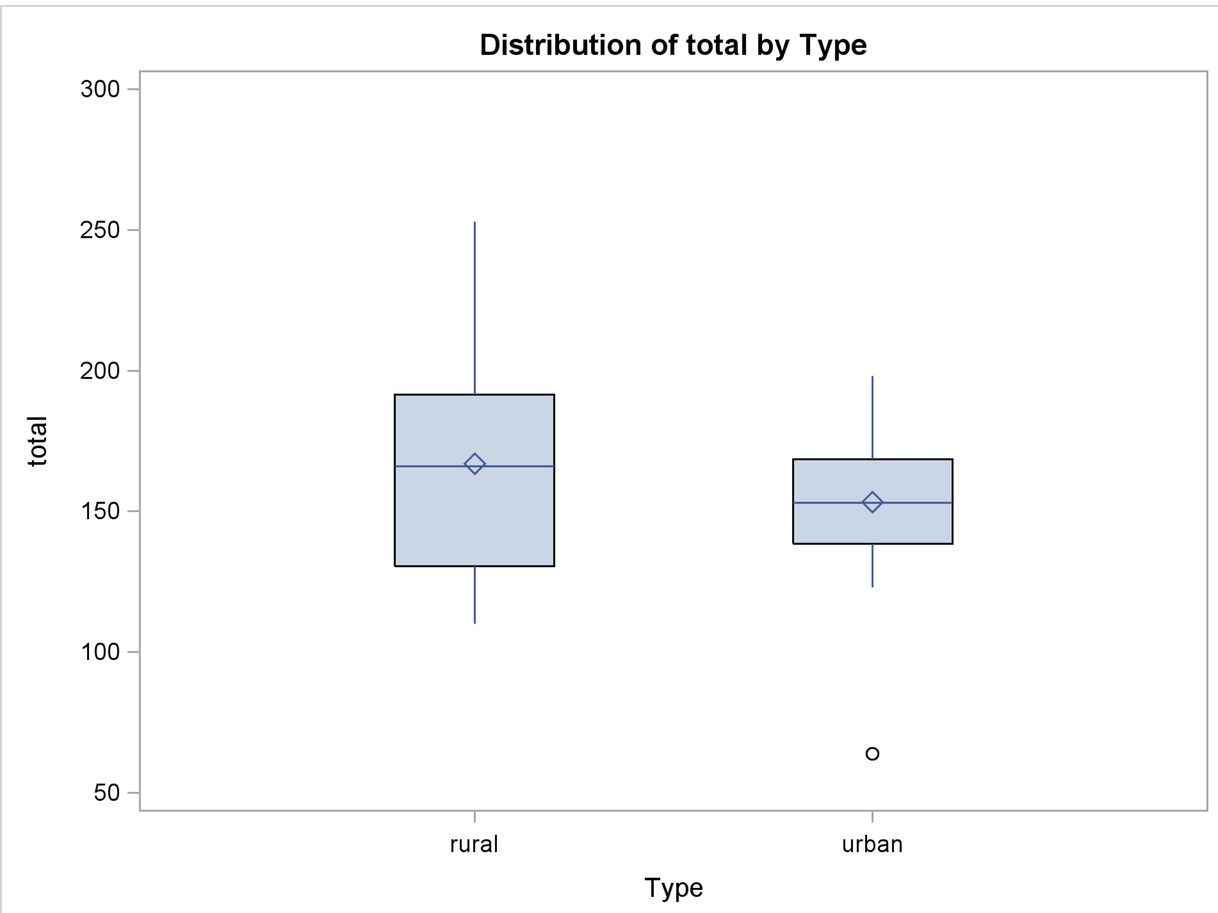
Suppose you now want to standardize the total scores in different types of schools prior to any further analysis. Before standardizing the total scores, you can use the box plot from PROC BOXPLOT to summarize the total scores for both types of schools.

To request this graph, you must specify the ODS GRAPHICS statement as follows. For more information about the ODS GRAPHICS statement, see Chapter 21, "Statistical Graphics Using ODS."

```
ods graphics on;
proc boxplot data=TotalScores;
   plot total*Type / boxstyle=schematic noserifs;
run;
ods graphics off;
```

The PLOT statement in the PROC BOXPLOT statement creates the schematic plots (without the serifs) when you specify **boxstyle=schematic noserifs**. Figure 82.1 displays a box plot for each type of school.

Figure 82.1 Schematic Plots from PROC BOXPLOT



Inspection reveals that one urban score is a low outlier. Also, if you compare the lengths of two box plots, there seems to be twice as much dispersion for the rural scores as for the urban scores.

The following PROC UNIVARIATE statement reports the information about the extreme values of the Score variable for each type of school:

```
proc univariate data=TotalScores;
   var total;
   by Type;
run;
```

Figure 82.2 displays the table from PROC UNIVARIATE for the lowest and highest five total scores for urban schools. The outlier (Obs = 3), marked in Figure 82.2 by the symbol ‘0’, has a score of 64.

Figure 82.2 Table for Extreme Observations When Type=urban

```
                          High School Scores Data

---------------------------------- Type=urban ----------------------------------

                          The UNIVARIATE Procedure
                              Variable:  total

                            Extreme Observations

                      ----Lowest----        ----Highest---

                      Value      Obs        Value      Obs

                         64       23          170       39
                        123       40          186       22
                        127       37          189       29
                        130       27          192       21
                        133       25          198       32
```

The following PROC STDIZE procedure requests the METHOD=STD option for computing the location and scale measures:

```
proc stdize data=totalscores method=std pstat;
   title2 'METHOD=STD';
   var total;
   by Type;
run;
```

Figure 82.3 displays the table of location and scale measures from the PROC STDIZE statement. PROC STDIZE uses the sample mean as the location measure and the sample standard deviation as the scale measure for standardizing. The PSTAT option displays a table containing these two measures.

Figure 82.3 Location and Scale Measures Table When METHOD=STD

```
                        High School Scores Data
                              METHOD=STD

---------------------------------- Type=rural ----------------------------------

                          The STDIZE Procedure

              Location and Scale Measures

              Location = mean     Scale = standard deviation

              Name         Location           Scale           N

              total      167.050000       41.956713          20

                        High School Scores Data
                              METHOD=STD

---------------------------------- Type=urban ----------------------------------

                          The STDIZE Procedure

              Location and Scale Measures

              Location = mean     Scale = standard deviation

              Name         Location           Scale           N

              total      153.300000       30.066768          20
```

The ratio of the scale of rural scores to the scale of urban scores is approximately 1.4 (41.96/30.07). This ratio is smaller than the dispersion ratio observed in the previous schematic plots.

The STDIZE procedure provides several location and scale measures that are resistant to outliers. The following statements invoke three different standardization methods and display the tables for the location and scale measures:

```
proc stdize data=totalscores method=mad pstat;
   title2 'METHOD=MAD';
   var total;
   by Type;
run;

proc stdize data=totalscores method=iqr pstat;
   title2 'METHOD=IQR';
   var total;
   by Type;
run;
```

```
proc stdize data=totalscores method=abw(4) pstat;
   title2 'METHOD=ABW(4)';
   var total;
   by Type;
run;
```

Figure 82.4 displays the table of location and scale measures when the standardization method is median absolute deviation (MAD). The location measure is the median, and the scale measure is the median absolute deviation from the median. The ratio of the scale of rural scores to the scale of urban scores is approximately 2.06 (32.0/15.5) and is close to the dispersion ratio observed in the previous schematic plots.

Figure 82.4 Location and Scale Measures Table When METHOD=MAD

```
                              High School Scores Data
                                    METHOD=MAD

---------------------------------- Type=rural ----------------------------------

                               The STDIZE Procedure

               Location and Scale Measures

            Location = median     Scale = median abs dev from median

               Name          Location            Scale            N

               total       166.000000        32.000000           20

                              High School Scores Data
                                    METHOD=MAD

---------------------------------- Type=urban ----------------------------------

                               The STDIZE Procedure

               Location and Scale Measures

            Location = median     Scale = median abs dev from median

               Name          Location            Scale            N

               total       153.000000        15.500000           20
```

Figure 82.5 displays the table of location and scale measures when the standardization method is IQR. The location measure is the median, and the scale measure is the interquartile range. The ratio of the scale of rural scores to the scale of urban scores is approximately 2.03 (61/30) and is, in fact, the dispersion ratio observed in the previous schematic plots.

Figure 82.5 Location and Scale Measures Table When METHOD=IQR

```
                    High School Scores Data
                           METHOD=IQR

---------------------------------- Type=rural ----------------------------------

                      The STDIZE Procedure

          Location and Scale Measures

          Location = median     Scale = interquartile range

          Name          Location            Scale            N

          total      166.000000       61.000000           20

                    High School Scores Data
                           METHOD=IQR

---------------------------------- Type=urban ----------------------------------

                      The STDIZE Procedure

          Location and Scale Measures

          Location = median     Scale = interquartile range

          Name          Location            Scale            N

          total      153.000000       30.000000           20
```

Figure 82.6 displays the table of location and scale measures when the standardization method is ABW, for which the location measure is the biweight 1-step M-estimate, and the scale measure is the biweight A-estimate. Note that the initial estimate for ABW is MAD. The following steps help to decide the value of the tuning constant:

1. For rural scores, the location estimate for MAD is 166.0, and the scale estimate for MAD is 32.0. The maximum of the rural scores is 253 (not shown), and the minimum is 110 (not shown). Thus, the tuning constant needs to be 3 so that it does not reject any observation that has a score between 110 to 253.
2. For urban scores, the location estimate for MAD is 153.0, and the scale estimate for MAD is 15.5. The maximum of the rural scores is 198, and the minimum (also an outlier) is 64. Thus, the tuning constant needs to be 4 so that it rejects the outlier (64) but includes the maximum (198) as an normal observation.
3. The maximum of the tuning constants, obtained in steps 1 and 2, is 4.

See Goodall (1983, Chapter 11) for details about the tuning constant. The ratio of the scale of rural scores to the scale of urban scores is approximately 2.06 (32.0/15.5). It is also close to the dispersion ratio observed in the previous schematic plots.

Figure 82.6 Location and Scale Measures Table When METHOD=ABW

```
                         High School Scores Data
                              METHOD=ABW(4)

---------------------------------- Type=rural ----------------------------------

                           The STDIZE Procedure

                Location and Scale Measures

      Location = biweight 1-step M-estimate     Scale = biweight A-estimate

                Name          Location            Scale            N

                total       162.889603       56.662855           20

                         High School Scores Data
                              METHOD=ABW(4)

---------------------------------- Type=urban ----------------------------------

                           The STDIZE Procedure

                Location and Scale Measures

      Location = biweight 1-step M-estimate     Scale = biweight A-estimate

                Name          Location            Scale            N

                total       156.014608       28.615980           20
```

The preceding analysis shows that METHOD=MAD, METHOD=IQR, and METHOD=ABW all provide better dispersion ratios than METHOD=STD does.

