Paper 4400-2015

SAS® Analytics plus Warren Buffett's Wisdom Beats Berkshire Hathaway! Huh?

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ABSTRACT

Individual investors face a daunting challenge. They must select a portfolio of securities comprised of a manageable number of individual stocks, bonds and/or mutual funds. An investor might initiate their portfolio selection process by choosing the number of unique securities to hold in their portfolio. This is both a practical matter and a matter of risk management. It is practical because there are tens of thousands of actively traded securities from which to choose, and it is impractical for an individual investor to own every available security. It is also a risk management measure because investible securities bring with them the potential of financial loss – to the point of becoming valueless in some cases. Increasing the number of securities in a portfolio decreases the probability that an investor will suffer drastically from corporate bankruptcy, for instance. However, holding too many securities in a portfolio can restrict performance. After deciding the number of securities to hold, the investor must determine *which* securities they will include in their portfolio and what proportion of available cash they will allocate to *each* security. Once their portfolio is constructed, the investor must manage the portfolio over time. This generally entails periodically reassessing the proportion of each security to maintain as time advances, but may also involve the elimination of some securities and the initiation of positions in new securities. This paper introduces an analytically driven method for portfolio security selection based on minimizing the mean correlation of returns across the portfolio. It also introduces a method for determining the proportion of each security that should be maintained within the portfolio. The methods for portfolio selection and security weighting described herein work in conjunction to maximize expected portfolio return, while minimizing the probability of loss over time. This involves a re-visioning of Nobel Laureate Harry Markowitz's prize-winning concept known as the "Efficient Frontier". Resultant portfolios are assessed via Monte Carlo simulation and results are compared to the Standard & Poor's 500 Index and Warren Buffett's Berkshire Hathaway, which has a well-establish history of beating the Standard & Poor's 500 Index over a long period. To those familiar with Dr. Markowitz's Modern Portfolio Theory this paper may appear simply as a repackaging of old ideas. It is not.

INTRODUCTION

The golden goose lays scrambled eggs! What does it mean to diversify investments and why is diversification important? In his 2014 annual letter to shareholders of Berkshire Hathaway stock, CEO Warren Buffett (2015) had the following to say about diversification and risk:

> Stock prices will always be far more volatile than cash-equivalent holdings. Over the long term, however, currency-denominated instruments are riskier investments – far riskier investments – than widely-diversified stock portfolios that are bought over time and that are owned in a manner invoking only token fees and commissions. That lesson has not customarily been taught in business schools, where volatility is almost universally used as a proxy for risk. Though this pedagogic assumption makes for easy teaching, it is dead wrong: Volatility is far from synonymous with risk.

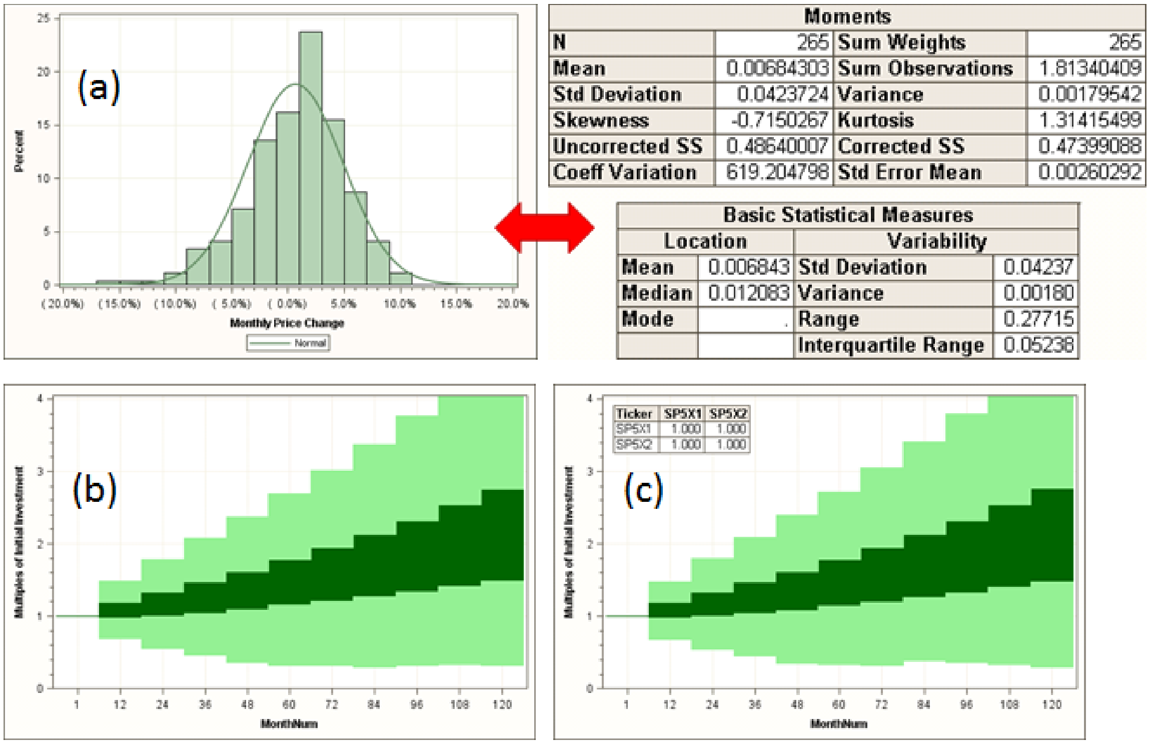
A SIMPLE THOUGHT EXPERIMENT

Diversification, if done well, serves to limit investment risk and boost portfolio performance. Effective portfolio diversification involves the assemblage of securities with low correlation of periodic returns among each pair. In other words, a well-diversified portfolio is one that has a low average correlation, where the average is taken over all constituent pairings. Simply adding more securities to a portfolio does not guarantee any better diversification. The power of minimizing average portfolio correlation is easily illustrated by a thought experiment involving comparison of two hypothetical portfolios. Consider Portfolio A, which is comprised of two *identical* instances of a mutual fund that tracks the S&P 500. An investment is split equally among the two instances of this fund. For example, a $1,000 initial investment is made in such a way that $500 is invested in the fund held in one account and simultaneously $500 is invested in another account holding exactly the same fund. How will Portfolio A perform? It should be obvious that it will perform *identically* to a portfolio that has the full $1,000 invested in a single instance of the fund held in a single account. This is true because the investment strategy of Portfolio A results in a portfolio with *perfect correlation* of returns among the securities held. No effective diversification has

been realized, though the investment is split among two holdings. Though this is obvious in this trivial example, many investors assemble portfolios that effectively attempt to “diversify” in a similar fashion, i.e., by including several funds that have very high correlation among them.

Now consider Portfolio B, which is similarly comprised of two instances of the S&P 500 tracking fund. This time, however, the correlation between the two instances is broken. This is accomplished (hypothetically) by again splitting the $1,000 initial investment equally, with $500 invested in the fund held in one account and simultaneously $500 is invested in a different strange account. In the strange account the tracking fund that has the same mean and the same standard deviation of monthly returns as the S&P 500, but the month-to-month price movement of the fund held in the strange account differs in a random way from the first. In other words, the price movement between these funds in this hypothetical portfolio occurs in such a way that if one fund’s price increases, the other fund’s price may increase, decrease or remain unchanged without any correlation between the two. However, over a long period of time, the two funds exhibit *identical* histograms of return. How will Portfolio B perform? Remarkably, Portfolio B will substantially outperform Portfolio A and the outperformance will continually increase with time. Even more intriguing, performance will be enhanced further in a Portfolio C containing five funds (again, with the initial investment split equally across each fund), but also constructed like Portfolio B such that no correlation exists between the funds and each fund exhibits an identical histogram of returns.

Figures 1-3 illustrate this point by showing results of a Monte Carlo simulation process that samples monthly returns of the S&P 500 with replacement (PROC SURVEYSELECT with METHOD=URS and SAMPRATE=1). In each figure the data sampled are derived from monthly S&P 500 returns for the 265 month period spanning February 1982 through February 2015. Figure 1(a) shows the histogram of monthly returns for the S&P 500 (PROC SGPLOT) and basic distribution statistics (PROC UNIVARIATE). The distribution is non-normal with negative skew. The plots shown in Figure 1(b-c) are box and whisker plots (PROC SGPLOT with a VBOX statement), for which the whiskers are the same width as the box and have been made to extend through all extreme values. The dark green band shows the interquartile range. Figure 1(b) shows the result of sampling the distribution of Figure 1(a) 50,000 times with replacement. Figure 1(c) illustrates the outcome of Portfolio A, which is based on the data of Figure 1(a) after being sampled 50,000 with replacement, with two instances of the S&P 500 fund sampled as a pair. Results of sampling Portfolio B are shown in Figure 2, with the two instances of the S&P 500 sampled independently, thus breaking the correlation. Figure 3 shows results for Portfolio C with 10 instances of the S&P 500 sampled independently. Pearson Correlation matrices (PROC CORR) are also shown within the images. All price data for Figures 1-3, as well as for all securities discussed throughout this paper, were obtained freely at https://finance.yahoo.com. In every case corporate share splits are accounted for and dividend and gains distributions are included and assumed to be reinvested in the share of the underlying security.



