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Multivariate time series modeling using VARMAX

Anders Milhøj, University of Copenhagen

ABSTRACT

Two examples of Vector Autoregressive Moving Average modeling with exogenous variables are given in this presentation. Data is from the real world. One example is about a two-dimensional time series for wages and prices in Denmark that spans more than a hundred years. The other is about the market for agricultural products, especially eggs! These examples give a general overview of the many possibilities offered by PROC VARMAX, such as handling of seasonality, causality testing and Bayesian modeling, and so on.

INTRODUCTION

The aim of this paper is to illustrate some of the many features in PROC VARMAX which is dedicated multivariate time series analysis. The name VARMAX is an abbreviation for Vector Autoregressive Moving Average models with eXogenous variables. This procedure is updated in the most recent version of Analytic Products 13.1 as of December 2013.

These models are multivariate generalizations of univariate ARMA models which have been popular since the book by Box and Jenkins (1976). For multivariate problems more interesting hypotheses could be tested in a dynamic model setup. But on the other hand the model building is more complicated. Some of the many facilities offered by PROC VARMAX are demonstrated by two examples in this paper. For a more examples the reader is referred to an upcoming book, Milhøj(2014).

TIME SERIES ANALYSIS OF THE DANISH WAGE-PRICE RELATION FOR 150 YEARS

In the first example two time series are considered: A yearly price index, denoted P, and a wage index 1818 to 1981, denoted W, for the years 1818 to 1981. The series of course exhibit a form of exponential growth, so it is natural to consider the logarithmically transformed series, LP = log(P) and LW = log(W). The basic application of the procedure VARMAX for modeling the two series simultaneously is simple as given by the next program. The option DIF=.. indicates that both series are differenced in order to achieve stationarity.

```
proc varmax data=sasmts.wageprice print=all plots=all;
model lp lw/dif=(lp(1) lw(1));
run;
```

This output has three main parts – basic statistical properties of the series, estimation of a particular model and checks of the fitted model. If the option PRINTALL is included, many more features of the model are presented leading to a huge output. The series of first differences passes the Dickey Fuller testing rejecting that a second order difference is needed.

By an algorithm to choose an appropriate order of a VARMA model an ARMA(4,1) model is selected. But it turns out that all parameters of the moving average term are insignificant, so an AR(4) will be used in the following. This model is applied by adding the option P=4 to the model statement. Far from all autoregressive parameters are significant, as seen by the schematic plot, Output 1, of the autoregressive coefficients.

```
                 Schematic Representation
                  of Parameter Estimates

                         Variable/
Lag            C     AR1     AR2     AR3     AR4

lp             .     ++      .-      ..      .+
lw             +     ++      +-      ..      ..

     + is > 2*std error,  - is < -2*std
       error,  . is between,  * is N/A
```

Output 1. A quick view of significant parameters

In the next program all these insignificant coefficients are restricted to zero and only the significant coefficients from the full fourth order autoregressive model are estimated. The coefficient 2-2-1, which is the + in the second order matrix in Output 1, is also restricted to zero as the p-value 4.2% is rather high.

```
proc varmax data=sasmts.wageprice printall plots=all;
model lp lw/dif=(lp(1) lw(1)) p=4;
restrict ar(2,1,1)=0,ar(2,2,1)=0,ar(3,1,1)=0,
ar(3,1,2)=0,ar(3,2,1)=0,ar(3,2,2)=0,ar(4,1,1)=0,ar(4,2,1),ar(4,2,2)=0;
run;
```

The resulting model is perhaps easiest understood by looking at its matrix representation, Output 2. Here the four two by two matrices are printed on top of each other. The zeroes represent all the restricted parameters while the remaining estimates are all significant.

The VARMAX Procedure

AR Coefficient Estimates

Lag	Variable	lp	lw
1	lp	0.24474	0.34273
	lw	0.24588	0.58371
2	lp	0.00000	-0.26004
	lw	-0.00000	-0.27528
3	lp	0.00000	-0.00000
	lw	-0.00000	-0.00000
4	lp	-0.00000	0.16878
	lw	0.00000	-0.00000

Output 2. Estimated autoregressive parameters in the restricted model

The model is a first order autoregressive model for the price series and a second order autoregressive model for the wage series. Moreover the prices affect the wages with lags up to two, while the wages affects the prices with lags up to four. An obvious possibility is however to exclude the fourth order effect as the third order effect is insignificant in order to have a more compact second order model. The t-static for this fourth order parameter is 3.03, but at high lags even significant coefficients are often set to zero in order to obtain parameter parsimony.

BAYESIAN ESTIMATION

A way to reduce the number of parameters in the autoregressive model is to consider Bayesian estimation. The idea is that an informative prior is applied to the autoregressive parameters where the prior reflects that the intuitive feeling that the parameters for lag 1 are more natural to include in the model than autoregressive parameters for higher lags. In Bayesian terms this is obtained by a prior that concentrates more around zero for higher lags so that the evidence from data has to be stronger in order to have a significant estimate at a high lag than for lag 1. This class of models is denoted BVAR models

In PROC VARMAX the prior covariance of the autoregressive matrices is as proposed by Litterman (1986). It includes two parameters, θ and λ, which could be specified by the user. The PRIOR option to the MODEL statement specifies the BVAR model with default values are $\theta = 0.1$ and $\lambda = 1$.

```
proc varmax data=sasmts.wageprice printall plots=all;
model lp lw/dif=(lp(1) lw(1)) p=4 prior;
run;
```

The result is that only one entry in the second order autoregressive parameter matrix is significant while no entries of the lag three and lag four matrices show significance.

```
                    Schematic Representation
                     of Parameter Estimates

           Variable/
           Lag           C     AR1     AR2     AR3     AR4

           lp            +     ++      ..      ..      ..
           lw            +     ++      .-      ..      ..

               + is > 2*std error,  - is < -2*std
                error,  . is between,  * is N/A
```

Output 3. A view of significant parameters in a Bayesian estimation

This application of the BVAR model seems to have done the job by reducing the order of the autoregressive model. The only significant parameter at lag two is found at the entry (2,2) which is the second order autoregressive parameter model for the univariate wage series. By this result it is natural to consider the second order autoregressive model, P =2, with the three remaining entries in the second order matrix restricted to zero.

