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Modeling Actuarial Risk using SAS® Enterprise Guide®: a Study on Mortality tables and Interest Rates

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ABSTRACT

This paper has the objective to present a methodology for interest rates, life tables and actuarial calculations using generational mortality tables and forward structure of interest rate for pension funds for making an analysis of long-term actuarial projections and their impacts on the actuarial liability. It was developed a computational algorithm in SAS® Enterprise Guide® and SAS Base for structuring the actuarial projections and then it was analysed the impacts of this new methodology. There was heavy use of PROC IML and SQL.

Keywords: Pension funds. Mortality Tables. Interest Rates. Mathematical Reserves. Actuarial Risk.

INTRODUCTION

The purpose of this paper is to present an approach to interest rates and mortality tables and a new way of measuring the actuarial obligations in pension plans, with the values of the impact of this implementation.

It is clear that the Supervisory Body wanted to align the convergence of the practices of Risk Based Supervision with regard to actuarial risk. Therefore, the objective of this work is to discuss the following points using as statistical support the SAS Enterprise Guide Statistical Software:

- The Lee-Carter model (LEE and CARTER, 1992) as an alternative for calculating the improvement of the central rate of mortality of a population and consequently obtaining the generational probabilities that vary over the years.
- The redefinition of the discount rate to be used in the pricing of the actuarial liabilities, changing from the current practice of adopting a fixed real rate of 6% for the pricing of actuarial liabilities at market value (thus allowing for real variable interest rates over time).
- The impact of adopting these methodologies and assumptions on actuarial liabilities with the population experience of a large pension fund.

MORTALITY TABLE

THE LEE CARTER METHOD

We present the Lee-Carter model for the calculation of the improvement of the central rate of a population and thus reach the generational probabilities of survival, i.e., probabilities that vary from year to year, thus capturing the trend of longevity gain that Brazilian population has been passing. American scholars Ronald Lee and Lawrence Carter developed this method of predicting mortality in 1992. Through the recorded mortality rates of the American population between the years 1933 and 1989, they predicted the evolution of the rates until the year 2065. In this work, Lee and Carter verified the evolution of the life expectancy of American population that increased from forty-seven years at the beginning of the century to seventy-five years in the year 1988. Using their method they found that by 2065 life expectancy could reach the incredible mark of 100 years. The Lee-Carter method is now one of the most widely used methods for estimating mortality in a population and several scholars have already applied the method to predict mortality trends in different countries. As an example, we mention Canada (LEE and NAULT, 1993), Chile (LEE and ROFMAN, 1994), Japan (WILMOTH, 1996), Brazil (FÍGOLI, 1998), Austria (CARTER and PRSKWETZ, 2001) among other countries. This method can be studied more deeply in (SANTOS and DUARTE, 2009). Our objective is not to deepen the study of the method, but to present its application in practice with the help of statistical software SAS Enterprise Guide.

The Lee-Carter model is one of the most influential studies in predicting mortality. It is based on historical experiences of mortality and makes no consideration for the inclusion of information regarding the advancement of medicine, behavioral changes or social influences that affect the behavior of mortality.

A positive point of the method is that it combines a demographic model with a time series model, allowing probabilistic intervals to be obtained for the respective forecasts. The method also allows for mortality rates to decline exponentially without setting an arbitrary threshold, or without any need to rationalize the eventual deceleration of life expectancy gains.

Let be the central rate of mortality for year *t* be adjusted by the matrix of mortality rates by the model:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t} \text{ ,with } \varepsilon_{x,t} \sim N(0,\sigma^2)$$

Where α_x , β_x and κ_t are parameters of the model and $\varepsilon_{x,t}$ is an error term with normal distribution of zero mean and variance constants equals to σ^2.

The first part of the adjustment consists in estimating the parameters α_x, β_x and κ_t. To obtain a single solution, the constraints are applied that the sum of the coefficients β_x is equal to 1 and the sum of the parameters κ_t is equal to 0. Under these hypotheses the coefficients must be simply the mean values in time of ln ($m_{x,t}$) for each x.

$$\alpha_x = \frac{\sum_{t=1}^{n} \ln(m_{x,t})}{n}$$

Where *n* is the total of years between the first year and the last year used in the estimation.

In addition, κ_t should be equal to the sum over the ages of the logarithms of the mortality rates, subtracted the coefficients α_x.

$$\kappa_t = \sum_x (\ln(m_{x,t}) - \alpha_x)$$

The estimation of the coefficients β_x is made by the equation:

$$\ln(m_{x,t}) - \alpha_x = \beta_x \kappa_t^1 + \varepsilon_{x,t}$$

Where κ_t^1 refers to previously estimated coefficients κ_t by least squares method. In short, we chose β_x to minimize:

$$\sum_{x,t} (\ln(m_{x,t}) - \alpha_x - \beta_x \kappa_t^*)^2 \rightarrow \beta_x = \frac{\sum_{t=t1}^{tn} \kappa_t^* (\ln(m_{x,t}) - \alpha_x)}{\sum_{t=t1}^{tn} \kappa_t^{*2}}$$

The second part of the fit of the Lee-Carter model is to re-estimate the parameter κ_t based on already estimated values α_x and β_x. In this step of the estimation of κ_t, we look for a parameter such that:

$$d_t = \sum_x \{\exp(\alpha_x + \beta_x \kappa_t) e_{x,t}\}$$

Where d_t is the total number of deaths observed in year *t* and $e_{x,t}$ corresponds to the population of age x at time *t*.

After we get the values of α_x, β_x and κ_t, the next step is to predict the values of κ_t for future years. According to the Lee-Carter model, we predict the values κ_t by selecting an ARIMA model (p, d, q). With the expected values of κ_t, we can describe the value of $m_{x,t}$ in term of its improvement factor (F.I), as indicated below:

Between 0 and t:

$$FI(x,t) = \exp(\beta_x(\kappa_t - \kappa_0))$$

Between t -1 and t:

$$FI(x,t) = \exp(\beta_x(\kappa_t - \kappa_{t-1}))$$

Thus, we find the value of $m_{x,t}$ and of q_x, t through the equations:

$$m_{x,t} = m_{x,0} \cdot FI(x,t)$$

$$q_{x,t} = q_{x,0} \cdot FI(x,t)$$

In CALCULATION THROUGH SAS® ENTERPRISE GUIDE® STATISTICAL SOFTWARE, the parameters of the Lee-Carter model will be estimated based on the values of the North American male and female central rates of mortality between 1933 and 2005 obtained from the Human Mortality Database (www.mortality.org), using the American Board AT-2000, as the base board for projection, since we used the mortality data of the American population as a data base for the application of the method. In this way, the two populations are basically the same. With the estimation of the parameters, we arrive at the estimation of the improvement factors, which are applied to the probabilities to obtain the projected mortality table over 30 years.

INTEREST RATE AGREEMENT STRUCTURE

Another focus of Brazilian Supervisor's concern with the mapping of actuarial risks is the trend of reducing interest rates in Brazil in the last decade.

A fixed income security is a government, municipal or private liability that generates a pre-established payment flow. These are representative securities of loans contracted by companies or governments, which promise to pay investors certain future flow of income.
The market value of a security is represented as the present value of future cash flows:

$$VP = \sum_{j=1}^{k} (1 + \hat{R}^t_{t,t+\tau_j})^{(-\tau_j/252)} . F_j$$

Where:

- VP represents the expected present value of cash flows.
- k, the number of payments of the contract
- Fj, the j-th payment of the contract.
- τ_j is the term on working days in which the j-th payment of this title occurs.
- $R^t_{t,t+\tau_j}$ is the annual effective rate, based on 252 business days, in t for the period between t and t + $\tau_{j,i}$

INTERPOLATION AND EXTRAPOLATION OF THE TERM STRUCTURE OF THE INTEREST RATE

In Brazil, the Insurance Supervisor through Franklin *et al* (2011) guided the insurance market to use the model of Nelson and Siegel (1987) and Svensson (1994). For the estimation of the parameters, the article proposes a traditional nonlinear optimization algorithm and a genetic algorithm developed for this purpose.

