SAS/STAT® 9.22 User's Guide

The ROBUSTREG Procedure

(Book Excerpt)

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Chapter 75

The ROBUSTREG Procedure

Contents

Overview: ROBUSTREG Procedure

The main purpose of robust regression is to detect outliers and provide resistant (stable) results in the presence of outliers. In order to achieve this stability, robust regression limits the influence of outliers. Historically, three classes of problems have been addressed with robust regression techniques:

- problems with outliers in the y-direction (response direction)
- problems with multivariate outliers in the x-space (that is, outliers in the covariate space, which are also referred to as leverage points)
- problems with outliers in both the y-direction and the x-space

Many methods have been developed in response to these problems. However, in statistical applications of outlier detection and robust regression, the methods most commonly used today are Huber M estimation, high breakdown value estimation, and combinations of these two methods. The ROBUSTREG procedure provides four such methods: M estimation, LTS estimation, S estimation, and MM estimation.

- M estimation was introduced by Huber (1973), and it is the simplest approach both computationally and theoretically. Although it is not robust with respect to leverage points, it is still used extensively in data analysis when contamination can be assumed to be mainly in the response direction.
- Least trimmed squares (LTS) estimation is a high breakdown value method introduced by Rousseeuw (1984). The breakdown value is a measure of the proportion of contamination that an estimation method can withstand and still maintain its robustness. The performance of this method was improved by the FAST-LTS algorithm of Rousseeuw and Van Driessen (2000).
- S estimation is a high breakdown value method introduced by Rousseeuw and Yohai (1984). With the same breakdown value, it has a higher statistical efficiency than LTS estimation.
- MM estimation, introduced by Yohai (1987), combines high breakdown value estimation and M estimation. It has both the high breakdown property and a higher statistical efficiency than S estimation.

Features

The main features of the ROBUSTREG procedure are as follows:

- offers four estimation methods: M, LTS, S, and MM
- provides 10 weight functions for M estimation

- provides robust R-square and deviance for all estimates
- provides asymptotic covariance and confidence intervals for regression parameter with the M, S, and MM methods
- provides robust Wald and F tests for regression parameters with the M and MM methods
- provides Mahalanobis distance and robust Mahalanobis distance with generalized minimum covariance determinant (MCD) algorithm
- provides outlier and leverage-point diagnostics
- supports parallel computing for S and LTS estimates
- supports constructed effects including spline and multimember effects
- produces fit plots and diagnostic plots by using ODS Graphics

Getting Started: ROBUSTREG Procedure

The following examples demonstrate how you can use the ROBUSTREG procedure to fit a linear regression model and obtain outlier and leverage-point diagnostics.

M Estimation

This example shows how you can use the ROBUSTREG procedure to do M estimation, which is a commonly used method for outlier detection and robust regression when contamination is mainly in the response direction.

```
data stack;
   input  x1 x2 x3 y exp$ @@;
datalines;
80  27   89  42   e1   80  27   88  37   e2
75  25   90  37   e3   62  24   87  28   e4
62  22   87  18   e5   62  23   87  18   e6
62  24   93  19   e7   62  24   93  20   e8
58  23   87  15   e9   58  18   80  14  e10
58  18   89  14  e11   58  17   88  13  e12
58  18   82  11  e13   58  19   93  12  e14
50  18   89   8  e15   50  18   86   7  e16
50  19   72   8  e17   50  19   79   8  e18
50  20   80   9  e19   56  20   82  15  e20
70  20   91  15  e21
;
```

The data set stack is the well-known stack-loss data set presented by Brownlee (1965). The data describe the operation of a plant for the oxidation of ammonia to nitric acid and consist of 21 four-dimensional observations. The explanatory variables for the response stack-loss (y) are the rate of operation (x1), the cooling water inlet temperature (x2), and the acid concentration (x3).

The following ROBUSTREG statements analyze the data:

```
proc robustreg data=stack;
   model y = x1 x2 x3 / diagnostics leverage;
   id    exp;
   test  x3;
run;
```

By default, the procedure does M estimation with the bisquare weight function, and it uses the median method for estimating the scale parameter. The MODEL statement specifies the covariate effects. The DIAGNOSTICS option requests a table for outlier diagnostics, and the LEVERAGE option adds leverage-point diagnostic results to this table for continuous covariate effects. The ID statement specifies that the variable exp is used to identify each observation (experiment) in this table. If the ID statement is omitted, the observation number is used to identify the observations. The TEST statement requests a test of significance for the covariate effects specified. The results of this analysis are displayed in the following figures.

Figure 75.1 Model Fitting Information and Summary Statistics

The ROBUSTREG Procedure

Model Information

Data Set	WORK.STACK
Dependent Variable	y
Number of Independent Variables	3
Number of Observations	21
Method	M Estimation

Summary Statistics

Variable	Q1	Median	Q3	Mean	Standard Deviation	MAD
x1	53.0000	58.0000	62.0000	60.4286	9.1683	5.9304
x2	18.0000	20.0000	24.0000	21.0952	3.1608	2.9652
x3	82.0000	87.0000	89.5000	86.2857	5.3586	4.4478
y	10.0000	15.0000	19.5000	17.5238	10.1716	5.9304

Figure 75.1 displays the model fitting information and summary statistics for the response variable and the continuous covariates. The columns labeled Q1, Median, and Q3 provide the lower quantile, median, and upper quantile, respectively. The column labeled MAD provides a robust estimate of the univariate scale, which is computed as the standardized median absolute deviation (MAD). See Huber (1981, p. 108) for more details about the standardized MAD. The columns labeled Mean and Standard Deviation provide the usual mean and standard deviation. A large difference between the standard deviation and the MAD for a variable indicates some extreme values for this variable. In the stack-loss data, the stack-loss (response y) has the biggest difference between the standard deviation and the MAD.

Figure 75.2 Model Parameter Estimates

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	-42.2854	9.5045	-60.9138	-23.6569	19.79	<.0001
x1	1	0.9276	0.1077	0.7164	1.1387	74.11	<.0001
x2	1	0.6507	0.2940	0.0744	1.2270	4.90	0.0269
x3	1	-0.1123	0.1249	-0.3571	0.1324	0.81	0.3683
Scale	1	2.2819					

Figure 75.2 displays the table of robust parameter estimates, standard errors, and confidence limits. The row labeled Scale provides a point estimate of the scale parameter in the linear regression model, which is obtained by the median method. See the section "M Estimation" on page 6388 for more information about scale estimation methods. For the stack-loss data, M estimation yields the fitted linear model:

$$\hat{y} = -42.2845 + 0.9276\text{x1} + 0.6507\text{x2} - 0.1123\text{x3}$$

Figure 75.3 Diagnostics

Diagnostics

Obs	exp	Mahalanobis Distance	Robust MCD Distance	Leverage	Standardized Robust Residual	Outlier
1	e1	2.2536	5.5284	*	1.0995	
2	e2	2.3247	5.6374	*	-1.1409	
3	e3	1.5937	4.1972	*	1.5604	
4	e4	1.2719	1.5887		3.0381	*
21	e21	2.1768	3.6573	*	-4.5733	*

Figure 75.3 displays outlier and leverage-point diagnostics. Standardized robust residuals are computed based on the estimated parameters. Both the Mahalanobis distance and the robust MCD distance are displayed. Outliers and leverage points, identified with asterisks, are defined by the standardized robust residuals and robust MCD distances that exceed the corresponding cutoff values displayed in the diagnostics summary. Observations 4 and 21 are outliers because their standardized robust residuals exceed the cutoff value in absolute value. The procedure detects four observations with high leverage. Leverage points (points with high leverage) with smaller standardized robust residuals than the cutoff value in absolute value are called good leverage points; others are called bad leverage points. Observation 21 is a bad leverage point.

Two particularly useful plots for revealing outliers and leverage points are a scatter plot of the standardized robust residuals against the robust distances (RDPLOT) and a scatter plot of the robust distances against the classical Mahalanobis distances (DDPLOT).

For the stack-loss data, the following statements produce the RDPLOT in Figure 75.4 and the DDPLOT in Figure 75.5. The histogram and the normal quantile-quantile plots (shown in Figure 75.6 and Figure 75.7, respectively) for the standardized robust residuals are also created with the HISTOGRAM and QQPLOT suboptions of the PLOTS= option.

```
ods graphics on;

proc robustreg data=stack
        plots=(rdplot ddplot histogram qqplot);
   model y = x1 x2 x3;
run;

ods graphics off;
```

These plots are helpful in identifying outliers in addition to good and bad high leverage points.

These graphical displays are requested by specifying the ODS GRAPHICS statement and the PLOTS= option in the PROC ROBUSTREG statement. For general information about ODS Graphics, see Chapter 21, "Statistical Graphics Using ODS." For specific information about the graphics available in the ROBUSTREG procedure, see the section "ODS Graphics" on page 6413.

Figure 75.4 RDPLOT for Stackloss Data

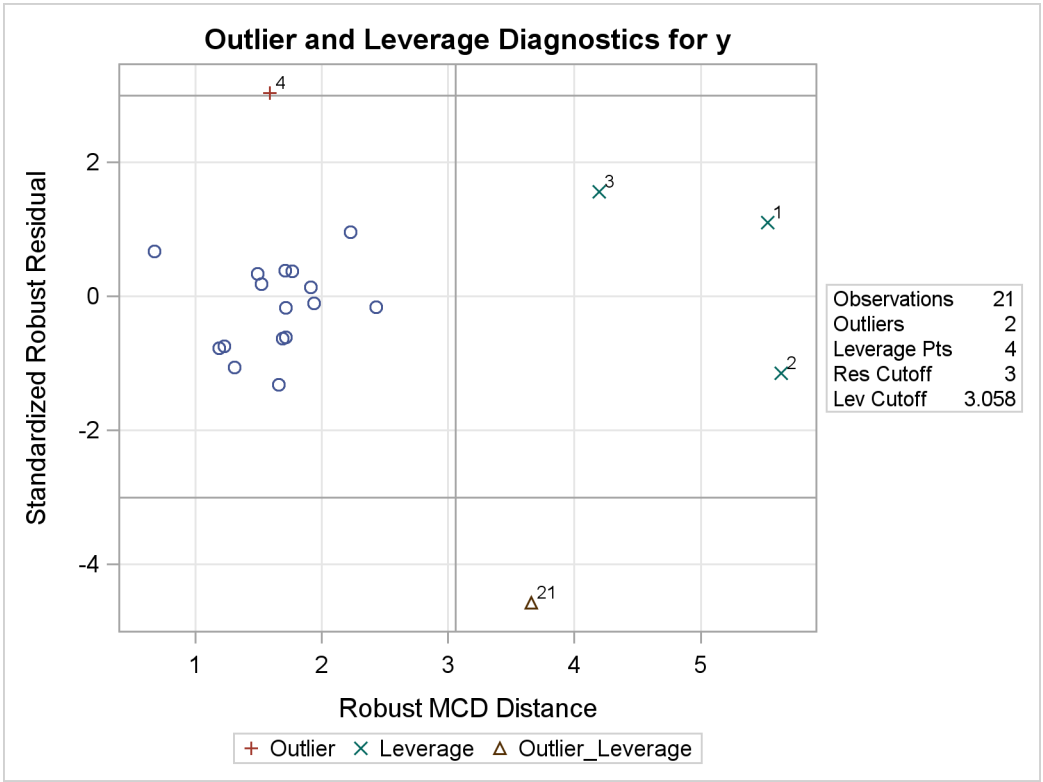


Figure 75.5 DDPLOT for Stackloss Data

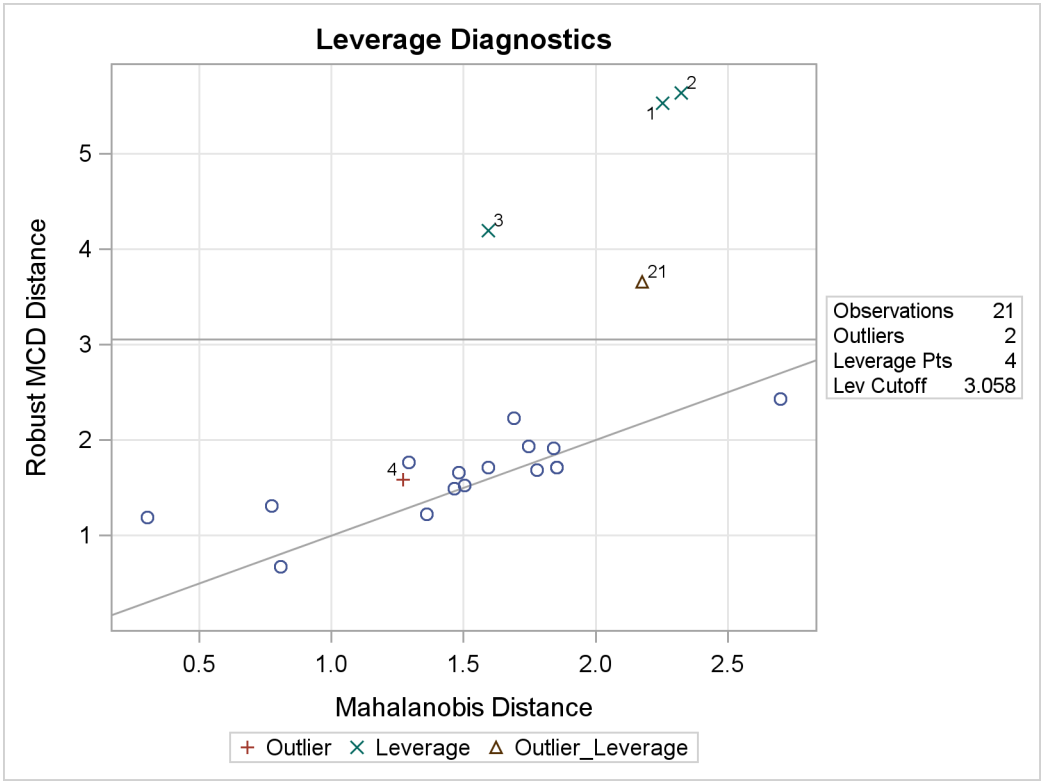


Figure 75.6 Histogram

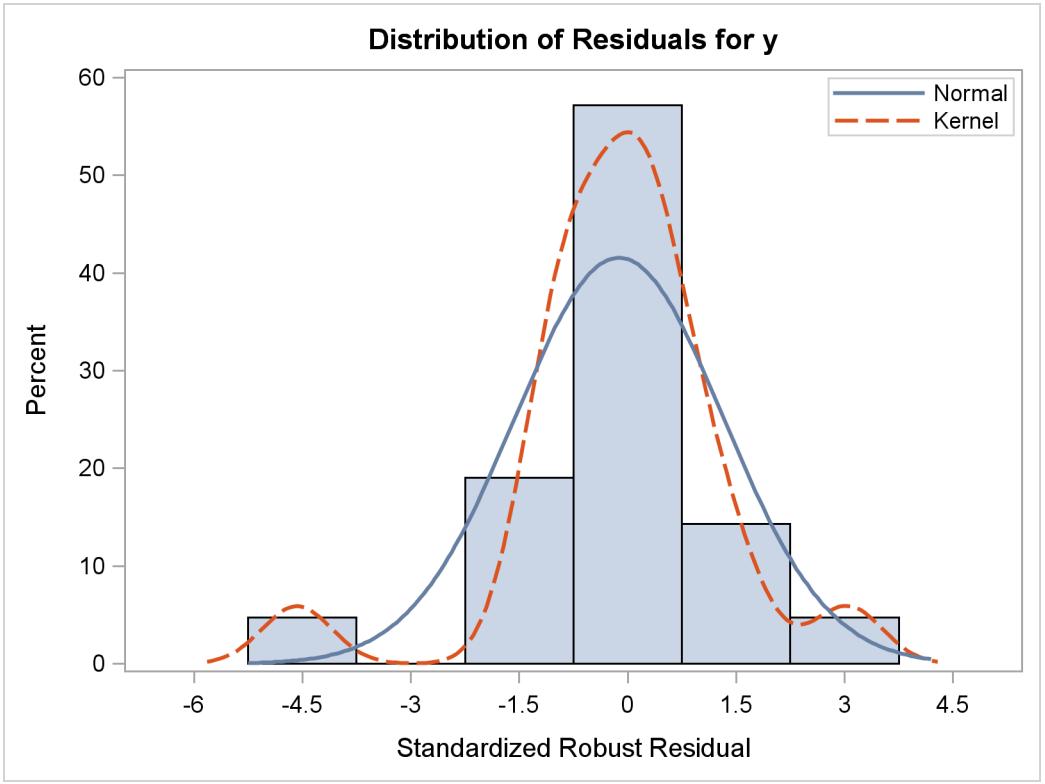


Figure 75.7 Q-Q Plot

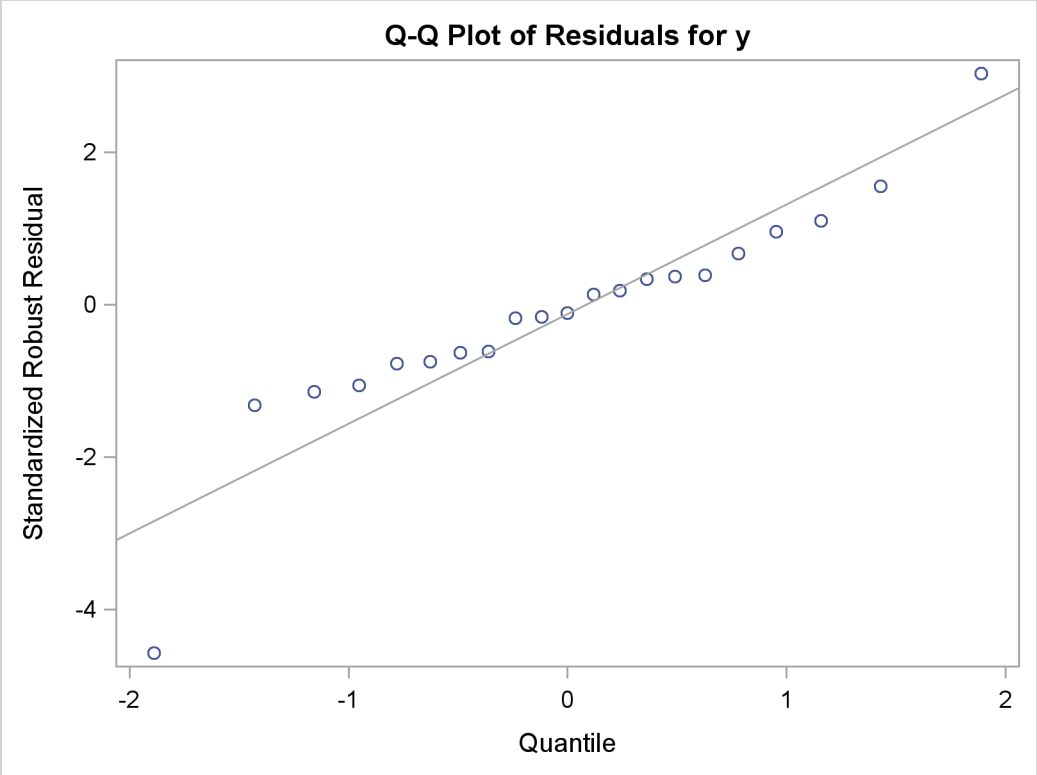


Figure 75.8 displays robust versions of goodness-of-fit statistics for the model. You can use the robust information criteria, AICR and BICR, for model selection and comparison. For both AICR and BICR, the lower the value, the more desirable the model.

Figure 75.8 Goodness-of-Fit Statistics

Goodness-of-Fit

Statistic	Value
R-Square	0.6659
AICR	29.5231
BICR	36.3361
Deviance	125.7905

Figure 75.9 displays the test results requested by the TEST statement. The ROBUSTREG procedure conducts two robust linear tests, the ρ test and the R_n^2 test. See the section "Linear Tests" on page 6393 for information about how the procedure computes test statistics and the correction factor lambda. Due to the large p-values for both tests, you can conclude that the effect x3 is not significant at the 5% level.

Figure 75.9 Test of Significance

Robust Linear Test

Test	Test Statistic	Lambda	DF	Chi-Square	Pr > ChiSq
Rho	0.9378	0.7977	1	1.18	0.2782
Rn2	0.8092		1	0.81	0.3683

For the bisquare weight function, the default tuning constant, $c = 4.685$, is chosen to yield a 95% asymptotic efficiency of the M estimates with the Gaussian distribution. See the section "M Estimation" on page 6388 for details. The smaller the constant c, the lower the asymptotic efficiency but the sharper the M estimate as an outlier detector. For the stack-loss data set, you could consider using a sharper outlier detector.

In the following invocation of the ROBUSTREG procedure, a smaller constant, $c = 3.5$, is used. This tuning constant corresponds to an efficiency close to 85%. See Chen and Yin (2002) for the relationship between the tuning constant and asymptotic efficiency of M estimates.

```
proc robustreg method=m(wf=bisquare(c=3.5)) data=stack;
   model y = x1 x2 x3 / diagnostics leverage;
   id    exp;
   test  x3;
run;
```

Figure 75.10 displays the table of robust parameter estimates, standard errors, and confidence limits with the constant $c = 3.5$.

The refitted linear model is

$$\hat{y} = -37.1076 + 0.8191x1 + 0.5173x2 - 0.0728x3$$

Figure 75.10 Model Parameter Estimates

The ROBUSTREG Procedure

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	-37.1076	5.4731	-47.8346	-26.3805	45.97	<.0001
x1	1	0.8191	0.0620	0.6975	0.9407	174.28	<.0001
x2	1	0.5173	0.1693	0.1855	0.8492	9.33	0.0022
x3	1	-0.0728	0.0719	-0.2138	0.0681	1.03	0.3111
Scale	1	1.4265					

Figure 75.11 displays outlier and leverage-point diagnostics with the constant $c = 3.5$. Besides observations 4 and 21, observations 1 and 3 are also detected as outliers.

Figure 75.11 Diagnostics

Diagnostics

Obs	exp	Mahalanobis Distance	Robust MCD Distance	Leverage	Standardized Robust Residual	Outlier
1	e1	2.2536	5.5284	*	4.2719	*
2	e2	2.3247	5.6374	*	0.7158	
3	e3	1.5937	4.1972	*	4.4142	*
4	e4	1.2719	1.5887		5.7792	*
21	e21	2.1768	3.6573	*	-6.2727	*

LTS Estimation

If the data are contaminated in the x-space, M estimation does not do well. The following example shows how you can use LTS estimation to deal with this situation.

```
data hbk;
   input index$ x1 x2 x3 y @@;
datalines;
1   10.1  19.6  28.3   9.7     39  2.1  0.0  1.2  -0.7
2    9.5  20.5  28.9  10.1     40  0.5  2.0  1.2  -0.5
3   10.7  20.2  31.0  10.3     41  3.4  1.6  2.9  -0.1
4    9.9  21.5  31.7   9.5     42  0.3  1.0  2.7  -0.7
5   10.3  21.1  31.1  10.0     43  0.1  3.3  0.9   0.6
6   10.8  20.4  29.2  10.0     44  1.8  0.5  3.2  -0.7
7   10.5  20.9  29.1  10.8     45  1.9  0.1  0.6  -0.5
8    9.9  19.6  28.8  10.3     46  1.8  0.5  3.0  -0.4
9    9.7  20.7  31.0   9.6     47  3.0  0.1  0.8  -0.9
10   9.3  19.7  30.3   9.9     48  3.1  1.6  3.0   0.1
11  11.0  24.0  35.0  -0.2     49  3.1  2.5  1.9   0.9
12  12.0  23.0  37.0  -0.4     50  2.1  2.8  2.9  -0.4
13  12.0  26.0  34.0   0.7     51  2.3  1.5  0.4   0.7
14  11.0  34.0  34.0   0.1     52  3.3  0.6  1.2  -0.5
15   3.4   2.9   2.1  -0.4     53  0.3  0.4  3.3   0.7
16   3.1   2.2   0.3   0.6     54  1.1  3.0  0.3   0.7
17   0.0   1.6   0.2  -0.2     55  0.5  2.4  0.9   0.0
18   2.3   1.6   2.0   0.0     56  1.8  3.2  0.9   0.1
19   0.8   2.9   1.6   0.1     57  1.8  0.7  0.7   0.7
20   3.1   3.4   2.2   0.4     58  2.4  3.4  1.5  -0.1
21   2.6   2.2   1.9   0.9     59  1.6  2.1  3.0  -0.3
22   0.4   3.2   1.9   0.3     60  0.3  1.5  3.3  -0.9
23   2.0   2.3   0.8  -0.8     61  0.4  3.4  3.0  -0.3
24   1.3   2.3   0.5   0.7     62  0.9  0.1  0.3   0.6
25   1.0   0.0   0.4  -0.3     63  1.1  2.7  0.2  -0.3
26   0.9   3.3   2.5  -0.8     64  2.8  3.0  2.9  -0.5
27   3.3   2.5   2.9  -0.7     65  2.0  0.7  2.7   0.6
28   1.8   0.8   2.0   0.3     66  0.2  1.8  0.8  -0.9
```

```
29   1.2   0.9   0.8   0.3     67  1.6  2.0  1.2  -0.7
30   1.2   0.7   3.4  -0.3     68  0.1  0.0  1.1   0.6
31   3.1   1.4   1.0   0.0     69  2.0  0.6  0.3   0.2
32   0.5   2.4   0.3  -0.4     70  1.0  2.2  2.9   0.7
33   1.5   3.1   1.5  -0.6     71  2.2  2.5  2.3   0.2
34   0.4   0.0   0.7  -0.7     72  0.6  2.0  1.5  -0.2
35   3.1   2.4   3.0   0.3     73  0.3  1.7  2.2   0.4
36   1.1   2.2   2.7  -1.0     74  0.0  2.2  1.6  -0.9
37   0.1   3.0   2.6  -0.6     75  0.3  0.4  2.6   0.2
38   1.5   1.2   0.2   0.9
;
```

The data set hbk is an artificial data set generated by Hawkins, Bradu, and Kass (1984). Both ordinary least squares (OLS) estimation and M estimation (not shown here) suggest that observations 11 to 14 are outliers. However, these four observations were generated from the underlying model, whereas observations 1 to 10 were contaminated. The reason that OLS estimation and M estimation do not pick up the contaminated observations is that they cannot distinguish good leverage points (observations 11 to 14) from bad leverage points (observations 1 to 10). In such cases, the LTS method identifies the true outliers.

