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Parameter Estimation of Cognitive Attributes using the Crossed Random-Effects Linear Logistic Test Model with PROC GLIMMIX

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ABSTRACT

A simulation study is designed to explore accuracy and precision of parameter estimation of cognitive attributes using the crossed random effects linear logistic test model (CRELLTM) with PROC GLIMMIX. The CRELLTM characterizes the weight of the cognitive attributes for item difficulties as random coefficients. The SAS® GLIMMIX package is applied to obtain parameter estimates. A series of simulations are conducted. Manipulated factors in this study include the sample size, the number of items, the population distribution shape, and the Q-matrix density. The number of cognitive attributes is fixed as 8 attributes. The results indicate that the CRELLTM using the SAS GLIMMIX procedure yields negligible bias and high correlations between estimated and true cognitive attributes; however, the RMSE is relatively large, especially with small sample size (i.e., 25 in this study).

INTRODUCTION

The linear logistic test model (LLTM) that incorporates cognitive task characteristics into the Rasch model has been widely used for various purposes in educational contexts, such as item selection, test development, construct validation, and so on. It is, therefore, crucial to identify well-defined task characteristics (or cognitive attributes or item stimulus features) and establish the appropriate relationships between test items and cognitive attributes (i.e., the weight design or the Q-matrix) in addition to determining the psychometric model for the probability of the correct responses. The LLTM is built based on the Rasch model so it is often considered an extension of the Rasch model (Chen, MacDonald, & Leu, 2011). To express the mathematical equation for the LLTM, the Rasch is presented first as follows:

$$P(X_{ij}=1|\theta_j,\beta_i)=\exp(\theta_j-\beta_i)/[1+\exp(\theta_j-\beta_i)]$$

, where *$P(X_{ij}=1)$* represents the probability of the correct response to item *i* for person *j*, θ_j is the ability level of person *j*, and β_i is the item difficulty of item *i*. When a linear combination of cognitive attributes replaces item difficulty, β_i, the Rasch model can be written into the LLTM formula as follows:

$$P(X_{ij}=1|\theta_j,q_{ik},\eta_k)=\exp(\theta_j-\sum_{j=1}^{p}q_{ik}\eta_k)/[1+\exp(\theta_j-\sum_{k=1}^{K}q_{ik}\eta_k)]$$

That is, in the LLTM the item difficulty parameters that are replaced by cognitive attributes can be conceptualized as

$$\hat{\beta}_i=\sum_{k=1}^{K}q_{ik}\eta_k$$

, where q_{ik}, an entry in a so-called Q-matrix, is the fixed and known weight (occurrence) assgined to cognitive attribute *k* that is involved in item *i* and η_k is the estimated parameter for cognitive attribute *k*.

However, the LLTM model assumes that the variance of item difficulties is completely accounted for by cognitive attributes (i.e., $\beta_i=\hat{\beta}_i$) . This can be considered a disadvantage of the LLTM for the purpose of decomposing item difficulty into cognitive attributes (Hartig, Frey, Nold, & Klieme, 2012). Janssen and De Boeck (2002) also observed that even though the Q-matrix is well designed and cognitive attributes well explain the majority of the variation in item difficulties, the LLTM often does not fit the data (Rijmen & de Boeck, 2002). To overcome the disadvantages of the LLTM, Janssen and colleagues (2004) proposed the crossed random-effects LLTM by adding the error term on item difficulty (i.e., $\beta_i=\hat{\beta}_i+\varepsilon_i$). Unlike the LLTM that has been widely examined if the model provides accurate parameter estimates for cognitive attributes (e.g., Baker, 1993; Green & Smith, 1987), the accuracy and precision of parameter estimation for cognitive attributes using the crossed random-effects LLTM has not been examined yet, to our knowledge.

Thus, this study was intended to examine the accuracy and precision of the CRELLTM in terms of parameter estimation of cognitive attributes. The SAS package called GLIMMIX was applied to obtain parameter estimates for the CRELLTM. The exploration of the performance of the CRELLTM in terms of the cognitive attribute estimates was under normal, slightly positively-skewed, positively-skewed, slightly negatively-skewed, negatively-skewed, leptokurtic,and platykurtic population distributions. Sparse and dense Q-matrices were used for the CRELLTM to examine how consistent the performance of the CRELLTM can be. The specific research questions are listed as follows: (a) How well does the CRELLTM perform in terms of parameter estimation of cognitive attributes under various shapes of the population distributions? (b) Does sparse or dense Q-matrix affect the performance of the CRELLTM on parameter estimation of cognitive attributes?

THE SAS GLIMMIX PROCEDURE

The NLMIXED procedure in SAS for the non-linear mixed modeling (NLMM) has been widely used to formulate diverse item response theory (IRT) models (e.g., De Boeck & Wilson, 2004; Wang & Jin, 2010). This approach becomes more and more popular for researchers and practitioners because of its availability of software and flexibility of modeling. However, the NLMIXED procedure cannot be used in this study because of the requirement of random effects for person ability and item difficulty simultaneously, which are called crossed random effects (De Boeck & Wilson, 2004). The GLIMMIX procedure is applicable for the estimation of crossed random effects (Wang & Jin, 2010).

The GLIMMIX procedure is a newly-developed package in SAS. Historically, this procedure could be applied from a SAS macro and then was an add-on product in SAS 9.1. Now as an individual package, a lot of improvements have been made in SAS 9.3 (Li, Chen, & Kromrey, 2013). The SAS PROC GLIMMIX performs estimation and statistical inference for generalized linear mixed models (GLMMs) that extends the class of generalized linear models (GLMs) by incorporating normally distributed random effects. To our knowledge, the GLIMMIX procedure has been applied to formulate few IRT-related models compared to PROC NLMIXED and there are even fewer examinations of its efficiency in parameter recovery. It is still not clear to us if the GLIMMIX procedure produces accurate and precise parameter estimates for the CRELLTM. Thus, a series of simulations were conducted in this study.

