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SAS/STAT[®] 9.22 User's Guide

The ANOVA Procedure

(Book Excerpt)



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Chapter 23

The ANOVA Procedure

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Overview: ANOVA Procedure

The ANOVA procedure performs *analysis of variance* (ANOVA) for balanced data from a wide variety of experimental designs. In analysis of variance, a continuous response variable, known as a *dependent variable*, is measured under experimental conditions identified by classification variables, known as *independent variables*. The variation in the response is assumed to be due to effects in the classification, with random error accounting for the remaining variation.

The ANOVA procedure is one of several procedures available in SAS/STAT software for analysis of variance. The ANOVA procedure is designed to handle balanced data (that is, data with equal numbers of observations for every combination of the classification factors), whereas the GLM procedure can analyze both balanced and unbalanced data. Because PROC ANOVA takes into account the special structure of a balanced design, it is faster and uses less storage than PROC GLM for balanced data.

Use PROC ANOVA for the analysis of balanced data only, with the following exceptions: one-way analysis of variance, Latin square designs, certain partially balanced incomplete block designs, completely nested (hierarchical) designs, and designs with cell frequencies that are proportional to each other and are also proportional to the background population. These exceptions have designs in which the factors are all orthogonal to each other.

For further discussion, see Searle (1971, p. 138). PROC ANOVA works for designs with block diagonal $\mathbf{X}'\mathbf{X}$ matrices where the elements of each block all have the same value. The procedure partially tests this requirement by checking for equal cell means. However, this test is imperfect: some designs that cannot be analyzed correctly might pass the test, and designs that can be analyzed correctly might not pass. If your design does not pass the test, PROC ANOVA produces a warning message to tell you that the design is unbalanced and that the ANOVA analyses might not be valid; if your design is not one of the special cases described here, then you should use PROC GLM instead. Complete validation of designs is not performed in PROC ANOVA since this would require the whole $\mathbf{X}'\mathbf{X}$ matrix; if you're unsure about the validity of PROC ANOVA for your design, you should use PROC GLM.

CAUTION: If you use PROC ANOVA for analysis of unbalanced data, you must assume responsibility for the validity of the results.

The ANOVA procedure automatically produces graphics as part of its ODS output. For general information about ODS graphics, see the section “[ODS Graphics](#)” on page 849 and Chapter 21, “[Statistical Graphics Using ODS](#).”

Getting Started: ANOVA Procedure

The following examples demonstrate how you can use the ANOVA procedure to perform analyses of variance for a one-way layout and a randomized complete block design.

One-Way Layout with Means Comparisons

A one-way analysis of variance considers one treatment factor with two or more treatment levels. The goal of the analysis is to test for differences among the means of the levels and to quantify these differences. If there are two treatment levels, this analysis is equivalent to a t test comparing two group means.

The assumptions of analysis of variance (Steel and Torrie 1980) are that treatment effects are additive and experimental errors are independently random with a normal distribution that has mean zero and constant variance.

The following example studies the effect of bacteria on the nitrogen content of red clover plants. The treatment factor is bacteria strain, and it has six levels. Five of the six levels consist of five different *Rhizobium trifolii* bacteria cultures combined with a composite of five *Rhizobium meliloti* strains. The sixth level is a composite of the five *Rhizobium trifolii* strains with the composite of the *Rhizobium meliloti*. Red clover plants are inoculated with the treatments, and nitrogen content is later measured in milligrams. The data are derived from an experiment by Erdman (1946) and are analyzed in Chapters 7 and 8 of Steel and Torrie (1980). The following DATA step creates the SAS data set Clover:

```

title1 'Nitrogen Content of Red Clover Plants';
data Clover;
    input Strain $ Nitrogen @@;
    datalines;
3DOK1  19.4 3DOK1  32.6 3DOK1  27.0 3DOK1  32.1 3DOK1  33.0
3DOK5  17.7 3DOK5  24.8 3DOK5  27.9 3DOK5  25.2 3DOK5  24.3
3DOK4  17.0 3DOK4  19.4 3DOK4   9.1 3DOK4  11.9 3DOK4  15.8
3DOK7  20.7 3DOK7  21.0 3DOK7  20.5 3DOK7  18.8 3DOK7  18.6
3DOK13 14.3 3DOK13 14.4 3DOK13 11.8 3DOK13 11.6 3DOK13 14.2
COMPOS 17.3 COMPOS 19.4 COMPOS 19.1 COMPOS 16.9 COMPOS 20.8
    ;

```

The variable Strain contains the treatment levels, and the variable Nitrogen contains the response. The following statements produce the analysis.

```

proc anova data = Clover;
    class strain;
    model Nitrogen = Strain;
run;

```

The classification variable is specified in the **CLASS** statement. Note that, unlike the GLM procedure, PROC ANOVA does not allow continuous variables on the right-hand side of the model. [Figure 23.1](#) and [Figure 23.2](#) display the output produced by these statements.

Figure 23.1 Class Level Information

Nitrogen Content of Red Clover Plants							
The ANOVA Procedure							
Class Level Information							
Class	Levels	Values					
Strain	6	3DOK1	3DOK13	3DOK4	3DOK5	3DOK7	COMPOS
Number of Observations Read					30		
Number of Observations Used					30		

The “Class Level Information” table shown in [Figure 23.1](#) lists the variables that appear in the **CLASS** statement, their levels, and the number of observations in the data set.

[Figure 23.2](#) displays the ANOVA table, followed by some simple statistics and tests of effects.

Figure 23.2 ANOVA Table

Nitrogen Content of Red Clover Plants					
The ANOVA Procedure					
Dependent Variable: Nitrogen					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	847.046667	169.409333	14.37	<.0001
Error	24	282.928000	11.788667		
Corrected Total	29	1129.974667			
	R-Square	Coeff Var	Root MSE	Nitrogen Mean	
	0.749616	17.26515	3.433463	19.88667	
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Strain	5	847.046667	169.4093333	14.37	<.0001

The degrees of freedom (DF) column should be used to check the analysis results. The model degrees of freedom for a one-way analysis of variance are the number of levels minus 1; in this case, $6 - 1 = 5$. The Corrected Total degrees of freedom are always the total number of observations minus one; in this case $30 - 1 = 29$. The sum of Model and Error degrees of freedom equal the Corrected Total.

The overall F test is significant ($F = 14.37, p < 0.0001$), indicating that the model as a whole accounts for a significant portion of the variability in the dependent variable. The F test for Strain is significant, indicating that some contrast between the means for the different strains is different from zero. Notice that the Model and Strain F tests are identical, since Strain is the only term in the model.

The F test for Strain ($F = 14.37, p < 0.0001$) suggests that there are differences among the bacterial strains, but it does not reveal any information about the nature of the differences. Mean comparison methods can be used to gather further information. The interactivity of PROC ANOVA enables you to do this without re-running the entire analysis. After you specify a model with a **MODEL** statement and execute the ANOVA procedure with a **RUN** statement, you can execute a variety of statements (such as **MEANS**, **MANOVA**, **TEST**, and **REPEATED**) without PROC ANOVA recalculating the model sum of squares.

The following command requests means of the Strain levels with Tukey's studentized range procedure.

```
means strain / tukey;
```

Results of Tukey's procedure are shown in [Figure 23.3](#).

Figure 23.3 Tukey's Multiple Comparisons Procedure

Nitrogen Content of Red Clover Plants				
The ANOVA Procedure				
Tukey's Studentized Range (HSD) Test for Nitrogen				
Alpha				0.05
Error Degrees of Freedom				24
Error Mean Square				11.78867
Critical Value of Studentized Range				4.37265
Minimum Significant Difference				6.7142
Means with the same letter are not significantly different.				
Tukey Grouping		Mean	N	Strain
	A	28.820	5	3DOK1
	A			
B	A	23.980	5	3DOK5
B				
B	C	19.920	5	3DOK7
B	C			
B	C	18.700	5	COMPOS
	C			
	C	14.640	5	3DOK4
	C			
	C	13.260	5	3DOK13

Examples of implications of the multiple comparisons results are as follows:

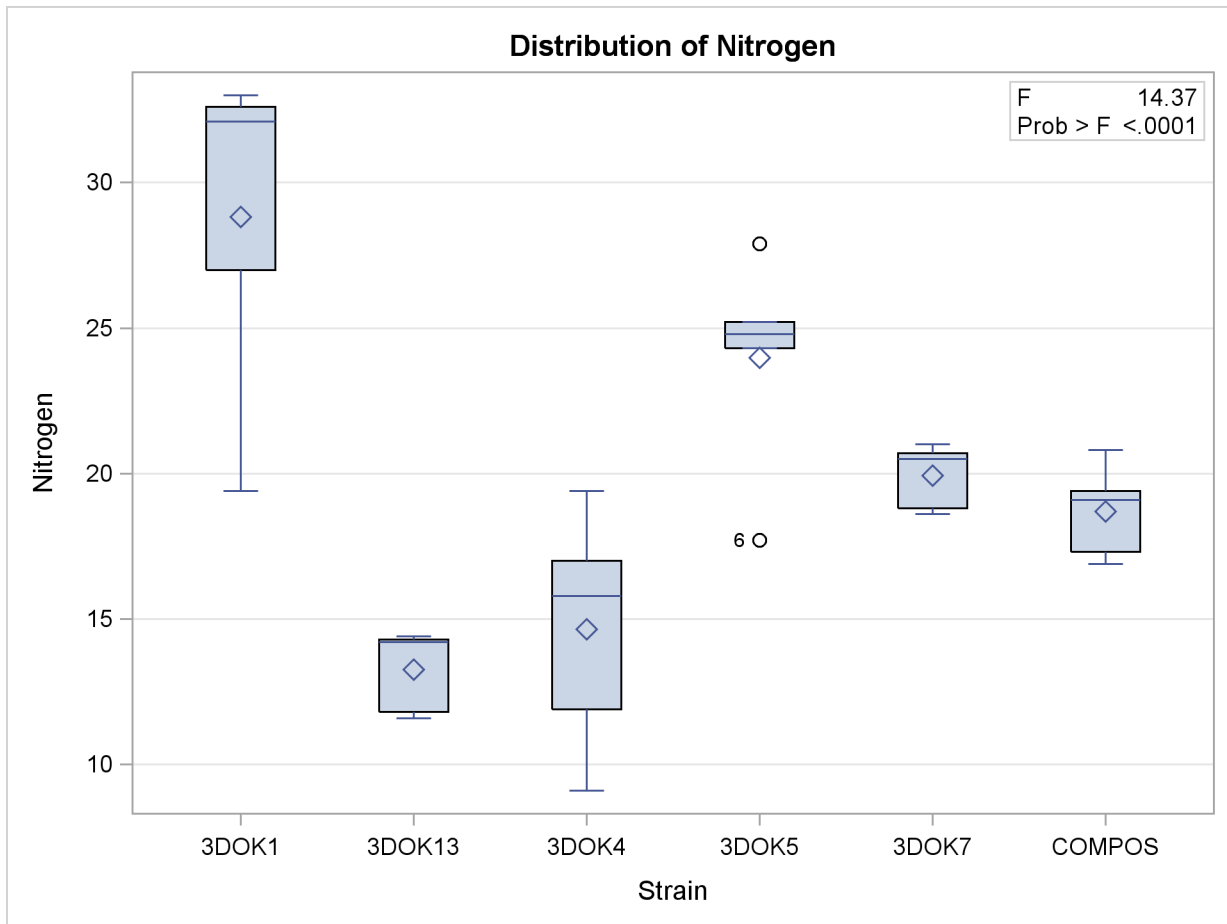
- Strain 3DOK1 fixes significantly more nitrogen than all but 3DOK5.
- While 3DOK5 is not significantly different from 3DOK1, it is also not significantly better than all the rest, though it is better than the bottom two groups.

Although the experiment has succeeded in separating the best strains from the worst, more experimentation is required in order to clearly distinguish the very best strain.

If you enable ODS Graphics, ANOVA also displays by default a plot that enables you to visualize the distribution of nitrogen content for each treatment. The following statements, which are the same as the previous analysis but with ODS graphics enabled, additionally produce [Figure 23.4](#).

```
ods graphics on;
proc anova data = Clover;
  class strain;
  model Nitrogen = Strain;
run;
ods graphics off;
```

When you specify the ODS GRAPHICS statement and fit a one-way analysis of variance model, the ANOVA procedure output includes a box plot of the dependent variable values within each classification level of the independent variable. For general information about ODS Graphics, see Chapter 21, “[Statistical Graphics Using ODS](#).” For specific information about the graphics available in the ANOVA procedure, see the section “[ODS Graphics](#)” on page 849.

Figure 23.4 Box Plot of Nitrogen Content for each Treatment

Randomized Complete Block with One Factor

This example illustrates the use of PROC ANOVA in analyzing a randomized complete block design. Researchers are interested in whether three treatments have different effects on the yield and worth of a particular crop. They believe that the experimental units are not homogeneous. So, a blocking factor is introduced that allows the experimental units to be homogeneous within each block. The three treatments are then randomly assigned within each block.