You can recompute the standard deviation after deleting the outlier from the original data set for comparison. The following statements create a data set NoOutlier that excludes the outlier from the TotalScores data set and invoke PROC STDIZE with METHOD=STD.

```
data NoOutlier;
   set totalscores;
   if (total = 64) then delete;
run;

proc stdize data=NoOutlier method=std pstat;
   title2 'After Removing Outlier, METHOD=STD';
   var total;
   by Type;
run;
```

Figure 82.7 displays the location and scale measures after deleting the outlier. The lack of resistance of the standard deviation to outliers is clearly illustrated: if you delete the outlier, the sample standard deviation of urban scores changes from 30.07 to 22.09. The new ratio of the scale of rural scores to the scale of urban scores is approximately 1.90 (41.96/22.09).

Figure 82.7 Location and Scale Measures Table When METHOD=STD without the Outlier

```
                           High School Scores Data
                      After Removing Outlier, METHOD=STD

---------------------------------- Type=rural ----------------------------------

                             The STDIZE Procedure

                Location and Scale Measures

                Location = mean     Scale = standard deviation

                Name         Location           Scale           N

                total      167.050000       41.956713          20

                           High School Scores Data
                      After Removing Outlier, METHOD=STD

---------------------------------- Type=urban ----------------------------------

                             The STDIZE Procedure

                Location and Scale Measures

                Location = mean     Scale = standard deviation

                Name         Location           Scale           N

                total      158.000000       22.088207          19
```

Syntax: STDIZE Procedure

The following statements are available in the STDIZE procedure:

PROC STDIZE < *options* > ;
  BY *variables* ;
  FREQ *variable* ;
  LOCATION *variables* ;
  SCALE *variables* ;
  VAR *variables* ;
  WEIGHT *variable* ;

The PROC STDIZE statement is required. The BY, LOCATION, FREQ, VAR, SCALE, and WEIGHT statements are described in alphabetical order following the PROC STDIZE statement.

PROC STDIZE Statement

PROC STDIZE < *options* > **;**

The PROC STDIZE statement invokes the procedure. You can specify the following options in the PROC STDIZE statement. Table 82.1 summarizes the options.

Table 82.1 Summary of PROC STDIZE Statement Options

Option	Description
Specify standardization methods	
METHOD=	specifies the name of the standardization method
INITIAL=	specifies the method for computing initial estimates for the A estimates
Unstandardize variables	
UNSTD	unstandardizes variables when you also specify the METHOD=IN option
Process missing values	
NOMISS	omits observations with any missing values from computation
MISSING=	specifies the method or a numeric value for replacing missing values
REPLACE	replaces missing data with zero in the standardized data
REPONLY	replaces missing data with the location measure (does not standardize the data)
Specify data set details	
DATA=	specifies the input data set
KEEPLEN	specifies that output variables inherit the length of the analysis variable
OUT=	specifies the output data set
OUTSTAT=	specifies the output statistic data set
Specify computational settings	
VARDEF=	specifies the variances divisor
NMARKERS=	specifies the number of markers when you also specify PCTLMTD=ONEPASS
MULT=	specifies the constant to multiply each value by after standardizing
ADD=	specifies the constant to add to each value after standardizing and multiplying by the value specified in the MULT= option
FUZZ=	specifies the relative fuzz factor for writing the output
Specify percentiles	
PCTLDEF=	specifies the definition of percentiles when you also specify the PCTLMTD=ORD_STAT option
PCTLMTD=	specifies the method used to estimate percentiles

Table 82.1 *continued*

Option	Description
PCTLPTS=	writes observations containing percentiles to the data set specified in the OUTSTAT= option
Normalize scale estimators	
NORM	normalizes the scale estimator to be consistent for the standard deviation of a normal distribution
SNORM	normalizes the scale estimator to have an expectation of approximately 1 for a standard normal distribution
Specify output	
PSTAT	displays the location and scale measures

These options and their abbreviations are described (in alphabetical order) in the remainder of this section.

ADD=*c*

specifies a constant, c, to add to each value after standardizing and multiplying by the value you specify in the MULT= option. The default value is 0.

DATA=*SAS-data-set*

specifies the input data set to be standardized. If you omit the DATA= option, the most recently created data set is used.

FUZZ=*c*

specifies the relative fuzz factor. The default value is 1E–14. For the OUT= data set, the score is computed as follows:

$$\text{if } |\text{Result}| < m \times c, \text{ then Result} = 0$$

where m is the constant specified in the MULT= option, or 1 if MULT= option is not specified.

For the OUTSTAT= data set and the Location and Scale table, the scale and location values are computed as follows:

$$\text{if Scale} < |\text{Location}| \times c, \text{ Scale} = 0$$

Otherwise,

$$\text{if } |\text{Location}| < m \times c, \text{ Location} = 0$$

INITIAL=*method*

specifies the method for computing initial estimates for the A estimates (ABW, AWAVE, and AHUBER). You cannot specify the following methods for initial estimates: INITIAL=ABW, INITIAL=AHUBER, INITIAL=AWAVE, and INITIAL=IN. The default is INITIAL=MAD.

KEEPLEN

specifies that output variables inherit the length of the analysis variable that PROC STDIZE uses to derive them. PROC STDIZE stores numbers in double-precision without this option.

Caution: The KEEPLEN option causes the output variables to permanently lose numeric precision by truncating or rounding the value. However, the precision of the output variables will match that of the input.

METHOD=*name*

specifies the name of the method for computing location and scale measures. Valid values for *name* are as follows: MEAN, MEDIAN, SUM, EUCLEN, USTD, STD, RANGE, MIDRANGE, MAXABS, IQR, MAD, ABW, AHUBER, AWAVE, AGK, SPACING, L, and IN.

For details about these methods, see the descriptions in the section "Standardization Methods" on page 6984. The default is METHOD=STD.

MISSING=*method* | *value*

specifies the method (or a numeric value) for replacing missing values. If you omit the MISSING= option, the REPLACE option replaces missing values with the location measure given by the METHOD= option. Specify the MISSING= option when you want to replace missing values with a different value. You can specify any name that is valid in the METHOD= option except the name IN. The corresponding location measure is used to replace missing values.

If a numeric value is given, the value replaces missing values after standardizing the data. However, you can specify the REPONLY option with the MISSING= option to suppress standardization for cases in which you want only to replace missing values.

MULT=*c*

specifies a constant, *c*, by which to multiply each value after standardizing. The default value is 1.

NMARKERS=*n*

specifies the number of markers used when you specify the one-pass algorithm (PCTLMTD=ONEPASS). The value *n* must be greater than or equal to 5. The default value is 105.

NOMISS

omits observations with missing values for any of the analyzed variables from calculation of the location and scale measures. If you omit the NOMISS option, all nonmissing values are used.

NORM

normalizes the scale estimator to be consistent for the standard deviation of a normal distribution when you specify the option METHOD=AGK, METHOD=IQR, METHOD=MAD, or METHOD=SPACING.

OUT=*SAS-data-set*

specifies the name of the SAS data set created by PROC STDIZE. The output data set is a copy of the DATA= data set except that the analyzed variables have been standardized. Note that analyzed variables are those specified in the VAR statement or, if there is no VAR statement, all numeric variables not listed in any other statement. See the section "Output Data Sets" on page 6989 for more information.

If you want to create a permanent SAS data set, you must specify a two-level name. See *SAS Language Reference: Concepts* for more information about permanent SAS data sets.

If you omit the OUT= option, PROC STDIZE creates an output data set named according to the DATA*n* convention.

OUTSTAT=*SAS-data-set*

specifies the name of the SAS data set containing the location and scale measures and other computed statistics. See the section "Output Data Sets" on page 6989 for more information.

PCTLDEF=*percentiles*

specifies which of five definitions is used to calculate percentiles when you specify the option PCTLMTD=ORD_STAT. By default, PCTLDEF=5. Note that the option PCTLMTD=ONEPASS implies PCTLDEF=5. See the section "Computational Methods for the PCTLDEF= Option" on page 6987 for details about percentile definition.

You cannot use PCTLDEF= when you compute weighted quantiles.

PCTLMTD=ORD_STAT | ONEPASS | P2

specifies the method used to estimate percentiles. Specify the PCTLMTD=ORD_STAT option to compute the percentiles by the order statistics method.

The PCTLMTD=ONEPASS option modifies an algorithm invented by Jain and Chlamtac (1985). See the section "Computing Quantiles" on page 6987 for more details about this algorithm.

PCTLPTS=*n*

writes percentiles to the OUTSTAT= data set. Values of *n* can be any decimal number between 0 and 100, inclusive.

A requested percentile is identified by the _TYPE_ variable in the OUTSTAT= data set with a value of P*n*. For example, suppose you specify the option PCTLPTS=10, 30. The corresponding observations in the OUTSTAT= data set that contain the 10th and the 30th percentiles would then have values _TYPE_=P10 and _TYPE_=P30, respectively.

PSTAT

displays the location and scale measures.

REPLACE

replaces missing data with the value 0 in the standardized data (this value corresponds to the location measure before standardizing). To replace missing data by other values, see the preceding description of the MISSING= option. You cannot specify both the REPLACE and REPONLY options.

REPONLY

replaces missing data only; PROC STDIZE does not standardize the data. Missing values are replaced with the location measure unless you also specify the MISSING=*value* option, in which case missing values are replaced with *value*. You cannot specify both the REPLACE and REPONLY options.

SNORM

normalizes the scale estimator to have an expectation of approximately 1 for a standard normal distribution when you specify the METHOD=SPACING option.

UNSTD

UNSTDIZE

unstandardizes variables when you specify the METHOD=IN(*ds*) option. The location and scale measures, along with constants for addition and multiplication that the unstandardization is based on, are identified by the _TYPE_ variable in the ds data set.

The ds data set must have a _TYPE_ variable and contain the following two observations: a _TYPE_= 'LOCATION' observation and a _TYPE_= 'SCALE' observation. The variable _TYPE_ can also contain the optional observations, 'ADD' and 'MULT'; if these observations are not found in the ds data set, the constants specified in the ADD= and MULT= options (or their default values) are used for unstandardization.

See the section "OUTSTAT= Data Set" on page 6989 for details about the statistics that each value of _TYPE_ represents. The formula used for unstandardization is as follows: If the final output value from the previous standardization is calculated as

$$result = add + multiply \times \frac{original - location}{scale}$$

The unstandardized variable is computed as

$$original = scale \times \frac{result - add}{multiply} + location$$

VARDEF=DF | N | WDF | WEIGHT | WGT

specifies the divisor to be used in the calculation of variances. By default, VARDEF=DF. The values and associated divisors are as follows.