Even though this study is a simulation study, it would be beneficial to the reader to introduce the required data format and execution processes. To execute the GLIMMIX procedure, the dataset needs to be imported into SAS and converted into a format that fits multilevel analyses. The commonly-used dataset format is shown in Figure 1, including 3 examinees and 5 items. As you can see, each row includes person ID and observations (5 items). An example of the SAS code that imports the data into SAS is listed below.

```
data timss;
infile 'C:\timss.dat';
input ID 1-3 @4 (i01 - i05) (2.);
person=_N_;
drop ID;
```

```
001 1 1 0 0 1
002 0 1 1 0 0
003 0 0 1 1 1
```

Figure 1. Example of the Commonly-Used Data Format

The required data format for the GLIMMIX is to have each row representing only one observation. For instance, an examinee answering 5 items would have 5 rows to represent his/her responses. Figure 2 shows an example of the GLIMMIX required data format. The first two columns are transferred from the dataset shown in Figure 1. The PROC TRANSPOS procedure can convert a wide data format (Figure 1) into a vertically long string format (the first three columns in Figure 2). An example of the SAS code for data transformation is listed below.

```
proc transpose data=TIMSS out=longform name=item prefix=resp;
by person;
```

Person	Item	Resp1	Q-Matrix A1	A2	A3	A4
001	i01	1	1	1	0	0
001	i02	1	1	0	1	0
001	i03	0	0	1	0	1
001	i04	0	1	0	0	1
001	i05	1	0	0	1	1
002	i01	0	1	1	0	0
002	i02	1	1	0	1	0
002	i03	1	0	1	0	1
002	i04	0	1	0	0	1
002	i05	0	0	0	1	1
003	i01	0	1	1	0	0
003	i02	0	1	0	1	0
003	i03	1	0	1	0	1
003	i04	1	1	0	0	1
003	i05	1	0	0	1	1

Figure 2. Example of the GLIMMIX Required Data Format

In addition to the item and response variables (the second and third columns in Figure 2), the designed matrix (or Q-matrix) is also needed in the dataset for SAS to estimate parameters for the CRELLTM. An example of the Q-matrix is presented from column 3 to column 6 in Figure 2. As you can see, there are four columns representing 4 cognitive attributes required by a test with 5 items. To answer these items correctly, Item 1 requires Attributes 1 and 2; Item 2 requires Attributes 1 and 3; Item 3 requires Attributes 2 and 4; Item 4 requires Attributes 1 and 4; Item 5 requires Attributes 3 and 4. This matrix is used repeatedly for persons 2 and 3. Figure 2 shows the final dataset needed for GLIMMIX to estimate the parameters for the CRELLTM.

```
data combination;
set longform;
a1=0;
a2=0;
a3=0;
a4=0;
if item ='i01' or item='i02' or item='i04' then a1=1;
if item ='i01' or item='i03' then a2=1;
if item ='i02' or item='i05' then a3=1;
if item ='i03' or item='i04' or item='i05'  then a4=1;
```

Below is the SAS code that you can obtain cognitive attribute estimates from PROC GLIMMIX:

```
proc glimmix data = combine method = laplace;
class item person;/*item is item name and person is person name;*/
model resp1 (descending) = a1 a2 a3 a4/s noint link=logit dist = binary
error=binomial; /*a1-Ma4 are four cognitive attributes;*/
random int / subject = person s;
random int / subject = item s;
run;
```

The SAS code above indicates that the GLIMMIX procedure is executed. The dataset name is “combine”. The estimation method is called LAPLACE, which utilizes maximum likelihood estimation based on a Laplace approximation of the marginal log likelihood (see SAS Institute Inc., 2011). Two classification variables are “item” and “person” in the CLASS statement. The dependent variable, resp1, and Q-matrix variables, a1 to a4, as fixed effects, are specified in the MODEL statement. There are two RANDOM statements that are required for the CRELLTM, one for person and the other for item.

THE SIMULATION STUDY

DATA GENERATION

The current study was conducted using a simulation approach. Data for this simulation study were generated using a random number generator, RANNOR, in SAS/IML statistical software. For each condition, 1000 replications were generated. The manipulated sample sizes were 25, 50, 100, 250, 500, and 1000. The population distributions investigated in the study included normal (skewness=0, kurtosis=0), negatively-skewed distribution (skewness=-2, Kurtosis=6), slightly negatively-skewed (skewness=-1, Kurtosis=3), slightly positively-skewed (skewness=1, kurtosis=3), positively-skewed (skewness=2, kurtosis=6), highly leptokurtic (skewness=0, kurtosis=25), and slight

platykurtic (skewness=0, kurtosis=-1) distributions. The numbers of items were 21 and 42 items. The number of cognitive attributes was fixed to be 8 attributes. The forms of the Q-matrix density were sparse (21.23% 1s) and dense (54.76% 1s).

Q-MATRICES

Like Baker's (1993) study, the sparse and dense Q-matrices that were extracted from Fischer and Formman (1972) and Medina-Diaz (1993), respectively, were used in this study. In the sparse Q-matrix, a total of 21 items with 8 cognitive attributes was involved in the original sparse Q-matrix in Baker's (1992) study. The true cognitive attribute parameters for the sparse Q-matrix were $\eta_1 = 2.152$, $\eta_2 = 1.229$, $\eta_3 = -.468$, $\eta_4 = 1.907$, $\eta_5 = 1.051$, $\eta_6 = .086$, $\eta_7 = .141$, and $\eta_8 = -.474$. As seen in Table 1, the sparse Q-matrix had only 34 out of 168 entries that contained 1s (approximately 21.23%). In contrast, the dense Q-matrix had 92 out of 168 entries that contained 1s (approximately 54.76%) shown in Table 2. The true cognitive attribute parameters for the dense Q-matrix were $\eta_1 = -.75$, $\eta_2 = -.3$, $\eta_3 = -.1$, $\eta_4 = -.08$, $\eta_5 = -.05$, $\eta_6 = .2$, $\eta_7 = .6$, and $\eta_8 = 1.2$. For 42 items with 8 attributes, the Q-matrix for items 1 to 21 was duplicated for items 22 to 42 so that the density of the Q-matrix keeps the same.

