# Practical Data Analysis with JMP ${ }^{\circ}$ 

 Second EditionRobert H. Carver

## Student Solutions

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Practical Data Analysis with JMP ${ }^{\circledR}$, Second Edition
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# Student Solutions to Application Scenarios 

## Scenario 1

Student answers will vary. Answers will depend on data set student selects to input into a new JMP data table

## Scenario 2

Quantity of cement (component 1 ), expressed as kg in a $\mathrm{m}^{\wedge} 3$ mixture.
Quantity of Superplasticizer (component 5), expressed as kg in a $\mathrm{m}^{\wedge} 3$ mixture.
Quantity of Fine Aggregate (component 7), expressed as kg in a $\mathrm{m}^{\wedge} 3$ mixture.

## Scenario 3

Columns that need to be corrected: DMDMARTL, RIDEXPRG, BPQ150A

## Scenario 4

NHANES does not contain experimental data because the experimenters are not manipulating any of the variables. The data was not obtained through a designed experiment but through observation.

Scenario 5
Open the Military table, and select Rows Row Selection Select Randomly and specify a sample size of $\mathbf{5 0 0}$. Then choose Tables Subset.

Scenario 6
This data table contains monthly stock values and volume from the FTSE 100 index, from1 January 2003 through 1 December 2007. Data were collected by observation on
the first day of each month. The date column is ordinal because it is a chronological variable. Open, High, Low, Close, Volume, and change\% are all Continuous columns containing numeric measurements. Open is the FTSE 100 index's opening price. High represents the high price for the day. Low is the low price for that day. Close is the closing price for that day. Volume is the number of shares exchanged during the day. change $\%$ is how much the index changed from open to close.

## Scenario 7

This data table contains statistics from earthquakes recorded worldwide between August 20, 2009 and September 19, 2009. Data was collected by observation on the first day of each month. The date column is ordinal because it is a chronological variable. Latitude is a continuous variable indicating the latitudinal coordinate of where the earthquake took place. Longitude is also a continuous variable indicating the longitudinal coordinate of where the earthquake took place. Magnitude is a continuous measurement of how strong the earthquake was, while depth is a continuous variable describing how far from the surface the epicenter was. Time is an ordinal column describing when the earthquake took place. This data was found by observation.

## Scenario 8

This table contains observational data from the WHO regarding tobacco use, cardiovascular disease and cancer rates. Code is a nominal variable uniquely identifying each nation. Country is a nominal variable that provides the name of the country relating to the data. Region is also a nominal variable indicating the region where the country is located in. TobaccoUse is a continuous variable observed describing the prevalence of tobacco use in that country. Female and Male are both continuous variables that were found observationally which describe the prevalence of tobacco use for both genders. CVmort is the mortality rate from cardiovascular disease for this country and CancerMort is the cancer mortality rate for this country. Both are continuous.

## Scenario 10

The variables are Activity (travel, feed, social), Period (morning, noon, afternoon, evening), and Groups (numeric). The observational units were groups of dolphins. Activity and Period are nominal and Groups (\# dolphins in each group) is continuous.
Scenario 11
The columns are as follows:
marst : marital status (nominal). Respondent's marital status, one of six levels empstat: employment status (nominal). Respondent's employment status, one of five levels.
sleeping minutes spent sleeping each day (continuous).
telff minutes spent on the telephone with family and friends each day (continuous).

## Scenario 11

This data table appears to contain demographic, economic, crime and other statistics for the 50 US states and the District of Columbia. The three specific variables are all continuous, and represent the following:
smoke is the percentage of the state population that smokes.
fed_spend is the per capita amount of federal spending in the state (dollars)
nuclear is the percentage of power coming from nuclear sources


## Student Solutions to Application Scenarios

## Scenario 1

a.Using the grabber tool, click and drag upwards to increase the number of bars in the histogram. A second peak near 80 appears when as the number of bars increases, while the peak at 75 remains.

c. Scale can be manipulated in order to change the center, shape, and spread of a histogram, so it is important to carefully analyze and think critically about the choice of scale on an axis.

## Scenario 2

a. This histogram has a shape that is skewed to the left, has a mean of about 70, and a spread described by a range from 35 to 80. It has one peak

Distributions
LifeExp


| Quantiles |  |  | Summary Statistics |  |
| :---: | :---: | :---: | :---: | :---: |
| 100.0 | maximu | 78.3 | Mean | 64.364077 |
| 99.5\% |  | 78.3 | Std Dev | 9.8722116 |
| 97.5\% |  | 76.9976 | Std Err Mean | 0.7069638 |
| 90.0\% |  | 75.1846 | Upper 95\% Mea | 65.758399 |
| 75.0\% | quartile | 71.77 | Lower 95\% Mea | 62.969755 |
| 50.0\% | median | 67.231 | N | 195 |
| 25.0\% | quartile | 57.251 |  |  |
| 10.0\% |  | 48.8434 |  |  |
| 2.5\% |  | 42.6994 |  |  |
| 0.5\% |  | 39.906 |  |  |
| 0.0\% | minimu | 39.906 |  |  |

c.The standard deviation is 9.87 in the 1985 data compared to 10.4 in the 2010 data.

Scenario 3
a. The points furthest to the left and right indicate the minimum and maximum respectively. In each boxplot, the ends of the box represent the first and third quartiles, and the line within the box represents the median. The diamond shows the location of the mean. We see a handful of outlying points in the LifeSpan boxplot, but not in the TotalSleep plot.

## LifeSpan



## TotalSleep


c. $99.5 \%$ of the species have a life span less than 100 years.
e.The animals that get the most sleep tend to be relatively small animals and have low predation, exposure, and danger values.
g.The animals that sleep in the most exposed locations are also the largest in terms of body weight. This may be because larger animals cannot hide as easily, or due to sheer size, they can sleep in exposed locations safely.

## Scenario 4

a....Volume has a nearly symmetrical and normal distribution. It ranges from 1043.49 to 2115.33 with a median of 1726.22 and a mean of 1710.49.
c.The FTSE declines approximately $25 \%$ of the time.
e.This line graph shows fluctuation without any obvious pattern. The monthly percentage change seems to vary at random from month to month, typically remaining approximately between $-3 \%$ and $+3 \%$. There is no obvious growth over the five years, in contrast to the closing index value.


## Scenario 5

a.The histogram has four bars. DFW, LAX, and ORD all have high with counts of over 4000 while LAS is low with only a count of around 400.

c. Because airlines attempt to schedule arrivals accurately, it is unlikely that very many flights would be extraordinarily early. However, given the many possible reasons for delays and the nature of travel, some flights can be exceptionally late. The practical minimum sets a lower bound for this variable, but there isn't a comparable upper bound. As such, a few flights with very long delays will tend to skew the data.

## Scenario 6

a.TobaccoUse is somewhat symmetrical with a mean of 24.77 and median of 25.6 . It ranges from 4.3 to 51.8 .
c.CVMort has two peaks at around 150 and 400 . It is skewed to the right. It has a mean of 355.5 and a median of 375 . It ranges from 106 to 713 .
e.Europe \& Central Asia and Sub-Saharan Africa have the highest count of countries in this data table. South Asia has the lowest count and America, East Asia \& Pacific and Middle East \& North Africa all fall in the middle.

Scenario 7


This is a strongly right-skewed distribution with 4 outliers (California, Texas, New York, and Florida). The mean population was $6,053,834$ people and the median was just $4,339,367$. States range from approximately 563,000 people in Wyoming to more than 37 million in California. The largest number of states have fewer than 5 million residents.

## Student Solutions to Application Scenarios

## Scenario 1



Public provision is most common by far in the Americas and Europe \& Central Asia.
Provide provision seems to be the norm in the rest of the world. Most areas have relatively few countries with both public and private, though such arrangements are fairly common (more than $25 \%$ of countries) in the Americas.


The fitted line and RSquare value are shown above. There is a strong, positive linear relationship between the two variables.

Scenario 2


The plot, equation and Rsquare are shown above. The correlation coefficient is 0.17395 . There is a weak negative relationship between mileage and price: the higher the mileage, the lower the price.


The distribution of price across the three cities seems to be fairly uniform. The box plot shows similar middle $50 \%$ with varying means. They also have very similar spreads.


As individuals get older, blood pressure increases.
c. Men have a higher average systolic blood pressure. Both genders have similar shape, being skewed to the right. Women have a far greater range, spanning from 70 to 270 while men have readings from 80 to about 210.


It appears there is little evidence of a relationship between pulse and blood pressure, as the $r$ squared statistic is .2 , which is very low.

Scenario 4
a.Tobacco is most heavily used in Europe and Central Asia and to a lesser extent in East Asia and the Pacific. There is a moderate use in the Middle East and North Africa as well as the Americas while South Asia and Sub-Saharan Africa has the lowest tobacco use.


Here again, we find scant evidence of a relationship.

## Scenario 5



More males were employed (at work) than females while more females were not in the labor force. About the same amount of males and females were unemployed and looking or employed and absent.
c.People employed had the lowest mean time spent sleeping. All employment statuses had nearly normal distributions with some like employed at work being more skewed to the right than others. Nearly all the spreads of employment categories ranged across the same amount of time.

Scenario 6
a. $\triangle$ Contingency Table

| Predation |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Count <br> Total \% <br> Col \% <br> Row \% |  | 2 | 3 | 4 | 5 |  |
| 1 | 10 | 7 | 7 | 2 | 1 | 27 |
|  | 16.13 | 11.29 | 11.29 | 3.23 | 1.61 | 43.55 |
|  | 71.43 | 46.67 | 58.33 | 28.57 | 7.14 |  |
|  | 37.04 | 25.93 | 25.93 | 7.41 | 3.70 |  |
| 2 | 2 | 7 | 2 | 0 | 2 | 13 |
|  | 3.23 | 11.29 | 3.23 | 0.00 | 3.23 | 20.97 |
|  | 14.29 | 46.67 | 16.67 | 0.00 | 14.29 |  |
|  | 15.38 | 53.85 | 15.38 | 0.00 | 15.38 |  |
| $\stackrel{i}{x}^{3}$ | 1 | 1 | 0 | 1 | 1 | 4 |
|  | 1.61 | 1.61 | 0.00 | 1.61 | 1.61 | 6.45 |
|  | 7.14 | 6.67 | 0.00 | 14.29 | 7.14 |  |
|  | 25.00 | 25.00 | 0.00 | 25.00 | 25.00 |  |
| 4 | 1 | 0 | 0 | 3 | 1 | 5 |
|  | 1.61 | 0.00 | 0.00 | 4.84 | 1.61 | 8.06 |
|  | 7.14 | 0.00 | 0.00 | 42.86 | 7.14 |  |
|  | 20.00 | 0.00 | 0.00 | 60.00 | 20.00 |  |
| 5 | 0 | 0 | 3 | 1 | 9 | 13 |
|  | 0.00 | 0.00 | 4.84 | 1.61 | 14.52 | 20.97 |
|  | 0.00 | 0.00 | 25.00 | 14.29 | 64.29 |  |
|  | 0.00 | 0.00 | 23.08 | 7.69 | 69.23 |  |
|  | 14 | 15 | 12 | 7 | 14 | 62 |
|  | 22.58 | 24.19 | 19.35 | 11.29 | 22.58 |  |

Animals with lower exposure values seem to have lower predation ratings. Conversely, creatures with higher exposure values also had higher predation ratings.
c. $\Delta$ Bivariate Fit of LifeSpan By TotalSleep


- Linear Fit
$\triangle$ Linear Fit LifeSpan $=36.932623-1.6403373^{*}$ TotalSleep
$\triangle$ Summary of Fit

| RSquare | 0.168266 |
| :--- | :--- |
| RSquare Adj | 0.152271 |
| Root Mean Square Error | 17.31945 |
| Mean of Response | 19.85185 |

Observations (or Sum Wgts) 54

There seems to be evidence of a weak negative relationship between lifespan and total sleep. The Rsquare statistic is only 0.168 .

## Scenario 7

a.
$\Delta$ Contingency Table

| Count | 1 |  | 2 to 10 | 11 to |
| :--- | ---: | ---: | ---: | ---: |
| Total \% |  |  | 100 |  |
| Col \% |  |  |  |  |
| Row \% |  |  |  |  |
| No Cloud | 468 | 121 | 12 | 601 |
|  | 45.53 | 11.77 | 1.17 | 58.46 |
|  | 59.47 | 55.50 | 52.17 |  |
|  | 77.87 | 20.13 | 2.00 |  |
| Overcast | 144 | 42 | 7 | 193 |
|  | 14.01 | 4.09 | 0.68 | 18.77 |
|  | 18.30 | 19.27 | 30.43 |  |
| Some Cloud | 74.61 | 21.76 | 3.63 |  |
|  | 175 | 55 | 4 | 234 |
|  | 17.02 | 5.35 | 0.39 | 22.76 |
|  | 22.24 | 25.23 | 17.39 |  |
|  | 74.79 | 23.50 | 1.71 |  |
|  | 787 | 218 | 23 | 1028 |
|  | 76.56 | 21.21 | 2.24 |  |

Neither the mosaic plot nor the contingency table show much evidence of large differences in number of birds struck across different sky conditions. Regardless of conditions, for example, it appears that about $75 \%$ of incidents involve a single bird.


There are various ways to approach this question. One simple way is to explore the relationship using Graph Builder. In this graph we see that median speed is approximately the same regardless of the number of birds struck. However, single-bird incidents occur at a wide variety of speed; as the number of birds involved increases, the variability of speed decreases.

Scenario 8


Using Graph Builder to investigate this relationship we find a positive but inconsistent relationship between income and percentage of population with a bachelor's degree. There is a clear upward pattern with a lot of scatter, indicating that a relationship exists but it is not very strong.


There are very few counties lying below the 45-degree diagonal line, indicating that the percentage of homes where a foreign language is spoken almost always exceeds the percentage of homes with a foreign-born member. This makes sense, assuming that homes with no foreign-born members would be less inclined to speak a foreign language.