The data from this study are input into the SAS data set RCB:

```

title1 'Randomized Complete Block';
data RCB;
  input Block Treatment $ Yield Worth @@;
  datalines;
1 A 32.6 112    1 B 36.4 130    1 C 29.5 106
2 A 42.7 139    2 B 47.1 143    2 C 32.9 112
3 A 35.3 124    3 B 40.1 134    3 C 33.6 116
;

```

The variables *Yield* and *Worth* are continuous response variables, and the variables *Block* and *Treatment* are the classification variables. Because the data for the analysis are balanced, you can use PROC ANOVA to run the analysis.

The statements for the analysis are

```
proc anova data=RCB;
  class Block Treatment;
  model Yield Worth=Block Treatment;
run;
```

The *Block* and *Treatment* effects appear in the **CLASS** statement. The **MODEL** statement requests an analysis for each of the two dependent variables, *Yield* and *Worth*.

Figure 23.5 shows the “Class Level Information” table.

Figure 23.5 Class Level Information

Randomized Complete Block			
The ANOVA Procedure			
Class Level Information			
Class	Levels	Values	
Block	3	1 2 3	
Treatment	3	A B C	
Number of Observations Read			9
Number of Observations Used			9

The “Class Level Information” table lists the number of levels and their values for all effects specified in the **CLASS** statement. The number of observations in the data set are also displayed. Use this information to make sure that the data have been read correctly.

The overall ANOVA table for *Yield* in Figure 23.6 appears first in the output because it is the first response variable listed on the left side in the **MODEL** statement.

Figure 23.6 Overall ANOVA Table for Yield

Randomized Complete Block					
The ANOVA Procedure					
Dependent Variable: Yield					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	225.2777778	56.3194444	8.94	0.0283
Error	4	25.1911111	6.2977778		
Corrected Total	8	250.4688889			
	R-Square	Coeff Var	Root MSE	Yield Mean	
	0.899424	6.840047	2.509537	36.68889	

The overall F statistic is significant ($F = 8.94$, $p = 0.0283$), indicating that the model as a whole accounts for a significant portion of the variation in Yield and that you can proceed to evaluate the tests of effects.

The degrees of freedom (DF) are used to ensure correctness of the data and model. The Corrected Total degrees of freedom are one less than the total number of observations in the data set; in this case, $9 - 1 = 8$. The Model degrees of freedom for a randomized complete block are $(b - 1) + (t - 1)$, where b = number of block levels and t = number of treatment levels. In this case, this formula leads to $(3 - 1) + (3 - 1) = 4$ model degrees of freedom.

Several simple statistics follow the ANOVA table. The R-Square indicates that the model accounts for nearly 90% of the variation in the variable Yield. The coefficient of variation (C.V.) is listed along with the Root MSE and the mean of the dependent variable. The Root MSE is an estimate of the standard deviation of the dependent variable. The C.V. is a unitless measure of variability.

The tests of the effects shown in [Figure 23.7](#) are displayed after the simple statistics.

Figure 23.7 Tests of Effects for Yield

Source	DF	Anova SS	Mean Square	F Value	Pr > F
Block	2	98.1755556	49.0877778	7.79	0.0417
Treatment	2	127.1022222	63.5511111	10.09	0.0274

For Yield, both the Block and Treatment effects are significant ($F = 7.79$, $p = 0.0417$ and $F = 10.09$, $p = 0.0274$, respectively) at the 95% level. From this you can conclude that blocking is useful for this variable and that some contrast between the treatment means is significantly different from zero.

Figure 23.8 shows the ANOVA table, simple statistics, and tests of effects for the variable Worth.

Figure 23.8 ANOVA Table for Worth

Randomized Complete Block					
The ANOVA Procedure					
Dependent Variable: Worth					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	1247.333333	311.833333	8.28	0.0323
Error	4	150.666667	37.666667		
Corrected Total	8	1398.000000			
	R-Square	Coeff Var	Root MSE	Worth Mean	
	0.892227	4.949450	6.137318	124.0000	
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Block	2	354.6666667	177.3333333	4.71	0.0889
Treatment	2	892.6666667	446.3333333	11.85	0.0209

The overall F test is significant ($F = 8.28$, $p = 0.0323$) at the 95% level for the variable Worth. The Block effect is not significant at the 0.05 level but is significant at the 0.10 confidence level ($F = 4.71$, $p = 0.0889$). Generally, the usefulness of blocking should be determined before the analysis. However, since there are two dependent variables of interest, and Block is significant for one of them (Yield), blocking appears to be generally useful. For Worth, as with Yield, the effect of Treatment is significant ($F = 11.85$, $p = 0.0209$).

Issuing the following command produces the Treatment means.

```
means Treatment;
run;
```

Figure 23.9 displays the treatment means and their standard deviations for both dependent variables.

Figure 23.9 Means of Yield and Worth

Randomized Complete Block					
The ANOVA Procedure					
Level of Treatment	N	-----Yield-----		-----Worth-----	
		Mean	Std Dev	Mean	Std Dev
A	3	36.8666667	5.22908532	125.000000	13.5277493
B	3	41.2000000	5.43415127	135.666667	6.6583281
C	3	32.0000000	2.19317122	111.333333	5.0332230

Syntax: ANOVA Procedure

The following statements are available in PROC ANOVA.

```

PROC ANOVA < options > ;
  CLASS variables < / option > ;
  MODEL dependents=effects < / options > ;
  ABSORB variables ;
  BY variables ;
  FREQ variable ;
  MANOVA < test-options > < / detail-options > ;
  MEANS effects < / options > ;
  REPEATED factor-specification < / options > ;
  TEST < H=effects > E=effect ;

```

The **PROC ANOVA**, **CLASS**, and **MODEL** statements are required, and they must precede the first **RUN** statement. The **CLASS** statement must precede the **MODEL** statement. If you use the **ABSORB**, **FREQ**, or **BY** statement, it must precede the first **RUN** statement. The **MANOVA**, **MEANS**, **REPEATED**, and **TEST** statements must follow the **MODEL** statement, and they can be specified in any order. These four statements can also appear after the first **RUN** statement.

Table 23.1 summarizes the function of each statement (other than the **PROC** statement) in the ANOVA procedure:

Table 23.1 Statements in the ANOVA Procedure

Statement	Description
ABSORB	absorbs classification effects in a model
BY	specifies variables to define subgroups for the analysis
CLASS	declares classification variables
FREQ	specifies a frequency variable
MANOVA	performs a multivariate analysis of variance
MEANS	computes and compares means
MODEL	defines the model to be fit
REPEATED	performs multivariate and univariate repeated measures analysis of variance
TEST	constructs tests that use the sums of squares for effects and the error term you specify

PROC ANOVA Statement

```

PROC ANOVA < options > ;

```

The **PROC ANOVA** statement starts the ANOVA procedure.

You can specify the following options in the **PROC ANOVA** statement:

DATA=SAS-data-set

names the SAS data set used by the ANOVA procedure. By default, PROC ANOVA uses the most recently created SAS data set.

MANOVA

requests the multivariate mode of eliminating observations with missing values. If any of the dependent variables have missing values, the procedure eliminates that observation from the analysis. The MANOVA option is useful if you use PROC ANOVA in interactive mode and plan to perform a multivariate analysis.

MULTIPASS

requests that PROC ANOVA reread the input data set, when necessary, instead of writing the values of dependent variables to a utility file. This option decreases disk space usage at the expense of increased execution times and is useful only in rare situations where disk space is at an absolute premium.

NAMELEN=*n*

specifies the length of effect names to be *n* characters long, where *n* is a value between 20 and 200 characters. The default length is 20 characters.

NOPRINT

suppresses the normal display of results. The NOPRINT option is useful when you want to create only the output data set with the procedure. Note that this option temporarily disables the Output Delivery System (ODS); see Chapter 20, “[Using the Output Delivery System](#),” for more information.

ORDER=DATA | FORMATTED | FREQ | INTERNAL

specifies the order in which to sort the levels of the classification variables (which are specified in the [CLASS](#) statement). This option applies to the levels for all classification variables, except when you use the (default) ORDER=FORMATTED option with numeric classification variables that have no explicit format. With this option, the levels of such variables are ordered by their internal value.

The ORDER= option can take the following values:

Value of ORDER=	Levels Sorted By
DATA	Order of appearance in the input data set
FORMATTED	External formatted value, except for numeric variables with no explicit format, which are sorted by their unformatted (internal) value
FREQ	Descending frequency count; levels with the most observations come first in the order
INTERNAL	Unformatted value

By default, ORDER=FORMATTED. For FORMATTED and INTERNAL, the sort order is machine-dependent.

OUTSTAT=SAS-data-set

names an output data set that contains sums of squares, degrees of freedom, F statistics, and probability levels for each effect in the model. If you use the CANONICAL option in the MANOVA statement and do not use an M= specification in the MANOVA statement, the data set also contains results of the canonical analysis. See the section “Output Data Set” on page 844 for more information.

PLOTS=NONE

controls the plots produced through ODS Graphics. When ODS Graphics is enabled, the ANOVA procedure can display a grouped box-plot of the input data with groups defined by an effect in the model. Such a plot is produced by default if you have a one-way model, with only a single classification variable, or if you use a MEANS statement. Specify the PLOTS=NONE option in order to prevent these plots from being produced when ODS Graphics is enabled.

ABSORB Statement

ABSORB variables ;

Absorption is a computational technique that provides a large reduction in time and memory requirements for certain types of models. The *variables* are one or more variables in the input data set.

For a main effect variable that does not participate in interactions, you can absorb the effect by naming it in an ABSORB statement. This means that the effect can be adjusted out before the construction and solution of the rest of the model. This is particularly useful when the effect has a large number of levels.

Several variables can be specified, in which case each one is assumed to be nested in the preceding variable in the ABSORB statement.

NOTE: When you use the ABSORB statement, the data set (or each BY group, if a BY statement appears) must be sorted by the variables in the ABSORB statement. Including an absorbed variable in the CLASS list or in the MODEL statement might produce erroneous sums of squares. If the ABSORB statement is used, it must appear before the first RUN statement or it is ignored.

When you use an ABSORB statement and also use the INT option in the MODEL statement, the procedure ignores the option but produces the uncorrected total sum of squares (SS) instead of the corrected total SS.

See the section “Absorption” on page 3063 in Chapter 39, “The GLM Procedure,” for more information.

BY Statement

BY *variables* ;

You can specify a BY statement with PROC ANOVA to obtain separate analyses on observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the ANOVA procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

Since sorting the data changes the order in which PROC ANOVA reads observations, the sorting order for the levels of the classification variables might be affected if you have also specified the ORDER=DATA option in the PROC ANOVA statement.

If the BY statement is used, it must appear before the first RUN statement, or it is ignored. When you use a BY statement, the interactive features of PROC ANOVA are disabled.

When both a BY and an ABSORB statement are used, observations must be sorted first by the variables in the BY statement, and then by the variables in the ABSORB statement.

For more information about BY-group processing, see the discussion in *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the discussion in the *Base SAS Procedures Guide*.

CLASS Statement

CLASS *variables* < / **TRUNCATE** > ;

The CLASS statement names the classification variables to be used in the model. Typical classification variables are Treatment, Sex, Race, Group, and Replication. If you use the CLASS statement, it must appear before the MODEL statement.

Classification variables can be either character or numeric. By default, class levels are determined from the entire set of formatted values of the CLASS variables.

NOTE: Prior to SAS 9, class levels were determined by using no more than the first 16 characters of the formatted values. To revert to this previous behavior, you can use the TRUNCATE option in the CLASS statement.

In any case, you can use formats to group values into levels. See the discussion of the FORMAT procedure in the *Base SAS Procedures Guide* and the discussions of the FORMAT statement and SAS formats in *SAS Language Reference: Dictionary*. You can adjust the order of CLASS variable levels with the ORDER= option in the PROC ANOVA statement. You can specify the following option in the CLASS statement after a slash (/):

TRUNCATE

specifies that class levels should be determined by using only up to the first 16 characters of the formatted values of CLASS variables. When formatted values are longer than 16 characters, you can use this option to revert to the levels as determined in releases prior to SAS 9.

FREQ Statement

FREQ *variable* ;

The FREQ statement names a variable that provides frequencies for each observation in the DATA= data set. Specifically, if n is the value of the FREQ variable for a given observation, then that observation is used n times.

The analysis produced by using a FREQ statement reflects the expanded number of observations. For example, means and total degrees of freedom reflect the expanded number of observations. You can produce the same analysis (without the FREQ statement) by first creating a new data set that contains the expanded number of observations. For example, if the value of the FREQ variable is 5 for the first observation, the first 5 observations in the new data set would be identical. Each observation in the old data set would be replicated n_i times in the new data set, where n_i is the value of the FREQ variable for that observation.

If the value of the FREQ variable is missing or is less than 1, the observation is not used in the analysis. If the value is not an integer, only the integer portion is used.

If the FREQ statement is used, it must appear before the first RUN statement or it is ignored.

