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Get Tangency Portfolio by SAS/IML

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USERS PROGRAM



ABSTRACT

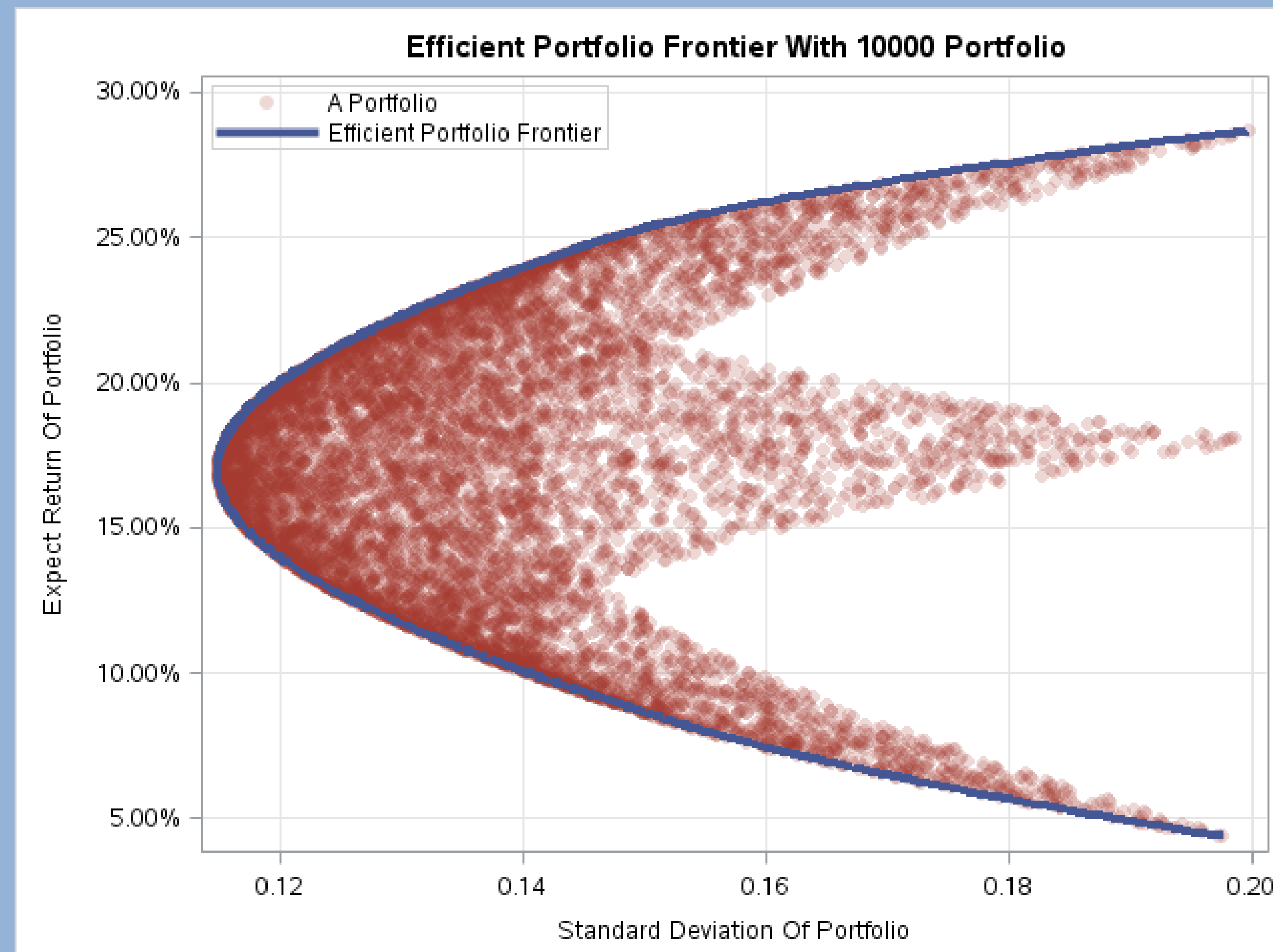
- Mean-Variance Model (Modern portfolio theory) maybe the most famous model in financial field.
- It assesses a portfolio which's the expected return(mean) is maximized under a given risk(variance). It comes from assumption that investor want as high as return while as low as risk as he could when he invested a couple of assets(a portfolio is the collection of many assets).This model could give us the many optimal portfolio(efficient portfolio frontier) when we know every asset's expect return and their covariance matrix. The tangency portfolio is one of such kind of optimal portfolio after we included a riskless rate(a long-term government bond rate), which means it has the maximal expect return (mean) and the minimal risk (variance) among all the optimal portfolios.
- This paper is trying to simulate some dummy data and get this tangency portfolio by IML code.