Value	Divisor	Formula
DF	degrees of freedom	$n - 1$
N	number of observations	n
WDF	sum of weights minus 1	$(\sum_i w_i) - 1$
WEIGHT | WGT	sum of weights	$\sum_i w_i$

BY Statement

BY *variables* **;**

You can specify a BY statement with PROC STDIZE to obtain separate analyses on observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the STDIZE procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

When you specify the option METHOD=IN(ds), the following rules are applied to BY-group processing:

- If the ds data set does not contain any of the BY variables, the entire DATA= data set is standardized by the location and scale measures (along with the constants for addition and multiplication) in the ds data set.
- If the ds data set contains some, but not all, of the BY variables or if some BY variables do not have the same type or length in the ds data set that they have in the DATA= data set, PROC STDIZE displays an error message and stops.
- If all of the BY variables appear in the ds data set with the same type and length as in the DATA= data set, each BY group in the DATA= data set is standardized using the location and scale measures (along with the constants for addition and multiplication) from the corresponding BY group in the ds data set. The BY groups in the ds data set must be in the same order in which they appear in the DATA= data set. All BY groups in the DATA= data set must also appear in the ds data set. If you do not specify the NOTSORTED option, some BY groups can appear in the ds data set but not in the DATA= data set; such BY groups are not used in standardizing data.

For more information about BY-group processing, see the discussion in *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the discussion in the *Base SAS Procedures Guide*.

FREQ Statement

FREQ *variable* **;**

If one variable in the input data set represents the frequency of occurrence for other values in the observation, specify the variable name in a FREQ statement. PROC STDIZE treats the data set as if each observation appeared *n* times, where *n* is the value of the FREQ variable for the observation. Nonintegral values of the FREQ variable are truncated to the largest integer less than the FREQ value. If the FREQ variable has a value that is less than 1 or is missing, the observation is not used in the analysis.

NOTRUNCATE
NOTRUNC

specifies that frequency values are not truncated to integers.

The nonintegral values of the FREQ variable can be used for the following standardization methods: AGK, ABW, AHUBER, AWAVE, EUCLEN, IQR, L, MAD, MEAN, MEDIAN, SPACING, STD, SUM, and USTD. Note only when PCTLMTD=ORD_STAT is specified that the nonintegral frequency values will be used for the MAD, MEDIAN, or IQR method; if PCTLMTD=ONEPASS is specified, the NOTRUNCATE option is ignored.

LOCATION Statement

LOCATION *variables* **;**

The LOCATION statement specifies a list of numeric variables that contain location measures in the input data set specified by the METHOD=IN option.

SCALE Statement

SCALE *variables* **;**

The SCALE statement specifies the list of numeric variables containing scale measures in the input data set specified by the METHOD=IN option.

VAR Statement

VAR *variable* **;**

The VAR statement lists numeric variables to be standardized. If you omit the VAR statement, all numeric variables not listed in the BY, FREQ, and WEIGHT statements are used.

WEIGHT Statement

WEIGHT *variable* **;**

The WEIGHT statement specifies a numeric variable in the input data set with values that are used to weight each observation. Only one variable can be specified.

The WEIGHT variable values can be nonintegers. An observation is used in the analysis only if the value of the WEIGHT variable is greater than zero.

The WEIGHT variable applies only when you specify the following standardization methods: AGK, EUCLEN, IQR, L, MAD, MEAN, MEDIAN, STD, SUM, and USTD. Note that weights are used for the METHOD=MAD, MEDIAN, or IQR only when PCTLMTD=ORD_STAT is specified; if PCTLMTD=ONEPASS is specified, the WEIGHT statement is ignored.

PROC STDIZE uses the value of the WEIGHT variable to calculate the following statistics:

The sample mean and sample variances are computed as

$$\overline{x}_w = \sum_i w_i x_i / \sum_i w_i \qquad \text{(sample mean)}$$

$$us_w^2 = \sum_i w_i x_i^2 / d \qquad \text{(uncorrected sample variances)}$$

$$s_w^2 = \sum_i w_i (x_i - \overline{x}_w)^2 / d \qquad \text{(sample variances)}$$

where w_i is the weight value of the ith observation, x_i is the value of the ith observation, and d is the divisor controlled by the VARDEF= option (see the VARDEF= option for details).

MEAN
: the weighted mean, $\overline{x}_w$

SUM
: the weighted sum, $\sum_i w_i x_i$

USTD
: the weighted uncorrected standard deviation, $\sqrt{us_w^2}$

STD
: the weighted standard deviation, $\sqrt{s_w^2}$

EUCLEN
: the weighted Euclidean length, computed as the square root of the weighted uncorrected sum of squares:

$$\sqrt{\sum_i w_i x_i^2}$$

MEDIAN
: the weighted median. See the section "Weighted Percentiles" on page 6988 for the formulas and descriptions.

MAD
: the weighted median absolute deviation from the weighted median. See the section "Weighted Percentiles" on page 6988 for the formulas and descriptions.

IQR
: the weighted median, 25th percentile, and the 75th percentile. See the section "Weighted Percentiles" on page 6988 for the formulas and descriptions.

AGK — the AGK estimate. This estimate is documented further in the ACECLUS procedure as the METHOD=COUNT option. See the discussion of the WEIGHT statement in Chapter 22, "The ACECLUS Procedure," for information about how the WEIGHT variable is applied to the AGK estimate.

L — the L_p estimate. This estimate is documented further in the FASTCLUS procedure as the LEAST= option. See the discussion of the WEIGHT statement in Chapter 34, "The FASTCLUS Procedure," for information about how the WEIGHT variable is used to compute weighted cluster means. Note that the number of clusters is always 1.

Details: STDIZE Procedure

Standardization Methods

The following table lists standardization methods and their corresponding location and scale measures available with the METHOD= option.

Table 82.2 Available Standardization Methods

Method	Location	Scale
MEAN	mean	1
MEDIAN	median	1
SUM	0	sum
EUCLEN	0	Euclidean length
USTD	0	standard deviation about origin
STD	mean	standard deviation
RANGE	minimum	range
MIDRANGE	midrange	range/2
MAXABS	0	maximum absolute value
IQR	median	interquartile range
MAD	median	median absolute deviation from median
ABW(c)	biweight 1-step M-estimate	biweight A-estimate
AHUBER(c)	Huber 1-step M-estimate	Huber A-estimate
AWAVE(c)	Wave 1-step M-estimate	Wave A-estimate
AGK(p)	mean	AGK estimate (ACECLUS)
SPACING(p)	mid-minimum spacing	minimum spacing
L(p)	L(p)	L(p)
IN(ds)	read from data set	read from data set

For METHOD=ABW(c), METHOD=AHUBER(c), or METHOD=AWAVE(c), c is a positive numeric tuning constant.

For METHOD=AGK(p), p is a numeric constant giving the proportion of pairs to be included in the estimation of the within-cluster variances.

For METHOD=SPACING(p), p is a numeric constant giving the proportion of data to be contained in the spacing.

For METHOD=L(p), p is a numeric constant greater than or equal to 1 specifying the power to which differences are to be raised in computing an L(p) or Minkowski metric.

For METHOD=IN(ds), ds is the name of a SAS data set that meets either of the following two conditions:

- contains a _TYPE_ variable. The observation that contains the location measure corresponds to the value _TYPE_= 'LOCATION', and the observation that contains the scale measure corresponds to the value _TYPE_= 'SCALE'. You can also use a data set created by the OUTSTAT= option from another PROC STDIZE statement as the ds data set. See the section "Output Data Sets" on page 6989 for the contents of the OUTSTAT= data set.
- contains the location and scale variables specified by the LOCATION and SCALE statements

PROC STDIZE reads in the location and scale variables in the ds data set by first looking for the _TYPE_ variable in the ds data set. If it finds this variable, PROC STDIZE continues to search for all variables specified in the VAR statement. If it does not find the _TYPE_ variable, PROC STDIZE searches for the location variables specified in the LOCATION statement and the scale variables specified in the SCALE statement.

The variable _TYPE_ can also contain the optional observations, 'ADD' and 'MULT'. If these observations are found in the ds data set, the values in the observation of _TYPE_ = 'MULT' are the multiplication constants, and the values in the observation of _TYPE_ = 'ADD' are the addition constants; otherwise, the constants specified in the ADD= and MULT= options (or their default values) are used.

For robust estimators, refer to Goodall (1983) and Iglewicz (1983). The MAD method has the highest breakdown point (50%), but it is somewhat inefficient. The ABW, AHUBER, and AWAVE methods provide a good compromise between breakdown and efficiency. The L(p) location estimates are increasingly robust as p drops from 2 (corresponding to least squares, or mean estimation) to 1 (corresponding to least absolute value, or median estimation). However, the L(p) scale estimates are not robust.

The SPACING method is robust to both outliers and clustering (Jannsen et al. 1995) and is, therefore, a good choice for cluster analysis or nonparametric density estimation. The mid-minimum spacing method estimates the mode for small p. The AGK method is also robust to clustering and more efficient than the SPACING method, but it is not as robust to outliers and takes longer to compute. If you expect g clusters, the argument to METHOD=SPACING or METHOD=AGK should be $\frac{1}{g}$ or less. The AGK method is less biased than the SPACING method for small samples. As a general guide, it is reasonable to use AGK for samples of size 100 or less and SPACING for samples of size 1000 or more, with the treatment of intermediate sample sizes depending on the available computer resources.

Computation of the Statistics

Formulas for statistics of METHOD=MEAN, METHOD=MEDIAN, METHOD=SUM, METHOD=USTD, METHOD=STD, METHOD=RANGE, and METHOD=IQR are given in the chapter “Elementary Statistics Procedures” (*Base SAS Procedures Guide*).

Note that the computations of median and upper and lower quartiles depend on the PCTLMTD= option.

The other statistics listed in Table 82.2, except for METHOD=IN, are described as follows:

EUCLEN
: Euclidean length.
$\sqrt{\sum_{i=1}^{n} x_i^2}$, where x_i is the ith observation and n is the total number of observations in the sample.

L(p)
: Minkowski metric. This metric is documented as the LEAST=p option in the PROC FASTCLUS statement of the FASTCLUS procedure (see Chapter 34, “The FASTCLUS Procedure”).

  If you specify METHOD=L(p) in the PROC STDIZE statement, your results are similar to those obtained from PROC FASTCLUS if you specify the LEAST=p option with MAXCLUS=1 (and use the default values of the MAXITER= option). The difference between the two types of calculations concerns the maximum number of iterations. In PROC STDIZE, it is a criterion for convergence on all variables; in PROC FASTCLUS, it is a criterion for convergence on a single variable.

  The location and scale measures for L(p) are output to the OUTSEED= data set in PROC FASTCLUS.

MIDRANGE
: (maximum + minimum)/2

ABW(c)
: Tukey’s biweight. Refer to Goodall (1983, pp. 376–378, p. 385) for the biweight 1-step M-estimate. Also refer to Iglewicz (1983, pp. 416-418) for the biweight A-estimate.

AHUBER(c)
: Hubers. Refer to Goodall (1983, pp. 371–374) for the Huber 1-step M-estimate. Also refer to Iglewicz (1983, pp. 416-418) for the Huber A-estimate of scale.

