Technical Paper

Orthogonal Arrays

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Orthogonal Arrays

By Warren F. Kufeld

SAS provides a catalog of over 117,000 orthogonal arrays. Orthogonal arrays are frequently used as plans for conducting experiments. This site provides:

- a library of strength-two (main-effects only) orthogonal arrays
- lists and other information about orthogonal arrays
- a library of difference schemes
- tools for orthogonal array and efficient factorial design generation.

Every orthogonal array listed in ***Orthogonal Arrays*** (Hedayat, Sloane, and Stufken, 1999) (http://neilsloane.com/oadir/index.html) can be found here along with many arrays that were not known in 1999 and many other larger arrays as well. Thanks to **Warren F. Kuhfeld**, **Don Anderson**, **Warwick DeLauney**, **Nam-Ky Nguyen**, **Shanqi Pang**, **Neil Sloane**, **Chung-yi Suen**, **Randy Tobias**, **J.C. Wang**, and **Yingshan Zhang** who have all kindly helped with some of the arrays and difference schemes in this catalog.

Orthogonal array research is an active area today, and new arrays are being discovered all of the time. Also, there are some larger arrays that are known to exist, but are not included here. Most of the larger arrays are based on difference schemes, and there are a few that are currently not in the software. The orthogonal array construction code, with hundreds of methods implemented, is freely available on the web. Please contact me if you can help fill in any arrays that are not on this site or provide more elegant construction methods. This resource cannot continue to grow without your help.

Click here to see a library of orthogonal arrays. This flat file contains virtually all known strength-two orthogonal array parents (except full-factorial designs) up to 143 runs and a few of the known parents in 144 runs. These arrays can be used to construct a wide variety of other arrays using the methods described with the orthogonal array lists. Note however that the SAS MktEx macro can **automatically** create these and many more orthogonal arrays.

Click here to see lists of orthogonal arrays. This PDF file has a list of virtually all known strength-two orthogonal arrays up through 143 runs, a list of parent orthogonal arrays (including virtually all known parent orthogonal arrays up through 143 runs and a reasonably comprehensive set of parent orthogonal arrays for 144 through 513 runs), and a reference list.

Neil Sloane's orthogonal array site (http://www.research.att.com/~njas/oadir/) provides many strength-two orthogonal arrays including some arrays that are alternatives to the ones shown here. This site also contains arrays of strength greater than two, assorted Hadamard constructions, many other useful designs not covered here, and information about the 1999 book ***Orthogonal Arrays*** by Hedayat, Sloane, and Stufken.

Nam-Ky Nguyen's site (http://designcomputing.net/gendex/noa/) provides many near-orthogonal arrays made with his NOA tool.

SAS provides a set of free macros for making orthogonal arrays and *D*-efficient nonorthogonal designs. They are documented in the free book ***Marketing Research Methods in SAS***. While the book features marketing research examples, these macros have been widely used to make factorial designs for many other application areas.

- Click here for the general book and macro site.
- Click here for a direct link to the macro documentation.
- Click here for a direct link to the macros.
- Click here for a direct link to the full 1309-page book.

The MktEx Macro for Efficient Factorial Designs

The MktEx macro (pronounced "Mark Tex") generates factorial experimental designs. The "Mkt" or "market" part of the name comes from the fact that it was originally designed with marketing researchers in mind. MktEx is just one in a series of Mkt macros. Some would only be used by marketing researchers and others doing choice modeling. Others, like MktEx, MktBIBD (balanced incomplete block design), MktBSize (balanced incomplete block design sizes), MktBal (balanced designs), MktOrth (orthogonal array catalog), and MktRuns (suggest number of runs), MktBlock (block a design), MktEval (evaluate a design), MktDups (check for duplicate runs), and MktLab (reassign levels and names), are of interest to a much larger audience. Marketing researchers have extremely interesting design requirements. They often need designs that are larger and more complicated than is typically required in other sciences. However, researchers in many other areas use MktEx every day to make designs for a variety of purposes. MktEx was developed for everyone who wants to make efficient designs, not just marketing researchers. This full-featured macro can easily handle simple problems like main-effects designs and more complicated problems including designs with interactions and designs with restrictions on which levels can appear together. Over 117,000 orthogonal arrays are available in its catalog. Efficient nonorthogonal designs are quickly and easily found. Here are some of the MktEx and other design macro capabilities:

• easy to use	• Hadamard matrices	• balanced designs
• efficient designs	• Taguchi designs	• any number of factors
• orthogonal arrays	• main effects	• up to 144-level factors
• nonorthogonal designs	• interactions	• any mix of levels
• nearly orthogonal designs	• blocking factors	• level restrictions
• full-factorial designs	• design diagnostics	• general restrictions
• fractional-factorial designs	• design evaluations	• ensure no duplicate profiles
• tabled designs	• automatic randomization	• partial profiles

MktEx and the other macros are documented here, they are free from here, and the entire book is available from here. The MktEx macro is free, however it requires the following SAS products in order to run: BASE, SAS/STAT, SAS/QC, and SAS/IML.