The following statements invoke the ROBUSTREG procedure with the LTS estimation method:

```
proc robustreg data=hbk fwls method=lts;
   model y = x1 x2 x3 / diagnostics leverage;
   id index;
run;
```

Figure 75.12 displays the model fitting information and summary statistics for the response variable and independent covariates.

Figure 75.12 Model Fitting Information and Summary Statistics

The ROBUSTREG Procedure

Model Information

Data Set	WORK.HBK
Dependent Variable	y
Number of Independent Variables	3
Number of Observations	75
Method	LTS Estimation

Summary Statistics

Variable	Q1	Median	Q3	Mean	Standard Deviation	MAD
x1	0.8000	1.8000	3.1000	3.2067	3.6526	1.9274
x2	1.0000	2.2000	3.3000	5.5973	8.2391	1.6309
x3	0.9000	2.1000	3.0000	7.2307	11.7403	1.7791
y	-0.5000	0.1000	0.7000	1.2787	3.4928	0.8896

Figure 75.13 displays information about the LTS fit, which includes the breakdown value of the LTS estimate. The breakdown value is a measure of the proportion of contamination that an estimation method can withstand and still maintain its robustness. In this example the LTS estimate minimizes the sum of 57 smallest squares of residuals. It can still estimate the true underlying model if the remaining 18 observations are contaminated. This corresponds to the breakdown value around 0.25, which is set as the default.

Figure 75.13 LTS Profile

LTS Profile

Total Number of Observations	75
Number of Squares Minimized	57
Number of Coefficients	4
Highest Possible Breakdown Value	0.2533

Figure 75.14 displays parameter estimates for covariates and scale. Two robust estimates of the scale parameter are displayed. See the section "Final Weighted Scale Estimator" on page 6397 for how these estimates are computed. The weighted scale estimator (Wscale) is a more efficient estimator of the scale parameter.

Figure 75.14 LTS Parameter Estimates

LTS Parameter Estimates

Parameter	DF	Estimate
Intercept	1	−0.3431
x1	1	0.0901
x2	1	0.0703
x3	1	−0.0731
Scale (sLTS)	0	0.7451
Scale (Wscale)	0	0.5749

Figure 75.15 displays outlier and leverage-point diagnostics. The ID variable index is used to identify the observations. If you do not specify this ID variable, the observation number is used to identify the observations. However, the observation number depends on how the data are read. The first 10 observations are identified as outliers, and observations 11 to 14 are identified as good leverage points.

Figure 75.15 Diagnostics

Diagnostics

Obs	index	Mahalanobis Distance	Robust MCD Distance	Leverage	Standardized Robust Residual	Outlier
1	1	1.9168	29.4424	*	17.0868	*
3	2	1.8558	30.2054	*	17.8428	*
5	3	2.3137	31.8909	*	18.3063	*
7	4	2.2297	32.8621	*	16.9702	*
9	5	2.1001	32.2778	*	17.7498	*
11	6	2.1462	30.5892	*	17.5155	*
13	7	2.0105	30.6807	*	18.8801	*
15	8	1.9193	29.7994	*	18.2253	*
17	9	2.2212	31.9537	*	17.1843	*
19	10	2.3335	30.9429	*	17.8021	*
21	11	2.4465	36.6384	*	0.0406	
23	12	3.1083	37.9552	*	−0.0874	
25	13	2.6624	36.9175	*	1.0776	
27	14	6.3816	41.0914	*	−0.7875	

Figure 75.16 displays the final weighted least squares estimates. These estimates are least squares estimates computed after deleting the detected outliers.

Figure 75.16 Final Weighted LS Estimates

Parameter Estimates for Final Weighted Least Squares Fit

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	−0.1805	0.1044	−0.3852	0.0242	2.99	0.0840
x1	1	0.0814	0.0667	−0.0493	0.2120	1.49	0.2222
x2	1	0.0399	0.0405	−0.0394	0.1192	0.97	0.3242
x3	1	−0.0517	0.0354	−0.1210	0.0177	2.13	0.1441
Scale	0	0.5572					

Syntax: ROBUSTREG Procedure

The following statements are available in PROC ROBUSTREG:

PROC ROBUSTREG < *options* > **;**
BY *variables* **;**
CLASS *variables* **;**
EFFECT *name=effect-type (variables* < */options* > *)* **;**
ID *variables* **;**
MODEL *response=* < *effects* > < */options* > **;**
OUTPUT < *OUT=SAS-data-set* > < *options* > **;**
PERFORMANCE < *options* > **;**
TEST *effects* **;**
WEIGHT *variable* **;**

The PROC ROBUSTREG statement invokes the procedure. The METHOD= option in the PROC ROBUSTREG statement selects one of the four estimation methods, M, LTS, S, and MM. By default, Huber M estimation is used. The MODEL statement is required and specifies the variables used in the regression. Main effects and interaction terms can be specified in the MODEL statement, as in the GLM procedure (Chapter 39, "The GLM Procedure.") The CLASS statement specifies which explanatory variables are treated as categorical. The ID statement names variables to identify observations in the outlier diagnostics tables. The WEIGHT statement identifies a variable in the input data set whose values are used to weight the observations. The OUTPUT statement creates an output data set that contains final weights, predicted values, and residuals. The TEST statement requests robust linear tests for the model parameters. The PERFORMANCE statement tunes the performance of the procedure by using single or multiple processors available on the hardware. In one invocation of PROC ROBUSTREG, multiple OUTPUT and TEST statements are allowed.

PROC ROBUSTREG Statement

PROC ROBUSTREG < *options* > **;**

The PROC ROBUSTREG statement invokes the procedure. You can specify the following options in the PROC ROBUSTREG statement.

COVOUT
saves the estimated covariance matrix in the OUTEST= data set. This option is not supported for LTS estimation.

DATA=*SAS-data-set*
specifies the input SAS data set used by PROC ROBUSTREG. By default, the most recently created SAS data set is used.

FWLS

requests that final weighted least squares estimates be computed. These estimates are equivalent to the least squares estimates after the detected outliers are deleted.

INEST=*SAS-data-set*

specifies an input SAS data set that contains initial estimates for all the parameters in the model. See the section "INEST= Data Set" on page 6410 for a detailed description of the contents of the INEST= data set.

ITPRINT

displays the iteration history for the iteratively reweighted least squares algorithm used by M and MM estimation. You can also use this option in the MODEL statement.

NAMELEN=*n*

specifies the length of effect names in tables and output data sets to be n characters, where n is a value between 20 and 200. The default length is 20 characters.

ORDER=DATA | FORMATTED | FREQ | INTERNAL

specifies the order in which to sort the levels of the classification variables (which are specified in the CLASS statement). This option applies to the levels for all classification variables, except when you use the (default) ORDER=FORMATTED option with numeric classification variables that have no explicit format. With this option, the levels of such variables are ordered by their internal value.

The ORDER= option can take the following values:

Value of ORDER=	Levels Sorted By
DATA	Order of appearance in the input data set
FORMATTED	External formatted value, except for numeric variables with no explicit format, which are sorted by their unformatted (internal) value
FREQ	Descending frequency count; levels with the most observations come first in the order
INTERNAL	Unformatted value

By default, ORDER=FORMATTED. For FORMATTED and INTERNAL, the sort order is machine-dependent. For more information about sorting order, see the chapter on the SORT procedure in the *Base SAS Procedures Guide* and the discussion of BY-group processing in *SAS Language Reference: Concepts*.

OUTEST=*SAS-data-set*

specifies an output SAS data set that contains the parameter estimates, and, if the COVOUT option is specified, the estimated covariance matrix. See the section "OUTEST= Data Set" on page 6411 for a detailed description of the contents of the OUTEST= data set.

PLOT | PLOTS <(*global-plot-options*)> <=*plot-request*>

PLOT | PLOTS<(*global-plot-options*)> <=(*plot-request* <...*plot-request*>)>

specifies options that control details of the plots. If you have enabled ODS GRAPHICS but do not specify the PLOTS= option, then PROC ROBUSTREG produces the robust fit plot by default when the model includes a single continuous independent variable.

The *global-plot-options* apply to all plots generated by the ROBUSTREG procedure. The following *global-plot-option* is available:

ONLY

suppresses the default robust fit plot. Only plots specifically requested are displayed.

You can specify more than one *plot-request* within the parentheses after PLOTS=. For a single plot request, you can omit the parentheses. The following *plot-requests* are available.

ALL

creates all appropriate plots.

DDPLOT<(LABEL=ALL | LEVERAGE | NONE | OUTLIER)>

creates a plot of robust distance against Mahalanobis distance. See the section "Leverage Point and Outlier Detection" on page 6409 for details about robust distance. The LABEL= option specifies how the points on this plot are to be labeled, as summarized by the following table.

Table 75.1 Options for Label

Value of LABEL=	Label Method
ALL	Label all points
LEVERAGE	Label leverage points
NONE	No labels
OUTLIERS	Label outliers

By default, the ROBUSTREG procedure labels both outliers and leverage points.

If you specify ID variables in the ID statement, the values of the first ID variable are used as labels; otherwise, observation numbers are used as labels.

FITPLOT<(NOLIMITS)>

creates a plot of robust fit against the single independent continuous variable specified in the model. You can request this plot when only a single independent continuous variable is specified in the model. Confidence limits are added on the plot by default. The NOLIMITS option suppresses these limits.

HISTOGRAM

creates a histogram for the standardized robust residuals. The histogram is superimposed with a normal density curve and a kernel density curve.

NONE

suppresses all plots.

QQPLOT
creates the normal quantile-quantile plot for the standardized robust residuals.

RDPLOT<(LABEL=ALL | LEVERAGE | NONE | OUTLIER)>
creates the plot of standardized robust residual against robust distance. See the section "Leverage Point and Outlier Detection" on page 6409 for details about robust distance. The LABEL= option specifies a label method for points on this plot. These label methods are described in Table 75.1.

If you specify ID variables in the ID statement, the values of the first ID variable are used as labels; otherwise, observation numbers are used as labels.

SEED=*number*
specifies the seed for the random number generator used to randomly select the subgroups and subsets for LTS and S estimation. By default or if you specify zero, the ROBUSTREG procedure generates a random seed.

METHOD=*method type* **<(** *options* **)>**
specifies the estimation method and some additional *options* for the estimation method. PROC ROBUSTREG provides four estimation methods: M estimation, LTS estimation, S estimation, and MM estimation. The default method is M estimation.

NOTE: Since the LTS and S methods use subsampling algorithms, these methods are not suitable in an analysis with variables that have only a few unequal values or a few unequal values within one BY group. For example, indicator variables that correspond to a classification variable often fall into this type. The same issue also applies to the initial LTS and S estimates in the MM method. In case of a model that includes classification independent variables or continuous independent variables with a few unequal values, the M method is recommended.

Options with METHOD=M

With METHOD=M, you can specify the following additional *options*:

ASYMPCOV=H1 | H2 | H3
specifies the type of asymptotic covariance computed for the M estimate. The three types are described in the section "Asymptotic Covariance and Confidence Intervals" on page 6393. By default, ASYMPCOV= H1.

CONVERGENCE=*criterion* **<(EPS=***value***)>**
specifies a convergence criterion for the M estimate. The three criteria listed in the following table are available.

Table 75.2 Options to Specify Convergence Criteria

Type	Option
Coefficient	CONVERGENCE=COEF
Residual	CONVERGENCE=RESID
Weight	CONVERGENCE=WEIGHT

By default, CONVERGENCE = COEF. You can specify the precision of the convergence criterion with the EPS= option. By default, EPS=1.E−8.

MAXITER=*n*

sets the maximum number of iterations during the parameter estimation. By default, MAXITER=1,000.

SCALE=*scale type* | *value*

specifies the scale parameter or a method for estimating the scale parameter. These methods and options are summarized in the following table.

Table 75.3 Options to Specify Scale

Scale	Option	Default d
Fixed constant	SCALE=*value*	
Huber estimate	SCALE=HUBER<(D=d)>	2.5
Median estimate	SCALE=MED	
Tukey estimate	SCALE=TUKEY<(D=d)>	2.5

By default, SCALE = MED.

WF | WEIGHTFUNCTION=*function type*

specifies the weight function used for the M estimate. The ROBUSTREG procedure provides 10 weight functions, which are listed in the following table. You can specify the parameters in these functions with the A=, B=, and C= options. These functions are described in the section "M Estimation" on page 6388. The default weight function is bisquare.

Table 75.4 Options to Specify Weight Functions

Weight Function	Option	Default a, b, c
Andrews	WF=ANDREWS<(C=c)>	1.339
Bisquare	WF=BISQUARE<(C=c)>	4.685
Cauchy	WF=CAUCHY<(C=c)>	2.385
Fair	WF=FAIR<(C=c)>	1.4
Hampel	WF=HAMPEL<(<A=a> <B=b> <C=c>)>	2, 4, 8
Huber	WF=HUBER<(C=c)>	1.345
Logistic	WF=LOGISTIC<(C=c)>	1.205
Median	WF=MEDIAN<(C=c)>	0.01
Talworth	WF=TALWORTH<(C=c)>	2.795
Welsch	WF=WELSCH<(C=c)>	2.985

Options with METHOD=LTS

With METHOD=LTS, you can specify the following additional *options*:

CSTEP=*n*
specifies the number of concentration steps (C-steps) for the LTS estimate. See the section "LTS Estimate" on page 6395 for information about how the default value is determined.

H=*n*
specifies the quantile for the LTS estimate. See the section "LTS Estimate" on page 6395 for information about how the default value is determined.

IADJUST=ALL | NONE
requests (IADJUST=ALL) or suppresses (IADJUST=NONE) the intercept adjustment for all estimates in the LTS algorithm. By default, the intercept adjustment is used for data sets with fewer than 10,000 observations. See the section "Algorithm" on page 6395 for details.

NBEST=*n*
specifies the number of best solutions kept for each subgroup during the computation of the LTS estimate. The default number is 10, which is the maximum number allowed.

NREP=*n*
specifies the number of times to repeat least squares fit in subgroups during the computation of the LTS estimate. See the section "LTS Estimate" on page 6395 for information about how the default number is determined.

SUBANALYSIS
requests a display of the subgrouping information and parameter estimates within subgroups. This option generates the ODS tables shown in Table 75.5.

Table 75.5 ODS Tables Available with SUBANALYSIS Option

ODS Table Name	Description
BestEstimates	Best final estimates for LTS
BestSubEstimates	Best estimates for each subgroup
CStep	C-step information for LTS
Groups	Grouping information for LTS

SUBGROUPSIZE=*n*
specifies the data set size of the subgroups in the computation of the LTS estimate. The default number is 300.

Options with METHOD=S

With METHOD=S, you can specify the following additional *options*:

ASYMPCOV=H1 | H2 | H3 | H4
specifies the type of asymptotic covariance computed for the S estimate. The four types are described in the section "Asymptotic Covariance and Confidence Intervals" on page 6400. By default, ASYMPCOV= H4.

CHIF= TUKEY | YOHAI
specifies the χ function for the S estimate. PROC ROBUSTREG provides two χ functions, Tukey's bisquare function and Yohai's optimal function, which you can request with CHIF=TUKEY and CHIF=YOHAI, respectively. The default is Tukey's bisquare function.

EFF=*value*
specifies the efficiency (as a fraction) for the S estimate. The parameter k_0 in the χ function is determined by this efficiency. The default efficiency is determined such that the consistent S estimate has the breakdown value of 25%. This option is overwritten by the K0= option if both of them are used.

K0=*value*
specifies the $k0$ parameter in the χ function of the S estimate. For CHIF=TUKEY, the default is 1.548. For CHIF=YOHAI, the default is 0.66. These default values correspond to a 50% breakdown value of the consistent S estimate.

MAXITER=*n*
sets the maximum number of iterations for computing the scale parameter of the S estimate. By default, MAXITER=1000.

NREP=*n*
specifies the number of repeats of subsampling in the computation of the S estimate. See the section "Algorithm" on page 6398 for information about how the default number of repeats is determined.

NOREFINE
suppresses the refinement for the S estimate. See the section "Algorithm" on page 6398 for details.

SUBSETSIZE=*n*
specifies the size of the subset for the S estimate. See the section "Algorithm" on page 6398 for information about how the default value is determined.

TOLERANCE=*value*
specifies the tolerance for the S estimate of the scale. The default value is 0.001.

Options with METHOD=MM

With METHOD=MM, you can specify the following additional *options*:

ASYMPCOV=H1 | H2 | H3 | H4
specifies the type of asymptotic covariance computed for the MM estimate. The four types are described in the section "Details: ROBUSTREG Procedure" on page 6387. By default, ASYMPCOV= H4.

BIASTEST< (ALPHA=*number***) >**
requests the bias test for the final MM estimate. See the section "Bias Test" on page 6402 for details about this test.

CHIF= TUKEY | YOHAI

selects the χ function for the MM estimate. PROC ROBUSTREG provides two χ functions: Tukey's bisquare function and Yohai's optimal function, which you can request with CHIF=TUKEY and CHIF=YOHAI, respectively. The default is Tukey's bisquare function. This χ function is also used by the initial S estimate if you specify the INITEST=S option.

CONVERGENCE=*criterion***< (EPS=***number***) >**

specifies a convergence criterion for the MM estimate. The three criteria listed in Table 75.6 are available.

Table 75.6 Options to Specify Convergence Criteria

Type	Option
Coefficient	CONVERGENCE=COEF
Residual	CONVERGENCE=RESID
Weight	CONVERGENCE=WEIGHT

By default, CONVERGENCE = COEF. You can specify the precision of the convergence criterion with the EPS= option. By default, EPS=1.E−8.

EFF=*value*

specifies the efficiency (as a fraction) for the MM estimate. The parameter k_1 in the χ function is determined by this efficiency. The default efficiency is set to 0.85, which corresponds to $k_1 = 3.440$ for CHIF=TUKEY or $k_1 = 0.868$ for CHIF=YOHAI.

INITH=*h*

specifies the integer h for the initial LTS estimate used by the MM estimator. See the section "Algorithm" on page 6401 for how to specify h and how the default is determined.

INITEST=LTS | S

specifies the initial estimator for the MM estimator. By default, the LTS estimator with its default settings is used as the initial estimator for the MM estimator.

K0=*number*

specifies the parameter k_0 in the χ function for the MM estimate. For CHIF=TUKEY, the default is $k_0 = 2.9366$. For CHIF=YOHAI, the default is $k_0 = 0.7405$. These default values correspond to the 25% breakdown value of the MM estimator.

MAXITER=*n*

sets the maximum number of iterations during the parameter estimation. By default, MAXITER=1,000.

BY Statement

BY *variables* **;**

You can specify a BY statement with PROC ROBUSTREG to obtain separate analyses on observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the ROBUSTREG procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

For more information about BY-group processing, see the discussion in *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the discussion in the *Base SAS Procedures Guide*.

CLASS Statement

CLASS *variables* < / **TRUNCATE** > **;**

The CLASS statement names the classification variables to be used in the model. Typical classification variables are Treatment, Sex, Race, Group, and Replication. If you use the CLASS statement, it must appear before the MODEL statement.

Classification variables can be either character or numeric. By default, class levels are determined from the entire set of formatted values of the CLASS variables.

NOTE: Prior to SAS 9, class levels were determined by using no more than the first 16 characters of the formatted values. To revert to this previous behavior, you can use the TRUNCATE option in the CLASS statement.

In any case, you can use formats to group values into levels. See the discussion of the FORMAT procedure in the *Base SAS Procedures Guide* and the discussions of the FORMAT statement and SAS formats in *SAS Language Reference: Dictionary*. You can adjust the order of CLASS variable levels with the ORDER= option in the PROC ROBUSTREG statement. You can specify the following option in the CLASS statement after a slash (/):

TRUNCATE
specifies that class levels should be determined by using only up to the first 16 characters of the formatted values of CLASS variables. When formatted values are longer than 16 characters, you can use this option to revert to the levels as determined in releases prior to SAS 9.

EFFECT Statement (Experimental)

EFFECT *name = effect-type (variables < / options >)* **;**

The EFFECT statement enables you to construct special collections of columns for design matrices. These collections are referred to as *constructed effects* to distinguish them from the usual model effects formed from continuous or classification variables, as discussed in the section "GLM Parameterization of Classification Variables and Effects" on page 410 of Chapter 19, "Shared Concepts and Topics."

The following *effect-types* are available:

COLLECTION	is a collection effect that defines one or more variables as a single effect with multiple degrees of freedom. The variables in a collection are considered as a unit for estimation and inference.
LAG	is a classification effect in which the level that is used for a given period corresponds to the level in the preceding period.
MULTIMEMBER | MM	is a multimember classification effect whose levels are determined by one or more variables that appear in a CLASS statement.
POLYNOMIAL | POLY	is a multivariate polynomial effect in the specified numeric variables.
SPLINE	is a regression spline effect whose columns are univariate spline expansions of one or more variables. A spline expansion replaces the original variable with an expanded or larger set of new variables.

Table 75.7 summarizes important options for each type of EFFECT statement.

Table 75.7 Important EFFECT Statement Options

Option	Description
Options for Collection Effects	
DETAILS	Displays the constituents of the collection effect
Options for Lag Effects	
DESIGNROLE=	Names a variable that controls to which lag design an observation is assigned
DETAILS	Displays the lag design of the lag effect
NLAG=	Specifies the number of periods in the lag
PERIOD=	Names the variable that defines the period
WITHIN=	Names the variable or variables that define the group within which each period is defined

Table 75.7 *continued*

Option	Description
Options for Multimember Effects	
NOEFFECT	Specifies that observations with all missing levels for the multimember variables should have zero values in the corresponding design matrix columns
WEIGHT=	Specifies the weight variable for the contributions of each of the classification effects
Options for Polynomial Effects	
DEGREE=	Specifies the degree of the polynomial
MDEGREE=	Specifies the maximum degree of any variable in a term of the polynomial
STANDARDIZE=	Specifies centering and scaling suboptions for the variables that define the polynomial
Options for Spline Effects	
BASIS=	Specifies the type of basis (B-spline basis or truncated power function basis) for the spline expansion
DEGREE=	Specifies the degree of the spline transformation
KNOTMETHOD=	Specifies how to construct the knots for spline effects

For further details about the syntax of these *effect-types* and how columns of constructed effects are computed, see the section "EFFECT Statement (Experimental)" on page 418 of Chapter 19, "Shared Concepts and Topics."

ID Statement

ID *variables* **;**

When the diagnostics table is requested with the DIAGNOSTICS option in the MODEL statement, the variables listed in the ID statement are displayed in addition to the observation number. These variables can be used to identify each observation. If the ID statement is omitted, the observation number is used to identify the observations.

MODEL Statement

<label:> **MODEL** *response* = *< effects > < / options >* **;**

Main effects and interaction terms can be specified in the MODEL statement, as in the GLM procedure (Chapter 39, "The GLM Procedure.")

The optional *label*, which must be a valid SAS name, is used to label the model in the OUTEST data set.

You can specify the following options for the model fit.

ALPHA=*value*
specifies the significance level for the confidence intervals for regression parameters. The value must be between 0 and 1. By default, ALPHA=0.05.

CORRB
produces the estimated correlation matrix of the parameter estimates.

COVB
produces the estimated covariance matrix of the parameter estimates.

CUTOFF=*value*
specifies the multiplier of the cutoff value for outlier detection. By default, CUTOFF=3.

DIAGNOSTICS< (ALL) >
requests the outlier diagnostics. By default, only observations identified as outliers or leverage points are displayed. To request that all observations be displayed, specify the ALL option.

FAILRATIO=*value*
specifies the threshold of failure ratio for the subsampling algorithm of an LTS or S estimate. It also applies to the initial LTS or S step in an MM estimate. The threshold must be between 0 and 1. Its default value is 0.99. See the section "LTS Estimate" on page 6395 or "S Estimate" on page 6397 for details.

ITPRINT
displays the iteration history for the iteratively reweighted least squares algorithm used by M and MM estimation. You can also use this option in the PROC statement.