AWAVE(c)
: Andrews’ wave. Refer to Goodall (1983, p. 376) for the Wave 1-step M-estimate. Also refer to Iglewicz (1983, pp. 416-418) for the Wave A-estimate of scale.

AGK(p)
: The noniterative univariate form of the estimator described by Art, Gnanadesikan, and Kettenring (1982).

  The AGK estimate is documented in the section on the METHOD= option in the PROC ACECLUS statement of the ACECLUS procedure (also see the section “Background” on page 780 in Chapter 22, “The ACECLUS Procedure”). Specifying METHOD=AGK(p) in the PROC STDIZE statement is the same as specifying METHOD=COUNT and P=p in the PROC ACECLUS statement.

SPACING(p)
: The absolute difference between two data values. The minimum spacing for a proportion p is the minimum absolute difference between two data values that

contain a proportion p of the data between them. The mid-minimum spacing is the mean of these two data values.

Computing Quantiles

PROC STDIZE offers two methods for computing quantiles: the one-pass approach and the order-statistics approach (like that used in the UNIVARIATE procedure).

The one-pass approach used in PROC STDIZE modifies the P^2 algorithm for histograms proposed by Jain and Chlamtac (1985). The primary difference comes from the movement of markers. The one-pass method allows a marker to move to the right (or left) by more than one position (to the largest possible integer) as long as it does not result in two markers being in the same position. The modification is necessary in order to incorporate the FREQ variable.

You might obtain inaccurate results if you use the one-pass approach to estimate quantiles beyond the quartiles (that is, when you estimate quantiles < P25 or > P75). A large sample size (10,000 or more) is often required if the tail quantiles (quantiles <= P10 or >= P90) are requested. Note that, for variables with highly skewed or heavy-tailed distributions, tail quantile estimates might be inaccurate.

The order-statistics approach for estimating quantiles is faster than the one-pass method but requires that the entire data set be stored in memory. The accuracy in estimating the quantiles is comparable for both methods when the requested percentiles are between the lower and upper quartiles. The default is PCTLMTD=ORD_STAT if enough memory is available; otherwise, PCTLMTD=ONEPASS.

Computational Methods for the PCTLDEF= Option

You can specify one of five methods for computing quantile statistics when you use the order-statistics approach (PCTLMTD=ORD_STAT); otherwise, the PCTLDEF=5 method is used when you use the one-pass approach (PCTLMTD=ONEPASS).

Percentile Definitions Let n be the number of nonmissing values for a variable, and let $x_1, x_2, \ldots, x_n$ represent the ordered values of the variable. For the tth percentile, let $p = t/100$. In the following definitions numbered 1, 2, 3, and 5, let

$$np = j + g$$

where j is the integer part and g is the fractional part of np. For definition 4, let

$$(n + 1)p = j + g$$

Given the preceding definitions, the tth percentile, y, is defined as follows:

PCTLDEF=1 weighted average at x_{np}

$$y = (1 - g)x_j + gx_{j+1}$$

where x_0 is taken to be x_1

PCTLDEF=2 observation numbered closest to np

$$y = x_i$$

where i is the integer part of $np + 1/2$ if $g \neq 1/2$. If $g = 1/2$, then
$y = x_j$ if j is even, or
$y = x_{j+1}$ if j is odd

PCTLDEF=3 empirical distribution function

$$y = x_j \text{ if } g = 0$$

$$y = x_{j+1} \text{ if } g > 0$$

PCTLDEF=4 weighted average aimed at $x_{p(n+1)}$

$$y = (1-g)x_j + gx_{j+1}$$

where x_{n+1} is taken to be x_n

PCTLDEF=5 empirical distribution function with averaging

$$y = (x_j + x_{j+1})/2 \text{ if } g = 0$$

$$y = x_{j+1} \text{ if } g > 0$$

Weighted Percentiles

When you specify a WEIGHT statement, or specify the NOTRUNCATE option in a FREQ statement, the percentiles are computed differently. The $100p$th weighted percentile y is computed from the empirical distribution function with averaging

$$y = \begin{cases} \frac{1}{2}(x_i + x_{i+1}) & \text{if } \sum_{j=1}^{i} w_j = pW \\ x_{i+1} & \text{if } \sum_{j=1}^{i} w_j < pW < \sum_{j=1}^{i+1} w_j \end{cases}$$

where w_i is the weight associated with x_i, and where $W = \sum_{i=1}^{n} w_i$ is the sum of the weights.

For PCTLMTD= ORD_STAT, the PCTLDEF= option is not applicable when a WEIGHT statement is used, or when a NOTRUNCATE option is specified in a FREQ statement. However, in this case, if all the weights are identical, the weighted percentiles are the same as the percentiles that would be computed without a WEIGHT statement and with PCTLDEF=5.

For PCTLMTD= ONEPASS, the quantile computation currently does not use any weights.

Missing Values

Missing values can be replaced by the location measure or by any specified constant (see the REPLACE option and the MISSING= option). You can also suppress standardization if you want only to replace missing values (see the REPONLY option).

If you specify the NOMISS option, PROC STDIZE omits observations with any missing values in the analyzed variables from computation of the location and scale measures.

Output Data Sets

OUT= Data Set

The output data set is a copy of the DATA= data set except that the analyzed variables have been standardized. Analyzed variables are those listed in the VAR statement or, if there is no VAR statement, all numeric variables not listed in any other statement.

OUTSTAT= Data Set

The new data set contains the following variables:

- the BY variables, if any
- _TYPE_, a character variable
- the analyzed variables

Each observation in the new data set contains a type of statistic as indicated by the _TYPE_ variable. The values of the _TYPE_ variable are as follows:

LOCATION	location measure of each variable
SCALE	scale measure of each variable
ADD	constant specified in the ADD= option. This value is the same for each variable.
MULT	constant specified in the MULT= option. This value is the same for each variable.
N	total number of nonmissing positive frequencies of each variable
NORM	norm measure of each variable. This observation is produced only when you specify the NORM option with METHOD=AGK, METHOD=IQR, METHOD=MAD, or METHOD=SPACING or when you specify the SNORM option with METHOD=SPACING.
NObsRead	number of physical records read
NObsUsed	number of physical records used in the analysis
NObsMiss	number of physical records containing missing values
Pn	percentiles of each variable, as specified by the PCTLPTS= option. The argument n is any real number such that $0 \le n \le 100$
SumFreqsRead	sum of the frequency variable (or the sum of NObsUsed ones when there is no frequency variable) for all observations read
SumFreqsUsed	sum of the frequency variable (or the sum of NObsUsed ones when there is no frequency variable) for all observations used in the analysis

SumWeightsRead	sum of the weight variable (or the sum of NObsUsed ones when there is no weight variable) for all observations read
SumWeightsUsed	sum of the weight variable (or the sum of NObsUsed ones when there is no weight variable) for all observations used in the analysis

Displayed Output

If you specify the PSTAT option, PROC STDIZE displays the following statistics for each variable:

- the name of the variable, Name
- the location estimate, Location
- the scale estimate, Scale
- the norm estimate, Norm (when you specify the NORM option with METHOD=AGK, METHOD=IQR, METHOD=MAD, or METHOD=SPACING or when you specify the SNORM option with METHOD=SPACING)
- sum of nonmissing positive frequencies, N
- sum of nonmissing positive weights if the WEIGHT statement is specified, Sum of Weights

ODS Table Names

PROC STDIZE assigns a name to the single table it creates. You can use this name to reference the table when using the Output Delivery System (ODS) to select a table or create an output data set. This name is listed in Table 82.3. For more information about ODS, see Chapter 20, "Using the Output Delivery System."

Table 82.3 ODS Table Produced by PROC STDIZE

ODS Table Name	Description	Statement	Option
Statistics	Location and Scale Measures	PROC	PSTAT

Example: STDIZE Procedure

Example 82.1: Standardization of Variables in Cluster Analysis

To illustrate the effect of standardization in cluster analysis, this example uses the Fish data set described in the "Getting Started" section of Chapter 34, "The FASTCLUS Procedure." The numbers are measurements taken on 159 fish caught from the same lake (Laengelmavesi) near Tampere in Finland (Puranen 1917). The complete data set is displayed in Chapter 83, "The STEPDISC Procedure."

The species (bream, parkki, pike, perch, roach, smelt, and whitefish), weight, three different length measurements (measured from the nose of the fish to the beginning of its tail, the notch of its tail, and the end of its tail), height, and width of each fish are recorded. The height and width are recorded as percentages of the third length variable.

Several new variables are created in the Fish data set: Weight3, Height, Width, and logLengthRatio. The weight of a fish indicates its size—a heavier pike tends to be larger than a lighter pike. To get a one-dimensional measure of the size of a fish, take the cubic root of the weight (Weight3). The variables Height, Width, Length1, Length2, and Length3 are rescaled in order to adjust for dimensionality. The logLengthRatio variable measures the tail length.

Because the new variables Weight3–logLengthRatio depend on the variable Weight, observations with missing values for Weight are not added to the data set. Consequently, there are 157 observations in the SAS data set Fish.

Before you perform a cluster analysis on coordinate data, it is necessary to consider scaling or transforming the variables since variables with large variances tend to have a larger effect on the resulting clusters than variables with small variances do.

This example uses three different approaches to standardize or transform the data prior to the cluster analysis. The first approach uses several standardization methods provided in the STDIZE procedure. However, since standardization is not always appropriate prior to the clustering (refer to Milligan and Cooper (1987) for a Monte Carlo study on various methods of variable standardization), the second approach performs the cluster analysis with no standardization. The third approach invokes the ACECLUS procedure to transform the data into a within-cluster covariance matrix.

The clustering is performed by the FASTCLUS procedure to find seven clusters. Note that the variables Length2 and Length3 are eliminated from this analysis since they both are significantly and highly correlated with the variable Length1. The correlation coefficients are 0.9958 and 0.9604, respectively. An output data set is created, and the FREQ procedure is invoked to compare the clusters with the species classification.