Moments			
N	265	Sum Weights	265
Mean	0.00684303	Sum Observations	1.81340409
Std Deviation	0.0423724	Variance	0.00179542
Skewness	-0.7150267	Kurtosis	1.31415499
Uncorrected SS	0.48640007	Corrected SS	0.47399088
Coeff Variation	619.204798	Std Error Mean	0.00260292

Basic Statistical Measures			
Location		Variability	
Mean	0.006843	Std Deviation	0.04237
Median	0.012083	Variance	0.00180
Mode	.	Range	0.27715
		Interquartile Range	0.05238

Figure 1. Panel (a) shows the histogram of monthly returns or the S&P 500 fund. To the right of panel (a) basic statistics for the distribution are displayed. Panel (b) shows a single instance of the S&P 500 fund also sampled 50,000 times with replacement. Panel (c) shows two distinct instances of the S&P 500 tracking fund sampled 50,000 times as a pair, with replacement. The correlation matrix is also shown in the inlay. In both cases the expected value after 10 years (120 months) is 2.0x the initial investment and the probability of a nominal loss is P=0.07.

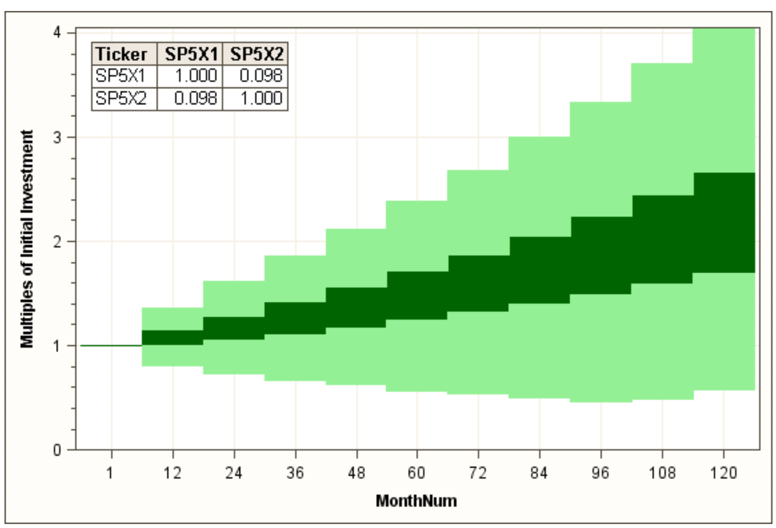


Figure 2. Two instances of the S&P 500 tracking fund sampled independently 50,000 times with replacement. The expected value after 10 years (120 months) is 2.1x the initial investment and the probability of a nominal loss after 10 years of P=0.01. The correlation matrix is also shown in the inlay.

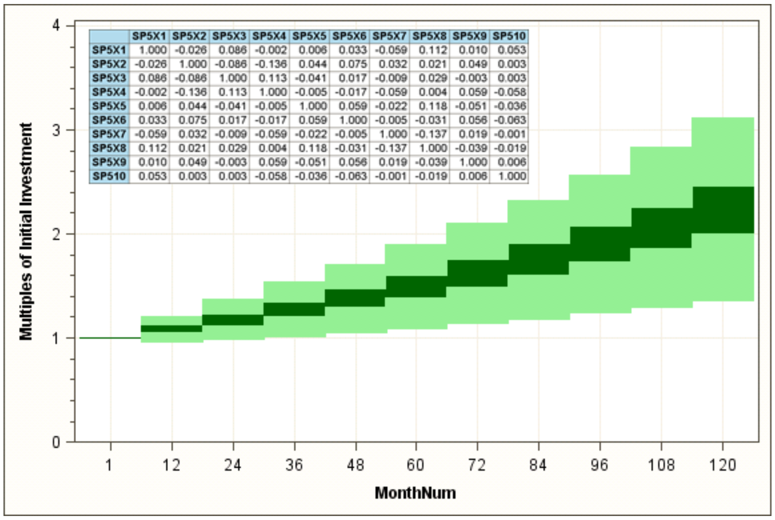


Figure 3. Ten instances of the S&P 500 tracking fund sampled independently 50,000 times with replacement. The expected value after 10 years (120 months) is 2.2x the initial investment and the probability of a nominal loss after 10 years of P=n/a. The correlation matrix is also shown in the inlay.

Given the tantalizing result of this thought experiment, as exemplified in Figure 3, one might feel a bit melancholy at this moment realizing that this is the outcome of a purely artificial portfolio construction that could never actually exist. However, from this thought experiement emerges a truly powerful idea that can be harnessed and used as the basis for construction of portfolios that not only outperform the S&P 500, but also outperform Berkshire Hathaway over 5 and 10 year periods. In particular, by systematically constructing correlation matricies of periodic (monthly) returns from a population of candidate securities and then identifying the portfolio that minimizes the average correlation among securities, *i.e.*, *by minimizing the average value of all non-diagnonal entries of the Pearson correlation matrix for any given number of securities, portfolios emerage with return vs. risk characteristics that prove extraordinarily attractive*.

EXPLORING MONTE CARLO SIMULATION FOR STOCK-BASED SECURITIES

Some may wonder if the Monte Carlo simulation approach of sampling with replacement is a valid method for analyzing the S&P 500 and other stock-based securities, *e.g.* individual stocks, stock indexes, exchange traded funds and mutual funds. One concern is that any autocorrelation exhibited by the actual time series is completely ignored by this simple resampling method. For example, resampling in this way could position the return associated with January 1996 adjacent to that for June 2004, which may be positioned adjacent to that for October 1985. If autocorrelation is of significance, then any conclusions drawn from this random reshuffling of the data would be suspect. To determine if autocorrelation is present in the S&P 500 time series sampled in monthly intervals an augmented Dickey-Fuller test (PROC AUTOREG) with m=3 is conducted to test the null hypothesis of unit root in the AR polynomial. Figure 4 shows the output from PROC AUTOREG employing the model statement: `model y = / stationarity = (adf=3).` The statistics Tau and Rho for the linear time trend both have p-values much greater then 0.05, providing no support for rejection of the null of unit root and suggesting that the output series may a difference-stationay process.

Augmented Dickey-Fuller Unit Root Tests							
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	F	Pr > F
Zero Mean	3	0.3658	0.7722	2.5499	0.9976		
Single Mean	3	-1.5159	0.8336	-1.2279	0.6641	4.4365	0.0606
▶ Trend	3	-5.9365	0.7477	-1.7645	0.7208	1.7460	0.8283

Figure 4. Augmented Dickey-Fuller Test Results.

With the null of unit root retained, we go on to look for evidence that AR terms are appropriate in a model of the differenced time series for S&P 500. Again, PROC AUTOREG is used, but now with a model statement: `model dy = / nlag=1 method=ml dwprob.` The dwprob option produces p-values for testing positive and negative autocorrelation from Durbin-Watson statistics. The output is shown in Figure 5. Both p-values are much greater than 0.05. Also shown in Figure 5 is the p-value of 0.2562 for the AR1 parameter estimate of -0.0581. Therefore, we can conclude that there is no statistically significant evidence of the S&P 500 exhibiting autocorrelation when sampled in monthly intervals over the 32 year period spanning February 1983 through February 2015.

Durbin-Watson Statistics			
Order	DW	Pr < DW	Pr > DW
1	1.9921	0.4692	0.5308

NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.

Parameter Estimates					
Variable	DF	Estimate	Standard Error	t Value	Approx Pr > |t|
Intercept	1	0.006952	0.002372	2.93	0.0036
AR1	1	-0.0581	0.0511	-1.14	0.2562

Figure 5. Estimating the differenced S&P 500 time series with an AR(1) error term.

Another concern with resampling might stem from the popular belief that certain months are special in the financial markets. For example, there is the "January Effect", which some believe brings exceptionally strong gains to investors in the month of January. Then there is the Wall Street saying, "As goes January, so goes the year." There is another bit of Wall Street wisdom that encourages investors to "Sell in May and go away." This expresses a belief that stock investments have limited upside during the summer vacation season as traders take time away. Finally, let's not ignore the common wisdom that October is the "treacherous month" for investors, as major downside risk seems to be ever-present in October. Fortunately, as shown in Figure 6, ANOVA analysis (PROC ANOVA) of the S&P 500 sampled in monthly intervals over the period between February 1983 and February 2015 reveals no statistically significant evidence that the series behaves differently from one month to the next, with a p-value of 0.5386.