LEVERAGE < (*leverage-options***) >**
requests an analysis of leverage points for the covariates. The results are added to the diagnostics table, which you can request with the DIAGNOSTICS option in the MODEL statement.

The following *leverage-options* are available:

CUTOFF=*value*
specifies the leverage cutoff value for leverage-point detection. See the section "Leverage Point and Outlier Detection" on page 6409 for details. The cutoff value can also be specified with the leverage cutoff α value by using the CUTOFFALPHA= option.

CUTOFFALPHA=*value*
specifies the leverage cutoff α value for leverage-point detection. The respective leverage cutoff value equals $\sqrt{\chi^2_{p;1-\alpha}}$ (or $\sqrt{\chi^2_{q;1-\alpha}}$ if projection is applied in the generalized MCD algorithm). By default, $\alpha = 0.025$.

MCDCUTOFF | MCDCUTOFF=*value*

specifies the MCD cutoff value for the final MCD reweighting step. See the section "Mahalanobis Distance versus Robust Distance" on page 6404 and Rousseeuw and Van Driessen (1999) for details. The cutoff value can also be specified with the MCD cutoff α value by using the MCDALPHA= option.

MCDALPHA=*value*

specifies the MCD cutoff α value for the final MCD reweighting step. The respective MCD cutoff value equals $\sqrt{\chi^2_{p;1-\alpha}}$ (or $\sqrt{\chi^2_{q;1-\alpha}}$ if projection is applied in the generalized MCD algorithm). By default, $\alpha = 0.025$.

OPC | OFFPLANECOEF

requests the ODS table of the coefficients for MCD-dropped components, when projection is applied in the generalized MCD algorithm. The OFFPLANECOEF option is ignored for the regular MCD algorithm.

PCUTOFF | PROJECTIONCUTOFF=*value*

specifies the projection cutoff value to be used to judge whether an observation is on or off the low-dimensional hyperplane identified by the projected MCD algorithm. See the section "Mahalanobis Distance versus Robust Distance" on page 6404 and Rousseeuw and Van Driessen (1999) for details. The projection cutoff value can also be specified with the projection cutoff α value by using the PALPHA= option.

PALPHA | PROJECTIONALPHA=*value*

specifies the projection cutoff α value to be used to judge whether an observation is on or off the low-dimensional hyperplane identified by the generalized MCD algorithm. The respective projection cutoff value equals $\sqrt{\chi^2_{1;1-\alpha}}$. By default, $\alpha = 0.001$.

PTOL | PROJECTIONTOLERANCE=*value*

specifies the projection tolerance value for the low-dimensional structure detection. See the section "Leverage Point and Outlier Detection" on page 6409 for details.

QUANTILE=*n*

specifies the quantile to be minimized for the MCD algorithm that is used for the leverage-point analysis. By default, QUANTILE=$[(3n + p + 1)/4]$, where n is the number of observations and p is the number of independent variables excluding the intercept.

NOGOODFIT

suppresses the computation of goodness-of-fit statistics.

NOINT

specifies no-intercept regression.

SINGULAR=*value*

specifies the tolerance for testing singularity of the information matrix and the crossproducts matrix for the initial least squares estimates. Roughly, the test requires that a pivot be at least *value* times the original diagonal value. By default, SINGULAR=1.E−12.

OUTPUT Statement

OUTPUT < *OUT=SAS-data-set* > *keyword=name* < *. . . keyword=name* > **;**

The OUTPUT statement creates an output SAS data set that contains statistics calculated after fitting the model. At least one specification of the form *keyword=name* is required.

All variables in the original data set are included in the new data set, along with the variables that are created with *keyword* options in the OUTPUT statement. These new variables contain fitted values and estimated quantiles. If you want to create a permanent SAS data set, you must specify a two-level name (refer to *SAS Language Reference: Concepts* for more information about permanent SAS data sets).

The following specifications can appear in the OUTPUT statement:

OUT=*SAS-data-set* specifies the new data set. By default, the procedure uses the DATA*n* convention to name the new data set.

keyword=name specifies the statistics to include in the output data set and gives names to the new variables. Specify a keyword for each desired statistic (see the following list), an equal sign, and the variable to contain the statistic.

The keywords allowed and the statistics they represent are as follows:

LEVERAGE specifies a variable to indicate leverage points. To include this variable in the OUTPUT data set, you must specify the LEVERAGE option in the PROC ROBUSTREG statement. See the section "Leverage Point and Outlier Detection" on page 6409 for how to define LEVERAGE.

MD specifies a variable to contain the Mahalanobis distances. See the section "Robust Distance" on page 6404 for the definition of Mahalanobis distance.

OUTLIER specifies a variable to indicate outliers. See the section "Leverage Point and Outlier Detection" on page 6409 for information about how to define OUTLIER.

PMD specifies a variable to contain the projected Mahalanobis distances. See the section "Robust Distance" on page 6404 for the definition of projected Mahalanobis distance.

POD specifies a variable to contain the projected off-plane distances. See the section "Robust Distance" on page 6404 for the definition of off-plane distance.

PRD specifies a variable to contain the projected robust MCD Mahalanobis distances. See the section "Robust Distance" on page 6404 for the definition of projected robust distance.

PREDICTED | P specifies a variable to contain the estimated responses

$$\hat{y}_i = \mathbf{x}_i^T \hat{\theta}$$

RD specifies a variable to contain the robust MCD Mahalanobis distances. See the section "Robust Distance" on page 6404 for the definition of robust distance.

RESIDUAL | R specifies a variable to contain the unstandardized residuals

$$y_i - \hat{y}_i \text{ or } y_i - \mathbf{x}_i^T \hat{\theta}$$

SRESIDUAL | SR specifies a variable to contain the standardized residuals

$$\frac{y_i - \hat{y}_i}{\hat{\sigma}} \text{ or } \frac{y_i - \mathbf{x}_i^T \hat{\theta}}{\hat{\sigma}}$$

STDP specifies a variable to contain the estimates of the standard errors of the estimated mean responses

$$\mathbf{x}_i^T \Sigma \mathbf{x}_i$$

where Σ denotes the covariance matrix of the parameter estimates. You can request the ODS table of this covariance matrix by using the COVB option of the MODEL statement. The STDP= option is applied to M, S, and MM estimation, but not to LTS estimation.

WEIGHT specifies a variable to contain the computed final weights.

PERFORMANCE Statement

The PERFORMANCE statement is used to change default options that affect the performance of PROC ROBUSTREG and to request tables that show the performance options in effect and timing details. See Chen (2002) for some empirical results.

PERFORMANCE < *options* > **;**

The following options are available:

CPUCOUNT=*n* | *ACTUAL*
specifies the number of processors to use for forming crossproduct matrices. You can specify any integer in the range 1-1024 for n. CPUCOUNT=ACTUAL sets CPUCOUNT to be the number of physical processors available. Note that this can be less than the physical number of CPUs if the SAS process has been restricted by system administration tools. Setting CPUCOUNT= to a number greater than the actual number of available CPUs might result in reduced performance. This option overrides the SAS system option CPUCOUNT=. If CPUCOUNT=1, then NOTHREADS is in effect, and PROC ROBUSTREG uses singly threaded code.

DETAILS
requests the "PerfSettings" table that shows the performance settings in effect and the "Timing" table that provides a broad timing breakdown of the PROC ROBUSTREG step.

THREADS
enables multithreaded computation. This option overrides the SAS system option THREADS | NOTHREADS.

NOTHREADS
disables multithreaded computation. This option overrides the SAS system option THREADS | NOTHREADS.

TEST Statement

<*label:*> **TEST** *effects* **;**

With M estimation and MM estimation, the TEST statement provides a means of obtaining a test for the canonical linear hypothesis concerning the parameters of the tested effects

$$\theta_j = 0, \quad j = i_1, \ldots, i_q$$

where q is the total number of parameters of the tested effects.

PROC ROBUSTREG provides two kinds of robust tests: the ρ test and the R_n^2 test. They are described in the section "Details: ROBUSTREG Procedure" on page 6387. No test is available for LTS and S estimation.

The optional *label*, which must be a valid SAS name, is used to label output from the corresponding TEST statement.

WEIGHT Statement

WEIGHT *variable* **;**

The WEIGHT statement specifies a weight variable in the input data set.

If you want to use fixed weights for each observation in the input data set, place the weights in a variable in the data set and specify the name in a WEIGHT statement. The values of the WEIGHT variable can be nonintegral and are not truncated. Observations with nonpositive or missing values for the weight variable do not contribute to the fit of the model.

Details: ROBUSTREG Procedure

This section describes the statistical and computational aspects of the ROBUSTREG procedure. The following notation is used throughout this section.

Let $X = (x_{ij})$ denote an $n \times p$ matrix, $y = (y_1, \ldots, y_n)^T$ denote a given n-vector of responses, and $\theta = (\theta_1, \ldots, \theta_p)^T$ denote an unknown p-vector of parameters or coefficients whose components are to be estimated. The matrix X is called the design matrix. Consider the usual linear model

$$y = X\theta + e$$

where $e = (e_1, \ldots, e_n)^T$ is an n-vector of unknown errors. It is assumed that (for a given X) the components e_i of e are independent and identically distributed according to a distribution $L(\cdot/\sigma)$, where σ is a scale parameter (usually unknown). Often $L(\cdot) \approx \Phi(\cdot)$, the standard normal distribution function. The vector of residuals for a given value of $\hat{\theta}$ is denoted by $r = (r_1, \ldots, r_n)^T$ and the ith row of the matrix X is denoted by x_i^T.

M Estimation

M estimation in the context of regression was first introduced by Huber (1973) as a result of making the least squares approach robust. Although M estimators are not robust with respect to leverage points, they are popular in applications where leverage points are not an issue.

Instead of minimizing a sum of squares of the residuals, a Huber-type M estimator $\hat{\theta}_M$ of θ minimizes a sum of less rapidly increasing functions of the residuals:

$$Q(\theta) = \sum_{i=1}^{n} \rho(\frac{r_i}{\sigma})$$

where $r = y - X\theta$. For the ordinary least squares estimation, ρ is the square function, $\rho(z) = z^2$.

If σ is known, then by taking derivatives with respect to θ, $\hat{\theta}_M$ is also a solution of the system of p equations:

$$\sum_{i=1}^{n} \psi(\frac{r_i}{\sigma}) x_{ij} = 0, \ j = 1, \ldots, p$$

where $\psi = \frac{\partial \rho}{\partial z}$. If ρ is convex, $\hat{\theta}_M$ is the unique solution.

The ROBUSTREG procedure solves this system by using iteratively reweighted least squares (IRLS). The weight function $w(x)$ is defined as

$$w(z) = \frac{\psi(z)}{z}$$

The ROBUSTREG procedure provides 10 kinds of weight functions through the WEIGHTFUNCTION= option in the MODEL statement. Each weight function corresponds to a ρ function. See the section "Weight Functions" on page 6390 for a complete discussion. You can specify the scale parameter σ with the SCALE= option in the PROC statement.

If σ is unknown, both θ and σ are estimated by minimizing the function

$$Q(\theta, \sigma) = \sum_{i=1}^{n} [\rho(\frac{r_i}{\sigma}) + a]\sigma, \ a > 0$$

The algorithm proceeds by alternately improving $\hat{\theta}$ in a location step and $\hat{\sigma}$ in a scale step.

For the scale step, three methods are available to estimate σ, which you can select with the SCALE= option.

1. (SCALE=HUBER<(D=d)>) Compute $\hat{\sigma}$ by the iteration

$$(\hat{\sigma}^{(m+1)})^2 = \frac{1}{nh} \sum_{i=1}^{n} \chi_d(\frac{r_i}{\hat{\sigma}^{(m)}})(\hat{\sigma}^{(m)})^2$$

   where

$$\chi_d(x) = \begin{cases} x^2/2 & \text{if } |x| < d \\ d^2/2 & \text{otherwise} \end{cases}$$

   is the Huber function and $h = \frac{n-p}{n}(d^2 + (1-d^2)\Phi(d) - 0.5 - d\sqrt{2\pi}e^{-\frac{1}{2}d^2})$ is the Huber constant (Huber 1981, p. 179). You can specify d with the D= option. By default, $d = 2.5$.

2. (SCALE=TUKEY<(D=d)>) Compute $\hat{\sigma}$ by solving the supplementary equation

$$\frac{1}{n-p} \sum_{i=1}^{n} \chi_d(\frac{r_i}{\sigma}) = \beta$$

   where

$$\chi_d(x) = \begin{cases} \frac{3x^2}{d^2} - \frac{3x^4}{d^4} + \frac{x^6}{d^6} & \text{if } |x| < d \\ 1 & \text{otherwise} \end{cases}$$

   Here $\psi = \frac{1}{6}\chi_1'$ is Tukey's bisquare function, and $\beta = \int \chi_d(s) d\Phi(s)$ is the constant such that the solution $\hat{\sigma}$ is asymptotically consistent when $L(\cdot/\sigma) = \Phi(\cdot)$ (Hampel et al. 1986, p. 149). You can specify d with the D= option. By default, $d = 2.5$.

3. (SCALE=MED) Compute $\hat{\sigma}$ by the iteration

$$\hat{\sigma}^{(m+1)} = \text{median}\{|y_i - x_i^T \hat{\theta}^{(m)}|/\beta_0, i = 1, \ldots, n\}$$

   where $\beta_0 = \Phi^{-1}(.75)$ is the constant such that the solution $\hat{\sigma}$ is asymptotically consistent when $L(\cdot/\sigma) = \Phi(\cdot)$ (Hampel et al. 1986, p. 312).

SCALE = MED is the default.

Algorithm

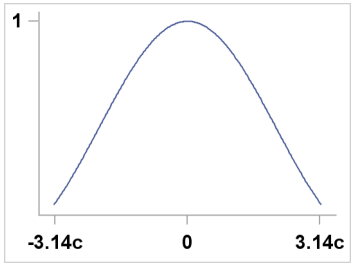
The basic algorithm for computing M estimates for regression is iteratively reweighted least squares (IRLS). As the name suggests, a weighted least squares fit is carried out inside an iteration loop. For each iteration, a set of weights for the observations is used in the least squares fit. The weights are constructed by applying a weight function to the current residuals. Initial weights are based on residuals from an initial fit. The ROBUSTREG procedure uses the unweighted least squares fit as a default initial fit. The iteration terminates when a convergence criterion is satisfied. The maximum number of iterations is set to 1,000. You can specify the weight function and the convergence criteria.

Weight Functions

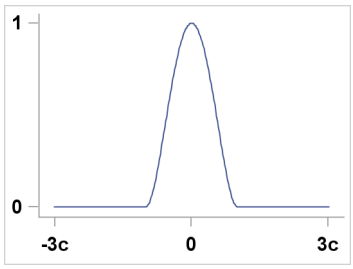
You can specify the weight function for M estimation with the WEIGHTFUNCTION= option. The ROBUSTREG procedure provides 10 weight functions. By default, the procedure uses the bisquare weight function. In most cases, M estimates are more sensitive to the parameters of these weight functions than to the type of the weight function. The median weight function is not stable and is seldom recommended in data analysis; it is included in the procedure for completeness. You can specify the parameters for these weight functions. Except for the Hampel and median weight functions, default values for these parameters are defined such that the corresponding M estimates have 95% asymptotic efficiency in the location model with the Gaussian distribution (Holland and Welsch 1977).

The following list shows the weight functions available. See Table 75.4 for the default values of the constants in these weight functions.

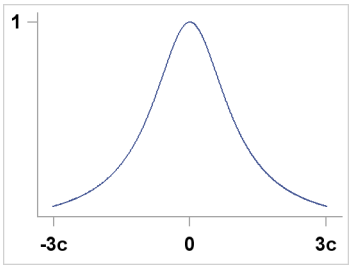
Andrews $$W(x,c) = \begin{cases} \frac{\sin(\frac{x}{c})}{\frac{x}{c}} & \text{if } |x| \leq \pi c \\ 0 & \text{otherwise} \end{cases}$$



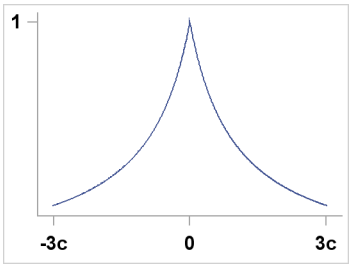
Bisquare $$W(x,c) = \begin{cases} (1-(\frac{x}{c})^2)^2 & \text{if } |x| < c \\ 0 & \text{otherwise} \end{cases}$$



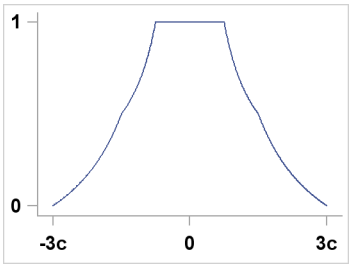
Cauchy $W(x,c) = \frac{1}{1+(\frac{|x|}{c})^2}$



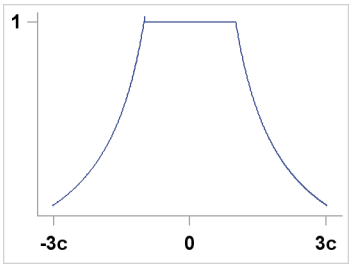
Fair $W(x,c) = \frac{1}{(1+\frac{|x|}{c})}$



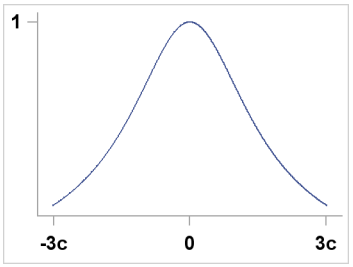
Hampel $$W(x,a,b,c) = \begin{cases} 1 & |x| < a \\ \frac{a}{|x|} & a < |x| \le b \\ \frac{a}{|x|}\frac{c-|x|}{c-b} & b < |x| \le c \\ 0 & \text{otherwise} \end{cases}$$



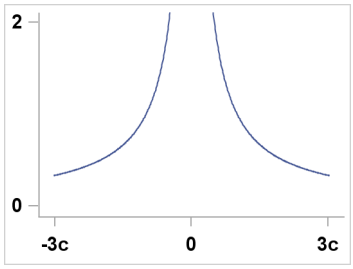
Huber $$W(x,c) = \begin{cases} 1 & \text{if } |x| < c \\ \frac{c}{|x|} & \text{otherwise} \end{cases}$$



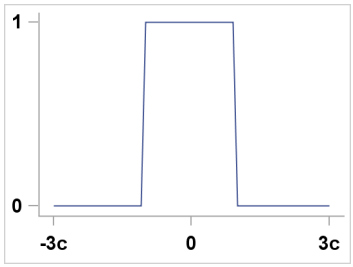
Logistic $W(x,c) = \frac{\tanh(\frac{x}{c})}{\frac{x}{c}}$



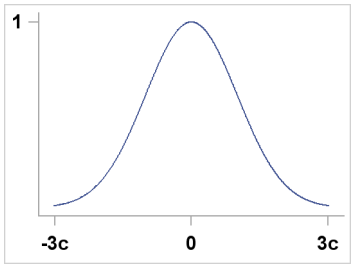
Median $$W(x,c) = \begin{cases} \frac{1}{c} & \text{if } x = 0 \\ \frac{1}{|x|} & \text{otherwise} \end{cases}$$



Talworth $$W(x,c) = \begin{cases} 1 & \text{if } |x| < c \\ 0 & \text{otherwise} \end{cases}$$



Welsch $$W(x,c) = \exp(-\tfrac{1}{2}(\tfrac{x}{c})^2)$$



Convergence Criteria

The following convergence criteria are available in PROC ROBUSTREG:

- relative change in the coefficients (CONVERGENCE= COEF)
- relative change in the scaled residuals (CONVERGENCE= RESID)
- relative change in weights (CONVERGENCE= WEIGHT)

You can specify the criteria with the CONVERGENCE= option in the PROC statement. The default is CONVERGENCE= COEF.

You can specify the precision of the convergence criterion with the EPS= suboption. The default is EPS=1.E−8.

In addition to these convergence criteria, a convergence criterion based on scale-independent measure of the gradient is always checked. See Coleman et al. (1980) for more details. A warning is issued if this criterion is not satisfied.

Asymptotic Covariance and Confidence Intervals

The following three estimators of the asymptotic covariance of the robust estimator are available in PROC ROBUSTREG:

$$\text{H1: } K^2 \frac{[1/(n-p)]\sum(\psi(r_i))^2}{[(1/n)\sum(\psi'(r_i))]^2}(X^TX)^{-1}$$

$$\text{H2: } K \frac{[1/(n-p)]\sum(\psi(r_i))^2}{[(1/n)\sum(\psi'(r_i))]}W^{-1}$$

$$\text{H3: } K^{-1}\frac{1}{(n-p)}\sum(\psi(r_i))^2 W^{-1}(X^TX)W^{-1}$$

where $K = 1 + \frac{p}{n}\frac{\text{Var}(\psi')}{(E\psi')^2}$ is a correction factor and $W_{jk} = \sum \psi'(r_i)x_{ij}x_{ik}$. Refer to Huber (1981, p. 173) for more details.

You can specify the asymptotic covariance estimate with the option ASYMPCOV= option. The ROBUSTREG procedure uses H1 as the default because of its simplicity and stability. Confidence intervals are computed from the diagonal elements of the estimated asymptotic covariance matrix.

R Square and Deviance

The robust version of R-square is defined as

$$R^2 = \frac{\sum \rho(\frac{y_i-\hat{\mu}}{\hat{s}}) - \sum \rho(\frac{y_i - x_i^T\hat{\theta}}{\hat{s}})}{\sum \rho(\frac{y_i-\hat{\mu}}{\hat{s}})}$$

and the robust deviance is defined as the optimal value of the objective function on the σ^2 scale

$$D = 2(\hat{s})^2 \sum \rho(\frac{y_i - x_i^T\hat{\theta}}{\hat{s}})$$

where $\rho' = \psi$, $\hat{\theta}$ is the M estimator of θ, $\hat{\mu}$ is the M estimator of location, and $\hat{s}$ is the M estimator of the scale parameter in the full model.

Linear Tests

Two tests are available in PROC ROBUSTREG for the canonical linear hypothesis

$$H_0: \quad \theta_j = 0, \quad j = i_1, \ldots, i_q$$

where q is the total number of parameters of the tested effects. The first test is a robust version of the F test, which is referred to as the ρ test. Denote the M estimators in the full and reduced models as $\hat{\theta}(0) \in \Omega_0$ and $\hat{\theta}(1) \in \Omega_1$, respectively. Let

$$\begin{aligned} Q_0 &= Q(\hat{\theta}(0)) = \min\{Q(\theta)|\theta \in \Omega_0\} \\ Q_1 &= Q(\hat{\theta}(1)) = \min\{Q(\theta)|\theta \in \Omega_1\} \end{aligned}$$

with

$$Q = \sum_{i=1}^{n} \rho(\frac{r_i}{\sigma})$$

The robust F test is based on the test statistic

$$S_n^2 = \frac{2}{q}[Q_1 - Q_0]$$

Asymptotically $S_n^2 \sim \lambda\chi_q^2$ under H_0, where the standardization factor is $\lambda = \int \psi^2(s)d\Phi(s) / \int \psi'(s)\ d\Phi(s)$ and Φ is the cumulative distribution function of the standard normal distribution. Large values of S_n^2 are significant. This test is a special case of the general τ test of Hampel et al. (1986, Section 7.2).

The second test is a robust version of the Wald test, which is referred to as R_n^2 test. The test uses a test statistic

$$R_n^2 = n(\hat{\theta}_{i_1}, \ldots, \hat{\theta}_{i_q}) H_{22}^{-1} (\hat{\theta}_{i_1}, \ldots, \hat{\theta}_{i_q})^T$$

where $\frac{1}{n}H_{22}$ is the $q \times q$ block (corresponding to $\theta_{i_1}, \ldots, \theta_{i_q}$) of the asymptotic covariance matrix of the M estimate $\hat{\theta}_M$ of θ in a p-parameter linear model.

Under H_0, the statistic R_n^2 has an asymptotic χ^2 distribution with q degrees of freedom. Large values of R_n^2 are significant. Refer to Hampel et al. (1986, Chapter 7) for more details.

Model Selection

When M estimation is used, two criteria are available in PROC ROBUSTREG for model selection. The first criterion is a counterpart of the Akaike (1974) information criterion for robust regression (AICR); it is defined as

$$\text{AICR} = 2\sum_{i=1}^{n} \rho(r_{i:p}) + \alpha p$$

where $r_{i:p} = (y_i - x_i^T\hat{\theta})/\hat{\sigma}$, $\hat{\sigma}$ is a robust estimate of σ and $\hat{\theta}$ is the M estimator with p-dimensional design matrix.

As with AIC, α is the weight of the penalty for dimensions. The ROBUSTREG procedure uses $\alpha = 2E\psi^2/E\psi'$ (Ronchetti 1985) and estimates it by using the final robust residuals.

The second criterion is a robust version of the Schwarz information criteria (BICR); it is defined as

$$\text{BICR} = 2\sum_{i=1}^{n} \rho(r_{i:p}) + p\log(n)$$

High-Breakdown-Value Estimation

The *breakdown value* of an estimator is defined as the smallest fraction of contamination that can cause the estimator to take on values arbitrarily far from its value on the uncontamined data.