The DATA step is as follows, after the initial PROC FORMAT step:

```
proc format;
   value specfmt
      1='Bream'
      2='Roach'
      3='Whitefish'
      4='Parkki'
      5='Perch'
      6='Pike'
      7='Smelt';
run;

data Fish (drop=HtPct WidthPct);
   title 'Fish Measurement Data';
   input Species Weight Length1 Length2 Length3 HtPct
         WidthPct @@;

   if Weight <= 0 or Weight=. then delete;
   Weight3=Weight**(1/3);
   Height=HtPct*Length3/(Weight3*100);
   Width=WidthPct*Length3/(Weight3*100);
   Length1=Length1/Weight3;
   Length2=Length2/Weight3;
   Length3=Length3/Weight3;
   logLengthRatio=log(Length3/Length1);

   format Species specfmt.;
   symbol = put(Species, specfmt2.);
datalines;
1  242.0 23.2 25.4 30.0 38.4 13.4 1  290.0 24.0 26.3 31.2 40.0 13.8
1  340.0 23.9 26.5 31.1 39.8 15.1 1  363.0 26.3 29.0 33.5 38.0 13.3
1  430.0 26.5 29.0 34.0 36.6 15.1 1  450.0 26.8 29.7 34.7 39.2 14.2
1  500.0 26.8 29.7 34.5 41.1 15.3 1  390.0 27.6 30.0 35.0 36.2 13.4
1  450.0 27.6 30.0 35.1 39.9 13.8 1  500.0 28.5 30.7 36.2 39.3 13.7
1  475.0 28.4 31.0 36.2 39.4 14.1 1  500.0 28.7 31.0 36.2 39.7 13.3
1  500.0 29.1 31.5 36.4 37.8 12.0 1     .  29.5 32.0 37.3 37.3 13.6
1  600.0 29.4 32.0 37.2 40.2 13.9 1  600.0 29.4 32.0 37.2 41.5 15.0
1  700.0 30.4 33.0 38.3 38.8 13.8 1  700.0 30.4 33.0 38.5 38.8 13.5
1  610.0 30.9 33.5 38.6 40.5 13.3 1  650.0 31.0 33.5 38.7 37.4 14.8
1  575.0 31.3 34.0 39.5 38.3 14.1 1  685.0 31.4 34.0 39.2 40.8 13.7
1  620.0 31.5 34.5 39.7 39.1 13.3 1  680.0 31.8 35.0 40.6 38.1 15.1
1  700.0 31.9 35.0 40.5 40.1 13.8 1  725.0 31.8 35.0 40.9 40.0 14.8

   ... more lines ...

7   19.7 13.2 14.3 15.2 18.9 13.6 7   19.9 13.8 15.0 16.2 18.1 11.6
;
```

The following macro, Std, standardizes the Fish data. The macro reads a single argument, mtd, which selects the METHOD= specification to be used in PROC STDIZE.

```
/*--- macro for standardization ---*/

%macro Std(mtd);
   title2 "Data are Standardized by PROC STDIZE with METHOD= &mtd";
   proc stdize data=fish out=sdzout method=&mtd;
      var Length1 logLengthRatio Height Width Weight3;
   run;
%mend Std;
```

The following macro, FastFreq, includes a PROC FASTCLUS statement for performing cluster analysis and a PROC FREQ statement for crosstabulating species with the cluster membership information that is derived from the previous PROC FASTCLUS statement. The macro reads a single argument, ds, which selects the input data set to be used in PROC FASTCLUS.

```
/*--- macro for clustering and crosstabulating ---*/
/*--- cluster membership with species          ---*/
%macro FastFreq(ds);
   proc fastclus data=&ds out=clust maxclusters=7 maxiter=100 noprint;
      var Length1 logLengthRatio Height Width Weight3;
   run;

   proc freq data=clust;
      tables species*cluster;
   run;
%mend FastFreq;
```

The following analysis (labeled 'Approach 1') includes 18 different methods of standardization followed by clustering. Since there is a large amount of output from this approach, only results from METHOD=STD, METHOD=RANGE, METHOD=AGK(0.14), and METHOD=SPACING(0.14) are shown. The following statements produce Output 82.1.1 through Output 82.1.4.

```
/***********************************************************/
/*                                                         */
/*     Approach 1: data are standardized by PROC STDIZE   */
/*                                                         */
/***********************************************************/

%Std(MEAN);
%FastFreq(sdzout);

%Std(MEDIAN);
%FastFreq(sdzout);

%Std(SUM);
%FastFreq(sdzout);

%Std(EUCLEN);
%FastFreq(sdzout);

%Std(USTD);
```

```
%FastFreq(sdzout);

%Std(STD);
%FastFreq(sdzout);

%Std(RANGE);
%FastFreq(sdzout);

%Std(MIDRANGE);
%FastFreq(sdzout);

%Std(MAXABS);
%FastFreq(sdzout);

%Std(IQR);
%FastFreq(sdzout);

%Std(MAD);
%FastFreq(sdzout);

%Std(AGK(.14));
%FastFreq(sdzout);

%Std(SPACING(.14));
%FastFreq(sdzout);

%Std(ABW(5));
%FastFreq(sdzout);

%Std(AWAVE(5));
%FastFreq(sdzout);

%Std(L(1));
%FastFreq(sdzout);

%Std(L(1.5));
%FastFreq(sdzout);

%Std(L(2));
%FastFreq(sdzout);
```

Output 82.1.1 Data Are Standardized by PROC STDIZE with METHOD=STD

```
                    Fish Measurement Data
      Data are Standardized by PROC STDIZE with METHOD= STD

                      The FREQ Procedure

                 Table of Species by CLUSTER

      Species     CLUSTER(Cluster)

      Frequency |
      Percent   |
      Row Pct   |
      Col Pct   |       1|       2|       3|       4|  Total
      ----------+--------+--------+--------+--------+
      Bream     |      0 |      0 |      0 |      0 |     34
                |   0.00 |   0.00 |   0.00 |   0.00 |  21.66
                |   0.00 |   0.00 |   0.00 |   0.00 |
                |   0.00 |   0.00 |   0.00 |   0.00 |
      ----------+--------+--------+--------+--------+
      Roach     |      0 |      0 |      0 |      0 |     19
                |   0.00 |   0.00 |   0.00 |   0.00 |  12.10
                |   0.00 |   0.00 |   0.00 |   0.00 |
                |   0.00 |   0.00 |   0.00 |   0.00 |
      ----------+--------+--------+--------+--------+
      Whitefish |      0 |      2 |      0 |      1 |      6
                |   0.00 |   1.27 |   0.00 |   0.64 |   3.82
                |   0.00 |  33.33 |   0.00 |  16.67 |
                |   0.00 |  10.53 |   0.00 |   7.69 |
      ----------+--------+--------+--------+--------+
      Parkki    |      0 |      0 |      0 |      0 |     11
                |   0.00 |   0.00 |   0.00 |   0.00 |   7.01
                |   0.00 |   0.00 |   0.00 |   0.00 |
                |   0.00 |   0.00 |   0.00 |   0.00 |
      ----------+--------+--------+--------+--------+
      Total           17       19       13       13      157
                   10.83    12.10     8.28     8.28   100.00
      (Continued)
```

Output 82.1.1 *continued*

```
                Fish Measurement Data
 Data are Standardized by PROC STDIZE with METHOD= STD

                  The FREQ Procedure

             Table of Species by CLUSTER

 Species     CLUSTER(Cluster)

 Frequency |
 Percent   |
 Row Pct   |
 Col Pct   |       1|       2|       3|       4|  Total
 ----------+--------+--------+--------+--------+
 Perch     |      0 |     17 |      0 |     12 |     56
           |   0.00 |  10.83 |   0.00 |   7.64 |  35.67
           |   0.00 |  30.36 |   0.00 |  21.43 |
           |   0.00 |  89.47 |   0.00 |  92.31 |
 ----------+--------+--------+--------+--------+
 Pike      |     17 |      0 |      0 |      0 |     17
           |  10.83 |   0.00 |   0.00 |   0.00 |  10.83
           | 100.00 |   0.00 |   0.00 |   0.00 |
           | 100.00 |   0.00 |   0.00 |   0.00 |
 ----------+--------+--------+--------+--------+
 Smelt     |      0 |      0 |     13 |      0 |     14
           |   0.00 |   0.00 |   8.28 |   0.00 |   8.92
           |   0.00 |   0.00 |  92.86 |   0.00 |
           |   0.00 |   0.00 | 100.00 |   0.00 |
 ----------+--------+--------+--------+--------+
 Total           17       19       13       13      157
              10.83    12.10     8.28     8.28   100.00
 (Continued)
```

Output 82.1.1 *continued*

```
                    Fish Measurement Data
     Data are Standardized by PROC STDIZE with METHOD= STD

                      The FREQ Procedure

                 Table of Species by CLUSTER

     Species     CLUSTER(Cluster)

     Frequency |
     Percent   |
     Row Pct   |
     Col Pct   |       5|       6|       7|  Total
     ----------+--------+--------+--------+
     Bream     |      0 |     34 |      0 |     34
               |   0.00 |  21.66 |   0.00 |  21.66
               |   0.00 | 100.00 |   0.00 |
               |   0.00 | 100.00 |   0.00 |
     ----------+--------+--------+--------+
     Roach     |      0 |      0 |     19 |     19
               |   0.00 |   0.00 |  12.10 |  12.10
               |   0.00 |   0.00 | 100.00 |
               |   0.00 |   0.00 |  38.00 |
     ----------+--------+--------+--------+
     Whitefish |      0 |      0 |      3 |      6
               |   0.00 |   0.00 |   1.91 |   3.82
               |   0.00 |   0.00 |  50.00 |
               |   0.00 |   0.00 |   6.00 |
     ----------+--------+--------+--------+
     Parkki    |     11 |      0 |      0 |     11
               |   7.01 |   0.00 |   0.00 |   7.01
               | 100.00 |   0.00 |   0.00 |
               | 100.00 |   0.00 |   0.00 |
     ----------+--------+--------+--------+
     Total           11       34       50      157
                   7.01    21.66    31.85   100.00
     (Continued)
```

Output 82.1.1 *continued*

```
                 Fish Measurement Data
Data are Standardized by PROC STDIZE with METHOD= STD

                  The FREQ Procedure

               Table of Species by CLUSTER

Species     CLUSTER(Cluster)

Frequency |
Percent   |
Row Pct   |
Col Pct   |       5|       6|       7|  Total
----------+--------+--------+--------+
Perch     |      0 |      0 |     27 |     56
          |   0.00 |   0.00 |  17.20 |  35.67
          |   0.00 |   0.00 |  48.21 |
          |   0.00 |   0.00 |  54.00 |
----------+--------+--------+--------+
Pike      |      0 |      0 |      0 |     17
          |   0.00 |   0.00 |   0.00 |  10.83
          |   0.00 |   0.00 |   0.00 |
          |   0.00 |   0.00 |   0.00 |
----------+--------+--------+--------+
Smelt     |      0 |      0 |      1 |     14
          |   0.00 |   0.00 |   0.64 |   8.92
          |   0.00 |   0.00 |   7.14 |
          |   0.00 |   0.00 |   2.00 |
----------+--------+--------+--------+
Total           11       34       50      157
              7.01    21.66    31.85   100.00
```