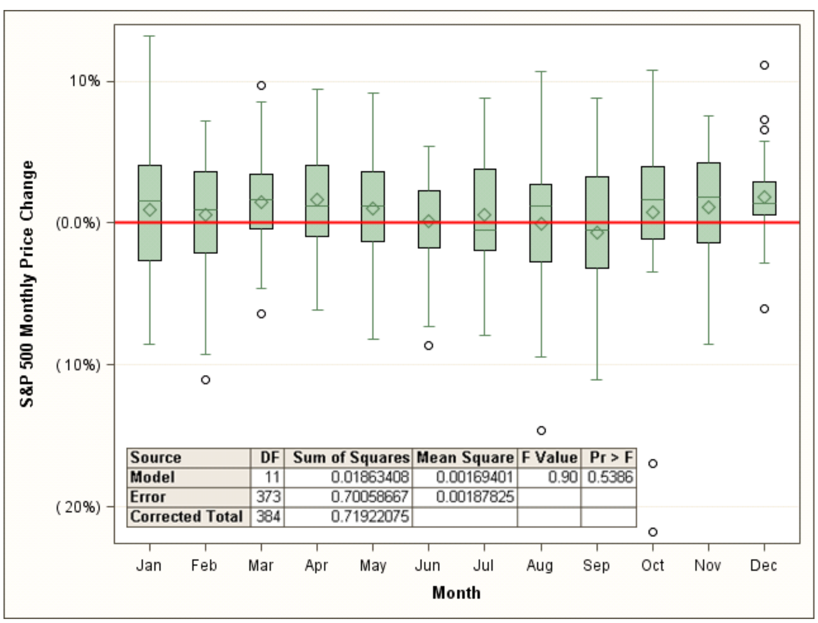


Figure 6. A Box and Whisker plot with ANOVA results displayed in the inlay.

WHAT IS AND WHAT IS NOT DESCRIBED IN THIS PAPER

Except in rare cases, the points illustrated in Figures 5 and 6 extend to most stocks, exchange traded funds and mutual funds. Unless otherwise stated, these points may be assumed true for all securities referenced throughout this paper. This paper focuses attention on portfolios assembled from among a wide variety of stock mutual funds from Vanguard®, T. Rowe Price®, American Funds®, Fidelity® and others. While the portfolio selection technique detailed in this paper extends directly to individual stocks, individual stock portfolios are not described. To be sure, neither individual corporate or government bonds nor bond mutual funds are considered for inclusion among the portfolios described in this paper.

This paper offers nothing to those looking to forecast the next minute, hour, day or year of return in a security. To be clear, this paper does not describe any new near-term trading strategy. To that end, consider Warren Buffett's (2015) observation in his 2014 annual letter to shareholder:

> Investors, of course, can, by their own behavior, make stock ownership highly risky. And many do. Active trading, attempts to "time" market movements, inadequate diversification, the payment of high and unnecessary fees to managers and advisors, and the use of borrowed money can destroy the decent returns that a life-long owner of equities would otherwise enjoy. Indeed, borrowed money has no place in the investor's tool kit: Anything can happen anytime in markets. And no advisor, economist, or TV commentator – and definitely not Charlie nor I – can tell you when chaos will occur. Market forecasters will fill your ear but will never fill your wallet.

Instead, this paper details an analytically-driven technique for constructing portfolios of mutual funds that offer a high reward-to-risk ratio. In this context, the long-term (e.g., 5 or more years) expected return (*i.e.*, reward), is optimized against the *probability of a nominal loss of invested capital over the period of investment* (*i.e.*, risk). It is important to note that, as Warren Buffett (2015) decried in his 2014 annual letter to shareholders, risk is *not* defined in this paper as variance of return, as Markowitz defined it. The process of portfolio development is broken into three distinct steps: (1) determine the number of securities to hold, (2) determine which securities to hold from among a population of possible options, and (3) determine the proper allocation of an initial investment to devote to each security held.

Implicitly embodied in the technique described in this paper are the eight basic rules for investors espoused by John Bogle (1999), Founder and retired CEO of The Vanguard Group:

1. Select low-cost funds
2. Consider carefully the added cost of advice
3. Do not overrate past fund performance
4. Use past performance to determine consistency and risk
5. Beware of stars (star fund managers)
6. Beware of asset size
7. Don't own too many funds
8. Buy your fund portfolio – and hold it

Finally, consider the use of "long-term" in the context of investment timeframes assessed in this paper. In his 2014 annual letter to shareholders, Warren Buffett (2015) advised investors, "Since I know of no way to reliably predict market movements, I recommend that you purchase Berkshire shares only if you expect to hold them for at least five years. Those who seek short-term profits should look elsewhere." To be sure, Buffett's advice is wisely heeded in regard to *any* stock or stock mutual fund investment. It is with this in mind that the label "long-term" is applied to investment horizons of ten years throughout this paper. In particular, portfolio performances are evaluated against a 10 year horizon. This window is particularly valuable, since it includes the disastrous Great Recession and bear market of 2007-2009 and the strong extended bull market that followed. This period is unprecedented in history in that it is the first bull market driven by a series of previously untested "quantitative easing" policies implemented by the U.S. Federal Reserve. (Board of Governors of the Federal Reserve System, 2013)

PORTFOLIO CONSTRUCTION

POPULATION OF SECURITIES FROM WHICH TO CONSTRUCT A PORTFOLIO

Every stock mutual fund is a candidate for selection in the construction of a portfolio of mutual funds. However, that broad universe of candidates can (and should) be winnowed down based on the wisdom of Buffett and the eight basic rules of Bogle. Certainly funds that carry high management fees, high front- or back-end load fees, excessive 12b-1 marketing expenses or other excessive fees should be immediately considered for exclusion. Though, in some cases of specialized funds certain load fees may be justified. Also, funds that have a short trading history can be excluded. Generally, a fund should be old enough to have produced results through two full economic cycles. The specific criterion used in this paper is that a fund must have an inception date of June 1, 1993 or earlier in order to be considered. Also, a fund must be open to new investors and must not require an initial investment of more than $5,000. With these broad filters in place and only for the purpose of demonstrating the power of the portfolio construction algorithm described in this paper, all appropriate stock mutual funds offered by Vanguard, T. Rowe Price, American Funds and Fidelity are initially considered to be within the population of candidate funds. Additionally, the 10 oldest actively traded funds are included along with others. The full list of funds considered for portfolio construction is shown in Appendix Table A. In order to acquire historical price data for the securities listed in Appendix Table A, SAS code was adapted from the work of DeVenezia (2010), who developed a SAS MACRO to download data from Yahoo! Finance web pages.

DETERMINATION OF PORTFOLIO SIZE AND CONSTITUENT SECURITIES

In determining the optimal number of securities to hold in the portfolio, recall rule 7 of Bogle's eight basic rules for investors, in which Bogle (1999) encourages investors to avoid owning too many funds. In his book *Common Sense on Mutual Funds*, Bogle provides the guidance, "I truly believe that it is generally unnecessary to go much beyond four or five equity funds. Too large a number can easily result in overdiversification. The net result: a portfolio whose performance inevitably comes to resemble that of an index fund." However, it is possible to work out an analytical process that provides guidance to selecting the right number. As far as the author is able to determine, this paper is the only place that such an analytical process is described. In the Introduction section the reader was acquainted with the core idea explored in this paper. In particular, the significance of selecting the portfolio that minimizes the average value of all non-diagonal entries of the Pearson correlation matrix for any given number of securities cannot be overstated in relation to producing desireable return vs. risk characteristics. This idea guides the investor in both determining the right number of securities to hold in her portfolio *and* in identifying precisely which securities to include from the population of candidate securities.

PortSize, AveCorr and ExpRtn Defined

Throughout this paper the variable name ***PortSize*** refers to the number of securities held within a given portfolio. The variable name ***AveCorr*** refers to the average value of all non-diagonal entries of the Pearson correlation matrix for any portfolio of size ***PortSize***. Think of ***AveCorr*** as a measure of how well the eggs in a portfolio are "scrambled." Finally, the variable name ***ExpRtn*** refers to the median value of the historical distribution of monthly returns exhibited by a portfolio of size ***PortSize***. For the results shown in Figure 7 and the associated discusssion, ***AveCorr*** and ***ExpRtn*** are determined assuming that ***PortSize*** securities enter into the portfolio *in equal proportion*. To be abundently clear, however, in Figure 8 the Monthly Return values shown along the abscisa are fully weighted returns, not returns based on an assumption of equal proportioned securities.

Determination of the Number of Securities to Hold Within a Portfolio

The variables ***AveCorr*** and ***ExpRtn*** are used to determine the optimal value of ***PortSize***. This is illustrated in Figure 7. With the reader's first exposure to this concept, the process may seem complicated and a bit confusing, primarily due to nomenclature. If that proves true, the reader should take time to carefully read through this section a second or third time. It will become clear with careful and uninterrupted reading. Nonetheless, the reader seeking only an overview of the main ideas captured in this paper may elect to skip this section and trust that the process yields an optimal value of ***PortSize*** in the range of four to seven mutual funds. This range will be different when the portfolio is constructed

exclusively from individual stocks, but that is a topic for a different paper. In later sections of this paper the optimal value of ***PortSize*** is set at five as a result of the process illustrated in Figure 7.

The process of determining the optimal value of ***PortSize*** considers the outcome of selecting a portfolio based on two completely different approaches. The first approach seeks to identify the portfolio that results in the *minimum* value of ***AveCorr*** for each value of ***PortSize***. This shall be referred to as the AveCorr Approach. The second approach seeks to identify the portfolio that results in the *maximum* value of ***ExpRtn*** for each value of ***PortSize***. This shall be referred to as the ExpRtn Approach. In Figure 7(a-c) results of the AveCorr Approach are represented by the green lines and results of the ExpRtn Approach are represented by the orange lines.

