Internal Draft Document:
Standardized Solution in CALIS, EQS, and LISREL

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Using the LINEQS or RAM input form of CALIS, standardized coefficients and standardized covariances (correlations) among the exogenous variables can be printed. The scaling in CALIS is done corresponding to the Bentler-Weeks model (see LINEQS model in Appendix A of the *CALIS: Extended User's Guide*).

The Bentler-Weeks Model

$$\eta = \beta\eta + \gamma\xi \quad ,$$

η : vector of endogenous variables,
ξ : vector of exogenous and error variables,
β : nonsingular coefficient matrix among endogenous variables,
γ : coefficients between endogenous and exogenous variables.

Variables of η and ξ can be manifest (observed) or latent (not observed) variables.

The covariance matrix C of the manifest variables in η and ξ is then:

$$C = J(I - B)^{-1}, \Phi, {}^T(I - B)^{-T} J^T \quad ,$$
$$J : \text{selection matrix}, \quad \Phi = \mathcal{E}\{\xi\xi^T\},$$

$$B = \begin{pmatrix} \beta & 0 \\ 0 & 0 \end{pmatrix} \qquad \text{and} \quad , = \begin{pmatrix} \gamma \\ I \end{pmatrix} \quad .$$

That means the standardized solutions of the LINEQS and RAM model of CALIS and the one computed by the EQS program (Bentler, 1989) are the same. However, this standardized solution is different from the one computed by the LISREL 7 program (Jöreskog, K.G. & Sörbom, D., 1988) for the Keesling-Wiley-Jöreskog model.

The Keesling-Wiley-Jöreskog Model

The general nonrecursive LISREL model assumes linear relationship among the latent variables

$$\eta = B\eta + \,,\ \xi + \zeta$$

$\eta_1, \ldots, \eta_m$: endogeneous (latent) variables,
$\xi_1, \ldots, \xi_n$: predetermined (latent) variables,
$\zeta_1, \ldots, \zeta_m$: disturbance terms,

where B is an m by m and , is an m by n coefficient matrix; and assumes also

$$\mathcal{E}\{\xi\zeta^T\} = 0, \quad \mathcal{E}\{\xi\} = 0.$$

It also assumes a linear relationship between manifest indicators and latent constructs

$$y = \Lambda_y \eta + \epsilon$$
$$x = \Lambda_x \xi + \delta$$

$y_1, \ldots, y_p$: manifest indicators of endogeneous variables,
$x_1, \ldots, x_q$: manifest indicators of predetermined variables,
$\epsilon_1, \ldots, \epsilon_p$: disturbance terms,
$\delta_1, \ldots, \delta_m$: disturbance terms,

where Λ_y is an p by m and Λ_x is an q by n coefficient matrix and assuming (as in factor analysis)

$$\mathcal{E}\{\epsilon\} = 0, \quad \mathcal{E}\{\delta\} = 0, \quad \mathcal{E}\{\eta\epsilon^T\} = 0, \quad \mathcal{E}\{\xi\delta^T\} = 0, \quad \mathcal{E}\{\eta\} = 0, \quad \mathcal{E}\{\xi\} = 0.$$

The covariance matrix C of the p + q manifest variables y and x is then:

$$C = J(I-A)^{-1}P(I-A)^{-T}J^T \quad ,$$

$$A = \begin{pmatrix} 0 & 0 & \Lambda_y & 0 \\ 0 & 0 & 0 & \Lambda_x \\ 0 & 0 & B & , \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \text{and} \quad P = \begin{pmatrix} \Theta_\epsilon & & & \\ & \Theta_\delta & & \\ & & \Psi & \\ & & & \Phi \end{pmatrix} \quad ,$$

with selection matrix J, $\Phi = \mathcal{E}\{\xi\xi^T\}$, $\Psi = \mathcal{E}\{\zeta\zeta^T\}$, $\Theta_\delta = \mathcal{E}\{\delta\delta^T\}$, and $\Theta_\epsilon = \mathcal{E}\{\epsilon\epsilon^T\}$.