In this example, MktEx produces an orthogonal array with 1 two-level factor and 6 three-level factors in 18 runs:

```
%mktex( 2 3 ** 6, n=18 )
```

This next example requests a nearly orthogonal design with 15 three-level factors in 36 runs. MktEx uses a combination of an orthogonal array and a computerized search algorithm to find an efficient design.

```
%mktex( 3 ** 15, n=36, seed=17 )
```

This next example illustrates finding a design with restrictions and interactions. You can write a SAS macro that prevents certain level combinations from occuring together or defines any type of restriction that you want. MktEx can find designs with very complicated sets of restrictions. Here is an example with a simple set of restrictions.

```
%macro resmac;
  avail = (x1 < 4) + (x2 < 4) + (x5 < 3) + (x6 < 3) + (x8 < 3);
  if (avail < 2) | (avail > 4) then bad = abs(avail - 3);
  else bad = 0;
  %mend;

%mktex( 4 4 2 2 3 3 2 3, n=36, interact=x2*x3 x2*x4 x3*x4 x6*x7,
        restrictions=resmac, seed=104)
```

The user defines a badness function for MktEx to minimize. More details on this example are available starting on page 431 of ***Marketing Research Methods in SAS***.

Hadamard Matrices That MktEx Can Make

Hadamard matrices are binary matrices, usually consisting of (1, -1) or (0, 1). Hadamard matrices are useful for making orthogonal arrays with two-level factors. MktEx can make Hadamard matrices for all of the following sizes up to 1000:

2	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76
80	84	88	92	96	100	104	108	112	116	120	124	128	132	136	140	144	148	152	156
160	164	168	172	176	180	184	188	192	196	200	204	208	212	216	220	224	228	232	236
240	244	248	252	256	260	264	268	272	276	280	284	288	292	296	300	304	308	312	316
320	324	328	332	336	340	344	348	352	356	360	364	368	372	376	380	384	388	392	396
400	404	408	412	416	420	424	428	432	436	440	444	448		456	460	464	468	472	
480	484	488	492	496	500	504		512	516	520	524	528		536	540	544	548	552	556
560	564	568	572	576	580	584	588	592	596	600	604	608	612	616	620	624	628	632	636
640	644	648		656	660	664	---	672	676	680	684	688	692	696	700	704	708	712	---
720	724	728	732	736	740	744	748	752	756	760	---	768		776	780	784	788	792	796
800	804	808	812	816	820	824	828	832		840	844	848		856	860	864	868	872	
880	884	888	---	896	900		908	912	916	920	924	928		936		944	948		---
960		968	972	976		984		992		1000									

It can make a number of larger Hadamard matrices as well. Every size up to 448 is in the list, with good but not complete coverage beyond that. The MktEx macro can construct these matrices when *n* is a multiple of 4 and one or more of the following hold:

- $n \le 448$ or n = 580, 596, 604, 612, 724, 732, 756, or 1060,
- $n - 1$ is prime,
- $n / 2 - 1$ is prime power and mod(n / 2, 4) = 2,
- n is a power of 2 (2, 4, 8, 16, ...) times the size of a smaller Hadamard matrix that is available.

Entries of "---" indicate sizes where there is currently no known construction method. Help in making the remaining Hadamard matrices would be welcome. The seed vectors for Williamson and other constructions can be found in MktEx. Other sources of Hadamard matrices on the web include the extensive site maintained by V.K. Gupta, A. Dhandapani, and Rajender Parsad (http://iasri.res.in/design/WebHadamard/WebHadamard.htm).