Figure 7(a) shows results for the variable ***AveCorr*** vs. ***PortSize*** for each of these approaches. It should not be a surprise that the AveCorr Approach results in the minimum value of ***AveCorr*** for each value of ***PortSize***. Notice that as ***PortSize*** increases, ***AveCorr*** increases monotonically. In other words, when the AveCorr Approach is employed, the apparent diversification benefit derived by adding more securities to a portfolio becomes relatively *less* with each additional security added to the portfolio. Be aware that the portfolios at each value of ***PortSize*** are not arbitrary. Rather, they are precisely known and are comprised of the securities that minimize ***AveCorr*** at any given value ***PortSize***. In other words, though we are only attempting to determine to optimal value of ***PortSize*** at present, once the optimal value is identified, the actual set of securities will also be fully specified, leaving one only with the problem of determining how to allocate capital to each security in order to fully define the complete portfolio. The orange line in Figure 7(a) shows the value of ***AveCorr*** for each value of ***PortSize*** when the ExpRtn Approach is employed. This line is not the focal point of Figure 7(a). It is shown only to aid in the interpretation of Figure 7(c) to follow. Nonetheless, notice that ***AveCorr*** does not increase monotonically when the ExpRtn Approach is employed. Also notice that the ExpRtn Approach results in higher values of ***AveCorr*** for each value of ***PortSize***, as compared to the AveCorr Approach, indicating that each portfolio identified in the ExpRtn selection process is less diversified than a portfolio identified by the AveCorr Approach at any given value of ***PortSize***.

Figure 7(b) shows results for the variable ***ExpRtn*** vs. ***PortSize*** for each of these approaches. It should not be a surprise that the ExpRtn Approach results in the maximum value of ***ExpRtn*** for each value of ***PortSize***. The most important element of this panel is the plateau on the green line (for the AveCorr Approach) in the range of ***PortSize*** four to seven, which is highlighted within the light red box. This region shows the highest values of ***ExpRtn*** for the AveCorr Approach at small values of ***AveCorr***. This is much easier to grasp in Figure 7(c). Figure 7(c) shows values for the ***ExpRtn*** data of Figure 7(b) divided by the values of the ***AveCorr*** data of Figure 7(a) at each value of ***PortSize***. In Figure 7(c) the portion of the AveCorr Approach line (green) residing above the ExpRtn Approach line (orange), represents acceptable values of ***PortSize.*** This region is highlighted within the light red box. At this point any portfolio within the indicated size range may be selected, so the value of ***PortSize***=5 is chosen. It is noteworthy that the outcome of this process coincides with the Bogle's guidance of four or five funds, which he stated only as opinion, unsupported by analysis[1].

[1] For the inquisitive reader, it is well worth the time to review Figure 4.6 on page 102 and the associated text in John Bogle's excellent book, *Common Sense on Mutual Funds: New Imperatives for the Intelligent Investor*. In fact, this author considers the entire book a must-read for every investor.

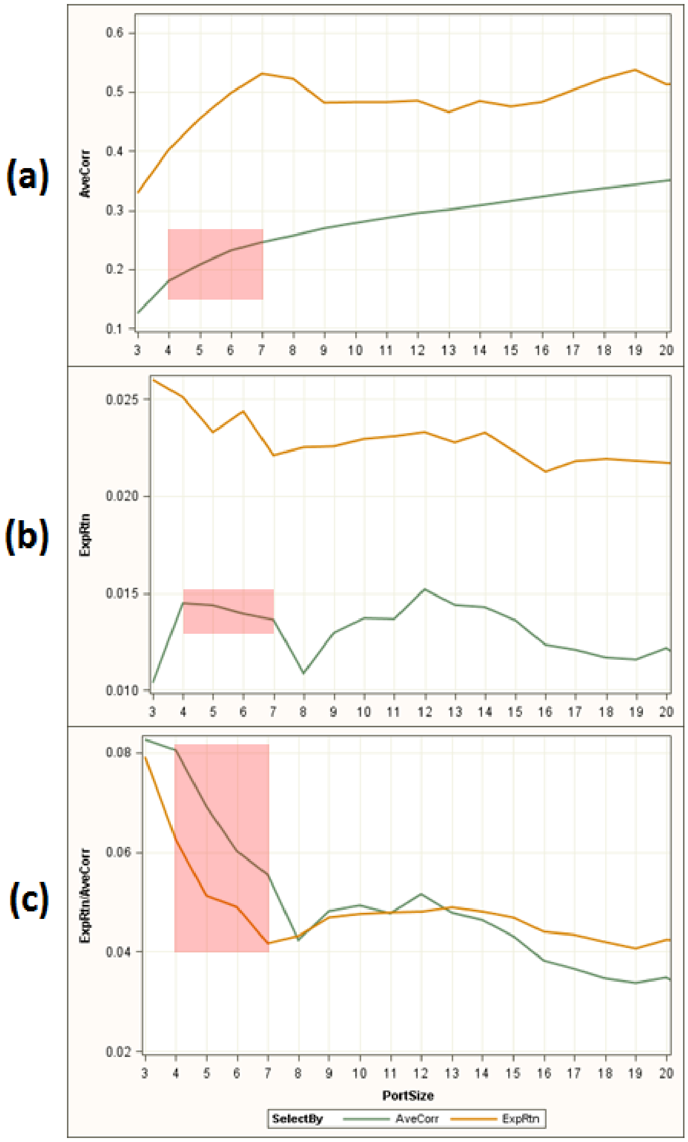


Figure 7. Illustration of the process for determining the optimal number of securities to hold in a portfolio for greatest return vs. risk characteristics. In each panel the abscissa (labled PortSize) is the number of securities held in a given portfolio. The light red highlighted region of panel (c) corresponds to desireable values of PortSize. Only the results for PortSize between 3 – 20 are shown. For the population shown in Appendix Table A, PortSize could be as large as 80.

A Note Regarding the Portfolio Selection Algorithm

The portfolio sizing process shown in Figure 7 yields not only the number of securities to hold within a portfolio. It also identifies the actual subset of securities emerging from the population of candidate securities shown in Appendix Table A, for example. This is a remarkably beneficial outcome when one considers how many combinations exist when selecting 5 securities from a population of 80 candidates. The math is 80!/5!75!=24,040,016 possible 5-component portfolios. It's noteworthy that this process works equally well with individual stocks. Consider what that means if a 20-component stock portfolio is selected from the population of 500 stocks comprising the S&P 500 index. In that case the math is 500!/20!480!=2.7x10^35 possible 20-component portfolios. That would be a big do-loop. To put it into better perspective, if each iteration of the loop were to take 1 microsecond to complete (*i.e.*, 1 million iterations per second), the entire process would take 8.5x10^21 years to run, whereas The Universe is estimated to be 13.77x10^9 years old (Universe 101, 2012). The algorithm behind Figure 7 works within minutes to select a portfolio of any size from the S&P 500, as well as from the funds in Appendix Table A.

The Resultant Portfolio Constituents

Because it is desirable to show validation results for the portfolio selected through the process described by Figure 7, the data involved only span the period February 1993 – February 2005[2]. This leaves the long-term 10-year period of March 2005 – February 2015 over which the performance of the selected portfolio can be observed for validation. It is important to understand this when reviewing the resultant portfolio, which is shown in Table 1.

Fund Name	Symbol
Fidelity Real Estate Investment	FRESX
Fidelity Select Biotechnology	FBIOX
Fidelity Select Consumer Staples	FDFAX
Fidelity Select Gold	FSAGX
T. Rowe Price International Discovery	PRIDX

Table 1. Portfolio selected by the process described in Figure 7 from the population of candidate mutual funds shown in Appendix Table A. This portfolio is the result of minimizing the value of AveCorr (0.208) for Portsize=5.

Had the portfolio for the ExpRtn process been selected, the resultant portfolio would include the securities identified in Table 2.

Fund Name	Symbol
Fidelity Select Brokerage & Invmt Mgmt	FSLBX
Fidelity Select Communications	FSDCX
Fidelity Select Technology	FSPTX
Fidelity Select Gold	FSAGX
Fidelity Select Materials	FSDPX

Table 2. Portfolio not selected by the process described in Figure 7. This portfolio is the result of maximizing the value of *ExpRtn* for *Portsize*=5, which results in a value of *AveCorr*=0.455.

ALLOCATION OF CAPITAL TO EACH SECURITY HELD IN THE PORTFOLIO

The Nobel Prize winning concept of the "efficient frontier" was introduced by Dr. Harry Markowitz in the 1950s (The Royal Swedish Academy of Sciences, 1990). As defined by Investopedia (2015), the efficient frontier is "a set of optimal portfolios that offers the highest expected return for a defined level of risk or the lowest risk for a given level of expected return. Portfolios that lie below the efficient frontier are sub-optimal, because they do not provide enough return for the level of risk. Portfolios that cluster to the right of the efficient frontier are also sub-optimal, because they have a higher level of risk for the defined rate of return." Defining "risk" simply as variance of returns, Dr. Markowitz termed portfolios residing along the efficient frontier as efficient portfolios.

