Paper 10729-2016

Using SAS® to Conduct Multivariate Statistical Analysis in Educational Research: Exploratory Factor Analysis and Confirmatory Factor Analysis

Jun Xu, Kevin Petway, Lili Yao, Steven Holtzman, Educational Testing Service

ABSTRACT

Multivariate statistical analysis plays an increasingly important role as the number of variables being measured increases in educational research. In both cognitive and noncognitive assessments, many instruments that researchers aim to study contain a large amount of variables where each measured variable can be assigned to a specific factor of a bigger construct. The factor of an instrument typically emerges through the use of educational theories or empirical research. Two types of factor analysis are widely used in order to understand the latent relationships among multiple variables based on different scenarios: (1) exploratory factor analysis (EFA), which is performed by using the SAS® PROC FACTOR procedure, is an advanced statistical method to probe deeply into the relationship among the variables and the larger construct and then develop a tailored model for the specific assessment; (2) confirmatory factor analysis (CFA), which is used after a model is established, is conducted by using the SAS® PROC CALIS procedure to examine the model fit of specific data and then make adjustment for the model as needed. This paper presents the application of SAS® to conduct these two types of factor analysis to fulfill various research purposes. Examples using noncognitive assessment data are demonstrated, and the interpretation of the fit statistics is discussed.

INTRODUCTION

Historically, cognitive skills have been believed to be the key factors for students' academic achievement and life success (Brody, 1992; Bartels, Rietveld, Van Baal, & Boomsma, 2002). Over the last few decades, noncognitive skills have drawn more attention and have been found to be equally, if not more, important factors in predicting students' academic achievement (Tracey & Sedlacek, 1982; Parker & Summerfeldt, 2004). In order to measure students' noncognitive skills, a commonly used method is to ask students to complete a questionnaire to report their self-perceived feelings towards the items. The Mission Skills Assessment (MSA), used as our example, asks middle-school students to answer Likert-type questions to measure six facets of noncognitive skills: teamwork, creativity, ethics, resilience, curiosity, and time management.

When studying noncognitive skills, many validity and reliability concerns need to be considered for four reasons: (1) the nature of their original complexity; (2) responses are self-perceived; (3) the easiness to fake responses, and (4) the assessments are relatively low-stakes. Therefore, careful examinations to identify the measurement issues of noncognitive items before digging into the real research questions are of great significance. Common techniques include the following: (1) zero-variance checks to see if students paid attention while answering, (2) outlier checks for unreasonable response patterns, and (3) factor analysis to see if specified items measure the intended skill. The third method is the focus of this paper.

This paper introduces concept of factor analysis, demonstrates main differences between EFA and CFA, and provides an example to illustrate implementation of these methods in SAS and interpretation of SAS output for research studies.

FACTOR ANALYSIS

Factor analysis is a multivariate statistical method that is implemented to detect the correlations between the variables observed and the potentially latent factors that are unobserved (Knott & Bartholomew, 1999). It is commonly used in noncognitive assessments as researchers aim to measure students' noncognitive attributes/skills without them being able to concretely measure those skills that are being detected by that specific question. Therefore, a series of carefully designed questions/prompts are given to the students to measure a variety of different noncognitive skills. Additionally, due to the fact that noncognitive skills are complex in nature, items sometimes measure unintended factors or the same assessment may produce different results when used for different groups of participants (Douglas, McDaniel & Snell, 1996). Therefore, factor analysis is a crucial and effective statistical method, functioning as the gate keeper before proceeding to more complex research questions, to ensure that the assessment is working correctly.

Generally, there are two types of commonly used factor analysis: exploratory (EFA) and confirmatory (CFA) (Thompson, 2004). EFA is used to identify complex correlations among items and group items that are part of unified constructs. The researchers make no a priori assumptions about the relationships among factors. CFA, on the other hand, tests the hypothesis that the items are associated with specific factors. Hypothesized models are tested against actual data and the analysis would demonstrate loadings of observed variables on the latent variables, as well as the relationships between the latent variables. To summarize, researchers usually use EFA with the goal to establish a theory or a model, while they more often conduct CFA to validate a hypothesized model with actual data.