Nelson and Siegel Model (1987)

The model establishes a functional form of 4 parameters that seeks to approximate the curve of the forward rates by a sum of exponential functions. The forward rates are given by the equation:

$$f_t(\tau) = \beta_{0,t} + \beta_{1,t}.e^{-\lambda_t.\tau} + \beta_{2,t}.\lambda_t.\tau.e^{-\lambda_t.\tau}$$

And spot rates can be calculated by:

$$y_t(\tau) = \beta_{0,t} + \beta_{1,t}.(\frac{1-e^{-\lambda_t.\tau}}{\lambda_t\tau}) + \beta_{2,t}.(\frac{1-e^{-\lambda_t.\tau}}{\lambda_t.\tau} - e^{-\lambda_t.\tau})$$

Where λ_t is the parameter that measures the decay velocity of the ETTJ and the parameters $\beta_{0,t}$, $\beta_{1,t}$ and $\beta_{2,t}$ can be interpreted as long, short and medium term latent dynamic factors, respectively. In addition, the parameters shall satisfy the following restrictions:

$$\beta_{0,t} > 0$$
$$\beta_{0,t} + \beta_{1,t} > 0$$

Figure 1

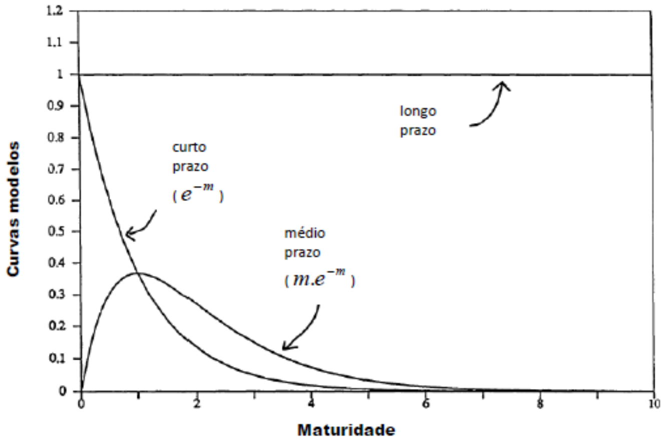

Figure 1 Components of the forward rate curve (Nelson and Siegel, 1987)

Svensson Model (1994)

The Svensson model extends to the Nelson and Siegel model with the addition of a new term exponential to the forward rate curve, containing 2 additional parameters:

$$f_t(\tau) = \beta_{0,t} + \beta_{1,t}.e^{-\lambda_{1,t}.\tau} + \beta_{2,t}.\lambda_{1,t}.\tau.e^{-\lambda_{1,t}.\tau} + \beta_{3,t}.\lambda_{2,t}.\tau.e^{-\lambda_{2,t}.\tau}$$

Besides, spot rates can be calculated by:

$$y_t(\tau) = \beta_{0,t} + \beta_{1,t}.(\frac{1-e^{-\lambda_{1,t}.\tau}}{\lambda_{1,t}.\tau}) + \beta_{2,t}.(\frac{1-e^{-\lambda_{1,t}.\tau}}{\lambda_{1,t}.\tau} - e^{-\lambda_{1,t}.\tau}) + \beta_{3,t}.(\frac{1-e^{-\lambda_{2,t}.\tau}}{\lambda_{2,t}.\tau} - e^{-\lambda_{2,t}.\tau})$$

Where $\beta_{0,t}$, $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ can be interpreted as latent long, short and the last two medium term dynamic factors, respectively. In fact, Svensson model becomes identical to the model of Nelson and Siegel when $\beta_{3,t} = 0$ or $\lambda_{1,t} = \lambda_{2,t}$.

In addition, the parameters of the Svensson model must follow the following constraints:

$$\{\lambda_{1,t} > 0; \lambda_{2,t} > 0; \beta_{0,t} > 0; \beta_{0,t} + \beta_{1,t} > 0\}$$

EXTRAPOLATION

Pension fund commitments generally have long-term characteristics, with terms that are longer than the longer term government bonds currently offered in the market. In this work, the extrapolation is done only by repeating the interest rate value in the last maturity, considering the linearity observed in the last maturities of the proposed ETTJ.

MATHEMATICAL RESERVE IN CLOSED SUPPLEMENTARY PENSION

$$V = \sum_{t=c}^{f} (probben(t) \times vpben(t) + \sum_{i=1}^{n} probdesp_i(t) \times desp_i(t) - probprem(t) \times prem(t)) \times desc(t)$$

Where:

- c - Initial period of flow of future commitments.
- f - Final period of the flow of future commitments.
- n - Quantity of expenses to be considered.
- $probben(t)$ - Probability of a claim occurring at the time t.
- $vpben(t)$ - Present value at the time of the claims generated at the time t.
- $desp_i(t)$ - Value of expenses with the i-th general expense (administrative expenses, taxes, financial, profit margin, etc.) at the time t.
- $probdesp_i(t)$ - Probability of the i-th general expense to be paid at the time t.
- $prem(t)$ - Amount of the premium to be paid at the time t.
- $probprem(t)$ - Probability of the premium to be paid at the time t.
- $desc(t)$ - Factor of financial discount to bring the values of the time t to the time in which the mathematical provision is calculated.

The formula above is generic and can be used both to calculate the pure provision and to calculate the charged provision. Besides, it can be applied to any type of long-term coverage (survival, death, disability, health, etc.).

This methodology is based on the calculation of the impact of the new assumptions proposed by SBR and is presented in the previous chapters for actuarial risk.

However, the actuarial switching resource will not be used. This is because, as the mortality probabilities are variable over the years, this disables using commutations to a fixed board. Thus, the core formula for bringing actuarial present value to the obligations is as follows:

$${}_nE_x = {}_np_x \times v^n$$

Where:

- ${}_np_x$ - Probability of survival of an individual of age *x* for *n* years.
- V^n - Financial discount factor.

In practice, where the financial discount factor V^n is given by the formula proposed by the Brazilian Insurance Supervisor in the practical example of utilization of ETTJ:

$$VP = \sum_{j=1}^{k} (1 + \hat{R}^t_{t,t+\tau_j})^{(-\tau_j/252)} . F_j$$

Where:

- VP represents the expected present value of cash flows.
- K, the number of payments of the contract.
- Fj, the j-th payment of the contract.
- τ_j is the term on working days in which the j-th payment of this title occurs;

- $R^{t}_{t,t+\tau_j}$ is the annual effective rate, based on 252 business days, in *t* for the period between *t* and *t* + $\tau_{j,i}$.

In this methodology, ETTJ maturities are used to mark maturities and discount the flows.

The probabilities ${}_n p_x$ is computed year by year, that is, in each year of the actuarial projection there is a certain probability of survival thus categorizing the generational actuarial calculation.

CALCULATION THROUGH SAS® ENTERPRISE GUIDE® STATISTICAL SOFTWARE

Through the Licensing Agreement signed between SAS® Institute Brasil LTDA and the author of this paper, it was possible to develop a computational solution for the estimation of the mortality table by Lee-Carter methodology and also for calculating the actuarial obligations with generational tables and ETTJ to discount the flows. The License Agreement is in possession with the author and available to be read. The technical bases of the calculation are as follows:

- Projection of 30 years (maturity date of chosen ETTJ).
- Only calculation of Mathematical Provision of Benefits Granted.
- AT-2000 Table was used for the estimation base using the methodology proposed in LEE and CARTER (1992)
- Term Structure of Interest Rate: Zero Coupon Curve (IPCA), SUSEP position at December 12, 2011.
- For comparison purposes, mathematical provisions were also calculated using the AT 83, AT 2000 and RP 2000 mortality tables and at constant interest rates of 6% per year.