MANOVA Statement

MANOVA < *test-options* > < *detail-options* > ;

If the MODEL statement includes more than one dependent variable, you can perform multivariate analysis of variance with the MANOVA statement. The *test-options* define which effects to test, while the *detail-options* specify how to execute the tests and what results to display.

When a MANOVA statement appears before the first RUN statement, PROC ANOVA enters a multivariate mode with respect to the handling of missing values; in addition to observations with

missing independent variables, observations with *any* missing dependent variables are excluded from the analysis. If you want to use this mode of handling missing values but do not need any multivariate analyses, specify the MANOVA option in the [PROC ANOVA](#) statement.

Test Options

You can specify the following options in the MANOVA statement as *test-options* in order to define which multivariate tests to perform.

H=effects | INTERCEPT | _ALL_

specifies effects in the preceding model to use as hypothesis matrices. For each SSCP matrix **H** associated with an effect, the H= specification computes an analysis based on the characteristic roots of $\mathbf{E}^{-1}\mathbf{H}$, where **E** is the matrix associated with the error effect. The characteristic roots and vectors are displayed, along with the Hotelling-Lawley trace, Pillai's trace, Wilks' lambda, and Roy's greatest root. By default, these statistics are tested with approximations based on the *F* distribution. To test them with exact (but computationally intensive) calculations, use the [MSTAT=EXACT](#) option.

Use the keyword INTERCEPT to produce tests for the intercept. To produce tests for all effects listed in the [MODEL](#) statement, use the keyword [_ALL_](#) in place of a list of effects.

For background and further details, see the section “[Multivariate Analysis of Variance](#)” on page 3087 in Chapter 39, “[The GLM Procedure](#).”

E=effect

specifies the error effect. If you omit the E= specification, the ANOVA procedure uses the error SSCP (residual) matrix from the analysis.

M=equation,...,equation | (row-of-matrix,...,row-of-matrix)

specifies a transformation matrix for the dependent variables listed in the [MODEL](#) statement. The equations in the M= specification are of the form

$$\begin{aligned} c_1 \times \text{dependent-variable} &\pm c_2 \times \text{dependent-variable} \\ &\dots \pm c_n \times \text{dependent-variable} \end{aligned}$$

where the c_i values are coefficients for the various *dependent-variables*. If the value of a given c_i is 1, it can be omitted; in other words $1 \times Y$ is the same as Y . Equations should involve two or more dependent variables. For sample syntax, see the section “[Examples](#)” on page 828.

Alternatively, you can input the transformation matrix directly by entering the elements of the matrix with commas separating the rows, and parentheses surrounding the matrix. When this alternate form of input is used, the number of elements in each row must equal the number of dependent variables. Although these combinations actually represent the columns of the **M** matrix, they are displayed by rows.

When you include an M= specification, the analysis requested in the MANOVA statement is carried out for the variables defined by the equations in the specification, not the original

dependent variables. If you omit the **M=** option, the analysis is performed for the original dependent variables in the **MODEL** statement.

If an **M=** specification is included without either the **MNAMES=** or the **PREFIX=** option, the variables are labeled MVAR1, MVAR2, and so forth by default.

For further information, see the section “[Multivariate Analysis of Variance](#)” on page 3087 in Chapter 39, “[The GLM Procedure](#).”

MNAMES=names

provides names for the variables defined by the equations in the **M=** specification. Names in the list correspond to the **M=** equations or the rows of the **M** matrix (as it is entered).

PREFIX=name

is an alternative means of identifying the transformed variables defined by the **M=** specification. For example, if you specify **PREFIX=DIFF**, the transformed variables are labeled DIFF1, DIFF2, and so forth.

Detail Options

You can specify the following options in the MANOVA statement after a slash as *detail-options*:

CANONICAL

produces a canonical analysis of the **H** and **E** matrices (transformed by the **M** matrix, if specified) instead of the default display of characteristic roots and vectors.

MSTAT=FAPPROX

MSTAT=EXACT

specifies the method of evaluating the multivariate test statistics. The default is **MSTAT=FAPPROX**, which specifies that the multivariate tests are evaluated by using the usual approximations based on the *F* distribution, as discussed in the “Multivariate Tests” section in Chapter 4, “[Introduction to Regression Procedures](#).” Alternatively, you can specify **MSTAT=EXACT** to compute exact *p*-values for three of the four tests (Wilks’ lambda, the Hotelling-Lawley trace, and Roy’s greatest root) and an improved *F*-approximation for the fourth (Pillai’s trace). While **MSTAT=EXACT** provides better control of the significance probability for the tests, especially for Roy’s Greatest Root, computations for the exact *p*-values can be appreciably more demanding, and are in fact infeasible for large problems (many dependent variables). Thus, although **MSTAT=EXACT** is more accurate for most data, it is not the default method. For more information about the results of **MSTAT=EXACT**, see the section “[Multivariate Analysis of Variance](#)” on page 3087 in Chapter 39, “[The GLM Procedure](#).”

ORTH

requests that the transformation matrix in the **M=** specification of the MANOVA statement be orthonormalized by rows before the analysis.

PRINTE

displays the error SSCP matrix **E**. If the **E** matrix is the error SSCP (residual) matrix from the analysis, the partial correlations of the dependent variables given the independent variables are also produced.

For example, the statement

```
manova / printe;
```

displays the error SSCP matrix and the partial correlation matrix computed from the error SSCP matrix.

PRINTH

displays the hypothesis SSCP matrix **H** associated with each effect specified by the **H=** specification.

SUMMARY

produces analysis-of-variance tables for each dependent variable. When no **M** matrix is specified, a table is produced for each original dependent variable from the **MODEL** statement; with an **M** matrix other than the identity, a table is produced for each transformed variable defined by the **M** matrix.

Examples

The following statements give several examples of using a MANOVA statement.

```
proc anova;
  class A B;
  model Y1-Y5=A B(A);
  manova h=A e=B(A) / printh printe;
  manova h=B(A) / printe;
  manova h=A e=B(A) m=Y1-Y2, Y2-Y3, Y3-Y4, Y4-Y5
    prefix=diff;

  manova h=A e=B(A) m=(1 -1 0 0 0,
                      0 1 -1 0 0,
                      0 0 1 -1 0,
                      0 0 0 1 -1) prefix=diff;

run;
```

The first MANOVA statement specifies **A** as the hypothesis effect and **B(A)** as the error effect. As a result of the **PRINTH** option, the procedure displays the hypothesis SSCP matrix associated with the **A** effect; and, as a result of the **PRINTE** option, the procedure displays the error SSCP matrix associated with the **B(A)** effect.

The second MANOVA statement specifies **B(A)** as the hypothesis effect. Since no error effect is specified, PROC ANOVA uses the error SSCP matrix from the analysis as the **E** matrix. The **PRINTE** option displays this **E** matrix. Since the **E** matrix is the error SSCP matrix from the analysis, the partial correlation matrix computed from this matrix is also produced.

The third MANOVA statement requests the same analysis as the first MANOVA statement, but the analysis is carried out for variables transformed to be successive differences between the original dependent variables. The **PREFIX=DIFF** specification labels the transformed variables as DIFF1, DIFF2, DIFF3, and DIFF4.

Finally, the fourth MANOVA statement has the identical effect as the third, but it uses an alternative form of the **M=** specification. Instead of specifying a set of equations, the fourth MANOVA statement specifies rows of a matrix of coefficients for the five dependent variables.

As a second example of the use of the **M=** specification, consider the following:

```
proc anova;
  class group;
  model dose1-dose4=group / nouni;
  manova h = group
    m = -3*dose1 -    dose2 +    dose3 + 3*dose4,
        dose1 -    dose2 -    dose3 +    dose4,
        -dose1 + 3*dose2 - 3*dose3 +    dose4
    mnames = Linear Quadratic Cubic
    / printe;
run;
```

The **M=** specification gives a transformation of the dependent variables dose1 through dose4 into orthogonal polynomial components, and the **MNAMES=** option labels the transformed variables as **LINEAR**, **QUADRATIC**, and **CUBIC**, respectively. Since the **PRINTE** option is specified and the default residual matrix is used as an error term, the partial correlation matrix of the orthogonal polynomial components is also produced.

For further information, see the section “[Multivariate Analysis of Variance](#)” on page 3087 in Chapter 39, “[The GLM Procedure](#).”

MEANS Statement

MEANS *effects* </ options> ;

PROC ANOVA can compute means of the dependent variables for any effect that appears on the right-hand side in the **MODEL** statement.

You can use any number of **MEANS** statements, provided that they appear after the **MODEL** statement. For example, suppose A and B each have two levels. Then, if you use the following statements

```
proc anova;
  class A B;
  model Y=A B A*B;
  means A B / tukey;
  means A*B;
run;
```

means, standard deviations, and Tukey’s multiple comparison tests are produced for each level of the

main effects A and B, and just the means and standard deviations for each of the four combinations of levels for A*B. Since multiple comparisons options apply only to main effects, the single MEANS statement

```
means A B A*B / tukey;
```

produces the same results.

Options are provided to perform multiple comparison tests for only main effects in the model. PROC ANOVA does not perform multiple comparison tests for interaction terms in the model; for multiple comparisons of interaction terms, see the LSMEANS statement in Chapter 39, “[The GLM Procedure](#).”

Table 23.2 summarizes categories of options available in the MEANS statement.

Table 23.2 Options Available in the MEANS Statement

Task	Available options
Perform multiple comparison tests	BON
	DUNCAN
	DUNNETT
	DUNNETTL
	DUNNETTU
	GABRIEL
	GT2
	LSD
	REGWQ
	SCHEFFE
	SIDAK
Perform multiple comparison tests	SMM
	SNK
	T
	TUKEY
	WALLER
Specify additional details for multiple comparison tests	ALPHA=
	CLDIFF
	CLM
	E=
	KRATIO=
Test for homogeneity of variances	LINES
	NOSORT
	HOVTEST
Compensate for heterogeneous variances	WELCH

Descriptions of these options follow. For a further discussion of these options, see the section “[Multiple Comparisons](#)” on page 3070 in Chapter 39, “[The GLM Procedure](#).”

ALPHA=*p*

specifies the level of significance for comparisons among the means. By default, ALPHA=0.05. You can specify any value greater than 0 and less than 1.

BON

performs Bonferroni *t* tests of differences between means for all main effect means in the MEANS statement. See the **CLDIFF** and **LINES** options, which follow, for a discussion of how the procedure displays results.

CLDIFF

presents results of the **BON**, **GABRIEL**, **SCHEFFE**, **SIDAK**, **SMM**, **GT2**, **T**, **LSD**, and **TUKEY** options as confidence intervals for all pairwise differences between means, and the results of the **DUNNETT**, **DUNNETTU**, and **DUNNETTL** options as confidence intervals for differences with the control. The CLDIFF option is the default for unequal cell sizes unless the **DUNCAN**, **REGWQ**, **SNK**, or **WALLER** option is specified.

CLM

presents results of the **BON**, **GABRIEL**, **SCHEFFE**, **SIDAK**, **SMM**, **T**, and **LSD** options as intervals for the mean of each level of the variables specified in the MEANS statement. For all options except **GABRIEL**, the intervals are confidence intervals for the true means. For the **GABRIEL** option, they are *comparison intervals* for comparing means pairwise: in this case, if the intervals corresponding to two means overlap, the difference between them is insignificant according to Gabriel's method.

DUNCAN

performs Duncan's multiple range test on all main effect means given in the MEANS statement. See the **LINES** option for a discussion of how the procedure displays results.

DUNNETT <(formatted-control-values)>

performs Dunnett's two-tailed *t* test, testing if any treatments are significantly different from a single control for all main effects means in the MEANS statement.

To specify which level of the effect is the control, enclose the formatted value in quotes in parentheses after the keyword. If more than one effect is specified in the MEANS statement, you can use a list of control values within the parentheses. By default, the first level of the effect is used as the control. For example,

```
means a / dunnett('CONTROL');
```

where CONTROL is the formatted control value of A. As another example,

```
means a b c / dunnett('CNTLA' 'CNTLB' 'CNTLC');
```

where CNTLA, CNTLB, and CNTLC are the formatted control values for A, B, and C, respectively.

DUNNETTL <(formatted-control-value)>

performs Dunnett's one-tailed *t* test, testing if any treatment is significantly less than the control. Control level information is specified as described previously for the **DUNNETT** option.

DUNNETTU <(formatted-control-value)>

performs Dunnett's one-tailed t test, testing if any treatment is significantly greater than the control. Control level information is specified as described previously for the **DUNNETT** option.

E=effect

specifies the error mean square used in the multiple comparisons. By default, PROC ANOVA uses the residual Mean Square (MS). The effect specified with the E= option must be a term in the model; otherwise, the procedure uses the residual MS.

GABRIEL

performs Gabriel's multiple-comparison procedure on all main effect means in the MEANS statement. See the **CLDIFF** and **LINES** options for discussions of how the procedure displays results.

GT2

see the **SMM** option.