EXAMPLES

DATASET

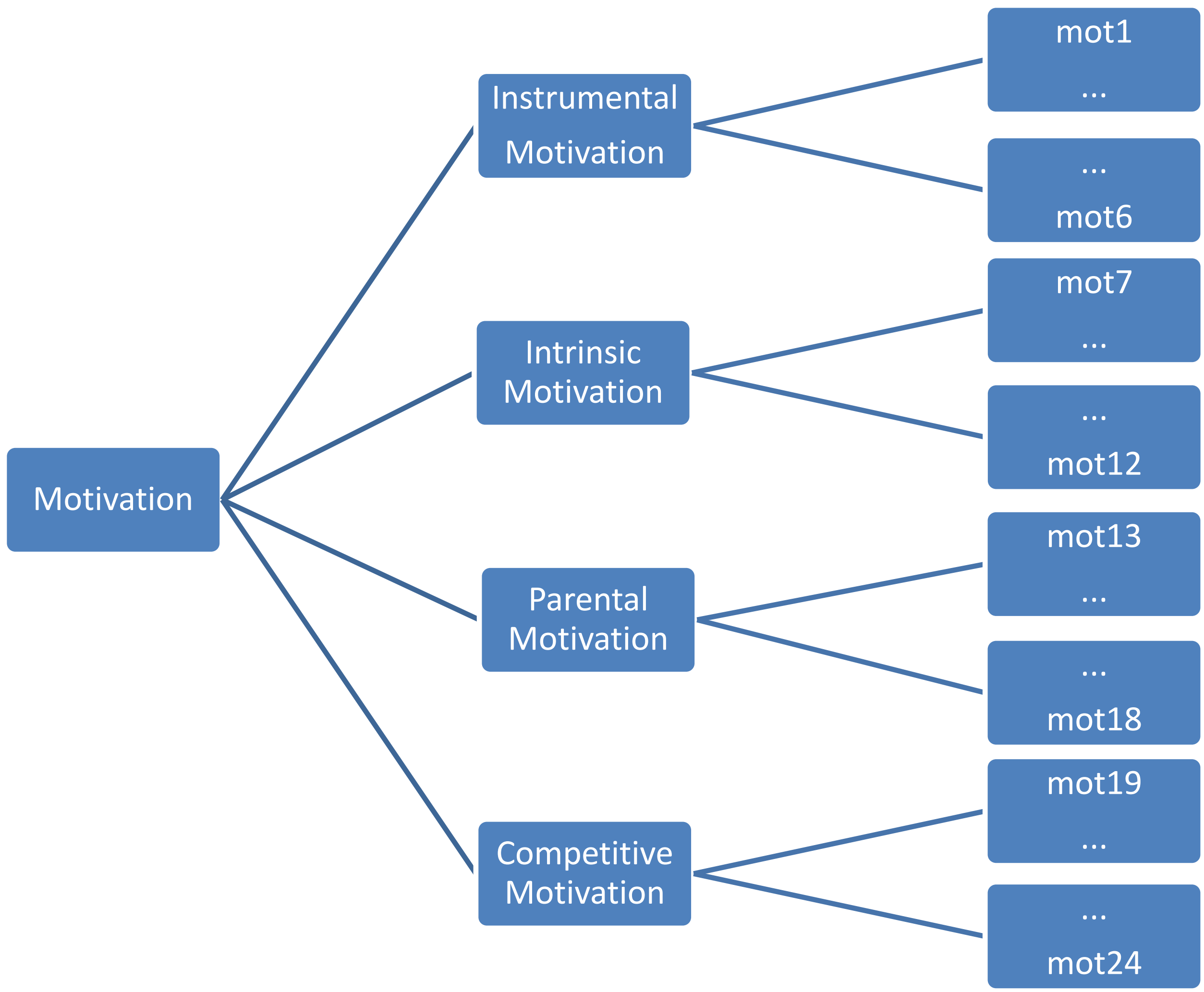
As previously mentioned, we use MSA data from a self-report Likert questionnaire administered in June 2012 to 1,675 students from 14 middle schools across the United States. This questionnaire contains the two assessments discussed below.