Standardized Solution for the Keesling-Wiley-Jöreskog Model

The formulas for the standardized solution computed by the LISREL 7 program are shown at page 38 of its manual (Jöreskog, K.G. & Sörbom, D., 1988):

$$\Lambda_y^* = \Lambda_y D_\eta$$
$$\Lambda_x^* = \Lambda_x D_\xi$$
$$B^* = D_\eta^{-1} B D_\eta$$
$$,^* = D_\eta^{-1} , D_\xi$$
$$\Phi^* = D_\xi^{-1} \Phi D_\xi^{-1}$$
$$\Psi^* = D_\eta^{-1} \Psi D_\eta^{-1}$$
$$\Theta_\epsilon^* = \Theta_\epsilon$$
$$\Theta_\delta^* = \Theta_\delta$$

with

$$D_\eta = diag((I - B)^{-1}(, \Phi, ^T + \Psi)(I - B)^{-T})^{1/2}$$
$$D_\xi = diag(\Phi)^{1/2}$$

Standardized Solution for the Bentler-Weeks Model

The formulas for the standardized solution of the Bentler-Weeks model can be written in corresponding notation:

$$B^* = D_\eta^{-1} B D_\eta$$
$$,^* = D_\eta^{-1} , D_\xi$$
$$\Phi^* = D_\xi^{-1} \Phi D_\xi^{-1}$$

with

$$D_\eta = diag((I - B)^{-1} , \Phi, ^T (I - B)^{-T})^{1/2}$$
$$D_\xi = diag(\Phi)^{1/2}$$

Since the variables ζ, ϵ, and δ of the Keesling-Wiley-Jöreskog model do not have coefficient matrices, LISREL cannot standardize "measured variables, errors in variables, or disturbances in equations" (Bentler, 1989, p.98). We implemented the Bentler-Weeks type of standardization in CALIS since we think it is more appropriate than that of the LISREL implementation. The *Equations with Standardized Coefficients* are the same for both the correlation matrix and the covariance matrix, so the manifest variables are being standardized as well as the latent variables, supporting the fact that the term "standardized" usually refers to the manifest variables.

Stability and Alienation Example

The following example is the example discussed on page 37 of the EQS (Version 3.0) Manual (Bentler, 1989):

```
DATA WHEATON(TYPE=COV);
   _TYPE_ = 'COV'; INPUT _NAME_ $ V1-V6;
   LABEL V1='Anomia (1967)' V2='Powerlessness (1967)'
         V3='Anomia (1971)' V4='Powerlessness (1971)'
         V5='Education' V6='Occupational Status Index';
   CARDS;
V1   11.834     .         .         .         .        .
V2    6.947    9.364      .         .         .        .
V3    6.819    5.091   12.532       .         .        .
V4    4.783    5.028    7.495    9.986        .        .
V5   -3.839   -3.889   -3.841   -3.625    9.610        .
V6  -21.899  -18.831  -21.748  -18.775  35.522  450.288
  ;


PROC CALIS DATA=WHEATON COV EDF=931 ALL NOMOD;
 LINEQS
      V1 =        F1                  + E1,
      V2 =    .833 F1                 + E2,
      V3 =        F2                  + E3,
      V4 =    .833 F2                 + E4,
      V5 =        F3                  + E5,
      V6 = LAMB (.5) F3               + E6,
      F1 = GAM1(-.5) F3               + D1,
      F2 = BETA (.5) F1 + GAM2(-.5) F3 + D2;
 STD
      E1-E6 = THE1-THE2 THE1-THE4 (6 * 3.),
      D1-D2 = PSI1-PSI2 (2 * 4.),
      F3    = PHI (6.) ;
 COV
      E1 E3 = THE5 (.2),
      E4 E2 = THE5 (.2);
RUN;
```

