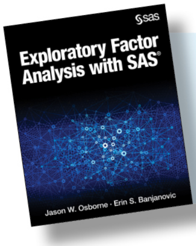

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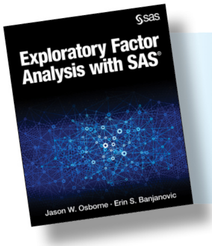

1

Introduction to Exploratory Factor Analysis

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A Tool for Exploration

Exploratory factor analysis (EFA) is a statistical tool used for exploring the underlying structure of data. It was originally developed in the early 1900s during the attempt to determine whether intelligence is a unitary or multidimensional construct (Spearman, 1904). It has since served as a general-purpose dimension reduction tool with many applications. In the modern social sciences it is often used to explore the psychometric properties of an instrument or scale. Exploratory factor analysis examines all the pairwise relationships between individual variables (e.g., items on a scale) and seeks to extract latent factors from the measured variables. During the 110 years since Spearman's seminal work in this area, few statistical techniques have been so widely used (or, unfortunately, misused).

The goal of this book is to explore best practices in applying EFA using SAS. We will review each of the major EFA steps (e.g., extraction, rotation), some associated practices (estimation of factor scores and higher-order factors), and some less common analyses that can inform the generalizability of EFA results (replication analyses and bootstrap analyses). We will review the SAS syntax for each task and highlight best practices according to research and practice. We will also demonstrate the procedures and analyses discussed throughout the book using real data, and we will occasionally survey some poor practices as a learning tool.

To get started in our exploration of EFA, we will first discuss the similarities and differences between EFA and principal components analysis (PCA), another technique that is commonly used for the same goal as EFA. We will then briefly summarize the steps to follow when conducting an EFA and conclude with a quick introduction to EFA in SAS.

EFA vs PCA

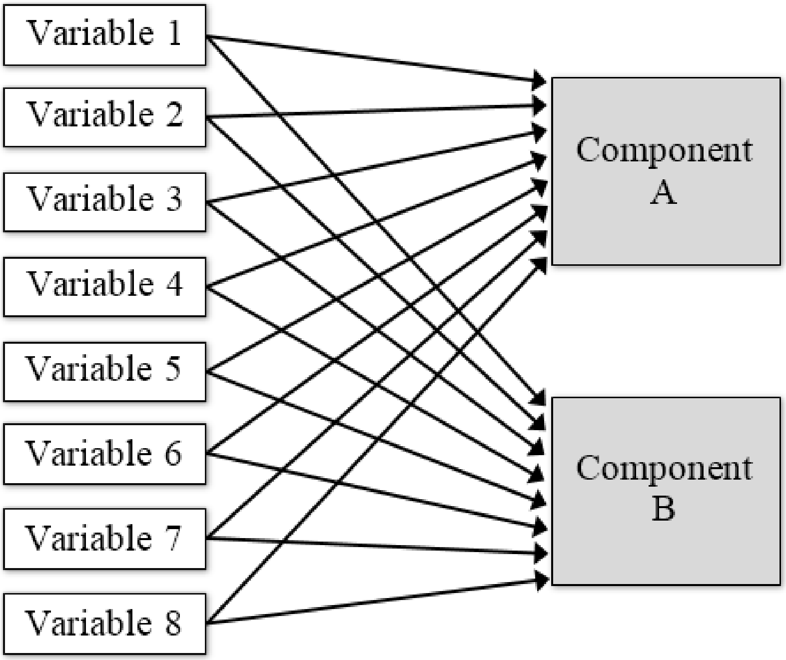
As you will come to learn, EFA is quite different from PCA. Unfortunately, there are many misconceptions about the two analyses, and one of the biggest is that PCA is part of, or synonymous with, EFA. This misconception probably has modern day roots from at least two factors:

1. Statistical software, including SAS, has PCA as the default extraction technique when performing exploratory factor analysis.
2. Many modern researchers use PCA and EFA interchangeably, or use PCA when performing an analysis that is more appropriate for EFA.

Although the two methods generally seem to do the same thing, they are different in some key ways. Principal components analysis is a computationally simplified version of a general class of dimension reduction analyses. EFA was developed before PCA (Hotelling, 1933), thanks to Spearman (1904). EFA was developed prior to the computer age when all statistical calculations were done by hand, often using matrix algebra. As such, these were significant undertakings requiring a great deal of effort. Because of the substantial effort required to perform EFA with hand calculations, significant scholarship and effort went into developing PCA as a legitimate alternative that was less computationally intense but that also provided similar results (Gorsuch, 1990). Computers became available to researchers at universities and industrial research labs later in the 20th century, but remained relatively slow and with limited memory until very late in the 20th century (about the time the first author was in graduate school using mainframes at the university). Our commentary on PCA is not intended to slight these scholars nor to minimize their substantial contributions, but rather to attempt to put PCA and EFA into context for the modern statistician and quantitative researcher. We will therefore focus on EFA, despite the popularity of PCA.