Motivation has been found to be a strong predictor of students' academic outcomes (Wong & Mihaly, 1991; Liu, Bridgeman & Adler, 2012). Based on psychological theories, researchers have divided motivation into four facets: instrumental, parental, and competitive motivation, which are three types of extrinsic motivation, and intrinsic motivation. The questionnaire asked participants six questions to measure each of the four facets within different situations of motivation. Each question is asked on a four-point response scale ranging from "Very Unlikely" to "Very Likely." A full list of the twenty-four items is shown in Table A in the Appendix. We implement CFA by using the SAS PROC CALIS procedure to examine if the four-factor model holds for the 1,675 students.

GRIT (equal to resilience in this case) has drawn increasing attention in the past decades. Duckworth (2007) invented GRIT. By her definition, GRIT is the resilience and passion that is crucial to achieve long-term goals. In MSA data, GRIT was measured by 36 items on a four-point Likert response scale ranging from "Never or Rarely," to "Usually or Always." Twenty-two items were negative statements and the responses were reverse-coded for this analysis. A full list of the thirty-six items is shown in Table B in the Appendix. A one-factor CFA was originally conducted for these items and the model fit is far below the acceptance level, which indicates these thirty-six items are measuring multiple factors rather than one merged factor as designed. Therefore, an additional EFA is necessary to identify the latent factors behind the items.

DIAGRAM OF CFA

On the next page is the diagram of the proposed four-factor CFA model for the twenty-four motivation items.



CODE OF CFA

We use the following SAS code to conduct a four-factor CFA for the twenty-four motivation variables. The PROC CALIS procedure is used.

```
proc calis data=msa;
 lineqs
  mot1   = p1  f_instru + e1,
  mot2   = p2  f_instru + e2,
  mot3   = p3  f_instru + e3,
  mot4   = p4  f_instru + e4,
  mot5   = p5  f_instru + e5,
  mot6   = p6  f_instru + e6,
  mot7   = p7  f_intrin + e7,
  mot8   = p8  f_intrin + e8,
  mot9   = p9  f_intrin + e9,
  mot10 = p10 f_intrin + e10,
  mot11 = p11 f_intrin + e11,
  mot12 = p12 f_intrin + e12,
  mot13 = p13 f_parent + e13,
  mot14 = p14 f_parent + e14,
  mot15 = p15 f_parent + e15,
```

```
  mot16 = p16 f_parent + e16,
  mot17 = p17 f_parent + e17,
  mot18 = p18 f_parent + e18,
  mot19 = p19 f_compet + e19,
  mot20 = p20 f_compet + e20,
  mot21 = p21 f_compet + e21,
  mot22 = p22 f_compet + e22,
  mot23 = p23 f_compet + e23,
  mot24 = p24 f_compet + e24;
 std
  e1-e24 = vare1-vare24,
  f_instru=1,
  f_intrin=1,
  f_parent=1,
  f_compet=1;
 cov
  f_instru f_intrin = covf1f2,
  f_instru f_parent = covf1f3,
  f_instru f_compet = covf1f4,
  f_intrin f_parent = covf2f3,
  f_intrin f_compet = covf2f4,
  f_parent f_compet = covf3f4;
 var
  mot1 mot2 mot3 mot4 mot5 mot6
  mot7 mot8 mot9 mot10 mot11 mot12
  mot13 mot14 mot15 mot16 mot17 mot18
  mot19 mot20 mot21 mot22 mot23 mot24;
run;
```

RESULTS OF CFA

By conducting the PROC CALIS procedure in SAS, we can see four sections in the output: (1) Model and Initial Values; (2) Descriptive Statistics; (3) Optimization; and (4) Maximum Likelihood Estimation. The Fit Summary table in the fourth section Maximum Likelihood Estimation contains the most important information regarding the model fit to our data.