- Linear Fit
$\Delta$ Linear Fit
pop2010 $=2857.8184+1.0650107^{*}$ pop 2000
The slope of the line is approximately 1.065 , indicating that on average, the population of US counties grew by $6.5 \%$ from 2000 to 2010 .


# Student Solutions to Application Scenarios 

## Scenario 1

NOTE: This contingency table provides the necessary information to respond to all parts:

| DMDMARTL |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Count <br> Total \% <br> Col \% <br> Row \% | Marrie <br> d | Widow ed | Divorc ed | Separat ed | Never <br> Marrie d | Living with Partner | Refuse <br> d | Don't <br> Know |  |
| Mexican American | 591 | 57 | 60 | 42 | 610 | 121 | 0 | 0 | 1481 |
|  | 9.19 | 0.89 | 0.93 | 0.65 | 9.49 | 1.88 | 0.00 | 0.00 | 23.03 |
|  | 22.57 | 13.26 | 13.36 | 26.58 | 26.38 | 26.54 | 0.00 | 0.00 |  |
|  | 39.91 | 3.85 | 4.05 | 2.84 | 41.19 | 8.17 | 0.00 | 0.00 |  |
| Other Hispanic | 86 | 4 | 10 | 2 | 73 | 20 | 0 | 1 | 196 |
|  | 1.34 | 0.06 | 0.16 | 0.03 | 1.14 | 0.31 | 0.00 | 0.02 | 3.05 |
|  | 3.28 | 0.93 | 2.23 | 1.27 | 3.16 | 4.39 | 0.00 | 100.00 |  |
|  | 43.88 | 2.04 | 5.10 | 1.02 | 37.24 | 10.20 | 0.00 | 0.51 |  |
| Non-Hispanic White | 1403 | 254 | 239 | 42 | 703 | 179 | 6 | 0 | 2826 |
|  | 21.82 | 3.95 | 3.72 | 0.65 | 10.93 | 2.78 | 0.09 | 0.00 | 43.95 |
|  | 53.59 | 59.07 | 53.23 | 26.58 | 30.41 | 39.25 | 100.00 | 0.00 |  |
|  | 49.65 | 8.99 | 8.46 | 1.49 | 24.88 | 6.33 | 0.21 | 0.00 |  |
| Non-Hispanic Black | 426 | 102 | 122 | 66 | 824 | 116 | 0 | 0 | 1656 |
|  | 6.63 | 1.59 | 1.90 | 1.03 | 12.81 | 1.80 | 0.00 | 0.00 | 25.75 |
|  | 16.27 | 23.72 | 27.17 | 41.77 | 35.64 | 25.44 | 0.00 | 0.00 |  |
|  | 25.72 | 6.16 | 7.37 | 3.99 | 49.76 | 7.00 | 0.00 | 0.00 |  |
| Other | 112 | 13 | 18 | 6 | 102 | 20 | 0 | 0 | 271 |
|  | 1.74 | 0.20 | 0.28 | 0.09 | 1.59 | 0.31 | 0.00 | 0.00 | 4.21 |
|  | 4.28 | 3.02 | 4.01 | 3.80 | 4.41 | 4.39 | 0.00 | 0.00 |  |
|  | 41.33 | 4.80 | 6.64 | 2.21 | 37.64 | 7.38 | 0.00 | 0.00 |  |
|  | 2618 | 430 | 449 | 158 | 2312 | 456 | 6 | 1 | 6430 |
|  | 40.72 | 6.69 | 6.98 | 2.46 | 35.96 | 7.09 | 0.09 | 0.02 |  |

a. $\operatorname{Pr}($ Mexican American $)=0.2303$
c. $\operatorname{Pr}($ Mexican American and Never Married $)=0.0949$.
e.No. In part e we found that $\operatorname{Pr}($ Never Married $\mid$ Mexican American $)=0.4114$. The marginal probability $\operatorname{Pr}($ Never Married $)=0.3596$. Because the probabilities are unequal, we find that the events are not independent.

## Scenario 2

For all of the questions that follow, we can use this contingency table:

a. $\operatorname{Pr}($ Binge at least once a week $)=0.1276$.
c. $\operatorname{Pr}($ Accident $)=0.0560$.
e. $\operatorname{Pr}($ Accident $\mid$ binge at least once a week $)=0.1443$.
g.No. Comparing the results in parts a and $f$ or parts c and e should lead to the conclusion that because the relevant marginal probabilities do not equal the corresponding conditionals, the events are not independent.

Scenario 3
NOTE: Different contingency tables are needed for different parts of this problem.
a. $\operatorname{Pr}($ Not in labor force $)=0.3154$

Parts cand d rely on this table:

| fullpart |  |  |  |  |
| :--- | :--- | :--- | ---: | ---: |
| Count Full NIU Part  <br> Total \% time (Not in time   <br> Col \%  univers   <br> Row \%  e)   <br> Female 2925 2884 1141 6950 <br>  23.88 23.55 9.32 56.74 <br>  46.44 66.30 71.31  <br>  42.09 41.50 16.42  <br> Male 3373 1466 459 5298 <br>  27.54 11.97 3.75 43.26 <br>  53.56 33.70 28.69  <br>  63.67 27.67 8.66  <br>  6298 4350 1600 12248 <br>  51.42 35.52 13.06  |  |  |  |  |

c. $\operatorname{Pr}($ Part-time or female $)=\operatorname{Pr}($ part-time $)+\operatorname{Pr}($ female $)-\operatorname{Pr}($ part-time and female $)=0.1306+0.5674-$ $0.0932=0.6048$
e. $\Delta$ Frequencies

| Level | Count | Prob |
| :--- | ---: | ---: |
| Divorced | 1683 | 0.13741 |
| Married - spouse absent | 169 | 0.01380 |
| Married - spouse present | 6085 | 0.49682 |
| Never married | 2905 | 0.23718 |
| Separated | 331 | 0.02702 |
| Widowed | 1075 | 0.08777 |
| Total | 12248 | 1.00000 |
| N Missing 20720 |  |  |
| $\quad$ Levels |  |  |

The marital status column identifies three types of respondents who are not married: those who are divorced, never married, or widowed. To find the probability of selecting a person who is not married, we sum the probabilities of these three categories:
$\operatorname{Pr}($ Not Married $)=0.13741+0.23718+0.08777=0.46236$.

This table can be used for Part f:

| Count <br> Total \% <br> Col \% <br> Row \% | Employ ed absent | Employ ed - at work | Not in <br> labor <br> force | Unemp loyed looking | Unemp loyed on layoff |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Divorced | 44 | 1086 | 488 | 56 | 9 | 1683 |
|  | 0.36 | 8.87 | 3.98 | 0.46 | 0.07 | 13.74 |
|  | 13.50 | 14.34 | 12.63 | 13.24 | 14.06 |  |
|  | 2.61 | 64.53 | 29.00 | 3.33 | 0.53 |  |
| Married - spouse absent | 5 | 99 | 57 | 6 | 2 | 169 |
|  | 0.04 | 0.81 | 0.47 | 0.05 | 0.02 | 1.38 |
|  | 1.53 | 1.31 | 1.48 | 1.42 | 3.13 |  |
|  | 2.96 | 58.58 | 33.73 | 3.55 | 1.18 |  |
| Married thouse present | 186 | 4107 | 1643 | 118 | 31 | 6085 |
|  | 1.52 | 33.53 | 13.41 | 0.96 | 0.25 | 49.68 |
|  | 57.06 | 54.24 | 42.53 | 27.90 | 48.44 |  |
|  | 3.06 | 67.49 | 27.00 | 1.94 | 0.51 |  |
| Never married | 70 | 1856 | 742 | 223 | 14 | 2905 |
|  | 0.57 | 15.15 | 6.06 | 1.82 | 0.11 | 23.72 |
|  | 21.47 | 24.51 | 19.21 | 52.72 | 21.88 |  |
|  | 2.41 | 63.89 | 25.54 | 7.68 | 0.48 |  |
| Separated | 10 | 214 | 90 | 12 | 5 | 331 |
|  | 0.08 | 1.75 | 0.73 | 0.10 | 0.04 | 2.70 |
|  | 3.07 | 2.83 | 2.33 | 2.84 | 7.81 |  |
|  | 3.02 | 64.65 | 27.19 | 3.63 | 1.51 |  |
| Widowed | 11 | 210 | 843 | 8 | 3 | 1075 |
|  | 0.09 | 1.71 | 6.88 | 0.07 | 0.02 | 8.78 |
|  | 3.37 | 2.77 | 21.82 | 1.89 | 4.69 |  |
|  | 1.02 | 19.53 | 78.42 | 0.74 | 0.28 |  |
|  | 326 | 7572 | 3863 | 423 | 64 | 12248 |
|  | 2.66 | 61.82 | 31.54 | 3.45 | 0.52 |  |

c.This problem is complicated by the fact that most cells in this column are blank and the remaining cells contain the label "Yes." There are 189 "Yes" values and 468 rows in all. Therefore $\operatorname{Pr}($ Evacuation $)=189 / 468=0.4038$.
e.

LRTYPE_TEXT

| Count <br> Total \% <br> Col \% <br> Row \% | LEAK | N/A | OTHER | $\begin{array}{\|l\|} \hline \text { RUPTU } \\ \text { RE } \end{array}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 42 | 18 | 139 | 59 | 258 |
|  | 10.29 | 4.41 | 34.07 | 14.46 | 63.24 |
|  | 50.60 | 72.00 | 66.51 | 64.84 |  |
|  | 16.28 | 6.98 | 53.88 | 22.87 |  |
| Yes | 41 | 7 | 70 | 32 | 150 |
|  | 10.05 | 1.72 | 17.16 | 7.84 | 36.76 |
|  | 49.40 | 28.00 | 33.49 | 35.16 |  |
|  | 27.33 | 4.67 | 46.67 | 21.33 |  |
|  | 83 | 25 | 209 | 91 | 408 |
|  | 20.34 | 6.13 | 51.23 | 22.30 |  |

$\operatorname{Pr}($ Rupture or Explosion $)=\operatorname{Pr}($ Rupture $)+\operatorname{Pr}($ Explosion $)-\operatorname{Pr}($ Rupture and Explosion $)=0.2230+$ $0.3676-0.0784=0.5122$.

## Scenario 5

a. $\operatorname{Pr}($ Registered $)=0.6295$
c.This table applies to questions c and d :

| subscription_type |  |  |  |
| :---: | :---: | :---: | :---: |
| Count <br> Total \% <br> Col \% <br> Row \% | Casual | Registe red |  |
| Female | 0 | 8634 | 8634 |
|  | 0.00 | 24.31 | 24.31 |
|  |  | 24.31 |  |
|  | 0.00 | 100.00 |  |
| Male | 0 | 26876 | 26876 |
|  | 0.00 | 75.69 | 75.69 |
|  |  | 75.69 |  |
|  | 0.00 | 100.00 |  |
|  | 0 | 35510 | 35510 |
|  | 0.00 | 100.00 |  |

$\operatorname{Pr}($ female who is registered $)=0.2431$.

Note: The full contingency table is too large to reproduce effectively here.
e. $\operatorname{Pr}($ began at South Station $)=0.0477=2,636 / 55,230$ trips .
g. $\operatorname{Pr}($ start at South Station and end at Library $)=0.0013=72 / 55,230$ trips.

## Scenario 6

a. A Fit $Y$ by $X$ contingency table shows that 19 of the 1,000 women were smokers with premature babies. Hence, the probability is 0.019 .
c.A Fit $Y$ by $X$ contingency table shows that 11 of the 1,000 women were smokers and mature moms. Hence, the probability is 0.011 .

Scenario 7
a. $\Delta$ Month


Birdstrikes occur most often July through October (months 7 through 10), and rather infrequently during December, January, and February. So, we would say that they do not occur with equal frequency through the year.

# Student Solutions to Application Scenarios 

## Scenario 1



The normal quantile plot appears to the left. The distribution is strongly skewed positively, and therefore the normal model is not suitable for this variable.
c. $\operatorname{Pr}(X>5.5)=1-0.9426=0.0574$. In comparison, based on the reported quantiles, we find that more than $10 \%$ of the observed data lies above 5.5 children per woman.

Scenario 2


In the shadowgram to the left we see a generally symmetric distribution that seems to be moundshaped. There may be some indication of a secondary peak at approximately 299,950 $\mathrm{km} / \mathrm{sec}$., but the overall impression is that the distribution might be well-described by the normal model.
c.The data set provides some support for the assumption. Michelson's various measurements of the speed of light seem to vary according to an approximate normal distribution.

## Scenario 3

a.


This distribution shows mild skewness. The lower tail is truncated and therefore shorter and thicker than a normal distribution would be.

## Scenario 4

a. Student answers will vary. Most will likely choose the weekly change column corresponding to the Hang Seng market index, but others might select a different column (e.g. Tel Aviv or S\&P). In these graphs, the points track most closely to the diagonal line.
c. The mean and standard deviation of the changes in Hang Seng for the weeks observed are 1.102065 and 5.242892. For a normal distribution with that mean and standard deviation, $\operatorname{Pr}(X<0)=0.5832$, or approximately 0.58 .

Scenario 5
Use these graphs to respond to all parts:


a. This histogram is mound-shaped with a single peak centered near 500 minutes. The large majority of respondents report between approximately 300 and 700 minutes of sleep per week.
c. The Age histogram is more skewed that the Sleeping histogram, with distinct secondary peaks in each tail. It appears to be centered near 40, but with the peaks in the tails it is difficult to generalize about the degree of dispersion. Again, the normal quantile plot casts doubts on using a normal model for this variable. The normal model seems to fit acceptably near the center of the distribution, but deviates quite dramatically in the tails.

## Scenario 6

These graphs can be used to respond to parts $a$ and $b$.

a. Closing values appear to be symmetric and bimodal, with peaks between 4000-5000 and 60006500. The center of the distribution is close to 5000 and it ranges from approximately 3500 to 7000.