[2] The author believes that 22 years is a more appropriate length of time for historical evaluation of security performance for portfolio construction. This is due to two primary considerations. First, under current law, it guarantees at least three Presidential administrations will occupy the Executive Branch of the U.S. government, with reasonable likelihood of at least one change in the dominant political party. Second, according to the National Bureau of Economic Research (NBER's Business Cycle Dating Committee, 2010) the average peak-to-peak length of the 11 documented U.S. business cycles between 1945 – June 2009 is 68.5 months (5.7 years), with a standard deviation of 36 months (3 years). The median length is 56 months (4.7 years), with an interquartile range of 48.5 months (4 years). Thus, 22 years provides a high likelihood that the U.S. economy will experience at least three full business cycles.

The process described in this paper for allocating capital to each security held in the portfolio of Table 1 resembles the efficient frontier, but it distinctly differs from the concept described by Markowitz. Most notably, the difference is in how risk is defined. The primary reason for seeking a different definition of investment risk is that use of variance of returns as a surrogate for risk seems to capture the *emotional* element of risk only about half of the time. Indeed, the author admits that there is inimitable irony in relying on an appeal to emotion as a foundational idea in a paper centered on analytics. Nonetheless, consider how an investor feels when her portfolio loses value over some time, *i.e.*, when her portfolio exhibits returns toward the left end of the distribution of historical returns. She may feel unsettled – she likely feels that her investment is risky. Now consider how her emotions might differ when events turn and the portfolio shows strong gains in value over a different period of time, *i.e.*, when her portfolio exhibits returns toward the right end of the distribution of historical returns. She might now feel happy, even elated. She might feel good about herself for making a smart choice in selecting her portfolio. She likely will not feel unsettled. Rather than running for cover and hiding from the "risk" of her investment, she might actually seek to add more capital to her portfolio. Such behavior is contrary to the behavior typically exhibited by those feeling that their investment is "risky". It seems that a definition of risk different from "variance of returns" would be beneficial. Indeed, Bogle (1999) also finds that variance of returns is an insufficient description of risk. He writes, "… risk is something far more difficult to quantify. It relates to how much you can afford to lose without excessive damage to your pocketbook or your psyche." Many others have raised objection to the use of variance of return as a measure of risk (Bernstein, 1996). However, to the best of the author's knowledge, the alternative proposed in this paper has not been used in calculations by others.

The idea behind the process of portfolio construction described herein is to assemble securities such that when capital is properly allocated among them, the portfolio generates the highest long-term expected return (reward) for a given probability of a nominal loss of invested capital (risk). The investor determines what degree of risk she is willing to accept. Then the portfolio is constructed in a manner that maximizes potential long-term return for that level of risk. In a sense, this is a re-envisioned form of the efficient frontier concept of Markowitz.

RISK CALCULATED AS THE PROBABILITY OF A NOMINAL LOSS OF INVESTED CAPITAL

An estimate of the probability distribution of losing capital in an increment of time is calculable by constructing a large number of portfolios and determining the distribution of zero returns across all portfolios for an increment of time. (Note: This has nothing to do with estimating the *magnitude* of a potential loss. It is strictly about assessing the probability that a loss of *any* magnitude will occur within an increment of time.) For example, a distribution of historical monthly returns for a single portfolio, $\boldsymbol{P^i}$, is constructed by multiplying the matrix, $\boldsymbol{R}$, of historical distributions of monthly returns for each security in Table 1 by a vector of weights, $\boldsymbol{w^i}$, where each component of $\boldsymbol{w^i}$ is a random number, but the components of $\boldsymbol{w^i}$ sum to 1. Certain additional constraints on $\boldsymbol{w^i}$ exist in order to ensure that a portfolio does not over- or underemphasize a security to an extreme. In particular, for all calculations described in this paper the components w_j^i of $\boldsymbol{w^i}$ are constrained as follows: $1/(2*\text{PortSize}) \le w_j^i \le 5/(2*\text{PortSize})$. In other words, for the portfolio constructed from the securities referenced in Table 1, the weighting given to each security is limited to between 10% and 50% of the total portfolio, as long as the combined weights sum to 100%.

For the results discussed in this paper one million such portfolios are calculated from one million distinct weight vectors, $\boldsymbol{w^i}$. The distribution of loss probabilities (risk) calculated in this fashion across all portfolios is shown in Figure 8(a). The histogram shows a discrete distribution because a finite number of months exist in the historical data. With 146 months used, the smallest increment possible between probability measurements is 1/146=0.006849. Of the one million portfolios calculated, the median probability of a loss in any given month is 0.342. Similarly, the expected monthly return can be calculated as the median return value for each portfolio. When the median monthly return is plotted against the probability of loss in any given month, a new sort of "efficient frontier" emerges. This is shown in Figure 8(b). Figure 8(b) is actually a scatter plot of one million open circles. Each circle represents a different portfolio construction. To be clear, the securities comprising each portfolio are those displayed in Table 1. The difference arises purely from the weighting given each security. As with the original Markowitz efficient frontier concept, the portfolios residing along the red line at the upper edge of Figure 8(b) represent the best choice for a given level of risk. The red arrow in Figure 8(b) signifies the portfolio construction selected as the best choice for very modest risk. As Figure 8(b) illustrates, there is

essentially no benefit in accepting more risk than this, since there is no point further to the right at which the expected monthly return increases as risk increases.

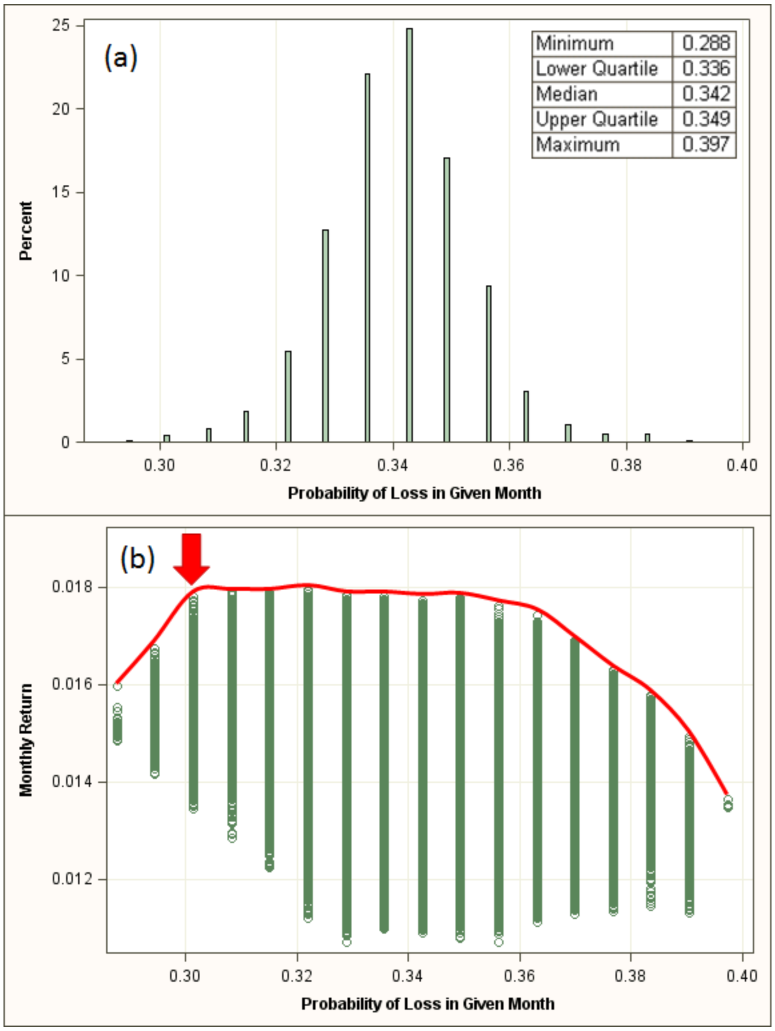


Figure 8. Panel (a) shows the histogram of the probability of negative returns in a given month for the full set of one million portfolio weightings applied to the monthy return distributions for the securities displayed in Table 1. Panel (b) shows a new type of efficient frontier for the same set of securities as in panel (a). The red line indicates the set of portfolios with the greatest expected monthly return for a given risk, where risk is defined as the probability of loss in a given month. The red arrow indicates the portfolio chosen for further assessment.

All that is left to do to fully specify the portfolio is to look up the values for the weight vector associated with the portfolio designated in Figure 8(b). The result is shown in Table 3.

Fund Name	Symbol	Weighting
Fidelity Real Estate Investment	FRESX	31.2%
Fidelity Select Biotechnology	FBIOX	14.6%
Fidelity Select Consumer Staples	FDFAX	10.6%
Fidelity Select Gold	FSAGX	12.0%
T. Rowe Price International Discovery	PRIDX	31.7%

Table 3. Final portfolio selected exclusively from month return data spanning the period February 1993 through February 2005.