It is possible to change the prior distribution by specifying the parameters of distribution. Other means of the priors distribution than the default value zero could be applied but in this application it is more interesting to change the variance of the prior. According to the definition of the prior the prior distribution become wider when θ and λ increase. For large values of θ and λ the result of the Bayesian estimation approaches the result of the maximum likelihood estimation and more parameters become significant. On the other hand the shrinkage toward zero is strengthened when smaller values of θ and λ are used. In the next application the rather small values of $\theta = 0.02$ and $\lambda = 0.5$ are applied.

```
proc varmax data=sasmts.wageprice printall plots=all;
model lp lw/dif=(lp(1) lw(1))
p=4 prior=(theta=0.02 lambda=0.5);
run;
```

The result, Output 4 is that all off diagonal entries of the autoregressive parameter matrices elements for all lags are insignificant meaning that the series show no lagged dependence. The only dependence between the series is the correlation 0.44 at lag zero.

```
                    Schematic Representation
                     of Parameter Estimates

           Variable/
           Lag           C     AR1     AR2     AR3     AR4

           lp            +     +.      ..      ..      ..
           lw            +     .+      .-      ..      ..

               + is > 2*std error,  - is < -2*std
                error,  . is between,  * is N/A
```

Output 4. A view of significant parameters in a Bayesian estimation using a prior with small variance

GRAPHICAL CONTROL OF MODEL FIT

In this section a second order model is considered for simplicity according to the conclusion in the precious sections. In the program the options PRINTALL and PLOTS=ALL give a huge output including statistics and graphs for a careful check of model fit.

```
proc varmax data=sasmts.wageprice printall plots=all;
model lp lw/dif=(lp(1) lw(1)) p=2;
restrict ar(2,1,1)=0,ar(2,2,1)=0;
run;
```

The output presents autocorrelation diagnostics for the white noise hypothesis for the residual series. The hypothesis is that all autocorrelations should be zero for both the wage and the price series. The estimated autocorrelations and cross correlations are presented as numbers in tables and they are displayed as graphs. Moreover their significance are shown in various ways. Here only the plots for the autocorrelation , the partial autocorrelation and the inverse autocorrelation. It is seen that all values are close to zero as they should be in order to accept the model fit. Moreover the fourth plot, p-values for portmanteau tests for the hypothesis are all above 5%.

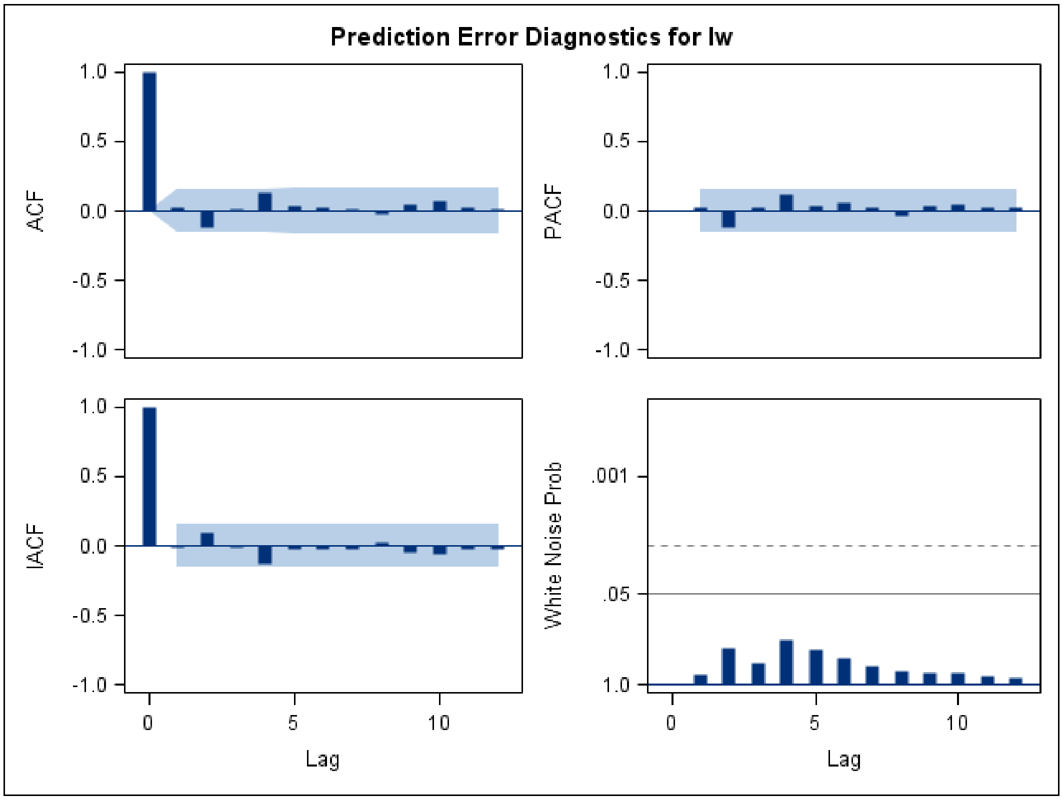

Figure 1. Residual autocorrelation functions for the wage series

GRAPHICAL PRESENTATIONS OF THE MODEL

The produced output by the code in the previous section also provides various graphics and tables that helps understanding the structure of the fitted model. The table of roots for the fitted autoregressive model, Output 5, tells that the polynomial has complex roots which is the same as saying that the observed series includes some oscillations. The wavelength is calculated by the reported radian 0.74, which corresponds to 42 degrees, as $2\pi/0.74 = 8.49$ years. The modulus is however only 0.54 so the periodic feature of the predictions are impossible to see on the forecast plot; see Figure 2 for the forecast plot of the wage series.

The VARMAX Procedure

Roots of AR Characteristic Polynomial

Index	Real	Imaginary	Modulus	Radian	Degree
1	0.39696	0.36335	0.5381	0.7412	42.4686
2	0.39696	-0.36335	0.5381	-0.7412	-42.4686
3	0.04929	0.00000	0.0493	0.0000	0.0000
4	0.00000	0.00000	0.0000	0.0000	0.0000