INTRODUCTION

- Suppose we have a portfolio which contains N assets and they are not independent. Each asset's expected return is r_i and standard deviation of the return is σ_i . Due to they are not independent, covariance between any two assets return is $COV(r_i, r_j) = \sigma_{ij}$.
- Now we need to calculate the mean and covariance of this portfolio :
(Suppose the percent of money invest into asset i is x_i , so $\sum x_i = 1$)
The expected return (mean) of this portfolio is : $\sum x_i * r_i$
The risk (variance) of this portfolio is : $X * COV * X'$
(Note: * in variance is matrix multiply operator unlike * in mean, X is row vector which represent x_i , X' is column vector – the transpose of X, COV is the covariance matrix of the N asset's return)
- Take for an example of two assets:
The expected return (mean) of the portfolio is : $x_1 * r_1 + x_2 * r_2$, $x_1 + x_2 = 1$
The variance of the portfolio is : $x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2 x_1 x_2 \sigma_{12}$, σ_1 is the standard deviation of asset one, σ_2 is the standard deviation of asset two, σ_{12} is the covariance of asset one and asset two.

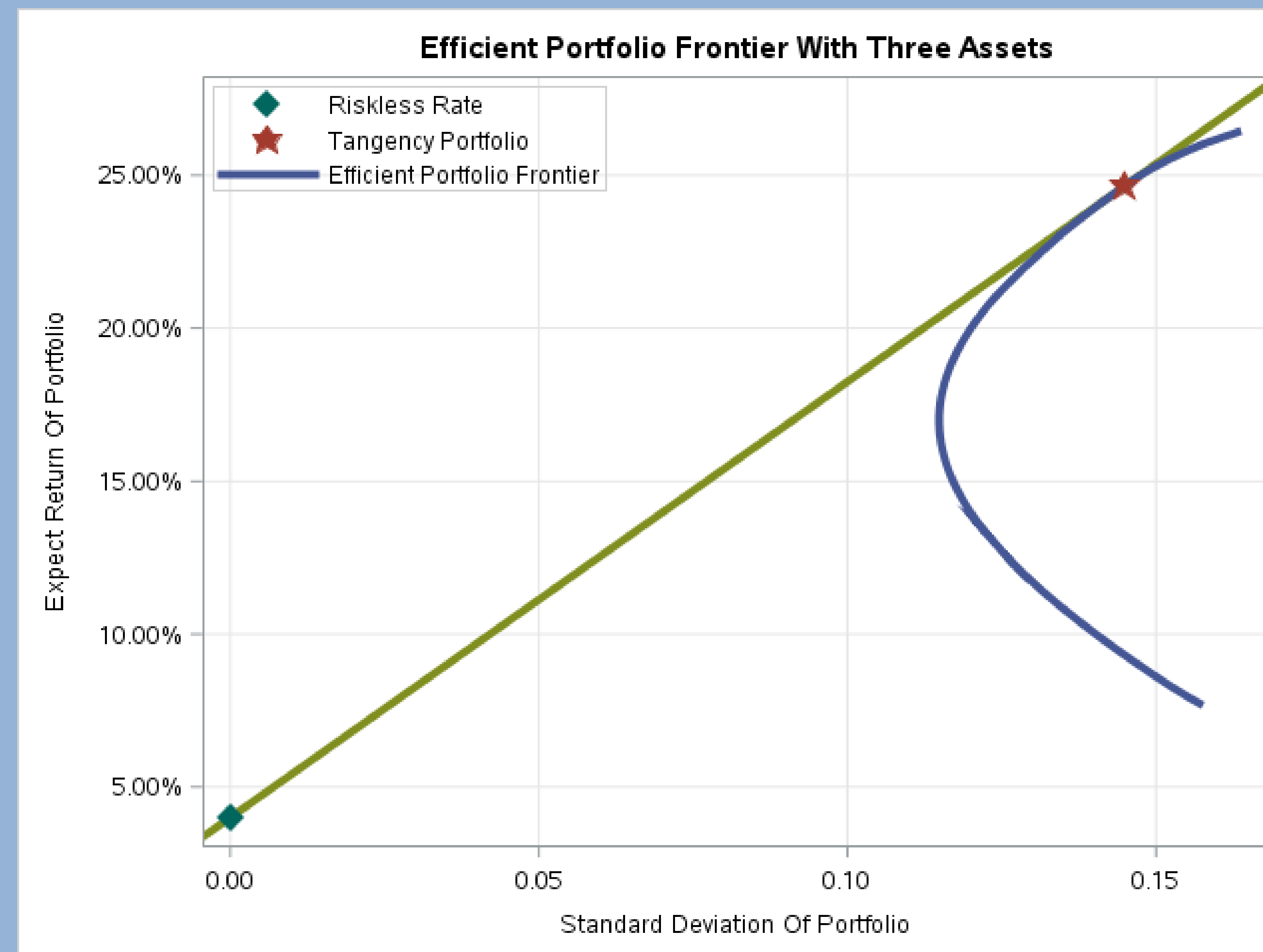
INTRODUCTION

- If we generate a lot of portfolios (i.e. a lot of x_i , $\sum x_i = 1$) and get its mean and variance, plot them into a graph, we can get the following picture. There is a frontier curve, which is a hyperbola. Any points on frontier curve represent a best portfolio, because for any variance (risk) of portfolio, we can find a point on frontier curve which can get the maximal expected return (mean) of portfolio.



INTRODUCTION

- How can we get tangency portfolio? Now we need a riskless rate (government long-term bond rate) r_f which have 0 variance. Tangency portfolio is a point on frontier curve, this point and r_f form a line which is tangency line to frontier curve. As long as we can get the line which has the maximal slope, then we can say we find this tangency portfolio. As the following picture showed:



EXAMPLE