Output 82.1.2 Data Are Standardized by PROC STDIZE with METHOD=RANGE

```
                        Fish Measurement Data
         Data are Standardized by PROC STDIZE with METHOD= RANGE

                          The FREQ Procedure

                     Table of Species by CLUSTER

          Species     CLUSTER(Cluster)

          Frequency |
          Percent   |
          Row Pct   |
          Col Pct   |       1|       2|       3|       4|  Total
          ----------+--------+--------+--------+--------+
          Bream     |      0 |      0 |     34 |      0 |     34
                    |   0.00 |   0.00 |  21.66 |   0.00 |  21.66
                    |   0.00 |   0.00 | 100.00 |   0.00 |
                    |   0.00 |   0.00 | 100.00 |   0.00 |
          ----------+--------+--------+--------+--------+
          Roach     |      0 |      0 |      0 |     19 |     19
                    |   0.00 |   0.00 |   0.00 |  12.10 |  12.10
                    |   0.00 |   0.00 |   0.00 | 100.00 |
                    |   0.00 |   0.00 |   0.00 |  61.29 |
          ----------+--------+--------+--------+--------+
          Whitefish |      0 |      0 |      0 |      3 |      6
                    |   0.00 |   0.00 |   0.00 |   1.91 |   3.82
                    |   0.00 |   0.00 |   0.00 |  50.00 |
                    |   0.00 |   0.00 |   0.00 |   9.68 |
          ----------+--------+--------+--------+--------+
          Parkki    |      0 |      0 |      0 |      0 |     11
                    |   0.00 |   0.00 |   0.00 |   0.00 |   7.01
                    |   0.00 |   0.00 |   0.00 |   0.00 |
                    |   0.00 |   0.00 |   0.00 |   0.00 |
          ----------+--------+--------+--------+--------+
          Total           17       14       34       31      157
                       10.83     8.92    21.66    19.75   100.00
          (Continued)
```

Output 82.1.2 *continued*

```
                      Fish Measurement Data
       Data are Standardized by PROC STDIZE with METHOD= RANGE

                       The FREQ Procedure

                    Table of Species by CLUSTER

        Species      CLUSTER(Cluster)

        Frequency |
        Percent   |
        Row Pct   |
        Col Pct   |       1|       2|       3|       4|  Total
        ----------+--------+--------+--------+--------+
        Perch     |      0 |      0 |      0 |      9 |     56
                  |   0.00 |   0.00 |   0.00 |   5.73 |  35.67
                  |   0.00 |   0.00 |   0.00 |  16.07 |
                  |   0.00 |   0.00 |   0.00 |  29.03 |
        ----------+--------+--------+--------+--------+
        Pike      |     17 |      0 |      0 |      0 |     17
                  |  10.83 |   0.00 |   0.00 |   0.00 |  10.83
                  | 100.00 |   0.00 |   0.00 |   0.00 |
                  | 100.00 |   0.00 |   0.00 |   0.00 |
        ----------+--------+--------+--------+--------+
        Smelt     |      0 |     14 |      0 |      0 |     14
                  |   0.00 |   8.92 |   0.00 |   0.00 |   8.92
                  |   0.00 | 100.00 |   0.00 |   0.00 |
                  |   0.00 | 100.00 |   0.00 |   0.00 |
        ----------+--------+--------+--------+--------+
        Total           17       14       34       31      157
                     10.83     8.92    21.66    19.75   100.00
        (Continued)
```

Output 82.1.2 *continued*

```
                         Fish Measurement Data
             Data are Standardized by PROC STDIZE with METHOD= RANGE

                           The FREQ Procedure

                      Table of Species by CLUSTER

              Species     CLUSTER(Cluster)

              Frequency |
              Percent   |
              Row Pct   |
              Col Pct   |       5|       6|       7|  Total
              ----------+--------+--------+--------+
              Bream     |      0 |      0 |      0 |     34
                        |   0.00 |   0.00 |   0.00 |  21.66
                        |   0.00 |   0.00 |   0.00 |
                        |   0.00 |   0.00 |   0.00 |
              ----------+--------+--------+--------+
              Roach     |      0 |      0 |      0 |     19
                        |   0.00 |   0.00 |   0.00 |  12.10
                        |   0.00 |   0.00 |   0.00 |
                        |   0.00 |   0.00 |   0.00 |
              ----------+--------+--------+--------+
              Whitefish |      3 |      0 |      0 |      6
                        |   1.91 |   0.00 |   0.00 |   3.82
                        |  50.00 |   0.00 |   0.00 |
                        |  13.04 |   0.00 |   0.00 |
              ----------+--------+--------+--------+
              Parkki    |      0 |     11 |      0 |     11
                        |   0.00 |   7.01 |   0.00 |   7.01
                        |   0.00 | 100.00 |   0.00 |
                        |   0.00 | 100.00 |   0.00 |
              ----------+--------+--------+--------+
              Total           23       11       27      157
                           14.65     7.01    17.20   100.00
              (Continued)
```

Output 82.1.2 *continued*

```
                     Fish Measurement Data
   Data are Standardized by PROC STDIZE with METHOD= RANGE

                       The FREQ Procedure

                  Table of Species by CLUSTER

    Species      CLUSTER(Cluster)

    Frequency |
    Percent   |
    Row Pct   |
    Col Pct   |       5|       6|       7|  Total
    ----------+--------+--------+--------+
    Perch     |     20 |      0 |     27 |     56
              |  12.74 |   0.00 |  17.20 |  35.67
              |  35.71 |   0.00 |  48.21 |
              |  86.96 |   0.00 | 100.00 |
    ----------+--------+--------+--------+
    Pike      |      0 |      0 |      0 |     17
              |   0.00 |   0.00 |   0.00 |  10.83
              |   0.00 |   0.00 |   0.00 |
              |   0.00 |   0.00 |   0.00 |
    ----------+--------+--------+--------+
    Smelt     |      0 |      0 |      0 |     14
              |   0.00 |   0.00 |   0.00 |   8.92
              |   0.00 |   0.00 |   0.00 |
              |   0.00 |   0.00 |   0.00 |
    ----------+--------+--------+--------+
    Total           23       11       27      157
                 14.65     7.01    17.20   100.00
```

Output 82.1.3 Data Are Standardized by PROC STDIZE with METHOD=AGK(0.14)

```
                         Fish Measurement Data
           Data are Standardized by PROC STDIZE with METHOD= AGK(.14)

                           The FREQ Procedure

                      Table of Species by CLUSTER

             Species     CLUSTER(Cluster)

             Frequency |
             Percent   |
             Row Pct   |
             Col Pct   |       1|       2|       3|       4|  Total
             ----------+--------+--------+--------+--------+
             Bream     |      0 |      0 |     34 |      0 |     34
                       |   0.00 |   0.00 |  21.66 |   0.00 |  21.66
                       |   0.00 |   0.00 | 100.00 |   0.00 |
                       |   0.00 |   0.00 | 100.00 |   0.00 |
             ----------+--------+--------+--------+--------+
             Roach     |      0 |      0 |      0 |     17 |     19
                       |   0.00 |   0.00 |   0.00 |  10.83 |  12.10
                       |   0.00 |   0.00 |   0.00 |  89.47 |
                       |   0.00 |   0.00 |   0.00 |  73.91 |
             ----------+--------+--------+--------+--------+
             Whitefish |      0 |      0 |      0 |      3 |      6
                       |   0.00 |   0.00 |   0.00 |   1.91 |   3.82
                       |   0.00 |   0.00 |   0.00 |  50.00 |
                       |   0.00 |   0.00 |   0.00 |  13.04 |
             ----------+--------+--------+--------+--------+
             Parkki    |     11 |      0 |      0 |      0 |     11
                       |   7.01 |   0.00 |   0.00 |   0.00 |   7.01
                       | 100.00 |   0.00 |   0.00 |   0.00 |
                       | 100.00 |   0.00 |   0.00 |   0.00 |
             ----------+--------+--------+--------+--------+
             Total           11       14       34       23      157
                           7.01     8.92    21.66    14.65   100.00
             (Continued)
```

Output 82.1.3 *continued*

```
                    Fish Measurement Data
Data are Standardized by PROC STDIZE with METHOD= AGK(.14)

                     The FREQ Procedure

                Table of Species by CLUSTER

  Species      CLUSTER(Cluster)

  Frequency |
  Percent   |
  Row Pct   |
  Col Pct   |       1|       2|       3|       4|  Total
  ----------+--------+--------+--------+--------+
  Perch     |      0 |      0 |      0 |      3 |     56
            |   0.00 |   0.00 |   0.00 |   1.91 |  35.67
            |   0.00 |   0.00 |   0.00 |   5.36 |
            |   0.00 |   0.00 |   0.00 |  13.04 |
  ----------+--------+--------+--------+--------+
  Pike      |      0 |      0 |      0 |      0 |     17
            |   0.00 |   0.00 |   0.00 |   0.00 |  10.83
            |   0.00 |   0.00 |   0.00 |   0.00 |
            |   0.00 |   0.00 |   0.00 |   0.00 |
  ----------+--------+--------+--------+--------+
  Smelt     |      0 |     14 |      0 |      0 |     14
            |   0.00 |   8.92 |   0.00 |   0.00 |   8.92
            |   0.00 | 100.00 |   0.00 |   0.00 |
            |   0.00 | 100.00 |   0.00 |   0.00 |
  ----------+--------+--------+--------+--------+
  Total           11       14       34       23      157
                7.01     8.92    21.66    14.65   100.00
  (Continued)
```

Output 82.1.3 *continued*

```
                        Fish Measurement Data
            Data are Standardized by PROC STDIZE with METHOD= AGK(.14)

                          The FREQ Procedure

                     Table of Species by CLUSTER

              Species     CLUSTER(Cluster)

              Frequency |
              Percent   |
              Row Pct   |
              Col Pct   |       5|       6|       7|  Total
              ----------+--------+--------+--------+
              Bream     |      0 |      0 |      0 |     34
                        |   0.00 |   0.00 |   0.00 |  21.66
                        |   0.00 |   0.00 |   0.00 |
                        |   0.00 |   0.00 |   0.00 |
              ----------+--------+--------+--------+
              Roach     |      0 |      0 |      2 |     19
                        |   0.00 |   0.00 |   1.27 |  12.10
                        |   0.00 |   0.00 |  10.53 |
                        |   0.00 |   0.00 |   5.71 |
              ----------+--------+--------+--------+
              Whitefish |      0 |      3 |      0 |      6
                        |   0.00 |   1.91 |   0.00 |   3.82
                        |   0.00 |  50.00 |   0.00 |
                        |   0.00 |  13.04 |   0.00 |
              ----------+--------+--------+--------+
              Parkki    |      0 |      0 |      0 |     11
                        |   0.00 |   0.00 |   0.00 |   7.01
                        |   0.00 |   0.00 |   0.00 |
                        |   0.00 |   0.00 |   0.00 |
              ----------+--------+--------+--------+
              Total           17       23       35      157
                           10.83    14.65    22.29   100.00
              (Continued)
```