Output 5. Roots of the autoregressive Polynomial

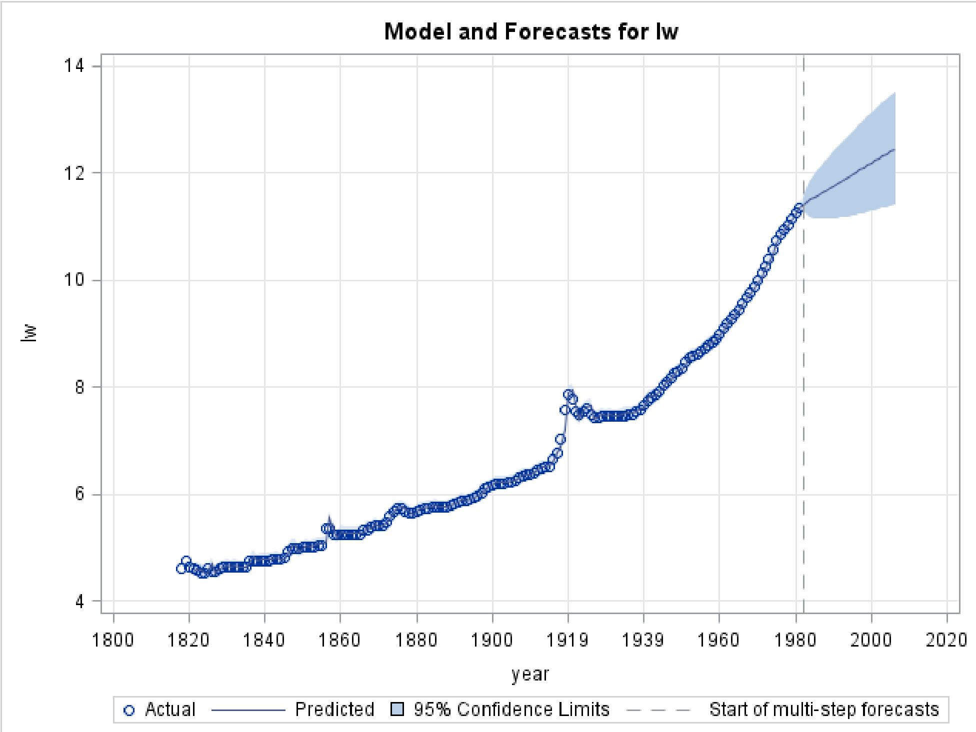

Figure 2. Forecasts fo the wage series

The impulse response function is defined as the coefficients of the infinite moving average representation of the model. The interpretation is that a large input error term to one of the series leads to changes in both series the same year and also the years to come. These coefficients are written to the output and presented as graphs, see Figure 3 and 4.

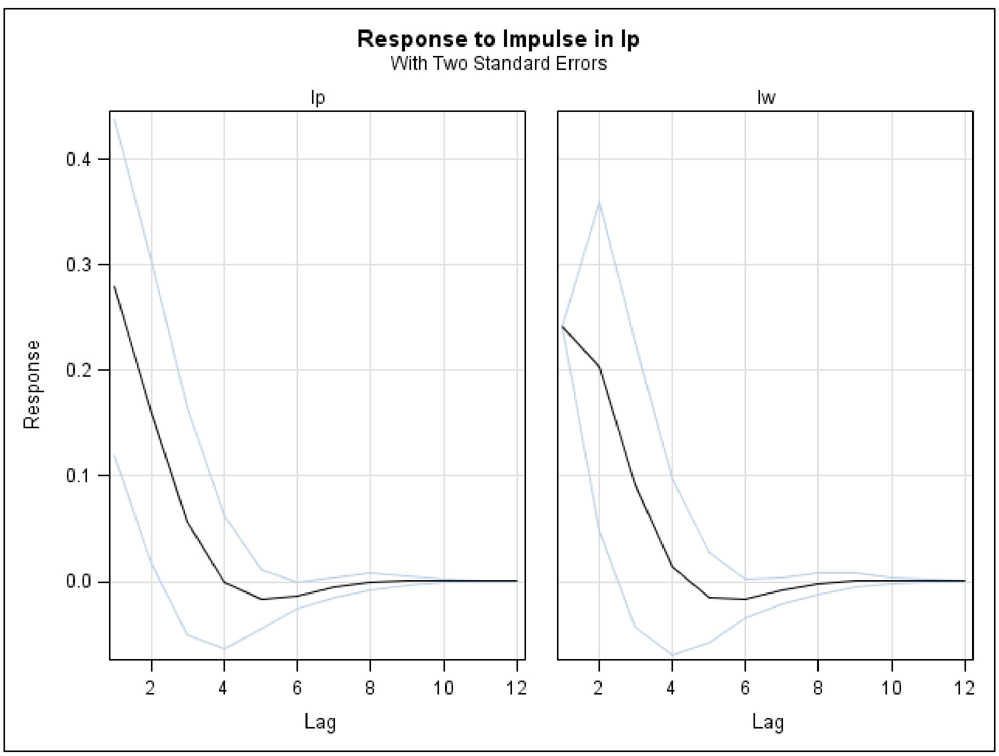

Figure 3. Impulse response function for the price series

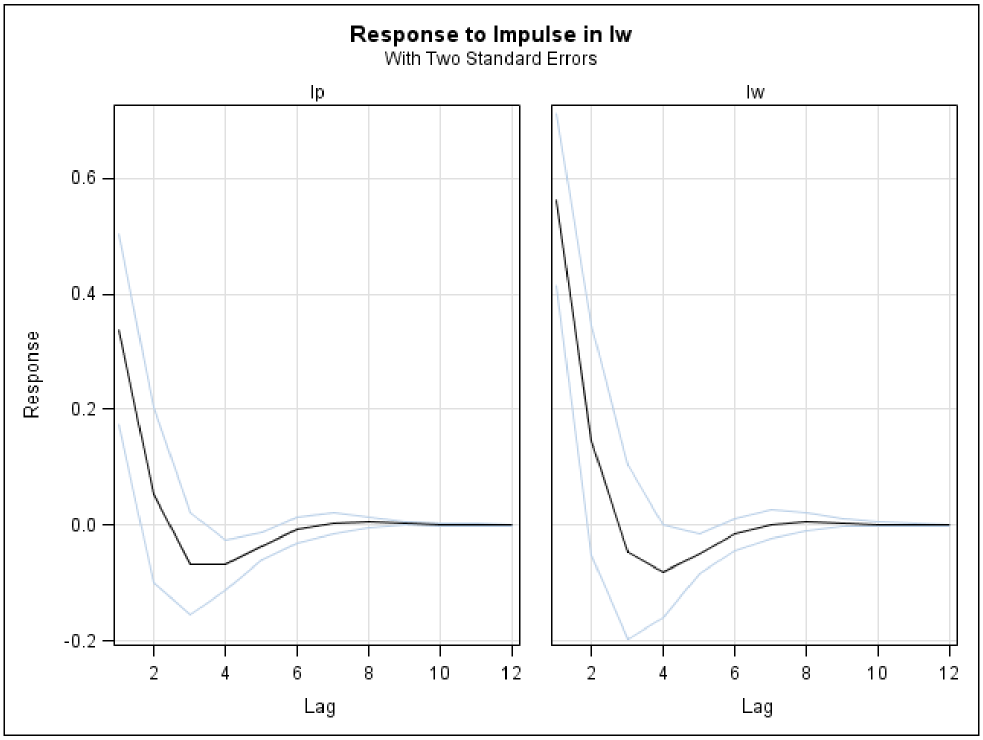

Figure 4. Impulse response function for the wage series

These effects are also accumulated, Figure 5 and 6, in order to show the total effects on each series are calculated as simply the sum of the effects up to a particular lead value.