The following ML estimates agree with those obtained by LISREL 7 and EQS 3:

```
                    Manifest Variable Equations

        V1      =     1.0000 F1   +  1.0000 E1

        V2      =     0.8330 F1   +  1.0000 E2

        V3      =     1.0000 F2   +  1.0000 E3

        V4      =     0.8330 F2   +  1.0000 E4

        V5      =     1.0000 F3   +  1.0000 E5

        V6      =     5.3641*F3   +  1.0000 E6
        Std Err       0.4334 LAMB
        t Value      12.3765


                     Latent Variable Equations

        F1      = -   0.6299*F3   +  1.0000 D1
        Std Err       0.0563 GAM1
        t Value     -11.1815

        F2      =     0.5932*F1   -  0.2405*F3   +  1.0000 D2
        Std Err       0.0468 BETA    0.0549 GAM2
        t Value      12.6769        -4.3825
```

Variances of Exogenous Variables

Variable	Parameter	Estimate	Standard Error	t Value
F3	PHI	6.621480	0.639495	10.354
E1	THE1	3.611932	0.200995	17.970
E2	THE2	3.591855	0.164428	21.845
E3	THE1	3.611932	0.200995	17.970
E4	THE2	3.591855	0.164428	21.845
E5	THE3	2.988520	0.498933	5.990
E6	THE4	259.745900	18.306936	14.188
D1	PSI1	5.668593	0.422996	13.401
D2	PSI2	4.515138	0.335373	13.463

Covariances among Exogenous Variables

Parameter			Estimate	Standard Error	t Value
E3	E1	THE5	0.905179	0.121660	7.440
E4	E2	THE5	0.905179	0.121660	7.440

The standardized solution consists of the *Equations with Standardized Coefficients* and the Correlations among Exogenous Variables. Since the variances of the exogenous variables are equal 1 these are not printed. The standardized solution obtained by CALIS is the same as obtained by EQS (see page 37 of the EQS 3.0 Manual) but differs from that of LISREL.

Equations with Standardized Coefficients

```
V1       =    0.8347 F1    + 0.5507 E1

V2       =    0.7847 F1    + 0.6199 E2

V3       =    0.8449 F2    + 0.5350 E3

V4       =    0.7969 F2    + 0.6041 E4

V5       =    0.8301 F3    + 0.5577 E5

V6       =    0.6505*F3    + 0.7595 E6
                     LAMB

F1       =  - 0.5628*F3    + 0.8266 D1
                     GAM1

F2       =    0.5693*F1    - 0.2062*F3    + 0.7080 D2
                     BETA          GAM2
```

Correlations among Exogenous Variables

Parameter			Estimate
E3	E1	THE5	0.250608
E4	E2	THE5	0.252009

References

[1] Bentler, P.M. (1989): *EQS, Structural Equations, Program Manual*, Program Version 3.0, Los Angeles: BMDP Statistical Software, Inc.

[2] Bentler, P.M. and Weeks, D.G. (1982), "Multivariate Analysis with Latent Variables", in *Handbook of Statistics, Vol. 2*, ed. P.R. Krishnaiah and L.N. Kanal, North Holland Publishing Company.

[3] Jöreskog, K.G. (1973), "A general method for estimating a linear structural equation system", in *Structural Equation Models in the Social Sciences*, eds. A.S. Goldberger & O.D. Duncan, New York: Academic Press.

[4] Jöreskog, K.G. and Sörbom, D. (1988), *LISREL 7: A Guide to the Program and Applications*, SPSS Inc., Chicago, Illinois.

[5] Keesling, J.W. (1972), "Maximum Likelihood Approaches to Causal Analysis", Ph.D. diss., Chicago, 1972.

[6] Wiley, D.E. (1973), "The Identification Problem for Structural Equation Models with Unmeasured Variables", in *Structural Equation Models in the Social Sciences*, eds. A.S. Goldberger and O.D. Duncan, New York: Academic Press.