In contrast, the \%change column is moderately symmetric with a single peak just above 0 . Most of the distribution lies between $-5 \%$ and $+5 \%$.


The volume column has a normal quantile plot that looks quite close to a normal distribution. It would be well described by a model $\sim \mathrm{N}(1710.4911,203.1369)$.

## Scenario 7

a.Here are the graphs, which very clearly show that the cell column is better modeled as normal than the broadband data. The broadband histogram is strongly skewed to the right and its probability plot does not track the diagonal line at all.



Using the normal model, we would estimate that approximately $8.4 \%$ of countries had 150 or more cell subscribers per 100 people.

## Scenario 8

a. Use Analyze > Distribution to obtain histograms, then red triangle Normal Quantile plot to produce these two plots:


Scenario 9


a. The graphs above show that Height is very strongly skewed to the right with many outliers. Speed is more closely normal, through there is a second mode at approximately 250 mph .
c.

| Summary Statistics |  |
| :--- | ---: |
| Mean | 143.10305 |
| Std Dev | 47.200664 |
| Std Err Mean | 0.3107123 |
| Upper 95\% Mean | 143.71206 |
| Lower 95\% Mean | 142.49403 |
| N | 23077 |

From the distribution platform, we find that the mean is 143.103 mph and the standard deviation is 47.2 .

Placing these values into the normal distribution calculator, we can approximate that the $90^{\text {th }}$ percentile of the normal distribution is 203.6 mph .

In this instance, the normal approximation comes reasonably close to the observed data.


# Student Solutions to Application Scenarios 

Scenario 1
a.Student answers will vary due to the operation of the random number generator.
c.The probability that a SRS of 250 households would include 25 or fewer homes without Internet service is 0.00031368 .

Scenario 2
a. The proportion of countries in Sub-Saharan Africa is 0.24227 .
c. $\Delta$ Frequencies

| Level | Count | Prob |  |  |
| :---: | :---: | :---: | :---: | :---: |
| America | 4 | 0.13333 | $\triangle$ Summary Statistics |  |
| Europe \& Central Asia | 12 | 0.40000 |  |  |
| Middle East \& North Africa | 4 | 0.13333 | Mean | 29.815526 |
| SESAP | 5 | 0.16667 | Std Dev | 32.151006 |
| Sub-Saharan Africa | 5 | 0.16667 | Std Err Mean | 5.8699438 |
| Total | 30 | 1.00000 | Upper 95\% Mean | 41.820909 |
| N Missing 0 |  |  | Lower 95\% Mean | 17.810143 |
| $5 \text { Levels }$ |  |  | N | 30 |

Student answers will vary due to random sampling. Above we find the results of one random sample-only 5 of the 30 countries are in Sub-Saharan Africa ( $16.7 \%$ ). The mean mortality rate in the sample is 29.82 (note that in this sample all 30 countries reported an infant mortality rate). In general students' results will not match the population values shown in parts a \& b due to sampling variation.

## Scenario 3

a.Student answers will vary. In general, the sampling distribution will be bell-shaped and symmetrical, centered very near 0.40 and ranging from about 0.35 to 0.45 .
c.Student answers will vary again. In general, the sampling distribution will be roughly bellshaped and possibly a little left skewed, centered very near 0.95 . Compare to the distribution in part c , this distribution will be steep and range only from about 0.90 to 1.00 .
e.In part c we notice that the population with a proportion of .95 generates samples with comparatively small standard errors. The risks associated with sampling variation tend to be smaller in more uniform populations.

## Scenario 4

a.Student responses will vary. In general, the sampling distribution will be bell-shaped and symmetrical, centered very near 15 with an overall standard error (std. deviation of the sample means) approximately equal to 0.10 and ranging from about 14.7 to 15.3.
c.Student responses will again vary. In general, the sampling distribution will be bell-shaped and symmetrical, centered very near 15 with an overall standard error (std. deviation of the sample means) approximately equal to 0.40 and ranging from about 13.8 to 16.2.
e.The results will be very similar to parts a and d though each student may have slightly different numerical results.

## Scenario 5

a.


4, 857 strikes occurred in California, for a proportion of 0.095 , or about 9.5\%.
c.Each student will obtain a different SRS, so these answers will vary. In general they will differ from the values in parts $a$ and $b$ due to the chance variation associated with random sampling.

| Scenario 6 |  |  |
| ---: | ---: | ---: |
| a. |  | Summary Statistics |
|  | Mean | 35.456326 |
| Std Dev | 10.999782 |  |
| Std Err Mean | 0.0585974 |  |
| Upper 95\% Mean | 35.571178 |  |
| Lower 95\% Mean | 35.341473 |  |
| N | 35238 |  |

The mean rider age is 35.46 years.
c.Using the CLT, we'd expect the sampling distribution of the sample mean to approach an approximately normal distribution as the sample size, $n$, grows large. The mean of the distribution should be 35.46 years with a standard error equal to approximately $11 /(\mathrm{sqrt}(\mathrm{n}))$.
e.Here are the results of one such simulation, rescaled for clarity:


The sampling distribution is symmetric and unimodal, with a mean at 35.47 years and a standard error of 1.936 . Note that in part c the CLT would have predicted a mean of 35.46 and a standard error of $11 / \sqrt{ } 50=1.56$

## Student Solutions to Application Scenarios

Scenario 1

| a.Confidence Intervals <br> Level <br> Count |
| :--- |
| Prob |

Based on the analysis shown to the left, 95 of 447 disruptions with known causes were ruptures. The estimated confidence interval is from 0.177 to 0.253 . We can be $95 \%$ confident that the true population proportion is somewhere between 0.177 and 0.253 .
c. When we lower the confidence level the interval becomes narrower.
a. Yes. We have a random sample of sufficient size to invoke the Central Limit Theorem.
c. $\triangle$ Test Probabilities

| Level | Estim Prob | Hypoth Prob |
| :--- | ---: | ---: |
| No | 0.14000 | 0.18000 |
| Yes | 0.86000 | 0.82000 |


|  |  | Hypoth <br> Prob (p1) | p-Value |
| :--- | :--- | ---: | :--- |
| Binomial Test | Level Tested | 0.18000 | 0.0556 |

e.A larger sample with the very same proportion provides more precision in the confidence interval (i.e. a narrower interval) and enhances the statistical significance of the test result.

## Scenario 3

a. Yes. We have a random sample of sufficient size to invoke the Central Limit Theorem.
c.We can be $99 \%$ confident that the population proportion is between 0.071 and 0.085 . Both intervals are centered at the same value, but the $99 \%$ interval is wider than the $95 \%$ interval.
e.The lower the confidence level, the narrower the interval.

## Scenario 4

a. Yes. We have a random sample of sufficient size to invoke the Central Limit Theorem.

| Test Probabilities |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Level <br> No <br> Yes | Estim Prob 0.87240 0.12760 | b Hypoth Prob |  |  |
|  |  | 0.90000 |  |  |
|  |  | 0.10000 |  |  |
| Test |  | ChiSquare | DF | Prob>Chisq |
| Likelih | ood Ratio | 29.8532 | 1 | <.0001* |
| Pearso |  | 32.1670 | 1 | <.0001* |
| Method: Fix hypothesized values, rescale omitted |  |  |  |  |

(For this question, it is simplest to create a small summary table). Create a Because of the question's wording, a two-tailed test is most appropriate here. Based on this random sample, we can confidently conclude that it is not credible to conclude that $10 \%$ of the population binge drinks at least once per week. If anything, this sample suggests a higher population proportion.

## Scenario 5

a.It depends The total sample size is 189 ; because some events or combination of events are relatively rare, it may be the case that $n p<5$, in which case we should not interpret the inferential results.
c. Although the observed relative frequency is 0.53 , and thus greater than 0.5 the p -Value is 0.362 which is quite high enough that we can readily attribute the result to sampling error. In other words, a null hypothesis that the population proportion is 0.50 or less is still plausible, so we fail to reject the null.

Scenario 6
a. $\Delta$ Confidence Intervals

| Level | Count | Prob | Lower CI | Upper CI | 1-Alpha |
| :--- | ---: | ---: | ---: | ---: | ---: |
| No | 63 | 0.50400 | 0.431141 | 0.57669 | 0.900 |
| Yes | 62 | 0.49600 | 0.42331 | 0.568859 | 0.900 |
| Total | 125 |  |  |  |  |

Note: Computed using score confidence intervals.
We can be $90 \%$ confident that the proportion of trading days on which McDonald's stock increases is somewhere between 0.423 and 0.569 .

## Scenario 7

a. Yes. We have very large samples, and can rely on the Central Limit Theorem.
c.The $99 \%$ confidence interval is ( 0.237 and $0 . .247$ ). Like the prior interval, this interval is centered at 0.237 , but is slightly wider.
e. $\Delta$ Confidence Intervals

| Level | Count | Prob | Lower CI | Upper CI | 1-Alpha |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Divorced | $7.782 \mathrm{e}+9$ | 0.09030 | 0.090303 | 0.090306 | 0.950 |
| Married - spouse absent | $9.234 \mathrm{e}+8$ | 0.01071 | 0.010714 | 0.010715 | 0.950 |
| Married - spouse present | $4.61 \mathrm{e}+10$ | 0.53530 | 0.535293 | 0.5353 | 0.950 |
| Never married | $2.51 \mathrm{e}+10$ | 0.29105 | 0.291049 | 0.291055 | 0.950 |
| Separated | $1.577 \mathrm{e}+9$ | 0.01830 | 0.018298 | 0.018299 | 0.950 |
| Widowed | $4.682 \mathrm{e}+9$ | 0.05433 | 0.054332 | 0.054335 | 0.950 |
| Total | $8.62 \mathrm{e}+10$ |  |  |  |  |

Note: Computed using score confidence intervals.

When we apply the sampling weights, the point estimate changes from $23.7 \%$ to $29.1 \%$, and the $95 \%$ confidence interval is approximately $29.1049 \%$ to $29.1055 \%$-- it shrinks dramatically in width, and is considerably higher than before.

## Scenario 8

a.It depends on which variables we examine. We have a random sample of sufficient size to invoke the Central Limit Theorem, but there is a considerable amount of missing data.

| Confidence Intervals |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Level | Count | Prob | Lower CI | Upper CI | 1-Alpha |
| 0 | 6 | 0.00010 | $3.662 \mathrm{e}-5$ | 0.000275 | 0.990 |
| 1 | 51692 | 0.86476 | 0.861118 | 0.868324 | 0.990 |
| 2 to 10 | 7583 | 0.12686 | 0.123392 | 0.130405 | 0.990 |
| 11 to 100 | 481 | 0.00805 | 0.007159 | 0.009044 | 0.990 |
| Over 100 | 14 | 0.00023 | 0.000119 | 0.00046 | 0.990 |
| Total | 59776 |  |  |  |  |

We can be $99 \%$ confident that, out of all instances where there is a bird strike, a single bird is struck somewhere between $86.1 \%$ and $86.8 \%$ of the time. The $99 \% \mathrm{CI}$ is slightly wider than the 95\% CI.

Scenario 9
a. Yes. We have a random sample of sufficient size to invoke the Central Limit Theorem, but there is a considerable amount of missing data.
c.This question is most easily done by creating a small summary table.

| Confidence Intervals |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Level | Count | Prob | Lower CI | Upper CI | 1-Alpha |
| No | 53094 | 0.96133 | 0.959685 | 0.962902 | 0.950 |
| Yes | 2136 | 0.03867 | 0.037098 | 0.040315 | 0.950 |
| Total | 55230 |  |  |  |  |

Note: Computed using score confidence intervals.
We can be $95 \%$ confident that between $3.9 \%$ and $4 \%$ start at the library.
e.The smaller sample makes the interval wider, but has no effect on the center of the interval.

# Student Solutions to Application Scenarios 

## Scenario 1

a.Probably. These columns contain continuous data, and though both distributions are strongly right-skewed, both have a sufficiently large number of observations to rely on the Central Limit Theorem. The critical question is whether we can view this particular time period as representative of the overall process of pipeline disruptions; if we can regard it as random, then we can proceed to make inferences.
c.The $90 \%$ interval is $-\$ 307,156$ to $\$ 2,979,847$. We can be $90 \%$ certain that the mean damage cost lies between these two values.

## e. $\Delta$ Confidence Intervals

| Parameter | Estimate | Lower CI | Upper CI | 1-Alpha |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Mean | 16384.57 | -6646.48 | 39415.62 | 0.990 |
| Std Dev | 192637.8 | 177593.4 | 210250.1 | 0.990 |

We can be $99 \%$ confident that the mean dollar cost of lost natural gas is between $-\$ 6646.48$ and $\$ 39,415.62$. NOTE: the distribution is so strongly rightskewed that we should be reluctant to draw conclusions from this sample, even with a sample size of 468 .


Student answers will vary but should conclude that if the null hypothesis were that $\mu=$ approximately $\$ 31,100$ then we would reject the null in favor of the one-sided alternative hypothesis.

Scenario 2

c.From the confidence interval in part b we can see that Michelson would probably have (erroneously) concluded that the value $300,000 \mathrm{kps}$ is not credible. The two-tailed hypothesis test yields a $P$-value $<0.0001$ and a test statistic equal to -13.898 ; Michelson would have rejected a null hypothesis that the constant speed of light is $300,000 \mathrm{kps}$.

## Scenario 3

a.Student answers will vary. On the one hand, because both measurements refer to the same child's height, we expect them to be quite similar. On the other hand, when a person stands the
spine may compress slightly, so that standing height measurements may be less than reclining measurements.

## Scenario 4

a.Yes. We do not know the population $\sigma$ so we will use the t -distribution. Because the sample is so large ( $n=1787$ ) we can rely on the Central Limit Theorem to proceed.
c.No. The interval is an estimate of the population mean, not the range of individual values. The interval provides an estimate of the location of the population mean acknowledging the uncertainty that arises from using a sample.
e.If the true population mean actually = 10 minutes the power of this test would be approximately 0.996 . In other words, if the reality were that the mean flight is delayed 10 minutes, this test would detect that the mean is less than 12 minutes.