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Fit Summary

	…	
Absolute Index	Fit Function	1.8286
	Chi-Square	3061.1126
	Chi-Square DF	246
	Pr > Chi-Square	<.0001
	…	
Parsimony Index	RMSEA Estimate	0.0827
	…	
Incremental Index	Bentler Comparative Fit Index	0.8351
	Bentler-Bonett NFI	0.8236

Bentler-Bonett Non-normed Index	0.8150
…	

Whether a model fit is satisfactory is determined by these parameters: the p value of the chi-square statistics, the Root Mean Square Error of Approximation (RMSEA), the comparative fit index (CFI) and Bentler-Bonett NFI. A satisfactory model fit is suggested by a nonsignificant chi square at a 0.05 level (Cheung & Rensvold, 2002), RMSEA values less than 0.08 (Browne & Cudeck, 1993), CFI values bigger than 0.95, and NFI greater than 0.90 (Byrne, 2006). However, due to the sensitive nature of the significance of chi square statistics, some literature suggested to exclude it from determining a model fit (Jöreskog, 1969).

From the Fit Summary table, we can see the p-value of the chi-square probability is smaller than 0.0001, which indicates the chi-square statistic is significant at the 0.05 level. The RMSEA is 0.0827, which means the residual is slightly greater than the cutoff of 0.08. The CFI and NFI results are 0.84 and 0.82, with none of them meeting the criterion. To summarize, from the resulting indices, we can claim that the model fit is less than ideal and therefore a further examination regarding these 24 items in this specific dataset will be needed before any actual analysis moves forward.

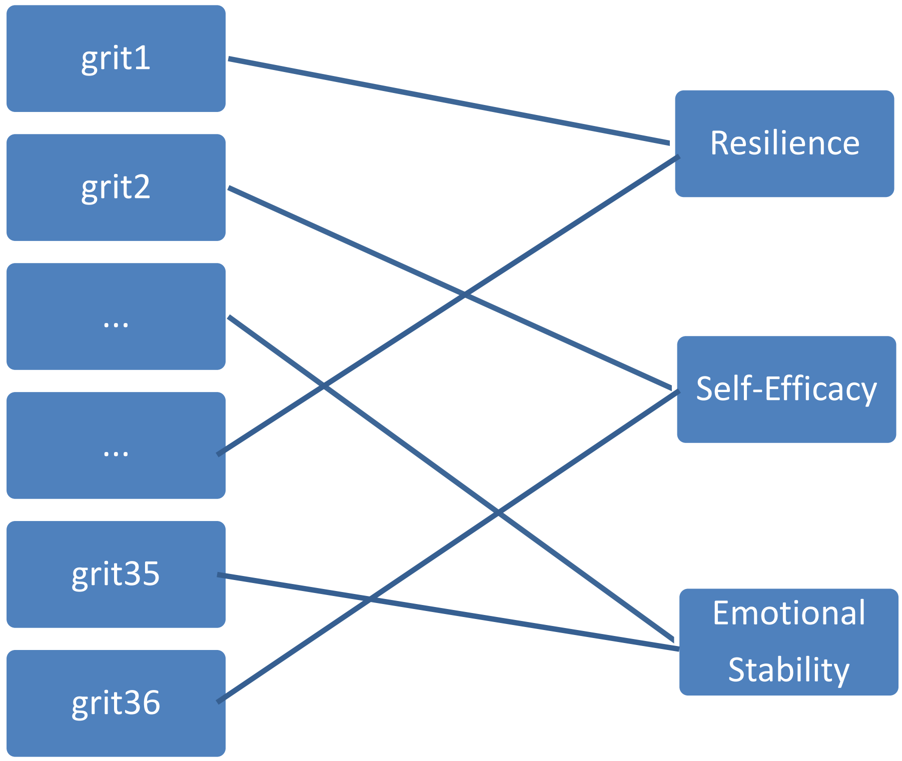
Standardized Results for Covariances Among Exogenous Variables

Var1	Var2	Parameter	Estimate	Standard Error	t Value
f_instru	f_intrin	covf1f2	0.47489	0.02389	19.87480
f_instru	f_parent	covf1f3	0.53200	0.02143	24.82774
f_instru	f_compet	covf1f4	0.82413	0.01293	63.75380
f_intrin	f_parent	covf2f3	0.10806	0.02899	3.72730
f_intrin	f_compet	covf2f4	0.42669	0.02564	16.64316
f_parent	f_compet	covf3f4	0.55098	0.02165	25.44929