Without getting into the technical details, which are available in other scholarly references on the topic, PCA computes the analysis without regard to the underlying latent structure of the variables, using all the variance in the manifest variables. What this means is that there is a fundamental assumption made when choosing PCA: that the measured variables are themselves of interest, rather than some hypothetical latent construct (as in EFA). This makes PCA similar to multiple regression in some ways, in that it seeks to create optimized weighted linear combinations of variables.

Figure 1.1 *Conceptual overview of principal components analysis*



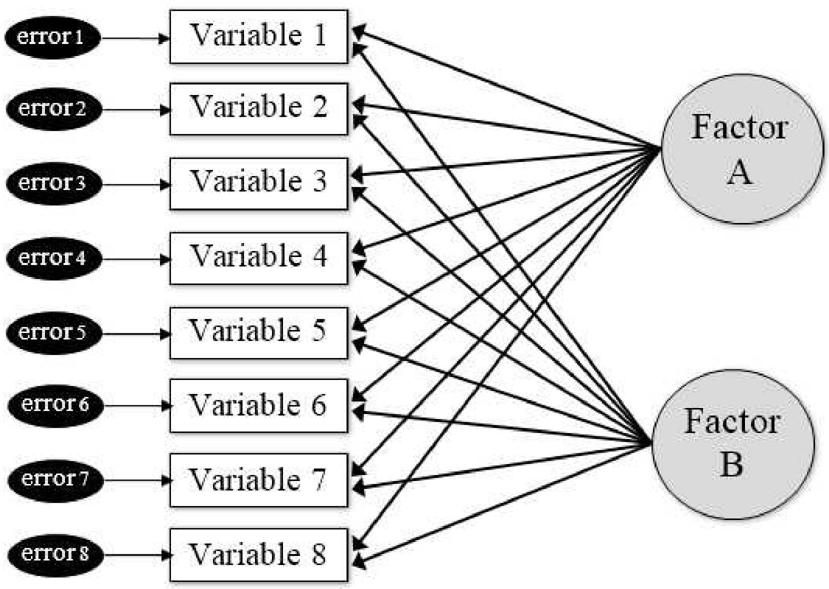
An example of a PCA model to extract two factors is presented in Figure 1.1 on page 3. We have already conducted some initial analyses (to be discussed in Chapter 3) that have convinced us of this two-component structure and led us to this model. Note that all PCA and EFA analyses extract as many components or factors as there are manifest variables, although not all are retained for interpretation; here we simplify for clarity to examine the first two components extracted and to see how they relate to the measured variables. Now the important thing to note in this figure is the direction of the arrows. Notice that they point from the variables to the components. This is because each component is formed as a *weighted linear combination* [1] of the predictor variables. One-hundred percent of what is in those variables ends up becoming the components. As analysts, we can then review the results and identify the primary component each variable loads on to create scales, we can create component scores, or we can do whatever else we would like with the results; but the components themselves are completely defined by the variables. In this way, principal components analysis combines manifest (observed) variables into components.

Exploratory factor analysis, on the other hand, is a group of extraction and rotation techniques that are all designed to model unobserved or latent constructs. It is referred to as common factor analysis or exploratory factor analysis. EFA assumes and asserts that there are latent variables that give rise to the manifest (observed) variables, and the calculations and results are interpreted very differently in light of this assumption.

1 Weighted linear combinations means that each variable has a different weight: the factor/component loading that determines how much or little each variable contributes to the composite. This is similar to multiple regression where a variable score is composed of different amounts (regression weights) of each variable in the equation.

You can see this very different conceptual vision of the same two-factor model reviewed above in Figure 1.2 on page 4 below. Notice the changed direction of the arrow between the variables and factors as well as the addition of error terms for each variable. Factor analysis recognizes that model variance contains both shared and unique variance across variables. EFA examines only the shared variance from the model each time a factor is created, while allowing the unique variance and error variance to remain in the model. The factors are then created as weighted linear combinations of the shared variance. When the factors are uncorrelated and communalities are moderate, PCA can produce inflated values of variance accounted for by the components (Gorsuch, 1997; McArdle, 1990). Since factor analysis analyzes only shared variance, factor analysis should yield the same general solution (all other things being equal) while also avoiding the illegitimate inflation of variance estimates.