Scenario 5
a. Yes. We do not know the population $\sigma$ so we will use the t -distribution. Because the sample is so large ( $n=1455$ ) we can rely on the Central Limit Theorem to proceed.
c. $\Delta$ Confidence Intervals We can be $95 \%$ confident that the mean time from scheduled Parameter Estimate Lower departure to wheels off is between 31.75 and 36.33 minutes.

| Mean | 34.0392 | 31.745 |
| :--- | :--- | :--- | :--- |

$\begin{array}{ll}\text { Std Dev } & 44.58922 \quad 43.025\end{array}$

Scenario 6
a.The speed column does seem to satisfy the conditions: it is moderately symmetric and the sample is very large ( $n=23,077$ ) so we can rely on the Central Limit Theorem to proceed. We do not know the population $\sigma$ so we will use the t -distribution.

The Cost of Repairs column is a smaller sample $(n=1716)$ and very strongly skewed. Even with the CLT, we should proceed with caution.
c. At the $99 \%$ confidence level, we can be $99 \%$ confident that the mean flight speed at impact is between 142.3 and 143.9 MPH.


The test results indicate that the sample provides convincing evidence to reject the null hypothesis, yielding a very small P-value of just 0.007 . The sample is, as noted, very right-skewed, but if anything that would overstate the population mean.
g.Student answers will vary, depending on which possible "True Mean" values they explore. It is useful to notice that the power of the test exceeds $90 \%$ for all true means below approximately $\$ 127,000$.

## Scenario 7

a.Yes. We do not know the population $\sigma$ so we will use the t -distribution. The sample is large enough ( $n=1000$; some mothers' gains are missing, $n=973$ ) and the distributions are reasonably symmetric so we can rely on the Central Limit Theorem to proceed.

\section*{c. $\Delta$ Confidence Intervals <br> | Parameter | Estimate | Lower CI | Upper CI | 1-Alpha |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Mean | 7.101 | 7.007368 | 7.194632 | 0.950 |
| Std Dev | 1.50886 | 1.445507 | 1.578065 | 0.950 |}

We can be $95 \%$ confident that the mean
birthweight of infants in NC for the year 2004 was between 7 and 7.2 pounds.

## Student Solutions to Application Scenarios

Scenario 1
a.

| Test Probabilities |
| :--- | ---: | ---: | ---: |
| Level Estim Prob Hypoth Prob  <br> CENTRAL 0.28632 0.20000  <br> EASTERN 0.29915 0.20000  <br> SOUTHERN 0.07479 0.20000  <br> SOUTHWEST 0.09188 0.20000  <br> WESTERN 0.24786 0.20000  <br> Test ChiSquare DF Prob>Chisq <br> Likelihood Ratio 122.9190 4 $<.0001^{*}$ <br> Pearson 109.8419 4 $<.0001^{*}$ |

Method: Fix hypothesized values, rescale omitted

| c. | Tests |  |  |
| :--- | :--- | ---: | ---: |
| $\mathbf{N}$ | DF | -LogLike | RSquare (U) |
| 425 | 4 | 2.4165731 | 0.0086 |
|  |  |  |  |
| Test | ChiSquare | Prob>ChiSq |  |
| Likelihood Ratio | 4.833 | 0.3049 |  |
| Pearson | 4.760 | 0.3128 |  |

No. At the 0.05 level of significance we reject the null hypothesis of equal probabilities.

Based on this sample, we would conclude that the variables are independent. We do not have sufficient evidence to conclude that the two variables are not independent (assuming a significance level of 0.05 ).
a.

| Contingency Table |
| :--- |
| Count Feed Soctivity   <br> Total \%     <br> Col \%     <br> Row \%     <br> Afternoon 0 9 14 23 <br>  0.00 4.76 7.41 12.17 <br>  0.00 14.52 35.90  <br>  0.00 39.13 60.87  <br> Evening 56 10 13 79 <br>  29.63 5.29 6.88 41.80 <br>  63.64 16.13 33.33  <br>  70.89 12.66 16.46  <br>  28 38 6 72 <br>      <br> Morning 24.81 20.11 3.17 38.10 <br>  31.82 61.29 15.38  <br>  38.89 52.78 8.33  <br> Noon 4 5 6 15 <br>  2.12 2.65 3.17 7.94 <br>  4.55 8.06 15.38  <br>  26.67 33.33 40.00  <br>  88 62 39 189 |


| $\triangle$ Tests |  |  |
| :---: | :---: | :---: |
| N D | DF -LogLike | RSquare (U) |
| 189 | $6 \quad 37.215041$ | 0.1880 |
| Test | ChiSquare | Prob>Chisq |
| Likelihood Ratio | - 74.430 | <.0001* |
| Pearson | 68.465 | <.0001* |

Because there are some cells with very small counts and expected counts, we should use caution making inferences from the ChiSquare test. However, we can note that the evidence points towards rejection of the null hypothesis of independence and we can also note (for example) that dolphins were regularly observed feeding in the morning and evening, but rarely if ever at other times.

Scenario 3
a.


| N | DF | -LogLike | RSquare (U) |
| ---: | ---: | ---: | ---: |
| 157 | 8 | 30.312288 | 0.1959 |


| Test | ChiSquare | Prob>ChiSq |
| :--- | ---: | ---: |
| Likelihood Ratio | 60.625 | $<.0001^{*}$ |
| Pearson | 54.842 | $<.0001^{*}$ |

c.


No. At the 0.05 level of significance we reject that null hypothesis that Provider and Region are independent.

No. At the 0.05 level of significance we reject that null hypothesis that MatLeave90+ and Region are independent.

## Scenario 4

$\triangle$ Tests

| N | DF | -Loglike | RSquare (U) |
| :---: | :---: | :---: | :---: |
| 6430 | 28 | 245.84627 | 0.0297 |
| Test |  | Chisquare | Prob>Chisq |
| Likelihood |  | 491.693 | <.0001* |
| Pearson |  | 496.462 | <.0001* |

Warning: 20\% of cells have expected count less than 5, ChiSquare suspect.

Because there are a substantial proportion of cells with very small expected counts, we should use caution making inferences from the ChiSquare test. However, we can note that the evidence points toward rejecting the null hypothesis of independence. We might observe (for example) that married respondents were disproportionately nonHispanic whites.

Scenario 5
a. $\Delta$ Contingency Table


[^0]No. At the 0.05 level of significance we reject that null hypothesis that binge drinking regularity and involvement in car accidents are independent. Students who report binging at least once a week are far more likely to have been involved in an accident than other students.

## Scenario 6

| $\triangle$ Test Probabilities |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Level | Estim Prob | Hypoth |  |  |
| 1 | 0.43548 |  |  |  |
| 2 | 0.20968 |  |  |  |
| 3 | 0.06452 |  |  |  |
| 4 | 0.08065 |  |  |  |
| 5 | 0.20968 |  |  |  |
| Test |  | ChiSquare | DF | Prob>Chisq |
| Likelih | od Ratio | 26.3429 | 4 | <.0001* |
| Pearso |  | 27.3548 | 4 | <.0001* |

The Chi-Square goodness-of-fit test indicates that the five categories are not equally distributed across mammalian species. We reject the null hypothesis that all proportions are equal at 0.20 .

Method: Fix hypothesized values, rescale omitted

```
c. }\triangle\mathrm{ Tests
\begin{tabular}{lrrrr}
\hline N & DF & -LogLike & RSquare (U) \\
\hline 62 & 16 & 24.460914 & 0.2498 \\
\hline Test & & & \\
\hline ChiSquare & Prob \(>\) ChiSq \\
Likelihood Ratio & 48.922 & \(<.0001^{*}\) \\
Pearson & 47.678 & \(<.0001^{*}\)
\end{tabular}
Warning: \(20 \%\) of cells have expected count less than 5, ChiSquare suspect. Warning: Average cell count less than 5, LR ChiSquare suspect.
```

The total sample size here leads to many cells with expected counts $<5$, makng the ChiSquare test unreliable. That said, the test results point in the direction of rejecting the null hypothesis.

Scenario 7
a.

| $\triangle$ Tests |  |  |  |
| :---: | :---: | :---: | :---: |
| N | DF | -LogLike | RSquare (U) |
| 12248 | 4 | 138.36581 | 0.0125 |
| Test |  | ChiSquare | Prob>ChiSq |
| Likelihoo |  | 276.732 | <.0001* |
| Pearson |  | 272.293 | <.0001* |

According to the Chi-Square test the two variables are not independent. There is sufficient evidence to reject a null hypothesis that they are independent.


Scenario 8

## a. $\quad \Delta$ Test Probabilities

| Level | Estim Prob | Hypoth Prob |  |
| :--- | ---: | ---: | ---: | ---: |
| female | 0.50300 | 0.50000 |  |
| male | 0.49700 | 0.50000 |  |
| Test | ChiSquare | DF | Prob>Chisq |
| Likelihood Ratio | 0.0360 | 1 | 0.8495 |
| Pearson | 0.0360 | 1 | 0.8495 |

Method: Fix hypothesized values, rescale omitted
c.

| $\triangle$ Tests |  |  |  |
| :---: | :---: | :---: | :---: |
| N | DF | -Loglike | RSquare (U) |
| 1000 | 2 | 2.9403290 | 0.0084 |
| Test |  | ChiSquare | Prob>ChiSq |
| Likelihoo |  | 5.881 | 0.0528 |
| Pearson |  | 9.584 | 0.0083* |

Warning: $20 \%$ of cells have expected count less than 5 , ChiSquare suspect.

According to the Chi-Square test the two variables are not independent. There is sufficient evidence to reject a null hypothesis that they are independent.

According to the Chi-Square test there is not sufficient evidence to reject a null hypothesis that mothers are equally likely to give birth to a male as a female baby.

We should be reluctant to draw inferences about this question because of the high number of cells with counts less than 5. That said, Pearson's test does indicate sufficient evidence to reject a null hypothesis that they are independent. It would be wise to obtain a larger sample before drawing a conclusion.

Scenario 9

c.


According to the Chi-Square test the two variables are not independent. There is sufficient evidence to reject a null hypothesis that they are independent. The distribution of phase of flight is different at different airports.

According to the Chi-Square test the number of birds struck per incident does vary by airport. There is sufficient evidence to reject a null hypothesis that they are independent.

## Student Solutions to Application Scenarios

## Scenario 1

NOTE: Complete answers should note that we have continuous data, independent samples, and that the samples in each part of the question are large enough to rely on the Central Limit Theorem.


## Scenario 2

a. We should first note that we have modest sample sizes ( $\mathrm{n}=35$ and $\mathrm{n}=43$ ) from strongly skewed distributions. Therefore, we should be reluctant to interpret the resulting interval at all. However, the reported $95 \%$ confidence interval is from $-\$ 11,026,606$ to $+\$ 32,748,087$.

Scenario 3


| Level Count | Std Dev | MeanAbsDif to Mean |  | MeanAbsDif to Median |
| :---: | :---: | :---: | :---: | :---: |
| LEAK 93 | 263657.6 |  | 8189.2 | 149673.5 |
| RUPTURE 95 | 399957.6 |  | 9527.7 | 210202.5 |
| Test | F Ratio | DFNum | DFDen | p -Value |
| O'Brien[.5] | 1.3173 | 1 | 186 | 0.2526 |
| Brown-Forsythe | 1.8915 | 1 | 186 | 0.1707 |
| Levene | 3.3319 | 1 | 186 | 0.0696 |
| Bartlett | 15.5717 | 1 |  | <.0001* |
| F Test 2-sided | 2.3012 | 94 | 92 | <.0001* |

## Scenario 4

a. Student answers will differ. We have only 8 individuals without PD, and for the baseline pitch and jitter, the distributions appear bimodal with few observations in the "center"; shimmer may be normally distributed for non-PD observations. Among individuals with PD ( $n=24$ ) the distributions tend to be skewed. As such, with non-normal distributions and small samples, this sample does not satisfy the conditions for the use of the $t$-test.
C. $\quad \Delta$ Wilcoxon / Kruskal-Wallis Tests (Rank Sums) Expected

| Level | Count | Score Sum | Score | Score Mean | (Mean-Mean0)/Std0 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 0 | 8 | 72.500 | 132.000 | 9.0625 | -2.569 |
| 1 | 24 | 455.500 | 396.000 | 18.9792 | 2.569 |

$\triangle$ 2-Sample Test, Normal Approximation s $\quad \mathbf{Z}$ Prob $>\mid \mathbf{Z}$ 72.5 -2.56929 0.0102
$\triangle$ 1-way Test, ChiSquare Approximation ChiSquare DF Prob>ChiSq

## Scenario 5

Based on the Wilcoxon test (assuming a significance level of $\alpha=0.05$ ) we reject the null hypothesis that the mean jitter measurement is equal for both groups. There is a statistically significant difference in this sample data.


If we rely on Levene's test, we conclude that there is insufficient evidence to conclude that the variances are different; the F Test 2sided leads to the opposite conclusion. To be safe we'll use the t-test assuming unequal variances for the next question.

## Scenario 6

a.



## Student Solutions to Application Scenarios

Scenario 1
a.



In this case we find that the regional variances are not equal but the residuals do appear to be approximately normal. According to Welch's test, the mean birthrate is not equal across the regions of the world. Strictly speaking we cannot rely on a formal test to determine which regions differ. Visual inspection of the means diamonds in the Oneway graph suggests that SubSaharan birth rates are unusually high, and that birth rates in Europe and Central Asia are unusually low.



| Effect Tests |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Source | Nparm | DF | Sum of |  |  |
| Squares | FRatio | Prob $>$ F |  |  |  |
| MatLeave90+ | 1 | 1 | 503.8 | 0.0032 | 0.9551 |
| Provider | 2 | 2 | 1529873.0 | 4.8346 | $0.0093^{\star}$ |
| MatLeave90+*Provider | 2 | 2 | 310435.1 | 0.9810 | 0.3774 |

We start by evaluating conditions. The Residual by Predicted Plot raises some question about the equality of variances, but it is not definitive. The residuals do not appear to be normally distributed, but we have reasonably large samples and can rely on the Central Limit Theorem.