HOVTEST**HOVTEST=BARTLETT****HOVTEST=BF****HOVTEST=LEVENE** <(TYPE=ABS | SQUARE)>**HOVTEST=OBRIEN** <(W=number)>

requests a homogeneity of variance test for the groups defined by the MEANS effect. You can optionally specify a particular test; if you do not specify a test, Levene's test (Levene 1960) with TYPE=SQUARE is computed. Note that this option is ignored unless your **MODEL** statement specifies a simple one-way model.

The HOVTEST=BARTLETT option specifies Bartlett's test (Bartlett 1937), a modification of the normal-theory likelihood ratio test.

The HOVTEST=BF option specifies Brown and Forsythe's variation of Levene's test (Brown and Forsythe 1974).

The HOVTEST=LEVENE option specifies Levene's test (Levene 1960), which is widely considered to be the standard homogeneity of variance test. You can use the TYPE= option in parentheses to specify whether to use the absolute residuals (TYPE=ABS) or the squared residuals (TYPE=SQUARE) in Levene's test. The default is TYPE=SQUARE.

The HOVTEST=OBRIEN option specifies O'Brien's test (O'Brien 1979), which is basically a modification of HOVTEST=LEVENE(TYPE=SQUARE). You can use the W= option in parentheses to tune the variable to match the suspected kurtosis of the underlying distribution. By default, W=0.5, as suggested by O'Brien (1979, 1981).

See the section "Homogeneity of Variance in One-Way Models" on page 3083 in Chapter 39, "The GLM Procedure," for more details on these methods. Example 39.10 in the same chapter illustrates the use of the HOVTEST and WELCH options in the MEANS statement in testing for equal group variances.

KRATIO=*value*

specifies the Type 1/Type 2 error seriousness ratio for the Waller-Duncan test. Reasonable values for KRATIO are 50, 100, and 500, which roughly correspond for the two-level case to ALPHA levels of 0.1, 0.05, and 0.01. By default, the procedure uses the default value of 100.

LINES

presents results of the **BON**, **DUNCAN**, **GABRIEL**, **REGWQ**, **SCHEFFE**, **SIDAK**, **SMM**, **GT2**, **SNK**, **T**, **LSD TUKEY**, and **WALLER** options by listing the means in descending order and indicating nonsignificant subsets by line segments beside the corresponding means. The **LINES** option is appropriate for equal cell sizes, for which it is the default. The **LINES** option is also the default if the **DUNCAN**, **REGWQ**, **SNK**, or **WALLER** option is specified, or if there are only two cells of unequal size. If the cell sizes are unequal, the harmonic mean of the cell sizes is used, which might lead to somewhat liberal tests if the cell sizes are highly disparate. The **LINES** option cannot be used in combination with the **DUNNETT**, **DUNNETTL**, or **DUNNETTU** option. In addition, the procedure has a restriction that no more than 24 overlapping groups of means can exist. If a mean belongs to more than 24 groups, the procedure issues an error message. You can either reduce the number of levels of the variable or use a multiple comparison test that allows the **CLDIFF** option rather than the **LINES** option.

LSD

see the **T** option.

NOSORT

prevents the means from being sorted into descending order when the **CLDIFF** or **CLM** option is specified.

REGWQ

performs the Ryan-Einot-Gabriel-Welsch multiple range test on all main effect means in the MEANS statement. See the **LINES** option for a discussion of how the procedure displays results.

SCHEFFE

performs Scheffé's multiple-comparison procedure on all main effect means in the MEANS statement. See the **CLDIFF** and **LINES** options for discussions of how the procedure displays results.

SIDAK

performs pairwise *t* tests on differences between means with levels adjusted according to Sidak's inequality for all main effect means in the MEANS statement. See the **CLDIFF** and **LINES** options for discussions of how the procedure displays results.

SMM**GT2**

performs pairwise comparisons based on the studentized maximum modulus and Sidak's uncorrelated-*t* inequality, yielding Hochberg's GT2 method when sample sizes are unequal, for all main effect means in the MEANS statement. See the **CLDIFF** and **LINES** options for discussions of how the procedure displays results.

SNK

performs the Student-Newman-Keuls multiple range test on all main effect means in the MEANS statement. See the [LINES](#) option for a discussion of how the procedure displays results.

T**LSD**

performs pairwise t tests, equivalent to Fisher's least-significant-difference test in the case of equal cell sizes, for all main effect means in the MEANS statement. See the [CLDIFF](#) and [LINES](#) options for discussions of how the procedure displays results.

TUKEY

performs Tukey's studentized range test (HSD) on all main effect means in the MEANS statement. (When the group sizes are different, this is the Tukey-Kramer test.) See the [CLDIFF](#) and [LINES](#) options for discussions of how the procedure displays results.

WALLER

performs the Waller-Duncan k -ratio t test on all main effect means in the MEANS statement. See the [KRATIO=](#) option for information about controlling details of the test, and see the [LINES](#) option for a discussion of how the procedure displays results.

WELCH

requests Welch's (1951) variance-weighted one-way ANOVA. This alternative to the usual analysis of variance for a one-way model is robust to the assumption of equal within-group variances. This option is ignored unless your [MODEL](#) statement specifies a simple one-way model.

Note that using the WELCH option merely produces one additional table consisting of Welch's ANOVA. It does not affect all of the other tests displayed by the ANOVA procedure, which still require the assumption of equal variance for exact validity.

See the section "[Homogeneity of Variance in One-Way Models](#)" on page 3083 in Chapter 39, "[The GLM Procedure](#)," for more details on Welch's ANOVA. [Example 39.10](#) in the same chapter illustrates the use of the [HOVTEST](#) and WELCH options in the MEANS statement in testing for equal group variances.

MODEL Statement

MODEL *dependents=effects* </ options> ;

The MODEL statement names the dependent variables and independent effects. The syntax of effects is described in the section "[Specification of Effects](#)" on page 840. For any model effect involving classification variables (interactions as well as main effects), the number of levels cannot exceed 32,767. If no independent effects are specified, only an intercept term is fit. This tests the hypothesis that the mean of the dependent variable is zero. All variables in effects that you specify in the MODEL statement must appear in the [CLASS](#) statement because PROC ANOVA does not allow for continuous effects.

You can specify the following options in the MODEL statement; they must be separated from the list of independent effects by a slash.

INTERCEPT

INT

displays the hypothesis tests associated with the intercept as an effect in the model. By default, the procedure includes the intercept in the model but does not display associated tests of hypotheses. Except for producing the uncorrected total SS instead of the corrected total SS, the INT option is ignored when you use an [ABSORB](#) statement.

NOUNI

suppresses the display of univariate statistics. You typically use the NOUNI option with a multivariate or repeated measures analysis of variance when you do not need the standard univariate output. The NOUNI option in a MODEL statement does not affect the univariate output produced by the [REPEATED](#) statement.

REPEATED Statement

REPEATED *factor-specification* < / options > ;

When values of the dependent variables in the [MODEL](#) statement represent repeated measurements on the same experimental unit, the REPEATED statement enables you to test hypotheses about the measurement factors (often called *within-subject factors*), as well as the interactions of within-subject factors with independent variables in the [MODEL](#) statement (often called *between-subject factors*). The REPEATED statement provides multivariate and univariate tests as well as hypothesis tests for a variety of single-degree-of-freedom contrasts. There is no limit to the number of within-subject factors that can be specified. For more details, see the section “[Repeated Measures Analysis of Variance](#)” on page 3089 in Chapter 39, “[The GLM Procedure](#).”

The REPEATED statement is typically used for handling repeated measures designs with one repeated response variable. Usually, the variables on the left-hand side of the equation in the [MODEL](#) statement represent one repeated response variable.

This does not mean that only one factor can be listed in the REPEATED statement. For example, one repeated response variable (hemoglobin count) might be measured 12 times (implying variables Y1 to Y12 on the left-hand side of the equal sign in the [MODEL](#) statement), with the associated within-subject factors treatment and time (implying two factors listed in the REPEATED statement). See the section “[Examples](#)” on page 839 for an example of how PROC ANOVA handles this case.

Designs with two or more repeated response variables can, however, be handled with the IDENTITY transformation; see [Example 39.9](#) in Chapter 39, “[The GLM Procedure](#),” for an example of analyzing a doubly-multivariate repeated measures design.

When a REPEATED statement appears, the ANOVA procedure enters a multivariate mode of handling missing values. If any values for variables corresponding to each combination of the within-subject factors are missing, the observation is excluded from the analysis.

The simplest form of the REPEATED statement requires only a *factor-name*. With two repeated factors, you must specify the *factor-name* and number of levels (*levels*) for each factor. Optionally, you can specify the actual values for the levels (*level-values*), a *transformation* that defines single-degree-of freedom contrasts, and *options* for additional analyses and output. When more than one within-subject factor is specified, *factor-names* (and associated level and transformation information) must be separated by a comma in the REPEATED statement. These terms are described in the following section, “Syntax Details.”

Syntax Details

You can specify the following terms in the REPEATED statement.

factor-specification

The *factor-specification* for the REPEATED statement can include any number of individual factor specifications, separated by commas, of the following form:

factor-name levels < (level-values) > < transformation >

where

<i>factor-name</i>	names a factor to be associated with the dependent variables. The name should not be the same as any variable name that already exists in the data set being analyzed and should conform to the usual conventions of SAS variable names. When specifying more than one factor, list the dependent variables in the MODEL statement so that the within-subject factors defined in the REPEATED statement are nested; that is, the first factor defined in the REPEATED statement should be the one with values that change least frequently.
<i>levels</i>	specifies the number of levels associated with the factor being defined. When there is only one within-subject factor, the number of levels is equal to the number of dependent variables. In this case, <i>levels</i> is optional. When more than one within-subject factor is defined, however, <i>levels</i> is required, and the product of the number of levels of all the factors must equal the number of dependent variables in the MODEL statement.
<i>(level-values)</i>	specifies values that correspond to levels of a repeated-measures factor. These values are used to label output; they are also used as spacings for constructing orthogonal polynomial contrasts if you specify a POLYNOMIAL transformation. The number of level values specified must correspond to the number of levels for that factor in the REPEATED statement. Enclose the <i>level-values</i> in parentheses.

The following *transformation* keywords define single-degree-of-freedom contrasts for factors specified in the REPEATED statement. Since the number of contrasts generated is always one less than the number of levels of the factor, you have some control over which contrast

is omitted from the analysis by which transformation you select. The only exception is the **IDENTITY** transformation; this transformation is not composed of contrasts, and it has the same degrees of freedom as the factor has levels. By default, the procedure uses the **CONTRAST** transformation.

CONTRAST<(ordinal-reference-level)>

generates contrasts between levels of the factor and a reference level. By default, the procedure uses the last level; you can optionally specify a reference level in parentheses after the keyword **CONTRAST**. The reference level corresponds to the ordinal value of the level rather than the level value specified. For example, to generate contrasts between the first level of a factor and the other levels, use

```
contrast (1)
```

HELMERT generates contrasts between each level of the factor and the mean of subsequent levels.

IDENTITY generates an identity transformation corresponding to the associated factor. This transformation is *not* composed of contrasts; it has n degrees of freedom for an n -level factor, instead of $n - 1$. This can be used for doubly-multivariate repeated measures.

MEAN<(ordinal-reference-level)>

generates contrasts between levels of the factor and the mean of all other levels of the factor. Specifying a reference level eliminates the contrast between that level and the mean. Without a reference level, the contrast involving the last level is omitted. See the **CONTRAST** transformation for an example.

POLYNOMIAL generates orthogonal polynomial contrasts. Level values, if provided, are used as spacings in the construction of the polynomials; otherwise, equal spacing is assumed.

PROFILE generates contrasts between adjacent levels of the factor.

For examples of the transformation matrices generated by these contrast transformations, see the section “[Repeated Measures Analysis of Variance](#)” on page 3089 in Chapter 39, “[The GLM Procedure](#).”

You can specify the following options in the **REPEATED** statement after a slash:

CANONICAL

performs a canonical analysis of the **H** and **E** matrices corresponding to the transformed variables specified in the **REPEATED** statement.

MSTAT=FAPPROX

MSTAT=EXACT

specifies the method of evaluating the multivariate test statistics. The default is **MSTAT=FAPPROX**, which specifies that the multivariate tests are evaluated by using the usual approximations based on the F distribution, as discussed in the “Multivariate Tests”

section in Chapter 4, “[Introduction to Regression Procedures](#).” Alternatively, you can specify `MSTAT=EXACT` to compute exact p -values for three of the four tests (Wilks’ lambda, the Hotelling-Lawley trace, and Roy’s greatest root) and an improved F-approximation for the fourth (Pillai’s trace). While `MSTAT=EXACT` provides better control of the significance probability for the tests, especially for Roy’s Greatest Root, computations for the exact p -values can be appreciably more demanding, and are in fact infeasible for large problems (many dependent variables). Thus, although `MSTAT=EXACT` is more accurate for most data, it is not the default method. For more information about the results of `MSTAT=EXACT`, see the section “[Multivariate Analysis of Variance](#)” on page 3087 in Chapter 39, “[The GLM Procedure](#).”

NOM

displays only the results of the univariate analyses.

NOU

displays only the results of the multivariate analyses.