The breakdown value of an estimator can be used as a measure of the robustness of the estimator. Rousseeuw and Leroy (1987) and others introduced the following high-breakdown-value estimators for linear regression.

LTS Estimate

The least trimmed squares (LTS) estimate proposed by Rousseeuw (1984) is defined as the p-vector

$$\hat{\theta}_{LTS} = \arg\min_{\theta} Q_{LTS}(\theta) \text{ with } Q_{LTS}(\theta) = \sum_{i=1}^{h} r^2_{(i)}$$

where $r^2_{(1)} \leq r^2_{(2)} \leq ... \leq r^2_{(n)}$ are the ordered squared residuals $r^2_i = (y_i - x_i^T\theta)^2$, $i = 1, \ldots, n$, and h is defined in the range $\frac{n}{2} + 1 \leq h \leq \frac{3n+p+1}{4}$.

You can specify the parameter h with the H= option in the PROC statement. By default, $h = [\frac{3n+p+1}{4}]$. The breakdown value is $\frac{n-h}{n}$ for the LTS estimate.

The ROBUSTREG procedure computes LTS estimates by using the FAST-LTS algorithm of Rousseeuw and Van Driessen (2000). The estimates are often used to detect outliers in the data, which are then downweighted in the resulting weighted LS regression.

Algorithm

Least trimmed squares (LTS) regression is based on the subset of h observations (out of a total of n observations) whose least squares fit possesses the smallest sum of squared residuals. The coverage h can be set between $\frac{n}{2}$ and n. The LTS method was proposed by Rousseeuw (1984, p. 876) as a highly robust regression estimator with breakdown value $\frac{n-h}{n}$. The ROBUSTREG procedure uses the FAST-LTS algorithm given by Rousseeuw and Van Driessen (2000). The intercept adjustment technique is also used in this implementation. However, because this adjustment is expensive to compute, it is optional. You can use the IADJUST option in the PROC statement to request or suppress the intercept adjustment. By default, PROC ROBUSTREG does intercept adjustment for data sets with fewer than 10,000 observations. The steps of the algorithm are described briefly as follows. Refer to Rousseeuw and Van Driessen (2000) for details.

1. The default h is $[\frac{3n+p+1}{4}]$, where p is the number of independent variables. You can specify any integer h with $[\frac{n}{2}] + 1 \leq h \leq [\frac{3n+p+1}{4}]$ with the H= option in the MODEL statement. The breakdown value for LTS, $\frac{n-h}{n}$, is reported. The default h is a good compromise between breakdown value and statistical efficiency.

2. If $p = 1$ (single regressor), the procedure uses the exact algorithm of Rousseeuw and Leroy (1987, p. 172).

3. If $p \geq 2$, the procedure uses the following algorithm. If $n < 2ssubs$, where $ssubs$ is the size of the subgroups (you can specify $ssubs$ by using the SUBGROUPSIZE= option in the PROC statement; by default, $ssubs = 300$), draw a random p-subset and compute the regression coefficients by using these p points (if the regression is degenerate, draw another p-subset). Compute the absolute residuals for all observations in the data set, and select the first h

points with smallest absolute residuals. From this selected h-subset, carry out *nsteps* C-steps (concentration steps; see Rousseeuw and Van Driessen (2000) for details). You can specify *nsteps* with the CSTEP= option in the PROC statement; by default, $nsteps = 2$. Redraw p-subsets and repeat the preceding computing procedure *nrep* times, and then find the *nbsol* (at most) solutions with the lowest sums of h squared residuals. You can specify *nrep* with the NREP= option in the PROC statement. By default, NREP=min{500, $\binom{n}{p}$}. For small n and p, all $\binom{n}{p}$ subsets are used and the NREP= option is ignored (Rousseeuw and Hubert 1996). You can specify *nbsol* with the NBEST= option in the PROC statement. By default, NBEST=10. For each of these *nbsol* best solutions, take C-steps until convergence and find the best final solution.

4. If $n \geq 5ssubs$, construct five disjoint random subgroups with size *ssubs*. If $2ssubs < n < 5ssubs$, the data are split into at most four subgroups with *ssubs* or more observations in each subgroup, so that each observation belongs to a subgroup and the subgroups have roughly the same size. Let *nsubs* denote the number of subgroups. Inside each subgroup, repeat the procedure in step 3 $[\frac{nrep}{nsubs}]$ times and keep the *nbsol* best solutions. Pool the subgroups, yielding the merged set of size n_{merged}. In the merged set, for each of the $nsubs \times nbsol$ best solutions, carry out *nsteps* C-steps by using n_{merged} and $h_{merged} = [n_{merged}\frac{h}{n}]$ and keep the *nbsol* best solutions. In the full data set, for each of these *nbsol* best solutions, take C-steps by using n and h until convergence and find the best final solution.

NOTE: At step 3 in the algorithm, a randomly selected p-subset might be degenerate (that is, its design matrix might be singular). If the total number of p-subsets from any subgroup is more than 4,000 and the ratio of degenerate p-subsets is more than the threshold specified in FAILRATIO option, the algorithm is terminated with a error message.

R-Square

The robust version of R-square for the LTS estimate is defined as

$$R^2_{LTS} = 1 - \frac{s^2_{LTS}(X, y)}{s^2_{LTS}(\mathbf{1}, y)}$$

for models with the intercept term and as

$$R^2_{LTS} = 1 - \frac{s^2_{LTS}(X, y)}{s^2_{LTS}(\mathbf{0}, y)}$$

for models without the intercept term, where

$$s_{LTS}(X, y) = d_{h,n}\sqrt{\frac{1}{h}\sum_{i=1}^{h} r^2_{(i)}}$$

Note that s_{LTS} is a preliminary estimate of the parameter σ in the distribution function $L(\cdot/\sigma)$.

Here $d_{h,n}$ is chosen to make s_{LTS} consistent, assuming a Gaussian model. Specifically,

$$\begin{aligned} d_{h,n} &= 1/\sqrt{1 - \frac{2n}{hc_{h,n}}\phi(1/c_{h,n})} \\ c_{h,n} &= 1/\Phi^{-1}(\frac{h+n}{2n}) \end{aligned}$$

with Φ and ϕ being the distribution function and the density function of the standard normal distribution, respectively.

Final Weighted Scale Estimator

The ROBUSTREG procedure displays two scale estimators, s_{LTS} and Wscale. The estimator Wscale is a more efficient scale estimator based on the preliminary estimate s_{LTS}; it is defined as

$$\text{Wscale} = \sqrt{\frac{\sum_i w_i {r_i}^2}{\sum_i w_i - p}}$$

where

$$w_i = \begin{cases} 0 & \text{if } |r_i|/s_{LTS} > k \\ 1 & \text{otherwise} \end{cases}$$

You can specify k with the CUTOFF= option in the MODEL statement. By default, $k = 3$.

S Estimate

The S estimate proposed by Rousseeuw and Yohai (1984) is defined as the p-vector

$$\hat{\theta}_S = \arg\min_{\theta} S(\theta)$$

where the dispersion $S(\theta)$ is the solution of

$$\frac{1}{n-p}\sum_{i=1}^{n}\chi(\frac{y_i - x_i^T\theta}{S}) = \beta$$

Here β is set to $\int \chi(s)d\Phi(s)$ such that $\hat{\theta}_S$ and $S(\hat{\theta}_S)$ are asymptotically consistent estimates of θ and σ for the Gaussian regression model. The breakdown value of the S estimate is

$$\frac{\beta}{\max_s \chi(s)}$$

The ROBUSTREG procedure provides two choices for χ: Tukey's bisquare function and Yohai's optimal function.

Tukey's bisquare function, which you can specify with the option CHIF=TUKEY, is

$$\chi_{k_0}(s) = \begin{cases} 3(\frac{s}{k_0})^2 - 3(\frac{s}{k_0})^4 + (\frac{s}{k_0})^6, & \text{if } |s| \le k_0 \\ 1 & \text{otherwise} \end{cases}$$

The constant k_0 controls the breakdown value and efficiency of the S estimate. If you specify the efficiency by using the EFF= option, you can determine the corresponding k_0. The default k_0 is 2.9366 such that the breakdown value of the S estimate is 0.25 with a corresponding asymptotic efficiency for the Gaussian model of 75.9%.

The Yohai function, which you can specify with the option CHIF=YOHAI, is

$$\chi_{k_0}(s) = \begin{cases} \frac{s^2}{2} & \text{if } |s| \le 2k_0 \\ k_0^2[b_0 + b_1(\frac{s}{k_0})^2 + b_2(\frac{s}{k_0})^4 & \\ \quad +b_3(\frac{s}{k_0})^6 + b_4(\frac{s}{k_0})^8] & \text{if } 2k_0 < |s| \le 3k_0 \\ 3.25k_0^2 & \text{if } |s| > 3k_0 \end{cases}$$

where $b_0 = 1.792$, $b_1 = -0.972$, $b_2 = 0.432$, $b_3 = -0.052$, and $b_4 = 0.002$. If you specify the efficiency by using the EFF= option, you can determine the corresponding k_0. By default, k_0 is set to 0.7405 such that the breakdown value of the S estimate is 0.25 with a corresponding asymptotic efficiency for the Gaussian model of 72.7%.

Algorithm

The ROBUSTREG procedure implements the algorithm by Marazzi (1993) for the S estimate, which is a refined version of the algorithm proposed by Ruppert (1992). The refined algorithm is briefly described as follows.

Initialize iter = 1.

1. Draw a random q-subset of the total n observations and compute the regression coefficients by using these q observations (if the regression is degenerate, draw another q-subset), where $q \ge p$ can be specified with the SUBSIZE= option. By default, $q = p$.

2. Compute the residuals: $r_i = y_i - \sum_{j=1}^{p} x_{ij}\theta_j$ for $i = 1, \ldots, n$. If iter $= 1$, set $s^* = 2\text{median}\{|r_i|, i = 1, \ldots, n\}$; if $s^* = 0$, set $s^* = \min\{|r_i|, i = 1, \ldots, n\}$;
   else while $\sum_{i=1}^{n} \chi(r_i/s^*) > (n-p)\beta$, set $s^* = 1.5s^*$; go to step 3.
   If iter > 1 and $\sum_{i=1}^{n} \chi(r_i/s^*) <= (n-p)\beta$, go to step 3; otherwise, go to step 5.

3. Solve for s the equation

$$\frac{1}{n-p}\sum_{i=1}^{n} \chi(r_i/s) = \beta$$

   using an iterative algorithm.

4. If iter > 1 and $s > s^*$, go to step 5. Otherwise, set $s^* = s$ and $\theta^* = \theta$. If $s^* <$ TOLS, return s^* and θ^*; otherwise, go to step 5.

5. If iter $<$ NREP, set iter = iter $+ 1$ and return to step 1; otherwise, return s^* and θ^*.

The ROBUSTREG procedure does the following refinement step by default. You can request that this refinement not be done by using the NOREFINE option in the PROC statement.

6. Let $\psi = \chi'$. Using the values s^* and θ^* from the previous steps, compute M estimates θ_M and σ_M of θ and σ with the setup for M estimation that is described in the section "M Estimation" on page 6388. If $\sigma_M > s^*$, give a warning and return s^* and θ^*; otherwise, return σ_M and θ_M.

You can specify TOLS with the TOLERANCE= option; by default, TOLERANCE=0.001. Alternately, you can specify NREP with the NREP= option. You can also use the options NREP=NREP0 or NREP=NREP1 to determine NREP according to the following table. NREP=NREP0 is set as the default.

Table 75.9 Default NREP

P	NREP0	NREP1
1	150	500
2	300	1000
3	400	1500
4	500	2000
5	600	2500
6	700	3000
7	850	3000
8	1250	3000
9	1500	3000
>9	1500	3000

NOTE: At step 1 in the algorithm, a randomly selected q-subset might be degenerate. If the total number of q-subsets from any subgroup is more than 4,000 and the ratio of degenerate q-subsets is more than the threshold specified in FAILRATIO option, the algorithm is terminated with a error message.

R-Square and Deviance

The robust version of R-square for the S estimate is defined as

$$R_S^2 = 1 - \frac{(n-p)S_p^2}{(n-1)S_\mu^2}$$

for the model with the intercept term and

$$R_S^2 = 1 - \frac{(n-p)S_p^2}{nS_0^2}$$

for the model without the intercept term, where S_p is the S estimate of the scale in the full model, S_μ is the S estimate of the scale in the regression model with only the intercept term, and S_0 is the S estimate of the scale without any regressor. The deviance D is defined as the optimal value of the objective function on the σ^2 scale:

$$D = S_p^2$$

Asymptotic Covariance and Confidence Intervals

Since the S estimate satisfies the first-order necessary conditions as the M estimate, it has the same asymptotic covariance as that of the M estimate. All three estimators of the asymptotic covariance for the M estimate in the section "Asymptotic Covariance and Confidence Intervals" on page 6393 can be used for the S estimate. Besides, the weighted covariance estimator H4 described in the section "Asymptotic Covariance and Confidence Intervals" on page 6403 is also available and is set as the default. Confidence intervals for estimated parameters are computed from the diagonal elements of the estimated asymptotic covariance matrix.

MM Estimation

MM estimation is a combination of high-breakdown-value estimation and efficient estimation, which was introduced by Yohai (1987). It has the following three steps:

1. Compute an initial (consistent) high-breakdown-value estimate $\hat{\theta}'$. The ROBUSTREG procedure provides two kinds of estimates as the initial estimate: the LTS estimate and the S estimate. By default, the LTS estimate is used because of its speed and high breakdown value. The breakdown value of the final MM estimate is decided by the breakdown value of the initial LTS estimate and the constant k_0 in the χ function. To use the S estimate as the initial estimate, you specify the INITEST=S option in the PROC statement. In this case, the breakdown value of the final MM estimate is decided only by the constant k_0. Instead of computing the LTS estimate or the S estimate as the initial estimate, you can also specify the initial estimate explicitly by using the INEST= option in the PROC statement. See the section "INEST= Data Set" on page 6410 for details.

2. Find $\hat{\sigma}'$ such that

$$\frac{1}{n-p}\sum_{i=1}^{n}\chi\left(\frac{y_i - x_i^T\hat{\theta}'}{\hat{\sigma}'}\right) = \beta$$

   where $\beta = \int \chi(s)d\Phi(s)$.

   The ROBUSTREG procedure provides two choices for χ: Tukey's bisquare function and Yohai's optimal function.

   Tukey's bisquare function, which you can specify with the option CHIF=TUKEY, is

$$\chi_{k_0}(s) = \begin{cases} 3(\frac{s}{k_0})^2 - 3(\frac{s}{k_0})^4 + (\frac{s}{k_0})^6 & \text{if } |s| \le k_0 \\ 1 & \text{otherwise} \end{cases}$$

   where k_0 can be specified with the K0= option. The default k_0 is 2.9366 such that the asymptotically consistent scale estimate $\hat{\sigma}'$ has the breakdown value of 25%.

   Yohai's optimal function, which you can specify with the option CHIF=YOHAI, is

$$\chi_{k_0}(s) = \begin{cases} \frac{s^2}{2} & \text{if } |s| \le 2k_0 \\ k_0^2[b_0 + b_1(\frac{s}{k_0})^2 + b_2(\frac{s}{k_0})^4 & \\ \quad +b_3(\frac{s}{k_0})^6 + b_4(\frac{s}{k_0})^8] & \text{if } 2k_0 < |s| \le 3k_0 \\ 3.25k_0^2 & \text{if } |s| > 3k_0 \end{cases}$$

where $b_0 = 1.792$, $b_1 = -0.972$, $b_2 = 0.432$, $b_3 = -0.052$, and $b_4 = 0.002$. You can specify k_0 with the K0= option. The default k_0 is 0.7405 such that the asymptotically consistent scale estimate $\hat{\sigma}'$ has the breakdown value of 25%.

3. Find a local minimum $\hat{\theta}_{MM}$ of

$$Q_{MM} = \sum_{i=1}^{n} \rho(\frac{y_i - x_i^T \theta}{\hat{\sigma}'})$$

such that $Q_{MM}(\hat{\theta}_{MM}) \leq Q_{MM}(\hat{\theta}')$. The algorithm for M estimation is used here.

The ROBUSTREG procedure provides two choices for ρ: Tukey's bisquare function and Yohai's optimal function.

Tukey's bisquare function, which you can specify with the option CHIF=TUKEY, is

$$\rho(s) = \chi_{k_1}(s) = \begin{cases} 3(\frac{s}{k_1})^2 - 3(\frac{s}{k_1})^4 + (\frac{s}{k_1})^6 & \text{if } |s| \leq k_1 \\ 1 & \text{otherwise} \end{cases}$$

where k_1 can be specified with the K1= option. The default k_1 is 3.440 such that the MM estimate has 85% asymptotic efficiency with the Gaussian distribution.

Yohai's optimal function, which you can specify with the option CHIF=YOHAI, is

$$\rho(s) = \chi_{k_1}(s) = \begin{cases} \frac{s^2}{2} & \text{if } |s| \leq 2k_1 \\ k_1^2[b_0 + b_1(\frac{s}{k_1})^2 + b_2(\frac{s}{k_1})^4 & \\ \quad + b_3(\frac{s}{k_1})^6 + b_4(\frac{s}{k_1})^8] & \text{if } 2k_1 < |s| \leq 3k_1 \\ 3.25k_1^2 & \text{if } |s| > 3k_1 \end{cases}$$

where k_1 can be specified with the K1= option. The default k_1 is 0.868 such that the MM estimate has 85% asymptotic efficiency with the Gaussian distribution.

Algorithm

The initial LTS estimate is computed using the algorithm described in the section "LTS Estimate" on page 6395. You can control the quantile of the LTS estimate with the option INITH=h, where h is an integer between $[\frac{n}{2}] + 1$ and $[\frac{3n+p+1}{4}]$. By default, $h = [\frac{3n+p+1}{4}]$, which corresponds to a breakdown value of around 25%.

The initial S estimate is computed using the algorithm described in the section "S Estimate" on page 6397. You can control the breakdown value and efficiency of this initial S estimate by the constant k_0, which can be specified with the K0 option.

The scale parameter σ is solved by an iterative algorithm

$$(\sigma^{(m+1)})^2 = \frac{1}{(n-p)\beta} \sum_{i=1}^{n} \chi_{k_0}(\frac{r_i}{\sigma^{(m)}})(\sigma^{(m)})^2$$

where $\beta = \int \chi_{k_0}(s) d\Phi(s)$.

Once the scale parameter is computed, the iteratively reweighted least squares (IRLS) algorithm with fixed scale parameter is used to compute the final MM estimate.

Convergence Criteria

In the iterative algorithm for the scale parameter, the relative change of the scale parameter controls the convergence.

In the iteratively reweighted least squares algorithm, the same convergence criteria for the M estimate used before are used here.

Bias Test

Although the final MM estimate inherits the high-breakdown-value property, its bias due to the distortion of the outliers can be high. Yohai, Stahel, and Zamar (1991) introduced a bias test. The ROBUSTREG procedure implements this test when you specify the BIASTEST option in the PROC statement. This test is based on the initial scale estimate $\hat{\sigma}'$ and the final scale estimate $\hat{\sigma}'_1$, which is the solution of

$$\frac{1}{n-p}\sum_{i=1}^{n}\chi\left(\frac{y_i - x_i^T\hat{\theta}_{MM}}{\hat{\sigma}'_1}\right) = \beta$$

Let $\psi_{k_0}(z) = \frac{\partial\chi_{k_0}(z)}{\partial z}$ and $\psi_{k_1}(z) = \frac{\partial\chi_{k_1}(z)}{\partial z}$. Compute

$$\begin{aligned}
\tilde{r}_i &= (y_i - x_i^T\hat{\theta}')/\hat{\sigma}' \quad \text{for } i = 1,\ldots,n \\
v_0 &= \frac{(1/n)\sum\psi'_{k_0}(\tilde{r}_i)}{(\hat{\sigma}'_1/n)\sum\psi_{k_0}(\tilde{r}_i)\tilde{r}_i}
\end{aligned}$$

$$\begin{aligned}
p_i^{(0)} &= \frac{\psi_{k_0}(\tilde{r}_i)}{(1/n)\sum\psi'_{k_0}(\tilde{r}_i)} \quad \text{for } i = 1,\ldots,n \\
p_i^{(1)} &= \frac{\psi_{k_1}(\tilde{r}_i)}{(1/n)\sum\psi'_{k_1}(\tilde{r}_i)} \quad \text{for } i = 1,\ldots,n \\
d^2 &= \frac{1}{n}\sum(p_i^{(1)} - p_i^{(0)})^2
\end{aligned}$$

Let

$$T = \frac{2n(\hat{\sigma}'_1 - \hat{\sigma}')}{v_0 d^2(\hat{\sigma}')^2}$$

Standard asymptotic theory shows that T approximately follows a χ^2 distribution with p degrees of freedom. If T exceeds the α quantile χ^2_α of the χ^2 distribution with p degrees of freedom, then the ROBUSTREG procedure gives a warning and recommends that you use other methods. Otherwise, the final MM estimate and the initial scale estimate are reported. You can specify α with the ALPHA= option following the BIASTEST option. By default, ALPHA=0.99.

Asymptotic Covariance and Confidence Intervals

Since the MM estimate is computed as a M estimate with a fixed scale in the last step, the asymptotic covariance for the M estimate can be used here for the asymptotic covariance of the MM estimate. Besides the three estimators H1, H2, and H3 as described in the section "Asymptotic Covariance and Confidence Intervals" on page 6393, a weighted covariance estimator H4 is available. H4 is calculated as

$$K^2 \frac{[1/(n-p)]\sum(\psi(r_i))^2}{[(1/n)\sum(\psi'(r_i))]^2} W^{-1}$$

where $K = 1 + \frac{p}{n}\frac{\mathrm{Var}(\psi')}{(E\psi')^2}$ is the correction factor and $W_{jk} = \frac{1}{\bar{w}}\sum w_i x_{ij} x_{ik}$, $\bar{w} = \frac{1}{n}\sum w_i$.

You can specify these estimators with the option ASYMPCOV= [H1 | H2 | H3 | H4]. The ROBUSTREG procedure uses H4 as the default. Confidence intervals for estimated parameters are computed from the diagonal elements of the estimated asymptotic covariance matrix.

R Square and Deviance

The robust version of R-square for the MM estimate is defined as

$$R^2 = \frac{\sum \rho(\frac{y_i - \hat{\mu}}{\hat{s}}) - \sum \rho(\frac{y_i - x_i^T\hat{\theta}}{\hat{s}})}{\sum \rho(\frac{y_i - \hat{\mu}}{\hat{s}})}$$

and the robust deviance is defined as the optimal value of the objective function on the σ^2 scale,

$$D = 2(\hat{s})^2 \sum \rho(\frac{y_i - x_i^T\hat{\theta}}{\hat{s}})$$

where $\rho' = \psi$, $\hat{\theta}$ is the MM estimator of θ, $\hat{\mu}$ is the MM estimator of location, and $\hat{s}$ is the MM estimator of the scale parameter in the full model.

Linear Tests

For MM estimation, the same ρ test and R_n^2 test used for M estimation can be used. See the section "Linear Tests" on page 6393 for details.

Model Selection

For MM estimation, the same two model selection methods used for M estimation can be used. See the section "Model Selection" on page 6394 for details.

Robust Distance

The ROBUSTREG procedure uses the robust multivariate location and scatter estimates for leverage-point detection. The procedure computes a robust version of the Mahalanobis distance by using a generalized minimum covariance determinant (MCD) method. The original MCD method was proposed by Rousseeuw (1984).

Algorithm

PROC ROBUSTREG implements a generalized MCD algorithm based on the fast-MCD algorithm formulated by Rousseeuw and Van Driessen (1999), which is similar to the algorithm for least trimmed squares (LTS).

Mahalanobis Distance versus Robust Distance

The canonical Mahalanobis distance is defined as

$$MD(x_i) = [(x_i - \bar{x})^T \bar{C}(X)^{-1}(x_i - \bar{x})]^{1/2}$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\bar{C}(X) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^T(x_i - \bar{x})$ are the empirical multivariate location and scatter, respectively. Here $x_i = (x_{i1}, \ldots, x_{ip})^T$ excludes the intercept. The relation between the Mahalanobis distance $MD(x_i)$ and the hat matrix $H = (h_{ij}) = X(X^T X)^{-1}X^T$ is

$$h_{ii} = \frac{1}{n-1}MD_i^2 + \frac{1}{n}$$

The canonical robust distance is defined as

$$RD(x_i) = [(x_i - T(X))^T C(X)^{-1}(x_i - T(X))]^{1/2}$$

where $T(X)$ and $C(X)$ are the robust multivariate location and scatter, respectively, obtained by MCD.

To achieve robustness, the MCD algorithm estimates the covariance of a multivariate data set mainly through an MCD h-point subset of the data set. This subset has the smallest sample-covariance determinant among all the possible h-subsets. Accordingly, the breakdown value for the MCD algorithm equals $\frac{(n-h)}{n}$. This means the MCD estimate is reliable, even if up to $\frac{100(n-h)}{n}\%$ observations in the data set are contaminated.