DATABASE ANALYSIS

The following is a brief analysis of the Database used in the calculation:

- Source: Retirement database of a large Brazilian Pension Fund

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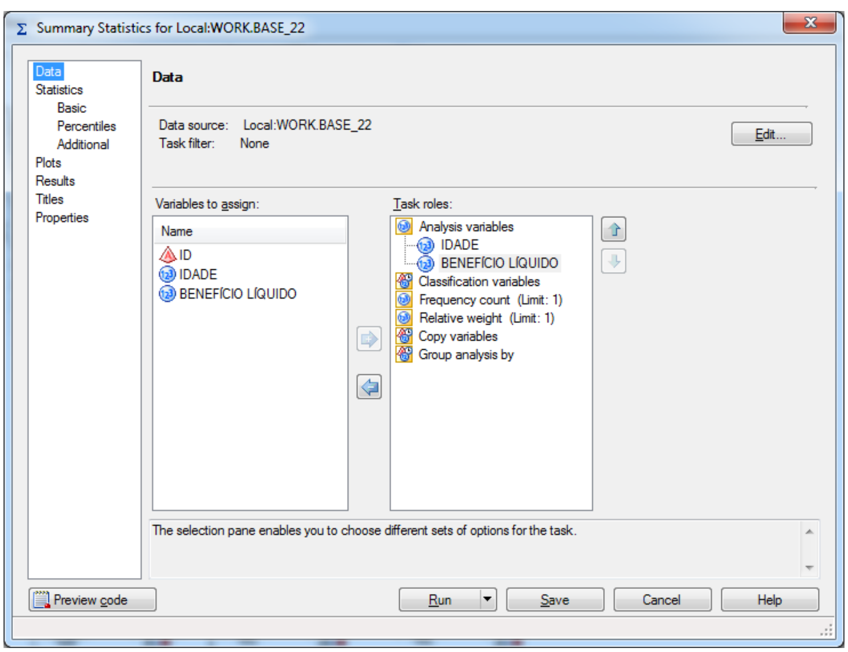

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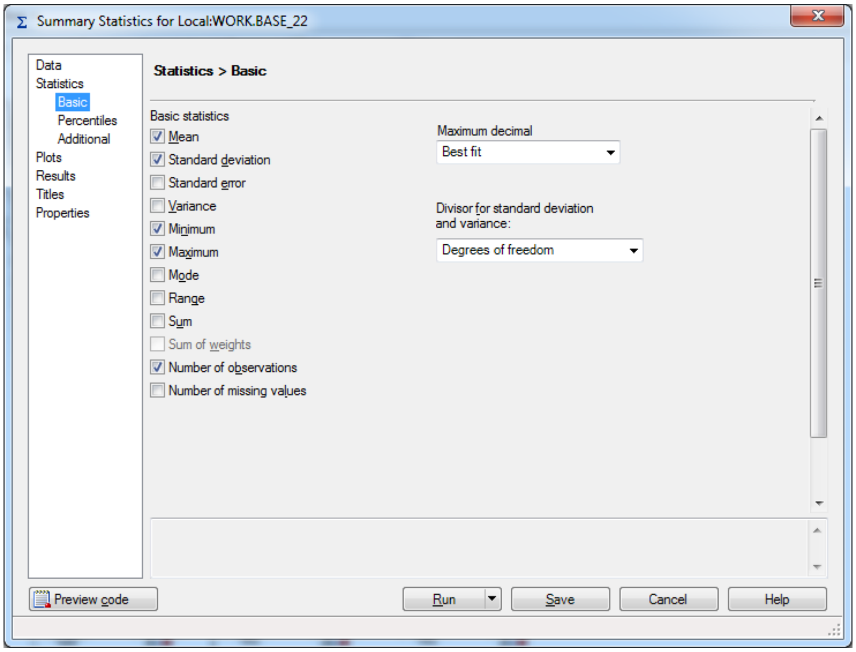

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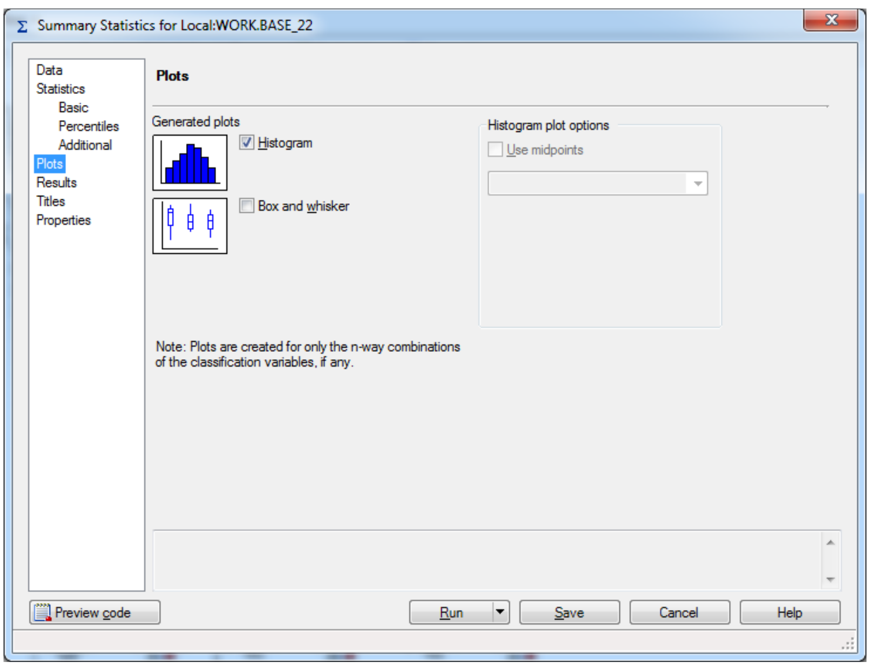

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Figure 2

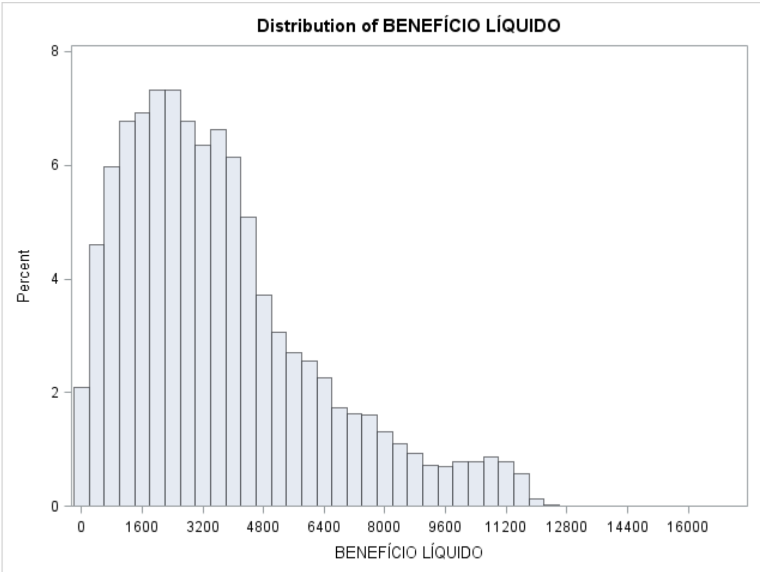

Figure 2 Distribution of Supplemental Benefit

Figure 3

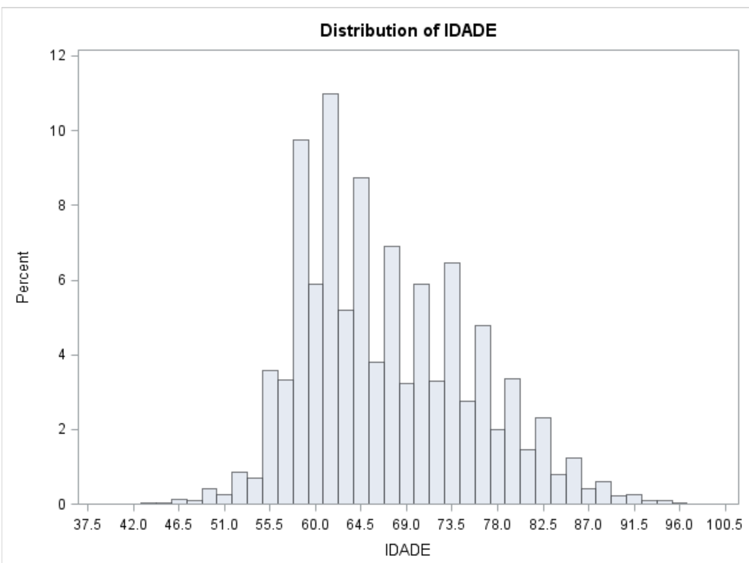

Figure 3 Distribution of Ages

LEE-CARTER ESTIMATE IN SAS ENTERPRISE GUIDE®

From now on we will show the results of the Lee-Carter estimation by the SAS software. The graph below shows the result of importing the base from the values of the North American Central Death Rates to the SAS Database format:

