



From *Bayesian Analysis of Item Response Theory Models Using SAS*<sup>®</sup>. Full book available for purchase [here](#).

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