Output 82.1.3 *continued*

```
                          Fish Measurement Data
          Data are Standardized by PROC STDIZE with METHOD= AGK(.14)

                            The FREQ Procedure

                       Table of Species by CLUSTER

            Species     CLUSTER(Cluster)

            Frequency |
            Percent   |
            Row Pct   |
            Col Pct   |       5|       6|       7|  Total
            ----------+--------+--------+--------+
            Perch     |      0 |     20 |     33 |     56
                      |   0.00 |  12.74 |  21.02 |  35.67
                      |   0.00 |  35.71 |  58.93 |
                      |   0.00 |  86.96 |  94.29 |
            ----------+--------+--------+--------+
            Pike      |     17 |      0 |      0 |     17
                      |  10.83 |   0.00 |   0.00 |  10.83
                      | 100.00 |   0.00 |   0.00 |
                      | 100.00 |   0.00 |   0.00 |
            ----------+--------+--------+--------+
            Smelt     |      0 |      0 |      0 |     14
                      |   0.00 |   0.00 |   0.00 |   8.92
                      |   0.00 |   0.00 |   0.00 |
                      |   0.00 |   0.00 |   0.00 |
            ----------+--------+--------+--------+
            Total           17       23       35      157
                         10.83    14.65    22.29   100.00
```

Output 82.1.4 Data Are Standardized by PROC STDIZE with METHOD=SPACING(0.14)

```
                        Fish Measurement Data
        Data are Standardized by PROC STDIZE with METHOD= SPACING(.14)

                          The FREQ Procedure

                     Table of Species by CLUSTER

            Species     CLUSTER(Cluster)

            Frequency |
            Percent   |
            Row Pct   |
            Col Pct   |       1|       2|       3|       4|  Total
            ----------+--------+--------+--------+--------+
            Bream     |      0 |      0 |      0 |      0 |     34
                      |   0.00 |   0.00 |   0.00 |   0.00 |  21.66
                      |   0.00 |   0.00 |   0.00 |   0.00 |
                      |   0.00 |   0.00 |   0.00 |   0.00 |
            ----------+--------+--------+--------+--------+
            Roach     |      0 |      0 |      0 |     17 |     19
                      |   0.00 |   0.00 |   0.00 |  10.83 |  12.10
                      |   0.00 |   0.00 |   0.00 |  89.47 |
                      |   0.00 |   0.00 |   0.00 |  85.00 |
            ----------+--------+--------+--------+--------+
            Whitefish |      3 |      0 |      0 |      3 |      6
                      |   1.91 |   0.00 |   0.00 |   1.91 |   3.82
                      |  50.00 |   0.00 |   0.00 |  50.00 |
                      |  13.04 |   0.00 |   0.00 |  15.00 |
            ----------+--------+--------+--------+--------+
            Parkki    |      0 |      0 |     11 |      0 |     11
                      |   0.00 |   0.00 |   7.01 |   0.00 |   7.01
                      |   0.00 |   0.00 | 100.00 |   0.00 |
                      |   0.00 |   0.00 | 100.00 |   0.00 |
            ----------+--------+--------+--------+--------+
            Total           23       17       11       20      157
                         14.65    10.83     7.01    12.74   100.00
            (Continued)
```

Output 82.1.4 *continued*

```
                    Fish Measurement Data
     Data are Standardized by PROC STDIZE with METHOD= SPACING(.14)

                      The FREQ Procedure

                   Table of Species by CLUSTER

      Species      CLUSTER(Cluster)

      Frequency |
      Percent   |
      Row Pct   |
      Col Pct   |       1|       2|       3|       4|  Total
      ----------+--------+--------+--------+--------+
      Perch     |     20 |      0 |      0 |      0 |     56
                |  12.74 |   0.00 |   0.00 |   0.00 |  35.67
                |  35.71 |   0.00 |   0.00 |   0.00 |
                |  86.96 |   0.00 |   0.00 |   0.00 |
      ----------+--------+--------+--------+--------+
      Pike      |      0 |     17 |      0 |      0 |     17
                |   0.00 |  10.83 |   0.00 |   0.00 |  10.83
                |   0.00 | 100.00 |   0.00 |   0.00 |
                |   0.00 | 100.00 |   0.00 |   0.00 |
      ----------+--------+--------+--------+--------+
      Smelt     |      0 |      0 |      0 |      0 |     14
                |   0.00 |   0.00 |   0.00 |   0.00 |   8.92
                |   0.00 |   0.00 |   0.00 |   0.00 |
                |   0.00 |   0.00 |   0.00 |   0.00 |
      ----------+--------+--------+--------+--------+
      Total           23       17       11       20      157
                   14.65    10.83     7.01    12.74   100.00
      (Continued)
```

Output 82.1.4 *continued*

```
                        Fish Measurement Data
        Data are Standardized by PROC STDIZE with METHOD= SPACING(.14)

                          The FREQ Procedure

                     Table of Species by CLUSTER

            Species      CLUSTER(Cluster)

            Frequency |
            Percent   |
            Row Pct   |
            Col Pct   |       5|       6|       7|  Total
            ----------+--------+--------+--------+
            Bream     |      0 |      0 |     34 |     34
                      |   0.00 |   0.00 |  21.66 |  21.66
                      |   0.00 |   0.00 | 100.00 |
                      |   0.00 |   0.00 | 100.00 |
            ----------+--------+--------+--------+
            Roach     |      0 |      2 |      0 |     19
                      |   0.00 |   1.27 |   0.00 |  12.10
                      |   0.00 |  10.53 |   0.00 |
                      |   0.00 |   5.26 |   0.00 |
            ----------+--------+--------+--------+
            Whitefish |      0 |      0 |      0 |      6
                      |   0.00 |   0.00 |   0.00 |   3.82
                      |   0.00 |   0.00 |   0.00 |
                      |   0.00 |   0.00 |   0.00 |
            ----------+--------+--------+--------+
            Parkki    |      0 |      0 |      0 |     11
                      |   0.00 |   0.00 |   0.00 |   7.01
                      |   0.00 |   0.00 |   0.00 |
                      |   0.00 |   0.00 |   0.00 |
            ----------+--------+--------+--------+
            Total           14       38       34      157
                          8.92    24.20    21.66   100.00
            (Continued)
```

Output 82.1.4 *continued*

```
                    Fish Measurement Data
    Data are Standardized by PROC STDIZE with METHOD= SPACING(.14)

                      The FREQ Procedure

                 Table of Species by CLUSTER

        Species     CLUSTER(Cluster)

        Frequency |
        Percent   |
        Row Pct   |
        Col Pct   |       5|       6|       7|  Total
        ----------+--------+--------+--------+
        Perch     |      0 |     36 |      0 |     56
                  |   0.00 |  22.93 |   0.00 |  35.67
                  |   0.00 |  64.29 |   0.00 |
                  |   0.00 |  94.74 |   0.00 |
        ----------+--------+--------+--------+
        Pike      |      0 |      0 |      0 |     17
                  |   0.00 |   0.00 |   0.00 |  10.83
                  |   0.00 |   0.00 |   0.00 |
                  |   0.00 |   0.00 |   0.00 |
        ----------+--------+--------+--------+
        Smelt     |     14 |      0 |      0 |     14
                  |   8.92 |   0.00 |   0.00 |   8.92
                  | 100.00 |   0.00 |   0.00 |
                  | 100.00 |   0.00 |   0.00 |
        ----------+--------+--------+--------+
        Total          14       38       34      157
                     8.92    24.20    21.66   100.00
```

The following analysis (labeled 'Approach 2') applies the cluster analysis directly to the original data. The following statements produce Output 82.1.5.

```
/*********************************************************/
/*                                                       */
/*         Approach 2: data are untransformed            */
/*                                                       */
/*********************************************************/

title2 'Data are Untransformed';
%FastFreq(fish);
```

Output 82.1.5 Untransformed Data

```
                    Fish Measurement Data
                    Data are Untransformed

                      The FREQ Procedure

                  Table of Species by CLUSTER

Species     CLUSTER(Cluster)

Frequency |
Percent   |
Row Pct   |
Col Pct   |       1|       2|       3|       4|  Total
----------+--------+--------+--------+--------+
Bream     |     13 |      0 |      0 |      0 |     34
          |   8.28 |   0.00 |   0.00 |   0.00 |  21.66
          |  38.24 |   0.00 |   0.00 |   0.00 |
          |  44.83 |   0.00 |   0.00 |   0.00 |
----------+--------+--------+--------+--------+
Roach     |      3 |      4 |      0 |      0 |     19
          |   1.91 |   2.55 |   0.00 |   0.00 |  12.10
          |  15.79 |  21.05 |   0.00 |   0.00 |
          |  10.34 |  25.00 |   0.00 |   0.00 |
----------+--------+--------+--------+--------+
Whitefish |      3 |      0 |      0 |      0 |      6
          |   1.91 |   0.00 |   0.00 |   0.00 |   3.82
          |  50.00 |   0.00 |   0.00 |   0.00 |
          |  10.34 |   0.00 |   0.00 |   0.00 |
----------+--------+--------+--------+--------+
Parkki    |      2 |      3 |      0 |      0 |     11
          |   1.27 |   1.91 |   0.00 |   0.00 |   7.01
          |  18.18 |  27.27 |   0.00 |   0.00 |
          |   6.90 |  18.75 |   0.00 |   0.00 |
----------+--------+--------+--------+--------+
Total           29       16       10       15      157
             18.47    10.19     6.37     9.55   100.00
 (Continued)
```

Output 82.1.5 *continued*

```
                Fish Measurement Data
                Data are Untransformed

                  The FREQ Procedure

              Table of Species by CLUSTER

Species     CLUSTER(Cluster)

Frequency |
Percent   |
Row Pct   |
Col Pct   |       1|       2|       3|       4|  Total
----------+--------+--------+--------+--------+
Perch     |      8 |      9 |      0 |      1 |     56
          |   5.10 |   5.73 |   0.00 |   0.64 |  35.67
          |  14.29 |  16.07 |   0.00 |   1.79 |
          |  27.59 |  56.25 |   0.00 |   6.67 |
----------+--------+--------+--------+--------+
Pike      |      0 |      0 |     10 |      0 |     17
          |   0.00 |   0.00 |   6.37 |   0.00 |  10.83
          |   0.00 |   0.00 |  58.82 |   0.00 |
          |   0.00 |   0.00 | 100.00 |   0.00 |
----------+--------+--------+--------+--------+
Smelt     |      0 |      0 |      0 |     14 |     14
          |   0.00 |   0.00 |   0.00 |   8.92 |   8.92
          |   0.00 |   0.00 |   0.00 | 100.00 |
          |   0.00 |   0.00 |   0.00 |  93.33 |
----------+--------+--------+--------+--------+
Total           29       16       10       15      157
             18.47    10.19     6.37     9.55   100.00
(Continued)
```