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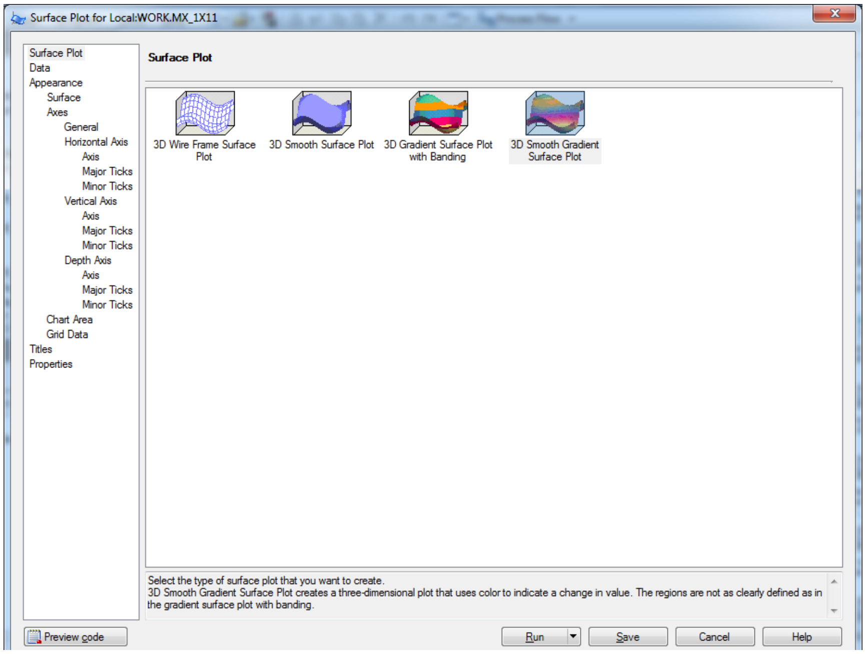

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Figure 4

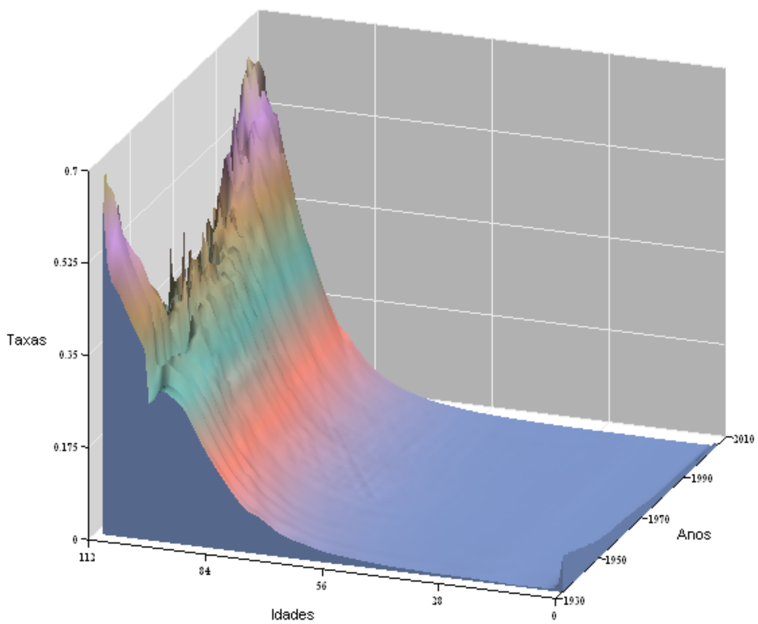

Figure 4 Graphs Ages x Year x Central Mortality Rate

In the above chart it is possible to see how the central rates of mortality gradually increase for each age. However, it is striking that central mortality rates have increased over the years between 1950 and 2010. Accessing the Human Mortality Database website (www.mortality.org) again, we extracted the files related to the number of exposed to the risk of death and to the number of deaths in the period in question (1933 to 2005), in order to analyze the behavior of these two variables, considering that the central mortality rate is composed of the relationship between the two variables. We noticed that the central rate of mortality extracted on the site was equal to the number of deaths on the number of exposed in the age, besides the corresponding year.

Figure 5

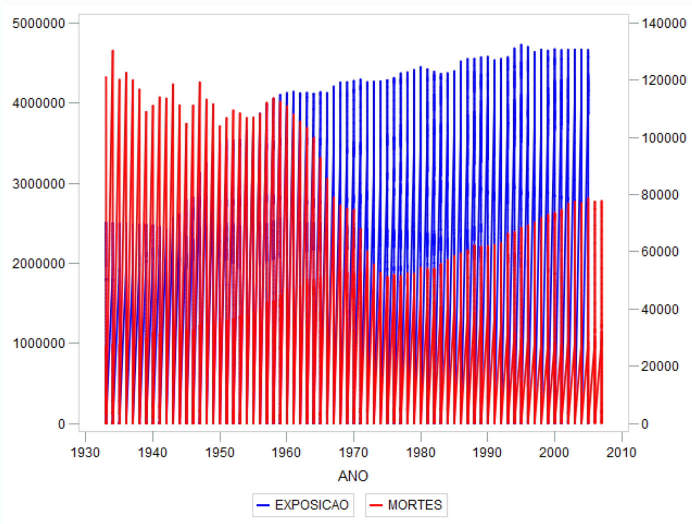

Figure 5 Graph Exposure and Deaths x Year

Analyzing comparatively the graph of figure 6 with that of figure 5, we notice that from the year of 1970 the inclination of the axis "Deaths" is greater than the inclination of the "Exposition" axis. In other words, even the exposure of the period gradually increased, deaths also increased, but at a higher rate, which explains the increase in the central mortality rates in the graph of Figure 5 (Ages x Year x Central Mortality Rate).

Graphic result of the $\alpha_x = \frac{\sum_{t=1}^{n} \ln(m_{x,t})}{n}$ estimation in SAS Enterprise Guide:

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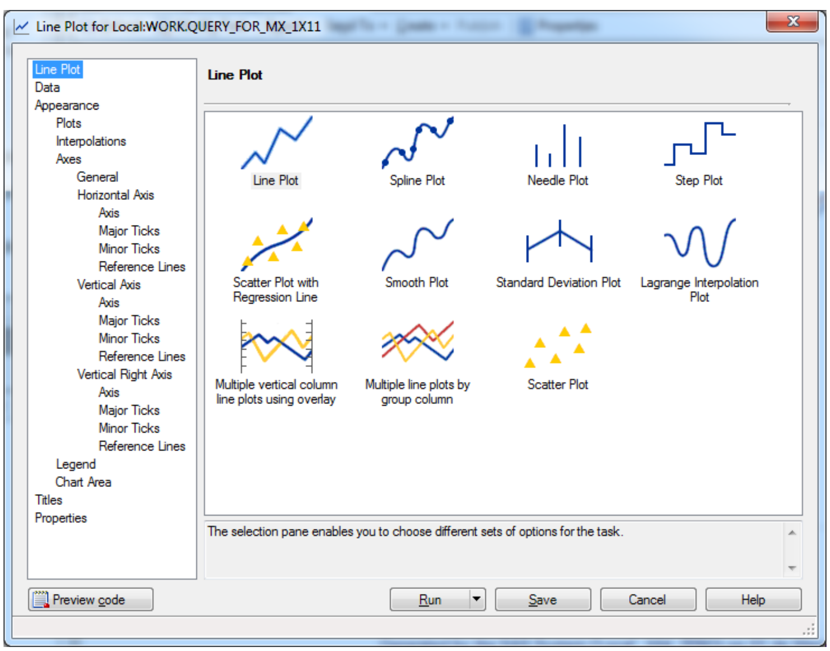

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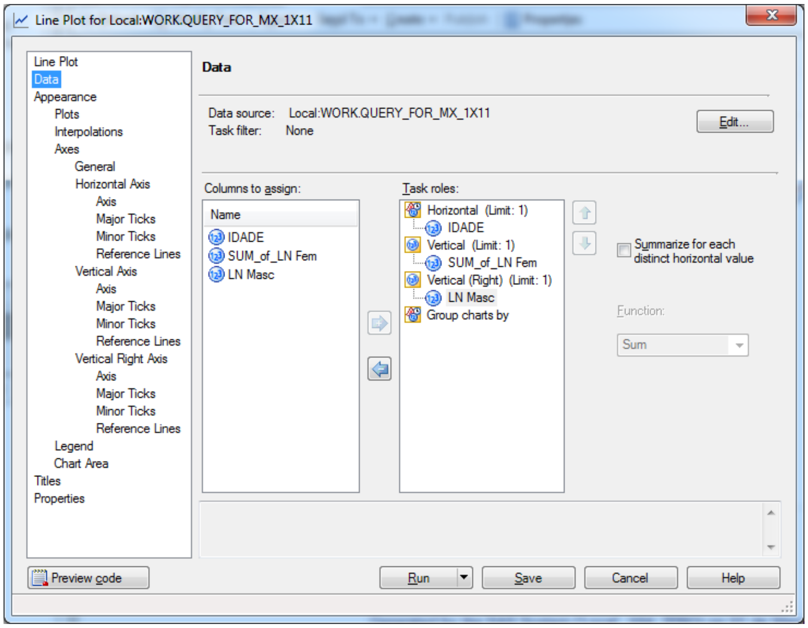