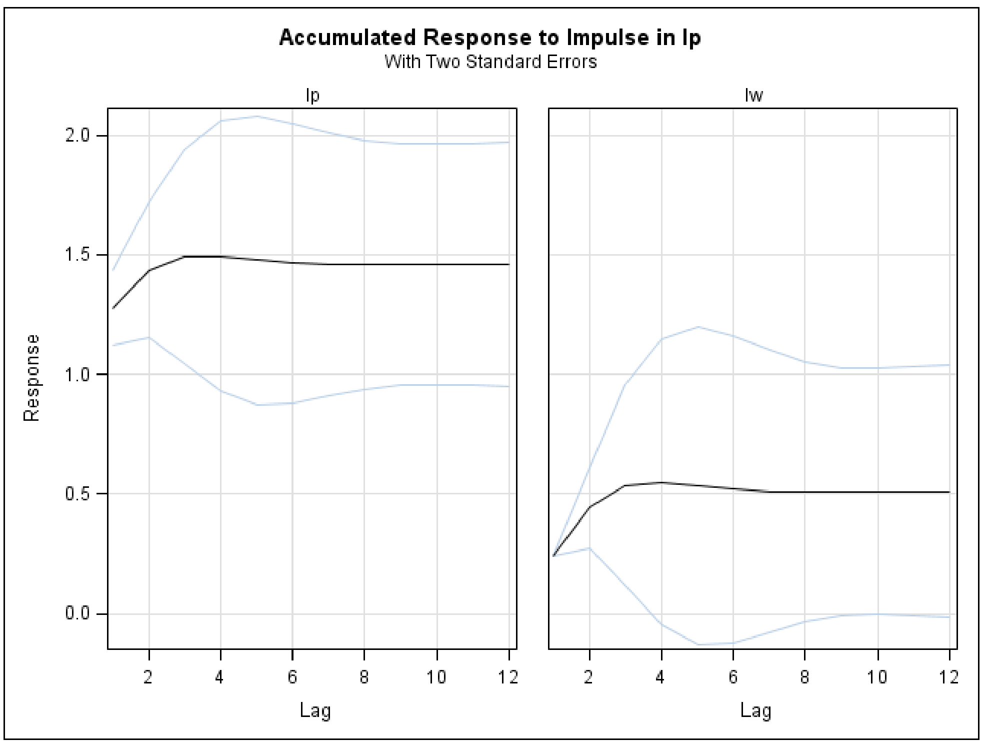

Figure 5. Accumulated impulse response function for the price series

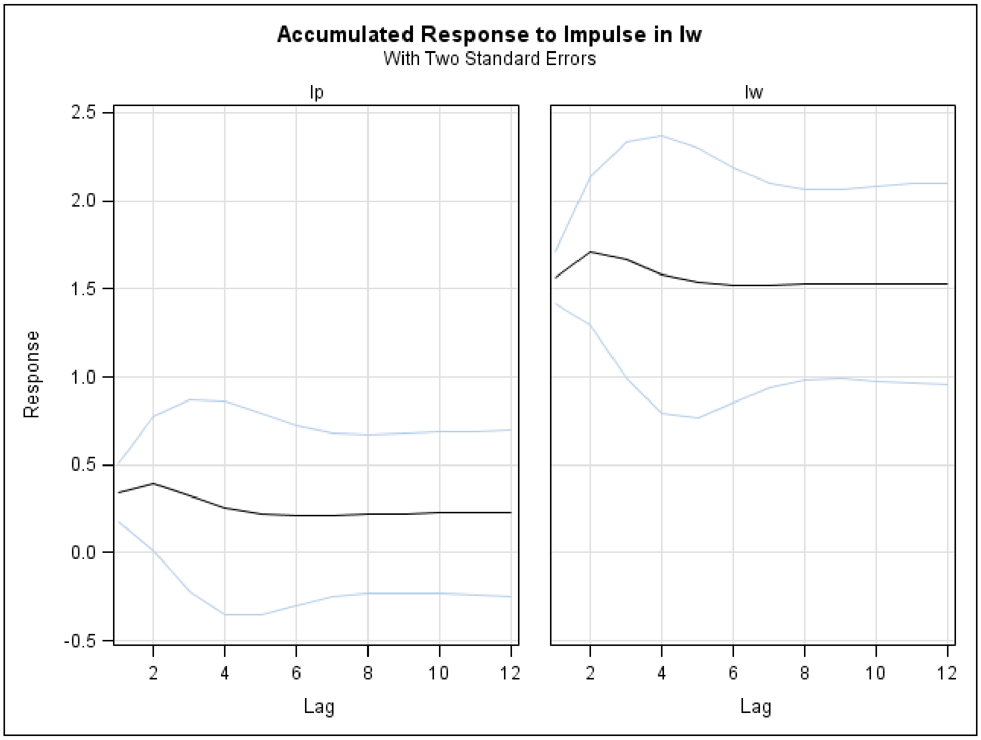

Figure 6. Accumulated impulse response function for the price series

A third type graphs, Figure 7 and 8, present the effect of a so called orthogonal shock to one of the series. The idea is that the error term is present in only one of the series and that it is not at all a part of the error for the other series as is usually the case as the error terms are correlated. In this plots the effect at lag zero, the immediate effect, is one for the series with the shock and zero for the other series. The effect for further years varies according to the damped periodic structure of the model.