	Cognitive Attribute								
Item	1	2	3	4	5	6	7	8	Total
1	0	0	0	0	0	0	1	0	1
2	0	0	0	1	0	0	1	0	2
3	0	0	0	0	0	1	1	0	2
4	1	0	0	0	0	0	0	1	2
5	0	1	0	0	0	0	0	1	2
6	0	0	0	0	1	0	0	1	2
7	0	0	1	0	0	0	0	1	2
8	0	1	0	0	0	0	1	0	2
9	1	0	0	0	0	0	1	0	2
10	0	0	0	0	0	0	0	0	0
11	0	0	0	1	0	0	0	0	1
12	0	0	0	0	0	1	0	0	1
13	0	0	1	0	0	0	1	0	2
14	0	0	0	0	0	0	0	1	1
15	0	0	0	1	0	0	0	1	2
16	0	0	0	0	1	0	1	0	2
17	0	1	0	0	0	0	0	0	1
18	0	0	1	0	0	0	0	0	1
19	0	0	0	0	0	1	0	1	2
20	1	0	0	0	0	0	0	1	2
21	0	0	0	0	1	0	0	1	2
Total	3	3	3	3	3	3	7	9	34

Table 1. Sparse Q-matrix

	Cognitive Attribute								
Item	1	2	3	4	5	6	7	8	Total
1	1	0	1	0	1	0	0	0	3
2	0	0	0	1	0	1	0	1	3
3	0	1	0	1	0	0	1	0	3
4	1	1	0	0	1	0	0	0	3
5	0	0	1	0	0	0	1	1	3
6	0	1	0	1	0	1	0	1	4
7	1	0	1	0	1	0	1	0	4
8	1	0	0	1	0	0	1	0	3
9	0	1	1	0	1	1	0	1	5
10	1	1	0	1	0	1	0	1	5
11	1	1	0	1	0	1	0	0	5
12	0	0	1	1	1	0	1	0	4
13	0	1	1	0	1	0	1	1	5
14	1	0	1	1	0	1	0	1	5
15	1	1	0	1	1	0	1	0	5
16	0	1	1	1	0	1	1	1	6
17	1	1	0	1	1	0	1	1	6
18	1	1	0	1	1	1	0	1	6
19	1	0	1	1	1	1	1	0	6
20	1	1	1	0	1	0	1	1	6
21	1	1	1	0	1	1	1	1	7
Total	12	12	11	12	12	9	12	12	92

Table 2. Dense Q-matrix

EVALUATION CRITERIA

Three decision criteria were used to assess the estimation performance of the CRE-LLTM, including bias, root mean square error (RMSE), and correlation. The estimation bias was computed as the average difference between the estimated and true parameters. The formula for estimation bias for cognitive attributes is as follows:

$$Bias = \frac{\sum \hat{\eta} - \eta}{n_{replication}}$$

The RMSE is the square root of the average squared difference between the estimated and true parameters and was used to detect the magnitude of estimation error. The RMSE formula is as follows:

$$RMSE = \sqrt{\frac{\sum (\hat{\eta} - \eta)^2}{n_{replication}}}$$

Person product-moment correlation was used to detect the consistency between the estimated and true sets of parameters. High correlation coefficients indicate that the set of estimated parameters is consistent with the true parameters. Finally, factorial ANOVA analyses with the generalized eta-squared effect size were used to examine what manipulated factors affect bias, RMSE, and correlation. The Cohen's moderate effect size of .0588 was applied as the practical significant level.

SIMULATION RESULTS

Overall, the CRELLTM with PROC GLIMMIX performed well in terms of parameter estimation of cognitive attributes. Bias was defined as the average deviation of the estimated parameter from the true parameters across replications. Bias rates in all conditions were negligible, and the correlations between the estimated and true parameters were high. However, the mean RMSE was relatively large, close to .60. The impact of the manipulated factors and their first degree interactions are to be discussed under each of the three evaluative criteria.

BIAS

The overall distribution of cognitive attributes estimation bias across all simulation conditions is presented as the box and whisker plot in Figure 3. The mean of estimate bias (0.0047) across all conditions was very close to zero, which indicated that cognitive attribute estimates approximate the true parameters. Based on the factorial ANOVA analyses with the generalized eta-squared effect size of .0588 as the significant level, the main effect for the population distribution shape (η^2=0.0880) and two interactions between population distribution shape and sample size (η^2=0.1115) as well as population distribution shape and Q-matrix density (η^2=0.0870), significantly affected the bias of cognitive attributes estimation. Two significantly interactive effects were discussed in more details.