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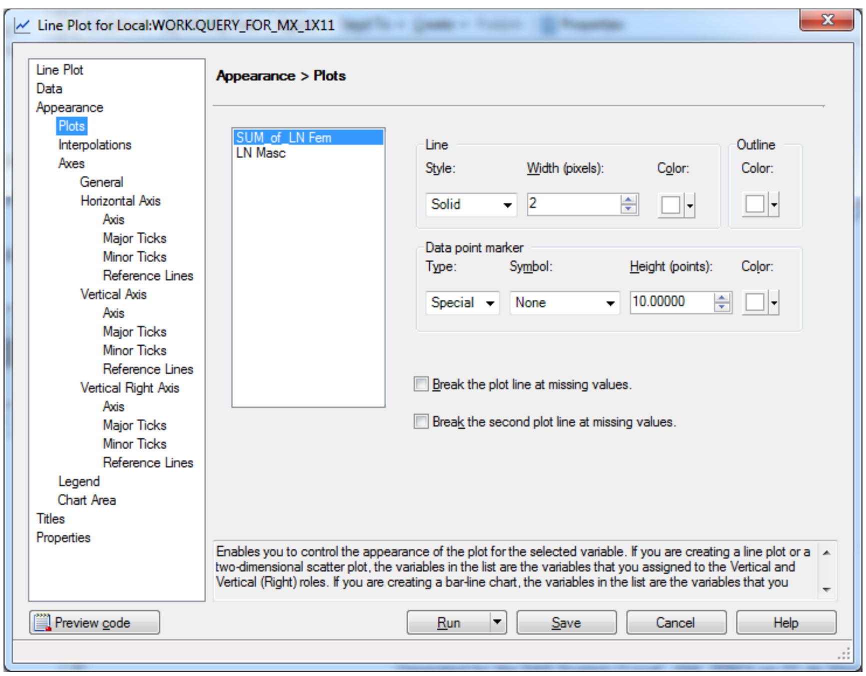

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Figure 6

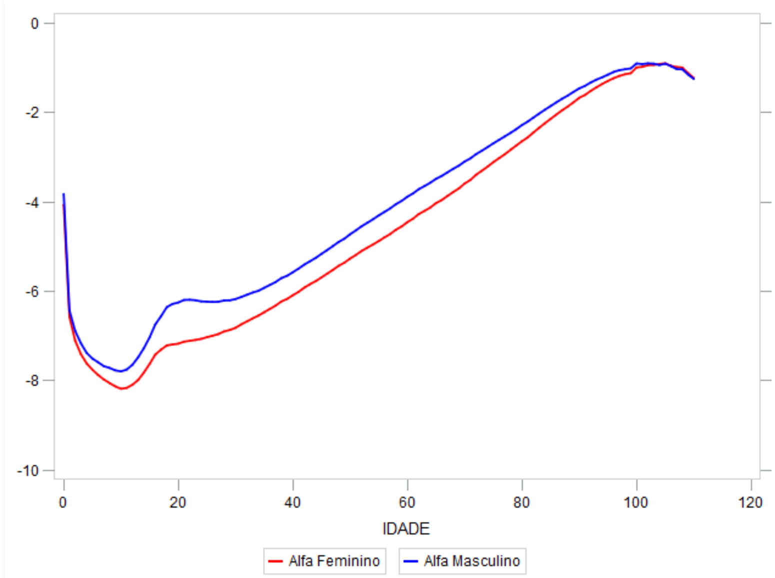

Figure 6 α_x **Calculated parameters graph**

In the chart above, given that α_x it is a specific parameter for each age group that describes the direct impact of age on the calculation of the central mortality rate (LEE and CARTER, 1992), we see that it has a significant impact on younger ages (children) and increases with the passing of the years, denoting their stronger relationship with mortality at younger and more advanced ages (mortality force). The Graphic result of the $\kappa_t = \sum_x (\ln(m_{x,t}) - \alpha_x)$ estimation in SAS Enterprise Guide:

Display 8

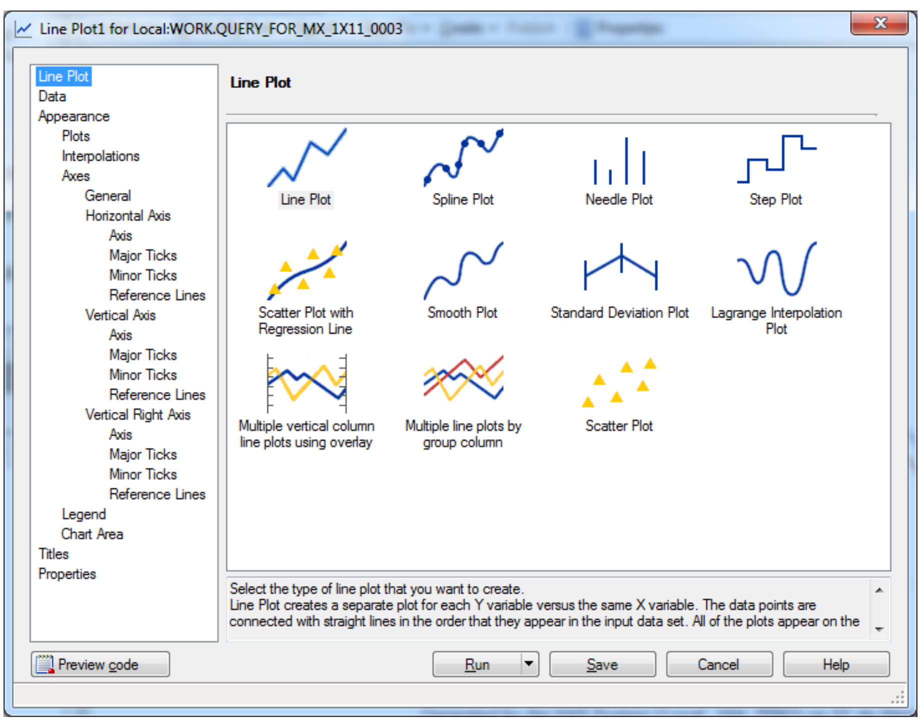

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Figure 7

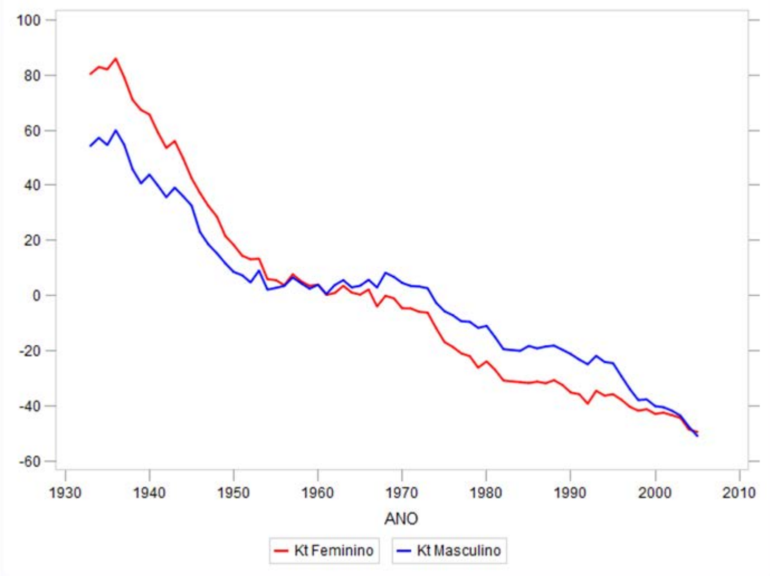

Figure 7 κ_t **calculated parameters graph**

In the graph above, we see the trend of decay of the κ_t parameter over the years.