Figure 1.2 *Conceptual overview of exploratory factor analysis*



There are two other issues with PCA that we will briefly note. First, PCA assumes that all variables are measured without error (an untenable assumption in almost any discipline), whereas EFA offers the option of acknowledging less than perfect reliability. Second, PCA parameters are selected in an attempt to reproduce sample, rather than population, characteristics (Thompson, 2004).

Thus, we have many similarities between PCA and some important conceptual and mathematical differences. Most authors agree that there is little compelling reason to choose PCA over other extraction methods, and that PCA can be limited and provide biased parameter estimates. Such a list of authors would include Bentler & Kano, 1990; Floyd & Widaman, 1995; Ford, MacCallum, & Tait, 1986; Gorsuch, 1990; Loehlin, 1990; MacCallum & Tucker, 1991; Mulaik, 1990; Widaman, 1993. If one is to seek best practices, one is hard pressed to conclude PCA is ever a best practice. Widman (1993) puts it very bluntly:

“principal components analysis should not be used if a researcher wishes to obtain parameters reflecting latent constructs or factors.” (p. 263). Unfortunately, it is still the default dimension reduction procedure in much statistical analysis software, even though it is usually not (in our opinion) the conceptually desirable choice, and usually has no clear advantage in modern quantitative methodology that we can detect.

This is a topic that arouses passions among statisticians, and the first author has rarely published a paper or given a talk on this topic without someone getting upset for taking this position so clearly and unapologetically. So let us sidestep this issue for the moment and summarize: PCA is not considered a factor analytic technique, and there is disagreement among statisticians about when it should be used, if at all. More often than not, researchers use PCA when EFA would be appropriate and preferable (for example, see Ford et al., 1986; Gorsuch, 1983; Widaman, 1993).

Steps to Follow When Conducting EFA

Exploratory factor analysis is meant to be exploratory in nature, and thus it is not desirable to prescribe a rigid formula or process for executing an EFA. The steps below are meant to be a loose guide, understanding that a factor analysis often requires returning to previous steps and trying other approaches to ensure the best outcome. The general pattern of performing an EFA falls into six general steps that will guide the discussion through the rest of the book:

1. Data cleaning
2. Deciding on an extraction method to use
3. Deciding how many factors to retain
4. Deciding on a method of rotation (if desired)
5. Interpreting results *(return to #3 if a solution is not ideal)*
6. Replication or evaluation of robustness *(return to the beginning if a solution is not replicable or robust)*

Step 1: Data cleaning. Without clean data, what follows in almost any analysis is moot. This is another point where passions run high among researchers and statisticians because there is considerable controversy about any manipulations of the sample and data (e.g., how to treat outliers, missing data). We have a clear position on the issue—data should be cleaned and issues (e.g., failing to meet assumptions) should be addressed. The first author wrote an entire book on the topic, in which he demonstrated repeatedly how clean data produces results that are better estimates of population parameters and, therefore, more accurate and replicable (Osborne, 2013). Instead of debating the point here, allow me to assert that data that is filled with errors or that fails to meet assumptions of the analysis being performed is likely to lead to poorer outcomes than data that is free of egregious errors and that meets assumptions. We will discuss some other data quality issues later in the book, including the importance of dealing appropriately with missing data.

Step 2: Deciding on an extraction method. An extraction technique is one of a group of methods that examines the correlation/covariation between all the variables and seeks to “extract” the latent variables from the measured/manifest variables.

There are several factor analysis extraction methods to choose from. SAS has seven *EFA extraction methods*: unweighted least squares (ULS), maximum likelihood (ML), principal axis factoring (PAF), iterated principal axis factoring (iterated PAF), alpha factoring, image factoring, and Harris factoring.[1] Information about the relative strengths and weaknesses of these techniques is not easy to obtain. To complicate matters further, naming conventions for some extraction techniques are not consistent, leaving it difficult to figure out which method a textbook or journal article author is describing, and whether or not it is actually available in the software the researcher is using. This probably explains the popularity of principal components analysis – not only is it the default in much statistical software, but it is one of the more consistent names researchers will see there.