We find no significant interaction term, and we do find a significant main effect associated with the Provider of benefits. It appears that countries with private provision of maternity benefits have significantly higher rates of maternal mortality.

## Scenario 2

a. We see no evidence that the ANOVA assumptions have been violated; variances across the three groups appear to be equal and residuals are approximately normal. The F Ratio of 4.6275
and corresponding P-value of 0.0187 indicate that we should reject the null hypothesis of equal means; there is compelling evidence that the different additives lead to different mean changes.


$\triangle$ Analysis of Variance

| Analysis of Variance |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  |  | Sum of |  |  |  |
| Source | DF | Squares | Mean Square | F Ratio | Prob > F |
| Additive | 2 | 34.41052 | 17.2053 | 4.6275 | $0.0187^{*}$ |
| Error | 27 | 100.38696 | 3.7180 |  |  |
| C. Total | 29 | 134.79748 |  |  |  |

$\triangle$ Means for Oneway Anova

| Means for Oneway Anova |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Level | Number | Mean | Std Error | Lower 95\% | Upper 95\% |
| extra | 10 | 24.8237 | 0.60976 | 23.573 | 26.075 |
| regular | 10 | 26.7721 | 0.60976 | 25.521 | 28.023 |
| super | 10 | 24.2766 | 0.60976 | 23.025 | 25.528 |
| Std Error uses a pooled estimate of error variance |  |  |  |  |  |

[^1]
c. We find that there is a significant improvement in insulation with the "super" additive-the temperature change is smallest with that additive. The company should switch from regular to super.

## Scenario 3

a.


We start by evaluating conditions, and find no signs that the sample data violate the conditions for inference.

| Effect Tests |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Source | Nparm | DF | Sum of <br> Squares | F Ratio | Prob $>$ F |
| City | 2 | 2 | 23253888.2 | 3.8819 | $0.0228^{*}$ |
| Model | 2 | 2 | 1168074996 | 194.9929 | $<.0001^{*}$ |
| City*Model | 4 | 4 | 34923934.8 | 2.9150 | $0.0235^{*}$ |

A review of the Effect Tests shows that we have a significant interaction effect as well as significant main effects. This tells us that prices vary by city and by model, and what's more the impact of model varies across the cities.

|  |  |  | Least <br> Sq Mean |
| :--- | ---: | ---: | ---: | ---: |
| Level |  |  | 15799.318 |
| Portland, Civic EX | A |  | 15531.000 |
| Raleigh, Civic EX | A |  | 15054.867 |
| Phoenix, Civic EX | A B |  | 150 |
| Portland, Corolla LE | B C |  | 13213.500 |
| Phoenix, Corolla LE | C D | 11400.231 |  |
| Raleigh,Corolla LE |  | D E | 10072.400 |
| Raleigh,PT Cruiser |  | D E | 9937.800 |
| Portland, PT Cruiser |  | E | 9154.538 |
| Phoenix,PT Cruiser |  | E | 8735.944 |

[^2]When we apply Tukey's HSD (output not shown fully here) we see the complexity of the interactions; we should not make statements about main effects but can use the connecting letters report to identify differences among the model-city combinations.


As usual we start by evaluating assumptions. We have a very large sample, so the Central Limit Theorem applies and we need not be concerned with normality (above we see the residuals are unimodal and symmetric, but depart from the normal model in the tails). We also see evidence that the variances are unequal. In practice, because of the very large sample it is not surprising that we find significant differences.

```
Welch's Test
Welch Anova testing Means Equal, allowing Std Devs Not Equal
    FRatio DFNum DFDen Prob > F
    180.4634 4 1071.1 <.0001*
```

Both Welch's test and the standard ANOVA results strongly indicate that there are significant differences in group means. There is no control group here. Tukey's HSD indicates that employed people at work get the least sleep and unemployed people who are looking report the most. All others are significantly different from those two groups, but indistinguishable from one another.

## Scenario 5

a.


As we can see from the output, the sample data seem to violate the assumptions of normality and equal variance. Each of the regional subsamples is large enough to rely on the Central Limit Theorem with respect to normality. Using Welch's test (below) we would conclude that the mean costs of property damage are not identical across the regions.
$\triangle$ Welch's Test

```
Welch Anova testing Means Equal, allowing Std Devs Not Equal
    FRatio DFNum DFDen Prob > F
    4.7168 4 157.95 0.0013*
```

c. The distribution of residuals (not shown here) raises questions about normality and the usual tests indicate that the variances of the different disruption-type subgroups are unequal.
According to Welch's test, there is at least one disruption type that differs from the others in terms of time required to make the area safe.

| Level | Count | Std Dev | MeanAbsDif to Mean |  | MeanAbsDif to Median |
| :---: | :---: | :---: | :---: | :---: | :---: |
| LEAK | 93 | 344.4257 |  | 1.8180 | 188.2473 |
| N/A | 24 | 59.0497 |  | 6.5347 | 42.2500 |
| OTHER | 226 | 130.7072 |  | 7.8671 | 80.4513 |
| RUPTURE | 94 | 193.1917 |  | 8.5523 | 97.7234 |
| Test |  | F Ratio | DFNum | DFDen | Prob $>$ F |
| O'Brien[.5] |  | 7.8111 | 3 | 433 | <.0001* |
| Brown-Fors | sythe | 8.3345 | 3 | 433 | <.0001* |
| Levene |  | 21.3279 | 3 | 433 | <.0001* |
| Bartlett |  | 58.8941 | 3 |  | <.0001* |

$\triangle$ Welch's Test
Welch Anova testing Means Equal, allowing Std Devs Not Equal
FRatio DFNum DFDen Prob > F
$14.35063121 .27<.0001^{*}$

Scenario 6
a.


| Test | FRatio | DFNum | DFDen | Prob $>$ F |
| :--- | ---: | ---: | ---: | ---: |
| O'Brien[.5] | 11.2126 | 3 | 116 | $<.0001^{*}$ |
| Brown-Forsythe | 9.7761 | 3 | 116 | $<.0001^{*}$ |
| Levene | 10.0397 | 3 | 116 | $<.0001^{*}$ |
| Bartlett | 7.0772 | 3 | . | $<.0001^{*}$ |

We start by examining assumptions. The residuals appear to be normally distributed (the sample sizes are large enough to rely on the Central Limit Theorem in this case), but the subsamples appear not to share a common variance.

Both Welch's test and the conventional ANOVA find no significant differences among group means.


The assumption of normality does appear to be satisfied; visual inspection of residuals vs. predicted values does not reveal any obvious differences in group variances.


The interaction plots indicate interaction effects between operator and machine, making it difficult to interpret the main effects of machine 옹 and operator separately.

Scenario 7: NOTE-- Due to the large amount of output required in this problem, only a few selected results are shown.
a. We begin by checking the normality and equal variance assumptions. These are particularly important with such a small sample.

Among the three analyses, we find that there are unequal variances for the analysis of Yield by Popcorn type and Yield by batch.
The residuals in the analysis by Batch appear to be approximately normal, but the others do not. Due to the non-normality, we should use the Wilcoxon approach for the other two.

Yield vs. Batch satisfies normality, but not Equal Variance. Hence, we should consult Welch's test for Yield vs. batch.

There are no significant main effects on yield for either Popcorn or Oil Amt. However, the Welch's test result does indicate a significant effect of batch size (small batches improve yield).

## Student Solutions to Application Scenarios

Scenario 1
a.
$\triangle$ Linear Fit
BMXBMI $=-8.367275+0.3846724^{*}$ BMXWAIST


Above are the regression results for adult females. We find a significant relationship between waist circumference and BMI, with the waist measurement accounting for about $88 \%$ of the variation in BMI. Each addition centimeter of waist circumference is associated with an increase of 0.3847 in BMI.
c.

| $\triangle$ Linear Fit |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| BMXBMI $=-3.120908+0.3436897 *$ BMXWAIST |  |  |  |  |
| $\triangle$ Summary of Fit |  |  |  |  |
| RSquare |  |  | 0.894795 |  |
| RSquare Adj |  |  | 0.893151 |  |
| Root Mean Square Error |  |  | 2.235914 |  |
| Mean of Response |  |  | 21.28939 |  |
| Observations (or Sum Wgts) |  |  | 66 |  |
| $\downarrow$ Lack Of Fit |  |  |  |  |
| $\triangle$ Analysis of Variance |  |  |  |  |
| Source | DF | Sum of Squares | Mean Square | F Ratio |
| Model | 1 | 2721.3139 | 2721.31 | 544.3378 |
| Error | 64 | 319.9559 | 5.00 | Prob $>$ F |
| C. Total | 65 | 3041.2698 |  | <.0001* |


$\triangle$ Parameter Estimates

| Term | Estimate | Std Error | t Ratio | Prob>\|t| |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Intercept | -3.120908 | 1.081852 | -2.88 | $0.0053^{*}$ |
| BMXWAIST | 0.3436897 | 0.014731 | 23.33 | $<.0001^{*}$ |

If we restrict the analysis to females under the age of 17 we find a slightly stronger relationship between Waist and BMI. The estimated slope is slightly smaller than before ( 0.344 vs .0 .385 ) but otherwise the regression models are very similar.

## Scenario 2

a.


In this regression we find a weak $\left(\mathrm{R}^{2}=0.31\right)$ but highly significant positive relationship. Subjects who differ in age by 1 year tend to have, on average, systolic BP that is approximately 0.47 points higher per year. This is not a strong relationship because age accounts for less than onethird of the variation in systolic BP.


Scenario 3


The estimated equation is appears beneath the graph, with $\mathrm{R}^{2}=0.979$ - indicating a very strong relationship and excellent fit.

Despite the strong summary statistics, the scatterplot very clearly indicates some doubt about the linear model: the points seem to bend around the line, suggesting that the relationship is not best described as a line.

Scenario 4


The equation appears beneath the graph, and $\mathrm{R}^{2}=0.03$.
This regression shows there is a weak, significant negative relationship between mileage and price for used cars. The further a car has been driven, on average the lower the price (about 4 cents per mile, on average). However there is considerable scatter around the line.

Scenario 5


In the scatterplot we see a moderately strong positive linear association.

| Parameter Estimates |  |  |  |  |
| :--- | :--- | :--- | ---: | ---: |
| Term | Estimate | Std Error | t Ratio | Prob $>\mid$ \|t $\mid$ |
| Intercept | 5.1911903 | 1.775282 | 2.92 | $0.0001^{*}$ |
| Prior Close | 0.9060521 | 0.031839 | 28.46 | $<.0001^{*}$ |


| $\triangle$ Custom Test |  |
| :---: | :---: |
| Random Walk | Walk |
| Parameter |  |
| Intercept | 0 |
| Prior Close | - 1 |
| $=$ | 1 |
| Value $\quad-0.09394$ | -0.093947855 |
| Std Error 0.03183 | 0.031838543 |
| t Ratio -2.95075 | -2.950758618 |
| Prob>\|t| 0.003796 | 0.0037964813 |
| SS 7.379005 | 7.3790050123 |
| Sum of Squares | Squares 7.3790050123 |
| Numerator DF | tor DF 1 |
| F Ratio | 8.7069764218 |
| Prob > F | F 0.0037964813 |

Although the estimated slope of 0.906 might appear to be approximately 1 , the custom test indicates a significant difference from $1(\mathrm{p}$-value $=.004)$. Moreover, we find that the Intercept is significantly different from 0 .

Scenario 6
a.


Using the Haydn data we find a similar story to the one we saw with Mozart. We again find the Golden Mean model plausible.

Scenario 7
a.

| $\triangle$ Linear Fit |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| TotalSleep $=13.305786-0.0207491 *$ Gestation |  |  |  |  |
| $\triangle$ Summary of Fit |  |  |  |  |
| RSquare |  |  | 0.398573 |  |
| RSquare Adj |  |  | 0.387007 |  |
| Root Mean Square Error |  |  | 3.619194 |  |
| Mean of Response |  |  | 10.48333 |  |
| Observations (or Sum Wgts) |  |  | 54 |  |
| $\checkmark$ Lack Of Fit |  |  |  |  |
| $\triangle$ Analysis of Variance |  |  |  |  |
| Source | DF | Sum of <br> Squares | Mean Square | F Ratio |
| Model | 1 | 451.3896 | 451.390 | 34.4610 |
| Error | 52 | 681.1254 | 13.099 | Prob $>$ F |
| C. Total | 53 | 1132.5150 |  | <.0001* |

$\triangle$ Parameter Estimates

| Term | Estimate | Std Error | t Ratio | Prob $>\|\mathbf{t}\|$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Intercept | 13.305786 | 0.688282 | 19.33 | $<.0001^{*}$ |
| Gestation | -0.020749 | 0.003535 | -5.87 | $<.0001^{*}$ |

Here we find a significant, but weak, negative relationship. On average, each additional day of gestation is associated with a reduction of 0.02 hours of sleep per night. Gestation accounts for only about $40 \%$ of the variation in total sleep, so it is a fair predictor of sleep hours.