Low-Dimensional Structure

It is possible that the original data is in p dimensional space, but the h-point subset that yields the minimum covariance determinant lies in a lower-dimensional hyperplane. Applying the canonical MCD algorithm to such a data set would result in a singular covariance problem (called exact fit in Rousseeuw and Van Driessen (1999)), so that the relevant robust distances cannot be computed. To

deal with the singularity problem and provide further leverage point analysis, PROC ROBUSTREG implements a generalized MCD algorithm. See the section "Generalized MCD Algorithm" on page 6408 for details. The algorithm distinguishes in-(hyper)plane points from off-(hyper)plane points, and performs MCD leverage point analysis in the dimension-reduced space by projecting all points onto the hyperplane.

Low-dimensional structure is often induced by classification covariates. Suppose, in a study with 25 female subjects and 5 male subjects, that *gender* is the only classification effect. If the breakdown setting is larger than $\frac{5}{(25+5)}$, the canonical MCD algorithm fails, and so does the relevant leverage point analysis. In this case, the MCD h-subset would contain only female observations and the constant *gender* in the h-subset would cause the relevant MCD estimate to be singular. The generalized MCD algorithm solves that problem by identifying all male observations as off-plane leverage points, and then carries out the leverage point analysis with all the other covariates being centered separately for female and male groups against their group means.

In general, low-dimensional structure is not necessarily due to classification covariates. Imagine that 80 children are supposed to play on a straight trail (denoted by $y = x$), but some adventurous children go off the trail. The following statements generate the children data and the relevant scatter plot.

```
data children;
  do i=1 to 80;
     off_trail=ranuni(321)>.9;
     x=rannor(111)*ranuni(321);
     trail_x=(i-40)/80*3;
     trail_y=trail_x;
     if off_trail=1 then y=x-1+rannor(321);
     else y=x;
     output;
  end;
run;

 ods graphics on;
 proc sgplot data=children;
    series   x=trail_x y=trail_y/lineattrs=(color="red" pattern=4);
    scatter  x=x y=y/group=off_trail;
    ellipse  x=x y=y/alpha=.05 lineattrs=(color="green" pattern=34);
 run;
```

Figure 75.17 Scatter Plot for Children Data

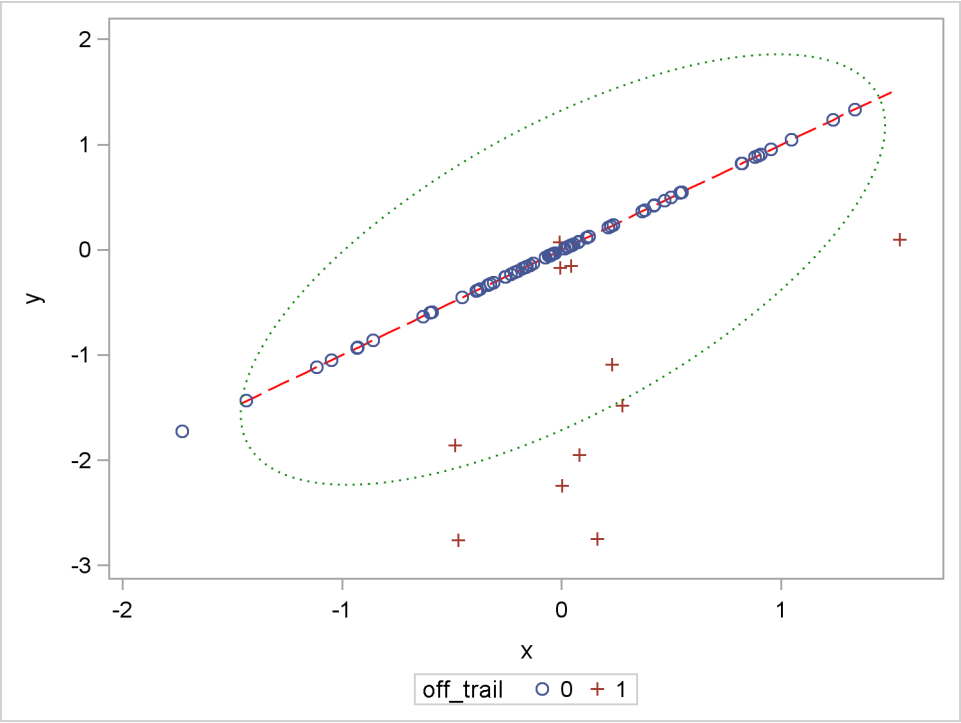


Figure 75.17 shows the positions of all the 80 children, the trail (as a red dashed line), and a contour curve of regular Mahalanobis distance centered at the mean position (as a green dotted ellipse). In terms of regular Mahalanobis distance, the associated covariance estimate is not singular, but its relevant leverage point analysis completely ignores the trail (which is the entity of the low-dimensional structure). The children outside of the ellipse are defined as leverage points, but the children off the trail would not be viewed as leverage points unless they have large Mahalanobis distances. As mentioned in Rousseeuw and Van Driessen (1999), the canonical MCD method can find the low-dimensional structure, but it does not provide further robust covariance estimation because the MCD covariance estimate is singular. As an improved version of the canonical MCD method, the generalized MCD method can find the trail, identify the children off the trail as off-plane leverage points, and further execute in-plane leverage analysis. The following statements apply the generalized MCD algorithm on the children data set.

```
proc robustreg data=children plots=ddplot(label=none);
   model i = x y/leverage;
run;
ods graphics off;
```

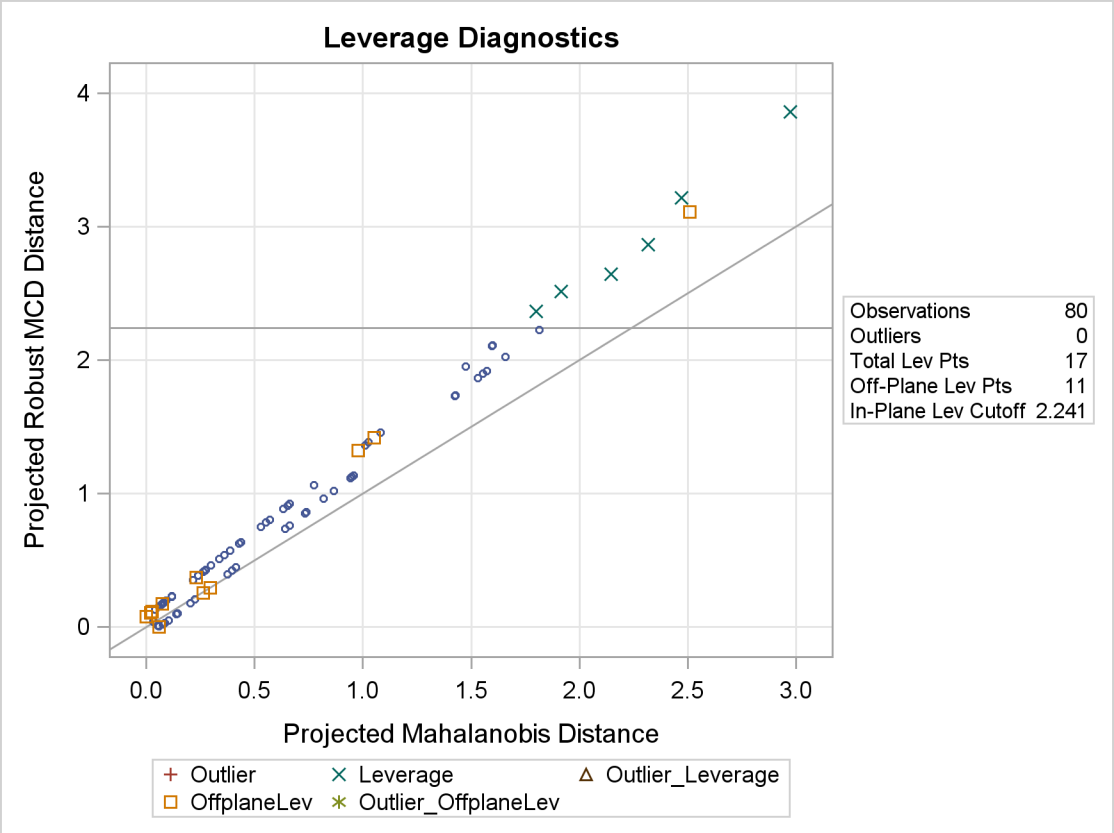
Figure 75.18 exactly identifies the equation underlying the trail. The analysis projects off-plane points onto the trail and computes their projected robust distances and projected Mahalanobis distances the same way as is done for the in-plane points.

Figure 75.18 Robust Dependence Equations

```
                    The ROBUSTREG Procedure

NOTE:  The following robust dependence equations simultaneously hold for 86.25%
       of the observations in the data set. The breakdown setting for the MCD
       algorithm is 25.00%.


                                    y =  x
```

Figure 75.19 shows the relevant distance-distance plot. Robust distance is typically larger than Mahalanobis distance because the sample covariance can be strongly influenced by unusual points that cause the sample covariance to be larger than the MCD covariance.

Figure 75.19 DDPlot for Children Data



NOTE: The PROC ROBUSTREG step in this example is used to obtain the leverage diagnostics; the response is not relevant for this analysis.

Through the off-plane and in-plane symbols and the horizontal cutoff line in Figure 75.19, you can separate all the children into four groups:

- on-trail and close to the MCD mean position

- on-trail but far away from the MCD mean position
- off-trail but close to the MCD mean position
- off-trail and far away from the MCD mean position

The children in the latter three groups are defined as leverage points in PROC ROBUSTREG.

Generalized MCD Algorithm

The generalized MCD algorithm follows the same resampling strategy as the canonical MCD algorithm by Rousseeuw and Van Driessen (1999) but with modifications in the following aspects.

1. Data are orthonormalized before further processing. The orthonormalized covariates, x_i^*, are defined by $x_i^* = (x_i - \bar{x})P\Lambda^{-1/2}$, where P and Λ are the eigenvector and eigenvalue matrices of $\bar{C}(X)$ (that is, $\bar{C}(X) = P\Lambda P^T$).

2. Let

$$S_h(X^*) = \frac{1}{h-1}\sum_{j=1}^{h}(x_{i_j}^* - \bar{x}^*)^T(x_{i_j}^* - \bar{x}^*) = \sum_{j=1}^{p-1}\lambda_j p_j {p_j}^T$$

   denote the covariance and eigendecomposition for a low-dimensional h-subset $\{x_{i_1}^*, \cdots, x_{i_h}^*\}$, where $\bar{x}^* = \frac{1}{h}\sum_{j=1}^{h} x_{i_j}^*$ and the eigenvalues satisfy

$$\lambda_1 \geq \cdots \geq \lambda_q > 0 = \lambda_{q+1} = \cdots = \lambda_p$$

   Then, the rank of $S_h(X^*)$ equals q, and the pseudo-determinant of $S_h(X^*)$ is defined as $\prod_{j=1}^{q}\lambda_j$. In finite precision arithmetic, q is defined as the number of λ's with $\frac{\lambda_i}{\lambda_1}$ being larger than a certain tolerance value. You can specify this tolerance with the PTOL suboption of the LEVERAGE option.

3. Given $S_h(X^*)$ and $\bar{x}^*$ as the covariance and center estimates, the projected Mahalanobis distance for x_i is defined as

$$\left[\sum_{j=1}^{q}\frac{\left((x_i^* - \bar{x}^*)p_j\right)^2}{\lambda_j}\right]^{1/2}$$

   The generalized algorithm also computes off-plane distance for each x_i as

$$\left[\sum_{j=q+1}^{p}\left((x_i^* - \bar{x}^*)p_j\right)^2\right]^{1/2}$$

   In finite precision arithmetic, $\left((x_i^* - \bar{x}^*)p_j\right)^2$ in the previous off-plane formula are truncated to zero if they satisfy

$$\frac{\left((x_i^* - \bar{x}^*)p_j\right)^2}{\lambda_j} \leq \text{cutoff}$$

You can tune this cutoff by using either the PCUTOFF or the PALPHA suboption of the LEVERAGE option. The points with zero off-plane distances are called in-plane points; otherwise, they are called off-plane points. Analogous to ordering all points in terms of their canonical Mahalanobis distances, with the generalized MCD algorithm the points are first sorted by their off-plane distances, and the points with the same off-plane distance values are further sorted by their projected Mahalanobis distances.

4. Instead of comparing the determinants of h-subset covariance matrices, the generalized algorithm compares both the ranks and pseudo-determinants of the h-subset covariance matrices. If the ranks of two matrices are different, the matrix with smaller rank is treated as if its determinant were smaller. If two matrices are of the same rank, they are compared in terms of their pseudo-determinants.

5. Suppose that the $S_h(X^*)$ of the minimum determinant is singular. Then the relevant low-dimensional structure or hyperplane can be identified by using the eigendecomposition of $S_h(X^*)$. The eigenvectors that correspond to the nonzero eigenvalues form a basis for the low-dimensional hyperplane. The projected off-plane distance (POD) for x_i is defined as the off-plane distance associated with the $S_h(X^*)$. To provide further leverage analysis on the low-dimensional hyperplane, every x_i^* is transformed into $(x_i^* p_1, \cdots, x_i^* p_q)$, where p_j are the eigenvectors of the $S_h(X^*)$. The projected robust distance (PRD) is then computed as the reweighted Mahalanobis distance on all the transformed in-plane points. The off-plane points are assigned zero weights at the reweighting stage, because they are leverage points by definition. The in-plane points are classified into two groups, the normal group and the in-plane leverage group. This classification is made by comparing their projected robust distances with a leverage cutoff value. See the section "Leverage Point and Outlier Detection" on page 6409 for details. This reweighting process mirrors the one proposed by Rousseeuw and Van Driessen (1999). However, the degree of freedom p for the reweighting critical χ^2 value is replaced by q. You can control the χ^2 critical value with the MCDCUTOFF or the MCDALPHA option.

If the data set under investigation has a low-dimensional structure, you can use two ODS objects, "DependenceEquations" and "MCDDependenceEquations," to identify the regressors that are linear combinations of other regressors plus certain constants. The equations in "DependenceEquations" hold for the entire data set, while the equations in "MCDDependenceEquations" apply only to the majority of the observations.

By using the OPC suboption of the LEVERAGE option, you can request an ODS table called "DroppedComponents". This table contains a set of coefficient vectors for regressors, which form a basis of the complementary space for the relevant low-dimensional structure.

Leverage Point and Outlier Detection

The regular variable LEVERAGE is defined as

$$\text{LEVERAGE} = \begin{cases} 0 & \text{if } RD(x_i) \leq C(p) \\ 1 & \text{otherwise} \end{cases}$$

where $C(p) = \sqrt{\chi^2_{p;1-\alpha}}$ is the cutoff value. $C(p)$ can be set with the leverage CUTOFF option, and α can be set with the leverage CUTOFFALPHA option.

If projected robust distances are computed for a data set that has a low-dimensional structure, the default cutoff value is $C(q) = \sqrt{\chi^2_{q;1-\alpha}}$ where q is the dimensionality of the low-dimensional space. The LEVERAGE is then defined as

$$\text{LEVERAGE} = \begin{cases} 0 & \text{if POD}(x_i) = 0 \text{ and PRD}(x_i) \le C(q) \\ 1 & \text{if POD}(x_i) = 0 \text{ and PRD}(x_i) > C(q) \text{ (called in-plane leverage)} \\ 1 & \text{if POD}(x_i) > 0 \text{ (called off-plane leverage)} \end{cases}$$

where POD is the projected off-plane distance and PRD denotes the projected robust distance. You can specify a cutoff value with the CUTOFF or the CUTOFFALPHA suboptions of the LEVERAGE option in the MODEL statement.

Residuals $r_i, i = 1, \ldots, n$, based on robust regression estimates are used to detect vertical outliers. The variable OUTLIER is defined as

$$\text{OUTLIER} = \begin{cases} 0 & \text{if } |r_i| \le k\hat{\sigma} \\ 1 & \text{otherwise} \end{cases}$$

where $\hat{\sigma}$ is the estimated scale in the model and the multiplier k of the cutoff value is specified by the CUTOFF= option in the MODEL statement. By default, $k = 3$.

An ODS table called "Diagnostics" contains these two variables (LEVERAGE and OUTLIER).

INEST= Data Set

When you use the M or MM estimation, you can use the INEST= data set to specify initial estimates for all the parameters in the model. The INEST= option is ignored if you specify LTS or S estimation by using the METHOD=LTS or METHOD=S option or if you specify the INITEST= option after the METHOD=MM option in the PROC statement. The INEST= data set must contain the intercept variable (named Intercept) and all independent variables in the MODEL statement.

If BY processing is used, the INEST= data set should also include the BY variables, and there must be at least one observation for each BY group. If there is more than one observation in a BY group, the first one read is used for that BY group.

If the INEST= data set also contains the _TYPE_ variable, only observations with _TYPE_ value "PARMS" are used as starting values.

You can specify starting values for the iteratively reweighted least squares algorithm in the INEST= data set. The INEST= data set has the same structure as the OUTEST= data set but is not required to have all the variables or observations that appear in the OUTEST= data set. One simple use of the INEST= option is passing the previous OUTEST= data set directly to the next model as an INEST= data set, assuming that the two models have the same parameterization.

OUTEST= Data Set

The OUTEST= data set contains parameter estimates for the model. You can specify a label in the MODEL statement to distinguish between the estimates for different models used by the ROBUSTREG procedure. If the COVOUT option is specified, the OUTEST= data set also contains the estimated covariance matrix of the parameter estimates. If the ROBUSTREG procedure does not converge, the parameter estimates are set to missing in the OUTEST data set.

The OUTEST= data set contains all variables specified in the MODEL statement and the BY statement. One observation consists of parameter values for the model with the dependent variable having the value -1. If the COVOUT option is specified, there are additional observations that contain the rows of the estimated covariance matrix. For these observations, the dependent variable contains the parameter estimate for the corresponding row variable. The following variables are also added to the data set:

MODEL is a character variable that contains the label of the MODEL statement, if present. Otherwise, the variable's value is blank.

NAME is a character variable that contains the name of the dependent variable for the parameter estimates or the name of the row for the covariance matrix estimates.

TYPE is a character variable that contains the type of the observation, either PARMS for parameter estimates or COV for covariance estimates.

METHOD is a character variable that contains the type of estimation method: either M estimation, LTS estimation, S estimation, or MM estimation.

STATUS is a character variable that contains the status of model fitting: either Converged, Warning, or Failed.

INTERCEPT is a numeric variable that contains the intercept parameter estimates and covariances.

SCALE is a numeric variable that contains the scale parameter estimates.

Any BY variables specified are also added to the OUTEST= data set.

Computational Resources

The algorithms for the various estimation methods need a different amount of memory for working space. Let p be the number of parameters estimated and n be the number of observations used in the model estimation.

For M estimation, the minimum working space (in bytes) needed is

$$3n + 2p^2 + 30p$$

If sufficient space is available, the input data set is also kept in memory; otherwise, the input data set is read again for computing the iteratively reweighted least squares estimates and the execution

time of the procedure increases substantially. For each reweighted least squares, $O(np^2 + p^3)$ multiplications and additions are required for computing the crossproduct matrix and its inverse. The $O(v)$ notation means that, for large values of the argument, v, $O(v)$ is approximately a constant times v.

Since the iteratively reweighted least squares algorithm converges very quickly (normally within fewer than 20 iterations), the computation of M estimates is fast.

LTS estimation is more expensive in computation. The minimum working space (in bytes) needed is

$$np + 12n + 4p^2 + 60p$$

The memory is mainly used to store the current data used by LTS for modeling. The LTS algorithm uses subsampling and spends much of its computing time on resampling and computing estimates for subsamples. Since it resamples if singularity is detected, it might take more time if the data set has serious singularities.

The MCD algorithm for leverage-point diagnostics is similar to the LTS algorithm.

ODS Table Names

The ROBUSTREG procedure assigns a name to each table it creates. You can specify these names when using the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in the following table.

Table 75.10 ODS Tables Produced by PROC ROBUSTREG

ODS Table Name	Description	Statement	Option
BestEstimates	Best final estimates for LTS	PROC	SUBANALYSIS
BestSubEstimates	Best estimates for each subgroup	PROC	SUBANALYSIS
BiasTest	Bias test for MM estimation	PROC	BIASTEST
ClassLevels	Classification variable levels	CLASS	Default
CorrB	Parameter estimate correlation matrix	MODEL	CORRB
CovB	Parameter estimate covariance matrix	MODEL	COVB
CStep	C-step for LTS fitting	PROC	SUBANALYSIS
DependenceEquations	Design dependence equations	MODEL	LEVERAGE
Diagnostics	Outlier diagnostics	MODEL	DIAGNOSTICS
DiagSummary	Summary of the outlier diagnostics	MODEL	Default
DroppedComponents	Coefficients for MCD-dropped components	MODEL	LEVERAGE
GoodFit	R square, deviance, AIC, and BIC	MODEL	METHOD
InitLTSProfile	Profile for initial LTS estimate	PROC	METHOD
InitSProfile	Profile for initial S estimate	PROC	METHOD
IterHistory	Iteration history	PROC	ITPRINT
LTSEstimates	LTS parameter estimates	PROC	METHOD

Table 75.10 (continued)

ODS Table Name	Description	Statement	Option
LTSLocationScale	Location and scale for LTS	PROC	METHOD
LTSProfile	Profile for LTS estimator	PROC	METHOD
LTSRsquare	R square for LTS estimate	PROC	METHOD
MCDDependenceEquations	Robust dependence equations	MODEL	LEVERAGE
MMProfile	Profile for MM estimator	PROC	METHOD
ModelInfo	Model information	MODEL	Default
NObs	Observations summary	PROC	Default
ParameterEstimates	Parameter estimates	MODEL	Default
ParameterEstimatesF	Final weighted LS estimates	PROC	FWLS
ParameterEstimatesR	Reduced parameter estimates	TEST	Default
ParmInfo	Parameter indices	MODEL	Default
SProfile	Profile for S estimator	PROC	METHOD
Groups	Groups for LTS fitting	PROC	SUBANALYSIS
SummaryStatistics	Summary statistics for model variables	MODEL	Default
Tests	Results for tests	TEST	Default

ODS Graphics

Graphical displays are important in robust regression and outlier detection. This section provides information about the basic ODS statistical graphics produced by the ROBUSTREG procedure.

If the model includes a single continuous independent variable, a plot of robust fit against this variable (FITPLOT) is provided by default. For diagnostics, two plots are particularly useful in revealing outliers and leverage points. The first is a scatter plot of the standardized robust residuals against the robust distances (RDPLOT). The second is a scatter plot of the robust distances against the classical Mahalanobis distances (DDPLOT). In addition to these two plots, a histogram and a quantile-quantile plot of the standardized robust residuals are also helpful.

These plots are controlled by the PLOTS= option in the PROC ROBUSTREG statement. You can specify more than one plot request with the PLOTS= option. Table 75.11 summarizes these requests.

In addition to the PLOTS= option, you must specify the ODS GRAPHICS statement. For more information about the ODS GRAPHICS statement, see Chapter 21, "Statistical Graphics Using ODS."

The names of the graphs that PROC ROBUSTREG generates are listed in Table 75.12, along with the required statements and options. The subsequent subsections provide information about these graphs.

Table 75.11 Options for Plots

Option	Plot
ALL	All appropriate plots
DDPLOT	Robust distance versus Mahalanobis distance
FITPLOT	Robust fit versus independent variable
HISTOGRAM	Histogram of standardized robust residuals
NONE	No plot
QQPLOT	Q-Q plot of standardized robust residuals
RDPLOT	Standardized robust residual versus robust distance

Fit Plot

When the model has a single independent continuous variable (with or without the intercept), the ROBUSTREG procedure automatically creates a plot of robust fit against this independent variable.

The following simple example shows the fit plot. The data, from Rousseeuw and Leroy (1987, Table 3), include the logarithm of surface temperature and the logarithm of light intensity for 47 stars in the direction of the constellation Cygnus.