PRINTE

displays the **E** matrix for each combination of within-subject factors, as well as partial correlation matrices for both the original dependent variables and the variables defined by the transformations specified in the `REPEATED` statement. In addition, the `PRINTE` option provides sphericity tests for each set of transformed variables. If the requested transformations are not orthogonal, the `PRINTE` option also provides a sphericity test for a set of orthogonal contrasts.

PRINTH

displays the **H** (SSCP) matrix associated with each multivariate test.

PRINTM

displays the transformation matrices that define the contrasts in the analysis. `PROC ANOVA` always displays the **M** matrix so that the transformed variables are defined by the rows, not the columns, of the displayed **M** matrix. In other words, `PROC ANOVA` actually displays **M'**.

PRINTRV

produces the characteristic roots and vectors for each multivariate test.

SUMMARY

produces analysis-of-variance tables for each contrast defined by the within-subjects factors. Along with tests for the effects of the independent variables specified in the `MODEL` statement, a term labeled `MEAN` tests the hypothesis that the overall mean of the contrast is zero.

UEPSDEF=unbiased-epsilon-definition

specifies the type of adjustment for the univariate F test that is displayed in addition to the Greenhouse-Geisser adjustment. The default is `UEPSDEF=HFL`, corresponding to the corrected form of the Huynh-Feldt adjustment (Huynh and Feldt 1976; Lecoutre 1991). Other alternatives are `UEPSDEF=HF`, the uncorrected Huynh-Feldt adjustment (the only available method in previous releases of SAS/STAT software), and `UEPSDEF=CM`, the adjustment of Chi and Muller (2009). See the section “[Hypothesis Testing in Repeated Measures Analysis](#)” on page 3091 in Chapter 39, “[The GLM Procedure](#),” for details about these adjustments.

Examples

When specifying more than one factor, list the dependent variables in the **MODEL** statement so that the within-subject factors defined in the **REPEATED** statement are nested; that is, the first factor defined in the **REPEATED** statement should be the one with values that change least frequently. For example, assume that three treatments are administered at each of four times, for a total of twelve dependent variables on each experimental unit. If the variables are listed in the **MODEL** statement as Y1 through Y12, then the following **REPEATED** statement

```
repeated trt 3, time 4;
```

implies the following structure:

	Dependent Variables											
	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12
Value of trt	1	1	1	1	2	2	2	2	3	3	3	3
Value of time	1	2	3	4	1	2	3	4	1	2	3	4

The **REPEATED** statement always produces a table like the preceding one.

For more information about repeated measures analysis and about using the **REPEATED** statement, see the section “[Repeated Measures Analysis of Variance](#)” on page 3089 in Chapter 39, “[The GLM Procedure](#).”

TEST Statement

```
TEST <H= effects> E= effect ;
```

Although an F value is computed for all SS in the analysis by using the residual MS as an error term, you can request additional F tests that use other effects as error terms. You need a **TEST** statement when a nonstandard error structure (as in a split plot) exists.

CAUTION: The ANOVA procedure does not check any of the assumptions underlying the F statistic. When you specify a **TEST** statement, you assume sole responsibility for the validity of the F statistic produced. To help validate a test, you might want to use the GLM procedure with the **RANDOM** statement and inspect the expected mean squares. In the GLM procedure, you can also use the **TEST** option in the **RANDOM** statement.

You can use as many **TEST** statements as you want, provided that they appear after the **MODEL** statement.

You can specify the following terms in the **TEST** statement.

- | | |
|------------------|--|
| H=effects | specifies which effects in the preceding model are to be used as hypothesis (numerator) effects. |
| E=effect | specifies one, and only one, effect to use as the error (denominator) term. The E= specification is required. |

The following example uses two TEST statements and is appropriate for analyzing a split-plot design.

```
proc anova;
  class a b c;
  model y=a|b(a)|c;
  test h=a e=b(a);
  test h=c a*c e=b*c(a);
run;
```

Details: ANOVA Procedure

Specification of Effects

In SAS analysis-of-variance procedures, the variables that identify levels of the classifications are called *classification variables*, and they are declared in the **CLASS** statement. Classification variables are also called *categorical*, *qualitative*, *discrete*, or *nominal variables*. The values of a classification variable are called *levels*. Classification variables can be either numeric or character. This is in contrast to the *response* (or *dependent*) *variables*, which are continuous. Response variables must be numeric.

The analysis-of-variance model specifies *effects*, which are combinations of classification variables used to explain the variability of the dependent variables in the following manner:

- Main effects are specified by writing the variables by themselves in the **CLASS** statement: A B C. Main effects used as independent variables test the hypothesis that the mean of the dependent variable is the same for each level of the factor in question, ignoring the other independent variables in the model.
- Crossed effects (interactions) are specified by joining the **CLASS** variables with asterisks in the **MODEL** statement: A*B A*C A*B*C. Interaction terms in a model test the hypothesis that the effect of a factor does not depend on the levels of the other factors in the interaction.
- Nested effects are specified by following a main effect or crossed effect with a **CLASS** variable or list of **CLASS** variables enclosed in parentheses in the **MODEL** statement. The main effect or crossed effect is nested within the effects listed in parentheses: B(A) C*D(A B). Nested effects test hypotheses similar to interactions, but the levels of the nested variables are not the same for every combination within which they are nested.

The general form of an effect can be illustrated by using the **CLASS** variables A, B, C, D, E, and F:

$$A * B * C(D E F)$$

The crossed list should come first, followed by the nested list in parentheses. Note that no asterisks appear within the nested list or immediately before the left parenthesis.

Main Effects Models

For a three-factor main effects model with A, B, and C as the factors and Y as the dependent variable, the necessary statements are

```
proc anova;
  class A B C;
  model Y=A B C;
run;
```

Models with Crossed Factors

To specify interactions in a factorial model, join effects with asterisks as described previously. For example, these statements specify a complete factorial model, which includes all the interactions:

```
proc anova;
  class A B C;
  model Y=A B C A*B A*C B*C A*B*C;
run;
```

Bar Notation

You can shorten the specifications of a full factorial model by using bar notation. For example, the preceding statements can also be written

```
proc anova;
  class A B C;
  model Y=A|B|C;
run;
```

When the bar (|) is used, the expression on the right side of the equal sign is expanded from left to right by using the equivalents of rules 2–4 given in Searle (1971, p. 390). The variables on the right- and left-hand sides of the bar become effects, and the cross of them becomes an effect. Multiple bars are permitted. For instance, A | B | C is evaluated as follows:

$$\begin{aligned} A | B | C &\rightarrow \{ A | B \} | C \\ &\rightarrow \{ A \ B \ A*B \} | C \\ &\rightarrow A \ B \ A*B \ C \ A*C \ B*C \ A*B*C \end{aligned}$$

You can also specify the maximum number of variables involved in any effect that results from bar evaluation by specifying that maximum number, preceded by an @ sign, at the end of the bar effect. For example, the specification A | B | C@2 results in only those effects that contain two or fewer variables; in this case, A B A*B C A*C and B*C.

The following table gives more examples of using the bar and at operators.

$A \mid C(B)$	is equivalent to	$A \ C(B) \ A * C(B)$
$A(B) \mid C(B)$	is equivalent to	$A(B) \ C(B) \ A * C(B)$
$A(B) \mid B(D \ E)$	is equivalent to	$A(B) \ B(D \ E)$
$A \mid B(A) \mid C$	is equivalent to	$A \ B(A) \ C \ A * C \ B * C(A)$
$A \mid B(A) \mid C @ 2$	is equivalent to	$A \ B(A) \ C \ A * C$
$A \mid B \mid C \mid D @ 2$	is equivalent to	$A \ B \ A * B \ C \ A * C \ B * C \ D \ A * D \ B * D \ C * D$

Consult the section “[Specification of Effects](#)” on page 3043 in Chapter 39, “[The GLM Procedure](#),” for further details on bar notation.

Nested Models

Write the effect that is nested within another effect first, followed by the other effect in parentheses. For example, if A and B are main effects and C is nested within A and B (that is, the levels of C that are observed are not the same for each combination of A and B), the statements for PROC ANOVA are

```
proc anova;
  class A B C;
  model y=A B C(A B);
run;
```

The identity of a level is viewed within the context of the level of the containing effects. For example, if City is nested within State, then the identity of City is viewed within the context of State.

The distinguishing feature of a nested specification is that nested effects never appear as main effects. Another way of viewing nested effects is that they are effects that pool the main effect with the interaction of the nesting variable.

See the “Automatic Pooling” section, which follows.

Models Involving Nested, Crossed, and Main Effects

Asterisks and parentheses can be combined in the **MODEL** statement for models involving nested and crossed effects:

```
proc anova;
  class A B C;
  model Y=A B(A) C(A) B * C(A);
run;
```

Automatic Pooling

In line with the general philosophy of the GLM procedure, there is no difference between the statements

```
model Y=A B(A);
```

and

```
model Y=A A*B;
```

The effect B becomes a nested effect by virtue of the fact that it does not occur as a main effect. If B is not written as a main effect in addition to participating in A*B, then the sum of squares that is associated with B is pooled into A*B.

This feature allows the automatic pooling of sums of squares. If an effect is omitted from the model, it is automatically pooled with all the higher-level effects containing the **CLASS** variables in the omitted effect (or within-error). This feature is most useful in split-plot designs.

Using PROC ANOVA Interactively

PROC ANOVA can be used interactively. After you specify a model in a **MODEL** statement and run PROC ANOVA with a **RUN** statement, a variety of statements (such as **MEANS**, **MANOVA**, **TEST**, and **REPEATED**) can be executed without PROC ANOVA recalculating the model sum of squares.

the section “[Syntax: ANOVA Procedure](#)” on page 821 describes which statements can be used interactively. You can execute these interactive statements individually or in groups by following the single statement or group of statements with a **RUN** statement. Note that the **MODEL** statement cannot be repeated; the ANOVA procedure allows only one **MODEL** statement.

If you use PROC ANOVA interactively, you can end the procedure with a **DATA** step, another PROC step, an **ENDSAS** statement, or a **QUIT** statement. The syntax of the **QUIT** statement is

```
quit;
```

When you use PROC ANOVA interactively, additional **RUN** statements do not end the procedure but tell PROC ANOVA to execute additional statements.

When a **WHERE** statement is used with PROC ANOVA, it should appear before the first **RUN** statement. The **WHERE** statement enables you to select only certain observations for analysis without using a subsetting **DATA** step. For example, the statement `where group ne 5` omits observations with **GROUP**=5 from the analysis. See *SAS Language Reference: Dictionary* for details about this statement.

When a **BY** statement is used with PROC ANOVA, interactive processing is not possible; that is, once the first **RUN** statement is encountered, processing proceeds for each **BY** group in the data set, and no further statements are accepted by the procedure.

Interactivity is also disabled when there are different patterns of missing values among the dependent variables. For details, see the section “Missing Values,” which follows.

Missing Values

For an analysis involving one dependent variable, PROC ANOVA uses an observation if values are nonmissing for that dependent variable and for all the variables used in independent effects.

For an analysis involving multiple dependent variables without the **MANOVA** or **REPEATED** statement, or without the **MANOVA** option in the PROC ANOVA statement, a missing value in one dependent variable does not eliminate the observation from the analysis of other nonmissing dependent variables. For an analysis with the **MANOVA** or **REPEATED** statement, or with the **MANOVA** option in the PROC ANOVA statement, the ANOVA procedure requires values for all dependent variables to be nonmissing for an observation before the observation can be used in the analysis.

During processing, PROC ANOVA groups the dependent variables by their pattern of missing values across observations so that sums and cross products can be collected in the most efficient manner.

If your data have different patterns of missing values among the dependent variables, interactivity is disabled. This could occur when some of the variables in your data set have missing values and either of the following conditions obtain:

- You do not use the **MANOVA** option in the PROC ANOVA statement.
- You do not use a **MANOVA** or **REPEATED** statement before the first RUN statement.

Output Data Set

The OUTSTAT= option in the PROC ANOVA statement produces an output data set that contains the following:

- the BY variables, if any
- **_TYPE_**, a new character variable. This variable has the value 'ANOVA' for observations corresponding to sums of squares; it has the value 'CANCORR', 'STRUCTUR', or 'SCORE' if a canonical analysis is performed through the **MANOVA** statement and no M= matrix is specified.
- **_SOURCE_**, a new character variable. For each observation in the data set, **_SOURCE_** contains the name of the model effect from which the corresponding statistics are generated.
- **_NAME_**, a new character variable. The variable **_NAME_** contains the name of one of the dependent variables in the model or, in the case of canonical statistics, the name of one of the canonical variables (CAN1, CAN2, and so on).
- four new numeric variables, SS, DF, F, and PROB, containing sums of squares, degrees of freedom, *F* values, and probabilities, respectively, for each model or contrast sum of squares generated in the analysis. For observations resulting from canonical analyses, these variables have missing values.