Graphic result of the $\beta_x = \dfrac{\sum_{t=t1}^{tn} \kappa_t^{*}(\ln(m_{x,t}) - \alpha_x)}{\sum_{t=t1}^{tn} \kappa_t^{*2}}$ estimation in SAS:

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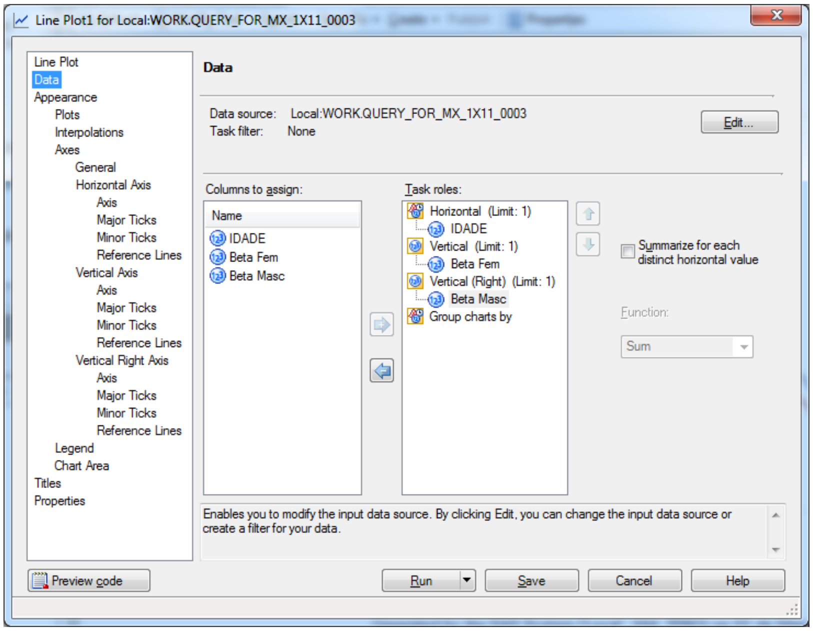

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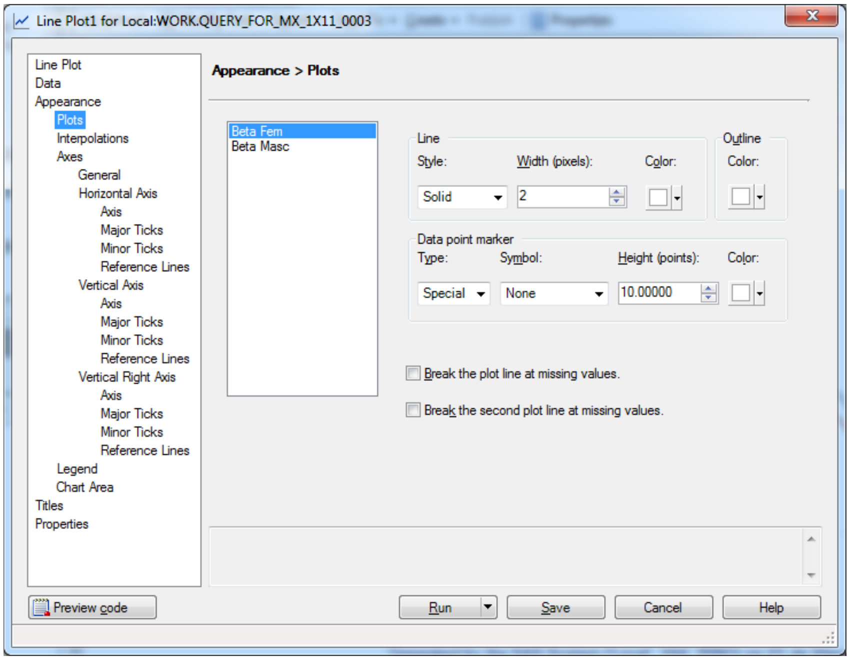

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Figure 8

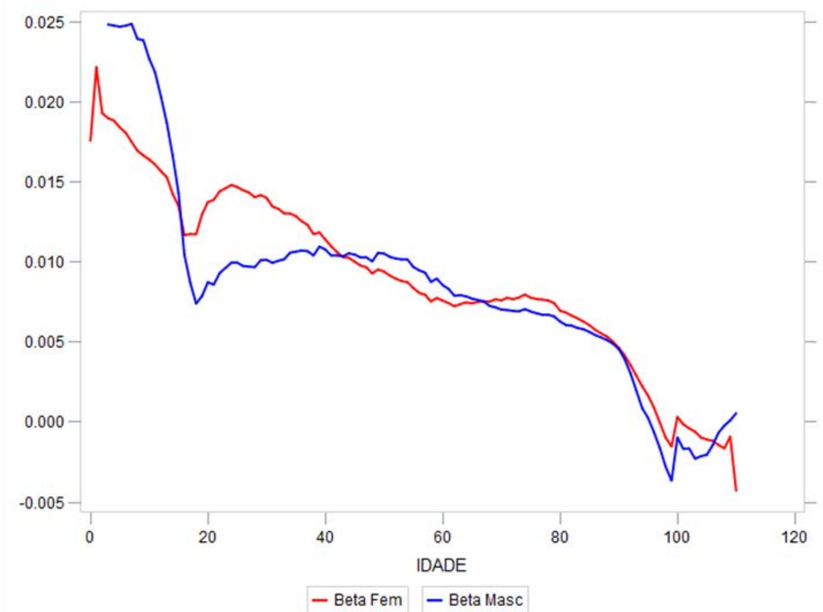

Figure 8 Graph of the calculated β_x parameters

For the estimation of *kt* up to 2042, we used, as proposed by SANTOS and DUARTE(2009) the Box-Jenkins ARIMA time-series prediction method with parameters (p, d, q) equal to (1,1,1). An ARIMA time series (p, d, q) stands for integrated autoregressive and moving averages, where p denotes the number of autoregressive terms, d the number of times we must differentiate the series before it becomes stationary, and q the number of terms of moving averages.

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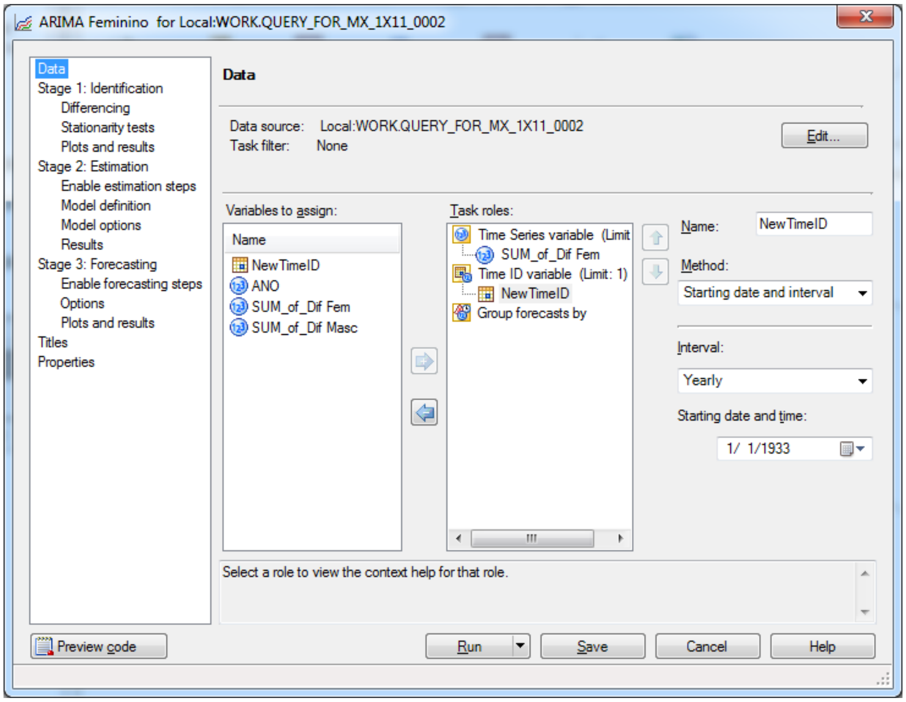