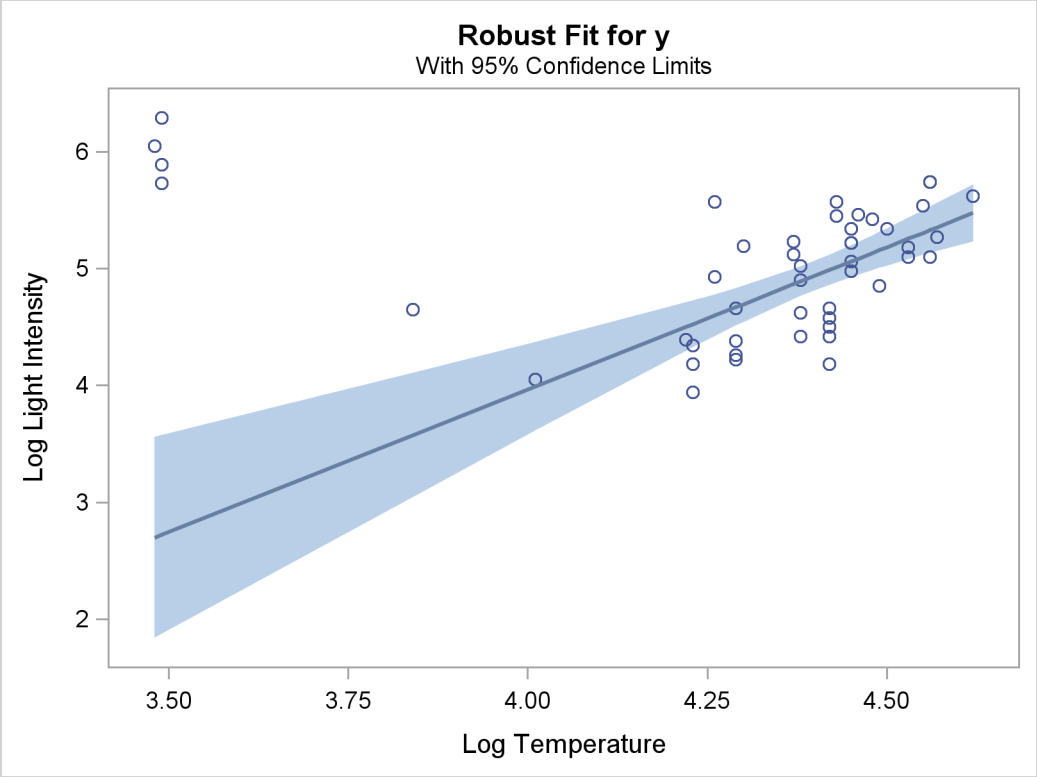
```
data star;
   input index x y @@;
   label x = 'Log Temperature'
         y = 'Log Light Intensity';
datalines;
1   4.37  5.23     25  4.38  5.02
2   4.56  5.74     26  4.42  4.66
3   4.26  4.93     27  4.29  4.66
4   4.56  5.74     28  4.38  4.90
5   4.30  5.19     29  4.22  4.39
6   4.46  5.46     30  3.48  6.05
7   3.84  4.65     31  4.38  4.42
8   4.57  5.27     32  4.56  5.10
9   4.26  5.57     33  4.45  5.22
10  4.37  5.12     34  3.49  6.29
11  3.49  5.73     35  4.23  4.34
12  4.43  5.45     36  4.62  5.62
13  4.48  5.42     37  4.53  5.10
14  4.01  4.05     38  4.45  5.22
15  4.29  4.26     39  4.53  5.18
16  4.42  4.58     40  4.43  5.57
17  4.23  3.94     41  4.38  4.62
18  4.42  4.18     42  4.45  5.06
19  4.23  4.18     43  4.50  5.34
20  3.49  5.89     44  4.45  5.34
21  4.29  4.38     45  4.55  5.54
22  4.29  4.22     46  4.45  4.98
23  4.42  4.42     47  4.42  4.50
24  4.49  4.85
;
```

The following statements plot the robust fit of the logarithm of light intensity with the MM method against the logarithm of the surface temperature.

```
ods graphics on;

proc robustreg data=star method=mm ;
    model y = x;
run;

ods graphics off;
```

Figure 75.20 shows the fit plot. Confidence limits are added on the plot by default.

Figure 75.20 Robust Fit



You can suppress the confidence limits with the NOLIMITS option, as shown in the following statements:

```
ods graphics on;

proc robustreg data=star method=mm plot=fitplot(nolimits);
    model y = x;
run;

ods graphics off;
```

Distance-Distance Plot

The distance-distance plot (DDPLOT) is mainly used for leverage-point diagnostics. It is a scatter plot of the robust distances (or projected robust distances) against the classical Mahalanobis distances (or projected classical Mahalanobis distances) for the independent variables. See the section "Leverage Point and Outlier Detection" on page 6409 for details about the robust distance.

You can use the PLOT=DDPLOT option to request this plot. The following statements use the stack data set in the section "M Estimation" on page 6361 to create the single plot shown in Figure 75.5.

```
ods graphics on;

proc robustreg data=stack plot=ddplot;
   model y = x1 x2 x3;
run;

ods graphics off;
```

The reference lines represent the cutoff values. The diagonal line is also drawn to show the distribution of the distances. By default, all outliers and leverage points are labeled with observation numbers. To change the default, you can use the LABEL= option as described in Table 75.1.

If you specify ID variables in the ID statement, the values of the first ID variable instead of observation numbers are used as labels.

Residual-Distance Plot

The residual-distance plot (RDPLOT) is used for both outlier and leverage-point diagnostics. It is a scatter plot of the standardized robust residuals against the robust distances. See the section "Leverage Point and Outlier Detection" on page 6409 for details about the robust distance.

You can use the PLOT=RDPLOT option to request this plot. The following statements use the stack data set in the section "M Estimation" on page 6361 to create the plot shown in Figure 75.4.

```
ods graphics on;

proc robustreg data=stack plot=rdplot;
   model y = x1 x2 x3;
run;

ods graphics off;
```

The reference lines represent the cutoff values. By default, all outliers and leverage points are labeled with observation numbers. To change the default, you can use the LABEL= option as described in Table 75.1.

If you specify ID variables in the ID statement, the values of the first ID variable instead of observation numbers are used as labels.

Histogram and Q-Q Plot

PROC ROBUSTREG produces a histogram and a Q-Q plot for the standardized robust residuals. The histogram is superimposed with a normal density curve and a kernel density curve. Using the stack data set in the section "M Estimation" on page 6361, the following statements create the plots in Figure 75.6 and Figure 75.7.

```
ods graphics on;

proc robustreg data=stack plots=(histogram qqplot);
   model y = x1 x2 x3;
run;

ods graphics off;
```

ODS Graph Names

PROC ROBUSTREG assigns a name to each graph it creates using ODS. You can use these names to reference the graphs when using ODS. The names are listed in Table 75.12.

To request these graphs, you must specify the ODS GRAPHICS statement in addition to the PLOTS= option described in Table 75.11. For more information about the ODS GRAPHICS statement, see Chapter 21, "Statistical Graphics Using ODS."

Table 75.12 ODS Graphics Produced by PROC ROBUSTREG

ODS Graph Name	Plot Description	Statement	PLOTS= Option
DDPlot	Robust distance versus Mahalanobis distance (or projected robust distance versus Projected Mahalanobis distance)	PROC	DDPLOT
FitPlot	Robust fit versus independent variable	PROC	FITPLOT
Histogram	Histogram of standardized robust residuals	PROC	HISTOGRAM
QQPlot	Q-Q plot of standardized robust residuals	PROC	QQPLOT
RDPlot	Standardized robust residual versus robust distance (or projected robust distance)	PROC	RDPLOT

Examples: ROBUSTREG Procedure

Example 75.1: Comparison of Robust Estimates

This example contrasts several of the robust methods available in the ROBUSTREG procedure.

The following statements generate 1,000 random observations. The first 900 observations are from a linear model, and the last 100 observations are significantly biased in the y-direction. In other words, 10% of the observations are contaminated with outliers.

```
data a (drop=i);
   do i=1 to 1000;
      x1=rannor(1234);
      x2=rannor(1234);
      e=rannor(1234);
      if i > 900 then y=100 + e;
      else y=10 + 5*x1 + 3*x2 + .5 * e;
      output;
   end;
run;
```

The following statements invoke PROC REG and PROC ROBUSTREG with the data set a.

```
proc reg data=a;
   model y = x1 x2;
run;

proc robustreg data=a method=m ;
   model y = x1 x2;
run;

proc robustreg data=a method=mm seed=100;
   model y = x1 x2;
run;

proc robustreg data=a method=s seed=100;
   model y = x1 x2;
run;

proc robustreg data=a method=lts seed=100;
   model y = x1 x2;
run;
```

The tables of parameter estimates generated by using M estimation, MM estimation, S estimation, and LTS estimation in the ROBUSTREG procedure are shown in Output 75.1.2, Output 75.1.3, Output 75.1.4, and Output 75.1.5, respectively. For comparison, the ordinary least squares (OLS) estimates produced by the REG procedure (Chapter 74, "The REG Procedure") are shown in Output 75.1.1. The four robust methods, M, MM, S, and LTS, correctly estimate the regression

coefficients for the underlying model (10, 5, and 3), but the OLS estimate does not.

Output 75.1.1 OLS Estimates for Data with 10% Contamination

The REG Procedure
Model: MODEL1
Dependent Variable: y

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	19.06712	0.86322	22.09	<.0001
x1	1	3.55485	0.86892	4.09	<.0001
x2	1	2.12341	0.83039	2.56	0.0107

Output 75.1.2 M Estimates for Data with 10% Contamination

The ROBUSTREG Procedure

Model Information

Data Set	WORK.A
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	M Estimation

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	10.0024	0.0174	9.9683	10.0364	331908	<.0001
x1	1	5.0077	0.0175	4.9735	5.0420	82106.9	<.0001
x2	1	3.0161	0.0167	2.9834	3.0488	32612.5	<.0001
Scale	1	0.5780					

Output 75.1.3 MM Estimates for Data with 10% Contamination

The ROBUSTREG Procedure

Model Information

Data Set	WORK.A
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	MM Estimation

Output 75.1.3 *continued*

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	10.0035	0.0176	9.9690	10.0379	323947	<.0001
x1	1	5.0085	0.0178	4.9737	5.0433	79600.6	<.0001
x2	1	3.0181	0.0168	2.9851	3.0511	32165.0	<.0001
Scale	0	0.6733					

Output 75.1.4 S Estimates for Data with 10% Contamination

The ROBUSTREG Procedure

Model Information

Data Set	WORK.A
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	S Estimation

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	10.0055	0.0180	9.9703	10.0408	309917	<.0001
x1	1	5.0096	0.0182	4.9740	5.0452	76045.2	<.0001
x2	1	3.0210	0.0172	2.9873	3.0547	30841.3	<.0001
Scale	0	0.6721					

Output 75.1.5 LTS Estimates for Data with 10% Contamination

The ROBUSTREG Procedure

Model Information

Data Set	WORK.A
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	LTS Estimation

Output 75.1.5 *continued*

LTS Parameter Estimates

Parameter	DF	Estimate
Intercept	1	10.0083
x1	1	5.0316
x2	1	3.0396
Scale (sLTS)	0	0.5880
Scale (Wscale)	0	0.5113

The next statements demonstrate that if the percentage of contamination is increased to 40%, the M method and the MM method with default options fail to estimate the underlying model. Output 75.1.6 and Output 75.1.7 display these estimates. However, by tuning the constant c for the M method and the constants INITH and K0 for the MM method, you can increase the breakdown values of the estimates and capture the right model. Output 75.1.8 and Output 75.1.9 display these estimates. Similarly, you can tune the constant EFF for the S method and the constant H for the LTS method and correctly estimate the underlying model with these methods. Results are not presented.

```
data b (drop=i);
   do i=1 to 1000;
      x1=rannor(1234);
      x2=rannor(1234);
      e=rannor(1234);
      if i > 600 then y=100 + e;
      else y=10 + 5*x1 +  3*x2 + .5 * e;
      output;
   end;
run;

proc robustreg data=b method=m ;
   model y = x1 x2;
run;

proc robustreg data=b method=mm;
   model y = x1 x2;
run;

proc robustreg data=b method=m(wf=bisquare(c=2));
   model y = x1 x2;
run;

proc robustreg data=b method=mm(inith=502 k0=1.8);
   model y = x1 x2;
run;
```

Output 75.1.6 M Estimates (Default Setting) for Data with 40% Contamination

The ROBUSTREG Procedure

Model Information

Data Set	WORK.B
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	M Estimation

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	44.8991	1.5609	41.8399	47.9584	827.46	<.0001
x1	1	2.4309	1.5712	-0.6485	5.5104	2.39	0.1218
x2	1	1.3742	1.5015	-1.5687	4.3171	0.84	0.3601
Scale	1	56.6342					

Output 75.1.7 MM Estimates (Default Setting) for Data with 40% Contamination

The ROBUSTREG Procedure

Model Information

Data Set	WORK.B
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	MM Estimation

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	43.0607	1.7978	39.5370	46.5844	573.67	<.0001
x1	1	2.7369	1.8140	-0.8185	6.2924	2.28	0.1314
x2	1	1.5211	1.7265	-1.8628	4.9049	0.78	0.3783
Scale	0	52.8496					

Output 75.1.8 M Estimates (Tuned) for Data with 40% Contamination

The ROBUSTREG Procedure

Model Information

Data Set	WORK.B
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	M Estimation

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	10.0137	0.0219	9.9708	10.0565	209688	<.0001
x1	1	4.9905	0.0220	4.9473	5.0336	51399.1	<.0001
x2	1	3.0399	0.0210	2.9987	3.0811	20882.4	<.0001
Scale	1	1.0531					

Output 75.1.9 MM Estimates (Tuned) for Data with 40% Contamination

The ROBUSTREG Procedure

Model Information

Data Set	WORK.B
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	MM Estimation

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	10.0103	0.0213	9.9686	10.0520	221639	<.0001
x1	1	4.9890	0.0218	4.9463	5.0316	52535.9	<.0001
x2	1	3.0363	0.0201	2.9970	3.0756	22895.5	<.0001
Scale	0	1.8992					

When there are bad leverage points, the M method fails to estimate the underlying model no matter what constant c you use. In this case, other methods (LTS, S, and MM) in PROC ROBUSTREG, which are robust to bad leverage points, correctly estimate the underlying model.

The following statements generate 1,000 observations with 1% bad high leverage points.

```
data c (drop=i);
   do i=1 to 1000;
      x1=rannor(1234);
      x2=rannor(1234);
      e=rannor(1234);
      if i > 600 then y=100 + e;
      else y=10 + 5*x1 + 3*x2 + .5 * e;
      if i < 11 then x1=200 * rannor(1234);
      if i < 11 then x2=200 * rannor(1234);
      if i < 11 then y= 100*e;
      output;
   end;
run;

proc robustreg data=c method=mm(inith=502 k0=1.8) seed=100;
   model y = x1 x2;
run;

proc robustreg data=c method=s(k0=1.8) seed=100;
   model y = x1 x2;
run;

proc robustreg data=c method=lts(h=502) seed=100;
   model y = x1 x2;
run;
```

Output 75.1.10 displays the MM estimates with initial LTS estimates, Output 75.1.11 displays the S estimates, and Output 75.1.12 displays the LTS estimates.

Output 75.1.10 MM Estimates for Data with 1% Leverage Points

The ROBUSTREG Procedure

Model Information

Data Set	WORK.C
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	MM Estimation

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	9.9820	0.0215	9.9398	10.0241	215369	<.0001
x1	1	5.0303	0.0206	4.9898	5.0707	59469.1	<.0001
x2	1	3.0222	0.0221	2.9789	3.0655	18744.9	<.0001
Scale	0	2.2134					

Output 75.1.11 S Estimates for Data with 1% Leverage Points

The ROBUSTREG Procedure

Model Information

Data Set	WORK.C
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	S Estimation

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	9.9808	0.0216	9.9383	10.0232	212532	<.0001
x1	1	5.0303	0.0208	4.9896	5.0710	58656.3	<.0001
x2	1	3.0217	0.0222	2.9782	3.0652	18555.7	<.0001
Scale	0	2.2094					

Output 75.1.12 LTS Estimates for Data with 1% Leverage Points

The ROBUSTREG Procedure

Model Information

Data Set	WORK.C
Dependent Variable	y
Number of Independent Variables	2
Number of Observations	1000
Method	LTS Estimation

LTS Parameter Estimates

Parameter	DF	Estimate
Intercept	1	9.9742
x1	1	5.0010
x2	1	3.0219
Scale (sLTS)	0	0.9952
Scale (Wscale)	0	0.5216

Example 75.2: Robust ANOVA

The classical analysis of variance (ANOVA) technique based on least squares assumes that the underlying experimental errors are normally distributed. However, data often contain outliers due to recording or other errors. In other cases, extreme responses occur when control variables in the experiments are set to extremes. It is important to distinguish these extreme points and determine whether they are outliers or important extreme cases. You can use the ROBUSTREG procedure for robust analysis of variance based on M estimation. Typically, there are no high leverage points in a well-designed experiment, so M estimation is appropriate.

The following example shows how to use the ROBUSTREG procedure for robust ANOVA.

An experiment was carried out to study the effects of two successive treatments (T1, T2) on the recovery time of mice with certain diseases. Sixteen mice were randomly assigned into four groups for the four different combinations of the treatments. The recovery times (time) were recorded (in hours) as shown in the following data set recover.

```
data recover;
   input  T1 $ T2 $ time @@;
datalines;
0 0 20.2  0 0 23.9  0 0 21.9  0 0 42.4
1 0 27.2  1 0 34.0  1 0 27.4  1 0 28.5
0 1 25.9  0 1 34.5  0 1 25.1  0 1 34.2
1 1 35.0  1 1 33.9  1 1 38.3  1 1 39.9
;
```

The following statements invoke the GLM procedure (Chapter 39, "The GLM Procedure") for a standard ANOVA:

```
proc glm data=recover;
    class T1 T2;
    model time = T1 T2 T1*T2;
run;
```

Output 75.2.1 Overall ANOVA

```
                          The GLM Procedure

Dependent Variable: time

                                   Sum of
 Source                     DF       Squares    Mean Square   F Value   Pr > F

 Model                       3    209.9118750     69.9706250      1.86   0.1905

 Error                      12    451.9225000     37.6602083

 Corrected Total            15    661.8343750

               R-Square     Coeff Var      Root MSE     time Mean

               0.317167      19.94488      6.136791      30.76875
```

Output 75.2.2 Model ANOVA

Source	DF	Type I SS	Mean Square	F Value	Pr > F
T1	1	81.4506250	81.4506250	2.16	0.1671
T2	1	106.6056250	106.6056250	2.83	0.1183
T1*T2	1	21.8556250	21.8556250	0.58	0.4609

Output 75.2.1 indicates that the overall model effect is not significant at the 10% level, and Output 75.2.2 indicates that neither treatment is significant at the 10% level.

The following statements invoke the ROBUSTREG procedure with the same model:

```
proc robustreg data=recover;
   class T1 T2;
   model time = T1 T2 T1*T2 / diagnostics;
   T1_T2: test T1*T2;
   output out=robout r=resid sr=stdres;
run;
```

Output 75.2.3 shows some basic information about the model and the response variable time.

Output 75.2.3 Model Fitting Information and Summary Statistics

The ROBUSTREG Procedure

Model Information

Data Set	WORK.RECOVER
Dependent Variable	time
Number of Independent Variables	2
Number of Continuous Independent Variables	0
Number of Class Independent Variables	2
Number of Observations	16
Method	M Estimation

Summary Statistics

Variable	Q1	Median	Q3	Mean	Standard Deviation	MAD
time	25.5000	31.2000	34.7500	30.7688	6.6425	6.8941

The "Parameter Estimates" table in Output 75.2.4 indicates that the main effects of both treatments are significant at the 5% level.

Output 75.2.4 Model Parameter Estimates

Parameter Estimates

Parameter			DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept			1	36.7655	2.0489	32.7497	40.7814	321.98	<.0001
T1	0		1	-6.8307	2.8976	-12.5100	-1.1514	5.56	0.0184
T1	1		0	0.0000	.	.	.	.	.
T2	0		1	-7.6755	2.8976	-13.3548	-1.9962	7.02	0.0081
T2	1		0	0.0000	.	.	.	.	.
T1*T2	0	0	1	-0.2619	4.0979	-8.2936	7.7698	0.00	0.9490
T1*T2	0	1	0	0.0000	.	.	.	.	.
T1*T2	1	0	0	0.0000	.	.	.	.	.
T1*T2	1	1	0	0.0000	.	.	.	.	.
Scale			1	3.5346					

The reason for the difference between the traditional ANOVA and the robust ANOVA is explained by Output 75.2.5, which shows that the fourth observation is an outlier. Further investigation shows that the original value of 24.4 for the fourth observation was recorded incorrectly.

Output 75.2.6 displays the robust test results. The interaction between the two treatments is not significant. Output 75.2.7 displays the robust residuals and standardized robust residuals.

Output 75.2.5 Diagnostics

Diagnostics

Obs	Standardized Robust Residual	Outlier
4	5.7722	*

Output 75.2.6 Test of Significance

Robust Linear Test T1_T2

Test	Test Statistic	Lambda	DF	Chi-Square	Pr > ChiSq
Rho	0.0041	0.7977	1	0.01	0.9431
Rn2	0.0041		1	0.00	0.9490

Output 75.2.7 ROBUSTREG Output

Obs	T1	T2	time	resid	stdres
1	0	0	20.2	−1.7974	−0.50851
2	0	0	23.9	1.9026	0.53827
3	0	0	21.9	−0.0974	−0.02756
4	0	0	42.4	20.4026	5.77222
5	1	0	27.2	−1.8900	−0.53472
6	1	0	34.0	4.9100	1.38911
7	1	0	27.4	−1.6900	−0.47813
8	1	0	28.5	−0.5900	−0.16693
9	0	1	25.9	−4.0348	−1.14152
10	0	1	34.5	4.5652	1.29156
11	0	1	25.1	−4.8348	−1.36785
12	0	1	34.2	4.2652	1.20668
13	1	1	35.0	−1.7655	−0.49950
14	1	1	33.9	−2.8655	−0.81070
15	1	1	38.3	1.5345	0.43413
16	1	1	39.9	3.1345	0.88679

Example 75.3: Growth Study of De Long and Summers

Robust regression and outlier detection techniques have considerable applications to econometrics. The following example from Zaman, Rousseeuw, and Orhan (2001) shows how these techniques substantially improve the ordinary least squares (OLS) results for the growth study of De Long and Summers.

De Long and Summers (1991) studied the national growth of 61 countries from 1960 to 1985 by using OLS with the following data set growth.

```
data growth;
   input country$ GDP LFG EQP NEQ GAP @@;
datalines;
Argentin  0.0089 0.0118 0.0214 0.2286 0.6079
Austria   0.0332 0.0014 0.0991 0.1349 0.5809
Belgium   0.0256 0.0061 0.0684 0.1653 0.4109
Bolivia   0.0124 0.0209 0.0167 0.1133 0.8634

   ... more lines ...

Venezuel  0.0120 0.0378 0.0340 0.0760 0.4974
Zambia   -0.0110 0.0275 0.0702 0.2012 0.8695
Zimbabwe  0.0110 0.0309 0.0843 0.1257 0.8875
;
```

The regression equation they used is

$$\text{GDP} = \beta_0 + \beta_1\text{LFG} + \beta_2\text{GAP} + \beta_3\text{EQP} + \beta_4\text{NEQ} + \epsilon$$

where the response variable is the growth in gross domestic product per worker (GDP) and the regressors are labor force growth (LFG), relative GDP gap (GAP), equipment investment (EQP), and nonequipment investment (NEQ).

The following statements invoke the REG procedure (Chapter 74, "The REG Procedure") for the OLS analysis:

```
proc reg data=growth;
   model GDP  = LFG GAP EQP NEQ ;
run;
```

The OLS analysis shown in Output 75.3.1 indicates that GAP and EQP have a significant influence on GDP at the 5% level.

Output 75.3.1 OLS Estimates

The REG Procedure
Model: MODEL1
Dependent Variable: GDP

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	−0.01430	0.01028	−1.39	0.1697
LFG	1	−0.02981	0.19838	−0.15	0.8811
GAP	1	0.02026	0.00917	2.21	0.0313
EQP	1	0.26538	0.06529	4.06	0.0002
NEQ	1	0.06236	0.03482	1.79	0.0787

The following statements invoke the ROBUSTREG procedure with the default M estimation.

```
ods graphics on;

proc robustreg data=growth plots=all;
   model GDP  = LFG GAP EQP NEQ / diagnostics leverage;
   id country;
run;

ods graphics off;
```

Output 75.3.2 displays model information and summary statistics for variables in the model.

Output 75.3.2 Model Fitting Information and Summary Statistics

The ROBUSTREG Procedure

Model Information

Data Set	WORK.GROWTH
Dependent Variable	GDP
Number of Independent Variables	4
Number of Observations	61
Method	M Estimation

Summary Statistics

Variable	Q1	Median	Q3	Mean	Standard Deviation	MAD
LFG	0.0118	0.0239	0.0281	0.0211	0.00979	0.00949
GAP	0.5796	0.8015	0.8863	0.7258	0.2181	0.1778
EQP	0.0265	0.0433	0.0720	0.0523	0.0296	0.0325
NEQ	0.0956	0.1356	0.1812	0.1399	0.0570	0.0624
GDP	0.0121	0.0231	0.0310	0.0224	0.0155	0.0150

Output 75.3.3 displays the M estimates. Besides GAP and EQP, the robust analysis also indicates that NEQ is significant. This new finding is explained by Output 75.3.4, which shows that Zambia, the 60th country in the data, is an outlier. Output 75.3.4 also identifies leverage points based on the robust MCD distances; however, there are no serious high-leverage points in this data set.