Additionally, we also examine the correlations among the four latent factors to see how they relate to each other. It is suggested to combine the factors if their correlations are extremely high. The covariances are presented above. The correlations are equivalent to the covariances in that the variances of the latent variables are equal to one. From the results table, we can find out that all correlations are small or medium, ranging from 0.10 to 0.60, expect for the correlation between instrumental motivation and competitive motivation, which is slighter higher than the cut off. Therefore, it is suggesting that instrumental motivation and competitive motivation are highly correlated and thus may need to be combined into one factor.

DIAGRAM OF EFA

Below is the diagram of the EFA model for the thirty-six GRIT variables. Unlike the CFA model, how many factors we have and which variable belongs to which factor is unknown and two consecutive EFAs will be used to determine these answers. The terminology will be assigned to each factor after the EFA analysis. In the diagram, resilience, self-efficacy, and emotional stability are three examples of GRIT that might evolve from the EFA.



CODE OF EFA WITHOUT ROTATION

```
proc factor data=msa method=ml scree priors=smc;
  var grit1 grit2 grit3 grit4 grit5 grit6
      grit7 grit8 grit9 grit10 grit11 grit12
      grit13 grit14 grit15 grit16 grit17 grit18
      grit19 grit20 grit21 grit22 grit23 grit24
      grit25 grit26 grit27 grit28 grit29 grit30
      grit31 grit32 grit33 grit34 grit35 grit36;
run;
```

RESULTS OF EFA WITHOUT ROTATION

The FACTOR Procedure
Initial Factor Method: Maximum Likelihood

Preliminary Eigenvalues: Total = 23.8025853
Average = 0.66118292

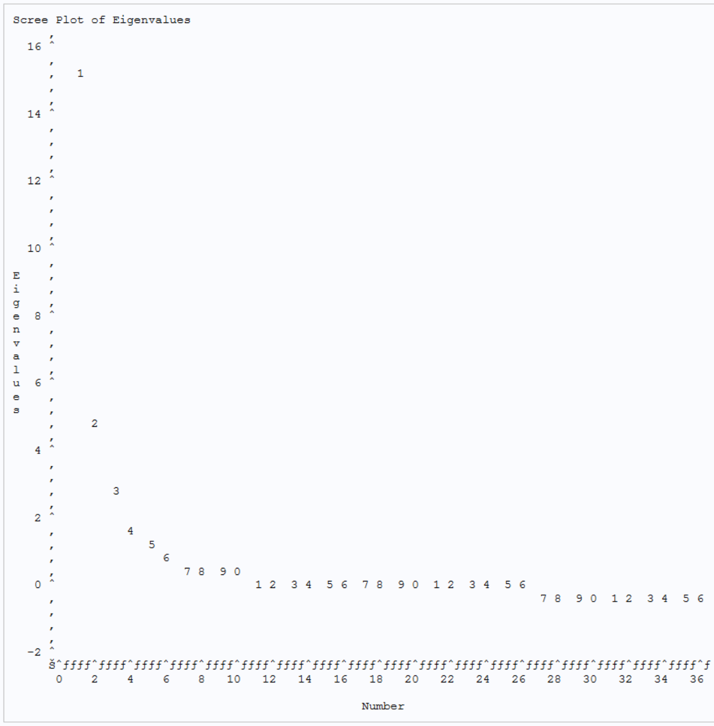
	Eigenvalue	Difference	Proportion	Cumulative
1	15.2760100	10.6504948	0.6418	0.6418
2	4.6255152	1.9186537	0.1943	0.8361
3	2.7068614	1.2735814	0.1137	0.9498
4	1.4332800	0.3989235	0.0602	1.0100
5	1.0343565	0.3382835	0.0435	1.0535
…	...	…	…	…

The FACTOR Procedure
Initial Factor Method: Maximum Likelihood

Preliminary Eigenvalues: Total = 23.8025853 Average = 0.66118292

	Eigenvalue	Difference	Proportion	Cumulative
32	-0.3023499	0.0197613	-0.0127	1.0601
33	-0.3221111	0.0284963	-0.0135	1.0465
34	-0.3506074	0.0106259	-0.0147	1.0318
35	-0.3612333	0.0349282	-0.0152	1.0166
36	-0.3961616		-0.0166	1.0000

4 factors will be retained by the PROPORTION criterion.