- Suppose we have three assets (oil bond stock), if we already have calculated their expect return and covariance matrix, what percent of one dollar should invest into oil, bond or stock? And we want this portfolio's expect return as maximal, and risk as minimal as it could be. That leads to a tangency line (to maximize the slope) which goes through the riskless rate (government long-term bond rate) and the tangency portfolio which is on the Efficient Portfolio Frontier. It looks like the previous picture.

- First simulate some dummy data:

```
/* the number of assets - a portfolio contains N assets */  
n=3;  
/* riskless rate - government long-term bond rate */  
f=0.04 ;  
  
/* simulated data - the return of N assets */  
asset=j(10000,n,.);  
call randseed(1234);  
call randgen(asset,'normal',0,0.2);
```

EXAMPLE

- Calculated the expected return and covariance of these three assets:

```
/* the expect return of N assets */  
return=j(1,n,.);  
call randgen(return,'uniform');
```

```
/* the covariance matrix of N assets */  
cov=cov(asset);
```

- Define a function to maximize the slope :

```
/* get the tangency portfolio - maximize slope of line*/  
start max_slope(x) global(f,cov,return);  
  portfolio_return=sum(return#x);  
  portfolio_std=sqrt(x*cov*x`);  
/*calculate slope - riskless rate's std is zero */  
  k=(portfolio_return-f)/portfolio_std;  
return (k);  
finish;
```

EXAMPLE

- Use CALL NLPNRA() to resolve the function to get the maximal slope, the constraint condition is:
 $x_1+x_2+x_3=1$, $0 \leq x_1 \leq 1$, $0 \leq x_2 \leq 1$, $0 \leq x_3 \leq 1$.

```
/* constraint conditions - x1+x2...=1 and 0<=x1<=1,0<=x2<=1,...*/
con=repeat({0,1,1},1,n)||{. ., .,0 1};
/* initial value of x1,x2,...*/
x=j(1,n,1/n);
/* maximize object function value*/
optn={1 1};
/* xres is a solution if rc>0 */
call nlpnra(rc,xres,"max_slope",x,optn,con);
/* print the solution if rc>0*/
print rc[l='Return Code'],xres[l='It is a Solution if Return Code > 0'];
```

- **Print the solution.**

```
ABSGCONV convergence criterion satisfied.
Return Code 3
It is a Solution if Return Code > 0
0.6293588  0.361734  0.0089071
```

It is convergent, so got the tangency portfolio (62.9% should invest into oil, 36.1% should invest into bond, and 0.9% should invest into stock).



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