Output 75.3.3 M Estimates

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	-0.0247	0.0097	-0.0437	-0.0058	6.53	0.0106
LFG	1	0.1040	0.1867	-0.2619	0.4699	0.31	0.5775
GAP	1	0.0250	0.0086	0.0080	0.0419	8.36	0.0038
EQP	1	0.2968	0.0614	0.1764	0.4172	23.33	<.0001
NEQ	1	0.0885	0.0328	0.0242	0.1527	7.29	0.0069
Scale	1	0.0099					

Output 75.3.4 Diagnostics

Diagnostics

Obs	country	Mahalanobis Distance	Robust MCD Distance	Leverage	Standardized Robust Residual	Outlier
1	Argentin	2.6083	4.0639	*	-0.9424	
5	Botswana	3.4351	6.7391	*	1.4200	
8	Canada	3.1876	4.6843	*	-0.1972	
9	Chile	3.6752	5.0599	*	-1.8784	
17	Finland	2.6024	3.8186	*	-1.7971	
23	HongKong	2.1225	3.8238	*	1.7161	
27	Israel	2.6461	5.0336	*	0.0909	
31	Japan	2.9179	4.7140	*	0.0216	
53	Tanzania	2.2600	4.3193	*	-1.8082	
57	U.S.	3.8701	5.4874	*	0.1448	
58	Uruguay	2.5953	3.9671	*	-0.0978	
59	Venezuel	2.9239	4.1663	*	0.3573	
60	Zambia	1.8562	2.7135		-4.9798	*
61	Zimbabwe	1.9634	3.9128	*	-2.5959	

Output 75.3.5 displays robust versions of goodness-of-fit statistics for the model.

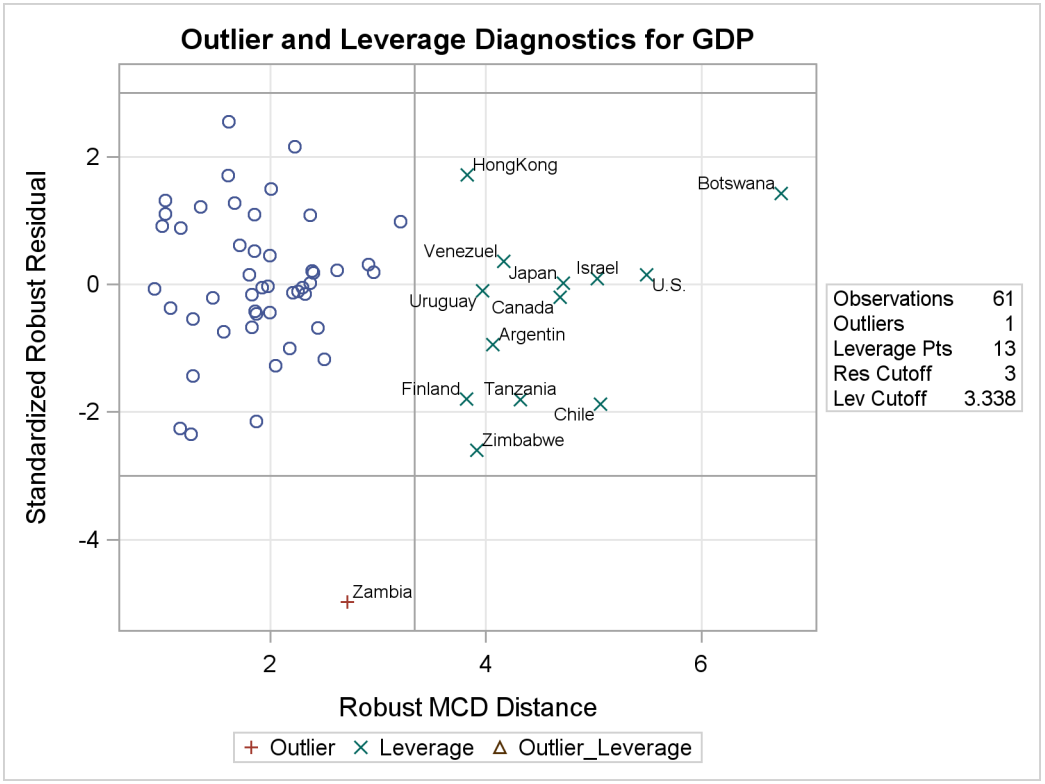
Output 75.3.5 Goodness-of-Fit Statistics

Goodness-of-Fit

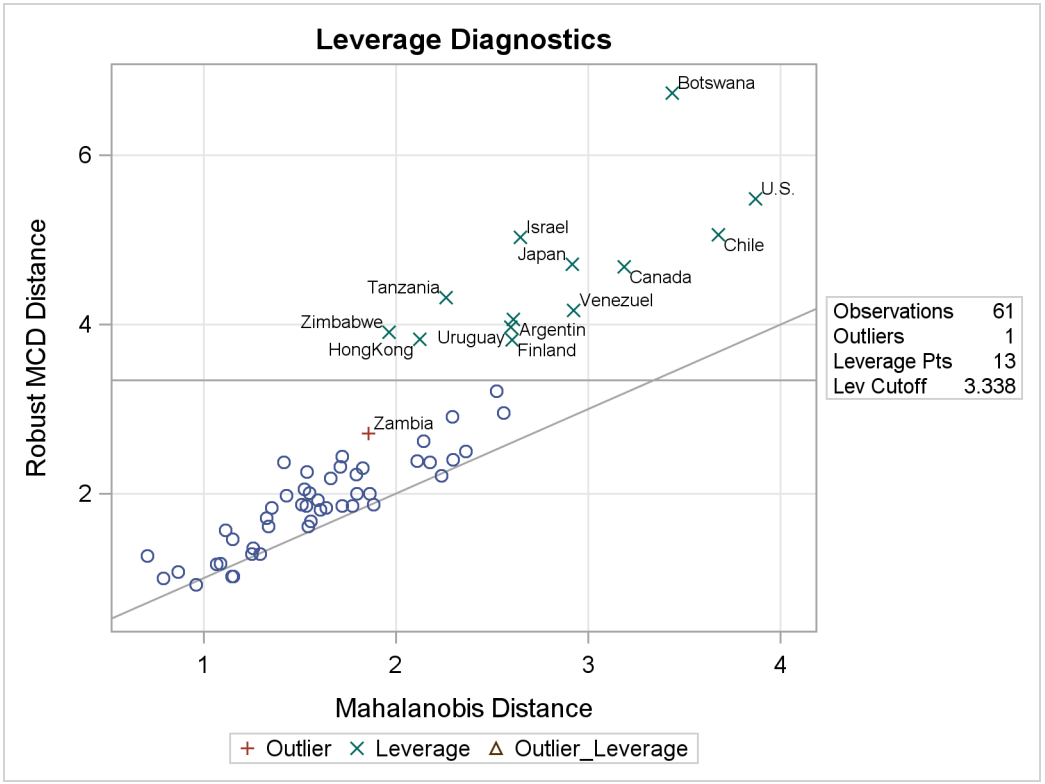
Statistic	Value
R-Square	0.3178
AICR	80.2134
BICR	91.5095
Deviance	0.0070

The PLOTS=ALL option generates four diagnostic plots. Output 75.3.6 and Output 75.3.7 are for outlier and leverage-point diagnostics. Output 75.3.8 and Output 75.3.9 are a histogram and a Q-Q plot of the standardized robust residuals, respectively.

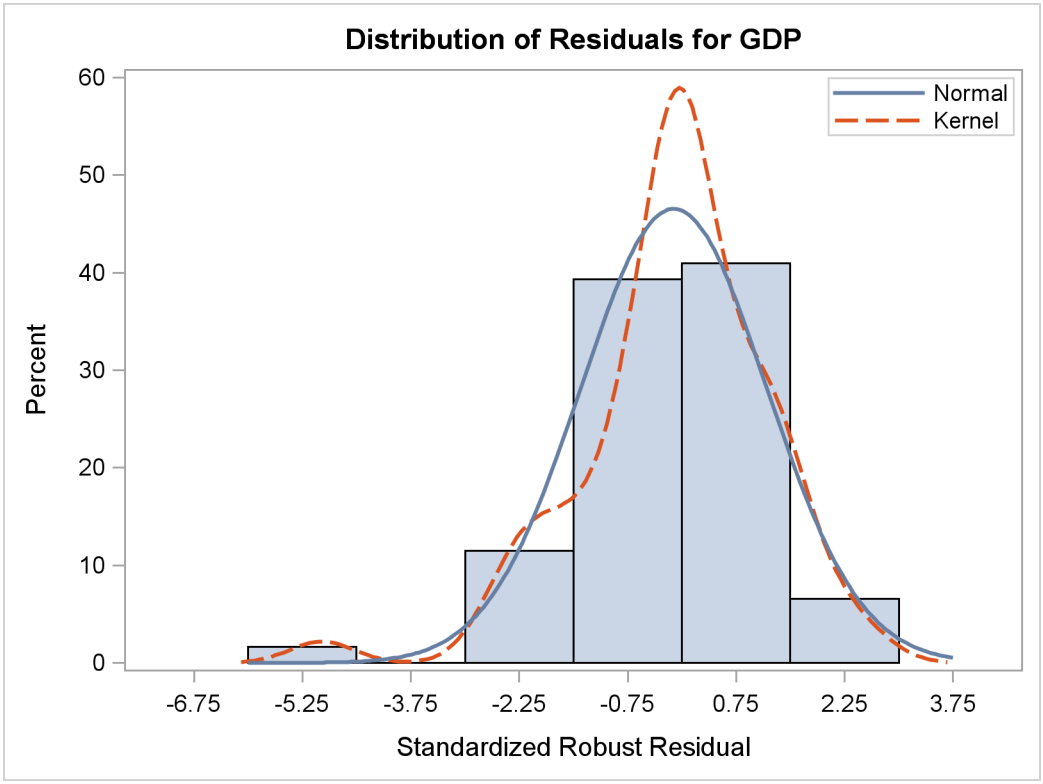
Output 75.3.6 RDPLOT for growth Data



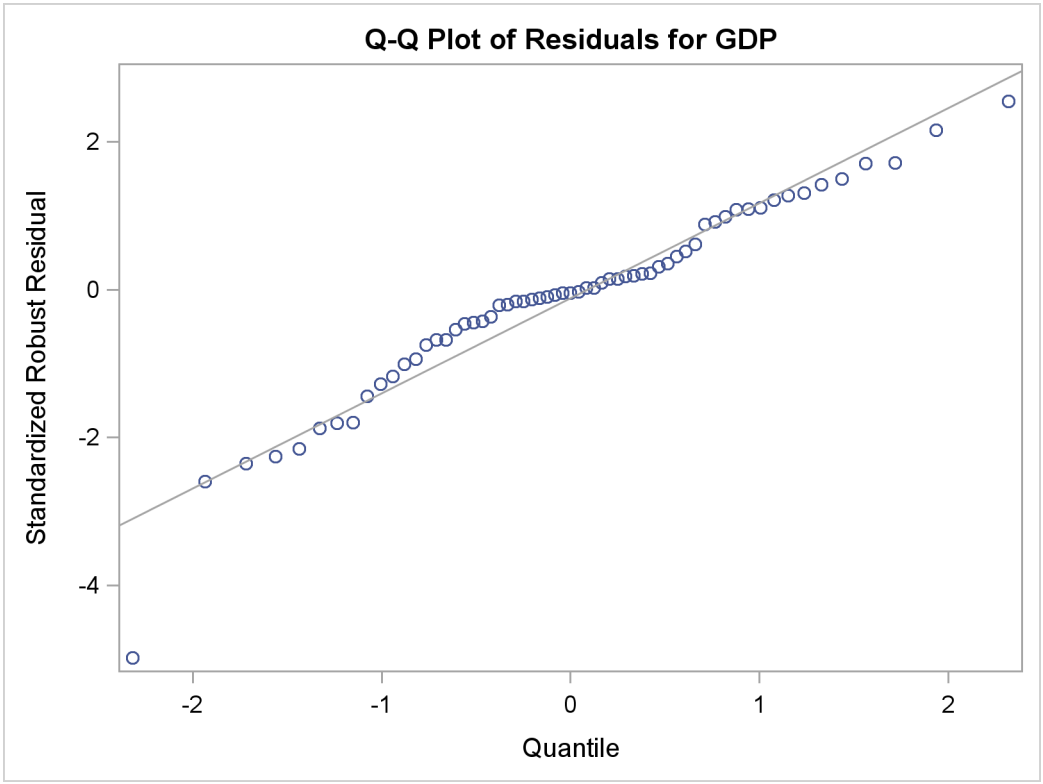
Output 75.3.7 DDPLOT for growth Data



Output 75.3.8 Histogram



Output 75.3.9 Q-Q Plot



The following statements invoke the ROBUSTREG procedure with LTS estimation, which was used by Zaman, Rousseeuw, and Orhan (2001). The results are consistent with those of M estimation.

```
proc robustreg method=lts(h=33) fwls data=growth seed=100;
   model GDP  = LFG GAP EQP NEQ / diagnostics leverage ;
   id country;
run;
```

Output 75.3.10 LTS Estimates and LTS R Square

The ROBUSTREG Procedure

LTS Parameter Estimates

Parameter	DF	Estimate
Intercept	1	-0.0249
LFG	1	0.1123
GAP	1	0.0214
EQP	1	0.2669
NEQ	1	0.1110
Scale (sLTS)	0	0.0076
Scale (Wscale)	0	0.0109

R-Square for LTS Estimation

R-Square	0.7418

Output 75.3.10 displays the LTS estimates and the LTS R Square.

Output 75.3.11 Diagnostics

Diagnostics

Obs	country	Mahalanobis Distance	Robust MCD Distance	Leverage	Standardized Robust Residual	Outlier
1	Argentin	2.6083	4.0639	*	-1.0715	
5	Botswana	3.4351	6.7391	*	1.6574	
8	Canada	3.1876	4.6843	*	-0.2324	
9	Chile	3.6752	5.0599	*	-2.0896	
17	Finland	2.6024	3.8186	*	-1.6367	
23	HongKong	2.1225	3.8238	*	1.7570	
27	Israel	2.6461	5.0336	*	0.2334	
31	Japan	2.9179	4.7140	*	0.0971	
53	Tanzania	2.2600	4.3193	*	-1.2978	
57	U.S.	3.8701	5.4874	*	0.0605	
58	Uruguay	2.5953	3.9671	*	-0.0857	
59	Venezuel	2.9239	4.1663	*	0.4113	
60	Zambia	1.8562	2.7135		-4.4984	*
61	Zimbabwe	1.9634	3.9128	*	-2.1201	

Output 75.3.11 displays outlier and leverage-point diagnostics based on the LTS estimates and the robust MCD distances.

Output 75.3.12 Final Weighted LS Estimates

Parameter Estimates for Final Weighted Least Squares Fit

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	-0.0222	0.0093	-0.0405	-0.0039	5.65	0.0175
LFG	1	0.0446	0.1771	-0.3026	0.3917	0.06	0.8013
GAP	1	0.0245	0.0082	0.0084	0.0406	8.89	0.0029
EQP	1	0.2824	0.0581	0.1685	0.3964	23.60	<.0001
NEQ	1	0.0849	0.0314	0.0233	0.1465	7.30	0.0069
Scale	0	0.0116					

Output 75.3.12 displays the final weighted least squares estimates, which are identical to those reported in Zaman, Rousseeuw, and Orhan (2001).

Example 75.4: Constructed Effects

The algorithms of PROC ROBUSTREG assume that a response variable is linearly dependent on the regressors. However, in practice, a response often depends on some factors in a nonlinear manner. This example demonstrates how a nonlinear response-factor relationship can be modeled by using constructed effects. (See the section "EFFECT Statement (Experimental)" on page 418 of Chapter 19, "Shared Concepts and Topics," for details.)

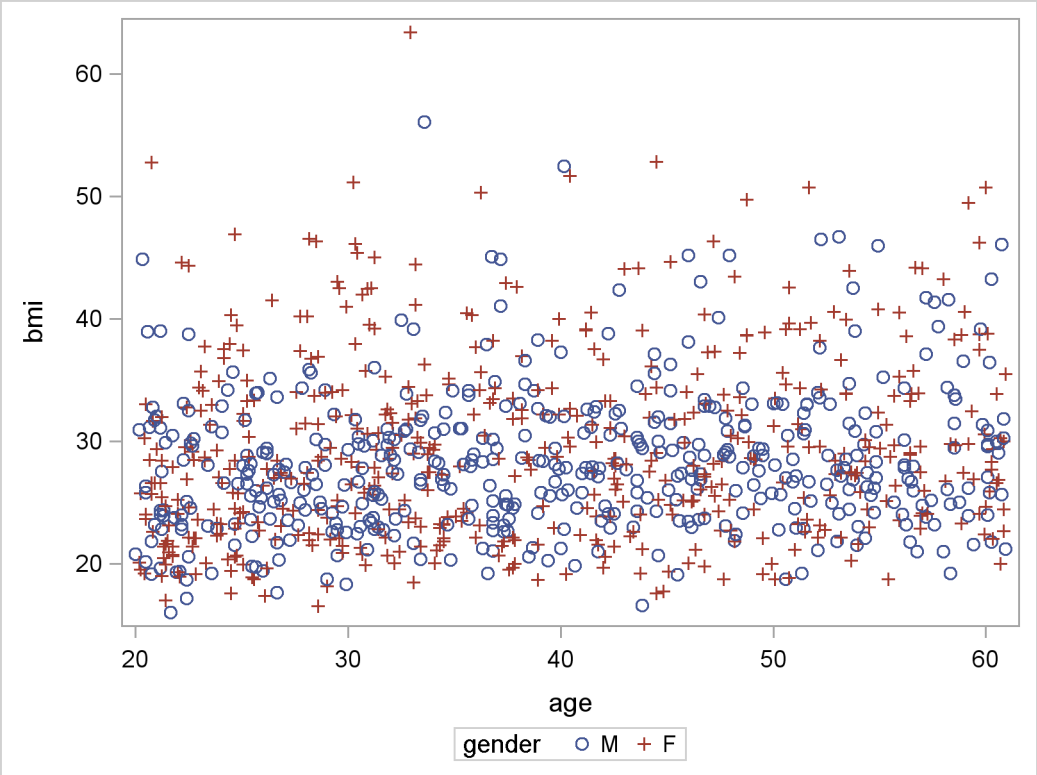
The following data set contains 526 female observations and 474 male observations sampled from 2003 National Health and Nutrition Examination Survey (NHANES). Each observation is composed of three values: *bmi* (body mass index), *age*, and *gender*, measured for subjects whose ages are between 20 and 60.

```
data one;
input bmi age gender$ @@;
datalines;
46.16 30.33 F 20.67 31.83 F 30.98 51.33 F 30.71 31.42 F
29.81 30.50 M 19.94 25.08 F 29.97 41.67 F 24.48 26.92 F
34.34 51.25 F 20.24 53.67 F 27.72 60.25 F 32.85 41.67 M
22.75 47.50 F 32.78 22.42 F 43.07 29.50 F 38.34 58.50 F
40.03 39.92 F 21.78 56.42 M 28.77 39.83 F 28.77 28.75 F

   ... more lines ...

26.53 27.92 F 23.77 29.00 F 29.86 60.58 M 25.41 44.08 M
26.53 24.83 M 33.33 42.08 F 30.52 32.50 F 31.89 38.17 F
32.20 35.92 F 21.73 26.67 M 32.10 39.33 M 25.13 51.75 M
;
run;
```

The goal of this analysis is to evaluate whether the *bmi-age* curves are different between women and men at a 5% significance level. In order to provide sufficient flexibility to model the effect of *age* on *bmi*, you can use regression splines that you define with an EFFECT statement. In this example, a regression spline of degree 2 with three knots is used for variable *age*. The knots are placed at the 25, 50, and 75 percentiles of *age*. This analysis assume that there is no interaction between *gender* and *age*, so that the *bmi-age* curves for women and men are the same up to a constant. The following statements produce the *bmi-age* scatter plot shown in Output 75.4.1:

```
proc sort data=one;
   by age;
run;

ods graphics on;
proc sgplot data=one;
   scatter x=age y=bmi/group=gender;
run;
```

Output 75.4.1 Scatter Plot for BMI Data



The observations with large *bmi* values (for example, *bmi*>40) are outliers that can substantially influence an ordinary least square (OLS) analysis. Output 75.4.1 shows that the distributions of *bmi* conditional on *age* are skewed toward the side of large *bmi*, and there are more observations with large *bmi* values (outliers) in the female group. Hence you can expect a significant *gender* difference in the *bmi-age* OLS regression analysis. This expectation is confirmed by the OLS *gender* p-value$=$ 0.0059 in Output 75.4.2, which is produced by the following statements:

```
proc glmselect data=one;
   class gender;
   effect age_sp=spl(age/degree=2 knotmethod=percentiles(3));
model bmi= gender age_sp /selection=none showpvalues;
output out=out_ols P=pred R=res;
run;
```

Output 75.4.2 OLS Estimates

The GLMSELECT Procedure
Least Squares Model (No Selection)

Parameter Estimates

Parameter		DF	Estimate	Standard Error	t Value	Pr > |t|
Intercept		1	29.890089	1.022825	29.22	<.0001
gender	F	1	1.167332	0.422565	2.76	0.0058
gender	M	0	0	.	.	.
age_sp	1	1	-4.404487	1.473761	-2.99	0.0029
age_sp	2	1	-3.329537	1.374096	-2.42	0.0156
age_sp	3	1	-0.966875	1.314964	-0.74	0.4623
age_sp	4	1	-1.611621	1.123854	-1.43	0.1519
age_sp	5	1	-0.484787	1.701281	-0.28	0.7757
age_sp	6	0	0	.	.	.

A robust regression method can reduce the outlier influence by automatically assigning smaller or even zero weights to outliers. For the *bmi* data, a robust regression method is likely to set less weight on observations with large *bmi*, so more female observations would receive smaller weights than male observations. The following statements invoke PROC ROBUSTREG with the *bmi* data set:

```
proc robustreg data=one method=s seed=100;
   class gender;
   effect age_sp=spl(age/degree=2 knotmethod=percentiles(3));
   model bmi = gender age_sp;
   output out=out_s P=pred R=res;
run;
```

Output 75.4.3 shows the parameter estimates and the diagnostics summary produced by PROC ROBUSTREG with the S method. In contrast to OLS, the robust p-value$= 0.5573$ of the *gender* coefficient indicates that the *gender* effect is not significant. The outlier diagnostics based on the S estimates find 19 outliers that are assigned lower weights by the S method than by the OLS method.