- if there is more than one dependent variable, then variables with the same names as the dependent variables represent
 - for `_TYPE_='ANOVA'`, the crossproducts of the hypothesis matrices
 - for `_TYPE_='CANCORR'`, canonical correlations for each variable
 - for `_TYPE_='STRUCTUR'`, coefficients of the total structure matrix
 - for `_TYPE_='SCORE'`, raw canonical score coefficients

The output data set can be used to perform special hypothesis tests (for example, with the IML procedure in SAS/IML software), to reformat output, to produce canonical variates (through the SCORE procedure), or to rotate structure matrices (through the FACTOR procedure).

Computational Method

Let \mathbf{X} represent the $n \times p$ design matrix. The columns of \mathbf{X} contain only 0s and 1s. Let \mathbf{Y} represent the $n \times 1$ vector of dependent variables.

In the GLM procedure, $\mathbf{X}'\mathbf{X}$, $\mathbf{X}'\mathbf{Y}$, and $\mathbf{Y}'\mathbf{Y}$ are formed in main storage. However, in the ANOVA procedure, only the diagonals of $\mathbf{X}'\mathbf{X}$ are computed, along with $\mathbf{X}'\mathbf{Y}$ and $\mathbf{Y}'\mathbf{Y}$. Thus, PROC ANOVA saves a considerable amount of storage as well as time. The memory requirements for PROC ANOVA are asymptotically linear functions of n^2 and nr , where n is the number of dependent variables and r the number of independent parameters.

The elements of $\mathbf{X}'\mathbf{Y}$ are cell totals, and the diagonal elements of $\mathbf{X}'\mathbf{X}$ are cell frequencies. Since PROC ANOVA automatically pools omitted effects into the next higher-level effect containing the names of the omitted effect (or within-error), a slight modification to the rules given by Searle (1971, p. 389) is used.

1. PROC ANOVA computes the sum of squares for each effect as if it is a main effect. In other words, for each effect, PROC ANOVA squares each cell total and divides by its cell frequency. The procedure then adds these quantities together and subtracts the correction factor for the mean (total squared over N).
2. For each effect involving two **CLASS** variable names, PROC ANOVA subtracts the SS for any main effect with a name that is contained in the two-factor effect.
3. For each effect involving three **CLASS** variable names, PROC ANOVA subtracts the SS for all main effects and two-factor effects with names that are contained in the three-factor effect. If effects involving four or more **CLASS** variable names are present, the procedure continues this process.

Displayed Output

PROC ANOVA first displays a table that includes the following:

- the name of each variable in the **CLASS** statement
- the number of different values or Levels of the **CLASS** variables
- the Values of the **CLASS** variables
- the Number of observations in the data set and the number of observations excluded from the analysis because of missing values, if any

PROC ANOVA then displays an analysis-of-variance table for each dependent variable in the **MODEL** statement. This table breaks down the Total Sum of Squares for the dependent variable into the portion attributed to the Model and the portion attributed to Error. It also breaks down the Mean Square term, which is the Sum of Squares divided by the degrees of freedom (DF). The analysis-of-variance table also lists the following:

- the Mean Square for Error (MSE), which is an estimate of σ^2 , the variance of the true errors
- the F Value, which is the ratio produced by dividing the Mean Square for the Model by the Mean Square for Error. It tests how well the model as a whole (adjusted for the mean) accounts for the dependent variable's behavior. This F test is a test of the null hypothesis that all parameters except the intercept are zero.
- the significance probability associated with the F statistic, labeled "Pr > F"
- R-Square, R^2 , which measures how much variation in the dependent variable can be accounted for by the model. The R^2 statistic, which can range from 0 to 1, is the ratio of the sum of squares for the model divided by the sum of squares for the corrected total. In general, the larger the R^2 value, the better the model fits the data.
- C.V., the coefficient of variation, which is often used to describe the amount of variation in the population. The C.V. is 100 times the standard deviation of the dependent variable divided by the Mean. The coefficient of variation is often a preferred measure because it is unitless.
- Root MSE, which estimates the standard deviation of the dependent variable. Root MSE is computed as the square root of Mean Square for Error, the mean square of the error term.
- the Mean of the dependent variable

For each effect (or source of variation) in the model, PROC ANOVA then displays the following:

- DF, degrees of freedom
- Anova SS, the sum of squares, and the associated Mean Square
- the F Value for testing the hypothesis that the group means for that effect are equal
- Pr > F, the significance probability value associated with the F Value

When you specify a **TEST** statement, PROC ANOVA displays the results of the requested tests. When you specify a **MANOVA** statement and the model includes more than one dependent variable, PROC ANOVA produces these additional statistics:

- the characteristic roots and vectors of $\mathbf{E}^{-1}\mathbf{H}$ for each **H** matrix
- the Hotelling-Lawley trace
- Pillai's trace
- Wilks' lambda
- Roy's greatest root

See [Example 39.6](#) in Chapter 39, “[The GLM Procedure](#),” for an example of the MANOVA results. These MANOVA tests are discussed in Chapter 4, “[Introduction to Regression Procedures](#).”

ODS Table Names

PROC ANOVA assigns a name to each table it creates. You can use these names to reference the table when using the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in [Table 23.3](#). For more information about ODS, see Chapter 20, “[Using the Output Delivery System](#).”

Table 23.3 ODS Tables Produced by PROC ANOVA

ODS Table Name	Description	Statement / Option
AltErrTests	Anova tests with error other than MSE	TEST /E=
Bartlett	Bartlett's homogeneity of variance test	MEANS / HOVTEST=BARTLETT
CLDiffs	Multiple comparisons of pairwise differences	MEANS / CLDIFF or DUNNETT or (Unequal cells and not LINES)
CLDiffsInfo	Information for multiple comparisons of pairwise differences	MEANS / CLDIFF or DUNNETT or (Unequal cells and not LINES)
CLMeans	Multiple comparisons of means with confidence/comparison interval	MEANS / CLM with (BON or GABRIEL or SCHEFFE or SIDAK or SMM or T or LSD)
CLMeansInfo	Information for multiple comparisons of means with confidence/comparison interval	MEANS / CLM
CanAnalysis	Canonical analysis	(MANOVA or REPEATED) / CANONICAL
CanCoef	Canonical coefficients	(MANOVA or REPEATED) / CANONICAL
CanStructure	Canonical structure	(MANOVA or REPEATED) / CANONICAL

Table 23.3 *continued*

ODS Table Name	Description	Statement / Option
CharStruct	Characteristic roots and vectors	(MANOVA / not CANONICAL) or (REPEATED / PRINTRV)
ClassLevels	Classification variable levels	CLASS statement
DependentInfo	Simultaneously analyzed dependent variables	default when there are multiple dependent variables with different patterns of missing values
Epsilons	Greenhouse-Geisser and Huynh-Feldt epsilons	REPEATED statement
ErrorSSCP	Error SSCP matrix	(MANOVA or REPEATED) / PRINTE
FitStatistics	R-Square, C.V., Root MSE, and dependent mean	default
HOVFTest	Homogeneity of variance ANOVA	MEANS / HOVTEST
HypothesisSSCP	Hypothesis SSCP matrix	(MANOVA or REPEATED) / PRINTE
MANOVATransform	Multivariate transformation matrix	MANOVA / M=
MCLines	Multiple comparisons LINES output	MEANS / LINES or ((DUNCAN or WALLER or SNK or REGWQ) and not (CLDIFF or CLM)) or (Equal cells and not CLDIFF)
MCLinesInfo	Information for multiple comparison LINES output	MEANS / LINES or ((DUNCAN or WALLER or SNK or REGWQ) and not (CLDIFF or CLM)) or (Equal cells and not CLDIFF)
MCLinesRange	Ranges for multiple range MC tests	MEANS / LINES or ((DUNCAN or WALLER or SNK or REGWQ) and not (CLDIFF or CLM)) or (Equal cells and not CLDIFF)
Means	Group means	MEANS statement
ModelANOVA	ANOVA for model terms	default
MultStat	Multivariate tests	MANOVA statement
NObs	Number of observations	default
OverallANOVA	Over-all ANOVA	default
PartialCorr	Partial correlation matrix	(MANOVA or REPEATED) / PRINTE
RepeatedTransform	Repeated transformation matrix	REPEATED (CONTRAST or HELMERT or MEAN or POLYNOMIAL or PROFILE)
RepeatedLevelInfo	Correspondence between dependents and repeated measures levels	REPEATED statement
Sphericity Tests	Sphericity tests Summary ANOVA for specified MANOVA H= effects	REPEATED / PRINTE MANOVA / H= SUMMARY
Welch	Welch's ANOVA	MEANS / WELCH

ODS Graphics

This section describes the use of ODS for creating statistical graphs with the ANOVA procedure. To request these graphs you must specify the ODS GRAPHICS statement with an appropriate model, as discussed in the following. For more information about the ODS GRAPHICS statement, see Chapter 21, “[Statistical Graphics Using ODS](#).”

When the ODS Graphics are in effect, if you specify a one-way analysis of variance model, with just one independent classification variable, or if you use a [MEANS](#) statement, then the ANOVA procedure will produce a grouped box plot of the response values versus the classification levels. For an example of the box plot, see the section “[One-Way Layout with Means Comparisons](#)” on page 813.

ODS Graph Names

PROC ANOVA produces a single graph, the name of which you can use for referencing it in ODS. The name is listed in [Table 23.4](#).

To request this graph you must specify the ODS GRAPHICS statement. For more information about the ODS GRAPHICS statement, see Chapter 21, “[Statistical Graphics Using ODS](#).”

Table 23.4 ODS Graphic Produced by PROC ANOVA

ODS Graph Name	Plot Description
BoxPlot	Box plot of observed response values by classification levels

Examples: ANOVA Procedure

Example 23.1: Randomized Complete Block With Factorial Treatment Structure

This example uses statements for the analysis of a randomized block with two treatment factors occurring in a factorial structure. The data, from Neter, Wasserman, and Kutner (1990, p. 941), are from an experiment examining the effects of codeine and acupuncture on post-operative dental pain in male subjects. Both treatment factors have two levels. The codeine levels are a codeine capsule or a sugar capsule. The acupuncture levels are two inactive acupuncture points or two active acupuncture points. There are four distinct treatment combinations due to the factorial treatment

structure. The 32 subjects are assigned to eight blocks of four subjects each based on an assessment of pain tolerance.

The data for the analysis are balanced, so PROC ANOVA is used. The data are as follows:

```

title1 'Randomized Complete Block With Two Factors';
data PainRelief;
    input PainLevel Codeine Acupuncture Relief @@;
    datalines;
1 1 1 0.0  1 2 1 0.5  1 1 2 0.6  1 2 2 1.2
2 1 1 0.3  2 2 1 0.6  2 1 2 0.7  2 2 2 1.3
3 1 1 0.4  3 2 1 0.8  3 1 2 0.8  3 2 2 1.6
4 1 1 0.4  4 2 1 0.7  4 1 2 0.9  4 2 2 1.5
5 1 1 0.6  5 2 1 1.0  5 1 2 1.5  5 2 2 1.9
6 1 1 0.9  6 2 1 1.4  6 1 2 1.6  6 2 2 2.3
7 1 1 1.0  7 2 1 1.8  7 1 2 1.7  7 2 2 2.1
8 1 1 1.2  8 2 1 1.7  8 1 2 1.6  8 2 2 2.4
;

```

The variable PainLevel is the blocking variable, and Codeine and Acupuncture represent the levels of the two treatment factors. The variable Relief is the pain relief score (the higher the score, the more relief the patient has).

The following statements invoke PROC ANOVA. The blocking variable and treatment factors appear in the **CLASS** statement. The bar between the treatment factors Codeine and Acupuncture adds their main effects as well as their interaction Codeine*Acupuncture to the model.

```

proc anova data=PainRelief;
    class PainLevel Codeine Acupuncture;
    model Relief = PainLevel Codeine|Acupuncture;
run;

```

The results from the analysis are shown in [Output 23.1.1](#), [Output 23.1.2](#), and [Output 23.1.3](#).

Output 23.1.1 Class Level Information

Randomized Complete Block With Two Factors						
The ANOVA Procedure						
Class Level Information						
Class	Levels	Values				
PainLevel	8	1	2	3	4	5 6 7 8
Codeine	2	1	2			
Acupuncture	2	1	2			
Number of Observations Read						32
Number of Observations Used						32

Output 23.1.2 ANOVA Table

Randomized Complete Block With Two Factors					
The ANOVA Procedure					
Dependent Variable: Relief					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	10	11.33500000	1.13350000	78.37	<.0001
Error	21	0.30375000	0.01446429		
Corrected Total	31	11.63875000			
	R-Square	Coeff Var	Root MSE	Relief Mean	
	0.973902	10.40152	0.120268	1.156250	

The Class Level Information and ANOVA table are shown in [Output 23.1.1](#) and [Output 23.1.2](#). The classification level information summarizes the structure of the design. It is good to check these consistently in search of errors in the DATA step. The overall F test is significant, indicating that the model accounts for a significant amount of variation in the dependent variable.