From the output, we can see four factors were retained by the proportion criterion. The preliminary eigenvalues are 15.2760100, 4.6255152, 2.7068614, and 1.4332800. Each of the four factors explains 64.18%, 19.43%, 11.37%, and 6.02% of the variance, respectively.

Factor analysis has an advantage that the axes of the factors can be rotated within the multidimensional variable space. The rotation method is often used to make the results of the factor analysis more easily interpretable (Browne, 2001). With the knowledge that there are four factors retained in mind, we will then conduct another EFA with the rotation method by specifying the number of factors in the model.

CODE OF EFA WITH FOUR FACTORS AND ROTATION

```
proc factor data=msa method=ml rotate=v n=4 reorder plot out=facsub4
priors=smc;
  var grit1 grit2 grit3 grit4 grit5 grit6
      grit7 grit8 grit9 grit10 grit11 grit12
      grit13 grit14 grit15 grit16 grit17 grit18
      grit19 grit20 grit21 grit22 grit23 grit24
```

```
      grit25 grit26 grit27 grit28 grit29 grit30
      grit31 grit32 grit33 grit34 grit35 grit36
      ;
RUN;
```

In this code, the varimax rotation is selected and four factors will be kept in the result.

RESULTS OF EFA WITH FOUR FACTORS AND ROTATION

Rotated Factor Pattern

	Factor1	**Factor2**	**Factor3**	**Factor4**
grit3	0.71378	0.11327	0.16013	0.08056
grit6	0.63243	0.13696	0.12069	-0.10226
grit9	0.63195	0.15119	0.08723	0.01850
grit1	0.58905	0.13210	0.08503	0.08759
grit10	0.55832	0.09571	0.13220	0.03099
grit2	0.55710	0.10377	0.16052	0.26539
grit7	0.54499	0.11591	0.14978	-0.01258
grit8	0.54431	0.06416	0.10458	0.06824
grit11	0.51178	0.03730	0.15530	0.02681
grit4	0.50774	-0.00863	0.11920	0.38322
grit22	0.49070	0.02115	0.27041	0.45230
grit5	0.47990	-0.02599	0.10864	0.21421
grit18	0.47328	0.06216	0.31439	0.33591
grit16	0.46778	0.19009	0.37246	-0.01841
grit14	0.41361	0.08643	0.40227	0.08337
grit32	0.21869	0.63444	0.07777	0.12400
grit28	0.07460	0.62015	0.22659	0.07478
grit27	0.14273	0.60679	0.03167	0.01584
grit26	0.18648	0.59979	0.00943	0.01836
grit29	0.10065	0.58177	0.40610	0.12202
grit25	0.09222	0.53775	-0.03127	-0.01405
grit31	0.10315	0.53210	0.09067	0.19550
grit33	0.06264	0.51546	-0.01308	0.14538
grit30	-0.01450	0.42668	-0.01673	0.00520
grit24	-0.11449	0.24730	-0.05812	0.19524
grit12	0.36884	0.06752	0.59511	0.15887
grit17	0.37983	0.10135	0.59407	0.11124
grit13	0.33741	0.02601	0.54371	0.24930

Rotated Factor Pattern				
	Factor1	**Factor2**	**Factor3**	**Factor4**
grit15	0.43628	0.11162	0.45303	0.07420
grit19	0.22706	-0.04806	0.37826	0.56023
grit34	-0.00256	0.30198	0.16246	0.53966
grit23	0.11254	0.40145	0.01999	0.49850
grit21	0.21337	-0.05383	0.43038	0.46632
grit20	0.43035	-0.00920	0.24766	0.44959
grit36	0.08005	0.37426	0.08522	0.41046
grit35	-0.01953	0.24231	-0.02897	0.40609