Scenario 8
a.

| $\triangle$ Linear Fit |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CancerMort $=125.91392+0.2954109 *$ TobaccoUse |  |  |  |  |  |
| $\triangle$ Summary of Fit |  |  |  |  |  |
|  |  | 0.009446 |  |  |  |
| RSquare Adj |  | 0.001646 |  |  |  |
| Root Mean Square Error |  |  | 31.46724 |  |  |
| Mean of Response |  |  | 133.2326 |  |  |
| Observations (or Sum Wgts) |  |  | 129 |  |  |
| Lack Of Fit |  |  |  |  |  |
| $\triangle$ Analysis of Variance |  |  |  |  |  |
| Source DFSum of <br> Squares |  |  | Mean Square |  | F Ratio |
| Model | 1 | 1199.21 |  | 1199.21 | 1.2111 |
| Error | 127 | 125753.81 |  | 990.19 | Prob $>$ F |
| C. Total | 128 | 126953.02 |  |  | 0.2732 |
| $\triangle$ Param | er Es | timates |  |  |  |
| Term |  | imate Std | Error | t Ratio | Prob> $\mid$ t $\mid$ |
| Intercept | 125 | 913927.2 | 204325 | 17.48 | <.0001* |
| Tobacco | e 0.29 | 541090.2 | 268434 | 1.10 | 0.2732 |

We find a non-significant relationship here - Tobacco Use is not a useful predictor of cancer deaths in a country.
c. The aggregate prevalence of tobacco use obscures the fine distinctions in the amount and length of tobacco use in individuals. We'd really want to look at data at the individual level in order to determine the degree to which increased tobacco use influences the risks of death from cancer or from cardiovascular disease.

## Scenario 9

a.


This is a highly significant, but weak, positive relationship. For each additional kg of cement in the mixture, compressive strength increases on average by 0.09 megapascals.

Scenario 10
a. There are slight differences, but when we round the major statistics we find that all four models are nearly identical: $Y_{i}=3+0.5 X_{i}$. All $R^{2}(0.66)$ and $p$-values ( 0.0022 for the slope) are the same.
c. In the other three graphs, the points do not fall in a linear pattern at all. This illustrates a substantial risk in running a linear regression without first examining the data visually. (In JMP we always see a scatterplot of the points either prior to fitting a model or in conjunction with fitting a model).
Scenario 11
a. The estimated equation is Price $=17625.688-0.054972^{*}$ Miles. On average, the price declines approximately 5.5 cents per mile driven, and a car that had never been driving would have an asking price of $\$ 17,625.69$.
c. The estimated equation is Price $=10659.169-0.0350164^{*}$ Miles. Due to the large $p$-value for the slope, we cannot be confident that the true slope differs from 0 , and hence should not venture an estimate of the price decline. The p-value for the intercept is significant, and we can estimate that a car that had never been driving would have an asking price of $\$ 10,659.17$.



The equation appears beneath the fitted line plot. In this case, the estimated slope is not significant $(p$-value $=0.253)$ and $R^{2}=0 . .03$.

Scenario 13
a.

| $\triangle$ Linear Fit |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| sales_per_capita $=1955.8357+0.3742659 *$ per_capita_income |  |  |  |  |
| $\triangle$ Summary of Fit |  |  |  |  |
| RSquare |  | 0.143168 |  |  |
| RSquare Adj 0 |  | 0.142892 |  |  |
| Root Mean Square Error 4 |  | 4941.592 |  |  |
| Mean of Response |  | 10374.8 |  |  |
| Observations (or Sum Wgts) |  | 3101 |  |  |
| $\checkmark$ Lack Of Fit |  |  |  |  |
| $\triangle$ Analysis of Variance |  |  |  |  |
| Source | DFSum of <br> Squares | Mean Square | e FR | atio |
| Model | $11.2645 \mathrm{e}+10$ | $1.264 \mathrm{e}+10$ | 517.81 |  |
| Error | $30997.5676 \mathrm{e}+10$ | 24419334 | 4 Prob |  |
| C. Total | $3100 \quad 8.832 \mathrm{e}+10$ |  | <. 00 |  |
| $\triangle$ Parameter Estimates |  |  |  |  |
| Term | Estimate | Std Error t | t Ratio | Prob> ${ }^{\text {\| }}$ \| |
| Intercep | 1955.8357 | 380.4682 | 5.14 | <.0001* |
| per_cap | income 0.3742659 | 0.016447 | 22.76 | <.0001* |

We first should note that the linear model is not particularly suitable for the cloud of points. There are a relatively small number of outlying points, but overall the trend is that higher per capita income is associated with higher retail spending. This makes logical sense because areas of higher incomes have residents who are in a position to
spend more, other things being equal. On average, each additional dollar in per capita income is associated with an increase of approximately 37 cents in spending. The estimated slope is highly significant, but the relationship is weak, with $\mathrm{R}^{2}=0.14$.

## Student Solutions to Application Scenarios

Scenario 1
a.
$\triangle$ Linear Fit
BMXBMI $=-8.367275+0.3846724^{*}$ BMXWAIST
$\triangle$ Summary of Fit

| RSquare | 0.880098 |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
| RSquare Adj | 0.879055 |  |  |  |
| Root Mean Square Error | 2.324656 |  |  |  |
| Mean of Response | 28.07205 |  |  |  |
| Observations (or Sum Wgts) | 117 |  |  |  |
| Lack Of Fit |  |  |  |  |
| Analysis of Variance |  |  |  |  |
| Sum of |  |  |  |  |
| Source | DF | Squares | Mean Square | F Ratio |
| Model | 1 | 4561.6239 | 4561.62 | 844.1161 |
| Error | 115 | 621.4628 | 5.40 | Prob $>$ F |
| C. Total | 116 | 5183.0867 |  | $<.0001^{*}$ |

$\triangle$ Parameter Estimates

| Term | Estimate | Std Error | t Ratio | Prob $>\|\mathbf{t}\|$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Intercept | -8.367275 | 1.272487 | -6.58 | $<.0001^{*}$ |
| BMXWAIST | 0.3846724 | 0.01324 | 29.05 | $<.0001^{*}$ |

We first performed this regression in the previous chapter. Above are the regression results for adult females. We find a significant relationship between waist circumference and BMI, with the waist measurement accounting for about $88 \%$ of the variation in BMI. Each addition centimeter of waist circumference is associated with an increase of 0.3847 in BMI. When we save the residuals and check their normality, we find the normality assumption seems to be reasonable. The graph of residuals vs. predicted values suggests that the dispersion of
residuals increases as predicted values increase, though it is not an overly dramatic tendency. We can probably trust this model for predictions.

c. Looking at the fitted line graph, it appears that the mean BMI for women with 68 cm . waists is approximately18 .
Scenario 2
a.


Once again we see the suggestion of heteroskedasticity on the left side of the graph. The residals are largely normal in shape, though somewhat right-skewed. We can probably use the model safely.
c.


The scatterplot to the left shows little or no relationship between pulse and systolic BP. If anything, there may be a very weak negative relationship here, contrary to the suspicion expressed in the question.
$\Delta$ Residual by Predicted Plot


The residuals graphs cast doubt on both normality and constant variance.

$\Delta \nabla$ Bivariate Fit of MortUnder5 By MortInfant


In Chapter 15 we noted that despite the strong summary statistics, the scatterplot very clearly indicates some doubt about the linear model: the points seem to bend around the line, suggesting that the relationship is not best described as a line.

The Residual by Predicted plot very clearly depicts both the non-linearity and the heteroskedasticity. Normality does not seem to present a serious problem.

Scenario 4
a.

(Note: it is wise to adjust the horizontal axis on the residual by predicted plot to more clearly see the pattern.) The residuals are not normally distributed, there may be a problem with constant variance on the left side of the graph. The sample size may be large enough to rely on the Central Limit Theorem.
c. Student answers will vary. The prediction bands on this graph are quite wide, and even with rescaling the axes it is difficult to read predicted values of Y. A reasonable response would be that the price should fall between $\$ 6200$ to $\$ 19,500$.
Scenario 5

```
a.
```



In the scatterplot we see a moderately strong positive linear association.
b.


Although the estimated slope of 0.906 might appear to be approximately 1 , the custom test indicates a significant difference from 1 ( p -value = .004). Moreover, we find that the Intercept is significantly different from 0 . Therefore, the Random Walk model does not
suit this set of data.

Scenario 6


With the Haydn data, in the Residual vs. Partb plot we find a heteroskedastic pattern; the residual do deviate slightly from normality, but the distribution is single peaked, so inference is probably appropriate.

## Scenario 7



Here we find a heteroskedastic pattern in which the variability of residuals increases as the Gestation period lengthens. Normality is not ideal, but the sample size may be enough to rely on the CLT. Given the non-constant variance, we should be reluctant to interpret or use the results of the regression.

## Scenario 8

## a. $\quad \Delta$ Residual by Predicted Plot


(Note: it is wise to adjust the horizontal axis on the residual by predicted plot to more clearly see the pattern.)
Recall that we find a non-significant relationship here - Tobacco Use is not a useful predictor of cancer deaths in a country. The residuals seem to show more variability in the middle range of tobacco use (non-constant variance), and residuals are nearly normal, with a long upper tail but large sample size. This model is not useful for inference.

Scenario 9
a.


These residuals look good... the Residual vs. Cement plot shows an even scatter above and below the 0-line and the normal quantile plot shows that the residuals follow a nearly normal distribution except for the lower tail. In any case, we have a large sample, so the CLT applies. We can safely interpret the results.

This is a highly significant, but weak, positive relationship. For each additional kg of cement in the mixture, compressive strength increases on average by 0.005 megapascals.

Scenario 10
a.


Above are the four plots of residuals vs. predicted. The residuals in the first regression are homoskedastic and approximately normal. The others indicate non-linearity and/or heteroskedasticity. Normality plots also indicate non-normal residuals in these small samples.

## Scenario 11

a.


Civic:

## $\triangle$ Residual by Predicted Plot



Corolla:


Cruiser
(Note: it is wise to adjust the horizontal axes on the residual by predicted plots to more clearly
see the pattern.)
Recall that we find a non-significant relationship for the Cruiser data. Each set of residuals would appear to have constant variance; the Civic data are most nearly normal, but normality is questionable for the others. Hence p-value estimates and confidence intervals may be inaccurate.

Scenario 12
a. $\quad \Delta$ Residual by Predicted Plot-- Latin America/ Caribbean


These residuals are unimodal and somewhat symmetric. With a small sample it is difficult to determine non-constant variance. No obvious violations, so inference is reasonable.
c.


This is a non-significant relationship. The residuals seem to show constant variance, but a skewed and flat distribution. Inference is safe.

Scenario 13
a.

$\triangle$ Residual Normal Quantile Plot


This is a very large sample, so the skewness to the right side may not be a major issue. The residuals do not appear to have a purely random pattern with constant variance, so judgments based on confidence intervals and p-values may be questionable.

## Student Solutions to Application Scenarios

Scenario 1
a.


The residual plots from this multiple regression model are very similar to those from the simple regression using Waist circumference as the only predictor (see those graphs below). We can use this set of data for estimation. The regression results themselves are shown below.

We find a strong relationship between BMI and the model, but this model is not much of an improvement over the previous model (shown again below). The intercept has changed dramatically, though in this model the intercept does not have much meaning. The effect size for the Waist measurement is almost equal to that of the single variable model, and the coefficient of height is not significant at the cusotmary .0 .05 level. The height variable is not significant at the 0.05 level, though it is significant at the 0.10 level.

The two-variable model has a very small improvement in goodness of fit in comparison to the single-variable model.


In short, the addition of the height data does not improve the model in any material way.
We first performed this regression in Chapter 15. At that time we found a significant relationship between waist circumference and BMI, with the waist measurement accounting for about $88 \%$ of the variation in BMI. Each addition centimeter of waist circumference is associated with an increase of 0.3847 in BMI. When we saved the residuals and check their normality, we find the normality assumption seems to be reasonable. The graph of residuals vs. predicted values suggested that the dispersion of residuals increases as predicted values increase, though it is not an overly dramatic tendency. We can probably trust this model for predictions.
c. NOTE: The scenario question mistakenly asks for you to use the Write Circumference as a predictor. The question should ask for thigh circumference.


In the model using waist and thigh circumference (note typographical error in early printings of the book that this is refrered to as wrist cicumference), we find residuals that are approximtely normal and more heteroskedastic than our prior models. In this sense, the model is less attractive than the earlier ones. On the other hand, the goodness of fit is improved (Adj. RSquare; see below) now equals 0.92 and both slopes are statistically signifiant and make logical sense.


## Scenario 2

a. Student answers will vary. One rotated scatterplot is shown here (including a density ellipsoid). We see a weak tendency for systolic BP to increase both as age and weight increase.

c.


Here again we find concerns about heteroskedasticity and normality; if we continue on to interpret the coefficient estimates, we see that the Diastolic BP adds little to the model. The estimated value is not significantly different from zero, and the adjusted $R^{2}$ is very nearly the same in the prior model using just 2 factors in the model. This model is no meaningful improvement over the prior one.

| Summary of Fit |  |
| :--- | ---: |
| RSquare | 0.194144 |
| RSquare Adj | 0.141589 |
| Root Mean Square Error | 8.379746 |
| Mean of Response | 110.56 |
| Observations (or Sum Wgts) | 50 |


| Analysis of Variance |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Source | DF | Sum of Squares | Mean Square | F Ratio |
| Model | 3 | 778.1931 | 259.398 | 3.6941 |
| Error | 46 | 3230.1269 | 70.220 | Prob $>\mathrm{F}$ |
| C. Total | 49 | 4008.3200 |  | 0.0183* |

$\triangle$ Parameter Estimates

| Term | Estimate | Std Error | t Ratio | Prob $>\|\mathbf{t}\|$ |
| :--- | ---: | ---: | ---: | ---: |
| Intercept | 95.121093 | 13.24218 | 7.18 | $<.0001^{*}$ |
| RIDAGEEX | -0.041445 | 0.041677 | -0.99 | 0.3252 |
| BMXWT | 0.2206758 | 0.068708 | 3.21 | $0.0024^{*}$ |
| BPXDII | 0.149581 | 0.142639 | 1.05 | 0.2998 |



The leverage plots immediately suggest a problem with collinearity, which is confirmed by the very high VIFs in the table of parameter estimates (below):

| Parameter Estimates |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Term | Estimate | Std Error | t Ratio | Prob $>\|\mathbf{t}\|$ | VIF |
| Intercept | 13.286898 | 0.702092 | 18.92 | $<.0001^{*}$ | . |
| MortMaternal | 0.0074981 | 0.002851 | 2.63 | $0.0093^{*}$ | 7.4325523 |
| MortUnder5 | 0.0316232 | 0.051616 | 0.61 | 0.5409 | 61.071947 |
| MortInfant | 0.1418103 | 0.076308 | 1.86 | 0.0649 | 48.623078 |

This model should not be used or interpreted.