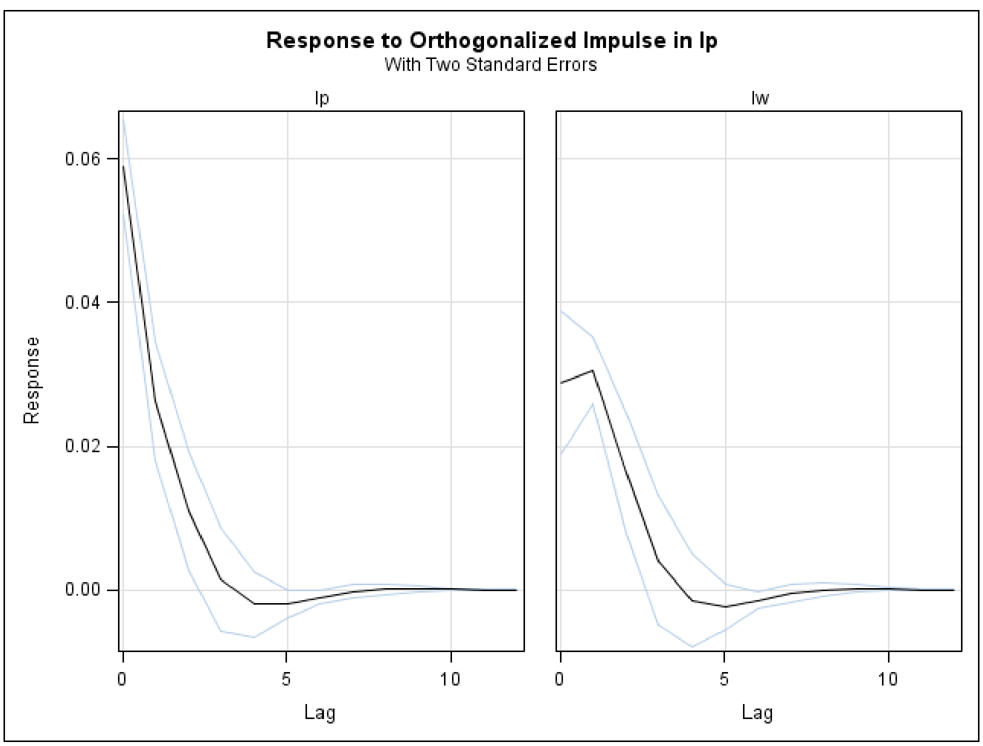

Figure 7. Response function for a specific shock to the price series

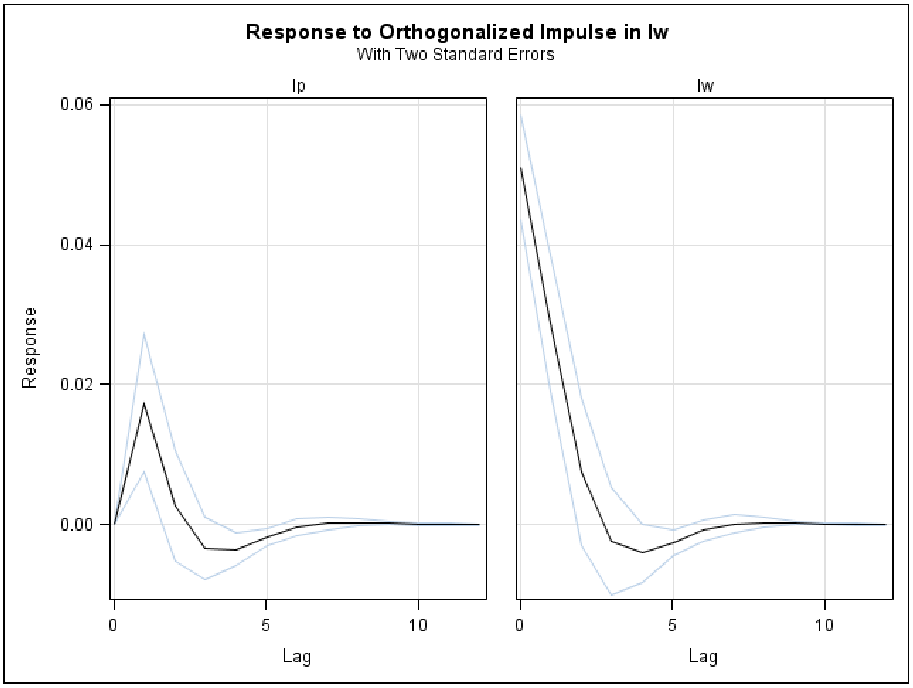

Figure 8. Response function for a specific shock to the wage series

INCLUDING A GARCH MODEL FOR THE RESIDUAL PROCESS

Plots of the prediction errors, Figure 9, for the wage series, clearly point at the presence of outliers. The Q-Q plot and the residual histogram, Figure 10, also make this point clear. These plots are also a part of the graphical output from an application of PROC VARMAX. The plots of the series of prediction errors tell that these outliers are for the years just following First World War for both series and also 1940 for the price series and 1856/57 for the wage series.

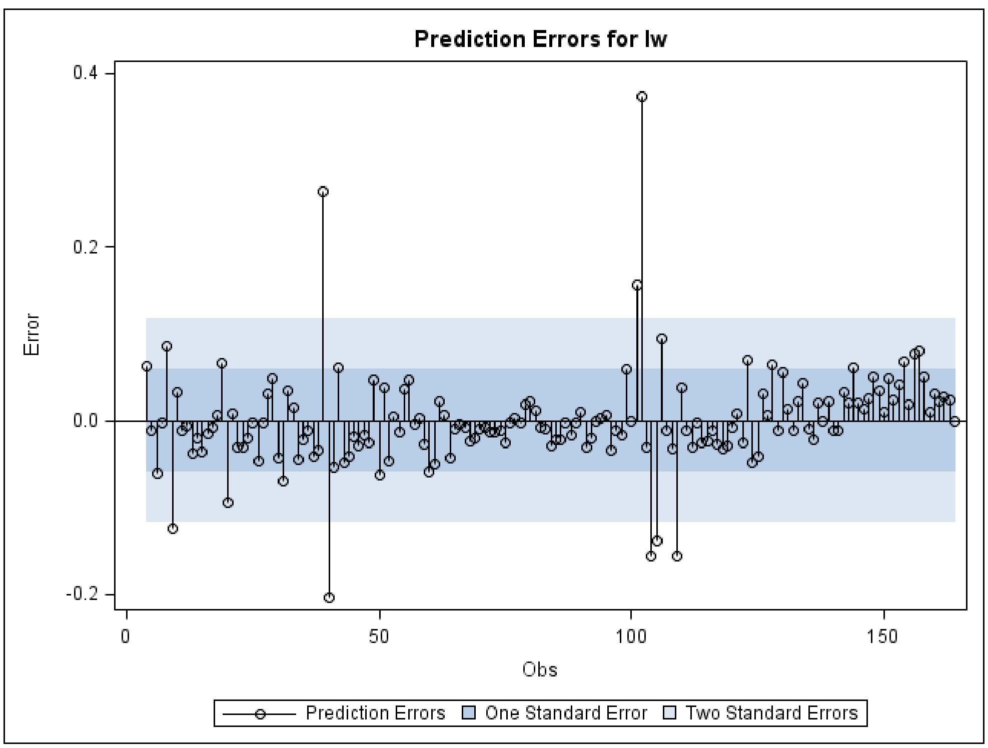

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Figure 9. Residuals for the wage series

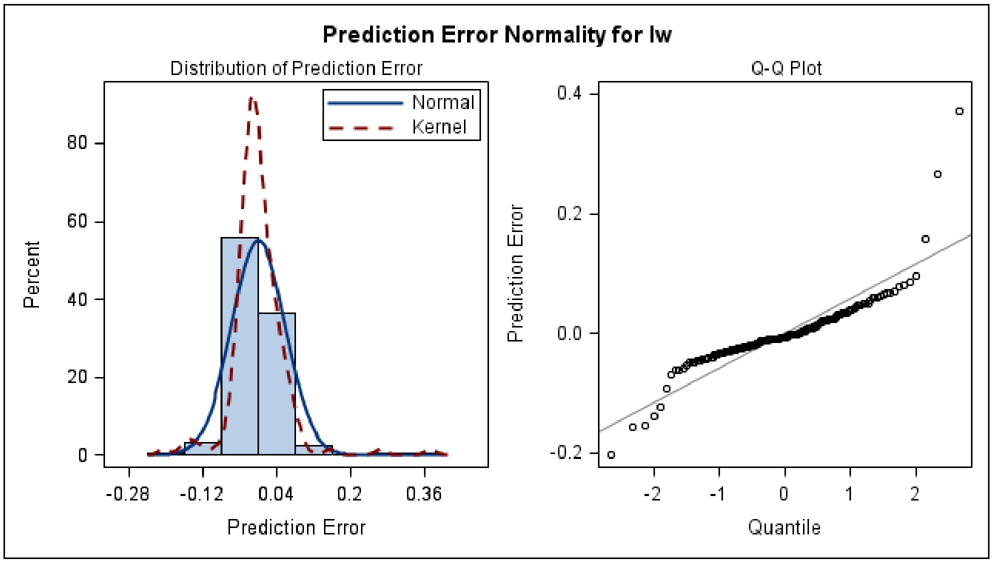

Figure 10. Graphical test for normality for the residuals of the wage series

These findings from the graphical output are confirmed by the tests for normality and ARCH effect, see Output 6.