COMPARATIVE LONG-TERM PERFORMANCE OF THE AVECORR PORTFOLIO RELATIVE TO THE S&P 500 INDEX AND BERKSHIRE HATHAWAY

The portfolio specified in Table 3 has been developed by a process designed to produce portfolios with exceptional long-term reward-to-risk characteristics. These portfolios tend to align strongly with Bogle's (1999) eight basic rules for investors. Most notably, these are expected to be portfolios that investors hold for the long-term. Without some sort of estimate of long-term expected results, investors may be reluctant to trust the outcome of this process for the period of time prescribed. We can address this by turning once again to the Monte Carlo simulation approach relied upon in the introduction of this paper. Figure 9 shows the results of sampling the portfolio of Table 3 with replacement 50,000 times. From this approach it is a simple matter to determine that the expected return after 10 years is 3.0x the initial investment and the probability of a loss after 10 years is 0.0018, with a 99% CI of 0.0015 – 0.0023. (Note: The probability of loss is taken as the mean probability of loss after 100 runs of the Monte Carlo simulation, where the MC simulation involves 50,000 cases per run. That is, the confidence intervals are derived via bootstrapping.) That seems decent, but a prudent investor will ask how it compares to alternatives.

EXPECTED PERFORMANCE ASSESED WITH MONTE CARLO SIMULATION

It is useful to compare results to the S&P 500 index, which is a common benchmark for mutual fund investing. As presented in Figure 1(b), recall that resampling of the S&P 500 return data yielded an expected return after 10 years of 2.0x the initial investment and a probability of loss after 10 years of 0.07. The reader should be careful not to use Figure 1(b) for comparison, however, as the data underlying that figure span the period February 1982 through February 2015, which includes the Black Monday market crash of October 19, 1987 and the Great Recession of 2007– 2009. Contrast that with the time period of February 1993 – February 2005 for the data underlying the results shown in Figure 9. For a true evaluation, it is essential that the comparative data for the S&P 500 span the same time period of the portfolio that will be compared to it. Figure 10 shows the Monte Carlo simulation of the S&P 500 corresponding to the same period as the data underlying Figure 9. Based on that data, the expected return for the S&P 500 after 10 years is 2.2x the initial investment and the probability of a loss after 10 years is 0.0434, with a 99% CI of 0.0409 – 0.0453. That is to say that the portfolio developed in this paper is expected to beat the return of the S&P 500 by more than 36% over 10 years, while exposing the investor to only 1/24 of the probability of a nominal loss compared to the S&P 500.

It is not often that performance of a mutual fund or a portfolio is compared to Berkshire Hathaway. A likely reason is that the overwhelming majority of stock mutual funds and individual stocks underperform against it over most multi-year periods. Given the strong influence that the teachings of Warren Buffett have had on framing the concepts described within this paper, though, it is appropriate to benchmark the resultant portfolio of Table 3 against Berkshire Hathaway. Figure 11 shows the Monte Carlo simulation of Berkshire Hathaway, again with underlying data spanning the same period as the data underlying Figure 9. The simulation reveals that the expected return for Berkshire Hathaway after 10 years is 4.9x the initial investment and the probability of a loss after 10 years is 0.0123, with a 99% CI of 0.0116 – 0.0137. Based on these results, the portfolio developed in this paper is expected to underperform Berkshire Hathaway by nearly 39% over 10 years, but exposing the investor to only about 1/7 of the probability of a nominal loss compared to Berkshire Hathaway.

PERFORMANCE BASED ON ACTUAL MARKET DATA

If validation data were unavailable, as would be the case if the portfolio had been developed with the complete market history from February 1993 through the current month, anticipated performance of the resultant portfolio could only be determined by Monte Carlo simulation or other time series forecasting techniques. However, validation data has been set aside in this case, so a comparison with actual performance is possible. To be clear, the resultant AveCorr portfolio was developed from 12 years of historical data spanning the period February 1993 – February 2005, and its performance is to be compared against 10 years of actual data spanning the period March 2005 – February 2015. In addition, the performance is compared to the actual histories of the S&P 500 and Berkshire Hathaway over the same time frame. The comparative results are shown in Figure 12.

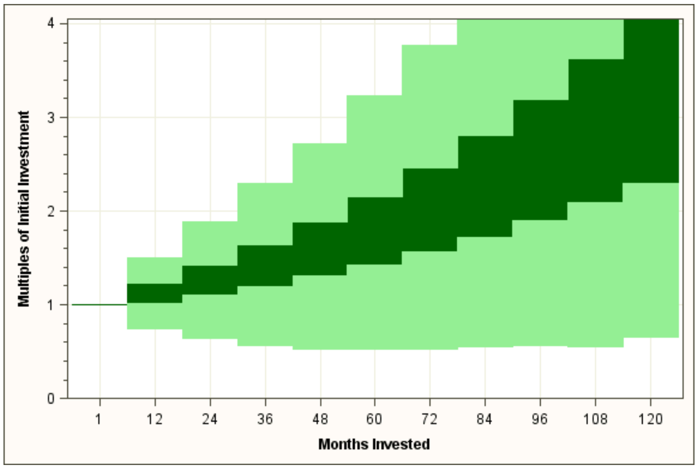


Figure 9. Box and whisker plot showing results of 50,000 simulations of performance of the portfolio specified in Table 3. Underlying data are from the period spanning February 1993 – February 2005.

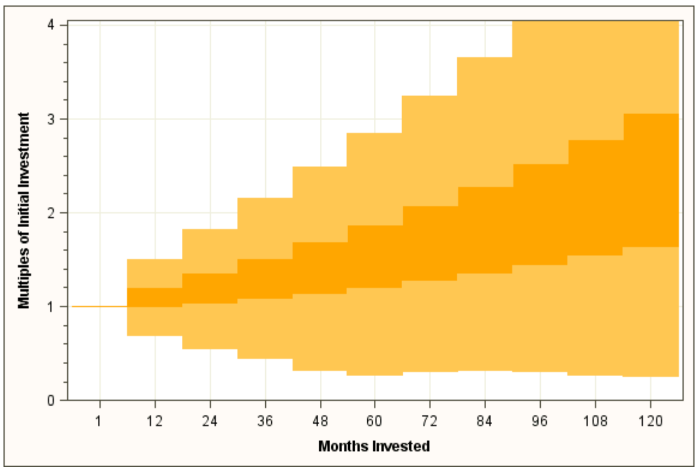


Figure 10. Box and whisker plot showing results of 50,000 simulations of performance of the S&P 500. Underlying data are from the period spanning February 1993 – February 2005.

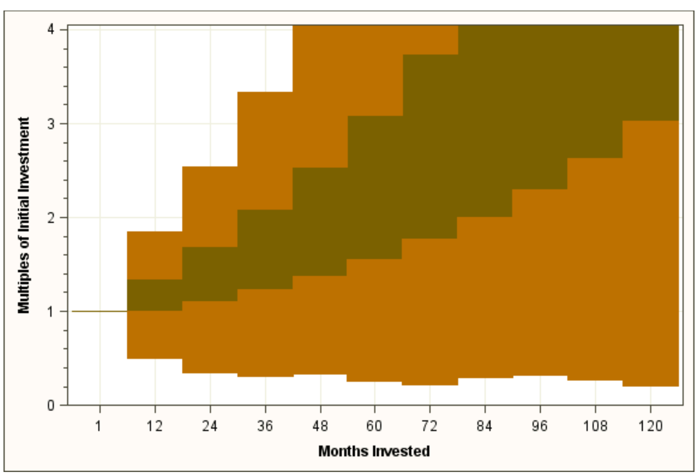


Figure 11. Box and whisker plot showing results of 50,000 simulations of performance of Berkshire Hathaway. Underlying data are from the period spanning February 1993 – February 2005.

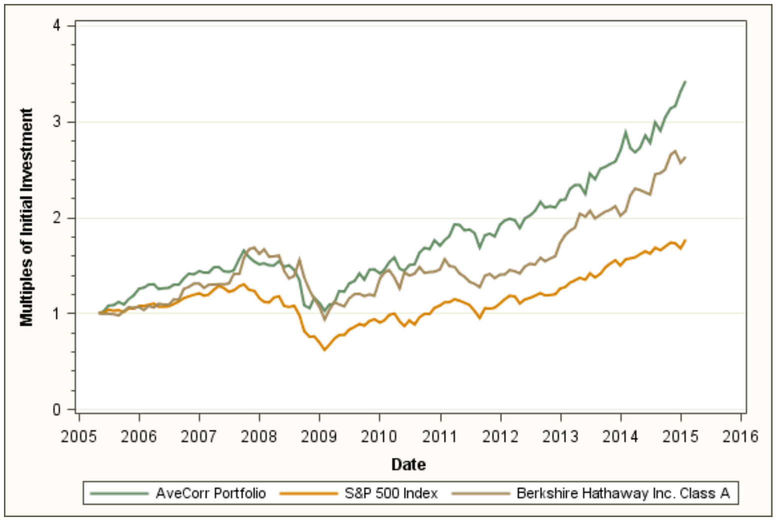


Figure 12. Comparative graph showing the 10-year performance of the portfolio specified in Table 3 (green line labeled "AveCorr Portfolio") compared against the S&P 500 (orange line) and Berkshire Hathaway (brown line). The AveCorr Portfolio line is based on constructing the portfolio exactly as specified in Table 3 and using the actual performance data of the underlying funds for the validation period of March 2005 – February 2015. All values are normalized to 1 as of March 1, 2005 for relative comparison. In every case dividends and distributions are reinvested in the security that produces them.