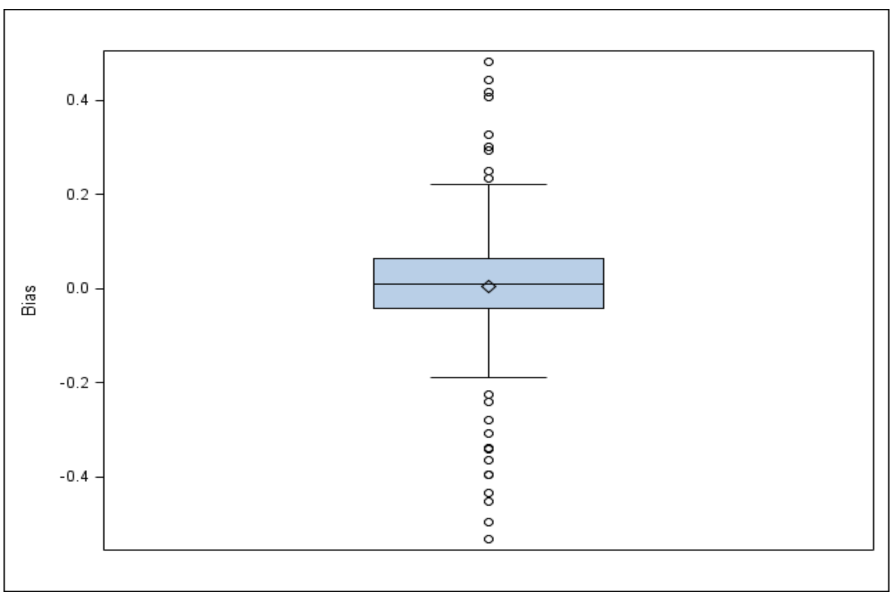

Figure 3: Distribution of Cognitive Attributes Estimation Bias

Figure 4 shows the distribution of mean attribute estimation bias with the interaction between sample size and population distribution shape. As can be seen in the figure, there was substantial variation in mean attribute bias across all sample sizes and population distribution shapes. Though there was no notable consistent overall pattern as to how bias changed with sample sizes and shapes, the normal distribution (skewness=0, kurtosis=0) seemed to yield smaller mean attribute bias across all sample sizes except 500. Similarly, the mean attribute bias was smaller for the approximation of normal distribution (skewness=0, kurtosis=-1) as well, compared with other population distribution shapes. Yet when the population distribution was symmetric and severely leptokurtic (skewness=0, kurtosis=25), negative bias of mean attribute estimation increased substantially as sample size decreased. In addition, more skewed and leptokurtic distributions (i.e., skewness = -2 or 2 and kurtosis = 6) with small sample sizes seemed yielding more bias, compared to less skewed and leptokurtic distributions.

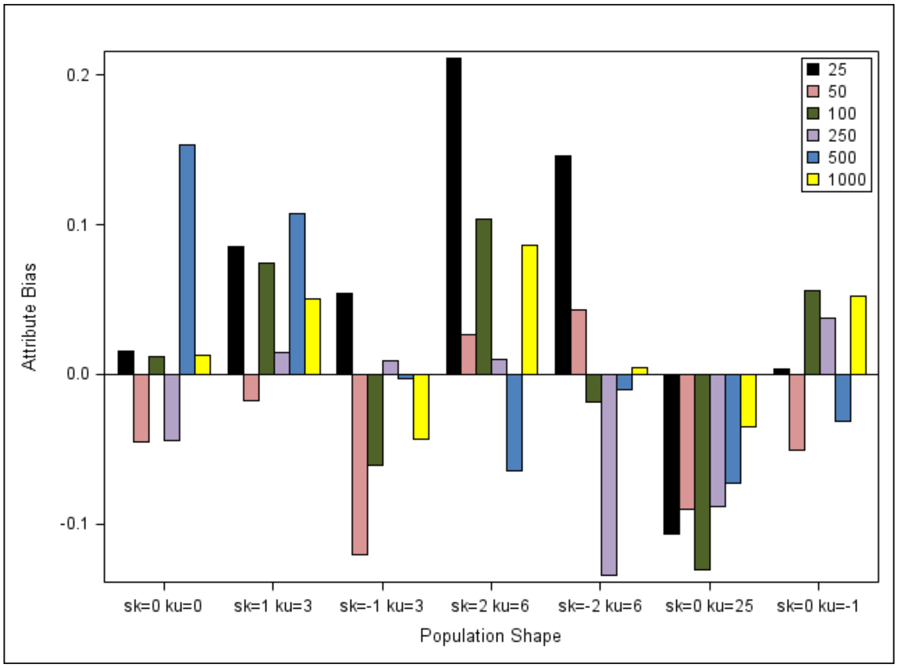

Figure 4: Distribution of Mean Attribute Estimation Bias by Sample Size and Shape

Evident in Figure 5 is that consistently dense Q-matrices tended to yield smaller mean attribute bias for skewed and leptokurtic distribution shapes. However, for the normal or slightly platykurtic distribution, the sparse Q-matrix seemed to produce less bias even though the bias they had for sparse and dense was rather small. Moreover, the population distribution with skewness of 2 and kurtosis of 25 (symmetric and severely leptokurtic) resulted in significant increase in negative mean attribute bias for the sparse Q-matrix.

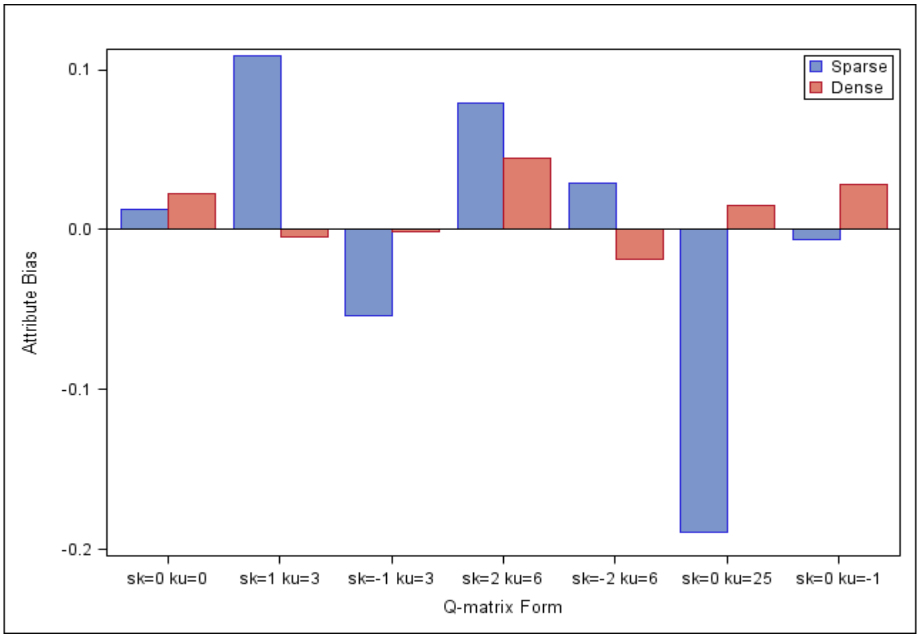

Figure 5: Distribution of Mean Attribute Estimation Bias by Shape and Q-matrix Density

RMSE

RMSE was computed as the square root of the average squared difference between the estimated and true parameters, and it was often used as an indicator of the precision of parameter estimation. The smaller the RMSE is, the more precise the parameter estimation is. In our study, the mean RMSE of cognitive attributes estimation in all conditions was quite large, compared with the negligible bias. As presented in Figure 6, the mean RMSE was .57.