Univariate Model White Noise Diagnostics

Variable	Durbin Watson	Normality Chi-Square	Normality Pr > ChiSq	ARCH F Value	ARCH Pr > F
lp	2.01516	29.51	<.0001	0.11	0.7452
lw	1.93824	1241.43	<.0001	7.99	0.0053

Output 6. Numerical tests for model fit

PROC VARMAX includes many possibilities for GARCH model fitting to the residuals - both univariate GARCH and multivariate GARCH. One problem when fitting multivariate GARCH models is the huge number of parameters that often are imposed to the model making the risk of over parameterization and numerical problems in the estimation procedure very likely. For this reason the GARC(1,1) model, which is commonly applied to univariate series, works badly in this example.

In this case the more sparsely parameterized CCC, Bollerslev(1990), formulation for a GARCH(1,1) model is used. It turns out that the estimated ARCH parameter for lag 1 at position (1,1) is zero. This element is restricted to zero by the further restriction ACH(1,1,1)=0 in the RESTRICT statement, so the we end up using the following code.

```
proc varmax data=sasmts.wageprice printall plots=all outest=est outstat=stat;
model dlp dlw/p=2 print=all;
garch q=1 form=ccc outht=outht;
restrict ar(2,1,1)=0,ar(2,2,1)=0,
ACH(1,1,1)=0;
output out=asdf ;
nloptions tech=qn maxiter=1000 maxfunc=200000;
id year interval=year;
run;
```

The resulting model has an ARCH parameter outside the parameter space as the roots of the ARCH polynomial is outside the unit circle. IIn order to get a more natural parameter value a further restriction is imposed by the RESTRICT statement. The parameter is bounded using a new feature in the December 2013 Analytical Update 13.1, that allows for limits to the estimated parameters and not just equalities. When the restriction ACH1_2_2<=0.9 is imposed this parameter is estimated to a value at the boundary, that is ACH1_2_2=0.9.

MULTIVARIATE MODELS AND CAUSALITY FOR THE DANISH MARKET FOR EGGS

In this section a vector ARIMA model is estimated for four monthly series related to the Danish market for eggs in the years 1965 - 76. The data set consists of four series of 144 observations

QEGG An index of the produced quantity of eggs.

PEGG An index of the price of eggs.

QTOT An index of the quantity of the total agricultural production

PTOT An index of the price of the total agricultural production

It is seen from the plots of the four series, Figure 11, that the series at least to some extend have trends and that seasonal effects are present, which is of course to be expected.

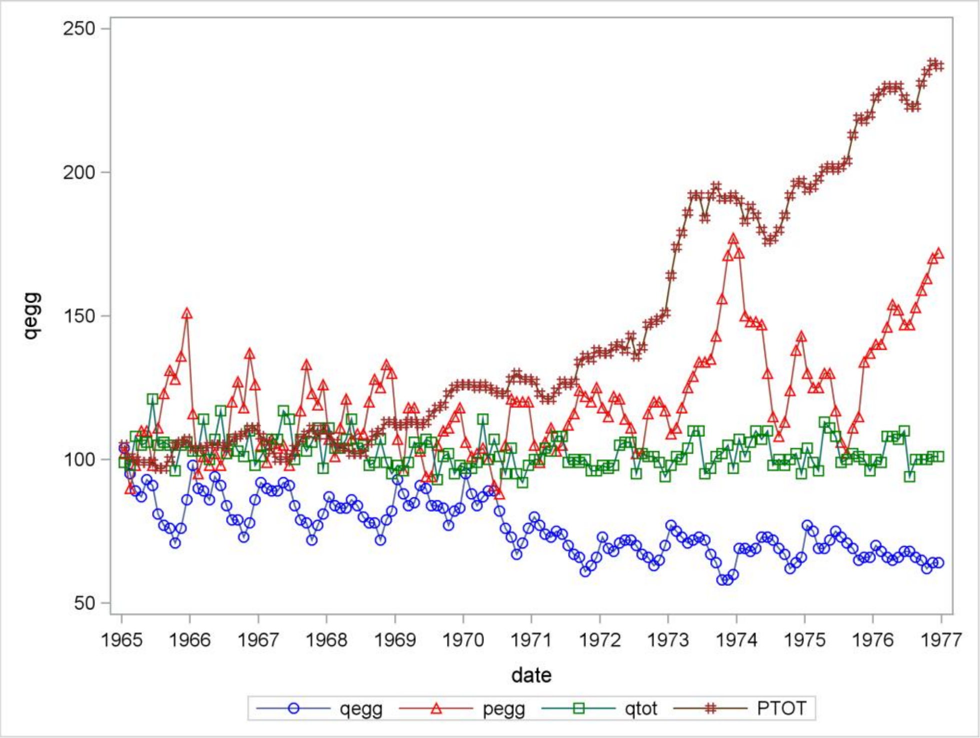

Figure 11. Plots of the four time series

An easy way to correct for the seasonality in the model is to include seasonal dummies. This gives a model for deterministic seasonality as an alternative to stochastic seasonal VARMA models. In PROC VARMAX this is done by the option NSEASON=12 to the model statement.

At the plot of the two price series it is obvious that the price raised rapidly the first months of 1973, which is a well established fact due to Denmark entering the European Union by January 1. 1973 mainly in order to help the farmers. Such events with a significant and easily understood impact on the time series are best modeled as exogenous. In this situation the effect is seen as positive changes of the price index series for the first months of 1973. Three months are specified by the option XLAG=3. One way to model this is simply to include dummy variables for the event, by the variable EUDUMMY. This variable is then only applied for the total price series as the four variables are separated by commas and the exogenous variable only applies for the independent variable PTOT.

A second order autoregressive model is acceptable except for the series QTOT for the produced quantity as seen by the schematic autocorrelation plot, Output 7.

```
data dummy;
set sasmts.egg;
eudummy=0;
if year(date)=1973 and month(date)=1 then eudummy=1;
run;
```

```
proc varmax data=dummy print=all plots=all ;
model qegg, pegg, qtot, ptot=eudummy/dif=(qegg(1) pegg(1) qtot(1) ptot(1))
nseason=12 p=2 lagmax=25 xlag=3;
id date interval=month;
run;
```