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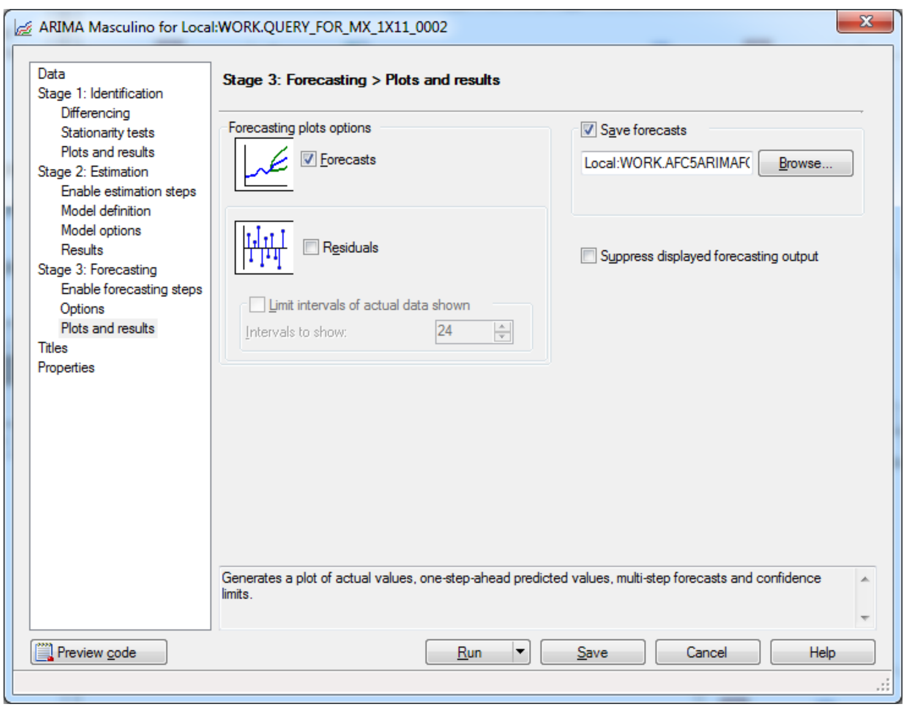

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Figure 9

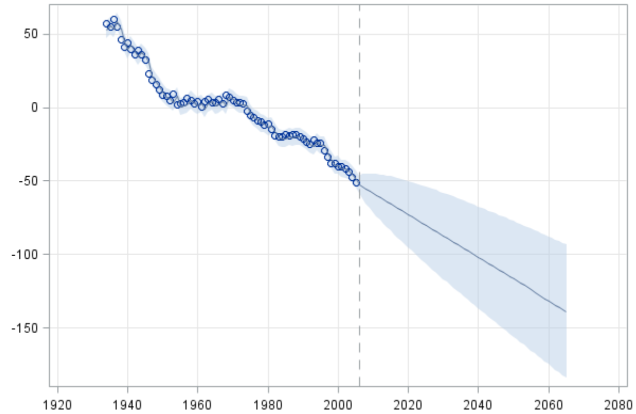


Figure 9 Graph of the prediction of the parameter by the ARIMA method (1,1,1)

A small part of the result of the estimation of the generational q_x, given that $q_{x,t} = q_{x,0} \cdot FI(x,t)$ and $FI(x,t) = \exp(\beta_x(\kappa_t - \kappa_{t-1}))$, which follows:

Display 13

	NewTimeID	0	1	2	3	4	5	6
1	2014	0.0020063026	0.0008968417	0.0004515548	0.0003331837	0.0002665095	0.0002171025	0.0001782516
2	2015	0.0019478084	0.0008636187	0.0004366431	0.0003222713	0.0002578194	0.0002101486	0.0001726426
3	2016	0.0018910195	0.0008316264	0.0004222238	0.0003117163	0.0002494127	0.0002034175	0.0001672102
4	2017	0.0018358864	0.0008008193	0.0004082808	0.000301507	0.0002412801	0.0001969019	0.0001619486
5	2018	0.0017823607	0.0007711534	0.0003947981	0.000291632	0.0002334126	0.0001905951	0.0001568527
6	2019	0.0017303955	0.0007425864	0.0003817607	0.0002820805	0.0002258017	0.0001844902	0.000151917
7	2020	0.0016799454	0.0007150777	0.0003691538	0.0002728418	0.000218439	0.0001785809	0.0001471367
8	2021	0.0016309662	0.0006885881	0.0003569633	0.0002639058	0.0002113163	0.0001728609	0.0001425068
9	2022	0.0015834149	0.0006630797	0.0003451753	0.0002552623	0.0002044259	0.0001673241	0.0001380226
10	2023	0.0015372501	0.0006385163	0.0003337766	0.000246902	0.0001977602	0.0001619646	0.0001336795
11	2024	0.0014924312	0.0006148628	0.0003227543	0.0002388155	0.0001913118	0.0001567768	0.0001294731
12	2025	0.001448919	0.0005920855	0.000312096	0.0002309938	0.0001850737	0.0001517552	0.000125399
13	2026	0.0014066754	0.0005701521	0.0003017896	0.0002234283	0.0001790389	0.0001468944	0.0001214531
14	2027	0.0013656634	0.0005490311	0.0002918236	0.0002161106	0.000173201	0.0001421893	0.0001176314
15	2028	0.0013258472	0.0005286925	0.0002821867	0.0002090326	0.0001675534	0.0001376349	0.00011393
16	2029	0.0012871918	0.0005091074	0.0002728681	0.0002021864	0.00016209	0.0001332264	0.000110345
17	2030	0.0012496634	0.0004902478	0.0002638572	0.0001955644	0.0001568047	0.0001289591	0.0001068728
18	2031	0.0012132291	0.0004720868	0.0002551438	0.0001891593	0.0001516917	0.0001248285	0.0001035099
19	2032	0.0011778571	0.0004545986	0.0002467182	0.000182964	0.0001467455	0.0001208302	0.0001002528
20	2033	0.0011435164	0.0004377583	0.0002385709	0.0001769715	0.0001419606	0.00011696	0.0000970982
21	2034	0.0011101769	0.0004215418	0.0002306925	0.0001711754	0.0001373316	0.0001132137	0.0000940428
22	2035	0.0010778094	0.000405926	0.0002230744	0.0001655691	0.0001328537	0.0001095874	0.0000910836
23	2036	0.0010463856	0.0003908887	0.0002157078	0.000160[illegible]	[illegible]1285217	0.0001060773	0.0000882175
24	2037	0.001015878	0.0003764084	0.0002085845	0.0001549013	0.000124331	0.0001026796	0.0000854416
25	2038	0.0009862598	0.0003624646	0.0002016964	0.0001498279	0.0001202769	0.0000993907	0.0000827531
26	2039	0.0009575052	0.0003490373	0.0001950358	0.0001449208	0.000116355	0.0000962072	0.0000801491
27	2040	0.0009295889	0.0003361074	0.0001885951	0.0001401744	0.000112561	0.0000931256	0.0000776271
28	2041	0.0009024865	0.0003236565	0.0001823672	0.0001355834	0.0001088907	0.0000901428	0.0000751844
29	2042	0.0008761743	0.0003116668	0.0001763448	0.0001311428	0.0001053401	0.0000872554	0.0000728186

Display 13

ESTIMATION OF THE STRUCTURE OF THE INTEREST RATE TERM

The estimation of the ETTJ was obtained from collecting the values of the parameters of Svensson model on the SUSEP Site in consultation made on December 12, 2011.