For the factor loadings, we use 0.40 as the cut off to decide if an item belongs to that factor. If one item has multiple loadings that are greater than 0.40, then it is a cross-loaded item. We will assign this item to the factor which has a bigger loading in our study, but it is recommended to further investigate the item as necessary. From the rotated factor pattern result, we can conclude that 15 items are correlated with factor 1, 10 items are correlated with factor 2, four items are correlated with factor 3, and seven items are correlated with factor 4. It is important to note that five items are associated with more than one factor. Their loadings are underlined in the table and should be assigned to one of the two factors based on psychological or educational theory. From careful review of all the thirty-six items, we can distinguish that we have four distinct factors and assigning them names based on related psychological theories. In our case, factor 1 can be called resilience, factor 2 can be called self-efficacy, factor 3 can be called emotional stability, and factor 4 can be named conscientiousness.

CONCLUSION

Factor analysis is an important multivariate statistical analysis to disentangle the relationships between the items and their latent factors. Researchers usually use EFA in order to establish a theory or a model, or in our example, to explore the latent factors that actually exist in the data in search of better model fit. On the other hand, CFA should be applied to validate an existing model with collected data. The procedures outlined in this paper show that PROC CALIS and PROC FACTOR have the analytic capabilities to conveniently implement CFAs and EFAs respectively and produce thorough and informative results to guide future research.

REFERENCES

Bartels, M., Rietveld, M. J. H., Van Baal, G. C. M., & Boomsma, D. I. (2002). Heritability of educational achievement in 12-year-olds and the overlap with cognitive ability. *Twin Research*, *5*, 544–553.

Brody, N. (1992). Intelligence (2nd edition). New York: Academic.

Browne, M. W. (2001). An overview of analytic rotation in exploratory factor analysis. *Multivariate Behavioral Research*, *36*(1), 111-150.

Browne, M. W., & Cudeck, R. (1993). Alternative ways of assessing model fit. In K. A. Bollen & J. S. Long (Eds.), *Testing structural equation models* (pp. 136–162). Newbury Park, CA: Sage Publications.

Byrne, B. M. (2006). Structural equation modeling with EQS: Basic concepts, applications, and programming (2nd ed.). Mahwah, NJ: Erlbaum.

Chamorro-Premuzic, T., & Furnham, A. (2003). Personality predicts academicperformance: Evidence from two longitudinal university samples. *Journal of Research in Personality*, *37*(4), 319-338.

Cheung, G. W., & Rensvold, R. B. (2002). Evaluating goodness –of-fit indexes for testing measurement invariance. *Structural Equating Modeling*, *9*(2), 233-255.

Douglas, E. F., McDaniel, M. A., & Snell, A. F. (1996). The validity of non-cognitive measures decays when applicants fake. *In Academy of Management Proceedings* (Vol. 1996, No. 1, pp. 127-131). Academy of Management.

Duckworth, A. L., Peterson, C., Matthews, M. D., & Kelly, D. R. (2007). Grit: Perseverance and passion for long-term goals. *Journal of Personality and Social Psychology, 92*(6), 1087.

Jöreskog, K.G. (1969). A general approach to confirmatory factor analysis, *Psychometrika, 34, 183-202.*

Knott, M., & Bartholomew, D. J. (1999). Latent variable models and factor analysis (No. 7). *Edward Arnold.*

Liu, O. L., Bridgeman, B. & Adler, R. M. (2012). Measuring learning outcomes assessment in higher education: Motivation matters. *Educational Researcher, 41(9), 352 - 362.*

Parker, J. D., Summerfeldt, L. J., Hogan, M. J., & Majeski, S. A. (2004). Emotional intelligence and academic success: Examining the transition from high school to university. *Personality and individual differences*, *36*(1), 163-172.

Thompson, B. (2004). Exploratory and confirmatory factor analysis: Understanding concepts and applications. *American Psychological Association.*

Tracey, T. J., & Sedlacek, W. E. (1982). Non-Cognitive Variables in Predicting Academic Success by Race.

Wong, M. M. & Mihaly, C. (1991). Motivation and academic achievement: The effects of personality traits and the quality of experience. *Journal of Personality*, *59*(3), 539-574.