This model is seen to give a satisfactory fit to the first 25 residual autocorrelations and cross correlations, as only very few elements are significantly different from zero, i.e. numerically larger than twice their standard error, as indicated by – or + at the schematic representation of the residual autocorrelations, see Output 7 The major problem is found at lag 11 for which some cross correlations numerically exceed 0.2. Portmanteau tests for cross correlations in the residuals also rejects the model fit due to many minor cross correlations which accumulated lead to significance. Of course the fit gets better if the model is extended by autoregressive parameters for lag 11 and 12, but still the portmanteau tests rejects the model.

```
                    Schematic Representation of Cross Correlations of Residuals
 Variable/
 Lag        0    1    2    3    4    5    6    7    8    9     10   11   12   13   14   15   16

 qegg      +-.- .... .... .... ..-. ...+ .... .... ..++ .... .... .-.. +... .... .... ....
....
 pegg      -+.+ .... .... .... .... .... ...- .... .... .... .... -+.+ .... .... .... ....
....
 qtot      ..+. .... .-.. ..-. .... .+.. +... -... .... .... .... .... .... .... .... ....
....
 PTOT      -+.+ .... .... .... .... .... .... .+.+ .... .... .... .... .... ...- ...+ ....
....

                      + is > 2*std error,  - is < -2*std error,  . is between

                    Schematic Representation of Cross Correlations of Residuals
         Variable/
         Lag        17       18       19       20       21       22       23       24       25

         qegg       ....     ....     ....     ....     ....     ....     ....     ....     ....
         pegg       ....     ....     ..-.     ....     ....     ...+     .-..     ....     ....

                      + is > 2*std error,  - is < -2*std error,  . is between
```

Output 7. The four dimension residual autocorrelation-crosscorrelation function

The model includes many parameters; 2 × 16 autoregressive parameters, 4 × 11 seasonal dummy parameters and a residual covariance matrix. The table of estimates is not shown here. It is evident, that the model is very heavily over-parameterized, when every series affects all series at two lags in the autoregressive part. Moreover the seasonal structure is not that significant. This leads to much too many non-significant parameters. In the next section the purpose of the model building is altered in order to reduce the model complexity in an immediate understandable way.

CAUSALITY TESTING

The Danish production of eggs is very small compared to the other sectors of the agricultural production which is dominated by bacon and diary producs. This means that it is impossible, that the size and pricing at the egg market could have any influence on the size and price of the total agricultural production. On the other hand it is of course natural to think that the egg market is influenced by the overall state of the total agricultural market. In econometric terms this means that the total agricultural production is probably exogenous to the egg market.

In the present context this means that it is of no use to set up a simultaneous model for all four series. And if the purpose is only to study the egg market, the two series for the total agricultural market could be included as right hand side variables in the model for the egg market.

However it has to be tested whether this is the case. According to the model structure the immediate impact from the total agricultural market of the egg market is modeled by the four dimensional covariance matrix for the four

remainder terms. Such correlations are by nature not directly interpreted as causal, as correlations could be directed both ways. If however one or more of the coefficients in the autoregressive model corresponding to effects from the egg market to the total agricultural market are significantly different from zero, then a present state of the egg market has some influence on future values of the total market for agricultural products. If this is the case, the total market for agricultural products cannot be exogenous. The test is testing whether a two by two block of every autoregressive coefficient matrix is zero.

This hypothesis is the same as testing so called Granger Causality. The idea of the original Granger, papers, Granger (1969 and 1980), is that causality is present if one group of series affect the other with a time delay, but not the other way around. The causality however depends upon what is known; i.e. which series the model includes besides the series of the causal relation. The following code performs the test for Granger causality.

```
proc varmax data=dummy print=all plots=all ;
model qegg, pegg, qtot, ptot=eudummy/dif=(qegg(1) pegg(1) qtot(1) ptot(1))
nseason=12 p=2 lagmax=25 xlag=3;
id date interval=month;
causal group1=(qegg pegg) group2=(qtot ptot);
nloptions maxiter=2000 maxfunc=20000;
run;
```

In the output element “Granger Causality Wald Test”, Output 8, it is seen, that the hypothesis is accepted with a significance probability p = 10%.

Granger-Causality Wald Test

Test	DF	Chi-Square	Pr > ChiSq
1	8	10.89	0.2083

```
Test 1:  Group 1 Variables:  qegg pegg
         Group 2 Variables:  qtot PTOT
```

Output 8. A Granger causality test

The conclusion of this part of the analysis is, that the two series relating to the total agricultural production in Denmark, QTOT and PTOT, do not Granger cause the series for the egg production QEGG and PEGG. For this reason they could be specified as explanatory variables in the models for the egg series, as their own statistical variation is of no interest for the models of the eggs. If the two series for total production are included as right hand side variables in a model for the two egg series they are considered deterministic in the model and the model tells nothing about their statistical variation. One could say that these two series for the total agricultural production are exogenous; for proper definitions of various forms of the concept of exogeneity see Engle, Hendry and Richard (1983).

FURTHER IMPROVEMENTS OF THE MODEL

In the following application of PROC VARMAX, the series QTOT and PTOT are used as right hand side variables in the model statement. The number of lags of the input series is specified as two by the option XLAG=2 in the model statement. This lag length applies to both input series as the left hand side variables are not separated by commas.

```
proc varmax data=sasmts.egg printall;
model qegg pegg = qtot ptot/dif=(qegg(1) pegg(1) qtot(1) ptot(1))
nseason=12 p=4 lagmax=25 xlag=2;
run;
```

Output 9 gives a schematic presentation of the significance of the estimated autoregressive parameters and their numerical values. The estimated autoregressive parameters could be given the interpretation, that the price series is influenced by the production series at lag one and two. The negative sign is expected as an increasing production in most cases leads to a lower price which persists for some months. But the schematic presentation also shows that no lagged influence is present for the price series to the production series. This argumentation says nothing about the correlation at lag zero, which is estimated to $\rho = -0.24$, but this correlation could be directed both ways as no lags are involved. This correlation matrix for the residual series is printed as the lag zero part of the cross correlation function.

```
                              The VARMAX Procedure

                  Schematic Representation of Parameter Estimates
            Variable/
            Lag           C     XL0     XL1     XL2     AR1     AR2     AR3     AR4

            qegg          -     .-      ..      ..      ..      ..      ..      ..
            pegg          +     .+      +.      ..      -.      --      ..      ..

                         + is > 2*std error,  - is < -2*std
                           error,  . is between,  * is N/A
```

Output 9. A quick view of significant parameters in the VARMAX causality model

This finding is in fact a Granger causality once again saying that the produced quantity of egg Granger causes the price of egg. This conclusion is drawn by a model where the total market for agricultural products is included. In the notation of Granger causality this is formulated as the casualty of the quantity to the price of egg is present in the information set consisting of the two series for the total market for agricultural products. The hypothesis of Granger Causality is formally tested again by a CAUSALITY statement. For comparison the opposite hypothesis that the production does not Granger cause the price is also tested.

```
proc varmax data=sasmts.egg;
model qegg pegg = qtot ptot/dif=(qegg(1) pegg(1) qtot(1) ptot(1))
nseason=12 p=2 xlag=2;
causal group1=(qegg) group2=(pegg);
run;
proc varmax data=sasmts.egg;
model qegg pegg = qtot ptot/dif=(qegg(1) pegg(1) qtot(1) ptot(1))
nseason=12 p=2 xlag=2;
causal group1=(pegg) group2=(qegg);
run;
```

Outputs 10 and 11 show that the significance probability for the for the first test is as high as 93%, while the hypothesis in the second test is below 0.0001. It is then concluded that the production series QEGG do in fact Granger cause the price series PEGG but not vice versa. This conclusion is drawn controlling for the effect of production and price series for the total agricultural market as they are used as right hand side variables.