Here are the parameters (IPCA Coupon):

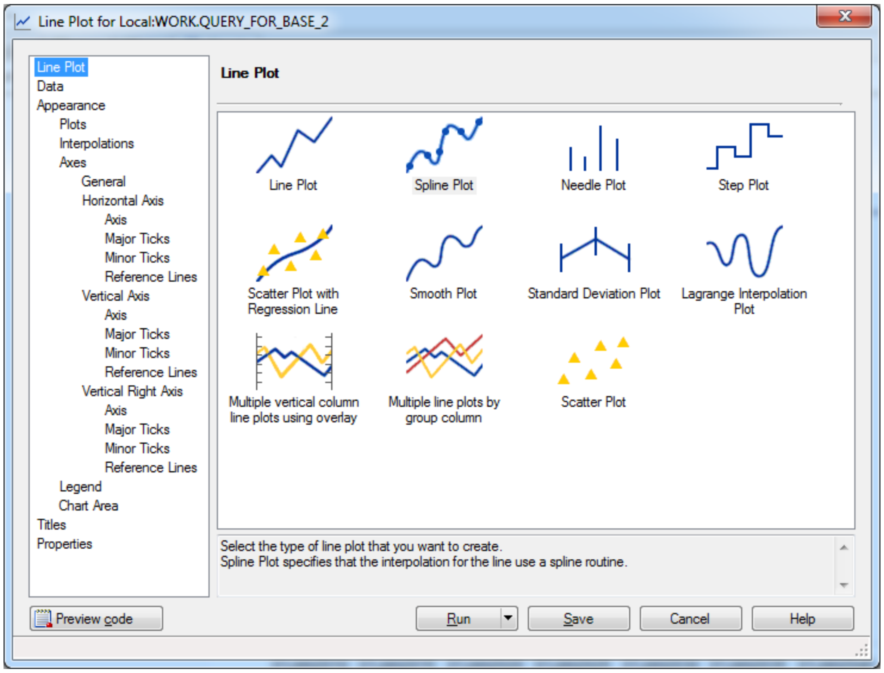
Table 1

	Betas	Lâmbda
1	0,0560	2,9574
2	0,0372	0,8053
3	-0,1081	
4	0,0116	

Table 1 Svensson Parameters

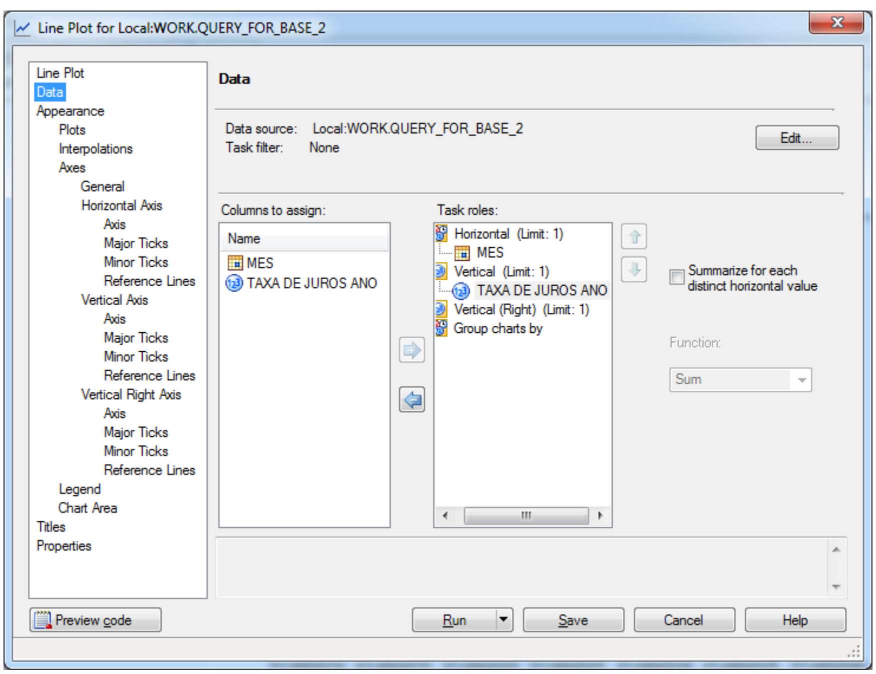
After adjusting the parameters to SAS Enterprise Guide, the figure below shows the interest rate values with 3 precision decimal places:

Display 14



Display 14

Display 15



Display 15

Figure 10

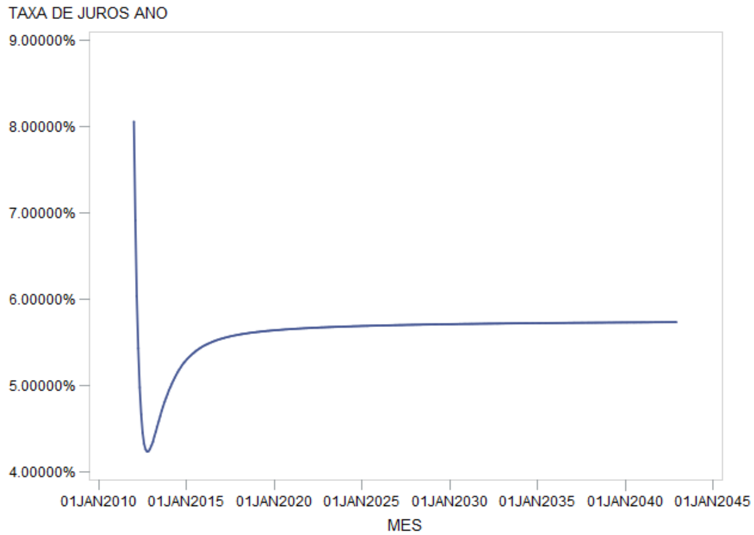


Figure 1 Chart of the Structure of the Interest Rate Term

To obtain maturities, we calculated the total number of business days as proposed.

Using actuarial calculation in SAS, the SQL PROC was utilized to estimate the actuarial obligation with relational algebra and the CASE function, which is analogous to the IF (Se) function in SQL language.

```
PROC SQL;
CREATE TABLE WORK.QUERY_FOR_BASE_22 AS
SELECT t3.MES,
       /* SUM_of_PROVISAO */
         (SUM(t1.'BENEFÍCIO LÍQUIDO'n * t2.DESCONTO * (CASE
            WHEN t1.IDADE = 0
            THEN t3.'0'n
         WHEN t1.IDADE = 1
            THEN t3.'1'n
         WHEN t1.IDADE = 2
            THEN t3.'2'n
         …
            THEN t3.'112'n
         WHEN t1.IDADE = 113
            THEN t3.'113'n
         ELSE t3.'114'n
         END))) AS SUM_of_PROVISAO
   FROM WORK.BASE_2 t2
        INNER JOIN WORK.BASE_21 t3 ON (t2.MES = t3.MES),WORK.BASE_22 t1
   GROUP BY t3.MES;
QUIT;
```

We considered 3 matrices: the base of the retirees, the interest rate, and the mortality table.

By multiplying them, as done in the multiplication of matrices in Linear Algebra, we have found not only the result of the Mathematical Reserve, but also the flow of benefits to present value without new retirees (new entrants) and already considering the generational probabilities aspects of survival. The whole calculation was made disregarding the inflationary effects. Here are the results:

As some pension funds still estimate their actuarial obligations with Table AT-2000 smoothed in 10%, minimum table provided by CNPC resolution no. 22 for the equalization of deficits, it was also analyzed its adoption as a technical basis for the purposes of comparison of the work.

Table 2

Provision AT2000 Table and interest of 6% per year	
Provision 2000 RP Table and IPCA Coupon	
Percent Increase	9,44%

Table 2 Comparison between Provision with AT 2000 and constant interest 6% a.a and Provision with Generation RP-2000 and ETTJ

Table 3

Provision AT2000 Table and interest of 6% per year	
Provision AT2000 Lee Carter and IPCA Coupon	
Percent Increase	7,73%

Table 3 Comparison between Provision with AT 2000 and constant interest 6% a.a and Provision with AT 2000 Lee-Carter and ETTJ

Table 4

Provision AT83 Table and interest of 6% per year	
Provision AT2000 Lee Carter and IPCA Coupon	
Percent Increase	7,90%

Table 4 Comparison between Provision with AT 83 and constant interest 6% a.a and Provision with AT 2000 Lee-Carter and ETTJ

CONCLUSION

Statistical software SAS Enterprise Guide proved to be an excellent tool for actuarial calculations in pension funding, given its simplicity and computational efficiency in the treatment of large databases.

The Lee-Carter methodology is an alternative for the closed supplementary pension market to estimate the longevity gain used in long-term actuarial projections.

The observed impacts may be even greater if provisions are stochastically estimated, since the maximum value for the provision, in the range of estimated results, may be even higher.

However, the estimation of mathematical reserves with the prevailing premises in the market may be undersized, given the trend of longevity gains that the Brazilian population has historically experienced.

The application of the methodology suggested by SBR and studied in this paper can generate different impacts on the different masses of Brazilian pension funds, whereas smaller pension funds may not support the impact of adopting more conservative assumptions.

Finally, it is suggested for those pension funds that need to equate technical-actuarial deficits to practice a test of adequacy of the assumptions presented in this article, namely, to discount the future estimated ETTJ flows and mainly using actuarial tables with expectancy of longevity gains, preferably with the experience of the fund's own population, in order to mitigate the risks of future actuarial deficits.

CONTACT INFORMATION

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