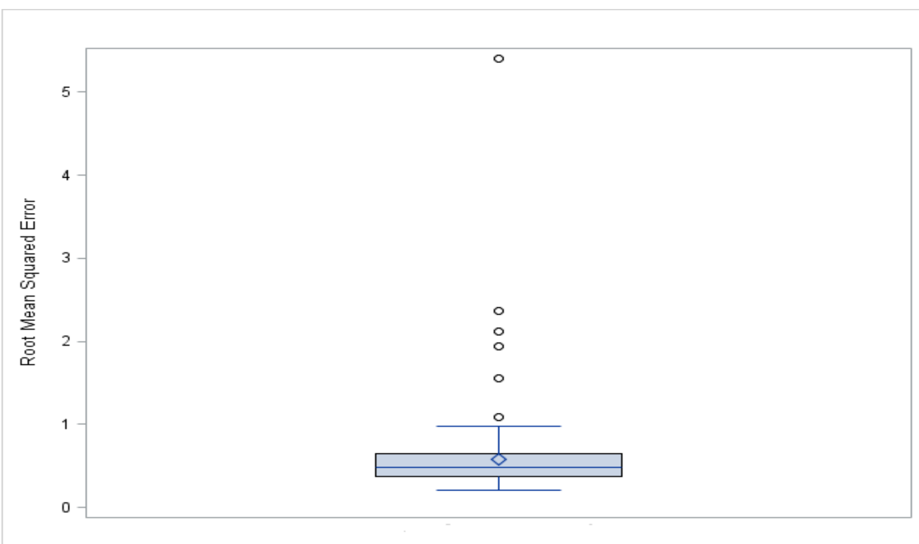

Figure 6: Distribution of RMSE for Cognitive Attributes Estimation

The results of factorial ANOVA analyses with generalized eta-squared effect sizes indicated that two main effects, sample size (η^2=0.1668) and number of items (η^2=0.0874), as well as 3 interactions between sample size and Q-matrix form (η^2=0.1123), sample size and population distribution shape (η^2=0.0838), as well as sample size and number of items (η^2=0.0743), were significantly associated with the RMSE of cognitive attributes estimation using the .0588 as the significant level.

As shown in Figure 6, mean RMSE decreased concomitantly with the increase of sample size. In the smallest sample size, 25, RMSE was the highest and had the widest range. In the sample sizes of 250, 500, and 1000, RMSEs were very similar. This finding was consistent with the conclusion in the existing literature that the larger the sample size, the more precise the estimation of the parameter. With regard to the number of items, RMSE of cognitive attributes estimates was higher in 21 items than in 42 items as presented in Figure 7. The parameter estimate of cognitive attribute was more precise with a larger number of items.