ACKNOWLEDGMENTS

The authors would like to thank Bruce Kaplan, Jennifer Minsky, Patrick Barnwell, Ted Blew, and Ed Kulick for their support and assistance with this paper.

CONTACT INFORMATION

Your comments and questions are valued and encouraged. Contact the first author at:

Jun Xu
Research Data Analyst
Educational Testing Service
660 Rosedale Road – Mail Stop 20-T
Princeton, NJ 08541
Telephone: 609-734-1989
Email: jxu@ets.org

TRADEMARKS

APPENDIX

Table A. Details of the 24 Motivation Variables

SAS Variable Name	Item/Prompt	Motivation Type
mot1	Studying hard is worth it because it will help me get into college one day.	Instrumental Motivation
mot2	Reading in my free time will help me to get into college.	
mot3	It will give me practice mastering the mathematics skills that I will need to get into college.	
mot4	Paying attention in science will be useful for high school.	
mot5	I study more for math class because I will learn many things in math that will help me get into the best college.	
mot6	Good grades in social studies will help me get into the college I want.	
mot7	Studying for quizzes is enjoyable because I learn while I am doing it.	Intrinsic Motivation
mot8	Reading in my free time is enjoyable.	
mot9	I am interested in learning how to solve math problems.	
mot10	I pay attention in science class because I enjoy it.	
mot11	I study more for math class because math is fun.	
mot12	I enjoy my social studies class.	
mot13	I will not go out because my parents expect me to do well on the quiz.	Parental Motivation
mot14	My parents want me to read as much as I can.	
mot15	My parents encouraged me to enroll in the club.	
mot16	I pay attention in science because my parents will be. disappointed if I do not do well in science.	
mot17	I study more for math class because my parents want me to.	
mot18	My parents expect me to do well in social studies.	
mot19	It is more important for to me to get good grades than to go out with my friends.	Competitive Motivation
mot20	Reading in my free time will help me get better grades than the other students in my class.	
mot21	I want to obtain the highest grade in my math classes and joining this club will help.	
mot22	Paying attention in science will help me do better on exams than the other students in my class.	
mot23	I study more for math class because I want to get higher grades than anyone else.	
mot24	I want to get better grades than the other students in my class.	

Table B. Details of the 36 GRIT Variables

SAS Variable Name	Item/Prompt
grit1	I give up easily when faced with an obstacle. (reversed)
grit2	I am easily discouraged. (reversed)
grit3	I give up easily. (reversed)
grit4	I am easily frustrated. (reversed)
grit5	I get annoyed by people. (reversed)
grit6	I avoid responsibilities. (reversed)
grit7	I forget to do things. (reversed)
grit8	I make careless mistakes. (reversed)
grit9	I quickly lose interest in the tasks I start. (reversed)
grit10	I react slowly. (reversed)
grit11	My interests change quickly. (reversed)
grit12	Sometimes I feel depressed. (reversed)
grit13	Sometimes when I fail I feel worthless. (reversed)
grit14	Sometimes, I do not feel in control of my school work. (reversed)
grit15	I am filled with doubts about my competence. (reversed)
grit16	I do not feel in control of my success in school. (reversed)
grit17	There are times when things look pretty bleak and hopeless to me. (reversed)
grit18	I get discouraged when things go wrong. (reversed)
grit19	I worry. (reversed)
grit20	I get stressed out easily when things don’t go my way. (reversed)
grit21	I worry about school. (reversed)
grit22	I get upset easily. (reversed)
grit23	I remain calm under pressure.
grit24	I seldom get mad.
grit25	I am diligent.
grit26	I complete tasks successfully.
grit27	When I try, I generally succeed.
grit28	I am confident I get the success I deserve in life.
grit29	Overall, I am satisfied with myself.
grit30	I determine what will happen in my life.
grit31	I am capable of coping with most of my problems.
grit32	I overcome challenges and setbacks.
grit33	I am resilient.
grit34	I am relaxed.
grit35	I am not easily bothered by things.
grit36	I remain calm when I have a lot of homework to do.