Granger-Causality Wald Test

Test	DF	Chi-Square	Pr > ChiSq
1	2	0.80	0.6705

```
Test 1:  Group 1 Variables:  qegg
         Group 2 Variables:  pegg
```

Output 10. Granger test accepting the hypothesis that the price of egg does not cause the production of eggs

Granger-Causality Wald Test

Test	DF	Chi-Square	Pr > ChiSq
1	2	42.35	<.0001

```
Test 1:  Group 1 Variables:  pegg
         Group 2 Variables:  qegg
```

Output 11. Test rejecting the hypothesis that the production of egg does not Granger cause the price of eggs

The independent variable QTOT for the total Danish agricultural production is unnecessary in the model as it affects none of the egg series. A final model is estimated where the series QTOT is excluded.

```
proc varmax data=sasmts.egg printall plots=all;
model pegg = qegg ptot/dif=(qegg(1) pegg(1) ptot(1))
nseason=12 p=2 q=0 xlag=2 lagmax=25;
run;
```

The estimated parameters of the resulting model are given in Output 12. The model fit is acceptable according to the the tests for non-normality and ARCH effect, Output 13. The autocorrelation functions for the residuals, see Figure 12, similar indicate no problems with the model fit.

Model Parameter Estimates

Equation	Parameter	Estimate	Standard Error	t Value	Pr > |t|	Variable
pegg	CONST1	5.03961	2.81339	1.79	0.0757	1
	SD_1_1	-2.68740	3.95314	-0.68	0.4979	S_1t
	SD_1_2	-9.87693	4.00431	-2.47	0.0150	S_2t
	SD_1_3	-7.52165	3.67157	-2.05	0.0427	S_3t
	SD_1_4	-5.67999	2.83333	-2.00	0.0472	S_4t
	SD_1_5	-6.85037	3.09041	-2.22	0.0285	S_5t
	SD_1_6	-9.04823	3.95502	-2.29	0.0239	S_6t
	SD_1_7	-6.73291	4.12107	-1.63	0.1049	S_7t
	SD_1_8	-13.03316	3.63028	-3.59	0.0005	S_8t
	SD_1_9	-5.22630	4.30014	-1.22	0.2266	S_9t
	SD_1_10	-0.12594	3.69095	-0.03	0.9728	S_10t
	SD_1_11	-4.42124	3.94961	-1.12	0.2652	S_11t
	XL0_1_1	-0.62633	0.23028	-2.72	0.0075	qegg(t)
	XL0_1_2	0.44563	0.17660	2.52	0.0129	PTOT(t)
	XL1_1_1	-1.54582	0.23513	-6.57	0.0001	qegg(t-1)
	XL1_1_2	0.14021	0.18796	0.75	0.4571	PTOT(t-1)
	XL2_1_1	-0.74888	0.25762	-2.91	0.0043	qegg(t-2)
	XL2_1_2	0.39013	0.18056	2.16	0.0327	PTOT(t-2)
	AR1_1_1	-0.04960	0.08666	-0.57	0.5681	pegg(t-1)
	AR2_1_1	-0.18502	0.07466	-2.48	0.0146	pegg(t-2)

Output 12. Estimated parameters of the final model

Univariate Model White Noise Diagnostics

	Durbin	Normality		ARCH	
Variable	Watson	Chi-Square	Pr > ChiSq	F Value	Pr > F
pegg	2.02483	3.91	0.1417	1.02	0.3141

Output 13. Numerical tests for fit of the final model

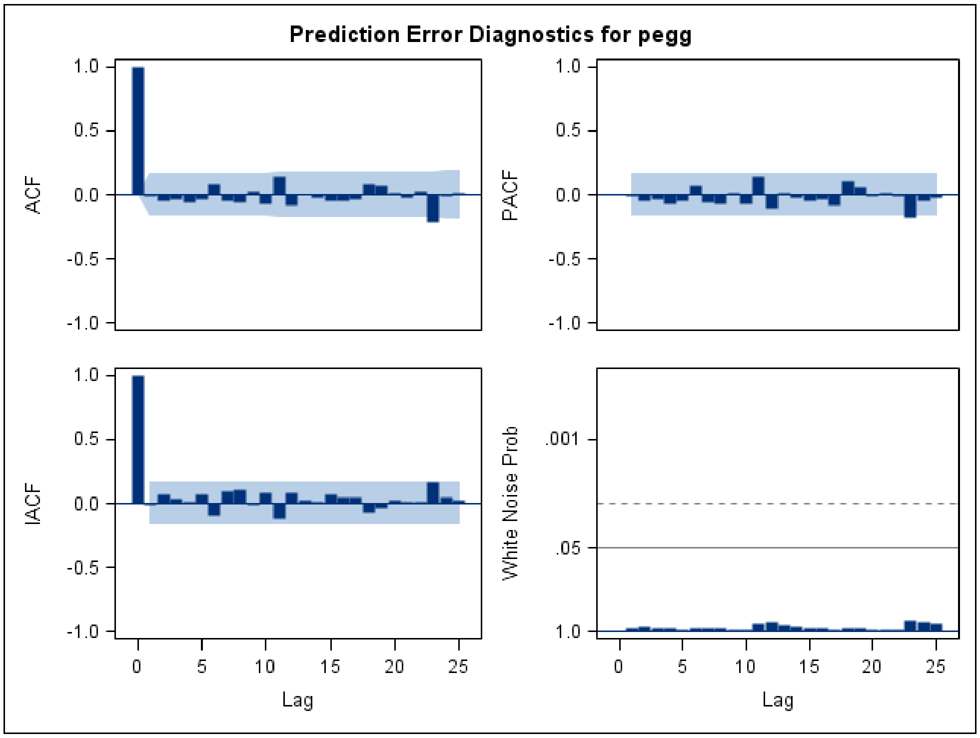

Figure 12. Autocorrelation functions for the residuals of the final model

CONCLUSION

PROC VARMAX is demonstrated to include many useful features which are useful for analyzing multivariate time series by parametric models. An alternative is to apply more descriptive models like models for unobserved components models for instance by PROC UCM, see Milhøj(2013). But parameterized models provides a better background setup for testing relevant hypotheses for the series. PROC VARMAX includes many tests for model fit and features for improving the fit if necessary so that conclusive testing could be performed.

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CONTACT INFORMATION

Your comments and questions are valued and encouraged. Contact the author at:

Name: Anders Milhøj
Organization: Department of Economics, University of Copenhagen
Address: Øster Farimagsgade 5
City, State ZIP: DK 1353, Copenhagen K, Denmark
Work Phone: +45 35323265
Email: anders.milhoj@econ.ku.dk