An article by Fabrigar, Wegener, MacCallum and Strahan (1999) argued that if data is relatively normally distributed, maximum likelihood is the best choice because “it allows for the computation of a wide range of indexes of the goodness of fit of the model [and] permits statistical significance testing of factor loadings and correlations among factors and the computation of confidence intervals.” (p. 277). If the assumption of multivariate normality is “severely violated” they recommend iterated PAF or ULS factoring (Fabrigar et al., 1999; Nunnally & Bernstein, 1994). Other authors have argued that in specialized cases, or for particular applications, other extraction techniques (e.g., alpha extraction) are most appropriate, but the evidence of advantage is slim. In general, ML, iterated PAF, or ULS will give you the best results, depending on whether your data is generally normally distributed or significantly non-normal. In Chapter 2, we will compare outcomes between the various factor extraction techniques.

Step 3: Deciding how many factors to retain for analysis. This, too, is an issue that suffers from anachronistic ideas and software defaults that are not always ideal (or even defensible). In this step, you (or the software) decide how many factors you are going to keep for analysis. The statistical software will always initially extract as many factors as there are variables (i.e., if you have 10 items in a scale, your software will extract 10 factors) in order to account for 100% of the variance. However, most of them will be meaningless. Remembering that the goal of EFA is to *explore* your data and *reduce* the number of variables being dealt with. There are several ways of approaching the decision of how many factors to extract and keep for further analysis. Our guide will always focus on the fact that extracted factors should make conceptual and theoretical sense, and be empirically defensible. We will explore guidelines for this later in Chapter 3.

Step 4: Deciding on a rotation method and rotating the factors. Rotation is often a source of some confusion. What exactly is rotation and what is happening when data is rotated? In brief, the goal is to clarify the factor structure and make the results of your EFA most interpretable. There are several rotation methodologies, falling into two general groups: orthogonal rotations and oblique rotations. Orthogonal rotations keep axes at a 90° angle,

1 Please note, there is one other extraction method, principal components analysis. There are also two additional extraction options to recognize and perform analyses on a set of scoring coefficients or a factor pattern matrix.

forcing the factors to be uncorrelated. Oblique rotations allow angles that are not 90°, thus allowing factors to be correlated if that is optimal for the solution. We argue that in most disciplines constructs tend to be at least marginally correlated with each other, and, as such, we should focus on oblique rotations rather than orthogonal. We will discuss these options in more detail in Chapter 4.

Step 5: Interpreting results. Remember that the goal of exploratory factor analysis is to explore whether your data fits a model that makes sense. Ideally, you have a conceptual or theoretical framework for the analysis—a theory or body of literature guiding the development of an instrument, for example. Even if you do not, the results should be sensible in some way. You should be able to construct a simple narrative describing how each factor, and its constituent variables, makes sense and is easily labeled. It is easy to get EFA to produce results. It is much harder to get sensible results.

Note also that EFA is an *exploratory* technique. As such, it should not be used, as many researchers do, in an attempt to *confirm* hypotheses or test competing models. That is what *confirmatory factor analysis* is for. It is a misapplication of EFA to use it in this way, and we need to be careful to avoid confirmatory language when describing the results of an exploratory factor analysis.

If your results do not make sense, it might be useful to return to an earlier step. Perhaps if you extract a different number of factors, the factors or solution will make sense. This is why it is an exploratory technique.

Step 6: Replication of results. One of the hallmarks of science is *replicability*, or the ability for other individuals, using the same materials or methods, to come to the same conclusions. We have not historically placed much emphasis on replication in the social sciences, but we should. As you will see in subsequent chapters, EFA is a slippery technique, and the results are often not clear. Even clear results often do not replicate exactly, even within an extremely similar data set. Thus, in our mind, this step is critical. If the results of your analysis do not replicate (or do not reflect the true nature of the variables in the "real world"), then why should anyone else care about them? Providing evidence that your factor structure is likely to replicate (either through another EFA or through CFA) makes your findings stronger and more relevant. In Chapter 6, we will explore a "traditional" method of replication[1] (similar to cross validation in regression models). In Chapter 7, we will play with the notion of applying a less traditional but perhaps more useful analysis using bootstrap analysis. Confirmatory factor analysis is outside the scope of this book, but is perhaps an even better method of replication.