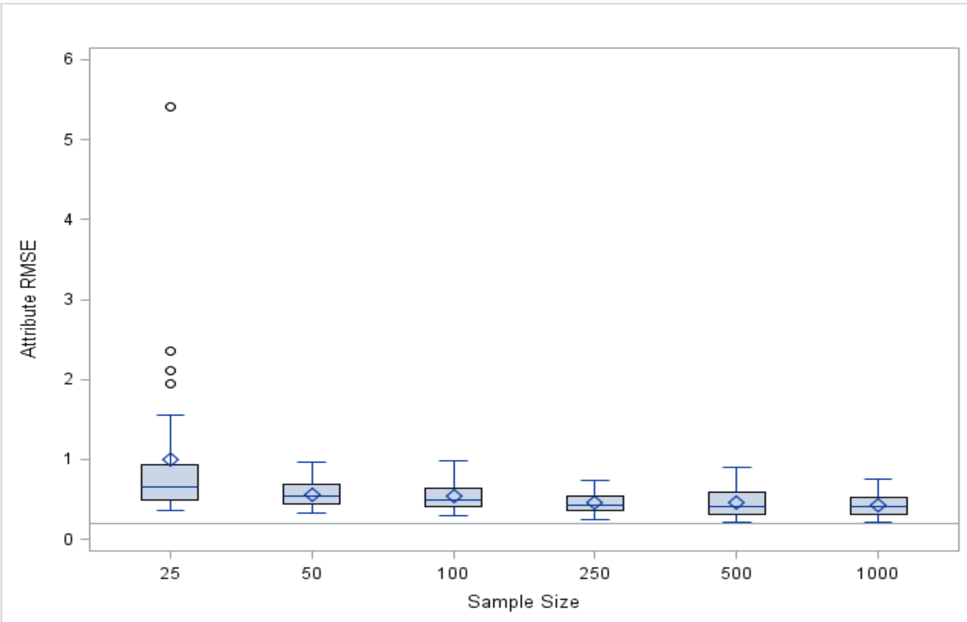

Figure 7: Distribution of Cognitive Attributes Estimation RMSE by Sample Size

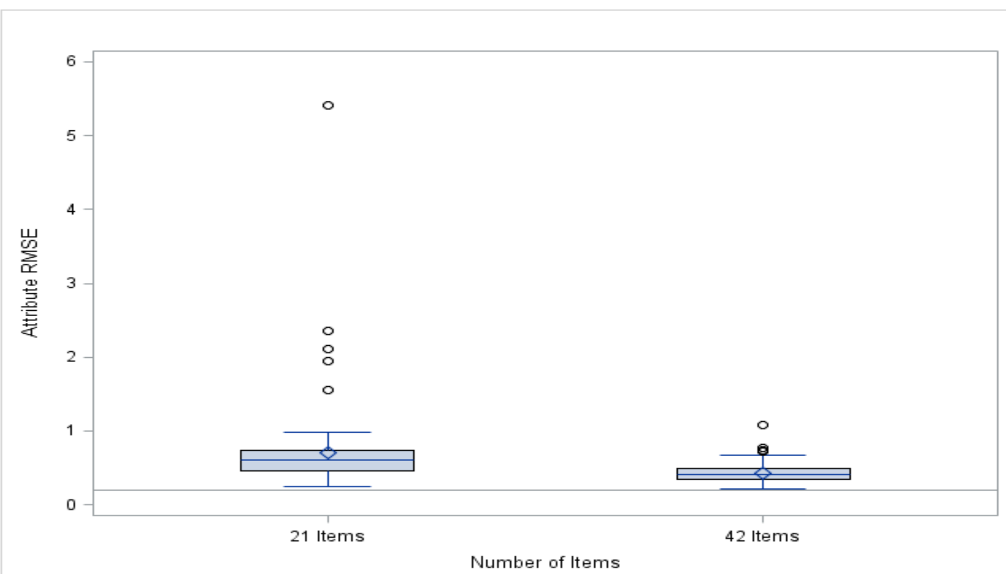

Figure 8: Distribution of Cognitive Attributes Estimation RMSE by Number of Items

The interaction of sample size by Q-matrix density had an effect on RMSE of cognitive attributes estimate as shown in Figure 8. In the sparse Q-matrix, the RMSE for the smallest sample size (i.e., n = 25) was a little bit higher than those for the larger sample sizes. On the contrary, in the dense Q-matrix, the RMSE for the sample size of 25 was

strikingly higher than those for larger sample sizes. In other words, the sparse and dense Q-matrices tended to yield the equivalent RMSE for sample sizes equal and above 50, but not for sample size of 25. Thus, researchers should be cautious when interpreting cognitive attribute estimates if the dense Q-matrix was used with very small sample size (e.g., below 50).

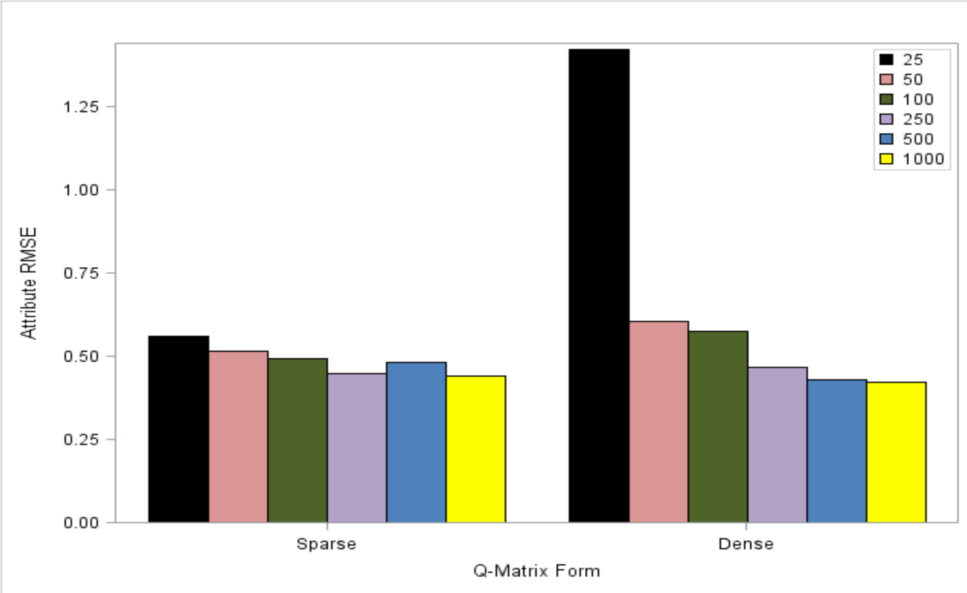

Figure 9: Distribution of Cognitive Attributes Estimation RMSE by Sample Size and Q-matrix density

The interaction effect of sample size by number of items on RMSE of cognitive attributes estimate, as presented in Figure 9, was similar to that between sample size and Q-matrix density. The RMSE for sample size of 25 was a little bit higher than those for larger sample sizes when there were 42 items on a test. However, when there were 21 items, the RMSE for sample size of 25 was much higher than those for larger sample sizes. Overall, the RMSE rates were slightly higher for 21 items than for 42 items. It seemed reasonable to say that the impact of number of items on RMSE decreases as sample size increases. This was reasonable, as more items provided more information to obtain a stable estimate of cognitive attributes.