Output 75.4.3 S Estimates and S Diagnostics Summary

The ROBUSTREG Procedure

Parameter Estimates

Parameter		DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept		1	28.2858	1.0081	26.3100	30.2616	787.33	<.0001
gender	F	1	0.2409	0.4114	−0.5654	1.0473	0.34	0.5581
gender	M	0	0.0000	.	.	.	.	.
age_sp	1	1	−3.8956	1.4376	−6.7133	−1.0779	7.34	0.0067
age_sp	2	1	−1.8692	1.3430	−4.5014	0.7630	1.94	0.1640
age_sp	3	1	−0.8336	1.2877	−3.3574	1.6903	0.42	0.5174
age_sp	4	1	−0.2329	1.1055	−2.3997	1.9338	0.04	0.8331
age_sp	5	1	0.0055	1.6632	−3.2543	3.2652	0.00	0.9974
age_sp	6	0	0.0000	.	.	.	.	.
Scale		0	6.1715					

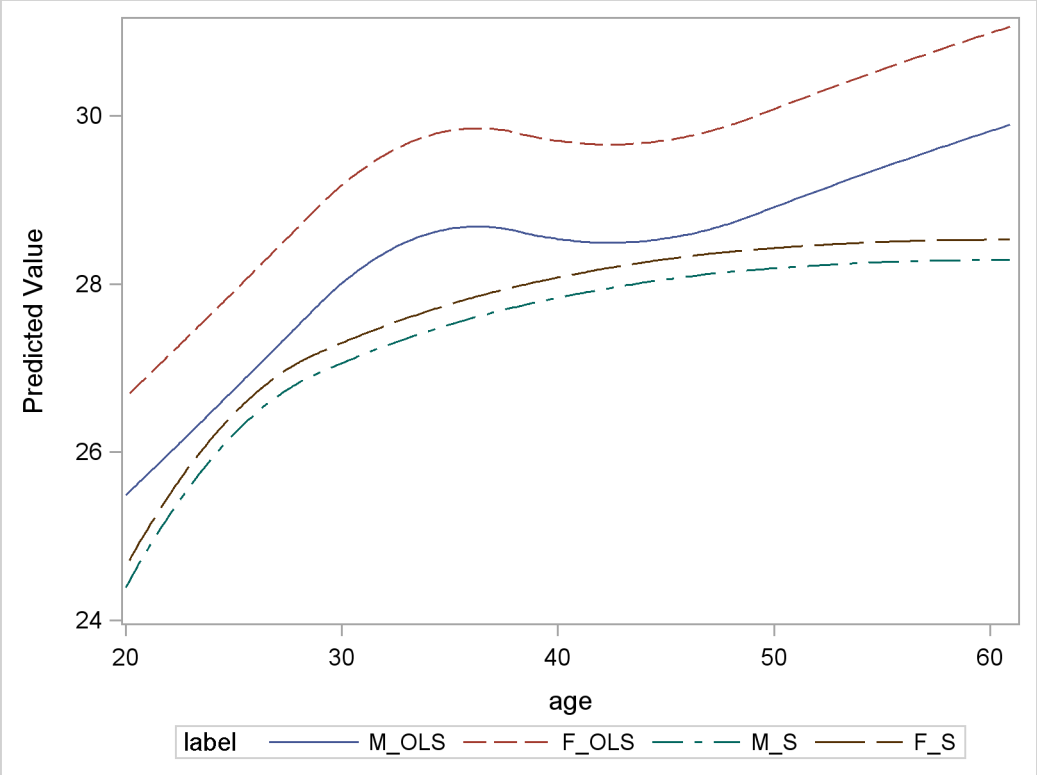
Diagnostics Summary

Observation Type	Proportion	Cutoff
Outlier	0.0190	3.0000

To further compare the OLS and S outputs, the following statements plot the *bmi* predictions in variable *age* for both methods in the same graph, which is shown in Output 75.4.4:

```
data out2_s;
   set out_s;
   if gender="F" then label="F_S  ";
   if gender="M" then label="M_S  ";
run;

data out2_ols;
   merge one out_ols;
   if gender='F' then label='F_OLS';
   if gender='M' then label='M_OLS';
   keep pred bmi gender age label;
run;

data out2;
   set out2_ols out2_s;
run;

proc sgplot data=out2;
   series x=age y=pred/group=label;
run;
```

Output 75.4.4 OLS and S Predictions



You can observe the following differences between the OLS and S predictions:

- The OLS prediction is larger
- The OLS curves have a local maximum near $age = 35$

Then, a question remains: is the significance of the *gender* effect for the OLS regression due solely to the outlying observations? To tentatively answer this question, the following statements drop the observations with the top 10% of *bmi* values from the original data set and reapply OLS and S methods on the reduced data set:

```
data three;
   set one;
   where bmi<38.315;
run;

proc robustreg data=three method=s seed=100;
   class gender;
   effect age_sp=spl(age/degree=2 knotmethod=percentiles(3));
   model bmi = gender age_sp;
   output out=out_s P=pred R=res;
run;

data out2_s;
   set out_s;
```

```
   if gender="F" then label="F_S  ";
   if gender="M" then label="M_S  ";
run;

proc glmselect data=three outdesign=four;
   class gender;
   effect age_sp=spl(age/degree=2 knotmethod=percentiles(3));
model bmi= gender age_sp /selection=none showpvalues;
output out=out_ols P=pred R=res;
run;

data out2_ols;
   merge three out_ols;
   if gender='F' then label='F_OLS';
   if gender='M' then label='M_OLS';
   keep pred bmi gender age label;
run;

data out2;
   set out2_ols out2_s;
run;

proc sgplot data=out2;
   series x=age y=pred/group=label;
run;
ods graphics off;
```

Output 75.4.5 S Estimates

The ROBUSTREG Procedure

Parameter Estimates

Parameter		DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept		1	27.6427	0.9741	25.7334	29.5520	805.23	<.0001
gender	F	1	-0.2650	0.4023	-1.0535	0.5234	0.43	0.5100
gender	M	0	0.0000	.	.	.	.	.
age_sp	1	1	-3.1859	1.4032	-5.9361	-0.4356	5.15	0.0232
age_sp	2	1	-1.5354	1.3051	-4.0934	1.0226	1.38	0.2394
age_sp	3	1	-0.3776	1.2499	-2.8273	2.0721	0.09	0.7626
age_sp	4	1	0.3299	1.0668	-1.7610	2.4208	0.10	0.7572
age_sp	5	1	0.0949	1.6221	-3.0845	3.2742	0.00	0.9534
age_sp	6	0	0.0000	.	.	.	.	.
Scale		0	4.9440					

Output 75.4.6 OLS Estimates

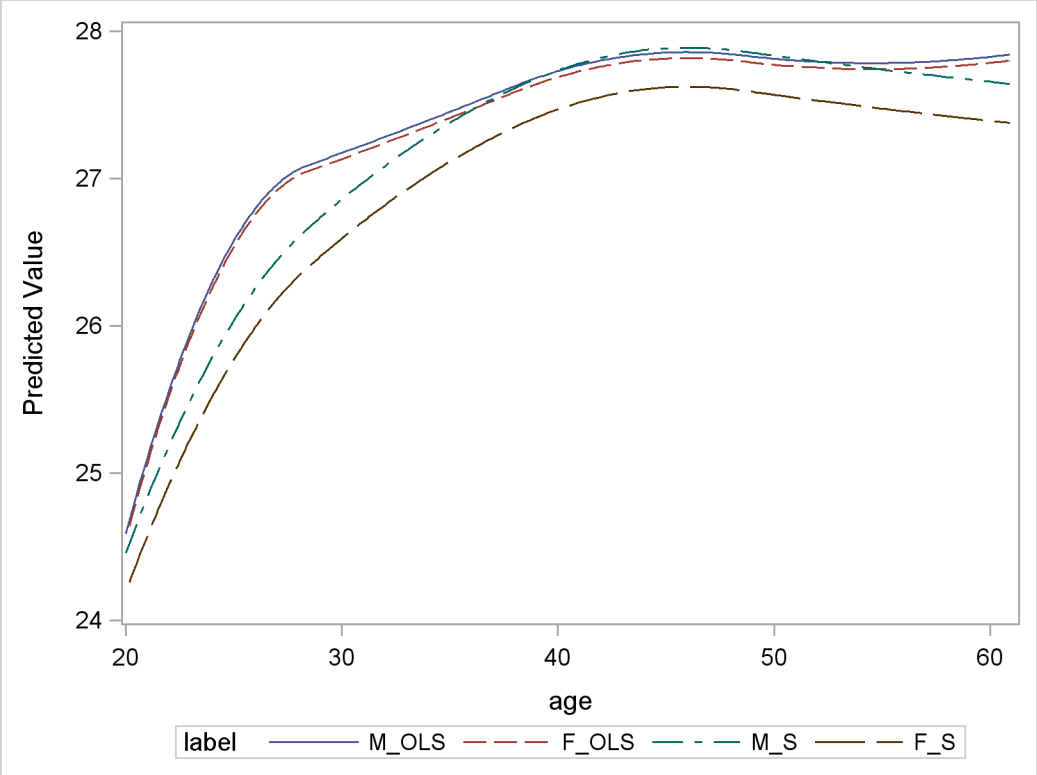
The GLMSELECT Procedure
Least Squares Model (No Selection)

Parameter Estimates

Parameter		DF	Estimate	Standard Error	t Value	Pr > |t|
Intercept		1	27.841568	0.780817	35.66	<.0001
gender	F	1	−0.040924	0.317749	−0.13	0.8976
gender	M	0	0	.	.	.
age_sp	1	1	−3.253964	1.121292	−2.90	0.0038
age_sp	2	1	−0.975273	1.034172	−0.94	0.3459
age_sp	3	1	−0.508979	0.999609	−0.51	0.6108
age_sp	4	1	0.089393	0.852774	0.10	0.9165
age_sp	5	1	−0.113706	1.298157	−0.09	0.9302
age_sp	6	0	0	.	.	.

In the reduced data set, 71 female observations and 29 male observations are dropped. Output 75.4.5 and Output 75.4.6 respectively show the refitted S and OLS parameter estimates, and Output 75.4.7 displays the fitted curves on the reduced data set. You can see that *gender* is no longer significant for the OLS model, and the OLS turning pattern has also disappeared, but the new S curves do not change much from the previous ones. The OLS *bmi-age* curves in Output 75.4.7 are closer to the S curves than to the OLS curves in Output 75.4.4. This suggests that indeed the difference between the OLS and S estimate results are due solely to the influence of the outlying observations.

Output 75.4.7 OLS and S Predictions on the Reduced Data Set



Example 75.5: Robust Diagnostics

This example models the selling price of a house as a function of several covariates. One of these covariates is a classification variable that indicates whether a house is located on a corner lot (called a corner house in this example). Because corner houses are relatively rare, the inclusion of this classification effect in the model introduces a low-dimensional structure (that is, the majority of the observations are located in a lower dimensional hyperplane defined by being non-corner houses) into the design matrix. As discussed in "Robust Distance" on page 6404, the presence of this low dimensional structure causes difficulties in the traditional computation of robust distances. This example illustrates how you can use the projected robust distance to address those difficulties and to obtain meaningful leverage diagnostics. It also shows how you can use the RDPLOT and DDPLOT options to illustrate the outlier-leverage relationship.

The following house price data set contains 66 home resale records on seven variables from February 15 to April 30, 1993 (The Data and Story Library, 2005). The records are randomly selected from the database maintained by the Albuquerque Board of Realtors.

```
data house;
  input price sqft age feats ne cor tax @@;
  label price = "Selling price"
        sqft  = "Square feet of living space"
        age   = "Age of home in year"
        feats = "Number out of 11 features (dishwasher, refrigerator,
                 microwave, disposer, washer, intercom, skylight(s),
                 compactor, dryer, handicap fit, cable TV access)"
        ne    = "Located in northeast sector of city (1) or not (0)"
        cor   = "Corner location (1) or not (0)"
        tax   = "Annual taxes";
  sum = sqft+age+feats+ne+cor+tax;
  id  = _N_;
  datalines;
   2050 2650 13 7 1 0 1639
   2150 2664  6 5 1 0 1193
   2150 2921  3 6 1 0 1635
   1999 2580  4 4 1 0 1732

... more lines ...

    869 1165  7 4 0 0  694
    766 1200  7 4 0 1  634
    739  970  4 4 0 1  541
   ;
   run;
```

To illustrate the dependence detection ability of the generalized MCD algorithm, an extra variable sum is created such that all the observations satisfy

$$\text{sum} = \text{sqft} + \text{age} + \text{feats} + \text{ne} + \text{cor} + \text{tax}$$

Adding sum does not change the rank of the original design matrix, so that sum is expected to be ignored in the model and also in the diagnostics. The next statements apply the MM method and the generalized MCD algorithm to the house price data.

```
ods graphics on;
ods trace output notes;
proc robustreg data=house method=MM plots=all;
   model price= sqft age feats ne cor tax sum/leverage(opc) diagnostics;
run;
ods trace off;
```

As shown in Output 75.5.1 and Output 75.5.2, PROC ROBUSTREG finds the design dependence equation and forces the parameter estimate of variable sum to be zero.

Output 75.5.1 MM Estimates

The ROBUSTREG Procedure

Parameter Estimates

Parameter	DF	Estimate	Standard Error	95% Confidence Limits		Chi-Square	Pr > ChiSq
Intercept	1	46.4062	79.1714	-108.767	201.5792	0.34	0.5578
sqft	1	0.3809	0.0756	0.2327	0.5291	25.37	<.0001
age	1	-2.6067	1.7610	-6.0582	0.8449	2.19	0.1388
feats	1	8.3627	14.7107	-20.4697	37.1951	0.32	0.5697
ne	1	65.0081	40.1329	-13.6508	143.6671	2.62	0.1053
cor	1	-19.2997	38.1907	-94.1520	55.5526	0.26	0.6133
tax	1	0.4699	0.1260	0.2229	0.7170	13.90	0.0002
sum	0	0.0000	.	.	.	.	.
Scale	0	157.5593					

Output 75.5.2 Design Dependence Equations

NOTE: The following variables have been ignored in the MCD computation because of linear dependence.

sum = sqft + age + feats + ne + cor + tax

Moreover, PROC ROBUSTREG also identifies a robust dependence equation on cor in Output 75.5.3, which holds for 77.27% of the observations but not for the entire data set.

Output 75.5.3 Robust Dependence Equations

NOTE: The following robust dependence equations simultaneously hold for 77.27% of the observations in the data set. The breakdown setting for the MCD algorithm is 22.73%.

cor = 0

Another way to represent the low-dimensional structure is to specify the coefficients of the MCD-dropped components on the data (see Output 75.5.4), which form a basis of the complementary space to the relevant low-dimensional hyperplane.

Output 75.5.4 Coefficients for MCD-Dropped Components

Coefficients for MCD-Dropped Components

Parameter	Design Drop0	Robust Drop1
sqft	0	0
age	0	0
feats	0	0
ne	0	0
cor	0	1.0000
tax	0	0
sum	1.0000	0

By definitions of projected robust distance and leverage point, an observation is called an off-plane leverage point if at least one of the robust or design dependence equations does not apply to the observation. In this example, the observations with cor $= 1$ are all off-plane leverage points. Output 75.5.5 lists the leverage points and outliers along with the relevant distance measurements and standardized residuals.

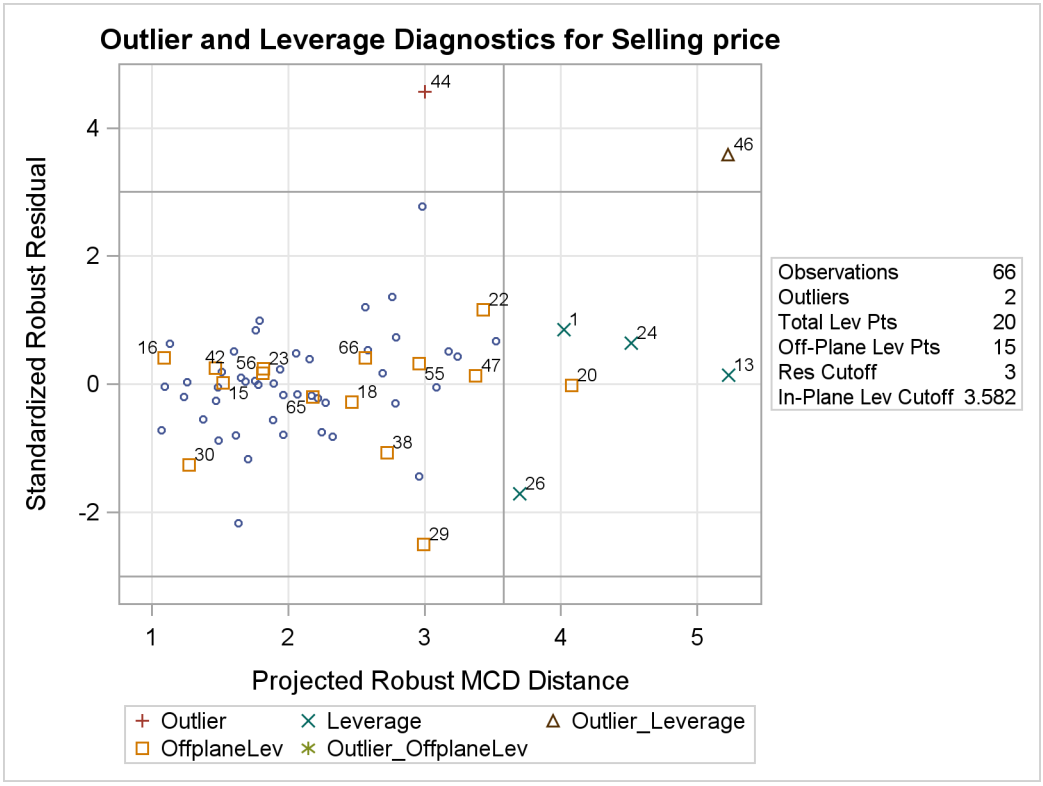
Output 75.5.5 Diagnostics

Diagnostics

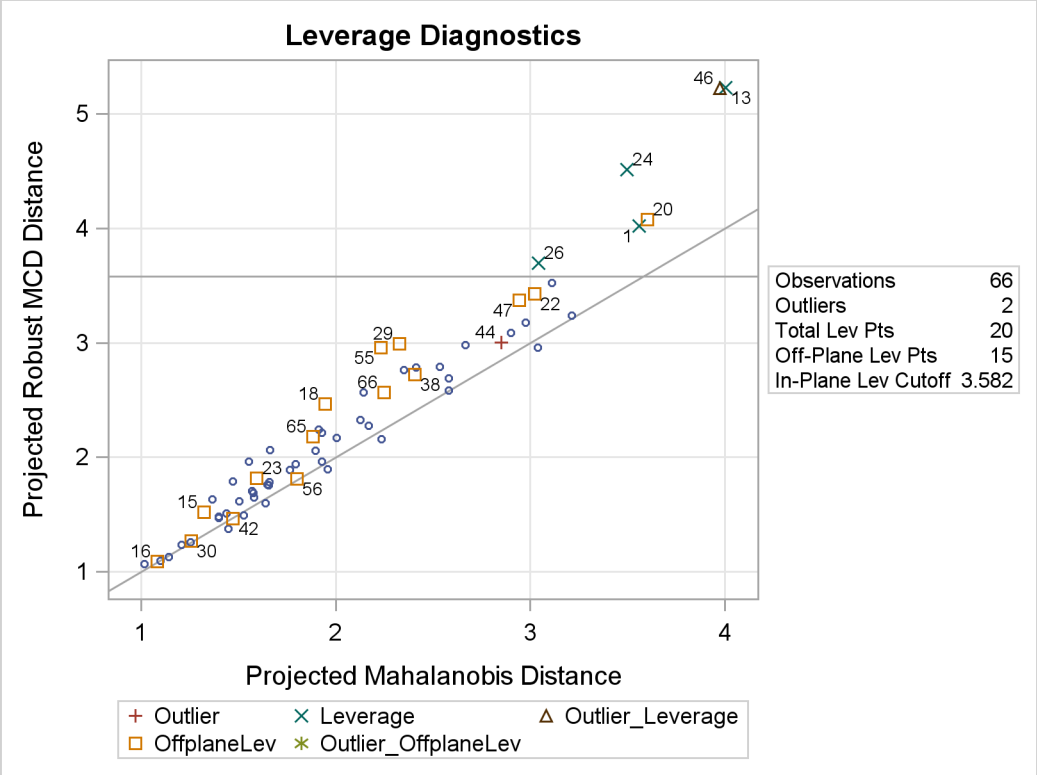
Obs	Projected Distance: Mahalanobis	Projected Distance: Robust	Projected Distance: Off-Plane	Leverage	Standardized Robust Residual	Outlier
1	3.5567	4.0211	0.0000	*	0.8522	
13	4.0034	5.2310	0.0000	*	0.1411	
15	1.3221	1.5219	2.3681	*	0.0226	
16	1.0839	1.0905	2.3681	*	0.4148	
18	1.9452	2.4655	2.3681	*	−0.2789	
20	3.6006	4.0771	2.3681	*	−0.0150	
22	3.0210	3.4307	2.3681	*	1.1664	
23	1.5920	1.8197	2.3681	*	0.2422	
24	3.4967	4.5154	0.0000	*	0.6464	
26	3.0420	3.6975	0.0000	*	−1.7068	
29	2.3264	2.9925	2.3681	*	−2.4980	
30	1.2587	1.2714	2.3681	*	−1.2558	
38	2.4064	2.7249	2.3681	*	−1.0620	
42	1.4722	1.4645	2.3681	*	0.2584	
44	2.8491	3.0019	0.0000		4.5665	*
46	3.9725	5.2271	0.0000	*	3.5835	*
47	2.9431	3.3728	2.3681	*	0.1365	
55	2.2325	2.9590	2.3681	*	0.3217	
56	1.7999	1.8119	2.3681	*	0.1715	
65	1.8831	2.1822	2.3681	*	−0.1990	
66	2.2483	2.5673	2.3681	*	0.4134	

From Output 75.5.6 and Output 75.5.7, you can see that there is no apparent corner-related difference for the houses in terms of standardized robust residual and projected MD versus projected RD, although all the corner houses are defined as off-plane leverage points.

Output 75.5.6 Projected RDPLOT



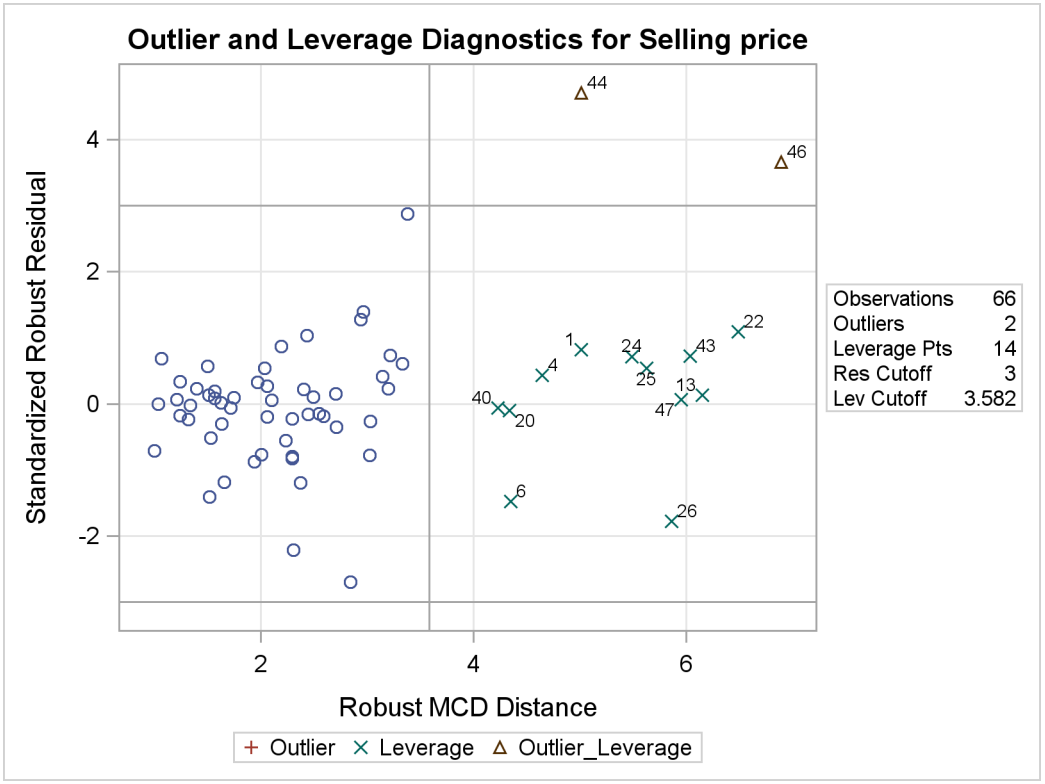
Output 75.5.7 Projected DDPLOT



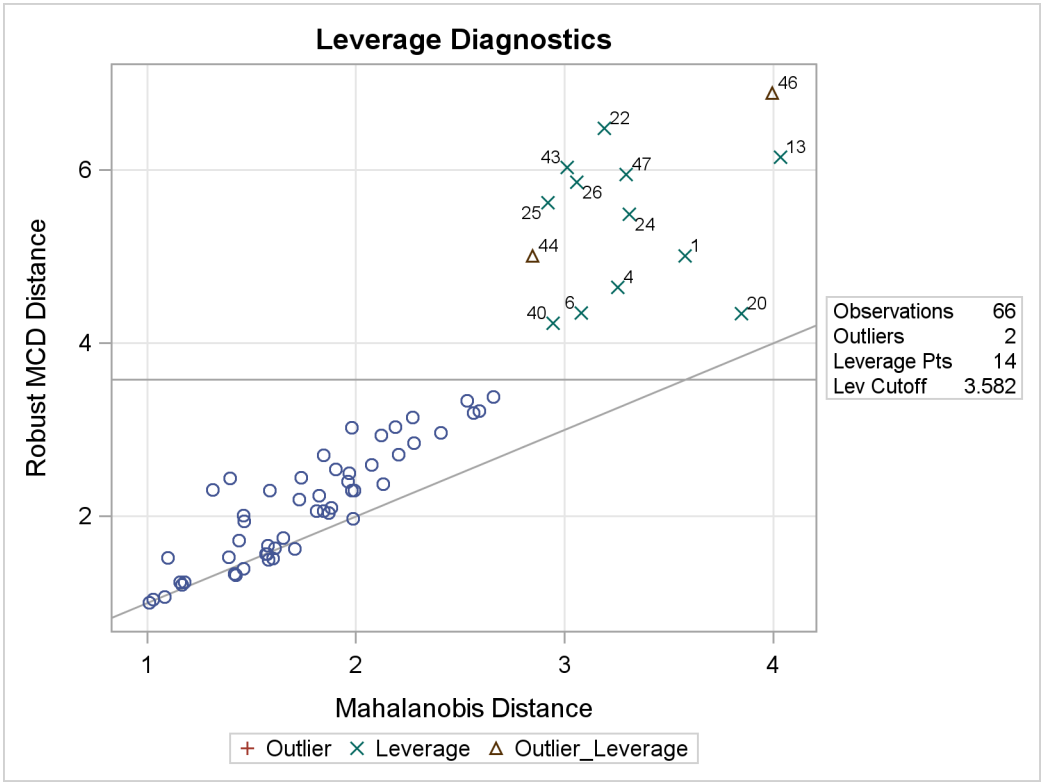
You might speculate that the projected MD and projected RD are equal to the regular MD and RD on the same data set without the variable cor. In fact, this is not true. (See Output 75.5.8 and Output 75.5.9 for the RDPLOT and DDPLOT on the data set without cor.) When included in the MODEL, cor is dropped in the distance calculation, but it is still used for the initial orthonormalization step and the h-subset searching. In this example, inclusion of cor causes all the other covariates to be centered separately for corner houses and non-corner houses. However, without cor, the centering process does not distinguish corner houses from non-corner houses, so that the MCD algorithm can still be influenced by cor through the correlation between cor and other covariates. The following statements drop the variable cor and produce the RDPLOT and DDPLOT for the reduced model, which are shown in Output 75.5.8 and Output 75.5.9:

```
proc robustreg data=house method=MM plots=all;
  model price= sqft age feats ne tax/leverage diagnostics;
run;
ods graphics off;
```

Output 75.5.8 RDPLOT on the Reduced Model



Output 75.5.9 DDPLOT on the Reduced Model



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Syntax Index

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