Output 23.1.3 Tests of Effects

Source	DF	Anova SS	Mean Square	F Value	Pr > F
PainLevel	7	5.59875000	0.79982143	55.30	<.0001
Codeine	1	2.31125000	2.31125000	159.79	<.0001
Acupuncture	1	3.38000000	3.38000000	233.68	<.0001
Codeine*Acupuncture	1	0.04500000	0.04500000	3.11	0.0923

[Output 23.1.3](#) shows tests of the effects. The blocking effect is significant; hence, it is useful. The interaction between codeine and acupuncture is significant at the 90% level but not at the 95% level. The significance level of this test should be determined before the analysis. The main effects of both treatment factors are highly significant.

Example 23.2: Alternative Multiple Comparison Procedures

The following is a continuation of the first example in the section “One-Way Layout with Means Comparisons” on page 813. You are studying the effect of bacteria on the nitrogen content of red clover plants, and the analysis of variance shows a highly significant effect. The following statements create the data set and compute the analysis of variance as well as Tukey’s multiple comparisons test for pairwise differences between bacteria strains; the results are shown in [Figure 23.1](#), [Figure 23.2](#), and [Figure 23.3](#)

```

title1 'Nitrogen Content of Red Clover Plants';
data Clover;
    input Strain $ Nitrogen @@;
    datalines;
3DOK1  19.4 3DOK1  32.6 3DOK1  27.0 3DOK1  32.1 3DOK1  33.0
3DOK5  17.7 3DOK5  24.8 3DOK5  27.9 3DOK5  25.2 3DOK5  24.3
3DOK4  17.0 3DOK4  19.4 3DOK4   9.1 3DOK4  11.9 3DOK4  15.8
3DOK7  20.7 3DOK7  21.0 3DOK7  20.5 3DOK7  18.8 3DOK7  18.6
3DOK13 14.3 3DOK13 14.4 3DOK13 11.8 3DOK13 11.6 3DOK13 14.2
COMPOS 17.3 COMPOS 19.4 COMPOS 19.1 COMPOS 16.9 COMPOS 20.8
;

proc anova data=Clover;
    class Strain;
    model Nitrogen = Strain;
    means Strain / tukey;
run;

```

The interactivity of PROC ANOVA enables you to submit further MEANS statements without re-running the entire analysis. For example, the following command requests means of the Strain levels with Duncan’s multiple range test and the Waller-Duncan k -ratio t test.

```

    means Strain / duncan waller;
run;

```

Results of the Waller-Duncan k -ratio t test are shown in [Output 23.2.1](#).

Output 23.2.1 Waller-Duncan K -ratio t Test

Nitrogen Content of Red Clover Plants	
The ANOVA Procedure	
Waller-Duncan K-ratio t Test for Nitrogen	
Kratio	100
Error Degrees of Freedom	24
Error Mean Square	11.78867
F Value	14.37
Critical Value of t	1.91873
Minimum Significant Difference	4.1665

Output 23.2.1 *continued*

Means with the same letter are not significantly different.				
Waller Grouping		Mean	N	Strain
	A	28.820	5	3DOK1
	B	23.980	5	3DOK5
	B			
C	B	19.920	5	3DOK7
C				
C	D	18.700	5	COMPOS
	D			
E	D	14.640	5	3DOK4
E				
E		13.260	5	3DOK13

The Waller-Duncan k -ratio t test is a multiple range test. Unlike Tukey's test, this test does not operate on the principle of controlling Type I error. Instead, it compares the Type I and Type II error rates based on Bayesian principles (Steel and Torrie 1980).

The Waller Grouping column in [Output 23.2.1](#) shows which means are significantly different. From this test, you can conclude the following:

- The mean nitrogen content for strain 3DOK1 is higher than the means for all other strains.
- The mean nitrogen content for strain 3DOK5 is higher than the means for COMPOS, 3DOK4, and 3DOK13.
- The mean nitrogen content for strain 3DOK7 is higher than the means for 3DOK4 and 3DOK13.
- The mean nitrogen content for strain COMPOS is higher than the mean for 3DOK13.
- Differences between all other means are not significant based on this sample size.

[Output 23.2.2](#) shows the results of Duncan's multiple range test. Duncan's test is a result-guided test that compares the treatment means while controlling the comparison-wise error rate. You should use this test for planned comparisons only (Steel and Torrie 1980). The results and conclusions for this example are the same as for the Waller-Duncan k -ratio t test. This is not always the case.

Output 23.2.2 Duncan's Multiple Range Test

	Alpha		0.05		
	Error Degrees of Freedom		24		
	Error Mean Square		11.78867		
Number of Means	2	3	4	5	6
Critical Range	4.482	4.707	4.852	4.954	5.031

Output 23.2.2 *continued*

Means with the same letter are not significantly different.				
Duncan Grouping		Mean	N	Strain
	A	28.820	5	3DOK1
	B	23.980	5	3DOK5
	B			
C	B	19.920	5	3DOK7
C				
C	D	18.700	5	COMPOS
	D			
E	D	14.640	5	3DOK4
E				
E		13.260	5	3DOK13

Tukey and Least Significant Difference (LSD) tests are requested with the following **MEANS** statement. The **CLDIFF** option requests confidence intervals for both tests.

```
means Strain/ lsd tukey cldiff ;
run;
```

The **LSD** tests for this example are shown in **Output 23.2.3**, and they give the same results as the previous two multiple comparison tests. Again, this is not always the case.

Output 23.2.3 T Tests (LSD)

Nitrogen Content of Red Clover Plants	
The ANOVA Procedure	
t Tests (LSD) for Nitrogen	
Alpha	0.05
Error Degrees of Freedom	24
Error Mean Square	11.78867
Critical Value of t	2.06390
Least Significant Difference	4.4818

Output 23.2.3 *continued*

Comparisons significant at the 0.05 level are indicated by ***.					
Strain Comparison	Difference Between Means	95% Confidence Limits			
3DOK1 - 3DOK5	4.840	0.358	9.322	***	
3DOK1 - 3DOK7	8.900	4.418	13.382	***	
3DOK1 - COMPOS	10.120	5.638	14.602	***	
3DOK1 - 3DOK4	14.180	9.698	18.662	***	
3DOK1 - 3DOK13	15.560	11.078	20.042	***	
3DOK5 - 3DOK1	-4.840	-9.322	-0.358	***	
3DOK5 - 3DOK7	4.060	-0.422	8.542		
3DOK5 - COMPOS	5.280	0.798	9.762	***	
3DOK5 - 3DOK4	9.340	4.858	13.822	***	
3DOK5 - 3DOK13	10.720	6.238	15.202	***	
3DOK7 - 3DOK1	-8.900	-13.382	-4.418	***	
3DOK7 - 3DOK5	-4.060	-8.542	0.422		
3DOK7 - COMPOS	1.220	-3.262	5.702		
3DOK7 - 3DOK4	5.280	0.798	9.762	***	
3DOK7 - 3DOK13	6.660	2.178	11.142	***	
COMPOS - 3DOK1	-10.120	-14.602	-5.638	***	
COMPOS - 3DOK5	-5.280	-9.762	-0.798	***	
COMPOS - 3DOK7	-1.220	-5.702	3.262		
COMPOS - 3DOK4	4.060	-0.422	8.542		
COMPOS - 3DOK13	5.440	0.958	9.922	***	
3DOK4 - 3DOK1	-14.180	-18.662	-9.698	***	
3DOK4 - 3DOK5	-9.340	-13.822	-4.858	***	
3DOK4 - 3DOK7	-5.280	-9.762	-0.798	***	
3DOK4 - COMPOS	-4.060	-8.542	0.422		
3DOK4 - 3DOK13	1.380	-3.102	5.862		
3DOK13 - 3DOK1	-15.560	-20.042	-11.078	***	
3DOK13 - 3DOK5	-10.720	-15.202	-6.238	***	
3DOK13 - 3DOK7	-6.660	-11.142	-2.178	***	
3DOK13 - COMPOS	-5.440	-9.922	-0.958	***	
3DOK13 - 3DOK4	-1.380	-5.862	3.102		

If you only perform the **LSD** tests when the overall model F test is significant, then this is called Fisher's protected **LSD** test. Note that the **LSD** tests should be used for planned comparisons.

The **TUKEY** tests shown in **Output 23.2.4** find fewer significant differences than the other three tests. This is not unexpected, as the **TUKEY** test controls the Type I experimentwise error rate. For a complete discussion of multiple comparison methods, see the section "**Multiple Comparisons**" on page 3070 in Chapter 39, "**The GLM Procedure**."

Output 23.2.4 Tukey's Studentized Range Test

Alpha	0.05
Error Degrees of Freedom	24
Error Mean Square	11.78867
Critical Value of Studentized Range	4.37265
Minimum Significant Difference	6.7142

Output 23.2.4 continued

Comparisons significant at the 0.05 level are indicated by ***.					
Strain Comparison	Difference Between Means	Simultaneous 95% Confidence Limits			
3DOK1 - 3DOK5	4.840	-1.874	11.554		
3DOK1 - 3DOK7	8.900	2.186	15.614	***	
3DOK1 - COMPOS	10.120	3.406	16.834	***	
3DOK1 - 3DOK4	14.180	7.466	20.894	***	
3DOK1 - 3DOK13	15.560	8.846	22.274	***	
3DOK5 - 3DOK1	-4.840	-11.554	1.874		
3DOK5 - 3DOK7	4.060	-2.654	10.774		
3DOK5 - COMPOS	5.280	-1.434	11.994		
3DOK5 - 3DOK4	9.340	2.626	16.054	***	
3DOK5 - 3DOK13	10.720	4.006	17.434	***	
3DOK7 - 3DOK1	-8.900	-15.614	-2.186	***	
3DOK7 - 3DOK5	-4.060	-10.774	2.654		
3DOK7 - COMPOS	1.220	-5.494	7.934		
3DOK7 - 3DOK4	5.280	-1.434	11.994		
3DOK7 - 3DOK13	6.660	-0.054	13.374		
COMPOS - 3DOK1	-10.120	-16.834	-3.406	***	
COMPOS - 3DOK5	-5.280	-11.994	1.434		
COMPOS - 3DOK7	-1.220	-7.934	5.494		
COMPOS - 3DOK4	4.060	-2.654	10.774		
COMPOS - 3DOK13	5.440	-1.274	12.154		
3DOK4 - 3DOK1	-14.180	-20.894	-7.466	***	
3DOK4 - 3DOK5	-9.340	-16.054	-2.626	***	
3DOK4 - 3DOK7	-5.280	-11.994	1.434		
3DOK4 - COMPOS	-4.060	-10.774	2.654		
3DOK4 - 3DOK13	1.380	-5.334	8.094		
3DOK13 - 3DOK1	-15.560	-22.274	-8.846	***	
3DOK13 - 3DOK5	-10.720	-17.434	-4.006	***	
3DOK13 - 3DOK7	-6.660	-13.374	0.054		
3DOK13 - COMPOS	-5.440	-12.154	1.274		
3DOK13 - 3DOK4	-1.380	-8.094	5.334		

Example 23.3: Split Plot

In some experiments, treatments can be applied only to groups of experimental observations rather than separately to each observation. When there are two nested groupings of the observations on the basis of treatment application, this is known as a *split plot design*. For example, in integrated circuit fabrication it is of interest to see how different manufacturing methods affect the characteristics of individual chips. However, much of the manufacturing process is applied to a relatively large wafer of material, from which many chips are made. Additionally, a chip's position within a wafer might also affect chip performance. These two groupings of chips—by wafer and by position-within-wafer—might form the *whole plots* and the *subplots*, respectively, of a split plot design for integrated circuits.

The following statements produce an analysis for a split-plot design. The **CLASS** statement includes the variables Block, A, and B, where B defines subplots within BLOCK*A whole plots. The **MODEL** statement includes the independent effects Block, A, Block*A, B, and A*B. The **TEST** statement asks for an F test of the A effect that uses the Block*A effect as the error term. The following statements produce [Output 23.3.1](#) and [Output 23.3.2](#):

```

title1 'Split Plot Design';
data Split;
    input Block 1 A 2 B 3 Response;
    datalines;
142 40.0
141 39.5
112 37.9
111 35.4
121 36.7
122 38.2
132 36.4
131 34.8
221 42.7
222 41.6
212 40.3
211 41.6
241 44.5
242 47.6
231 43.6
232 42.8
;

proc anova data=Split;
    class Block A B;
    model Response = Block A Block*A B A*B;
    test h=A e=Block*A;
run;

```

Output 23.3.1 Class Level Information and ANOVA Table

Split Plot Design			
The ANOVA Procedure			
Class Level Information			
Class	Levels	Values	
Block	2	1 2	
A	4	1 2 3 4	
B	2	1 2	
Number of Observations Read			16
Number of Observations Used			16

Output 23.3.1 *continued*

Split Plot Design					
The ANOVA Procedure					
Dependent Variable: Response					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	182.0200000	16.5472727	7.85	0.0306
Error	4	8.4300000	2.1075000		
Corrected Total	15	190.4500000			
R-Square	Coeff Var	Root MSE	Response Mean		
0.955736	3.609007	1.451723	40.22500		

First, notice that the overall F test for the model is significant.