Indeed, the performance of the portfolio specified in Table 3 not only outperforms the S&P 500 over the long-term 10-year period under consideration, it also outperforms Berkshire Hathaway! Hence, the title of this paper is derived from Figure 12. Interestingly, the results of Monte Carlo simulation presented in Figures 9 - 11 strongly suggest that the AveCorr portfolio would outperform the S&P 500, but it provides only modest support for an expectation that it would outperform Berkshire Hathaway. In fact, this is a shortcoming of the approach described in this paper. In particular, the usefulness of the approach outlined herein requires that the histogram of historical returns remains a valid representation of the portfolio over the unknown long-term future. If the distributions of future returns for the constituents of the portfolio substantially deviate from their historical distributions, then the Monte Carlo simulation results will prove inaccurate. In the comparison between Figure 9 -10 and Figure 12 for the AveCorr portfolio and the S&P 500, however, the expected return emerging from Monte Carlo simulation is dead on, with the validation data overlapping the Monte Carlo results with impressive accuracy. The outlying result comes from comparison between the Monte Carlo simulation for Berkshire Hathaway of Figure 11 and the actual results shown for Berkshire Hathaway in Figure 12. The fascinating thing about this comparison is that it actually highlights something that Warren Buffett (2015) eluded to in his 2014 annual letter to shareholders when he wrote about future prospects for the company's stock, "The bad news is that Berkshire's long-term gains – measured by percentages, not by dollars – cannot be dramatic and *will not come close* to those achieved in the past 50 years. The numbers have become too big. I think Berkshire will outperform the average American company, but our advantage, if any, won't be great." This prophetic expression seems already to be affecting the stock of Berkshire Hathaway. The reality is that the company's stock has substantially *underperformed* its own historical performance during the period March 2005 – February 2015, as compared to the period spanning February 1993 – February 2005. This is seen in Figure 13(a-b). Figure 13(a) shows the performance of Berkshire Hathaway relative to the S&P 500 for the earlier period and Figure 13(b) shows the same comparison for the more recent period – the period anticipated by Monte Carlo simulation in Figures 10 – 11. While Berkshire Hathaway outperformed the S&P 500 over both periods, the outperformance over the period shown in Figure 13(b) was notably less than that of Figure 13(a), even when accounting for the fact that the period for Figure 13(b) is two years less than that for Figure 13(a). The performance of the S&P 500 was about the same after 10 years in each case.

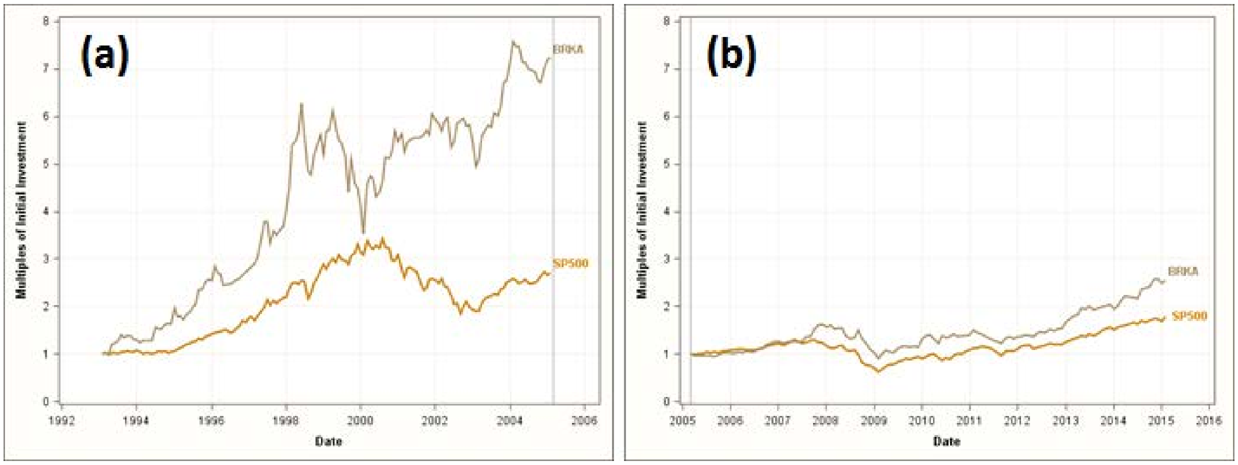


Figure 13. Comparison of the relative performance of Berkshire Hathaway (BRKA) to the S&P 500 over two different time periods. Panel (a) shows the comparison for the period spanning February 1993 – February 2005. Panel (b) shows the comparison for the period spanning March 2005 – February 2015.

COMPARATIVE LONG-TERM PERFORMANCE OF THE EXPRTN PORTFOLIO RELATIVE TO THE S&P 500 INDEX AND BERKSHIRE HATHAWAY

The process for portfolio selection described in this paper and illustrated in Figure 7 effectively pits two selection approaches against each other. To this point the emphasis has been placed on choosing the portfolio that results from the AveCorr Approach (*i.e.*, the 5-component portfolio that resulted in the minimum average value of the non-diagonal elements of the correlation matrix). Ignored until now has been the portfolio that emerged from the ExpRtn Approach (*i.e.*, the 5-component portfolio displayed in Table 2 that resulted from seeking to maximize the expected return of equally weighted portfolios at each value of ***PortSize***). Employing a process identical to that used to determine the most advantageous weightings for the securities displayed in Table 1; weightings have been calculated for the securities displayed in Table 2 for the ExpRtn Approach portfolio. These weightings are displayed in Table 4.

Fund Name	Symbol	Weighting
Fidelity Select Brokerage & Invmt Mgmt	FSLBX	33.1%
Fidelity Select Communications	FSDCX	10.2%
Fidelity Select Technology	FSPTX	11.9%
Fidelity Select Gold	FSAGX	31.2%
Fidelity Select Materials	FSDPX	13.6%

Table 4. The 5-component portfolio selected by the ExpRtn Approach. Weightings have been determined by the same process used to calculate weights for the portfolio appearing in Table 3.

One may have wondered if the AveCorr Approach is truly better than the ExpRtn Approach. After all, it seems that if a high expected return for a portfolio is desirable in the future, one would want to construct the portfolio from securities that have exhibited high returns in the past. As with the portfolio selected by the AveCorr Approach, it is useful to compare results between Monte Carlo simulation for the portfolio against comparable results for the S&P 500 and for Berkshire Hathaway.

Figure 14 shows the results of sampling the portfolio of Table 4 with replacement 50,000 times. This reveals that the expected return after 10 years is 3.2x the initial investment and the probability of a loss after 10 years is 0.0505, with a 99% CI of 0.0482 – 0.0530. (Again, the confidence intervals are derived via bootstrapping.) If one considers only the expected return, this portfolio seems slightly more attractive than the portfolio selected by the AveCorr Approach, with an expected return of only 3.0x over ten years. However, notice that the expected probability of a nominal loss after 10 years is higher than that for the S&P 500 (P=0.0434) and for Berkshire Hathaway (P=0.0123). In fact, probability of a nominal loss after

10 years is about 28x higher for the ExpRtn portfolio than for the AveCorr Approach portfolio (P=0.0018). Indeed, much is sacrificed in order to achieve a slightly higher expected return over 10 years. Moreover, the interquartile range of expected returns after 10 years for the portfolio spans 2.0x to 5.2x. By comparison, the interquartile range for the AveCorr portfolio is 2.3x to 4.1x, a much tighter range.

As with the AveCorr portfolio represented in Figure 9 and Figure 12, the ExpRtn portfolio was developed from 12 years of historical data spanning the period February 1993 – February 2005. In Figure 15 the ExpRtn portfolio's performance is compared against 10 years of actual data spanning the period March 2005 – February 2015. Further, like the AveCorr portfolio, the performance of the ExpRtn portfolio is compared to the actual histories of the S&P 500 and Berkshire Hathaway over the same time frame.

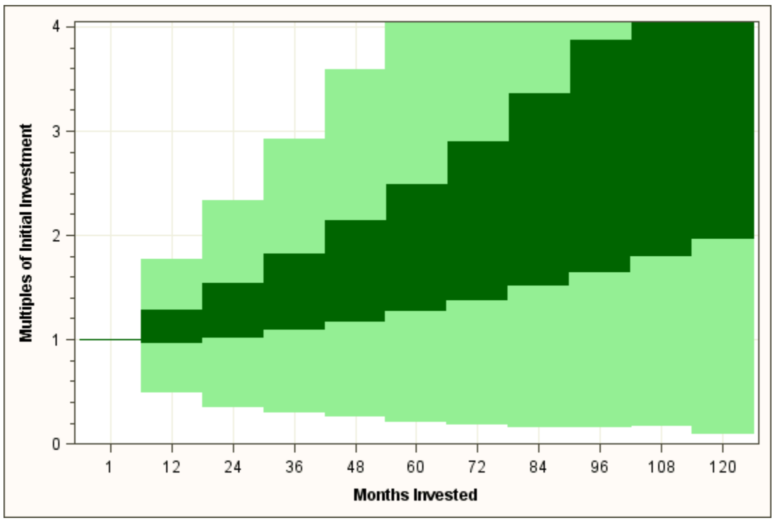


Figure 14. Box and whisker plot showing results of 50,000 simulations of performance of the portfolio specified in Table 4. Underlying data are from the period spanning February 1993 – February 2005.