Output 82.1.5 *continued*

```
                         Fish Measurement Data
                         Data are Untransformed

                           The FREQ Procedure

                      Table of Species by CLUSTER

            Species     CLUSTER(Cluster)

            Frequency |
            Percent   |
            Row Pct   |
            Col Pct   |       5|       6|       7|  Total
            ----------+--------+--------+--------+
            Bream     |      0 |      0 |     21 |     34
                      |   0.00 |   0.00 |  13.38 |  21.66
                      |   0.00 |   0.00 |  61.76 |
                      |   0.00 |   0.00 |  47.73 |
            ----------+--------+--------+--------+
            Roach     |     12 |      0 |      0 |     19
                      |   7.64 |   0.00 |   0.00 |  12.10
                      |  63.16 |   0.00 |   0.00 |
                      |  30.77 |   0.00 |   0.00 |
            ----------+--------+--------+--------+
            Whitefish |      0 |      0 |      3 |      6
                      |   0.00 |   0.00 |   1.91 |   3.82
                      |   0.00 |   0.00 |  50.00 |
                      |   0.00 |   0.00 |   6.82 |
            ----------+--------+--------+--------+
            Parkki    |      6 |      0 |      0 |     11
                      |   3.82 |   0.00 |   0.00 |   7.01
                      |  54.55 |   0.00 |   0.00 |
                      |  15.38 |   0.00 |   0.00 |
            ----------+--------+--------+--------+
            Total           39        4       44      157
                         24.84     2.55    28.03   100.00
            (Continued)
```

Output 82.1.5 *continued*

```
                    Fish Measurement Data
                   Data are Untransformed

                     The FREQ Procedure

                 Table of Species by CLUSTER

       Species     CLUSTER(Cluster)

       Frequency |
       Percent   |
       Row Pct   |
       Col Pct   |       5|       6|       7|  Total
       ----------+--------+--------+--------+
       Perch     |     20 |      0 |     18 |     56
                 |  12.74 |   0.00 |  11.46 |  35.67
                 |  35.71 |   0.00 |  32.14 |
                 |  51.28 |   0.00 |  40.91 |
       ----------+--------+--------+--------+
       Pike      |      1 |      4 |      2 |     17
                 |   0.64 |   2.55 |   1.27 |  10.83
                 |   5.88 |  23.53 |  11.76 |
                 |   2.56 | 100.00 |   4.55 |
       ----------+--------+--------+--------+
       Smelt     |      0 |      0 |      0 |     14
                 |   0.00 |   0.00 |   0.00 |   8.92
                 |   0.00 |   0.00 |   0.00 |
                 |   0.00 |   0.00 |   0.00 |
       ----------+--------+--------+--------+
       Total           39        4       44      157
                    24.84     2.55    28.03   100.00
```

The following analysis (labeled 'Approach 3') transforms the original data with the ACECLUS procedure and creates a TYPE=ACE output data set that is used as an input data set for the cluster analysis. The following statements produce Output 82.1.6.

```
/*********************************************************/
/*                                                       */
/*    Approach 3: data are transformed by PROC ACECLUS   */
/*                                                       */
/*********************************************************/

title2 'Data are Transformed by PROC ACECLUS';
proc aceclus data=fish out=ace p=.02 noprint;
   var Length1 logLengthRatio Height Width Weight3;
run;
%FastFreq(ace);
```

Output 82.1.6 Data Are Transformed by PROC ACECLUS

```
                     Fish Measurement Data
              Data are Transformed by PROC ACECLUS

                       The FREQ Procedure

                 Table of Species by CLUSTER

Species      CLUSTER(Cluster)

Frequency |
Percent   |
Row Pct   |
Col Pct   |       1|       2|       3|       4|  Total
----------+--------+--------+--------+--------+
Bream     |     13 |      0 |      0 |      0 |     34
          |   8.28 |   0.00 |   0.00 |   0.00 |  21.66
          |  38.24 |   0.00 |   0.00 |   0.00 |
          |  44.83 |   0.00 |   0.00 |   0.00 |
----------+--------+--------+--------+--------+
Roach     |      3 |      4 |      0 |      0 |     19
          |   1.91 |   2.55 |   0.00 |   0.00 |  12.10
          |  15.79 |  21.05 |   0.00 |   0.00 |
          |  10.34 |  25.00 |   0.00 |   0.00 |
----------+--------+--------+--------+--------+
Whitefish |      3 |      0 |      0 |      0 |      6
          |   1.91 |   0.00 |   0.00 |   0.00 |   3.82
          |  50.00 |   0.00 |   0.00 |   0.00 |
          |  10.34 |   0.00 |   0.00 |   0.00 |
----------+--------+--------+--------+--------+
Parkki    |      2 |      3 |      0 |      0 |     11
          |   1.27 |   1.91 |   0.00 |   0.00 |   7.01
          |  18.18 |  27.27 |   0.00 |   0.00 |
          |   6.90 |  18.75 |   0.00 |   0.00 |
----------+--------+--------+--------+--------+
Total           29       16       10       15      157
             18.47    10.19     6.37     9.55   100.00
(Continued)
```

Output 82.1.6 *continued*

```
                    Fish Measurement Data
             Data are Transformed by PROC ACECLUS

                     The FREQ Procedure

                Table of Species by CLUSTER

      Species     CLUSTER(Cluster)

      Frequency |
      Percent   |
      Row Pct   |
      Col Pct   |       1|       2|       3|       4|  Total
      ----------+--------+--------+--------+--------+
      Perch     |      8 |      9 |      0 |      1 |     56
                |   5.10 |   5.73 |   0.00 |   0.64 |  35.67
                |  14.29 |  16.07 |   0.00 |   1.79 |
                |  27.59 |  56.25 |   0.00 |   6.67 |
      ----------+--------+--------+--------+--------+
      Pike      |      0 |      0 |     10 |      0 |     17
                |   0.00 |   0.00 |   6.37 |   0.00 |  10.83
                |   0.00 |   0.00 |  58.82 |   0.00 |
                |   0.00 |   0.00 | 100.00 |   0.00 |
      ----------+--------+--------+--------+--------+
      Smelt     |      0 |      0 |      0 |     14 |     14
                |   0.00 |   0.00 |   0.00 |   8.92 |   8.92
                |   0.00 |   0.00 |   0.00 | 100.00 |
                |   0.00 |   0.00 |   0.00 |  93.33 |
      ----------+--------+--------+--------+--------+
      Total           29       16       10       15      157
                   18.47    10.19     6.37     9.55   100.00
      (Continued)
```

Output 82.1.6 *continued*

```
                   Fish Measurement Data
            Data are Transformed by PROC ACECLUS

                    The FREQ Procedure

                Table of Species by CLUSTER

Species     CLUSTER(Cluster)

Frequency |
Percent   |
Row Pct   |
Col Pct   |       5|       6|       7|  Total
----------+--------+--------+--------+
Bream     |      0 |      0 |     21 |     34
          |   0.00 |   0.00 |  13.38 |  21.66
          |   0.00 |   0.00 |  61.76 |
          |   0.00 |   0.00 |  47.73 |
----------+--------+--------+--------+
Roach     |     12 |      0 |      0 |     19
          |   7.64 |   0.00 |   0.00 |  12.10
          |  63.16 |   0.00 |   0.00 |
          |  30.77 |   0.00 |   0.00 |
----------+--------+--------+--------+
Whitefish |      0 |      0 |      3 |      6
          |   0.00 |   0.00 |   1.91 |   3.82
          |   0.00 |   0.00 |  50.00 |
          |   0.00 |   0.00 |   6.82 |
----------+--------+--------+--------+
Parkki    |      6 |      0 |      0 |     11
          |   3.82 |   0.00 |   0.00 |   7.01
          |  54.55 |   0.00 |   0.00 |
          |  15.38 |   0.00 |   0.00 |
----------+--------+--------+--------+
Total           39        4       44      157
             24.84     2.55    28.03   100.00
(Continued)
```

Output 82.1.6 *continued*

```
                     Fish Measurement Data
                Data are Transformed by PROC ACECLUS

                       The FREQ Procedure

                   Table of Species by CLUSTER

         Species     CLUSTER(Cluster)

         Frequency |
         Percent   |
         Row Pct   |
         Col Pct   |       5|       6|       7|  Total
         ----------+--------+--------+--------+
         Perch     |     20 |      0 |     18 |     56
                   |  12.74 |   0.00 |  11.46 |  35.67
                   |  35.71 |   0.00 |  32.14 |
                   |  51.28 |   0.00 |  40.91 |
         ----------+--------+--------+--------+
         Pike      |      1 |      4 |      2 |     17
                   |   0.64 |   2.55 |   1.27 |  10.83
                   |   5.88 |  23.53 |  11.76 |
                   |   2.56 | 100.00 |   4.55 |
         ----------+--------+--------+--------+
         Smelt     |      0 |      0 |      0 |     14
                   |   0.00 |   0.00 |   0.00 |   8.92
                   |   0.00 |   0.00 |   0.00 |
                   |   0.00 |   0.00 |   0.00 |
         ----------+--------+--------+--------+
         Total           39        4       44      157
                      24.84     2.55    28.03   100.00
```

Table 82.4 displays a table summarizing each classification results. In this table, the first column represents the standardization method, the second column represents the number of clusters that the seven species are classified into, and the third column represents the total number of observations that are misclassified.

Table 82.4 Summary of Clustering Results

Method of Standardization	Number of Clusters	Misclassification
MEAN	5	71
MEDIAN	5	71
SUM	6	51
EUCLEN	6	45
USTD	6	45
STD	5	33
RANGE	7	32
MIDRANGE	7	32
MAXABS	7	26
IQR	5	28
MAD	4	35
ABW(5)	6	34
AWAVE(5)	6	29
AGK(0.14)	7	28
SPACING(0.14)	7	25
L(1)	6	41
L(1.5)	5	33
L(2)	5	33
untransformed	5	71
PROC ACECLUS	5	71

Consider the results displayed in Output 82.1.1. In that analysis, the method of standardization is STD, and the number of clusters and the number of misclassifications are computed as shown in Table 82.5.

Table 82.5 Computations of Numbers of Clusters and Misclassification When Standardization Method Is STD

Species	Cluster Number	Misclassification in Each Species
Bream	6	0
Roach	7	0
Whitefish	7	3
Parkki	5	0
Perch	7	29
Pike	1	0
Smelt	3	1

In Output 82.1.1, the bream species is classified as cluster 6 since all 34 bream are categorized into cluster 6 with no misclassification. A similar pattern is seen with the roach, parkki, pike, and smelt species.

For the whitefish species, two fish are categorized into cluster 2, one fish is categorized into cluster 4, and three fish are categorized into cluster 7. Because the majority of this species is categorized into cluster 7, it is recorded in Table 82.5 as being classified as cluster 7 with 3 misclassifications. A similar pattern is seen with the perch species: it is classified as cluster 7 with 29 misclassifications.

In summary, when the standardization method is STD, seven species of fish are classified into only five clusters and the total number of misclassified observations is 33.

The result of this analysis demonstrates that when variables are standardized by the STDIZE procedure with methods including RANGE, MIDRANGE, MAXABS, AGK(0.14), and SPACING(0.14), the FASTCLUS procedure produces the correct number of clusters and less misclassification than it does when other standardization methods are used. The SPACING method attains the best result, probably because the variables Length1 and Height both exhibit marked groupings (bimodality) in their distributions.

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