When we estimate a simple linear model using gestation as the factor, we find a heteroskedastic pattern in which the variability of residuals diminishes as the Gestation period lengthens. Normality is not ideal, but the sample size is large enough to rely on the CLT. Given the non-constant variance, we should be reluctant to interpret or use the results of the regression.

This model is not an improvement over the prior two. We still see heteroskedasticity in the plot of residuals vs. fitted values (not shown here). We see evidence of collinearity in the large VIF for BrainWt, and only the Gestation variable is statistically significant.

## Scenario 5

## a. $\Delta$ Correlations

|  | GeneralIndex BasicGoods CapGoods IntermedGoods Consumer | Durables | NonDur |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | ---: | ---: |
| GeneralIndex | 1.0000 | 0.9932 | 0.9634 | 0.9716 | 0.9801 | 0.9254 | 0.9598 |
| BasicGoods | 0.9932 | 1.0000 | 0.9524 | 0.9675 | 0.9651 | 0.9237 | 0.9415 |
| CapGoods | 0.9634 | 0.9524 | 1.0000 | 0.9277 | 0.9119 | 0.8884 | 0.8850 |
| IntermedGoods | 0.9716 | 0.9675 | 0.9277 | 1.0000 | 0.9241 | 0.9060 | 0.8952 |
| Consumer | 0.9801 | 0.9651 | 0.9119 | 0.9241 | 1.0000 | 0.9021 | 0.9918 |
| Durables | 0.9254 | 0.9237 | 0.8884 | 0.9060 | 0.9021 | 1.0000 | 0.8396 |
| NonDur | 0.9598 | 0.9415 | 0.8850 | 0.8952 | 0.9918 | 0.8396 | 1.0000 |

In the correlation matrix we find that the Basic Goods index is most highly correlated with the General Index. The simple model that estimates monthly values of the General IIP from the Basic Goods IIP provides an excellent goodness of fit and the sample is large enough to invoke the CLT. However, we do see some evidence of non-linearity in the plot of residuals vs. fitted values (below):

```
\(\triangle\) Residual by Predicted Plot
```



Given the $\mathrm{R}^{2}$ value of nearly 0.99 , the non-linearity may not be a major problem. The estimation results are as follows:

| Parameter Estimates |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
| Term | Estimate | Std Error | t Ratio | Prob> $\mathbf{\| t \|}$ |
| Intercept | -45.42024 | 2.919964 | -15.56 | $<.0001^{*}$ |
| BasicGoods | 1.3979391 | 0.015697 | 89.06 | $<.0001^{*}$ |

An increase of 1 in the Basic Goods index will be accompanied on average by an increase of approximately 1.4 in the General Index.
c. See discussion in part (b) above. It is not surprising that these index variables are all highly correlated because they all measure different aspects of the fundamental production activity within the Indian economy, and all reflect the general level of economic activity.

Scenario 6
a. Student models will vary. Here is one plausible result using the Enfield and Orono columns:


The residuals appear to have a non-constant variance,
which raises a problem with using this model for prediction or estimation. The model adjusted $R^{2}$ is approximately 0.9 which indicates a very good fit. Both variables are statistically significant and we see no real evidence of collinearity.

| Parameter Estimates |  |  |  |  |  |  |
| :--- | ---: | :--- | ---: | :--- | ---: | ---: |
| Term | Estimate | Std Error | t Ratio | Prob $>\|\mathbf{t}\|$ | VIF |  |
| Intercept | -136.672 | 81.02309 | -1.69 | 0.1001 | . |  |
| Enfield | 1.1770766 | 0.332625 | 3.54 | $0.0011^{*}$ | 1.1065036 |  |
| Orono | 0.6331057 | 0.034694 | 18.25 | $<.0001^{*}$ | 1.1065036 |  |

## Student Solutions to Application Scenarios



Scenario 2
a.




| $\triangle$ Summary of Fit |  | $\triangle$ Parameter Estimates |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Term | Estimate | Std Error | t Ratio | Prob> $\|t\|$ | VIF |
| RSquare RSquare Adj | 0.243348 0.232382 | Intercept | 92.303784 | 5.273382 | 17.50 | <.0001* |  |
| Root Mean Square Error | 16.71694 | RIAGENDR[Male] | 0.0106928 | 1.187986 | 0.01 | 0.9928 | 1.0644213 |
| Mean of Response | 123.1848 | RIDAGEYR | 0.4634077 | 0.060513 | 7.66 | <.0001* | 1.0023125 |
| Observations (or Sum Wgts) | 211 | BMXWT | 0.1316212 | 0.055519 | 2.37 | 0.0187* | 1.0664799 |

The leverage plots indicate collinearity problems. We see that the model has rather poor fit, and only the Gender variable is not statistically significant.
Scenario 3
a. $\quad \Delta$ Regression Plot

$\triangle$ Parameter Estimates

| Term | Estimate | Std Error | t Ratio | Prob> \|t| | VIF |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Intercept | 15.612043 | 0.522589 | 29.87 | $<.0001^{*}$ | , |  |
| MortMaternal | 0.0315 | 0.001648 | 19.12 | $<.0001^{*}$ | 2.7075165 |  |
| (MortMaternal-310.039)*(MortMaternal-310.039) | $-1.144 \mathrm{e}-5$ | $2.304 \mathrm{e}-6$ | -4.97 | $<.0001^{*}$ | 2.6834511 |  |
| MatLeave90+[No] | 1.9731988 | 0.420709 | 4.69 | $<.0001^{*}$ | 1.101238 |  |

This model fits the data rather well, and all coefficients are significant. We find that other things equal higher rates of maternal mortality are associated with higher birthrates, and that after controlling for differences in maternal mortality, countries that do not offer lengthy maternity leaves have higher birthrates than countries with longer leaves. Residuals appear to be normally distributed with equal variances.

a.


- Transformed Fit Log
$\triangle$ Transformed Fit Log
Log(Subs per 100 Pop $)=-485.9273+0.2448519^{*}$ Year

For Denmark, the annual growth rate is $e^{0.2448519}-1=0.277$ or $27.7 \%$ per year.

For the U.S., the annual growth rate is $e^{0.2228187}-1=0.249$ or $24.9 \%$ per year.

Scenario 5
a.


The logistic regression results appear to the left. The regressor, PPE, is statistically significant and we see that patients with Parkinson's Disease have significantly lower PPE values than patients without PD. In the Logistic Plot, the dark markers are patients with PD; we see that the estimated curve distinguishes between PD and non-PD patients.


## Scenario 6

a.

| $\triangle$ Nominal Logistic Fit for Composer |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Converged in Gradient, 4 iterations |  |  |  |  |
| $\square$ Iterations |  |  |  |  |
| $\triangle$ Whole Model Test |  |  |  |  |
| Model <br> Difference <br> Full <br> Reduced | -Loglikelihood |  | DF ChiSquare | Prob>ChiSq |
|  | - 7.384012 |  | 214.76802 | 0.0006* |
|  | 36.695744 |  |  |  |
|  | 44.079756 |  |  |  |
| RSquare (U) |  | 0.1675 |  |  |
| AICC |  | 79.7915 |  |  |
| BIC |  | 85.8681 |  |  |
| Observations (or Sum Wgts) 64 |  |  | 4 |  |
| Measure | Training Definition |  |  |  |
| Entropy R | RSquare | 0.1675 1-Loglike(model)/Loglike(0) |  |  |
| Generalized RSquare |  | 0.2756 (1-(L) | (L $(\mathrm{l}) / \mathrm{L}(\mathrm{model})$ ) | $\wedge(2 / n)) /\left(1-L(0)^{\wedge}(2 / n)\right)$ |
| Mean - Log p |  | $0.5734 \sum-\log (\mathrm{p}[\mathrm{j}) / \mathrm{n}$ |  |  |
| RMSE |  | $0.4445 \sim \sum(y[j]-\rho[j])^{2} / n$ |  |  |
| Mean Abs Dev |  | $0.3928 \sum \mathrm{ly}[\mathrm{j}]-\rho[j] / / n$ |  |  |
| Misclassification Rate |  | $0.3281 \sum(\rho[\mathrm{j}] \neq \mathrm{\rho}$ Max)/n |  |  |
| N |  | 64 n |  |  |
| $\checkmark$ Lack Of Fit |  |  |  |  |
| $\triangle$ Parameter Estimates |  |  |  |  |
| Term | Estimate Std Error C |  | ChiSquare Prob>ChiSq |  |
| Intercept | 1.9248859 | 20.7482643 | 6.62 | 0.0101* |
| Parta | -0.124979 | 0.0505836 | 6.10 | 0.0135* |
| Partb | 0.0530201 | 30.0309365 | 2.94 | 0.0866 |
| For log odds of Haydn/Mozart |  |  |  |  |
| Covariance of Estimates |  |  |  |  |
| $\triangle$ Effect Likelihood Ratio Tests |  |  |  |  |
|  |  | L-R |  |  |
| Source | Nparm | DF ChiSquare | Prob>ChiSq |  |
| Parta | 1 | 18.23254153 | $30.0041^{*}$ |  |
| Partb | 1 | 13.46717072 | 20.0626 |  |

The results are to the left. We find that the whole model is significant with a rather poor fit, as measured by U. Other things being equal, the longer Part a is the lower the odds that it was composed by Haydn. Conversely, the longer Part b is (holding Part a constant) the higher the odds that it was composed by Haydn.

Scenario 7
a. Here are the results for the quadratic and linear fits:


| $\triangle$ Linear Fit |  |  |  |  |  |  | $\triangle$ Polynomial Fit Degree $=2$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cellular subscribers per 100 population $=17.42575+$ |  |  |  |  |  |  | Cellular subscribers per 100 population $=17.232324+$ |  |  |  |  |  |  |  |
| $1.3272881 *$ Telephone lines per 100 population |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | $0.0300629^{*}(\text { Telephone lines per } 100 \text { population- } 21.7652)^{\wedge} 2$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | $\triangle$ Summary of Fit |  |  |  |  |  |  |  |
| RSquare Adj 0.578965 |  |  |  |  |  |  | RSquare 0.75143 |  |  |  |  |  |  |  |
| Root Mean Square Error 23.92515 |  |  |  |  |  |  | Root Mean Square Error |  |  | 18.47915 |  |  |  |  |
| Mean of | spons |  | 46.15928 |  |  |  | Mean of Response |  |  | 46.15928 |  |  |  |  |
| Observa | (or | Sum Wgts) | 194 |  |  |  | Observations (or Sum Wgts) 194 |  |  |  |  |  |  |  |
| $\checkmark$ Lack Of Fit |  |  |  |  |  |  | Lack Of Fit |  |  |  |  |  |  |  |
| $\triangle$ Analysis of Variance |  |  |  |  |  |  | $\triangle$ Analysis of Variance |  |  |  |  |  |  |  |
|  |  | Sum of |  |  |  |  | Source | DF | Sum of Squares | Mean Square | FRatio |  |  |  |
| Source | DF | Squares | Mean Square | F Ratio |  |  | Model | 2 | 197168.00 | 98584.028 | 288.6972 |  |  |  |
| Model | 1 | 152487.19 | 152487 | 266.3937 |  |  | Error |  | 65222.47 | 341.5 Pr | Prob $>$ F |  |  |  |
| Error | 192 | 109903.27 | 572 | Prob $>$ F |  |  | C. Total | 193 | 262390.46 |  | <.0001* |  |  |  |
| C. Total | 193 | 262390.46 | <.0001* |  |  |  | $\triangle$ Parameter Estimates |  |  |  |  |  |  |  |
| $\triangle$ Parameter Estimates |  |  |  |  |  |  | Term Intercept |  |  |  | Estimate | Std Error | t Ratio | Prob> $\mid$ t |
|  |  |  |  |  |  |  | 17.232324 1.9558382 | 1.899833 0.083454 | 9.07 23.44 | <.0001* $<$ < $0001^{*}$ |
| Intercep |  |  | 17.42575 | 2.459637 | 7.08 | <,0001* |  |  |  |  | (Teleph | lines $p$ | per 100 popu | ulation-21.7652)^2 | $2-0.030063$ | 0.002628 | -11.44 | <.0001* |
| Telepho | lines $p$ | er 100 popu | lation 1.3272881 | 10.081321 | 16.32 | <.0001* |  |  |  |  |  |  |  |  |

We can see that the quadratic model has better goodness of fit statistics, and graphically it is clear the parabolic model fits the observed points better than the linear model.

## Scenario 8

a. Here are the results for the linear, quadratic and log- linear fits: The linear and log-linear are nearly indistinguishable. None of the models fit particularly well, which visual inspection makes clear. The quadratic model has the best fit of the three, but it is weak.


## Student Solutions to Application Scenarios

## Scenario 1

## a. $\quad \Delta$ Model Comparison



As shown above, using a 6-month season (top row) is a minor improvement over the 3-month season. The variance, MAPE, and MAE are smaller with this model than the earlier model, and RSquare is very slightly higher.
c. $\quad \triangle$ Model Comparison

| Report | Graph | Model | DF | Variance | AIC | SBC | RSquare | -2LoglH | Weights | . 2.4 .6.8 | MAPE | MAE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | $\square$ | - Winters Method (Additive) | 101 | 45.37764 | 711.39900 | 719.33217 | 0.942 | 705.399 | 0.999661 |  | 3.132245 | 5.846902 |
| - | 回 | - ARI $(2,1)$ | 107 | 42.046266 | 727.42716 | 735.52860 | 0.954 | 721.42716 | 0.000331 |  | 2.825473 | 5.258672 |
| - | 回 | - Winters Method (Additive) | 104 | 50.177156 | 735.50239 | 743.52087 | 0.940 | 729.50239 | 0.000006 | - . | 3.315726 | 6.109066 |
| - $\nabla$ | $\square$ | $-\operatorname{ARI}(1,1)$ | 108 | 46.320348 | 736.88434 | 742.28530 | 0.949 | 732.88434 | 0.000003 |  | 2.877901 | 5.367753 |
| - | $\square$ | - AR(1) | 109 | 99.891165 | 830.57245 | 835.99151 | 0.878 | 826.57245 | 0.000000 |  | 4.290917 | 7.913439 |

As shown above, the $\operatorname{AR}(2,1)$ model is an improvement as indicated by all measures of fit.