Output 23.3.2 Tests of Effects

Source	DF	Anova SS	Mean Square	F Value	Pr > F
Block	1	131.1025000	131.1025000	62.21	0.0014
A	3	40.1900000	13.3966667	6.36	0.0530
Block*A	3	6.9275000	2.3091667	1.10	0.4476
B	1	2.2500000	2.2500000	1.07	0.3599
A*B	3	1.5500000	0.5166667	0.25	0.8612
Tests of Hypotheses Using the Anova MS for Block*A as an Error Term					
Source	DF	Anova SS	Mean Square	F Value	Pr > F
A	3	40.19000000	13.39666667	5.80	0.0914

The effect of Block is significant. The effect of A is not significant: look at the F test produced by the **TEST** statement, not at the F test produced by default. Neither the B nor A*B effects are significant. The test for Block*A is irrelevant, as this is simply the main-plot error.

Example 23.4: Latin Square Split Plot

The data for this example is taken from Smith (1951). A Latin square design is used to evaluate six different sugar beet varieties arranged in a six-row (Rep) by six-column (Column) square. The data are collected over two harvests. The variable Harvest then becomes a split plot on the original Latin square design for whole plots. The following statements produce [Output 23.4.1](#), [Output 23.4.2](#), and [Output 23.4.3](#):

```

title1 'Sugar Beet Varieties';
title3 'Latin Square Split-Plot Design';
data Beets;
  do Harvest=1 to 2;
    do Rep=1 to 6;
      do Column=1 to 6;
        input Variety Y @;
        output;
      end;
    end;
  end;
  datalines;
3 19.1 6 18.3 5 19.6 1 18.6 2 18.2 4 18.5
6 18.1 2 19.5 4 17.6 3 18.7 1 18.7 5 19.9
1 18.1 5 20.2 6 18.5 4 20.1 3 18.6 2 19.2
2 19.1 3 18.8 1 18.7 5 20.2 4 18.6 6 18.5
4 17.5 1 18.1 2 18.7 6 18.2 5 20.4 3 18.5
5 17.7 4 17.8 3 17.4 2 17.0 6 17.6 1 17.6
3 16.2 6 17.0 5 18.1 1 16.6 2 17.7 4 16.3
6 16.0 2 15.3 4 16.0 3 17.1 1 16.5 5 17.6
1 16.5 5 18.1 6 16.7 4 16.2 3 16.7 2 17.3
2 17.5 3 16.0 1 16.4 5 18.0 4 16.6 6 16.1
4 15.7 1 16.1 2 16.7 6 16.3 5 17.8 3 16.2
5 18.3 4 16.6 3 16.4 2 17.6 6 17.1 1 16.5
;

proc anova data=Beets;
  class Column Rep Variety Harvest;
  model Y=Rep Column Variety Rep*Column*Variety
        Harvest Harvest*Rep
        Harvest*Variety;
  test h=Rep Column Variety e=Rep*Column*Variety;
  test h=Harvest          e=Harvest*Rep;
run;

```

Output 23.4.1 Class Level Information

Sugar Beet Varieties					
Latin Square Split-Plot Design					
The ANOVA Procedure					
Class Level Information					
Class	Levels	Values			
Column	6	1	2	3	4 5 6
Rep	6	1	2	3	4 5 6
Variety	6	1	2	3	4 5 6
Harvest	2	1	2		
Number of Observations Read					72
Number of Observations Used					72

Output 23.4.2 ANOVA Table

Sugar Beet Varieties					
Latin Square Split-Plot Design					
The ANOVA Procedure					
Dependent Variable: Y					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	46	98.9147222	2.1503200	7.22	<.0001
Error	25	7.4484722	0.2979389		
Corrected Total	71	106.3631944			
	R-Square	Coeff Var	Root MSE	Y Mean	
	0.929971	3.085524	0.545838	17.69028	
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Rep	5	4.32069444	0.86413889	2.90	0.0337
Column	5	1.57402778	0.31480556	1.06	0.4075
Variety	5	20.61902778	4.12380556	13.84	<.0001
Column*Rep*Variety	20	3.25444444	0.16272222	0.55	0.9144
Harvest	1	60.68347222	60.68347222	203.68	<.0001
Rep*Harvest	5	7.71736111	1.54347222	5.18	0.0021
Variety*Harvest	5	0.74569444	0.14913889	0.50	0.7729

First, note from [Output 23.4.2](#) that the overall model is significant.

Output 23.4.3 Tests of Effects

Tests of Hypotheses Using the Anova MS for Column*Rep*Variety as an Error Term					
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Rep	5	4.32069444	0.86413889	5.31	0.0029
Column	5	1.57402778	0.31480556	1.93	0.1333
Variety	5	20.61902778	4.12380556	25.34	<.0001

Tests of Hypotheses Using the Anova MS for Rep*Harvest as an Error Term					
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Harvest	1	60.68347222	60.68347222	39.32	0.0015

[Output 23.4.3](#) shows that the effects for Rep and Harvest are significant, while the Column effect is not. The average Ys for the six different Varieties are significantly different. For these four tests, look at the output produced by the two `TEST` statements, not at the usual ANOVA procedure output. The Variety*Harvest interaction is not significant. All other effects in the default output should either be tested by using the results from the `TEST` statements or are irrelevant as they are only error terms for portions of the model.

Example 23.5: Strip-Split Plot

In this example, four different fertilizer treatments are laid out in vertical strips, which are then split into subplots with different levels of calcium. Soil type is stripped across the split-plot experiment, and the entire experiment is then replicated three times. The dependent variable is the yield of winter barley. The data come from the notes of G. Cox and A. Rotti.

The input data are the 96 values of Y, arranged so that the calcium value (Calcium) changes most rapidly, then the fertilizer value (Fertilizer), then the Soil value, and, finally, the Rep value. Values are shown for Calcium (0 and 1); Fertilizer (0, 1, 2, 3); Soil (1, 2, 3); and Rep (1, 2, 3, 4). The following example produces [Output 23.5.1](#), [Output 23.5.2](#), [Output 23.5.3](#), and [Output 23.5.4](#).

```

title1 'Strip-split Plot';
data Barley;
  do Rep=1 to 4;
    do Soil=1 to 3; /* 1=d 2=h 3=p */
      do Fertilizer=0 to 3;
        do Calcium=0,1;
          input Yield @;
          output;
        end;
      end;
    end;
  end;
end;

```

```

      datalines;
4.91 4.63 4.76 5.04 5.38 6.21 5.60 5.08
4.94 3.98 4.64 5.26 5.28 5.01 5.45 5.62
5.20 4.45 5.05 5.03 5.01 4.63 5.80 5.90
6.00 5.39 4.95 5.39 6.18 5.94 6.58 6.25
5.86 5.41 5.54 5.41 5.28 6.67 6.65 5.94
5.45 5.12 4.73 4.62 5.06 5.75 6.39 5.62
4.96 5.63 5.47 5.31 6.18 6.31 5.95 6.14
5.71 5.37 6.21 5.83 6.28 6.55 6.39 5.57
4.60 4.90 4.88 4.73 5.89 6.20 5.68 5.72
5.79 5.33 5.13 5.18 5.86 5.98 5.55 4.32
5.61 5.15 4.82 5.06 5.67 5.54 5.19 4.46
5.13 4.90 4.88 5.18 5.45 5.80 5.12 4.42
;

proc anova data=Barley;
  class Rep Soil Calcium Fertilizer;
  model Yield =
    Rep
    Fertilizer Fertilizer*Rep
    Calcium Calcium*Fertilizer Calcium*Rep(Fertilizer)
    Soil Soil*Rep
    Soil*Fertilizer Soil*Rep*Fertilizer
    Soil*Calcium Soil*Fertilizer*Calcium
    Soil*Calcium*Rep(Fertilizer);
  test h=Fertilizer          e=Fertilizer*Rep;
  test h=Calcium calcium*fertilizer e=Calcium*Rep(Fertilizer);
  test h=Soil                e=Soil*Rep;
  test h=Soil*Fertilizer      e=Soil*Rep*Fertilizer;
  test h=Soil*Calcium
    Soil*Fertilizer*Calcium    e=Soil*Calcium*Rep(Fertilizer);
  means Fertilizer Calcium Soil Calcium*Fertilizer;
run;

```

Output 23.5.1 Class Level Information

Strip-split Plot			
The ANOVA Procedure			
Class Level Information			
Class	Levels	Values	
Rep	4	1 2 3 4	
Soil	3	1 2 3	
Calcium	2	0 1	
Fertilizer	4	0 1 2 3	
Number of Observations Read			96
Number of Observations Used			96

Output 23.5.2 ANOVA Table

Strip-split Plot					
The ANOVA Procedure					
Dependent Variable: Yield					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	95	31.89149583	0.33569996	.	.
Error	0	0.00000000	.		
Corrected Total	95	31.89149583			
	R-Square	Coeff Var	Root MSE	Yield Mean	
	1.000000	.	.	5.427292	
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Rep	3	6.27974583	2.09324861	.	.
Fertilizer	3	7.22127083	2.40709028	.	.
Rep*Fertilizer	9	6.08211250	0.67579028	.	.
Calcium	1	0.27735000	0.27735000	.	.
Calcium*Fertilizer	3	1.96395833	0.65465278	.	.
Rep*Calcium(Fertili)	12	1.76705833	0.14725486	.	.
Soil	2	1.92658958	0.96329479	.	.
Rep*Soil	6	1.66761042	0.27793507	.	.
Soil*Fertilizer	6	0.68828542	0.11471424	.	.
Rep*Soil*Fertilizer	18	1.58698125	0.08816563	.	.
Soil*Calcium	2	0.04493125	0.02246562	.	.
Soil*Calcium*Fertili	6	0.18936042	0.03156007	.	.
Rep*Soil*Calc(Ferti)	24	2.19624167	0.09151007	.	.

Notice in [Output 23.5.2](#) that the default tests against the residual error rate are all unavailable. This is because the Soil*Calcium*Rep(Fertilizer) term in the model takes up all the degrees of freedom, leaving none for estimating the residual error rate. This is appropriate in this case since the [TEST](#) statements give the specific error terms appropriate for testing each effect. [Output 23.5.3](#) displays the output produced by the various [TEST](#) statements. The only significant effect is the Calcium*Fertilizer interaction.

Output 23.5.3 Tests of Effects

Tests of Hypotheses Using the Anova MS for Rep*Fertilizer as an Error Term					
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Fertilizer	3	7.22127083	2.40709028	3.56	0.0604

Output 23.5.3 *continued*

Tests of Hypotheses Using the Anova MS for Rep*Calcium(Fertili) as an Error Term					
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Calcium	1	0.27735000	0.27735000	1.88	0.1950
Calcium*Fertilizer	3	1.96395833	0.65465278	4.45	0.0255
Tests of Hypotheses Using the Anova MS for Rep*Soil as an Error Term					
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Soil	2	1.92658958	0.96329479	3.47	0.0999
Tests of Hypotheses Using the Anova MS for Rep*Soil*Fertilizer as an Error Term					
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Soil*Fertilizer	6	0.68828542	0.11471424	1.30	0.3063
Tests of Hypotheses Using the Anova MS for Rep*Soil*Calc(Ferti) as an Error Term					
Source	DF	Anova SS	Mean Square	F Value	Pr > F
Soil*Calcium	2	0.04493125	0.02246562	0.25	0.7843
Soil*Calcium*Fertili	6	0.18936042	0.03156007	0.34	0.9059

Output 23.5.4 Results of MEANS statement

Level of Fertilizer	N	-----Yield-----	
		Mean	Std Dev
0	24	5.18416667	0.48266395
1	24	5.12916667	0.38337082
2	24	5.75458333	0.53293265
3	24	5.64125000	0.63926801
Level of Calcium	N	-----Yield-----	
		Mean	Std Dev
0	48	5.48104167	0.54186141
1	48	5.37354167	0.61565219
Level of Soil	N	-----Yield-----	
		Mean	Std Dev
1	32	5.54312500	0.55806369
2	32	5.51093750	0.62176315
3	32	5.22781250	0.51825224

Output 23.5.4 *continued*

Level of Calcium	Level of Fertilizer	N	-----Yield-----	
			Mean	Std Dev
0	0	12	5.34666667	0.45029956
0	1	12	5.08833333	0.44986530
0	2	12	5.62666667	0.44707806
0	3	12	5.86250000	0.52886027
1	0	12	5.02166667	0.47615569
1	1	12	5.17000000	0.31826233
1	2	12	5.88250000	0.59856077
1	3	12	5.42000000	0.68409197

Output 23.5.4 shows the results of the **MEANS** statement, displaying for various effects and combinations of effects, as requested. You can examine the Calcium*Fertilizer means to understand the interaction better.

In this example, you could reduce memory requirements by omitting the Soil*Calcium*Rep(Fertilizer) effect from the model in the **MODEL** statement. This effect then becomes the ERROR effect, and you can omit the last **TEST** statement in the statements shown earlier. The test for the Soil*Calcium effect is then given in the Analysis of Variance table in the top portion of output. However, for all other tests, you should look at the results from the **TEST** statement. In large models, this method might lead to significant reductions in memory requirements.

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