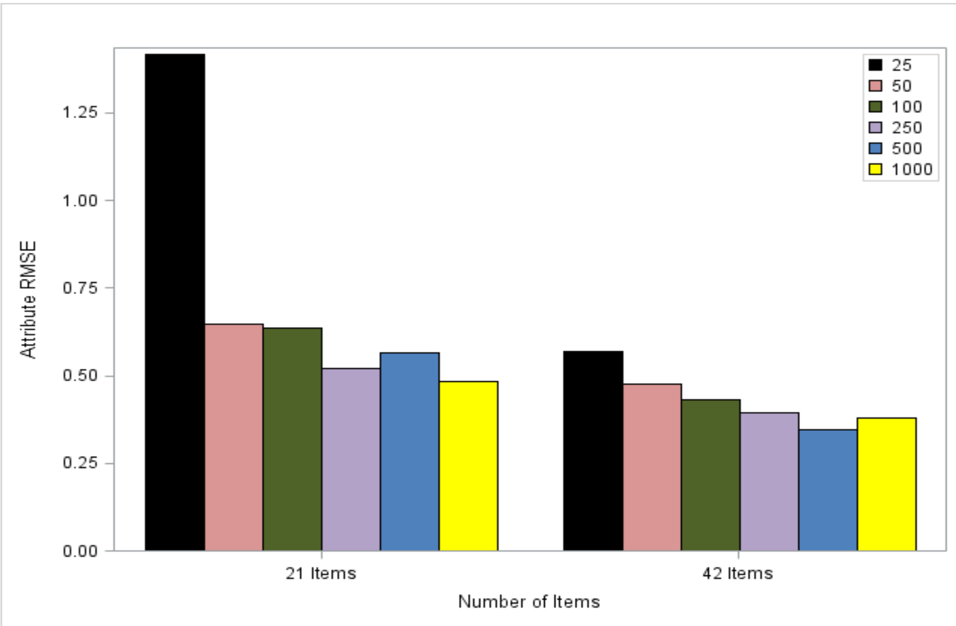

Figure 10: RMSE Distribution of Cognitive Attributes Estimation by Sample Size and Number of Items

The effect of population distribution shape on RMSE depends on the sample size. As shown in Figure 10, in all population shapes except in the slightly negatively skewed distribution (sk=-1, ku=3), the RMSE rates were the highest when sample size was 25. In some population distribution shapes, the pattern of observing a decreasing RMSE in larger sample sizes was very obvious (e.g., sk=0, ku=0; sk=1, ku=3; sk=2, ku=6); while in the other population shapes, the trend was not that clear.

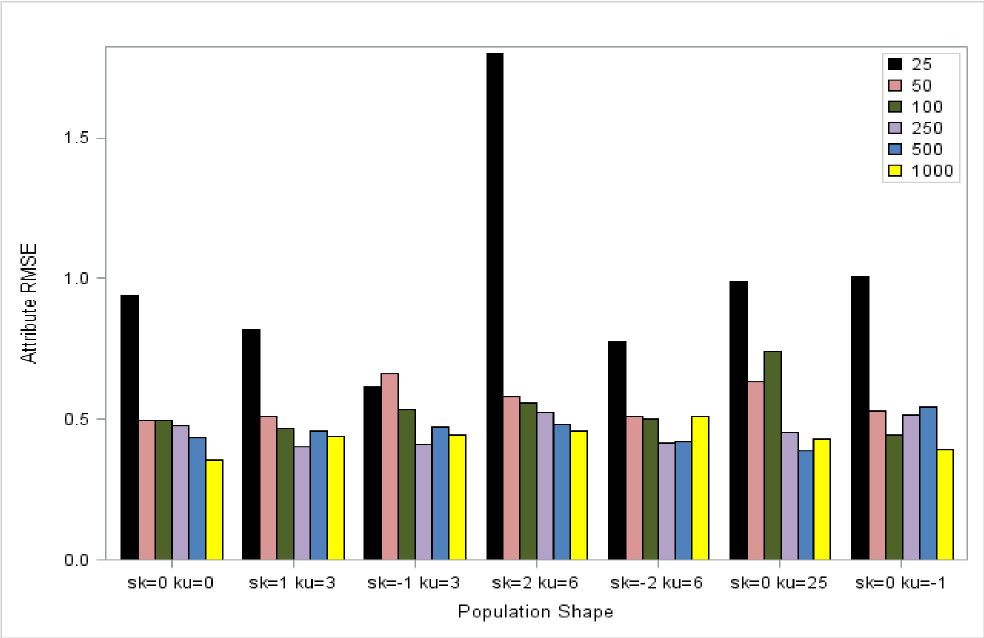

Figure 11: RMSE Distribution of Cognitive Attributes Estimation by Sample Size and Population Distribution Shape

CORRELATION

Pearson product-moment correlation was computed to detect the consistency between the estimated and true sets of cognitive attributes parameters. The high correlation coefficients imply that the estimated and true parameters are consistent. As shown in Figure 11 the correlations in this study were rather high overall (*r* = 0.89), representing that the estimated cognitive attributes were pretty consistent with the true cognitive attributes.

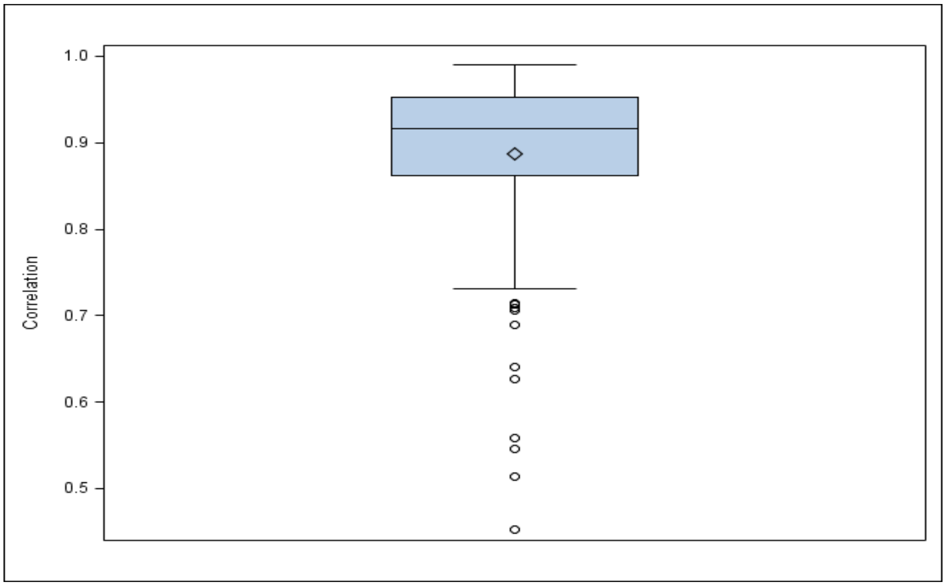

Figure 12: Distribution of Correlations between the Estimated and True Cognitive Attribute Parameters

The results of factorial ANOVA analyses with generalized eta-square effect sizes showed that number of items ($\eta^2 = 0.10053$) and the interaction of population distribution shape by sample size ($\eta^2 = 0.15922$) had an impact on the correlations. As shown in Figure 12, the mean correlation for 42 items was higher than that for 21 items. And the range of correlations for 42 items was also smaller.