## Scenario 2

a. Student answers will vary. Responses should note that Durables show a marked upward trend with likely seasonal component. Below are summary results for several reasonable approaches. Among the methods available through the Time Series platform, Linear Exponential Smoothing outperforms the others according to the measures we have studied. The adjusted RSquare statistics for the regression-based models are inferior to all but the $\operatorname{AR}(1)$ model, as follows: Linear, (.854), Quadratic (.855), LogLinear (.867).

| $\triangle$ Model Comparison |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Report | Graph | Model | DF | Variance | AIC | SBC | RSquare | -2Logl | Weights | . 2.4 . 6.8 | MAPE | MAE |
| - | $\square$ | - Winters Method (Additive) | 104 | 528.09261 | 989.97456 | 997.99305 | 0.873 | 983.97456 | 0.995917 |  | 5.876557 | 18.367321 |
| - | $\square$ | - Linear (Holt) Exponential Smoothing | 107 | 519.71387 | 1000.9686 | 1006.3513 | 0.875 | 996.96864 | 0.004082 | : : | 5.741653 | 17.872988 |
| - | 回 | - ARI $(1,1)$ | 108 | 597.81201 | 1017.5667 | 1022.9677 | 0.870 | 1013.5667 | 0.000001 |  | 5.840147 | 18.695112 |
| - | 回 | - AR(1) | 109 | 689.3022 | 1044.5854 | 1050.0045 | 0.832 | 1040.5854 | 0.000000 |  | 6.783594 | 21.001116 |

## Scenario 3

a. This is an annual series and therefore there can be no seasonal component.
c. Student answers will vary. For the Malaysia data, a 3 ${ }^{\text {rd }}$ degree polynomial (cubic) model provides a very good fit:

e. These countries are all best approximated by different models. Effective time-series modeling requires the use of a variety of approaches.

## Scenario 4

a. The fertility rate in Brazil has declined following an S-shaped curve:


An $\operatorname{AR}(1,1)$ model fits modertately well, with relatively high $R S q u a r e ~(0.969)$, low variance ( 0.077 ) and MAPE and MAE of $5.35 \%$ and 0.20 respectively.
c.


India's decline is very regular, especially since 1960. Linear Exponential Smoothing (Holt's method) and $\operatorname{AR}(1,1)$ models both fit extremely well.
e.


Saudi Arabia's decline is very regular, especially since 1980. Linear Exponential Smoothing (Holt's method) and AR( 1,1 ) models both fit extremely well.

## Scenario 5

a. $\Delta$ Time Series Bangor
c.

e.


Bangor: For this series, an $\operatorname{AR}(4,1)$ works moderately well. The strong seasonal element here suggests that points are correlated with the observation 4 quarters earlier.

Winslow: Here we see the dramatic change occurring roughly half-way through the time series. Simple exponential smoothing provides are reasonably good model.

Scenario 6
a. $\Delta \oplus$ Time Series CO2


For this series, a log-linear model fits quite well ( $\mathrm{Rsqr}=0.905$ ). The other time series methods do not fit quite as well, though an $\operatorname{AR}(1,1)$ provides a good fit.
C. $\Delta \triangle$ Time Series CO2
e.


In sharp contrast to the prior two graphs, China's CO 2 emissions have been rapidly rising. A $3^{\text {rd- }}$ degree polynomial (cubic) provides a moderately good fit, as does $\operatorname{AR}(1,1)$.

CO 2 emissions in the US rose for much of the period and seem to have leveled off, presenting a quite different pattern from the prior 4 nations. A $2^{\text {nd }}$ degree polynomial fits best.


Scenario 8
a. $\triangle$ Correlations

|  | NIK225 | FTSE | SP500 HangSeng |  | IGBM | TA100 |
| :--- | ---: | ---: | :--- | ---: | ---: | ---: |
| NIK225 | 1.0000 | 0.9674 | 0.9812 | 0.9688 | 0.9379 | 0.9506 |
| FTSE | 0.9674 | 1.0000 | 0.9810 | 0.9770 | 0.9795 | 0.9305 |
| SP500 | 0.9812 | 0.9810 | 1.0000 | 0.9652 | 0.9637 | 0.9498 |
| HangSeng | 0.9688 | 0.9770 | 0.9652 | 1.0000 | 0.9731 | 0.9468 |
| IGBM | 0.9379 | 0.9795 | 0.9637 | 0.9731 | 1.0000 | 0.9281 |
| TA100 | 0.9506 | 0.9305 | 0.9498 | 0.9468 | 0.9281 | 1.0000 |

There are 1 missing values. The correlations are estimated by REML method.

The Nikkei225 has the highest correlation with the S\&P500 (0.9812) and the FTSE100 is close behind with $r=0.9810$ )
c. Yes. Both markets are engaged in competition in the same global markets, and move very closely together as indicatd by their very high correlation.

## Scenario 9

a.


This is a non-stationary annual series with a curvilinear trend since approximately 1800 . We saw in the previous chapter that a quadratic trend model fit to a degree, but this pattern is probably better modeled as an autoregressive process.
c.


This non-stationary annual series shows considerable variability. A trend model won't capture the year-to year oscillations, but an autoregressive model will.

Looking at the summary of 4 different AR models (below) it appears that the AR $(2,1)$ model performs best. The estimates using that model are shown below the Model Comparison table.

| $\triangle$ Model Comparison |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Report | Graph Model | DF | Variance | AIC | SBC | RSquare | -2LogLH | Weights | . 2.4 . 6.8 | MAPE | MAE |
| - | $\square \quad-\operatorname{ARI}(2,1)$ | 129 | 0.0300717 | -84.75350 | -76.10509 | 0.890 | -90.7535 | 0.977823 |  | 0.245075 | 0.140145 |
| - | $\square-\operatorname{ARI}(1,1)$ | 130 | 0.0322389 | -76.70080 | -70.93519 | 0.881 | -80.7008 | 0.017444 |  | 0.254696 | 0.145682 |
| - $\square$ | $\square-\mathrm{AR}(2)$ | 130 | 0.0322338 | -74.01722 | -65.34617 | 0.876 | -80.01722 | 0.004559 |  | 0.257463 | 0.147277 |
| - | $\square \quad-\mathrm{AR}(1)$ | 131 | 0.03416 | -67.47918 | -61.69849 | 0.869 | -71.47918 | 0.000173 |  | 0.261399 | 0.149573 |
| 2013 | 58.272704408 |  |  |  |  |  |  |  |  |  |  |
| 2014 | 58.257001165 |  |  |  |  |  |  |  |  |  |  |
| 2015 | 58.262675668 |  |  |  |  |  |  |  |  |  |  |
| 2016 | 58.28208585 |  |  |  |  |  |  |  |  |  |  |
| 2017 | 58.290882083 |  |  |  |  |  |  |  |  |  |  |

## Student Solutions to Application <br> Scenarios

Scenario 1
a.

| $\sigma$ | Pattern |
| ---: | :--- |
| 1 | ---1 |
| 2 | ---1 |
| 3 | +--1 |
| 4 | --+2 |
| 5 | ---2 |

The first 5 rows are shown to the left.
c. Assuming we follow the example presented in the chapter, we now have 50 experimental runs, the first 10 of which are assigned to team member \#1. Each team member will perform 10 of the 16 possible runs, with each member having a slightly different pattern assigned randomly.

## Scenario 2

a. There will be 32 runs in a Resolution IV, full-factorial design.
c. [NOTE: The question should read: "Briefly explain what happens when we move from a twofactor screening design to a five-factor design."]

In a two-factor screening design there would be just four runs $\left(2^{2}\right)$ and the five-factor model has $2^{5}=32$ runs.

## Scenario 3

| a. | Pattern | Gender | TestCondition | Interruptions |  |
| ---: | :--- | :--- | :--- | :--- | :--- |
| 1 | 212 | Male | AwakeFirst | Interrupted |  |
| 2 | 112 | Female | AwakeFirst | Interrupted |  |
| 3 | 111 | Female | AwakeFirst | Fullnight |  |
| 4 | 112 | Female | AwakeFirst | Interrupted |  |
| 5 | 212 | Male | AwakeFirst | Interrupted |  |

The first five rows of the data table, including Patterns, are shown above.
c. With 72 subjects, the prediction profiler shows that the variance ranges from approximately 0.042 to approximately 0.056 . With 144 subjects, the corresponding variance range is reduced by half, ranging from approximately 0.021 to 0.028 .

## Scenario 4

a. Categorical factors: type of incentive, timing of incentive, survey mode, guarantee vs. lottery. Continuous factors: Duration of survey, number of contacts made, and amount of money offered.
[some students might classify "burden" of survey as categorical.]
c. Assuming that we use minimal number of factor levels described in b, and two factor levels for the continuous factors, we would have four dichotomous categorical factors and three continuous factors. This would, then, require $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2=2^{7}=128$ runs.

## Scenario 5

a. Here are the first five rows of the table:

|  | Pattern | ImpactModifier | Thermal Stabilizer | AntiUV |
| ---: | :--- | :--- | :--- | ---: |
| 1 | 413 | MBS | PdBaCd | 10 |
| 2 | 331 | ABS | BaCd | 3 |
| 3 | 233 | CPE | BaCd | 10 |
| 4 | 421 | MBS | Pb | 3 |
| 5 | 411 | MBS | PdBaCd | 3 |

c. The full-factorial design has 528 runs and the fractional custom design has 35 . In the initial design, the Anti-UV additive is tested at levels of 3,5 and 10 with each of the three tested in
one-third of the runs. In the revised design, the levels are $3,6.5$, and 10 . The profiler for the custom design reveals substantial interactions among the three factors; the fractional design can detect these, but the loss of resolution in this design could be costly.
Scenario 6
a. This table has 72,072 rows. Here are the first five:

|  | Pattern | Subject | Call to Action | Promotion | Salutation | Closing |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 12231 | Crayola | Because | Prodcut | User | Crayola |  |
| 2 | 12232 | Crayola | Because | Prodcut | User | Education |  |
| 3 | 22122 | Help | Because | None | Greetings | Education |  |
| 4 | 21312 | Help | As Crayola | Amazon | Hi | Education |  |
| 5 | 22231 | Help | Because | Prodcut | User | Crayola |  |

c. In the full factorial design, every combination of all levels the five factors $(2 \times 3 \times 2 \times 3 \times 2=72)$ is tested whereas in the reduced custom design, far fewer are tested because interactions are limited to two factors at a time.

## Student Solutions to Application Scenarios


DIAMETER Limit Summaries

| Points plotted | MACHINE | LCL | Avg | UCL Limits Sigma | Sample Size |  |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: |
| Average | A386 | 3.998813 | 4.42727 | 4.855727 | Standard Deviation | 6 |
| Average | A455 | 4.150375 | 4.490839 | 4.831304 | Standard Deviation | 6 |
| Average | C334 | 3.825622 | 4.130796 | 4.43597 | Standard Deviation | 6 |
| Standard Deviation A386 | 0.010107 | 0.332878 | 0.655649 | Standard Deviation | 6 |  |
| Standard Deviation A455 | 0.008032 | 0.264515 | 0.520998 | Standard Deviation | 6 |  |
| Standard Deviation C334 | 0.007199 | 0.237097 | 0.466995 | Standard Deviation | 6 |  |

As we can see in the graphs above, Machine C334 may have an unstable standard deviation and machine A455 shows two sample means beyond the control limits. These machines should be inspected closely for possible adjustment.
c.


This capability analysis shows that $5 \%$ of the observations lie outside the capability limits, indicating that the process is capable of producing tubing that is within .5 mm of 4.4 .

## Scenario 2

a.


This process is out of control at three points. Because a day with 0 cancellations is desirable, we should not be concerned about dates with values below the LCL. However, the chart shows 3 date well above the UCL.

Scenario 3
a.

c.


Production of basic goods has been rising steadily over time, which is a good thing. This is not a process designed for a constant target, but rather one of continuous growth.

Once again we see a steady pattern of growth, with clear seasonal variation. In contrast to the control chart for Basic Goods, the one for NonDurables may exhibit a more linear upward trend, and substantial growth in variability (the R Chart) in the most recent years.

Because the need for basic goods probably follows the growth in population we might expect steady growth akin to population trends.

Scenario 4
a. In most regions except for the Southwest the standard deviations are sufficiently unstable that we should not interpret the Xbar charts. In the Southwest, the standard deviations have been steadily increasing but the limited data (only five sample mean) indicates increasing mean times to restore the area to safety, but still within control limits.

## Scenario 5





Scenario 8
a.


It appears that the variability of the process standard deviation has increased over time, with one recent $S$ above the UCL. Nearly all of the sample means are within the control limits; early in the observation period (roughly the first 15 samples) the mean magnitudes remained quite close to 4.0 . Since that time, the fluctuations in mean magnitude have increased even as the mean appears to have remained stable.


[^0]:    $\Delta$ Tests

    | N | DF | -LogLike | RSquare (U) |
    | ---: | ---: | ---: | ---: |
    | 3801 | 3 | 33.676445 | 0.0410 |


    | Test | ChiSquare | Prob>ChiSq |
    | :--- | ---: | ---: |
    | Likelihood Ratio | 67.353 | $<.0001^{*}$ |

    Pearson $85.878<.0001^{*}$

[^1]:    Std Error uses a pooled estimate of error variance

[^2]:    Levels not connected by same letter are significantly different.