Difference Schemes That MktEx Can Make

Click here to see a library of difference schemes and generalized Hadamard matrices. Difference schemes provide the building blocks from which many orthogonal arrays are constructed. Let D(λs, c, s) denote a difference scheme with λs rows and c columns with entries 0, 1, ..., s - 1. When $c = \lambda s$, the difference scheme is square, and D(λs, λs, s) is called a generalized Hadamard matrix. MktEx can make the following difference schemes and generalized Hadamard matrices:

D(3,3,3)	D(6,6,3)	D(9,9,3)	D(12,12,3)	D(15,9,3)	D(18,18,3)	D(21,12,3)
D(24,24,3)	D(27,27,3)	D(30,30,3)	D(33,13,3)	D(36,36,3)	D(39,13,3)	D(42,21,3)
D(45,27,3)	D(48,48,3)	D(51,24,3)	D(54,54,3)	D(57,27,3)	D(60,30,3)	D(63,36,3)
D(66,30,3)	D(69,24,3)	D(72,72,3)	D(75,27,3)	D(78,30,3)	D(81,81,3)	D(84,42,3)
D(87,27,3)	D(90,90,3)	D(93,30,3)	D(96,96,3)	D(99,39,3)	D(102,48,3)	D(105,29,3)
D(108,108,3)	D(111,36,3)	D(114,30,3)	D(117,39,3)	D(120,48,3)	D(123,30,3)	D(126,72,3)
D(129,48,3)	D(132,132,3)	D(135,81,3)	D(138,48,3)	D(141,30,3)	D(144,144,3)	D(147,42,3)
D(150,150,3)	D(153,72,3)	D(156,48,3)	D(159,36,3)	D(162,162,3)	D(165,42,3)	D(168,84,3)
D(171,81,3)						

D(4,4,4)	D(8,8,4)	D(12,12,4)	D(16,16,4)	D(20,10,4)	D(24,20,4)	D(28,12,4)
D(32,32,4)	D(36,36,4)	D(40,16,4)	D(44,12,4)	D(48,48,4)	D(52,16,4)	D(56,56,4)
D(60,20,4)	D(64,64,4)	D(68,32,4)	D(72,36,4)	D(76,16,4)	D(80,40,4)	D(84,36,4)
D(88,32,4)	D(92,36,4)	D(96,96,4)	D(100,36,4)	D(104,48,4)	D(108,108,4)	D(112,56,4)
D(116,36,4)	D(120,56,4)	D(124,20,4)	D(128,128,4)			

D(5,5,5)	D(10,10,5)	D(15,8,5)	D(20,20,5)	D(25,25,5)	D(30,11,5)	D(35,10,5)
D(40,20,5)	D(45,20,5)	D(50,50,5)	D(55,11,5)	D(60,20,5)	D(65,20,5)	D(70,20,5)
D(75,40,5)	D(80,80,5)	D(85,20,5)	D(90,90,5)	D(95,20,5)	D(100,100,5)	

D(6,2,6)	D(12,6,6)	D(18,2,6)	D(24,6,6)	D(30,2,6)	D(36,7,6)	D(42,2,6)
D(48,10,6)	D(54,2,6)	D(60,8,6)	D(66,2,6)	D(72,12,6)	D(78,2,6)	D(84,8,6)

D(7,7,7)	D(14,14,7)	D(21,9,7)	D(28,28,7)	D(35,9,7)	D(42,18,7)	D(49,49,7)
D(56,28,7)	D(63,14,7)	D(70,18,7)				

D(8,8,8)	D(16,16,8)	D(24,8,8)	D(32,32,8)	D(40,10,8)	D(48,16,8)	D(56,56,8)
D(64,64,8)						

D(9,9,9)	D(18,18,9)	D(27,27,9)	D(36,36,9)	D(45,18,9)	D(54,54,9)
D(10,2,10)	D(20,5,10)	D(30,2,10)	D(40,6,10)	D(50,2,10)	
D(11,11,11)	D(22,22,11)	D(33,11,11)	D(44,44,11)		
D(12,6,12)	D(24,6,12)	D(36,6,12)			
D(13,13,13)	D(26,26,13)				
D(14,2,14)	D(28,5,14)				
D(15,5,15)	D(30,5,15)				
D(16,16,16)	D(32,32,16)				
D(17,17,17)					
D(18,2,18)					
D(19,19,19)					
D(20,4,20)					
D(21,6,21)					
D(22,2,22)					

Difference Schemes That MktEx Cannot Make

There are a few larger arrays that should be included in MktEx for completeness, but they rely on obscure difference schemes. MktEx still needs the following difference schemes:

```
D(60,36,3)   D(102,51,3)
D(60,21,4)   D(112,64,4)
D(30,15,5)   D(35,17,5)   D(40,25,5)   D(55,17,5)   D(60,25,5)   D(65,25,5)   D(85,35,5)
D(60,11,6)   D(84,16,6)
D(35,11,7)   D(63,28,7)
D(40,8,10)
D(30,7,15)
```

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