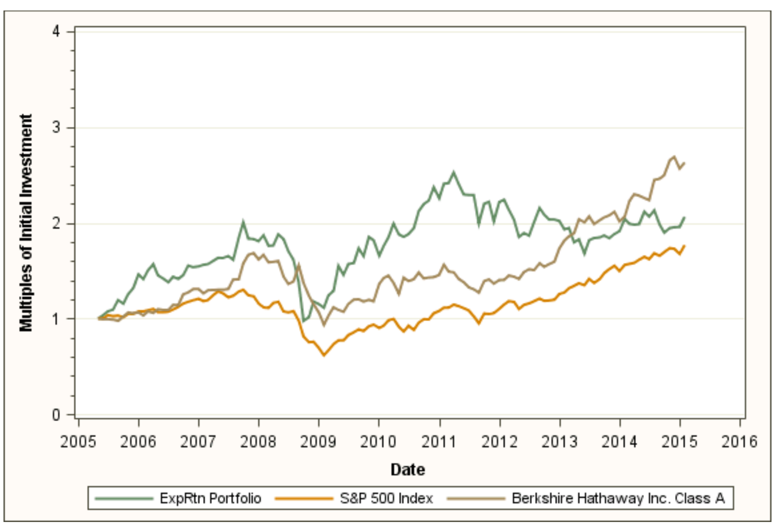


Figure 15. Comparative graph showing the 10-year performance of the portfolio specified in Table 4 (green line labeled "ExpRtn Portfolio") compared against the S&P 500 (orange line) and Berkshire Hathaway (brown line). The ExpRtn Portfolio line is based on constructing the portfolio exactly as specified in Table 4 and using the actual performance data of the underlying funds for the validation period of March 2005 – February 2015. As with Figure 12, all values are normalized to 1 as of March 1, 2005 for relative comparison. In every case dividends and distributions are reinvested in the security that produces them.

Figure 15 reveals the penalty for overvaluing the historical returns relative to the benefit of minimizing the portfolio's ***AveCorr*** value. To be sure, the ExpRtn portfolio does exhibit breathtaking performance for the 2005 – 2008 and 2009 – 2011 rough timeframes. However, the underperformance for the period 2011 – 2015 erased the advantage that the portfolio seemed to have over Berkshire Hathaway. In fact, for the approximately 4-year period following the 2011 peak, the ExpRtn portfolio displayed a sizable net loss, while both the S&P 500 and Berkshire Hathaway produced exceptional gains. In other words, the penalty for sacrificing a low ***AveCorr*** value in the portfolio selection process is seen in the form of long-term volatility in returns.

CONCLUSION

It is possible to analytically derive portfolios comprised entirely of easy-to-purchase, low-cost mutual funds that substantially outperform the S&P 500 index over a long-term time horizon. Furthermore, if the portfolio selection process focuses on choosing the set of funds that results in the minimum AveCorr value for the number of funds selected, the resultant portfolio is shown to outperform Berkshire Hathaway's stock performance over a recent 10-year period.

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DISCLAIMER AND FULL DISCLOSURE

Nothing presented in this paper constitutes financial advice, nor should it be construed in part or in whole as constituting financial advice. The author has no specific training as a financial advisor and is not certified in any capacity to provide financial advice. The author does not work in the financial industry, nor does he have any direct connection with the financial industry in any capacity other than as an individual retail investor. All results shown in this paper should only be considered examples of the techniques discussed herein and should not be us used by the reader in any way to direct financial decisions of any type.

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I wish to acknowledge the people who influenced the presentation of thought contained within this paper. I must begin with my oldest daughter, Shanaya, who inspired this entire project during an enchanting conversation that we shared as the two of us enjoyed a walk together one crisp autumn afternoon in

2014. At fifteen, her investment knowledge and interest in Warren Buffett surprised and delighted me. She asked insightful and challenging philosophical questions that led me to perform some calculations when we returned home. Had it not been for that conversation, the ideas conveyed in this paper might have never come to light.

Additionally, I must thank the many friends and colleagues who displayed great patience as they permitted me to think out loud with them. Their generosity of time, enthusiastic listening and discerning questions helped me to refine my articulation of several of the ideas described herein. I particularly want to acknowledge Pete Alle, Elizabeth Craig, Dan Rosier, Lance Trenary, Bill Sanford and Joe Oberweis. By carefully listening and asking questions, each of them contributed to the organization of my thoughts in ways that may not be readily clear to them. I thank each of them for their time, comments, and most of all for their friendship.

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Further, Dr. Gerald Bell, Founder of the Bell Leadership Institute, inspired me to continue pursuing this work when he encouraged me (and all of his students) to always remember that “80% of major discoveries in science are made by people who do not work directly in the field of the discovery.” It is the freedom to think without the intellectual shackles of “common knowledge” that makes this true. While the work described within this paper may not be of the magnitude of a major discovery, it is certainly far afield from my usual professional focus as a marketer of milk and ice cream.

Finally, I want to dedicate this work to my daughters, Shanaya and Ariana. One of my many wishes for them is that they develop investment wisdom and discipline sufficient to create for themselves lives of financial independence. May they always maintain their love for learning, always pursue their interests passionately, and always speak their minds freely, completely, and with confidence.

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APPENDIX

Fund Name	Symbol		Fund Name	Symbol
American Funds American Balanced A	ABALX		MFS Massachusetts Investors Fund	MITTX
American Funds Europacific A	AEPGX		Nicholas Fund	NICSX
American Funds Fndmntl Invst A	ANCFX		Nicholas II Fund	NCTWX
American Funds Growth Fund A	AGTHX		Pioneer Fund	PIODX
American Funds Income Fund A	AMECX		Putnam Investors Fund	PINVX
American Funds New Perspective A	ANWPX		T. Rowe Price Capital Appreciation	PRWCX
CGM Mutual Fund	LOMMX		T. Rowe Price Dividend Growth	PRDGX
Dodge & Cox Balance Fund	DODBX		T. Rowe Price Equity Income	PRFDX
Dodge & Cox Stock Fund	DODGX		T. Rowe Price Equity Index 500	PREIX
Fidelity Fund	FFIDX		T. Rowe Price European Stock	PRESX
Fidelity Real Estate Investment	FRESX		T. Rowe Price Growth & Income	PRGIX
Fidelity Select Air Trans	FSAIX		T. Rowe Price Growth Stock	PRGFX
Fidelity Select Automotive	FSAVX		T. Rowe Price International Discovery	PRIDX
Fidelity Select Banking	FSRBX		T. Rowe Price International Stock	PRITX
Fidelity Select Biotechnology	FBIOX		T. Rowe Price Japan	PRJPX
Fidelity Select Brokerage & Invmt Mgmt	FSLBX		T. Rowe Price Mid-Cap Growth	RPMGX
Fidelity Select Chemicals	FSCHX		T. Rowe Price New America Growth	PRWAX
Fidelity Select Communications	FSDCX		T. Rowe Price New Asia	PRASX
Fidelity Select Computers	FDCPX		T. Rowe Price New Era	PRNEX
Fidelity Select Constr & Housing	FSHOX		T. Rowe Price New Horizons	PRNHX
Fidelity Select Consumer Finance	FSVLX		T. Rowe Price Science & Technology	PRSCX
Fidelity Select Consumer Staples	FDFAX		T. Rowe Price Small-Cap Value	PRSVX
Fidelity Select Defense and Aero	FSDAX		T. Rowe Price Spectrum Growth	PRSGX
Fidelity Select Electronics	FSELX		T. Rowe Price Spectrum Income	RPSIX
Fidelity Select Energy	FSENX		Vanguard Dividend Growth	VDIGX
Fidelity Select Env & Alt Energy	FSLEX		Vanguard Energy	VGENX
Fidelity Select Gold	FSAGX		Vanguard Equity Income	VEIPX
Fidelity Select Health Care	FSPHX		Vanguard Explorer	VEXPX
Fidelity Select Industrial Equip	FSCGX		Vanguard Growth and Income	VQNPX
Fidelity Select Insurance	FSPCX		Vanguard Health Care	VGHCX
Fidelity Select Leisure	FDLSX		Vanguard International Growth	VWIGX
Fidelity Select Materials	FSDPX		Vanguard International Value	VTRIX
Fidelity Select Medical Delivery	FSHCX		Vanguard Morgan Growth	VMRGX
Fidelity Select Multimedia	FBMPX		Vanguard Precious Metals and Mining	VGPMX
Fidelity Select Retailing	FSRPX		Vanguard Small Cap Index	NAESX
Fidelity Select Software & Comp Srv	FSCSX		Vanguard Total Stock Market	VTSMX
Fidelity Select Technology	FSPTX		Vanguard U.S. Growth	VWUSX
Fidelity Select Telecomm	FSTCX		Vanguard Wellington Fund	VWELX
Fidelity Select Transportation	FSRFX		Vanguard Windsor	VWNDX
Fidelity Select Utilities	FSUTX		Vanguard Windsor II	VWNFX

Appendix Table A. Population of candidate mutual funds considered for inclusion in portfolio construction.