Basic Syntax for EFA

The `FACTOR` procedure is used to conduct EFA in SAS. You must be careful when conducting an EFA in SAS because the `FACTOR` procedure is also used to conduct PCA—if you do not specify an extraction method or if you just select one of the extraction options,

1 We put that in quotation marks because most researchers reporting results from an EFA fail to do any replication at all.

you could end up with a PCA and not an EFA! Below is a brief summary of the syntax and the basic options available. We will delve into further details and options in the chapters to come.

```
PROC FACTOR   DATA = dataset-name
         NFACTORS = number-of-factors-to-retain
         METHOD = factor-extraction-method
         ROTATE = rotation-method;
   VAR variables-to-include;
               RUN;
```

In the above syntax, you would specify all of the arguments that are highlighted. You would identify the data set to use for the analysis, tell SAS the number of factors to extract (we will discuss the multi-step procedure for this in Chapter 3), specify one of the seven EFA extraction methods (note there are other options, but they are not EFA extraction methods), specify one of the 25 rotation methods, and list the variables to be factored. If you do not specify the **NFACTORS**, **METHOD**, and **ROTATE** options along with the **VAR** statement, then the analysis will still run, but SAS will use its default options of retaining the number of factors identified by the Kaiser Criterion, performing PCA extraction, not conducting rotation, and using all of the variables in the data set. Thus, it is best practice to get into the habit of specifying all of the options, even if you choose to use one of the methods that is a default, so that you do not accidentally overlook a key component of your analysis.

Let's take a minute to quickly review the structure of the syntax above. Note that there is a semicolon on the end of some lines and not others. The placement of the semicolon is of insurmountable importance in SAS—if it is in the wrong place your code will not run and you will get errors galore! The semicolon signals the end of a *statement* and tells SAS how to parse what we are requesting. In the above syntax, we have a **PROC FACTOR** statement, a **VAR** statement, and a **RUN** statement. You will notice that there are some other key terms in the statement that allow users to specify the inputs for our analysis (e.g., **DATA =**, **NFACTORS =**). These are referred to as *options* within the statement. Finally, notice that the syntax starts with **PROC FACTOR**. This tells SAS that we are using the **FACTOR** procedure, and the following statements and options will specify the details of the analysis we would like to conduct. We will use this terminology throughout this book.

We hope most of what was just described was a review. If it was not, we recommend you first review some other excellent introductory texts to using SAS (e.g., Cody, 2007; Delwiche & Slaughter, 2012) before attempting to follow the syntax in this book. It is expected that readers have an understanding of basic data manipulation, the **DATA** step, and at least a few basic procedures (e.g., **MEANS**, **SORT**, **FREQ**). In addition, readers are encouraged to refer to the SAS documentation about **PROC FACTOR** for additional information throughout this book (SAS Institute Inc., 2015).

Summary

The goal of this book is to be a friendly, practical, applied overview of best practices in EFA, with applications to SAS. In the coming chapters, we will explore various aspects of EFA, and the best practices at each step. We will skip the matrix algebra and equations as much as possible. If you are seeking a more mathematical tome on the subject, there are many good ones already available.

Throughout the book, we describe and explain the SAS procedures for each analysis. We also review key syntax for our example analyses. The complete syntax for all of the analyses conducted throughout the book, the data sets that are used, and the solutions to exercises are available on the author's page at http://support.sas.com/publishing/authors.

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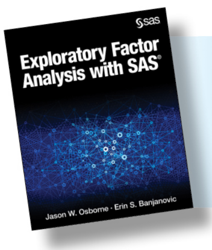

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About This Book

Purpose

This book is designed to present a modern overview of best practices in exploratory factor analysis. We present these concepts from an applied, reader-friendly perspective, guiding you through many different aspects of factor analysis and presenting evidence-based recommendations about how to get the most from your analysis (and how to present it to others clearly).

Is This Book for You?

Factor analysis is used in many disciplines, for many purposes. If you use SAS and also use factor analysis, this book will help you choose options that are defensible and help you meet your goals. If you have never used factor analysis and want to learn, this book is also written for you! We guide you through concepts and their application gently, using minimal technical jargon, and we suggest ways to think about the results that reflect modern practices.

Prerequisites

We wrote this book expecting readers to have a basic working knowledge of SAS and basic knowledge of statistics fundamentals. We do *not* assume you can do matrix algebra in your sleep, that you already know what an eigenvalue is, or that you know the binary language of moisture vaporators. We do, however, assume that most readers understand basic data manipulation in SAS, the DATA step, and at least a few procedures (e.g., MEANS, SORT, FREQ).

Scope of This Book

This book is designed to cover everything you need to know about exploratory factor analysis, and some closely related constructs such as reliability and replicability. Our goal was to stay focused on this one topic, as it is complex and confusing, with many options and steps. We chose, therefore, not to cover related topics like confirmatory factor analysis, construction and validation of scales, latent variable modeling such as structural equation modeling, and other measurement methodologies (for example, item response theory or Rasch measurement).

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SAS 9.4 was used for this book.

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Additional Resources

Although this book illustrates many analyses regularly performed in businesses across industries, questions specific to your aims and issues might arise. To fully support you, SAS Institute and SAS Press offer you the following help resources:

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