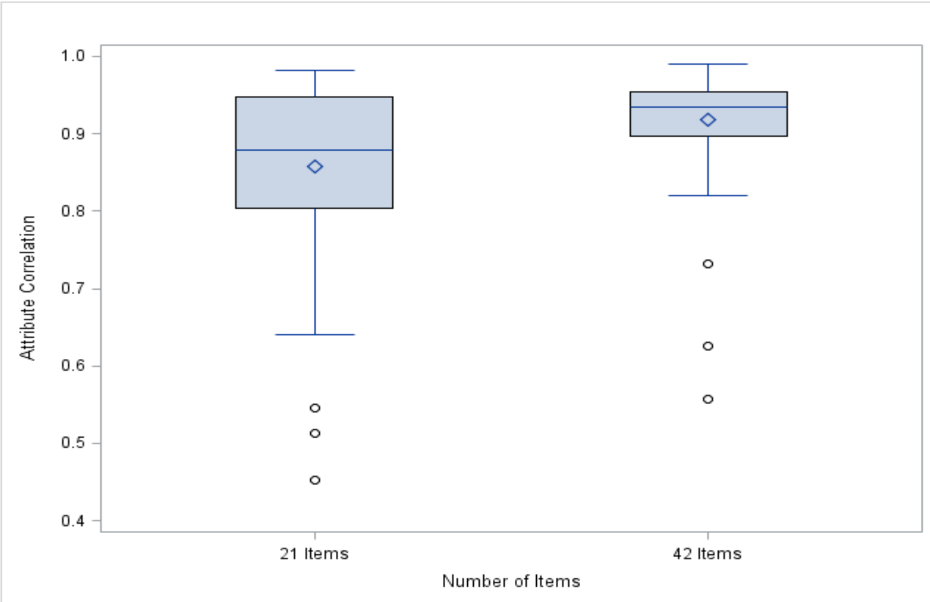

Figure 13: Distribution of Correlations between Estimated and True Cognitive Attributes by Number of Items

As shown in Figure 13, the variation of correlations also depended on the interaction between population distribution shape and sample size. In some population distribution shapes, correlations increased with the increase of sample size (e.g., sk=0, ku=0; sk=1, ku=3; sk=0, ku=25). However, in the other population distribution shapes, this pattern was not clear. In summary, correlations between estimated and true cognitive attributes were high in all conditions.

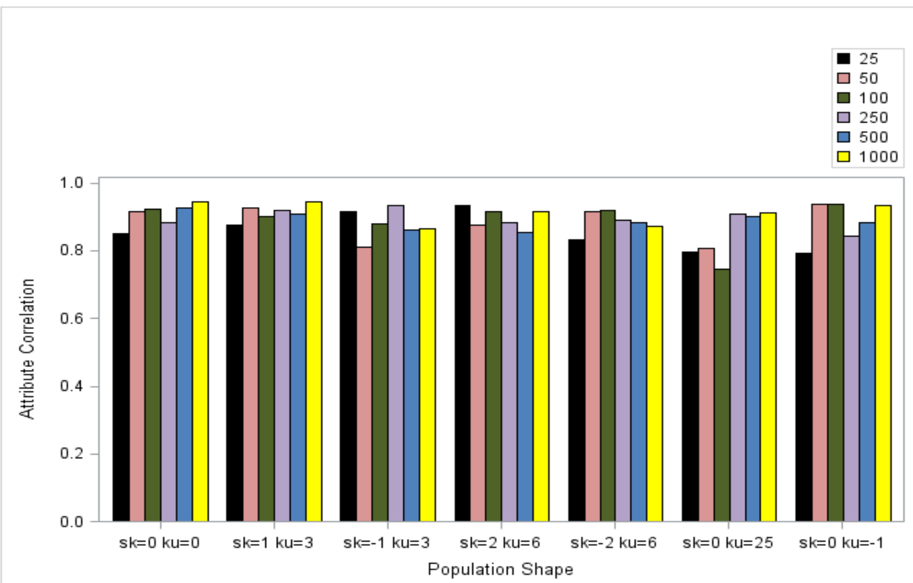

Figure 14: Distribution of Correlations between Estimated and True Cognitive Attributes by Sample Size and Population Distribution Shape

CONCLUSION

Overall, the SAS PROC GLIMMIX procedure used to estimate cognitive attributes for the cross random-effects LLTM performed well. The mean of bias across all conditions was negligible. But it should be cautioned that for the skewed (e.g., sk=2, ku=6) or heavily leptokurtic (e.g., sk=0, ku=25) distributions with small sample sizes (e.g., n=25), the bias became larger. The correlations between the estimated and true parameters were high across all conditions. Increasing the number of items would obtain a higher correlation. In addition, some population distributions showed the pattern of increasing the correlation when sample size increases, but the others did not have an observable pattern.

However, the mean RMSE was relatively large across all conditions. Two of the manipulated factors, sample size and number of items, had an impact on the RMSE of cognitive attribute estimates. The generalized eta-squared effect sizes in the factorial ANOVA analyses also suggested that the impact of Q-matrix form, number of items, and population distribution shape on RMSE depended on the sample size. The smallest sample size (i.e., 25) with the dense Q-matrix, fewer items, and almost all population distributions yielded larger RMSE.

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